Module-II

$\Rightarrow$ every periodic signal has its components as pulses and belongs to the ∞-dimensional V.S. spanned by $\{\cos(0.2\pi/T\,t+\phi_0), \cos(K\cdot 2\pi/T\,t+\phi_K)\ldots\}$ for $K\in\mathbb{N}$.

$\Rightarrow$ $\boxed{\langle x_1,x_2\rangle = \int_{(T)} x_1(t)\,x_2(t)\,dt}$ Complex signals: $\boxed{\langle x_1,x_2\rangle = \int_{(T)} x_1(t)\,\overline{x_2(t)}\,dt}$

$\Rightarrow$ $A_0\cos(\Omega_0 t+\phi_0) \rightarrow \boxed{\mathcal{S}} \longrightarrow \boxed{A_0|H(\Omega_0)|\cos(\Omega_0 t+\phi_0+\angle H(\Omega_0))}$

LSI, stable, real $h(t)$ where $H(\Omega_0)=\int h(t)\cdot e^{-j\Omega_0 t}\cdot dt$

$e^{j(\Omega_0 t+\phi_0)} \longrightarrow \boxed{\mathcal{S}} \rightarrow H(j\Omega_0)\,e^{j(\Omega_0 t+\phi_0)}$

$e^{-j(\Omega_0 t+\phi_0)} \longrightarrow \boxed{\mathcal{S}} \rightarrow H(-j\Omega_0)\,e^{j(\Omega_0 t+\phi_0)}$ real $h(t)$: $\rightarrow \boxed{H(\Omega_0) = \overline{H(-\Omega_0)}}$

$\Rightarrow$ BASIS-I :- $\cos\left(\frac{2\pi}{T}\cdot Kt\right)$; $K=1,2,\ldots$ $K=l \Rightarrow \boxed{T/2}$ $\boxed{K=l=0 \Rightarrow T}$ $\rightarrow$ we don't take this into the basis calc.

$\frac{1}{2}\int 2\cos\left(\frac{2\pi}{T}Kt\right)\cos\left(\frac{2\pi}{T}lt\right)dt \Rightarrow K\neq l \Rightarrow \boxed{0}$

$\Rightarrow$ For $p(t)$, assuming $p(t)$ is real, $p(t)=A_0+\sum_{K=1}^{\infty} A_K\cos\left(\frac{2\pi}{T}Kt+\phi_K\right)$

$\rightarrow$ we also have $\sin\left(\frac{2\pi}{T}Kt\right) \rightarrow \perp^{ar}$ to itself & other cosines for $K\neq l$

$\Rightarrow$ $A_K\cos\left(\frac{2\pi}{T}Kt+\phi_K\right) = \left\{\frac{2}{T}\int_{(T)} p(\lambda)\cdot\cos\left(\frac{2\pi}{T}K\lambda\right)d\lambda\right\}\cdot\cos\left(\frac{2\pi}{T}Kt\right)$
$+\left\{\frac{2}{T}\int_{(T)} p(\lambda)\cdot\sin\left(\frac{2\pi}{T}\cdot K\lambda\right)d\lambda\right\}\cdot\sin\left(\frac{2\pi}{T}\cdot Kt\right)$

$\Rightarrow$ $p(t)=A_0+\sum_{K=1}^{\infty} A_K\cos\left(\frac{2\pi}{T}\cdot Kt+\phi_K\right)$ $e^{st}\rightarrow H(s)e^{st}$

$\Rightarrow$ Complex functions are eigenfunctions of LTI systems $z^n \rightarrow H(z)z^n$

$\Rightarrow$ $\boxed{x(t)=\sum a_K e^{jK\omega_0 t}}$ $\Rightarrow$ if $x(t)$ is real $x(t)=\overline{x(t)}=\sum a_K^* e^{-jK\omega_0 t} = \sum a_{-K}^* e^{jK\omega_0 t}$

$\Rightarrow$ $\boxed{a_K=a_{-K}^* \text{ for real } x(t)} \longrightarrow x(t)=a_0+2\sum_{1}^{\infty} a_K\cos(\omega_0 tK+\phi_K)$

$\Rightarrow$ $x(t)e^{-jn\omega_0 t}=\sum_{-\infty}^{\infty} a_K e^{j(K-n)\omega_0 t}$ $\int_{(T)} e^{j(K-n)\omega_0 t}\cdot dt = \begin{cases} 0 & K\neq n \\ T & K=n\end{cases}$

$\Rightarrow$ $\boxed{a_K=\frac{1}{T}\int_{(T)} x(t)\,e^{-jK\omega_0 t}\cdot dt}$ (integrating both sides) $a_0=\frac{1}{T}\int_{(T)} x(t)\cdot dt$ $\hookrightarrow$ average/DC value

$\Rightarrow$ $x(t)=\begin{cases}1\ ; & |t|<T_1 \\ 0\ ; & T_1<|t|<T/2\end{cases}$ $(\omega_0=2\pi/T)$ $\longrightarrow$ $\boxed{a_K=\frac{\sin(K\omega_0 T_1)}{K\pi}}$

PROPERTIES:

1) Linearity : $x(t)\leftrightarrow a_K$ $y(t)\leftrightarrow b_K$ $\Rightarrow Ax(t)+y(t)\longleftrightarrow Aa_K+b_K$ [LC is also periodic]

2) Time shift :- $x(t)\leftrightarrow a_K$ $\boxed{x(t-t_0)\leftrightarrow e^{-jK\omega_0 t_0}\cdot a_K}$ magnitude $\rightarrow$ same

3) Time reversal :- $\sum_{-\infty}^{+\infty} a_K e^{j\omega_0 K(-t)}$ $K=-r \Rightarrow \sum a_{-r}\,e^{j\omega_0 rt}$ $\boxed{\begin{array}{l} x(t)\rightarrow a_K \\ x(-t)\rightarrow a_{-K}\end{array}}$

$\Rightarrow$ $x(t)$ is even $\Rightarrow a_K=a_{-K}$ $x(t)$ is real and even $\Rightarrow a_K=a_{-K}=\overline{a_K}$ $x(t)$ is odd $\Rightarrow a_K=-a_{-K}$ real & odd $\Rightarrow$ P. imag F. coeff

4) Time scaling :- F. coeff don't change but the series changes

5) Multiplication :- $x(t)y(t)\longrightarrow \boxed{h_K=\sum a_l b_{K-l}}$ (convolution)

6) $x(t)\longrightarrow a_K$ $\overline{x(t)}\longrightarrow \overline{a_{-K}}$ $\Rightarrow$ real : $a_K=\overline{a_{-K}}$ $\Rightarrow$ i) $a_0\in\mathbb{R}$ ii) $|a_K|=|a_{-K}|$

7) Differenciation :- $x(t)\rightarrow a_K$ $\frac{dx(t)}{dt}\rightarrow j\omega_0 K a_K$ $\int_{-\infty}^{t} x(t)\cdot dt \longrightarrow \left(\frac{1}{jK\omega_0}\right)\cdot a_K$

Impulse train : $x(t)=\sum_{-\infty}^{+\infty}\delta(t-KT)\longrightarrow a_K=\frac{1}{T}$ $\sum a_K e^{jK\omega_0 t}\longrightarrow \boxed{\mathcal{S}}\longrightarrow \sum a_K H(jK\omega_0)\,e^{jK\omega_0 t}$ (LSI, stable)

$\rightarrow$ same shift by $-\omega$ to

becomes narrower.

S. coeff

($\boxed{a_K = \frac{1}{T} X(jk\omega_0)}$)

$$\boxed{H(j\Omega) = \int_{-\infty}^{+\infty} h(t) e^{-j\Omega t} dt}$$ F.T. $\Rightarrow \langle h(t), e^{j\Omega t} \rangle_2$

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Aperiodic Signals

$\Rightarrow \langle x_1, x_2 \rangle = \int_{(T)} x_1 \overline{x_2} \, d\lambda$; $e^{j(\frac{2\pi}{T}kt)}$, $e^{-j(\frac{2\pi}{T}kt)} \rightarrow \perp$ to one another, T is the modulus.

$\rightarrow A_0 e^{j(\Omega_0 t + \phi_0)} \rightarrow \boxed{\delta} \rightarrow H(j\Omega_0) \cdot A_0 e^{j(\Omega_0 t + \phi_0)}$ $\boxed{H(j\Omega_0) = \langle h(t), e^{j\Omega_0 t} \rangle}$

$\vec{v} = \langle v, \hat{v} \rangle$

$\rightarrow$ To generate h(t) back, we add the inner products with all $e^{j\Omega t}$ of $H(j\Omega)$ $\rightarrow \boxed{h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(j\Omega) \cdot e^{j\Omega t} d\Omega} \Rightarrow$ inverse F.T.

$H(j\Omega)$

$\rightarrow$ periodic $\Rightarrow a_K = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t} dt \Rightarrow Ta_K = X(j\omega) = \lim_{T \to \infty} \int \ldots = \int x(t) e^{-j\omega t} dt$

PROPERTIES:

① Linearity :- $a x_1(t) + b x_2(t) \rightarrow a X_1(j\Omega) + b X_2(j\Omega)$

② Shift :- $x(t) \rightarrow X(j\Omega)$ $\boxed{x(t - t_0) \rightarrow e^{-j\Omega t_0} X(j\Omega)}$ (starting phase is changed)

③ Conjugation :- $x(t) \rightarrow X(j\Omega)$ $x(-t) \rightarrow \int x(t) e^{j\Omega t} dt \Rightarrow x(-t) \rightarrow X(-j\Omega)$

$\overline{x(t)} \rightarrow \overline{X(-j\Omega)}$ 1) real : $X(j\Omega) = \overline{X(-j\Omega)}$ 3) Odd : $X(j\Omega) = -\overline{X(j\Omega)}$

real $x(t) \Rightarrow Re\{X(j\Omega)\} = Re\{X(-j\Omega)\}$ 2) even : $X(j\Omega) = X(-j\Omega) = \overline{X(j\Omega)}$

$\Rightarrow Im\{X(j\Omega)\} = -Im\{X(-j\Omega)\}$

$\Rightarrow Ev(x(t)) \xrightarrow{F} Re\{X(j\Omega)\} \Rightarrow O_d(x(t)) \xrightarrow{F} j \cdot Im\{X(j\Omega)\}$

$\Rightarrow Re(x(t)) \xrightarrow{F} \frac{X(j\Omega) + \overline{X(-j\Omega)}}{2}$

④ Duality $\Rightarrow x(t) \rightarrow X(j\Omega)$ $\boxed{X(jt) \rightarrow 2\pi x(-\Omega)}$

⑤ Diff. and Integrn $\Rightarrow \frac{dx(t)}{dt} \rightarrow (j\omega) \cdot X(j\omega)$ $\int_{-\infty}^{t} x(t) \cdot dt \rightarrow \frac{1}{j\omega} X(j\omega) + \pi X(0) \delta(\omega)$

$e^{j\Omega_0 t} x(t) \rightarrow X(j(\Omega - \Omega_0))$ $-jt \, x(t) \rightarrow \frac{dX(j\omega)}{d\omega}$

$-\frac{1}{jt} x(t) + \pi x(0) \delta(t) \rightarrow \int_{-\infty}^{\omega} X(j\omega) d\omega$

⑥ Parseval's reln :- $\langle x_1, x_2 \rangle = \frac{1}{2\pi} \langle X_1, X_2 \rangle$ or $\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \underbrace{|X(j\Omega)|^2}_{\text{energy spectral density}} d\Omega$

⑦ Convolutions :- $x(t) * y(t) \rightarrow X(j\omega) \cdot Y(j\omega)$

$\rightarrow$ For a LSI system : $Y(j\Omega) = X(j\Omega) H(j\Omega)$

$\Rightarrow x_1(t) \cdot x_2(t) \longrightarrow X_1(j\omega) * X_2(j\omega)$ (duality)

1) $\left. \frac{dx(t)}{dt} \right|_{t=0} = \frac{1}{2\pi} \int X(j\omega) \cdot j\omega \, d\omega$ 2) $x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) \cdot d\omega$ & V.V.

3) $X(j\omega)$ is periodic $\Rightarrow x(t)$ is also periodic