FORMULAES

$ax(t)+by(t) \longrightarrow aX(j\Omega)+bY(j\Omega)$

$x(t-t_0) \longrightarrow e^{-j\Omega t_0}X(j\Omega)$

$e^{+j\Omega_0 t}x(t) \longrightarrow X(j(\Omega-\Omega_0))$

$\overline{x(t)} \longrightarrow \overline{X(-j\Omega)}$

$x(-t) \longrightarrow X(-j\Omega)$

$x(at) \longrightarrow \frac{1}{|a|}X(j\Omega/a)$

convolution: $x(t)*y(t) \longrightarrow X(j\omega)Y(j\omega)$

Multiplication: $x(t)\,y(t) \longrightarrow \frac{1}{2\pi}X(j\omega)*Y(j\omega)$

$\frac{d}{dt}x(t) \longrightarrow j\omega X(j\omega)$

$\int_{-\infty}^{t} x(t)\cdot dt \longrightarrow \frac{1}{j\omega}X(j\omega)+\pi X(0)\delta(\omega)$

$t\,x(t) \longrightarrow j\frac{d(X(j\omega))}{d\omega}$

$x(t)$ is real $\longrightarrow$
- $X(j\Omega)=\overline{X(-j\Omega)}$
- $Re\{X(j\Omega)\} = Re\{X(-j\Omega)\}$
- $Im\{X(j\Omega)\} = -Im\{X(-j\Omega)\}$
- $|X(j\Omega)| = |X(-j\Omega)|$
- $\measuredangle X(j\omega) = -\measuredangle X(-j\Omega)$

$x(t)$ is real & even $\longrightarrow$ $X(j\Omega)$ is real & even

$x(t)$ is real & odd $\longrightarrow$ $X(j\Omega)$ is real & odd

$x_e(t)=Ev\{x(t)\} \longrightarrow Re\{X(j\omega)\}$

$x_o(t)=Od\{x(t)\} \longrightarrow j\,Im\{X(j\Omega)\}$

Parseval's Theorem: $\int_{-\infty}^{+\infty}|x(t)|^2\cdot dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\Omega)|^2 d\Omega$

FOURIER TRANSFORM PAIRS

$\left(\text{sinc}(x) = \frac{\sin\pi x}{\pi x}\right)$

$e^{j\omega_0 t} \longrightarrow 2\pi\delta(\omega-\omega_0)$

$\cos\omega_0 t \longrightarrow \pi[\delta(\omega-\omega_0)+\delta[\omega+\omega_0]]$

$\sin\omega_0 t \longrightarrow \frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$

$x(t)=1 \longrightarrow 2\pi\delta(\Omega)$

$x(t)=\begin{cases}1 & |t|<T_1\\ 0 & T_1<|t|<T/2\end{cases} \longrightarrow a_k=\left(\frac{\sin k\omega_0 T_1}{k\pi}\right)=\frac{\omega_0 T_1}{\pi}\text{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right)$

and $x(t+T)=x(t)$ $\xrightarrow{F.T.}$ $\sum_{k=-\infty}^{+\infty}\frac{2\sin k\omega_0 T_1}{k}\delta(\omega-k\omega_0)$

if periodic

6) $\sum_{n=-\infty}^{+\infty} \delta(t-nT) \quad \frac{2\pi}{T}\sum_{K=-\infty}^{+\infty} \delta\left(\omega-\frac{2\pi K}{T}\right)$ $\quad a_K = 1/T$

IMP. 7) $x(t) = \begin{cases} 1, & |t|<T_1 \\ 0, & o/w \end{cases} \longrightarrow \frac{2\sin\Omega T_1}{\Omega} = \boxed{\text{sinc}\left(\frac{\Omega T_1}{\pi}\right)\cdot 2T_1}$

$T_1 = T/2 \Rightarrow \boxed{\text{sinc}(\Omega T/\pi)\cdot T}$

IMP. 8) $\frac{\sin Wt}{\pi t} \longrightarrow X(j\omega) = \begin{cases} 1 ; & |\omega|<W \\ 0 ; & o/w \end{cases}$

$\left(= \frac{W}{\pi}\,\text{sinc}\left(\frac{Wt}{\pi}\right)\right)$

9) $\delta(t) \longrightarrow 1$

10) $u(t)$: $\text{sgn}(t) \longrightarrow 2/j\Omega$

11) $u(t) = \frac{1}{2}[1+\text{sgn}(t)] \longrightarrow \pi\delta(\omega) + 1/j\Omega$

12) $\delta(t-t_0) \longrightarrow e^{-j\Omega t_0}$

13) $e^{-at}\cdot u(t) \longrightarrow \frac{1}{a+j\Omega}$ $\quad [\text{Re}\{a\}>0]$

14) $t e^{-at}\cdot u(t) \longrightarrow \left(\frac{1}{a+j\Omega}\right)^2$ $\quad \left[= j\frac{dX(j\Omega)}{d\Omega}\right]$

15) $\frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \longrightarrow \left(\frac{1}{a+j\Omega}\right)^n$ $\quad [\text{Re}(a)>0]$

$x(t-t_0) = \frac{1}{2\pi}\int X(j\omega) e^{j\omega(t-t_0)}\,dt = e^{-j\omega t_0} F(x(t))$