DEPARTMENT OF ELECTRICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

CLASS TEST 3 **November 6, 2023**

Course Number: EE 229 (Division I) **Course Name:** Signal Processing - I

Date: 6 November 2023 (Monday) **Time:** 08:40 – 09:20 (40 minutes)

Maximum marks = 30

Instructions:

1. This is a **closed book, closed notes** examination. Please **use your own stationery** to answer the questions, write your **name and roll number** on your answer sheet(s) and staple the sheets used by you together, in the correct order.
2. This question paper has **two questions.**
3. Please **show important steps** of working/ reasoning clearly.
4. Please **begin the answers** to each of the **main questions**: Q1, Q2 on a **fresh page** of the answer booklet.

Q1. (20 marks)

The signal $x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$ is subjected to ideal uniform sampling at the rate of 18 kilosamples per second. All frequencies, including f_1 and f_2 are expressed in kiloHertz (kHz) and time 't' is measured in milliseconds. The sampled signal is subjected to the action of an adjustable ideal bandpass filter, with a real valued impulse response, having a frequency response of 1 in its passband and 0 on the rest of the frequency axis (stopband). The phase response of the bandpass filter can be taken to be zero for all frequencies.

(a) In the first experiment, $f_1 = 3$, $f_2 = 5$; the passband is from 4 to 16 kHz. Obtain the output of the bandpass filter, expressing it as a sum of sinusoids, specifying the frequencies.

(b) In the second experiment, $f_1 = 7$, $f_2 = 10$; the passband is from 4 to 12 kHz. Obtain the output of the bandpass filter and express it as a product of two terms, the first term being a pure sinusoid of an appropriate frequency, the second, a sum of two sinusoids in which the frequency of one sinusoid is twice that of the other. Specify all frequencies clearly.

Q2. (10 marks)

The signal $y(t)$ is known to have the form $y(t) = \{A\ e^{B\,t} + C\}$ where A, B, C are real constants that need to be determined, by sampling the signal at appropriately chosen locations. Show that it is possible to obtain the values of A, B, C by sampling $y(t)$ at the three distinct points $t = 0, t_2, t_3,$ and obtain the values of A, B, C in terms of such $\{y(0), y(t_2), y(t_3)\}$. (Hint: you will have to suggest convenient choices of the points).

(End of question paper)

QUIZ

Class Test-3

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30/30

Q1) 18 Kilosamples/sec

Let $H(j\Omega)$ be the F.T. of the impulse response, then

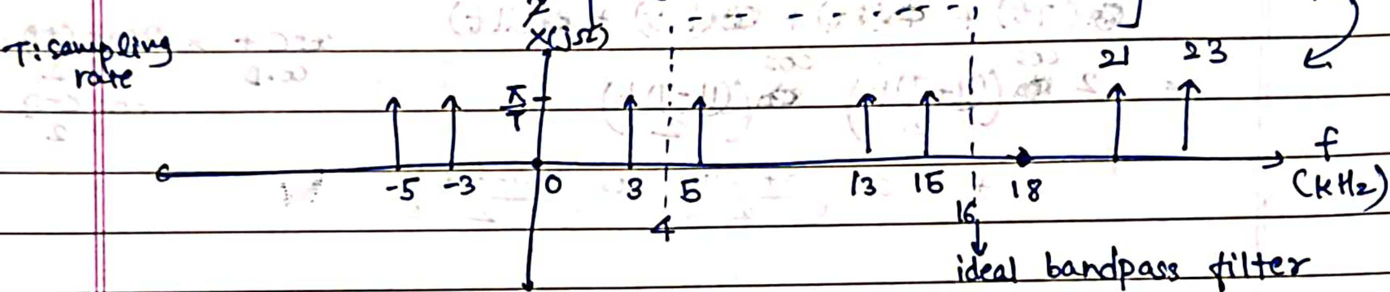
$\angle H(j\Omega) \longrightarrow$ phase response - 0

20

a) $f_1 = 3 \quad f_2 = 5$

$$\cos(2\pi f_1 t) = \frac{e^{-j(2\pi f_1)t} + e^{j(2\pi f_1 t)}}{2}$$

$$\text{F.T.}(\cos(\omega t)) = \frac{2\pi}{2}\left[\delta(\omega - 2\pi f_1) + \delta(\omega + 2\pi f_1)\right] \longrightarrow \text{sampling}$$



sampling rate = 18 Kilosamples/sec

So, the output in fourier domain is

$$\pi\left[\delta(\omega - 2\pi f_2) + \delta(\omega + (18 - 2\pi f_1)) + \delta(\omega + (18 - 2\pi f_2))\right]$$

$\downarrow$ I.F.T.

$$\frac{1}{2\pi}\int \pi\delta(\omega - 2\pi f_2)\cdot e^{+j\omega t}\,d\omega = \frac{1}{2\pi}\cdot\pi\, e^{j(2\pi f_2)t} \qquad (1)$$

$$= \frac{e^{j(2\pi f_2)t}}{2}$$

So, the signal, $y(t) = \frac{1}{2}\left[e^{j(2\pi f_2 t)} + e^{j(2\pi(18 - f_1)t)} + e^{j(2\pi(18 - f_2)t)}\right]$

$f_2 = 5$ KHz $\quad f_1 = 3$ KHz

the frequencies present are $\boxed{5\text{KHz}, 13\text{KHz}, 15\text{KHz}}$

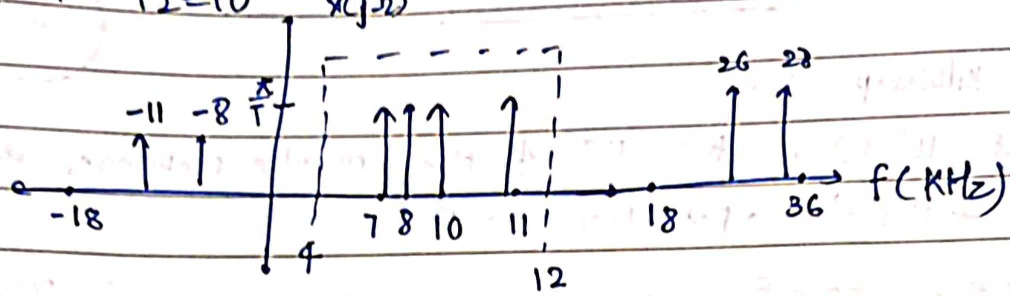
$$y(t) = \frac{1}{2}\Big[j\sin(2\pi f_2 t) + \cos(2\pi f_2 t) + j\sin(2\pi(18 - f_1)t) + \cos(2\pi(18 - f_1)t) + j\sin(2\pi(18 - f_2))t + \cos(2\pi(18 - f_2)t)\Big]$$

since there is NO phase response, and assuming similarity on LHS.

$$\boxed{y(t) = \cos(5t) + \cos(13t) + \cos(15t)}$$

(t: ms, f: KHz)

(b) $f_1 = 7$ $\quad f_2 = 10$ $\quad$ (T: sampling rate)



The output of bandpass filter in Fourier domain is $\quad$ (Ω: KHz)

$$\pi\left[\delta(\Omega-7)+\delta(\Omega-8)+\delta(\Omega-10)+\delta(\Omega-11)\right]$$

$\downarrow$

$$\cos(7t)+\cos(8t)+\cos(10t)+\cos(11t)$$

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

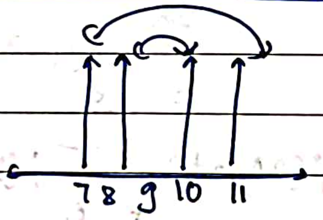
$$= 2\cos\left(\frac{(11+7)t}{2}\right)\cos\left(\frac{(11-7)t}{2}\right) + 2\cos\left(\frac{(8+10)t}{2}\right)\cos\left(\frac{(10-8)t}{2}\right)$$

$$= 2\cos 9t\left[\cos 2t + \cos t\right]$$

So, $\boxed{y(t) = (2\cos 9t)\left[\cos t + \cos 2t\right]}$

The appropriate frequency is 9 KHz, and for the sinusoids in the sum, it is 1KHz and 2KHz.

Frequencies :- $\boxed{9\text{KHz}, 1\text{KHz}, 2\text{KHz}}$



→ The average in pairs is 9KHz for these impulses, and the difference in frequencies b/w one pair is twice the other.

Q2) $y = \{Ae^{Bt} + C\}$ $A, B, C \rightarrow$ using sampling

at $t=0$ $y(0) = Ae^{B \cdot 0} + C = Ae^{0} + C = A + C$

$y(t_2) = Ae^{Bt_2} + C$ $y(t_3) = Ae^{Bt_3} + C$

Now,

$y(t_2) - y(t_3) = A(e^{Bt_2} - e^{Bt_3})$ — ①

$C = y(t_2) - Ae^{Bt_2} = y(0) - A$

$\boxed{y(t_2) - y(0) = A(e^{Bt_2} - 1)}$ — ②

dividing ① and ②, $K = \dfrac{y(t_2) - y(t_3)}{y(t_2) - y(0)} = \dfrac{e^{Bt_2} - e^{Bt_3}}{e^{Bt_2} - 1}$

$Ke^{Bt_2} - K = e^{Bt_2} - e^{Bt_3}$ → can be solved to find B

$\boxed{(K-1)e^{Bt_2} + e^{Bt_3} = K}$ ~~log(a+b)~~

This equation can be solved to find the value of B in terms of K

Let's say $B = f(K)$, then,

$A = \dfrac{y(t_2) - y(t_3)}{e^{Bf(K) \cdot t_2} - e^{f(K) \cdot t_3}}$

and, $\boxed{C = y(0) - A}$ → so, we have a ~~unique~~ set of solutions for A, B, C

→ a convenient choice of the points ⇒

$t_2 = 1$ $t_3 = 2$

$A + C = y(0)$

$Ae^{B} + C = y(1)$

$Ae^{2B} + C = y(2)$

$y(1) - y(0) = A(e^{B} - 1)$ ⇒ $A = \dfrac{y(1) - y(0)}{(e^{B} - 1)}$

$y(2) - y(1) = A(e^{2B} - e^{B})$

$\dfrac{y(2) - y(1)}{y(1) - y(0)} = \dfrac{Ae^{B}\cancel{(e^{B}-1)}}{\cancel{(e^{B}-1)}} = Ae^{B}$

So, $\dfrac{y(2) - y(1)}{y(1) - y(0)} = \left(\dfrac{y(1) - y(0)}{(e^{B} - 1)}\right) \cdot e^{B}$ Let $C = \dfrac{y(2) - y(1)}{(y(1) - y(0))^2}$

$$C = \frac{e^B}{e^B - 1} \qquad Ce^B - C = e^B$$

$$(C-1)e^B = C \qquad e^B = \frac{C}{C-1}$$

$$\boxed{B = \ln\left(\frac{C}{C-1}\right)} \quad — ①$$

$$A\left(\frac{C}{C-1} - 1\right) = y(1) - y(0)$$

$$A\left(\frac{1}{C-1}\right) = y(1) - y(0) \Rightarrow \boxed{A = (C-1)\,(y(1) - y(0))} \quad - ②$$

and $c = y(0) - A = y(0) - (C-1)(y(1) - y(0))$.

$$c = \boxed{Cy(0) - (C-1)\,y(1)} \quad — ③$$

Here, $C = \dfrac{y(2) - y(1)}{(y(1) - y(0))^2}$

So, we can obtain a set of solutions for A, B, C.

Alt. Thought → It is basically an equation in 3 unknowns A, B, C and so if we have 3 initial conditions, we can make a unique D.E. for the same (degree-2) and it will have a unique solⁿ, hence, we need at least 3 samples of y(t) and with that we will have a y(t) being defined.