EE 229 : SIGNAL PROCESSING - I : DIV: B.TECH.

AUTUMN SEMESTER JULY-NOVEMBER 2023

CLASS TEST 1 MAXIMUM MARKS : 20

INSTRUCTIONS - 1. Begin answering these questions from the reverse side and continue on additional sheets you have brought

2. Write your name and roll no. here :

NAME — NIMAY UPEN SHAH

ROLL No - 22B1232.

3. Staple the additional sheets to this main sheet correctly and return.

19.5 / 20

Q1 -

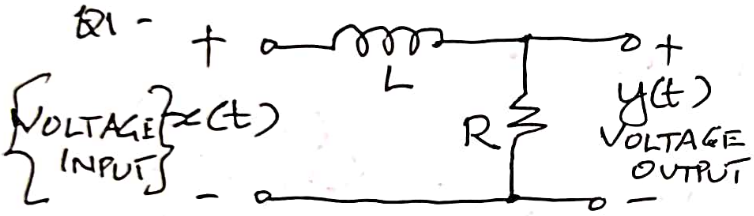


Fig. Q1

(a) For the circuit of Fig. Q1, obtain a linear constant coefficient differential equation relating $x(t)$ and $y(t)$

(b) Obtain the step response of the linear shift invariant system of Fig. Q1. Assume an initially unexcited inductor

(c) Hence obtain its impulse response.

{MARKS : 3 + 4 + 3 = 10}

Q2 - A discrete time, linear shift invariant system has the impulse response

$$h[n] = 0.5^n u[n]$$

(a) Is the system causal ? Explain.

(b) Is the system BIBO stable? Explain

(c) Obtain the output of the system when the input is $x[n] = 0.3^n u[n]$

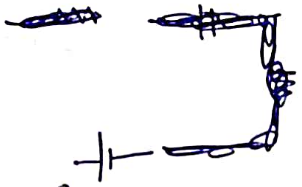
(d) Obtain a linear constant coefficient difference equation (LCCDE) relating the input $x[n]$ and output $y[n]$.

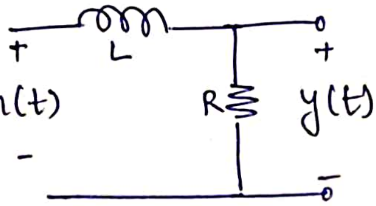
{MARKS: 1+2+4+3=10}

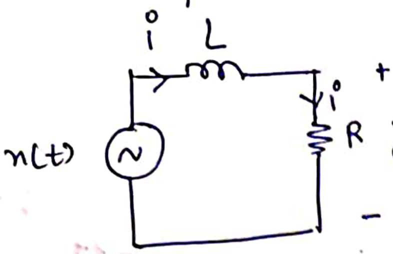
END OF QUESTION PAPER

Q1)

(a)







$y(t) = iR \Rightarrow i = \frac{1}{R} y(t)$

and $\frac{di}{dt} = \frac{1}{R} \frac{d(y(t))}{dt}$

Using KVL $\Rightarrow$ $x(t) - L\frac{di}{dt} - iR = 0$

$$x(t) = L\frac{di}{dt} + iR$$

3

using $i = \frac{1}{R} y(t)$ and $\frac{di}{dt} = \frac{1}{R}\frac{d\,y(t)}{dt}$,

$$x(t) = L\cdot\frac{1}{R}\frac{dy(t)}{dt} + \frac{1}{\cancel{R}}\cdot y(t)\cancel{R}$$

$\boxed{x(t) = \frac{L}{R}\frac{dy(t)}{dt} + y(t)}$ $\longrightarrow$ L.C.C.D.E. (differential) relating in and output.

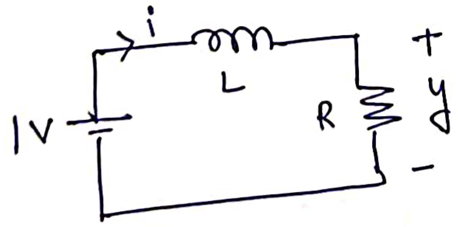
(b) If we have the step function as an input

NOTE: The system is LSI since differential operators and normal (buffer) adding operators are linear, i.e.,

$x_1(t) + x_2(t) \longrightarrow \frac{L}{R}\frac{d(y_1(t)+y_2(t))}{dt} + (y_1(t) + y_2(t))$ and

$cx_1(t) = \frac{L}{R}\, c\left(\frac{dy_1(t)}{dt}\right) + c\,y_1(t)$, $x_1(t-t_0) = \frac{L}{R}\frac{dy(t-t_0)}{dt} +$

$x(t) = u(t)$ $\Rightarrow$ For $t > 0 \rightarrow x(t) = 1$



[where $y \Rightarrow f^n c$ of t]

$1 = L\frac{di}{dt} + iR$ $\Rightarrow$ $L\frac{di}{dt} = 1 - iR$

$\frac{di}{1-iR} = \frac{1}{L}dt$ $\Rightarrow$ $\int_0^i \frac{di}{\frac{1}{R} - i} = \int_0^t \frac{R}{L}dt$

$\Rightarrow \left(-\ln\left[\frac{1}{R} - i\right]\right)_0^i = Rt/L$

since $x(t) = 0$ $\forall$ $i = 0$ for all $t <$ as the system is L [inductor is at initial rest].

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EE-229 : CLASS TEST

CONT.

$$\ln\left(\frac{\frac{1}{R}-i}{\frac{1}{R}}\right) = \frac{-Rt}{L}$$

(as $i=0$ for $t<0$)

4

$$\frac{1}{R} - i = \frac{1}{R} e^{-Rt/L} \Rightarrow \boxed{i = \frac{1}{R}\left(1-e^{-Rt/L}\right)} \cdot u(t)$$

as $y(t) = iR$, $y(t) = iR = \left(1-e^{-Rt/L}\right) \cdot u(t)$; $y(t)=0 \; \forall t<0$

Step response, $s(t) = \boxed{y(t) = \left(1-e^{-Rt/L}\right) \cdot u(t)}$ [initially unexcited inductor]

<iii> Now, impulse response, $h(t) = \frac{d}{dt}(s(t))$

$$u(t) \xrightarrow{\mathcal{S}} s(t)$$

$$u(t-\Delta) \xrightarrow{\mathcal{S}} s(t-\Delta) \quad \text{[Shift invariant]}$$

$$\frac{u(t)-u(t-\Delta)}{\Delta} \xrightarrow{\mathcal{S}} \frac{s(t)-s(t-\Delta)}{\Delta} \quad \text{[linear]}$$

so, $\delta(t) \xrightarrow{\mathcal{S}} \frac{d}{dt}(s(t))$

so, $h(t) = \frac{d}{dt}\left[\left(1-e^{-Rt/L}\right) \cdot u(t)\right]$

3

$$= +e^{-Rt/L}\left(\frac{+R}{L}\right) u(t) + \left(1-e^{-Rt/L}\right) \frac{d(u(t))}{dt}$$

$$= \frac{R}{L} e^{-Rt/L} u(t) + \left(1-e^{-Rt/L}\right) \delta(t-0)$$

as $x(t)\,\delta(t-\tau) = x(\tau)\,\delta(t-\tau)$

$$\left(1-e^{-Rt/L}\right)\delta(t-0) = \left(1-e^{-R\cdot 0/L}\right)\delta(t) = 0\cdot\delta(t) = 0$$

so, $\boxed{h(t) = \frac{R}{L}\left(e^{-Rt/L}\right) \cdot u(t)}$ (impulse response)

Date
Page

Q2) discrete-time, LSI system,

$$h[n] = \left(\frac{1}{2}\right)^n u[n]$$

⟨i⟩ - For causality, $h[n] = 0 \;\forall\; n < 0$

as $y[n] = \sum x[K]h[n-K]$
we want $y[n]$ independent of $x[K] \;\forall\; K > n$
$\Rightarrow h[n-K] = 0 \;\forall\; K > n \Rightarrow h[n$ … $\forall$

Here, as $u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$

1

$$h[n] = \begin{cases} (1/2)^n & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

as $h[n] = 0 \;\forall\; n < 0 \Rightarrow$ $\boxed{\text{SYSTEM IS CAUSAL}}$

⟨ii⟩ For BIBO stability, the system should be Absolutely Summ…

i.e. $\sum_{K=-\infty}^{\infty} |h[K]| < \infty$

Here, $|h[K]| = \begin{cases} 0 & \forall\; K < 0 \\ \left(\frac{1}{2}\right)^K & \forall\; K \geq 0 \end{cases}$

so, $\sum_{-\infty}^{\infty} |h[K]| = \sum_{0}^{\infty} |h[K]| = \sum_{K=0}^{\infty} \left(\frac{1}{2}\right)^K$

2

$$= \left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \cdots$$

$$= 1 + 1/2 + (1/2)^2 + \cdots$$

By Taylor's theorem, we know that

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots \quad \text{for } |x| < 1$$

so, $1 + 1/2 + (1/2)^2 + \cdots = \frac{1}{1 - 1/2} = 2$

Since the sum, $2 < \infty \longrightarrow$ $\boxed{\text{system is BIBO STab}}$

⟨iii⟩ $x[n] = 0.3^n\, u[n]$

as the system is LSI

$$y[n] = \sum_{-\infty}^{\infty} x[K]\, h[n-K] = \sum_{-\infty}^{\infty} (0.3)^K u[K]\, (0.5)^{n-K} u[$$

$u[K] = 0 \;\forall\; K < 0$ $\qquad u[n-K] = 0 \;\forall\; n-K < 0$
$= 1 \;\forall\; K \geq 0$ $\qquad = 1 \;\forall\; n-K \geq 0$

So, we need $K \geq 0$ and $n-K \geq 0 \Rightarrow K \leq n$ [for all other K, the sum is 0]

So, $y[n] = \sum_{0}^{n} (0.3)^K (0.5)^{n-K} = \sum \left(\frac{3}{10}\right)^K \left(\frac{5}{10}\right)^{n-K}$

$$= \sum_{0}^{n} \left(\frac{1}{10}\right)^{K+n-K} \cdot 3^K 5^{n-K} = \left(\frac{1}{10}\right)^n \sum_{0}^{n} (3^K 5^{n-K})$$

GP

$$y[n] = \left(\frac{1}{10}\right)^n \left[3^0 5^n + 3 \cdot 5^{n-1} + 3^2 \cdot 5^{n-2} + \dots + 5^0 \cdot 3^n\right]$$

$$\frac{5^n\left(1 - (3/5)^{n+1}\right)}{(1-3/5)} \qquad \left[\text{Sum of a GP} = \frac{a(1-r^n)}{1-r}\right]$$

$$= \frac{1}{(10)^n}\left[\frac{\cancel{5^n}\,[5^{n+1} - 3^{n+1}]}{\cancel{5^{n+1}}\,(2/\cancel{5})}\right]$$

~~4~~ 3.5

$$y[n] = \frac{1}{2}\left[5(0.5)^n - 3(0.3)^n\right] u[n]$$

So, $\boxed{y[n] = \frac{1}{2}\left[5(0.5)^n - 3(0.3)^n\right]}\ u[n]$

(d) $y[n] = \sum_{-\infty}^{+\infty} x[K]\, h[n-K] = \sum_{-\infty}^{+\infty} x[K]\,(0.5)^{n-K}\, u[n-K]$

$n-K \geq 0 \Rightarrow K \leq n$

$$y[n] = \sum_{-\infty}^{n} x[K]\,(0.5)^{n-K}$$

$$y[n-1] = \sum_{-\infty}^{n-1} x[K]\,(0.5)^{n-K-1}$$

$$\frac{1}{2} y[n-1] = \sum_{-\infty}^{n-1} x[K]\,(0.5)^{\cancel{(n-K-1)+1}} = \sum_{-\infty}^{n-1} x[K]\,(0.5)^{n-K}$$

So, $y[n] - \frac{1}{2} y[n-1] = \sum_{-\infty}^{n} x[K](0.5)^{n-K} - \sum_{-\infty}^{n-1} x[K](0.5)^{n-K}$

$$= \left(x[n](0.5)^{n-n} + \cancel{x[n-1](0.5)^1} + \dots + \cancel{x[n-r](0.5)^r}\right) - \left(\cancel{x[n-1](0.5)^1} + \dots + \cancel{x[n-r](0.5)^r} + \dots\right)$$

Date
Page

③ $y[n] - \frac{1}{2} y[n-1] = x[n] \cdot (0.5)^{n-n} = x[n]$

So, $\boxed{y[n] = x[n] + \frac{1}{2} y[n-1]}$ → Linear Constant Coeff. Difference equation.

For (iii) $\frac{1}{2}\left[5(0.5)^n - 3(0.3)^n\right] = y[n]$

$\frac{1}{2}\left[5(0.5)^n - 3(0.3)^{n-1}(0.5)\right] = \frac{1}{2} y[n-1]$

$y[n] - \frac{1}{2} y[n-1] = -\frac{3}{2}(0.3)^n + \frac{3}{2}(0.3)^{n-1}(0.5) = \frac{3(0.3)^{n-1}}{2}\left[\frac{+1}{2} - 0.3\right]$

$= \frac{3}{2}(0.3)^{n-1}\left[0.5 - 0.3\right] = \frac{3}{2}(0.3)^{n-1}\left[0.2\right]$

$= \frac{3}{\cancel{2}}(0.3)^{n-1}(\cancel{2}/10) = (0.3)^n = x[n]$