دانشگاه صنعتی امیرکبیر

دانشکده مهندسی هوافضا

عنوان

تکلیف شماره 6

نگارش

فاطمه مقدسیان

استاد :

دکتر سبزه پرور

1-Derive 0,1,2 and 3rdorder Least Square filters and draw the related graphs.
2-Derive 4th order Least Square filters and draw the graphs.
3-Analyze 4thorder residual, and compare it to the third order results

K	$t=(k-1)t_s$	x_k^*
1	0	1.2
2	1	0.2
3	2	2.9
4	3	2.1

$$R=\sum_{k=1}^{n}(\widehat{x_k}-x_k^*)^2$$

Zeroth Order System (One State Filter)

$$\widehat{x_k}=a \Rightarrow R=\sum_{k=1}^{n}(a-x_k^*)^2$$

$$R=(a-x_1^*)^2+(a-x_2^*)^2+\cdots+(a-x_n^*)^2 \Rightarrow R=(a-1.2)^2+(a-0.2)^2+(a-2.9)^2+(a-2.1)^2$$

$$\frac{\partial R}{\partial a}=0=2(a-x_1^*)+2(a-x_2^*)+\cdots+2(a-x_n^*) \Rightarrow \frac{\partial R}{\partial a}=2(a-1.2)+2(a-0.2)+2(a-2.9)+2(a-2.1)$$

$$\frac{\partial^2 R}{\partial a^2}=2n \Rightarrow \frac{\partial^2 R}{\partial a^2}=2\times 4=8$$

$$\frac{\partial R}{\partial a}\times\frac{1}{\frac{\partial^2 R}{\partial a^2}}=-(1.2+0.2+2.9+2.1)=6.4$$

$$\widehat{x_k}=a=\frac{\sum_{k=1}^{n}x_k^*}{n} \Rightarrow a=\frac{1.2+0.2+2.9+2.1}{4}=1.6$$

First Order System (Two State Filter)

$$\widehat{x_k}=a+bt \Rightarrow R=\sum_{k=1}^{n}(a+bt-x_k^*)^2 \quad \& \quad \dot{\widehat{x_k}}=b$$

$$R=(a-x_1^*)^2+(a+bt_s-x_2^*)^2+\cdots+(a+b(n-1)t_s-x_n^*)^2 \Rightarrow R$$
$$=(a-1.2)^2+(a+bt_s-0.2)^2+(a+2bt_s-2.9)^2+(a+3bt_s-2.1)^2$$

$$\frac{\partial R}{\partial a}=0=2(a-x_1^*)+2(a+bt_s-x_2^*)+\cdots+2(a+b(n-1)t_s-x_n^*) \Rightarrow \frac{\partial R}{\partial a}$$
$$=2(a-1.2)+2(a+bt_s-0.2)+2(a+2bt_s-2.9)+2(a+3bt_s-2.1)$$

$$\frac{\partial R}{\partial b}=0=2t_s(a+bt_s-x_2^*)+\cdots 2(n-1)t_s(a+b(n-1)t_s-x_n^*) \Rightarrow \frac{\partial R}{\partial a}$$
$$=2t_s(a+bt_s-0.2)+4t_s(a+2bt_s-2.9)+6t_s(a+3bt_s-2.1)$$

$$na+\sum_{k=1}^{n}b(k-1)t_s=\sum_{k=1}^{n}x_k^*$$

$$a\sum_{k=1}^{n}(k-1)t_s + b\sum_{k=1}^{n}\left((k-1)t_s\right)^2 = \sum_{k=1}^{n}(k-1)t_s\, x_k^*$$

$t_s = 1\ second\ \&\ n = 4$

$$\sum_{k=1}^{n}(k-1)t_s = 0+1+2+3 = 6$$

$$\sum_{k=1}^{n}x_k^* = 1.2+0.2+2.9+2.1 = 6.4$$

$$\sum_{k=1}^{n}(k-1)t_s\, x_k^* = 0.2+5.8+6.3 = 12.3$$

$$\begin{cases} 4a+6b = 6.4 \\ 6a+14b = 12.3 \end{cases} \Rightarrow\ a = 0.79\ \&\ b = 0.54$$

$\widehat{x_k} = 0.79 + 0.54t_s$

Second Order System (Three State Filter)

$\widehat{x_k} = a + bt + c\,t^2\ \ \&\ \dot{\widehat{x_k}} = b + 2ct\ \ \&\ \ddot{\hat{x}} = 2c$

$\widehat{x_k} = a + b(k-1)t_S + c[(k-1)t_S]^2$

$$R = \sum_{k=1}^{n}(a+bt+ct^2-x_k^*)^2\ \Rightarrow R$$
$$= (a-1.2)^2 + (a+bt_S+ct_S^2-0.2)^2 + (a+2bt_S+4ct_S^2-2.9)^2 + (a+3bt_S+9ct_S^2-2.1)^2$$

$$\frac{\partial R}{\partial a} = 0 = 2(a-1.2) + 2(a+bt_S+ct_S^2-0.2) + 2\,(a+2bt_S+4ct_S^2-2.9) + 2(a+3bt_S+9ct_S^2-2.1)$$

$$\frac{\partial R}{\partial b} = 0 = 0 + 2t_S(a+bt_S+ct_S^2-0.2) + 4t_S(a+2bt_S+4ct_S^2-2.9) + 6t_S(a+3bt_S+9ct_S^2-2.1)$$

$$\frac{\partial R}{\partial c} = 0 = 0 + 2t_S^2(a+bt_S+ct_S^2-0.2) + 8t_S^2\,(a+2bt_S+4ct_S^2-2.9) + 18t_S^2(a+3bt_S+9ct_S^2-2.1)$$

$t_s = 1\ second\ \ \&\ n = 4$

$$\begin{cases} 2(a-1.2)+2(a+b+c-0.2)+2(a+2b+4c-2.9)+2(a+3b+9c-2.1) = 0 \\ 2(a+b+c-0.2)+4(a+2b+4c-2.9)+6(a+3b+9c-2.1) = 0 \\ 2(a+b+c-0.2)+8(a+2b+4c-2.9)+18(a+3b+9c-2.1) = 0 \end{cases}$$

$\Rightarrow a = 0.84\ \&\ b = 0.39\ \&\ c = 0.05$

$\widehat{x_k} = 0.84 + 0.39t_S + 0.05\,t_S^2\ \ \&\ \dot{\widehat{x_k}} = 0.39 + 0.1t_S\ \ \&\ \ddot{\hat{x}} = 0.1$

Third Order System (Four State Filter)

$\widehat{x_k} = a + bt + ct^2 + dt^3$ & $\widehat{\dot{x}_k} = b + 2ct + 3dt^2$ & $\widehat{\ddot{x}_k} = 2c + 6dt$ & $\widehat{\dddot{x}_k} = 6d$

$$R = \sum_{k=1}^{n} (a + bt + ct^2 + dt^3 - x_k^*)^2 \Rightarrow R$$

$$= (a - 1.2)^2 + (a + bt_s + ct_s^2 + dt_s^3 - 0.2)^2 + (a + 2bt_s + 4ct_s^2 + 9dt_s^3 - 2.9)^2 + (a + 3bt_s + 9ct_s^2 + 27dt_s^3 - 2.1)^2$$

$$\frac{\partial R}{\partial a} = 0 = 2(a - 1.2) + 2(a + bt_s + ct_s^2 + dt_s^3 - 0.2) + 2(a + 2bt_s + 4ct_s^2 + 9dt_s^3 - 2.9) + 2(a + 3bt_s + 9ct_s^2 + 27dt_s^3 - 2.1)$$

$$\frac{\partial R}{\partial b} = 0 = 2t_s(a + bt_s + ct_s^2 + dt_s^3 - 0.2) + 4t_s(a + 2bt_s + 4ct_s^2 + 9dt_s^3 - 2.9) + 6t_s(a + 3bt_s + 9ct_s^2 + 27dt_s^3 - 2.1)$$

$$\frac{\partial R}{\partial c} = 0 = 2t_s^2(a + bt_s + ct_s^2 + dt_s^3 - 0.2) + 8t_s^2(a + 2bt_s + 4ct_s^2 + 9dt_s^3 - 2.9) + 18t_s^2(a + 3bt_s + 9ct_s^2 + 27dt_s^3 - 2.1)$$

$$\frac{\partial R}{\partial d} = 0 = 2t_s^3(a + bt_s + ct_s^2 + dt_s^3 - 0.2) + 18t_s^3(a + 2bt_s + 4ct_s^2 + 9dt_s^3 - 2.9) + 54t_s^3(a + 3bt_s + 9ct_s^2 + 27dt_s^3 - 2.1)$$

$t_s = 1\ second$ & $n = 4$

$$\begin{cases} 2(a - 1.2 + a + b + c + d - 0.2 + a + 2b + 4c + 9d - 2.9 + a + 3b + 9c + 27d - 2.1) = 0 \\ 2(a + b + c + d - 0.2) + 4(a + 2b + 4c + 9d - 2.9) + 6(a + 3b + 9c + 27d - 2.1) = 0 \\ 2(a + b + c + d - 0.2) + 8(a + 2b + 4c + 9d - 2.9) + 18(a + 3b + 9c + 27d - 2.1) = 0 \\ 2(a + b + c + d - 0.2) + 18(a + 2b + 4c + 9d - 2.9) + 54(a + 3b + 9c + 27d - 2.1) = 0 \end{cases}$$

$\Rightarrow a = 1.2$ & $b = -5.25$ & $c = 5.45$ & $d = -1.2$

$\widehat{x_k} = 1.2 - 5.25t + 5.25t^2 - 1.2t^3$ & $\widehat{\dot{x}_k} = -5.25 + 10.5t - 3.6t^2$ & $\widehat{\ddot{x}_k} = 10.5 - 7.2t$ & $\widehat{\dddot{x}_k} = -7.2$

Fourth Order System (five State Filter)

$\widehat{x_k} = a + bt + ct^2 + dt^3 + et^4$ & $\widehat{\dot{x}_k} = b + 2ct + 3dt^2 + 4et^3$ & $\widehat{\ddot{x}_k} = 2c + 6dt + 12et^2$ & $\widehat{\dddot{x}_k} = 6d + 24et$

$$R = \sum_{k=1}^{n} (a + bt + ct^2 + dt^3 + et^4 - x_k^*)^2 \Rightarrow R$$

$$= (a - 1.2)^2 + (a + bt_s + ct_s^2 + dt_s^3 + et_s^4 - 0.2)^2 + (a + 2bt_s + 4ct_s^2 + 9dt_s^3 + 16et_s^4 - 2.9)^2 + (a + 3bt_s + 9ct_s^2 + 27dt_s^3 + 81et_s^4 - 2.1)^2$$

$$\frac{\partial R}{\partial a} = 0 = 2(a - 1.2) + 2(a + bt_s + ct_s^2 + dt_s^3 + et_s^4 - 0.2) + 2(a + 2bt_s + 4ct_s^2 + 9dt_s^3 + 16et_s^4 - 2.9) + 2(a + 3bt_s + 9ct_s^2 + 27dt_s^3 + 81et_s^4 - 2.1)$$

$$\frac{\partial R}{\partial b} = 0 = 2t_s(a + bt_s + ct_s^2 + dt_s^3 + et_s^4 - 0.2) + 4t_s(a + 2bt_s + 4ct_s^2 + 9dt_s^3 + 16et_s^4 - 2.9) + 6t_s(a + 3bt_s + 9ct_s^2 + 27dt_s^3 + 81et_s^4 - 2.1)$$

$$\frac{\partial R}{\partial c} = 0 = 2t_s^2(a + bt_s + ct_s^2 + dt_s^3 + et_s^4 - 0.2) + 8t_s^2(a + 2bt_s + 4ct_s^2 + 9dt_s^3 + 16et_s^4 - 2.9) + 18t_s^2(a + 3bt_s + 9ct_s^2 + 27dt_s^3 + 81et_s^4 - 2.1)$$

$$\frac{\partial R}{\partial d} = 0 = 2t_s^3(a + bt_s + ct_s^2 + dt_s^3 + et_s^4 - 0.2) + 18t_s^3(a + 2bt_s + 4ct_s^2 + 9dt_s^3 + 16et_s^4 - 2.9) + 54t_s^3(a + 3bt_s + 9ct_s^2 + 27dt_s^3 + 81et_s^4 - 2.1)$$

$$\frac{\partial R}{\partial e} = 0 = 2t_s^4(a + bt_s + ct_s^2 + dt_s^3 + et_s^4 - 0.2) + 32t_s^4(a + 2bt_s + 4ct_s^2 + 9dt_s^3 + 16et_s^4 - 2.9) + 162t_s^4(a + 3bt_s + 9ct_s^2 + 27dt_s^3 + 81et_s^4 - 2.1)$$

$t_s = 1\ second\ \ \&\ n = 4$

$$\begin{cases} 2(a-1.2) + 2(a+b+c+d+e-0.2) + 2(a+2b+4c+9d+16e-2.9) + 2(a+3b+9c+27d+81e-2.1) = 0 \\ 2(a+b+c+d+e-0.2) + 4(a+2b+4c+9d+16e-2.9) + 6(a+3b+9c+27d+81e-2.1) = 0 \\ 2(a+b+c+d+e-0.2) + 8(a+2b+4c+9d+16e-2.9) + 18(a+3b+9c+27d+81e-2.1) = 0 \\ 2(a+b+c+d+e-0.2) + 18(a+2b+4c+9d+16e-2.9) + 54(a+3b+9c+27d+81e-2.1) = 0 \\ 2(a+b+c+d+e-0.2) + 32(a+2b+4c+9d+16e-2.9) + 162(a+3b+9c+27d+81e-2.1) = 0 \end{cases}$$

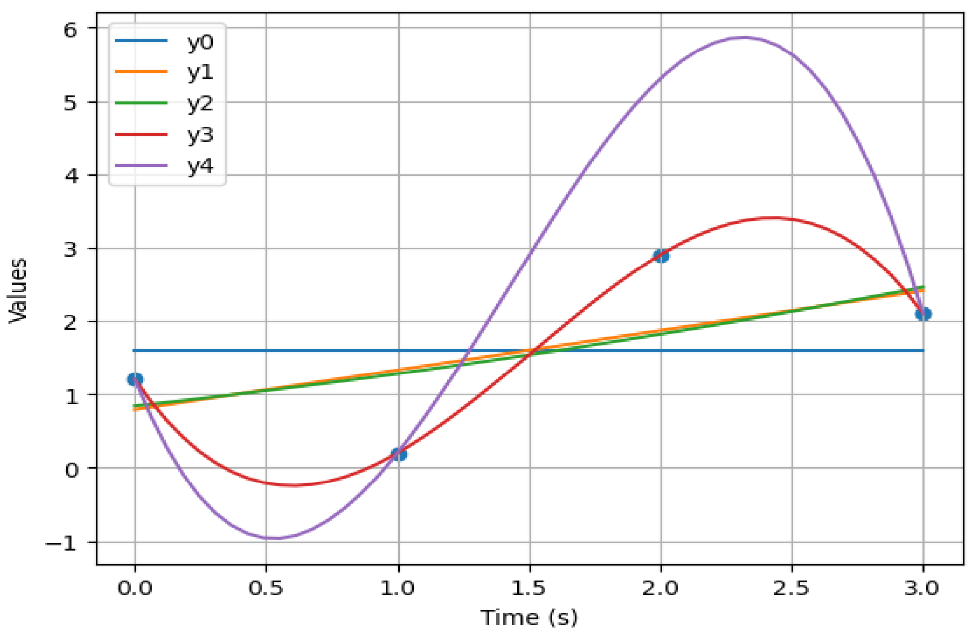
$\Rightarrow a = 1.2\ \&\ b = -8.85\ \&\ c = 10.25\ \&\ d = -2.4\ \&\ \mathrm{e} = -1.63790491\mathrm{e} - 16$

$\widehat{x_k} = 1.2 - 8.85t + 10.25t^2 - 2.4t^3 - 1.63790491\mathrm{e} - 16t^4$

$R_4 = 2.4257472835546113\mathrm{e} - 29$

$R_3 = 0$

اختلاف درجه 4 با درجه 3 نزدیک به صفر است.



نمودار تخمین مینیمم مربعات برای درجه صفر ، یک ، دو ، سه و چهار