Department of Electrical Engineering, IIT Bombay:

AUTUMN SEMESTER: JUL-NOV 2023

Semester-End Examination: EE 229 – Signal Processing I – (B. Tech. Division)

Maximum Marks: 80 (40 percent weight)

Date: Thursday 16 Nov. 2023 *Time: 08:30 to 11:30 hours*

Instructions:

1. This is a closed book, closed notes examination. This question paper has two pages.
2. Please begin the answer to each main question, Q1, Q2, Q3, Q4, Q5 on a fresh page of the answer booklet.
3. Show your reasoning and important steps clearly.

Q1. (16 marks): A discrete time, linear shift invariant (LSI) system S has the impulse response $h[n]$ given by $h[n] = 3\,\delta[n] + 2\,\delta[n-1]$, where $\delta[n]$ denotes the unit ~~step~~ impulse sequence.

(a) Let the following input sequence $x[n]$ be applied to the system S:
$x[n] = 2^n\,(\alpha + \beta\, n + \tau\, n^2)$, for all n, where α, β, τ are constants. Show that the output sequence $y[n]$ emerging from S, for this input, is also of the form
$y[n] = 2^n\,(\alpha_1 + \beta_1\, n + \tau_1\, n^2)$, for all n, for appropriate constants α_1, β_1 and τ_1.
Obtain the constants α_1, β_1, τ_1 in terms of the constants α, β, τ.

(b) Two identical 'copies' of this discrete system S are connected in cascade (series), that is the output of the first is given as the input to the second. Obtain the impulse response of this cascade system.

Q2. (16 marks): The system function of a continuous time rational system is given by:

$$H(s) = \frac{(s+2)}{(s+1)(s-3)},$$

(a) Obtain the possible regions of convergence of this system function. For each of the possible regions of convergence, obtain the impulse response of the corresponding system that emerges, for this system function and that region of convergence.

(b) Identify the causal system(s), if any, from among the systems in Q2(a). Explain.

(c) Identify the stable system(s), if any, from among the systems in Q2(a). Explain.

(d) Identify *causal and stable* systems, if any, in Q2(a). Explain.

(e) Identify the system(s) from among the systems in Q2(a), which are neither causal, nor stable, if any. Explain.

(f) Obtain the differential equation relating the output, $y(t)$ of this system to the input $x(t)$.

Q3. (16 marks): A discrete time, linear, shift invariant, causal, system S_1, with input sequence $x[n]$ and output sequence $y[n]$ is described by $y[n] = 0.5\,y[n-1] + x[n] - 0.3\,x[n-1]$.

(a) Obtain the system function of S_1. Specify its region of convergence clearly and obtain the impulse response of S_1. Obtain the absolute sum of the impulse response. Hence, show that the system is BIBO stable.

(b) Obtain the response of the system to the unit step input sequence $u[n]$.

(c) Identify an input $x[n]$, with only two nonzero samples: $x[0]$, $x[1]$, which results in the output $y[n]$, also with only two nonzero samples $y[0]$, $y[1]$. If $y[0] = 1$, obtain $x[0]$, $x[1]$, $y[1]$, for this particular input $x[n]$.

Q4. (16 marks): A continuous time signal $x(t)$ is said to be Gaussian, if it is of the form $x(t) = e^{-\beta t^2}$ for all t, for a positive, real, constant β.

(a) Show that the Fourier Transform of a Gaussian signal is also Gaussian and obtain the Fourier Transform of { $x(t) = e^{-\beta t^2}$, for all t}, explicitly. ***Hint:*** take the derivative of the Gaussian. Hence, write a differential equation involving the Gaussian. Use the properties of the Fourier Transform and the form of the differential equation.

(b) Obtain the Fourier Transform of the signal $y(t) = e^{-\beta t^2} \sin(\gamma t)$, for all t, given that β, γ are positive, real, constants. Show that this Fourier Transform is zero at zero frequency. Can it be zero at any frequency *other than* zero frequency?

Q5. (16 marks): In the scheme of Fig.Q5, an input continuous time signal $x(t)$, with time t measured in milliseconds, is ideally and uniformly sampled with a sampling rate of 10 kHz. We construct the sequence $x[n]$, given by: $x[n] = x(nT)$ for every integer n; where $T = 0.1$ millisecond (i.e. the sampling interval). The sequence $x[n]$ thus formed, is applied as the input to the discrete time, linear, shift invariant, stable system S_0 to produce the output sequence $y[n]$. The continuous time train of uniformly spaced, ideal impulses described by $y(t) = \sum_{n=-\infty}^{n=+\infty} y[n]\, \delta(t - nT)$, where $\delta(t)$ is the continuous time unit impulse, is now applied to the ideal continuous time lowpass filter F, with a cut-off frequency of 5 kHz, to produce the continuous time signal $y_1(t)$.

(a) If the input $x(t) = \cos(2\pi\alpha t) + \cos(2\pi\gamma t)$, where $0 < \alpha < 5$; $5 < \gamma < 10$; KHz
KHz
$\alpha \neq 10 - \gamma$, show that the output
$y_1(t) = A\cos(2\pi\alpha t + \phi_1) + C\cos(2\pi(10-\gamma)t + \phi_2)$
where $A \geq 0$, $C \geq 0$, ϕ_1, ϕ_2 are appropriate real constants, in general.

(b) Let $\alpha = 2, \gamma = 7$. Then, obtain the ratio A/C if the system S_0 is described by:

(i) $y[n] = x[n] + x[n-1]$ (ii) $y[n] = x[n] - x[n-1]$

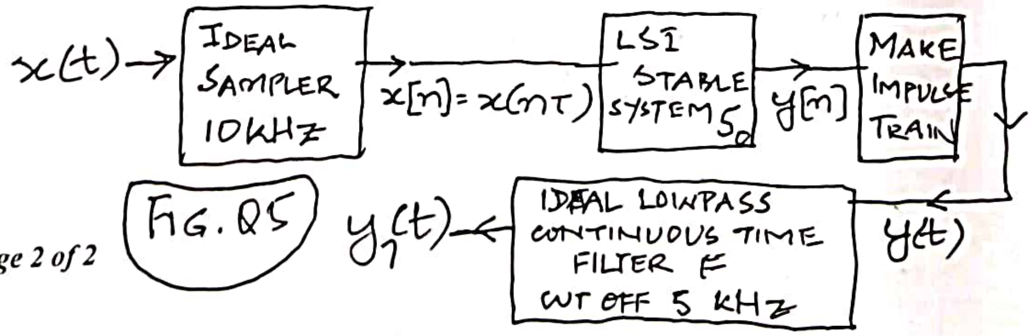

FIG. Q5