Q1)

(a) $y(t) = \frac{dx(t)}{dt}$

A: $x(t) = 10\cos(Bt)$ B is finite $|x(t)| \leq 10$

$y(t) = \frac{dx(t)}{dt} = -10B\sin(Bt)$

$|y(t)| \leq 10B$

Note :- I considered $B>0$ (assuming it is obvious here)
if $B<0$ the bound should be $10|B|$.

(i) So, the statement (A) is correct

R: For a system to be BIBO stable, the output must be bounded for all inputs that are bounded
i.e. $\forall\ x(t)$ such that $0 \leq |x(t)| < M$ (M is finite)
(response) $y(t)$ is $0 \leq |y(t)| < M_x$ (M_x is a finite no.)

For the case $x(t) = \sin(t^2)$
$|x(t)| = |\sin(t^2)| < 2$ (or ≤ 1) i.e. the input is bounded.

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But, $y(t) = \frac{d}{dt}(\sin t^2) = 2t\cos(t^2)$

and as $t \to \infty$ $y(t) \to \infty$ i.e. $y(t)$ is not bounded.
So, the system is NOT BIBO stable.
(ii) So, the statement R is incorrect.

(b) $y[n] = \begin{cases} \frac{1}{x[n]+x[n-1]} & ;\ x[n]+x[n-1] \neq 0 \\ 0 & ;\ x[n]+x[n-1] = 0 \end{cases}$

A: A periodic bounded input → e.g. x_1 is such that
$x_1[n+n_0] = x_1[n]$ where n_0 is the period
and $0 \leq |x_1[n]| < M$ (where M is a finite no.)

The responce to $x_1[n]$ is $y_1[n]$ given by

$$y_1[n] = \begin{cases} \dfrac{1}{x_1[n]+x_1[n-1]} & x_1[n]+x_1[n-1] \neq 0 \\ 0 & x_1[n]+x_1[n-1] = 0 \end{cases}$$

Now, $$y_1[n-n_0] = \begin{cases} \dfrac{1}{x_1[n-n_0]+x_1[n-n_0-1]} & x_1[n-n_0]+x_1[n-n_0-1] \neq 0 \\ 0 & x_1[n-n_0]+x_1[n-n_0-1] = 0 \end{cases}$$

as $x_1[n-n_0] = x_1[n]$

and $x_1[n-n_0-1] = x_1[n-1]$ as n_0 is the period

$$y_1[n-n_0] = \begin{cases} \dfrac{1}{x_1[n]+x_1[n-1]} & (x_1[n]+x_1[n-1]) \neq 0 \\ 0 & (x_1[n]+x_1[n-1]) = 0 \end{cases} = y_1[n]$$

so, the output is periodic.

[NOTE:- the period may not necessarily be fundamental but it will be a multiple of it]

Also, as $0 \leq |x_1[n]| < M$

$0 \leq |x_1[n-1]| < M$

and so $0 \leq |x_1[n]+x_1[n-1]| < M$ [By Δ inequality]

when $x_1[n]+x_1[n-1] = 0$ only then $|x_1[n]+x_1[n-1]|$ can be 0

so, if $x_1[n]+x_1[n-1] \neq 0$

$0 < |x_1[n]+x_1[n-1]| \leq M$

3 and so, $\dfrac{1}{M} < \dfrac{1}{|x_1[n]+x_1[n-1]|} < \infty$ i.e., it is bounded above.

Hence, $y_1[n]$ is bounded & has same period

i) A is correct

2)

system is shift invariant as proved above (n_0 can be any ~~natural~~ no.)

i)]

$$\delta[n] = \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases} \qquad \delta[n-1] = \begin{cases} 1 & n=1 \\ 0 & n\neq 1 \end{cases}$$

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~~when~~

i/2) ⇒

$$\delta[n]+\delta[n-1] = \begin{cases} 1 & n=0,1 \\ 0 & \text{else where} \end{cases}$$

so, the impulse response becomes

$$h[n] = \begin{cases} \frac{1}{\delta[n] + \delta[n-1]} & (\delta[n] + \delta[n-1]) \neq 0 \\ 0 & \delta[n] + \delta[n-1] = 0 \end{cases}$$

$$h[n] = \begin{cases} 1 & n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

as $h[n] = 0 \;\; \forall\, n < 0$ and also is absolutely summable

i.e. $\sum_{K=-\infty}^{+\infty} |h[K]| = 2 < \infty$ (finite), the system is causal and BIBO stable.

(ii) R is correct.

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Q2

$$h[n] = \begin{cases} 1 & n = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) As the system is LSI,

$$y_1[n] = x_1[n] * h[n]$$

$$y_1[n] = \sum_{K=-\infty}^{+\infty} x_1[K]\, h[n-K]$$

$$x_1[n] = \begin{cases} a_0 & ; n=0 \\ a_1 & ; n=1 \\ a_2 & ; n=2 \\ a_3 & ; n=3 \\ a_4 & ; n=4 \\ 0 & ; \text{otherwise} \end{cases} \qquad y_1[n] = \begin{cases} 3 & n=0 \\ 10 & n=1 \\ 1 & n=2 \\ 2 & n=3 \\ 3 & n=4 \end{cases}$$

For $n<0$

$$y_1[-n] = \sum_{-\infty}^{+\infty} x_1[K]\, h[n-K]$$

e.g. $n=-1$

$$y_1[-1] = \sum_{-\infty}^{+\infty} x_1[K]\, h[-1-K] = \dots \underbrace{x_1[-3]}_{\to 0} h[2] + \underbrace{x_1[-2]}_{\to 0} h[1] + \underbrace{x_1[-1]}_{\to 0} h[0] + x_1[0] \underbrace{h[-1]}_{\to 0} + \dots$$

$$= 0$$

and so, $y_1[n] = 0 \ \forall\ n<0$ —— (i)

For $n=0$

$$y_1[0] = \sum_{K=-\infty}^{+\infty} x_1[K]\, h[-K]$$

$$3 = \dots x_1[-3] \underbrace{h[3]}_{\to 0} + \underbrace{x_1[-2]}_{\to 0} h[2] + \underbrace{x_1[-1]}_{\to 0} h[1] + x_1[0]\, h[0] + x_1[1] \underbrace{h[-1]}_{\to 0} + \dots$$

$$3 = (x_1[0])(h[0]) \Rightarrow a_0 \cdot 1 = 3 \Rightarrow a_0 = x_1[0] = 3 \text{ —— (i)}$$

$$y_1[1] = \sum_{K=-\infty}^{+\infty} x_1[K]\, h[1-K] = 0 \dots \underbrace{x_1[-3]}_{\to 0} h[4] + \underbrace{x_1[-2]}_{\to 0} h[3] + \underbrace{x_1[-1]}_{\to 0} h[2] + x_1[0]\, h[1] + x_1[1]\, h[0] + x_1[2] \underbrace{h[-1]}_{\to 0}$$

$$10 = (a_0)(1) + (a_1)(1) = 3 + a_1 \Rightarrow a_1 = x_1[1] = 7 \text{ —— (ii)}$$

$$y_1[2] = \sum_{-\infty}^{+\infty} x_1[K]\, h[2-K] = \ldots x_1[0]h[2] + x_1[1]h[1] + x_1[2]h[0] + x_1[3]\, h[-1] + \ldots$$

$h[-1] \to 0$

$$1 = a_0(1) + a_1(1) + a_2(1)$$

$$1 = 3 + 7 + a_2 \Rightarrow a_2 = x_1[2] = -9 \quad \text{— (iii)}$$

$$y_1[3] = \sum_{-\infty}^{+\infty} x_1[K]\, h[3-K] = \ldots x_1[0]h[3] + x_1[1]h[2] + x_1[2]h[1] + x_1[3]h[0] + \ldots$$

$$2 = a_1 + a_2 + a_3 = 7 - 9 + a_3 \Rightarrow a_3 = x_1[3] = 4 \quad \text{— (iv)}$$

$$y_1[4] = \sum_{-\infty}^{+\infty} x_1[K]\, h[4-K] = \ldots x_1[0]h[4] + x_1[1]h[3] + x_1[2]h[2] + x_1[3]h[1] + x_1[4]h[0] + \ldots$$

$$3 = a_2 + a_3 + a_4$$

$$3 = -9 + 4 + a_4 \Rightarrow a_4 = x_1[4] = 8 \quad \text{— (v)}$$

so,
$$\boxed{x_1[n] = \begin{cases} 3, & n=0 \\ 7, & n=1 \\ -9, & n=2 \\ 4, & n=3 \\ 8, & n=4 \\ 0, & \text{otherwise} \end{cases}}$$

Ans.

so, $y_1[5] = \sum x_1[K]\, h[5-K] = x_1[2]h[3] + x_1[3]h[2] + x_1[4]h[1] + x_1[5]h[0] + \ldots$

$$y_1[5] = 0 + a_3 + a_4 + 0 \ldots = 4 + 8 = 12$$

$$y_1[6] = \sum x_1[K]\, h[6-K] = x_1[3]h[3] + x_1[4]h[2] + x_1[5]h[1] + \ldots$$

$$y_1[6] = a_4 = 8$$

For $n > 6$

$$y_1[n] = \sum x_1[K]\, h[n-K] = x_1[0]h[n] + \ldots + x_1[4]h[n-4] + x_1[5]h[n-5] + \ldots$$

as $n > 6$ or $n \geq 7$
$n - 4 \geq 3$
$h[K] = 0 \;\forall\; K < n-4$

and $x_1[K] = 0 \;\forall\; K \geq 5$

so, the whole summation becomes 0, and

$y_1[n] = 0 \quad \forall \; n \geq 7$

so,

$$\boxed{y_1[n] = \begin{cases} 12, & n=5 \\ 8, & n=6 \\ 0, & n \geq 7. \end{cases}}$$ Ans.

⑥ $h[n] = \begin{cases} 1 & ; n=0,1,2 \\ 0 & ; o/w \end{cases}$

as $\delta[n] = \begin{cases} 1 & ; n=0 \\ 0 & ; o/w \end{cases}$ $\qquad \delta[n-1] = \begin{cases} 1 & ; n=1 \\ 0 & ; o/w \end{cases}$

h[n] can be written as

$$h[n] = \delta[n] + \delta[n-1] + \delta[n-2]$$

~~So, the syste~~

So, if $\delta[n] + \delta[n-1] + \delta[n-2] \xrightarrow{\boxed{S}} h[n]$

$x[n] * (\delta[n] + \delta[n-1] + \delta[n-2]) \xrightarrow{\boxed{S}} x[n] * h[n]$

(LTI system)

as $x[n] * \delta[n] = \sum_{K=-\infty}^{+\infty} \delta[K]\, x[n-K] = \sum \delta[K] \cdot x[n-0]$

$= x[n] \sum \delta[K] = x[n]$

and $x[n] * \delta[n-1] = \sum x[n-K]\, \delta[K-1] = x[n-1]$

so,

$x[n] + x[n-1] + x[n-2] \xrightarrow{\boxed{S}} y[n] \; (= x[n] * h[n])$

i.e. $\boxed{y[n] = x[n] + x[n-1] + x[n-2]}$ (is the system LCCDE.)

when input, $x_2[n] = \cos(Bn)$

$y_2[n] = \cos(Bn) + \cos(B(n-1)) + \cos(B(n-2))$ $\quad (B>0).$

$= \cos(Bn) + \cos(B(n-2)) + \cos(B(n-1))$

$= 2\cos\left[\frac{B(n+n-2)}{2}\right] \cos\left[\frac{B(n-n+2)}{2}\right] + \cos(B(n-1)).$

$$y_2[n] = 2\cos(B(n-1))\cos B + \cos(B(n-1))$$

$$y_2[n] = (1+2\cos B)\left[\cos[B(n-1)]\right]$$

$$\boxed{y_2[n] = (1+2\cos B)\left(\cos(Bn-B)\right)} \Rightarrow \begin{array}{l} A = 1+2\cos B \\ D = -B \end{array}$$

$$y_2[n] = (1+2\cos B)\left(\cos(Bn-B)\right)$$

if we want $y_2[n] = 0 \quad \forall\, n$

$$1+2\cos B = 0 \quad \text{or} \quad \cos(B(n-B)) = 0$$

$$\boxed{\cos B = \frac{-1}{2}} \quad \text{or} \quad \underbrace{B(n-1) = (2m+1)\frac{\pi}{2}}_{\text{can't be true always}}$$

$$\left(B = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \dots\right)$$

A can always be made >0 depending on the particular value of B i.e.

if $B = \pi$

$$y_2[n] = [1+2\cos(\pi)]\cos(\pi n-\pi)$$

$$= -1\cos(\pi n-\pi) \quad = \cos(\pi+(\pi n-\pi)) = \underline{1\cdot\cos(\pi n)}$$

as $\cos(\pi+\theta) = -(\cos\theta)$

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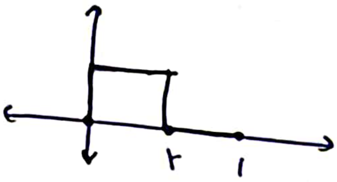
Q3) $x(t) = \begin{cases} u(t) - u(t-r) & 0<t<1 \\ \text{and } 0<r<1. \end{cases}$

$u(t) = \begin{cases} 1 & t>0 \end{cases}$

$u(t-r) = 1 \quad t>r$

$x(t+1) = x(t)$



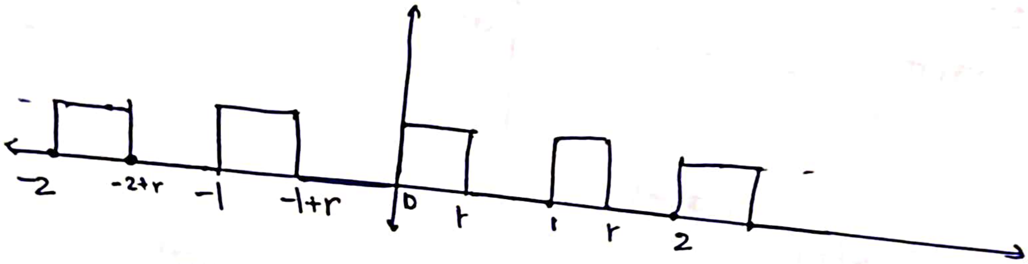
For $t<r$

$x(t) = \begin{cases} 1-0 = 1 \end{cases}$

$t>r$ and $t<1$

$x(t) = 1-1 = 0$

So, $x(t)$ is



$$x(t) = B_0 + \sum_{m=1}^{\infty} B_m \cos(2\pi m t + \alpha_m).$$

→ The period of the signal is "1".

→ The period of all the signals $\cos(2\pi m t + \alpha_m)$ is also 1.

Integrating both sides from $[0,1]$

$$\int_0^1 x(t)\cdot dt = \int_0^1 B_0\cdot dt + \sum_{m=1}^{\infty} B_m \int_0^1 \underbrace{\cos(2\pi m t + \alpha_m)\, dt}_{0 \text{ for all } m}$$

$$B_0(1) = \int_0^1 x(t)\cdot dt$$

$$= \int_0^r 1\cdot dt + \int_r^1 0\cdot dt \;\cancel{\Rightarrow \boxed{r = B_0}}$$

Let the series be expanded as

$$x(t) = B_0 + \sum_{k=-\infty}^{+\infty} a_k e^{j.2\pi k t}$$

where a_k may be complex

and here ~~it~~ is obtained by using

$$B_m \cos(2\pi m t + \alpha m) = B_m\left[e^{j[2m\pi t + \alpha m]} + e^{-j[2m\pi t + \alpha m]}\right]$$

$$= \left(B_m e^{j\alpha m}\right) e^{j2m\pi t} + \left(B_m e^{-jm\alpha}\right) e^{-j2m\pi t}$$

$$= a_m e^{jm\pi 2t} + a_{m'} e^{-jm2\pi t}$$

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$\frac{\pi}{2})$]

$\pi/2$

$\omega_0 = r$

$$x(t) = B_0 + \sum a_K e^{j \cdot 2\pi K t}$$

for some m

$$x(t) \cdot e^{-j 2\pi K t} = B_0 e^{-j \cdot 2\pi K t} + \sum a_K \cdot e^{j 2\pi K t} \cdot e^{-j 2\pi m t}$$

on integrating both sides,

$$\int_{(T)} x(t) e^{-j 2\pi m t} = a_m (1) \quad \left[\text{as} \int e^{j \cdot 2\pi K t} \cdot e^{-j 2\pi m t} \begin{array}{ll} = 0 & m \neq K \\ = 1 & m = K \end{array} \right]$$

$$a_m = \int_{(T)} x(t) \cdot e^{-j 2\pi m t}$$

$$a_m = \int_0^r 1 \cdot e^{-j \cdot 2\pi m t} \cdot dt + \int_r^1 0 \cdot e^{-j 2\pi m t} \cdot dt$$

$$a_m = \left(\frac{e^{-j \cdot 2\pi m t}}{(-2\pi j m)} \right)_0^r = \frac{1 - e^{-2\pi j m r}}{2\pi j m}.$$

$$a_{-m} = \frac{1 - e^{2\pi j m r}}{-2\pi j m} = \frac{e^{2\pi j m r} - 1}{2\pi j m} = \overline{a_m}$$

so, $$a_m e^{j 2\pi m t} + a_{-m} e^{-j 2\pi m t} = a_m e^{j \cdot 2\pi m t} + \overline{(a_m e^{j \cdot 2\pi m t})}$$

$$= \text{Re}\left\{ a_m \cdot e^{j 2\pi m t} \right\}$$

$$= \text{Re}\left\{ \left(\frac{1 - e^{-2\pi j m r}}{2\pi j m} \right) \cdot e^{j \cdot 2\pi m t} \right\}$$

$$= \text{Re}\left\{ \frac{e^{2\pi j m t}}{2\pi j m} - \frac{e^{2\pi j m (t-r)}}{2\pi j m} \right\}$$

$$= \frac{\sin(2\pi m t)}{2\pi m} - \frac{\sin(2\pi m (t-r))}{2\pi m}$$

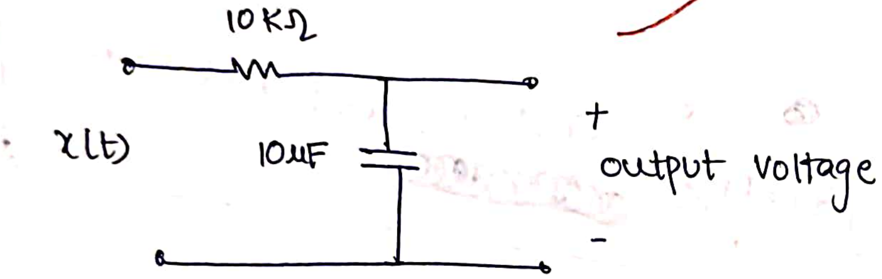
$$= \frac{\cancel{2}\sin\left(\frac{2\pi m t - 2\pi m(t-r)}{2}\right)\cos\left(\frac{2\pi m t + 2\pi m(t-r)}{2}\right)}{\cancel{2}\pi m}$$

$$= \frac{1}{\pi m}\sin(\pi m r)\cos(2\pi m t - \pi m r).$$ [coupling opposite terms]

So, the final expansion becomes

$$x(t) = r + \sum_{m=1}^{\infty}\left[\frac{\sin(\pi m r)}{\pi m}\right]\cos(2\pi m t - \pi m r)$$

(b)



$x(t) - iR = y(t)$ and ~~y(t)=~~ $i = C\frac{dy(t)}{dt}$

so, $x(t) = y(t) + RC\left(\frac{dy(t)}{dt}\right)$. $\left[RC = 10\times10^{3}\times10\times10^{-6} = 10^{-1} = \left(\frac{1}{10}\right).\right]$

~~taken~~ Taking Fourier transform on both sides,

$$X(j\Omega) = Y(j\Omega) + RC(j\Omega)\cdot Y(j\Omega)$$

$$\left[\begin{array}{l} x(t) = \frac{1}{2\pi}\int X(j\Omega)e^{j\Omega t}\cdot d\Omega \\ \frac{dx(t)}{dt} = \frac{1}{2\pi}\int j\Omega X(j\Omega)\cdot e^{j\Omega t}\,d\Omega \end{array}\right]$$

CONT.

So, $X(j\Omega) = (1 + RCj\Omega)\,Y(j\Omega)$

i.e. The $F(Y(t)) = \frac{1}{(1+RCj\Omega)}\cdot F(X(j\Omega))$. [Bo term is lost here]

M1 as here $X(j\Omega) \longleftrightarrow B_m\cos(2\pi m t + \alpha_m)$

i.e. ~~$X(m\Omega_0) \longrightarrow B_m\cos(2\pi m t + \alpha_m)$~~

So, the coefficients of ... will be

$$\sum C_m \cos(2\pi t m + \alpha_m) = \sum \frac{B_m \cos(2\pi m t + \alpha_m)}{(1+RCj(m\omega_0))}$$

i.e. the coefficients will become

$$\frac{B_m}{\left(1+RCjm\left(\frac{2\pi}{1}\right)\right)} = \frac{B_m}{(1+RCjm)} \qquad \left[\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{1}\right]$$

$$= \frac{\sin(\pi m r)}{(\pi m)(1+RCjm)} = \frac{(\sin \pi m r)}{\pi m(1+RCjm)}$$

$$\cancel{C_0 = \frac{B_0}{RCjm}}$$

$$\boxed{y(t) = r + \sum_{m=1}^{\infty} \frac{\sin(\pi m r)}{[illegible]} \cos(2\pi m t + \alpha_m)}$$

(c) $B_m = 0$ for all non-zero even integers m

$$B_m = \frac{\sin(\pi m r)}{\pi m} = 0$$

$$\pi m r = (k\pi)$$

$$m = 2n$$

$$2nr = k \qquad n = 1, 2, 3, \ldots$$

$\boxed{r = \frac{1}{2}}$ $\longrightarrow$ $B_m = 0$ for all non-zero even integers m

or $r = 1 \rightarrow$ not possible as $0 < r < 1$.

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भारतीय प्रौद्योगिकी संस्थान मुंबई
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उत्तर पुस्तिका/ Answer Book-4

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EE-BTECH/D2/T3
शाखा/प्रभाग/Branch/Div.शैक्षणिक बैच /Tutorial Batch

अनुभाग/Section

Q4) $x_1(t) = e^{-2t}u(t)$ $x_2(t) = e^{-5t}u(t)$

$$X_1(j\Omega) = \int_{-\infty}^{+\infty} x_1(t)e^{-j\Omega t}\,dt$$

$$= \int_{-\infty}^{+\infty} e^{-2t}u(t)\cdot e^{-j\Omega t}\cdot dt = \int_0^{\infty} e^{-2t}\cdot e^{-j\Omega t}\,dt$$

$$= \int_0^{\infty} e^{-(j\Omega+2)t}\cdot dt = \frac{1}{-(j\Omega+2)}\left[e^{-(j\Omega+2)t}\right]_0^{+\infty}$$

$$= \frac{-1}{j\Omega+2}\left[0-1\right]$$

$$\boxed{X_1(j\Omega) = \frac{1}{j\Omega+2}}$$

and $X_2(j\Omega) = \int_{-\infty}^{+\infty} e^{-5t}\cdot e^{-j\Omega t}\,dt\; u(t) = \int_0^{\infty} e^{-5t}e^{-j\Omega t}\,dt$

$$= \frac{1}{j\Omega+5}\left[1-0\right]$$

$$\boxed{X_2(j\Omega) = \frac{1}{j\Omega+5}}$$

a) $|X_1(j\Omega)| = \frac{1}{\sqrt{(\Omega)^2+(2)^2}} = \frac{1}{\sqrt{\Omega^2+4}}$ and $|X_1(j\Omega)|^2 = \frac{1}{\Omega^2+4}$

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So, $\int_{-\infty}^{+\infty} \frac{1}{\Omega^2+4}\cdot d\Omega = \frac{1}{2}\left[\tan^{-1}\left(\frac{\Omega}{2}\right)\right]_{-\infty}^{+\infty} = \frac{1}{2}\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]$

$= \frac{1}{2}[\pi] = \pi/2$

(b) $$\int_{-\infty}^{+\infty} X_1(j\Omega)\, X_2(j\Omega)\, d\Omega$$

$$\int_{-\infty}^{+\infty} x_1(t)\, x_2(t) \cdot dt = \int_{-\infty}^{+\infty}\left(x_1(t) \cdot \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_2(j\Omega)\, e^{-j\Omega t}\, d\Omega\right) \cdot dt$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_2(j\Omega)\left(\int_{-\infty}^{+\infty} x_1(t)\, e^{-j\Omega t}\, dt\right) d\Omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_1(j\Omega)\, X_2(j\Omega)\, d\Omega$$

(parseval's theorem) $\left[\langle x_1, x_2\rangle = \frac{1}{2\pi}\langle X_1, X_2\rangle\right]$

so, as $$\int_{-\infty}^{+\infty} x_1(t) \cdot x_2(t) = \int (e^{-2t} u(t)) \cdot (e^{-5t} u(t)) \cdot dt$$

$$= \int_{-\infty}^{+\infty} e^{-7t} u(t)\, dt$$

$$= \int_{0}^{\infty} e^{-7t}\, dt = \frac{1}{(-7)}[0-1]$$

$$= \frac{1}{7}$$

so, $\int X_1(j\Omega)\, X_2(j\Omega)\, d\Omega = 2\pi/7 \Rightarrow$ (B)

(c) $$|X_1(j\Omega)|^2 = \frac{1}{\Omega^2+4}$$

$$\left(\frac{1}{j\Omega+2}\right)\left(\frac{1}{j\Omega-2}\right) = \frac{1}{(j\Omega)^2 - (2)^2} = \frac{1}{-\Omega^2-4}$$

i.e. $$\frac{1}{\Omega^2+4} = -\underbrace{\left(\frac{1}{j\Omega+2}\right)}_{X_1(j\Omega)}\underbrace{\left(\frac{1}{j\Omega-2}\right)}_{X_2(j\Omega)}$$

By convolution property

$x_1(t) * x_2(t)$ has F.T. $X_1(j\Omega) \cdot X_2(j\Omega)$

as IFT of $\frac{1}{j\Omega+2}$ is $e^{-2t}u(t)$

and IFT of $\frac{1}{j\Omega-2}$ is $e^{2t}u(t)$

so, IFT of $(-X_1 \cdot X_2) = -x_1 * x_2$

$$= -\int_{-\infty}^{+\infty} e^{-2\cancel{K}} \cdot u(\cancel{K})\, e^{2(t-K)}\, u(t-K)\, dK$$

$K>0 \qquad t-K>0 \rightarrow K<t$

$$= -\int_{0}^{t} e^{-2K} \cdot e^{2t} \cdot e^{-2K}\, dK = -e^{2t}\int_0^t e^{-4K}\, dK$$

$$= +e^{-2t}\left[\frac{e^{-4K}}{(+4)}\right]_0^t$$

$$= \frac{1}{4}\left[e^{+2t}\left[e^{-4t}-1\right]\right]$$

$$= \frac{1}{4}\left[e^{-2t} - e^{+2t}\right]$$

$$\left(\frac{1}{j\Omega+2} \cdot \frac{1}{j\Omega-2} \quad \frac{j\Omega-2-j\Omega-2}{-\Omega^2-4}\right)$$

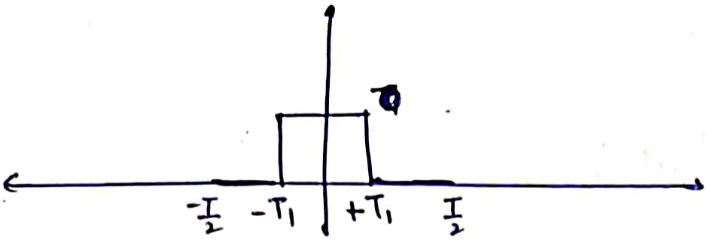
so, IFT of $|X_1(j\Omega)|^2$ is $\boxed{\frac{1}{4}\left[e^{-2t}-e^{2t}\right]}$

Ans.

4

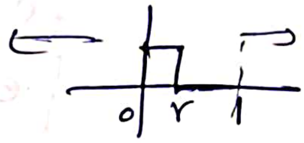
$\cos C + \cos D = 2\left(\cos\frac{C+D}{2}\cos\frac{C-D}{2}\right)$

$\cancel{\left(\frac{1}{j\Omega+2}\right)\left(\frac{1}{j\Omega+5}\right)}$



$\int_{-T_1}^{T_1} 1\cdot e^{-jk\omega_0 t}\,dt$

$\frac{2\sin(k\omega_0 t)}{k\omega_0}$ $\quad \omega_0 = \frac{2\pi}{T_0}$ $\quad \frac{(2\sin(k\omega_0 t))}{k\omega_0}$



$B_0 = \int_0^r 1\cdot dt = r$

$B_k = \int_0^r 1\cdot e^{-j2\pi k t}\,dt = \frac{[1-e^{-j\cdot 2\pi k r}]}{2\pi j k}$

$B_k = \frac{1-e^{-j\cdot 2\pi k r}}{2\pi k\cdot j}$ $\quad B_{-k} = \overline{B_k}$

$Re\left\{(B_k)e^{+2\pi j k t}\right\} \quad Re\left\{\left(\frac{1-e^{-j2\pi k r}}{2\pi k j}\right)\left(e^{2\pi j k t}\right)\right\}$

$= \frac{1}{2\pi k}\left[\sin(2\pi k t) - \sin(2\pi k(t-r))\right]$

$= \frac{1}{2\pi k}\left[2\cos\left(\frac{2\pi k(2t-r)}{2}\right)\sin\left(\frac{2\pi k r}{2}\right)\right]$

$= \frac{\sin(\pi k r)}{\pi k}\cdot\sin(2\pi k t - \pi k r)$

$y(t) \rightarrow \sum a_k e^{+jk\omega_0 t}$

$\frac{dy(t)}{dt} \rightarrow \sum a_k(+jk\omega_0)\cdot e^{+jk\omega_0 t}$

$y(t) + RC\frac{dy(t)}{dt} = \sum a_k(1+RCjk\omega_0)e^{+jk\omega_0 t}$

$a_k(1+RCjk\omega_0) = B_k.$

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उत्तर पुस्तिका/ Answer Book-4

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अनुभाग/Section

Q3) (b) as $x(t) = y(t) + RC\dfrac{dy(t)}{dt}$

MT.

Let $y(t) = a_0 + \sum a_K \cos(2\pi Kt + \alpha_K)$

$$\frac{dy(t)}{dt} = \sum a_K(-\sin(2\pi Kt + \alpha_K))(2\pi K)$$

$$y(t) + RC\frac{dy(t)}{dt} = a_0 + \sum a_K\left[\cos(2\pi Kt + \alpha_K) - 2\pi KRC\sin(2\pi Kt+\alpha_K)\right]$$

$$x(t) = a_0 + \sum a_K\left[\cos(2\pi Kt + \alpha_K) - 2\pi KRc\sin(2\pi Kt + \alpha_K)\right]$$

$$b_0 + \sum b_m \cos(2\pi mt + \alpha_m) = a_0 + \sum a_K\left[\cos\theta - C\cdot\sin\theta\right]$$

$$\cos\theta - C\sin\theta = \frac{1}{\sqrt{1+C^2}}\cos\theta - \frac{C}{\sqrt{1+C^2}}\sin\theta$$

$\cos\alpha$

$= \cos(\theta+\alpha)$ where $\left(\tan\alpha = C = \frac{1}{10} = RC\right)$

$$b_0 + \sum b_m \cos(2\pi mt + \alpha_m) \rightarrow a_0 + \sum a_K \cos(2\pi t + \alpha_K)$$

so, $a_0 = b_0 = r$

$a_K = b_K = \dfrac{\sin(\pi m r)}{\pi m}$

and $\alpha_K = \alpha_m + \alpha$ where $\alpha = \tan^{-1}(2\pi K)$

$$y(t) = r + \sum b_m \cos(2\pi mt + \alpha_m + \tan^{-1}(2\pi K))$$

$|H(j\Omega_0)| \cos(\cdots+\alpha)$

cos C + cos D

M2)

~~y(t) = x(t)~~

$$\boxed{Y(j\Omega) = \frac{X(j\Omega)}{1+RCj\Omega}}$$

- all expansion coeff. in exponential series are $\underline{\left(\frac{1}{1+RCj\omega_0}\right) a_K} \rightarrow x(t)$.

$$y(t) = \sum_{-\infty}^{+\infty} \frac{a_K \cdot e^{+j\Omega K t}}{1+RCj(\Omega = m\omega_0)}$$

so, we now have.

$$Re\left\{ \frac{a_K}{1+RCj\, m\omega_0} \cdot e^{jK\omega_0 t} \right\} = b_m$$ expansion coefficients of $X(t)$

$$y(t) = r + \sum a_K \cos(2\pi K t + \alpha_K)$$

$$|H(j\Omega_0)| = \left| e^{-t/\tau} u(t) \right| = 1$$

and $\measuredangle H(j\Omega_0) = \underline{\alpha}$

(4) Simplify to claim full marks