# PROPERTIES (list)

① **Linearity :-** $x_1(t) \xrightarrow{\mathcal{L}} X_1(s)$ , $R_1$

$x_2(t) \xrightarrow{\mathcal{L}} X_2(s)$ , $R_2$

$$a x_1(t) + b x_2(t) \xrightarrow{\mathcal{L}} aX_1(s) + bX_2(s) \quad , \quad ROC \supset (R_1 \cap R_2)$$

eg. - ($x_1(t) = x_2(t)$ and $a = -b$ → $X(s) = 0$ everywhere)

→ if there are poles at $\infty$ → $\infty$ can't itself be included although we go upto $\infty$

**Method :-** $R_1, R_2 \rightarrow R_1 \cap R_2 \longrightarrow$ extend it to the nearest poles (which may be at $\infty$).

(eg.- $\frac{1}{s+2} \rightarrow 0$ as $s \rightarrow \infty$ (zero at $\infty$) → included in the ROC)

② **Time-shifting :-** $x(t) \xrightarrow{\mathcal{L}} X(s)$ ROC = R

$x(t-t_0) \xrightarrow{\mathcal{L}} e^{-st_0} X(s)$ $\boxed{ROC = R}$

And, conversely, $e^{s_0 t} x(t) \longrightarrow \underbrace{X(s - s_0)}$ $\boxed{ROC = R + Re\{s_0\}}$

ROC is the ROC of X(s) shifted by $Re\{s_0\}$

when $s_0 = j\omega_0 \Rightarrow e^{j\omega_0 t} x(t) \longrightarrow X(s - j\omega_0)$ ROC = R

③ Time-scaling :- $x(t) \xrightarrow{L} X(s)$ ROC = R

$x(at) \xrightarrow{L} \frac{1}{|a|} X\left(\frac{s}{a}\right)$ ROC = aR = $R_1$

④ Conjugation :- $x(t) \xrightarrow{L} X(s)$

$x^*(t) \xrightarrow{L} X^*(s^*)$ ROC = R

$\boxed{X(s) = \overline{X(\bar{s})}}$ when x is real.

then, if X(s) has a pole/zero at $s = s_0$

↳ X(s) also has a pole/zero at $s = s_0^*$

eg.-

⑤ Convolution :- $x_1(t) \xrightarrow{L} X_1(s)$ $(R_1)$ $x_2(t) \xrightarrow{L} X_2(s)$ $(R_2)$

then, $x_1(t) * x_2(t) \xrightarrow{L} X_1(s) \cdot X_2(s)$ ROC $\supset (R_1 \cap R_2)$

⑥ Differenciation :- $\frac{d\,x(t)}{dt} \xrightarrow{L} s \cdot X(s)$ ROC $\supset$ R

ROC can increase if X(s) has a first-order pole at s=0 i.e. cancelled by multiplication with s

eg.- $x(t) = u(t) \rightarrow X(s) = \frac{1}{s}$ Re(s) > 0

$dx(t)/dt = \delta(t) \longrightarrow X(s) = 1$ Re(s) $\in \mathbb{R}$

IN s-domain :→

$$\boxed{-t\,x(t) \xrightarrow{L} \frac{dX(s)}{ds} \;,\quad \text{ROC} = R}$$

⑦ Integration :-

$$\int_{-\infty}^{t} x(\tau)\, d\tau \xrightarrow{L} \frac{1}{s} X(s) \;;\quad \text{ROC} \supset (R \cap \text{Re}(s) > 0)$$

using $\boxed{\int_{-\infty}^{t} x(\tau)d\tau = x(t) * u(t)}$ (IMP.)

IVT ⇒ $x(0^+) = \lim_{s\to\infty} sX(s)$

FVT ⇒ $\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$

# LAPLACE TRANSFORM PAIRS

| | Signal | Transform | ROC |
|---|---|---|---|
| 1. | $\delta(t)$ | $1$ | All $s$ |
| 2. | $u(t)$ | $1/s$ | $Re(s) > 0$ |
| 3. | $-u(-t)$ | $1/s$ | $Re(s) < 0$ |
| 4. | $\frac{t^{n-1}}{(n-1)!}u(t)$ | $1/s^n$ | $Re(s) > 0$ |
| 5. | $-\frac{t^{n-1}}{(n-1)!}u(-t)$ | $1/s^n$ | $Re(s) < 0$ |
| 6. | $e^{-at}u(t)$ | $1/(s+a)$ | $Re(s) > -a$ |
| 7. | $-e^{-at}u(-t)$ | $1/(s+a)$ | $Re(s) < -a$ |
| 8. | $\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$ | $\frac{1}{(s+a)^n}$ | $Re(s) > -a$ |
| 9. | $-\frac{t^{n-1}}{(n-1)!}e^{-at}u(-t)$ | $\frac{1}{(s+a)^n}$ | $Re(s) < -a$ |
| 10. | $\delta(t-T)$ | $e^{-sT}$ | all $s$ |
| 11. | $[\cos(\omega_0 t)]u(t)$ | $\frac{s}{s^2+\omega_0^2}$ | $Re(s) > 0$ |
| 12. | $[\sin(\omega_0 t)]u(t)$ | $\frac{\omega_0}{s^2+\omega_0^2}$ | $Re(s) > 0$ |
| 13. | $[e^{-\alpha t}\cos(\omega_0 t)]u(t)$ | $\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$ | $Re(s) > -\alpha$ |
| 14. | $[e^{-\alpha t}\sin(\omega_0 t)]u(t)$ | $\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$ | $Re(s) > -\alpha$ |
| 15. | $u_n(t) = \frac{d^n\delta(t)}{dt^n}$ | $s^n$ | All $s$ |
| 16. | $u_{-n}(t) = \underbrace{u(t) * \dots * u(t)}_{n\text{-times}}$ | $1/s^n$ | $Re(s) > 0$. |

Laplace Transforms and LTI Systems