AP Calculus AB

1. Limits and Continuity

- **Limit Definition**: $\lim_{x \to a} f(x) = L$
- **Limit Laws**:
  - Sum/Difference: $\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
  - Product: $\lim_{x \to a}[f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
  - Quotient: $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ (if $\lim_{x \to a} g(x) \neq 0$)
- **Squeeze Theorem**: If $f(x) \leq g(x) \leq h(x)$ and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} g(x) = L$.
- **Continuity**: A function $f(x)$ is continuous at $x = a$ if: $\lim_{x \to a} f(x) = f(a)$

2. Derivatives

- **Definition of Derivative**: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
- **Basic Derivative Rules**:
  - Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$
  - Constant Rule: $\frac{d}{dx}[c] = 0$
  - Sum/Difference: $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$
  - Product Rule: $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x)g(x) + f(x)g'(x)$
  - Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$
  - Chain Rule: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$
- **Derivatives of Trigonometric Functions**:
  - $\frac{d}{dx}[\sin x] = \cos x$
  - $\frac{d}{dx}[\cos x] = -\sin x$
  - $\frac{d}{dx}[\tan x] = \sec^2 x$
- **Derivatives of Exponential and Logarithmic Functions**:
  - $\frac{d}{dx}[e^x] = e^x$
  - $\frac{d}{dx}[\ln x] = \frac{1}{x}$
- **Implicit Differentiation**: Differentiate both sides of an equation with respect to x and solve for $\frac{dy}{dx}$.

3. Applications of Derivatives

- **Tangent Line Equation**: $y = f(a) + f'(a)(x - a)$
- **Mean Value Theorem**: If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there exists $c \in (a, b)$ such that: $f'(c) = \frac{f(b) - f(a)}{b - a}$
- **Increasing/Decreasing Functions**:
  - If $f'(x) > 0$, $f(x)$ is increasing.
  - If $f'(x) < 0$, $f(x)$ is decreasing.
- **Concavity**:
  - If $f''(x) > 0$, $f(x)$ is concave up.

- If $f''(x) < 0$, $f(x)$ is concave down.
- **Optimization**: Find critical points ($f'(x) = 0$ or undefined) and use the First or Second Derivative Test to determine maxima/minima.

4. Integrals

- **Antiderivatives**: $\int f(x)\,dx = F(x) + C$ where $F'(x) = f(x)$
- **Basic Integration Rules**:
  - Power Rule: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)
  - $\int e^x\,dx = e^x + C$
  - $\int \frac{1}{x}\,dx = \ln|x| + C$
- **Definite Integral**: $\int_a^b f(x)\,dx = F(b) - F(a)$
- **Fundamental Theorem of Calculus**: $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$
- **Substitution Rule**: $\int f(g(x))g'(x)\,dx = \int f(u)\,du$ where $u = g(x)$

5. Applications of Integrals

- **Area Under a Curve**: $\text{Area} = \int_a^b f(x)\,dx$
- **Area Between Two Curves**: $\text{Area} = \int_a^b [f(x) - g(x)]\,dx$ where $f(x) \geq g(x)$
- **Volume of Revolution**:
  - Disk Method: $V = \pi \int_a^b [f(x)]^2\,dx$
  - Washer Method: $V = \pi \int_a^b \left([f(x)]^2 - [g(x)]^2\right)dx$
- **Average Value of a Function**: $\text{Average Value} = \frac{1}{b-a} \int_a^b f(x)\,dx$

6. Differential Equations

- **Separation of Variables**: $\frac{dy}{dx} = f(x)g(y) \implies \int \frac{1}{g(y)}\,dy = \int f(x)\,dx$
- **Exponential Growth/Decay**: $\frac{dy}{dt} = ky \implies y = y_0 e^{kt}$

7. Miscellaneous

- **L'Hôpital's Rule**:
  If $\lim_{x\to a} \frac{f(x)}{g(x)}$ is $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then:
  $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$
- **Intermediate Value Theorem**: If $f(x)$ is continuous on $[a, b]$ and k is between $f(a)$ and $f(b)$, then there exists $c \in (a, b)$ such that $f(c) = k$.