Team Control Number: **13570**
Problem Chosen: **A**

2022 High School Mathematical Contest in Modeling (HiMCM)
Summary Sheet

Do not include the name of your school, advisor, or team members on this or any page.
Papers must be submitted as an Adobe PDF electronic file, and types in English, with a readable font of at least 12-point type. Papers must be within the 25-page limit.

Honey bees play a massive role in human existence. Albert Einstein, arguably the most influential physicist of the twentieth century, famously said "If the bee disappeared off the surface of the globe, then man would only have four years of life left." This statement can easily be proven—as bees are significant contributors to the pollination of plant species. This contributes to both the geographical and genetic diversity of plants, both of which are key ecological functions. However, human externalities such as the introduction of pesticides and the consumption of land negatively impact bee populations in a condition known as Colony Collapse Disorder (CCD), which destroys ecological function. Hence, this mathematical modeling paper aims to determine how the population and pollination of bees are affected by certain factors in order to improve conservation efforts.

We described in Task 1 a time-based stochastic model for the population of a single honey bee colony that takes into account the density of food sources in the environment, the roles that bees play inside a hive, and the impact of negative externalities (e.g., change in weather, the introduction of predators, pesticides, and viruses). Our model is able to justifiably use real-world data to accurately predict behavior and patterns in the population of the honey bee colony in response to several variables. Our object-oriented approach to programming the stochastic simulation allows for flexibility in the model. Although we are limited by long runtimes and computational intensity for visualization and the fact that no internal factors are accounted for, the model remains accurate due to its strengths.

Meanwhile, Task 2 implements our model from Task 1 by varying eight experimental variables to perform a sensitivity analysis. The population that resulted from the real-world data without negative externalities is at ~73,863 bees, which is justifiably acceptable. We defined the sensitivity of a variable as the average change in the ratio of the percentage change of the output to the percentage change of the input. After analyzing the results of 8 different variables, we concluded that the three variables that are the most influential on the honey bee population are (1) the rate at which queens produce eggs (sensitivity = 0.5591), (2) the probability of the pesticides killing the bees (sensitivity = 0.3008), and (3) the probability of the predators attacking each day (sensitivity = 0.2453).

In Task 3, we developed a geometric model that determines the number of hives necessary to completely pollinate an 81,000-m^2 field. We defined complete pollination according to a minimum number of bees foraging in each spatial region. Our model concludes that for hives with a population of 80,000, we require 23–34 hives to pollinate the 81,000-m^2 field. Other estimates for lower colony populations were also calculated and shown to require more hives to pollinate the same area.

Overall, the results from our paper can help beekeepers in keeping track of factors that may negatively impact the growth of honey bee populations, and how to strategically position honey bee hives in order to pollinate a given crop area. The primary suggestions that were developed in this model is through the investment in technology that improves egg-laying efficiency and by separating the predators and pesticides from bees. Our model produces accurate data with justifiable assumptions, and is beneficial to conservationists. Through this, we can protect our honey bees, and perhaps even human life, as we know it.

Table of Contents

Task 4: Infographic

Team 13570

Beecoming Endangered!

Lessons from Modeling the European Honey Bee (*Apis mellifera*)

1500 - 2000 eggs daily

Each colony has one **queen bee.** They lay eggs daily keeping up the population.

85% of the hive are **forager bees.** They scout & collect food for the colony.

Drone bees only mate with the queen and take up a small potion of the colony.

Guard bees attempt to protect the hive from predators. This includes wasps.

1st Egg Rate

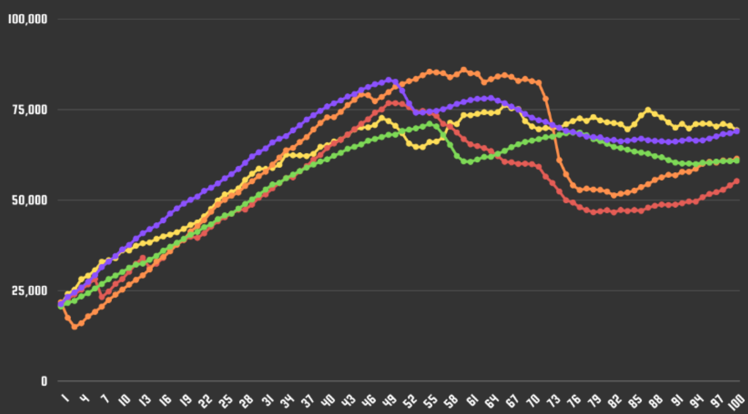
2nd Pesticides

3rd Predators

4th Weather

up to **72%** pesticides

These are the top four leading factors that affect the population of a bee colony. Factors 2-4 promote **Colony Collapse Disorder (CCD)**.



Bee population vs time graph (100 days)

Control Test

To be able to pollinate a 20-acre plot of land, 36 to 50 colonies with a population of **50,000** bees are needed. That is up to 2.5 million bees! **Pollination is essential** to keep ecosystems on our planet alive. We have to put our very best effort into preserving bees for the sake of humanity. Avoid using harmful **pesticides**. Next, reduce man-made **stresses** on bees as they reduce egg production. Finally, take actions to end **climate change** to prevent extreme weather conditions.

50 colonies / 20-acre

"If the bee disappeared off the surface of the globe, then man would only have four years of life left." - Albert Einstein

Introduction

Bees, most especially the European honey bee (*Apis mellifera*), are known to be the most frequent floral visitor of crops worldwide[1]. They pollinate plants and allow them to reproduce which allows agricultural industries to flourish. Additionally, honey bees maintain our environment's ecological status as the plants they pollinate also act as food and shelter for many other animals.

Unfortunately, despite their huge importance to our society, honey bees are one of the most vulnerable species on our planet. Various environmental stressors, such as pesticides, predators, pathogens, and parasites, can negatively impact honey bee populations drastically[2]. This negatively impacts the pollination of our crops and vegetation, leading to a decreased supply of vital flora. Conservation efforts are, therefore, important in controlling the effect of negative externalities on hive colonies. However, for such plans to be effectively implemented, we must understand how bee populations grow and how negative factors impact bee colonies.

As such, this paper aims to develop mathematical and computational models analyzing several aspects of a bee population, propagation, and pollination. Task 1 aims to identify the factors pertaining to a colony's population and develop a model that predicts its change over time. Task 2 then aims to use this model to determine which factors most significantly affect the population of a hive. Building upon the first two tasks, Task 3 aims to develop another model that predicts the number of bee colonies needed to support the pollination of an 81,000-square-meter section of land. Finally, we summarize all of our findings in Task 4 through a non-technical infographic.

Task 1: Modeling Honey Bee Colony Population

For Task 1, we have been tasked to formulate a model for the population of a single honey bee colony, accounting for various internal and external factors.

Factor Analysis

We begin the ideation for our model by first understanding which real-world phenomena are known to affect honey bee colonies. From our preliminary research, we found that several internal and external real-world factors influence the dynamics of the population of honey bees.

Types of Bees

In a real-world honey bee colony, there are primarily three types of bees: queen bees, worker bees, and drone bees. A colony only has one queen bee which is responsible for laying eggs that drives the growth of the colony's population. Worker bees, which are fertilized eggs that grow to become female bees, take on several responsibilities, such as foraging for food, attending to the queen, and guarding the hive, among many others. Meanwhile, drone bees, which are unfertilized eggs that grow to become male bees, serve the purpose of mating with the queen bee. Both queen bees and worker bees are females, which are differentiated by the amount of nutrition that they receive during the larval stage[3].

Seasonal Life Expectancy

On average, *A. mellifera* workers have a life expectancy of 15–38 days in the summer and 150–200 days in the winter, drones live on an average of 55 days, and queens live for 1–2 years. External factors (e.g., pesticides, predation, weather, etc.) and behavioral factors (e.g., foraging and nursing) can also influence the lifespan of an individual honey bee[3].

Production of Eggs

To grow the population, queen bees lay an average of 1500–2000 eggs per day, primarily during the growth of the hive. However, during honey production seasons, queens lay an average of 1000–1500 eggs per day. Fertilized eggs develop into workers; meanwhile, unfertilized eggs develop into drones[3].

Food Requirements

Honey bees use flower nectar as their primary source of food. The average worker bee requires around 22 microliters (µL) of 50% (1:1 w/w) sugar syrup per day[4]. Meanwhile, a single flower can only produce anywhere from 0.1 to 3.8 µL of sugar syrup on average per day[5]. This means that several worker bees would need to repeatedly travel and forage on several flowers to sustain the food requirements of their colony. Through this process, honey bees become the primary pollinators for many flowering plants[6].

Environmental Externalities

In ideal environmental conditions, food availability should be the only major limiting factor in the growth of populations of honey bees. Unfortunately, several environmental factors, such as habitat loss, pesticide presence, climate change, and pathogens and pests, can lead to a reduction in honey bee populations. More specifically, this leads to a phenomenon known as Colony Collapse Disorder (CCD) wherein worker bees disappear while the queen and young bees remain[7]. Due to the lack of worker bees, the supply of key resources such as food and water sharply decline, causing the collapse of the colony.

Model Variables

Based on the real-world factors that we have discovered from our preliminary research, our group selected several variables that contribute to and/or reflect the overall change in the population of the honey bee colony to be simulated in our model. We have summarized these different variables with their corresponding definitions in Table 1 (for colony variables), Table 2 (for individual bee variables), and Table 3 (for environmental externalities variables).

Table 1. Variables pertaining to the entire colony.

Variable	Definition
P	**Population of the colony.** This refers to the total number of honey bees that are present in the colony, which are worker bees, drone bees, larvae, and one queen bee.
F	**Food availability of the colony.** This refers to the total amount of remaining food supply (50% sugar) in terms of microliters (µL). This value can increase when the colony's worker bees forage for more food.
δP_{egg}	**Rate of production of eggs.** This refers to the number of eggs laid by the queen bee per day. Based on our previous literature search, queen bees will lay anywhere from 1500–2000 eggs per day on average during growth season.
η_{fert}	**Fraction of fertilized eggs.** This refers to the fraction of eggs (i.e., between 0 and 1) that are fertilized. A larger value of this variable would lead to a larger percentage of the population being female (i.e., worker bees).
η_{hatch}	**Fraction of eggs that hatch.** This refers to the fraction of eggs that hatch and grow to become a bee in the hive. For the purposes of our modeling process, we will assume that all eggs will hatch into larvae that will eventually grow into bees.

Table 2. Variables affecting individual bees.

Variable	Definition
B_{type}	**Type of bee.** This refers to the role of the bee in the colony. More information on the different types can be found in the Model Development section.
L	**Life expectancy.** This refers to the predetermined lifetime of a single honey bee in terms of days. The value of this variable is based on the bee's role and mathematically randomized according to a normal distribution where the mean is the average life
C_{cap}	**Carrying capacity.** All worker bees are assigned the same maximum amount of food that one can carry in terms of microliters (µL).
C_{extract}	**Extraction yield.** This refers to the maximum amount of food that they can extract per food source in terms of microliters (µL).
u	**Velocity of the bee.** This refers to the speed that the bee travels, which is homogenous regardless of its role.
$E_{\max}$	**Maximum energy of the bee.** This refers to the maximum amount of energy that a bee can possess. An energy property, which is initially equal to E_{max}, is defined to account for worker bees that may be overfatigued by their foraging tasks later on in the model proper.
m_{work}	**Fatigue penalty.** This refers to the reduction in the life expectancy of the bee whose energy property is less than or equal to 0 (i.e., over fatigued).
R_E	**Energy consumption rate.** This refers to the amount of energy that is consumed or regenerated per unit of time whenever the bee is working or resting, respectively.
T_{harvest}	**Harvesting time.** This refers to the total time that a bee can harvest food within a day. This is considered a portion of the bee's working time (i.e., $T_{harvest} \leq T_{work}$).
T_{work}	**Working time.** This refers to the amount of time that a worker bee can work in a day.
T_{rest}	**Resting time.** This refers to the amount of time that a worker bee rests in a day.

Table 3. Variables affecting environmental externalities.

Variable	Definition
K_{flower}	**Food capacity of each flowering region**. This refers to the amount of food (50% sugar) that each flowering region (i.e., food source) can maximally hold.
ρ_{flower}	**Abundance of flowering regions**. This refers to the fraction of the environment that are flowering regions, which can act as a food source for the honey bees. This variable will hold a value between 0 and 1.
p_{weather}	**Probability of dying due to weather**. This variable accounts for the negative impact of weather on the bees. A higher value for this variable means that the bee colony is affected by more severe weather conditions, which would increase the chance of bees dying to it.
$p_{\text{pesticide}}$	**Probability of dying due to pesticides.** This variable accounts for the negative impact of pest repellents by defining the probability that a pesticide-contaminated space would kill a honey bee. A higher probability value could be interpreted as a higher intensity of pesticide use in the environment surrounding the honey bee colony.
p_{predator}	**Probability of dying due to predators.** This variable accounts for the negative impact of predators, such as wasps, bears, and raccoons, by defining the probability of a predatory attack on

Variable	Definition
	of the honey bee hive. A higher probability value could be interpreted as a higher frequency of predator attacks on the colony's hive.
p_{viral}	**Probability of dying due to pathogens and parasites.** This variable accounts for the negative impact of transmittable externalities, such as pathogens and parasites, by defining the probability that a bee larva is infected at hatching. A higher probability value means a higher intensity of pathogen or parasite infection spread in the hive.

Assumptions and Justifications

It should be noted that while we have selected an extensive collection of model variables, the actual factors and mechanisms driving the real-world dynamics of bee populations are far more complex. As such, we have also made several assumptions to simplify the development of our model, and in turn, make its implementation much more feasible (see Table 4).

Table 4. Assumptions and justifications for Tasks 1 and 2.

#	Assumption	Justification
1)	The honey bees in our model all belong to the species *Apis mellifera*.	We chose the European honey bee *A. mellifera* as the representative species of honey bees since it is known to be the most common kind.
2)	The model will simulate only 100 days during the late spring to the summer period.	Flowers bloom primarily from spring to summer, which is an optimal condition for the food-foraging patterns of honey bees. We would also be able to constrain the average lifespan of our modeled bees to known values during summer.
3)	Food sources are homogenous (i.e., every flowering region will have the same land area and nectar amount).	Since the only purpose of flowering regions in the population model is to act as food sources, employing variations would only overcomplicate the model implementation.
4)	There is a sufficiently large number of different kinds of combs within the hive; in essence, honeycomb, brood combs, and structural combs.	The model focuses on drastic changes in the honey bee population and not changes in the growth of the hive. If there is sufficient food to gather from the environment, then there should be no hindrances for them to grow internally.
5)	There will always be one queen bee and no new queen bees can be spawned.	This task focuses on the population modeling of a single colony. This removes the necessity for modeling competition of food sources among different queens and interaction between bees of different colonies.
6)	Food regenerates to the maximum amount daily.	This aims to simplify the modeling of the real-world regeneration of nectar.
7)	There will always be at least one drone bee to mate with the queen bee.	This ensures that regardless of population decline, the queen bee will always be able to lay new eggs.
8)	Pathogens and parasites are equivalent negative externalities.	These two externalities are treated the same as both are transmittable entities within the honey bee colony.

Model Development

For this task, we decided to utilize a time-series stochastic approach for our model which will simulate the population fluctuations of a single honey bee colony. This is affected by the randomness of the actions of each honey bee and their collective interactions with environmental entities, such as food sources (i.e., flowering regions) and negative externalities (e.g., weather, predators, parasites, pesticides). In the process of developing and implementing the model, we have formulated several subsystems which we shall discuss in this section.

Definition of Tick System

Since the model simulates real-world dynamics, we found utmost importance in the development of a reliable "tick" system that defines and controls the time propagation in our data. Given that we will be looking at macro-scale time periods in Task 1, we decided that 1 tick in our data simulation will correspond to 1 real-world day. For example, to simulate 100 real-world days, our model will require 100 ticks to finish. It should be noted that a tick only controls the rate of the data output of our model. The amount of time it takes to complete 1 tick would be discussed later on in the Strengths and Limitations section.

Environment Space Mapping

A real-world bee colony's hive would normally be situated in an open field full of flowering plants so that their pollination capabilities would be maximized. To abstract this into our model, we used a discretized map representation wherein the smallest unit of space (i.e., a unit region) represents a 1-meter by 1-meter square region in the real world. Every unit region can then be classified as either flowering (1) or non-flowering (0). Computationally, the classification ***C(i)*** of the unit region ***i*** on the environment map is determined by:

$$C(i) = \begin{cases} 0 & \text{if } \texttt{random.uniform(0,1)} \leq \rho_{\text{flower}} \\ 1 & \text{if } \texttt{random.uniform(0,1)} > \rho_{\text{flower}} \end{cases} \quad \text{(Eq. 1)}$$

For the purposes of Task 1, we decided to set the overall environment's map as a 49 by 49 square unit region grid, with the honey bee hive situated at the center unit region of the environment map. A visual summary of our environmental mapping system is shown in Figure 1. This whole approach would also make it possible to model bee movements later on, which is vital for analyzing the impact of certain negative externalities that emerge from interactions with spatial regions on the foraging process of honey bee colonies.

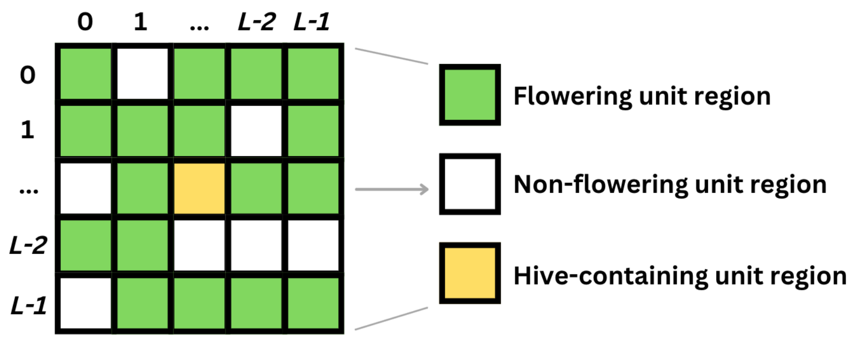


Figure 1. Visualization of our model's environment mapping system. ***L*** corresponds to the number of unit regions on one side of the environment map. In our case, ***L*** = 49. Non-flowering and flowering unit regions are assigned using the rules in Equation 1.

Types of Bees

From our research in the Factor Analysis section, we know that real-world honey bees have very specific and distinct roles. Additionally, newly laid eggs do not immediately hatch into fully-grown bees as they go through a larval stage first. Given this phenomenon, we decided to model six different types of honey bees, as summarized in Table 5. A visual summary of how we modeled the time intervals between the life cycle stages of bees and the relative proportion of fully-grown bee types is visualized in Figure 2.

Table 5. Honey bee type and descriptions

#	Honey Bee Type	Description and Tasks
1)	Egg	This is the first stage of a bee's life cycle. It has no tasks relevant to the colony.
2)	Larva	This is the second stage of a bee's life cycle. It only serves as a transitory state between eggs and fully-grown bees.
3)	Queen	There will only be 1 queen bee in our simulation. It lays new eggs per simulation tick which grows the colony population.
4)	Forager Bees (Worker)	A specific subtype of worker bees. This makes up the majority of the colony population. They scout the environment map for potential food sources and forage resources which will sustain the needs of the colony.
5)	Guard Bees (Worker)	A specific subtype of worker bees. This makes up a small portion of the colony population. They stay idle in the hive and attempt to protect it from invading predators.
6)	Drone Bees	This makes up the minority of the colony population. They serve to mate so that the queen bee is able to generate new eggs.

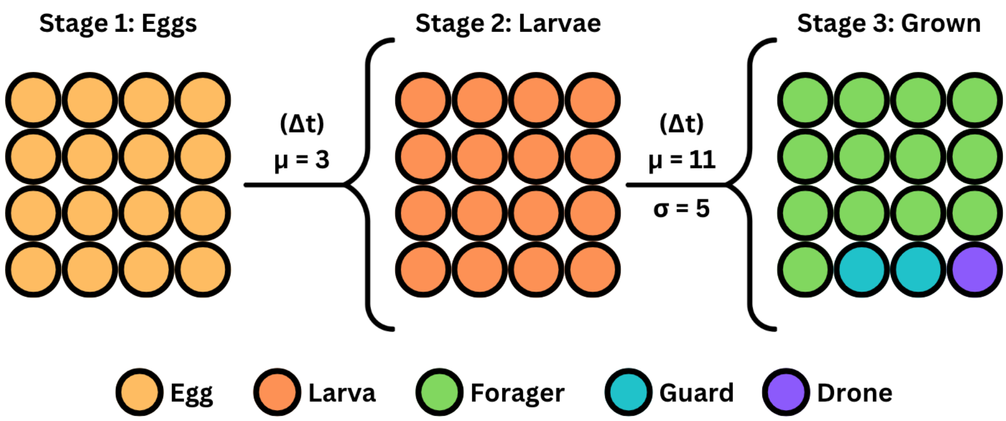


Figure 2. Summary of our subsystem for modeling the different bee roles and life cycle stages. ***Δt*** indicates the ticks elapsed between stages which are modeled as a normal distribution (with mean μ and standard deviation σ) for stochastic purposes. The relative proportion of fully-grown bees is not necessarily to scale.

Mechanisms for Population Variation

Our main motivation behind choosing a time-series stochastic model is its capability of simulating the varied and random behavior of individual bees in a colony while maintaining a specific order of tasks. Consequently, such a model would be limited by the sources of randomness in each tick, which would also account for variations in our population data. In line with this, we have chosen several such mechanisms while developing our model.

Mechanism 1: Scouting, Weighing, and Foraging Food Sources

In the real-world, honey bees are known to relay location information of flower patches with viable nectar supply to their colony through a phenomenon called waggle dancing[8]. In our stochastic model, employee bees (a subtype of forager bees) scout the vicinity of the hive via random movements until it detects the nearest food source/s that contains a positive amount of food in it. The employee bee then performs a waggle dance, which we modeled to relay information on the location and the amount of food within that food source back to the hive, similar to the real-world phenomenon. Once the hive has "determined" a list of all viable food sources and their properties, our model assigns a "weight" to each food source $\boldsymbol{i}$ based on $\boldsymbol{d}$ (distance of the food source from the hive) and $\boldsymbol{K}$ (amount of food available from the food source) through the following equation:

$$w_i = \begin{cases} 1 & \text{if } \overline{K} = 0 \text{ and } \overline{d} = 0 \\ \frac{1}{2}\left(1 + \frac{d_{\max} - d}{\overline{d}}\right) & \text{if } \overline{K} = 0 \text{ and } \overline{d} > 0 \\ \frac{1}{2}\left(1 + \frac{K - K_{\min}}{\overline{K}}\right) & \text{if } \overline{K} > 0 \text{ and } \overline{d} = 0 \\ \frac{1}{2}\left(\frac{K - K_{\min}}{\overline{K}} + \frac{d_{\max} - d}{\overline{d}}\right) & \text{otherwise} \end{cases} \tag{Eq. 2}$$

where $\boldsymbol{w_i}$ is the assigned weight for $\boldsymbol{i}$, $\overline{d}$ is the range of possible distances across all food sources, and $\overline{K}$ is the range of the possible values of the amount of food available from all food sources. The model then assigns the number of bees $\boldsymbol{P_w(w_i)}$ that will forage a food source $\boldsymbol{i}$ with an assigned weight $\boldsymbol{w_i}$ using the following function:

$$P_w(w_i) = P_{forager} \cdot \frac{K[w_i]}{\binom{n}{2}} \tag{Eq. 3}$$

where $\boldsymbol{P_{forager}}$ is the population of forager bees in the colony, $\boldsymbol{K[w_i]}$ is the position of the weight $\boldsymbol{w_i}$ on an ascendingly sorted list of all weights for all food sources ($1 \leq \boldsymbol{K[w_i]} \leq \boldsymbol{n}$), and $\boldsymbol{n}$ is the total number of food sources. Afterward, the worker bees will proceed in a straight line vector from the hive to the assigned food source. However, if a bee passes by a food source that has a positive amount of food, it procures food from that source instead of proceeding to the assigned food source. For a visual summary of Mechanism 1, see Figure 3.

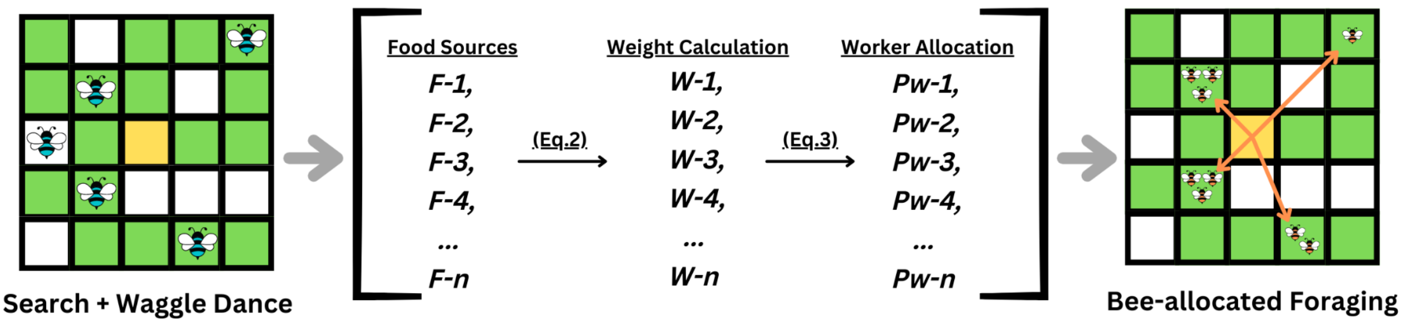


Figure 3. Scouting, weighing, and foraging mechanism for our population model.

<u>Mechanism 2: Energy Penalties</u>

Since worker bees travel and forage, they are prone to work fatigue. To account for this, we employ the following procedures in updating the energy of the bees. There are two cases of a bee's action for each tick: resting or not resting. For clarity, let us denote a bee's current energy as $\boldsymbol{E}$.

Case 1: When the Bee is Resting. If the bee is at the hive, any action it does increases its energy by $\boldsymbol{R_E}$ (defined in the Variables section). Otherwise, it decreases its energy by $\boldsymbol{R_E}$. If the bee has rested for longer than its resting period, it begins to work.

Case 2: When the Bee is Not Resting. For every tick, the energy decreases by $\boldsymbol{E}$. Whenever the bee has less energy than the maximum energy, there are three possible cases:

<u>Case 2a: If the bee is idle or depositing food</u>, the bee replenishes its energy fully.

<u>Case 2b: If the bee is harvesting food</u>, the bee replenishes its energy fully and removes that energy from the food source when the amount of food is a positive integer but moves back to the hive whenever there is no more food available.

<u>Case 2c: If the bee is hungry</u>, we define the required amount of food as the minimum value between (1) the difference of the maximum energy of the bee ($\boldsymbol{E_{max}}$) and the current energy of the bee ($\boldsymbol{E}$), and (2) the amount of food that is available in the hive. The energy of the bee increases by the required amount of food, while the amount of food in the hive decreases by the required amount of food by the bee.

When the amount of work is longer than the work duration of the bee, the bee begins to rest. However, at the end of the calculation, if the energy is zero or negative, the life span of the bee decreases by its fatigue penalty $\boldsymbol{m}_{work,}$ and its energy returns to the maximum amount of energy it can have.

Mechanism 3: Negative Environmental Externalities

There are several environmental factors that we have accounted for in this model: weather, pesticides, wasps, and parasites. Even though our model is limited to these, the object-oriented nature of the code written by our group allows for the addition of other environmental externalities.

Weather. When bees are exposed to harsh weather, such as rain, each bee has a chance $\boldsymbol{p_{weather}}$ of dying per tick. This remains constant regardless of the tick.

Pesticides. Pesticides are human-made chemicals that contaminate the crops that bear food for the bees. In this model, we assume that this hypothetical pesticide is sprayed throughout the entire map, except for the hive. Whenever a bee visits a contaminated unit region, it has a probability $\boldsymbol{p_{pesticide}}$ to die.

Predators. In the real world, predators attack bee colonies which kills the bees in the hive. In our model, there is a probability $\boldsymbol{p_{predator}}$ that a predatory attack occurs each day. If such occurs, the attack reduces the amount of food throughout the hive by a factor of $\boldsymbol{f}$ and decreases the number of bees in the hive by a factor of $\boldsymbol{g}$, where $\boldsymbol{f}$ and $\boldsymbol{g}$ are real numbers from 0 to 1, inclusive. In our model, we selected $\boldsymbol{f} = 0.7$ and $\boldsymbol{g} = 0.8$.

Pathogens and Parasites. According to numerous studies, parasites for bees inflict themselves during the larval stage and are transmittable, which is quite similar to pathogens. Therefore, we assign an intensity $\boldsymbol{p_{viral}}$ which refers to the fraction of the hatched eggs (i.e., new larvae) that were inflicted by some hypothetical parasite/pathogen. Once a bee larva is inflicted, this would decrease its lifespan from $\boldsymbol{L}$ to $\boldsymbol{kL}$, where $\boldsymbol{k}$ is a real number from 0 to 1, inclusive. In our model, we selected $\boldsymbol{k} = 0.5$.

Model Program

We implemented our time-series stochastic population model in C++ as it is known to be a fast programming language. An object-oriented programming approach was utilized as we were handling several classes of entities. For every tick in the simulation, several procedures are run, as better summarized in Figure 4. Selected code snippets are shown in Appendix A.

Strengths and Limitations

The major strength of this formulated model is its **accuracy**, as the values are extremely adjustable to model real-world data (as we will see in Task 2). By determining the actual values that are present in the real world from research, we can more or less accurately predict the correct behavior and patterns of the population growth for a single honey bee colony.

Another major strength of the model is the **reprogrammability** of the model to implement additional factors. As aforementioned in the previous section, there are four major negative environmental externalities that are implemented in the question: weather, pesticides, wasps, and parasites. Once the behavior of the environmental externality is accurately modeled, we can correctly implement this into the model.

However, a limitation of this model is its **computational intensity**, despite our program being implemented in C++, which is a relatively fast programming language. As we account for a number of ticks in the magnitude of 10^3 and a number of bees in the magnitude of 10^5, it is computationally intensive (i.e., 1 tick in our program takes approximately 1 second of runtime). Moreover, visualizing these actual factors (through a GUI) requires a lot of computational power to run.

Another limitation of this model is that it **does not account for internal factors** within the hive such as the amount of space that a bee has (i.e., our model does not account for nurse bees whose job it is to create more comb with the hive). However, due to our model's reprogrammability, additional features such as this could be implemented easily into the program.

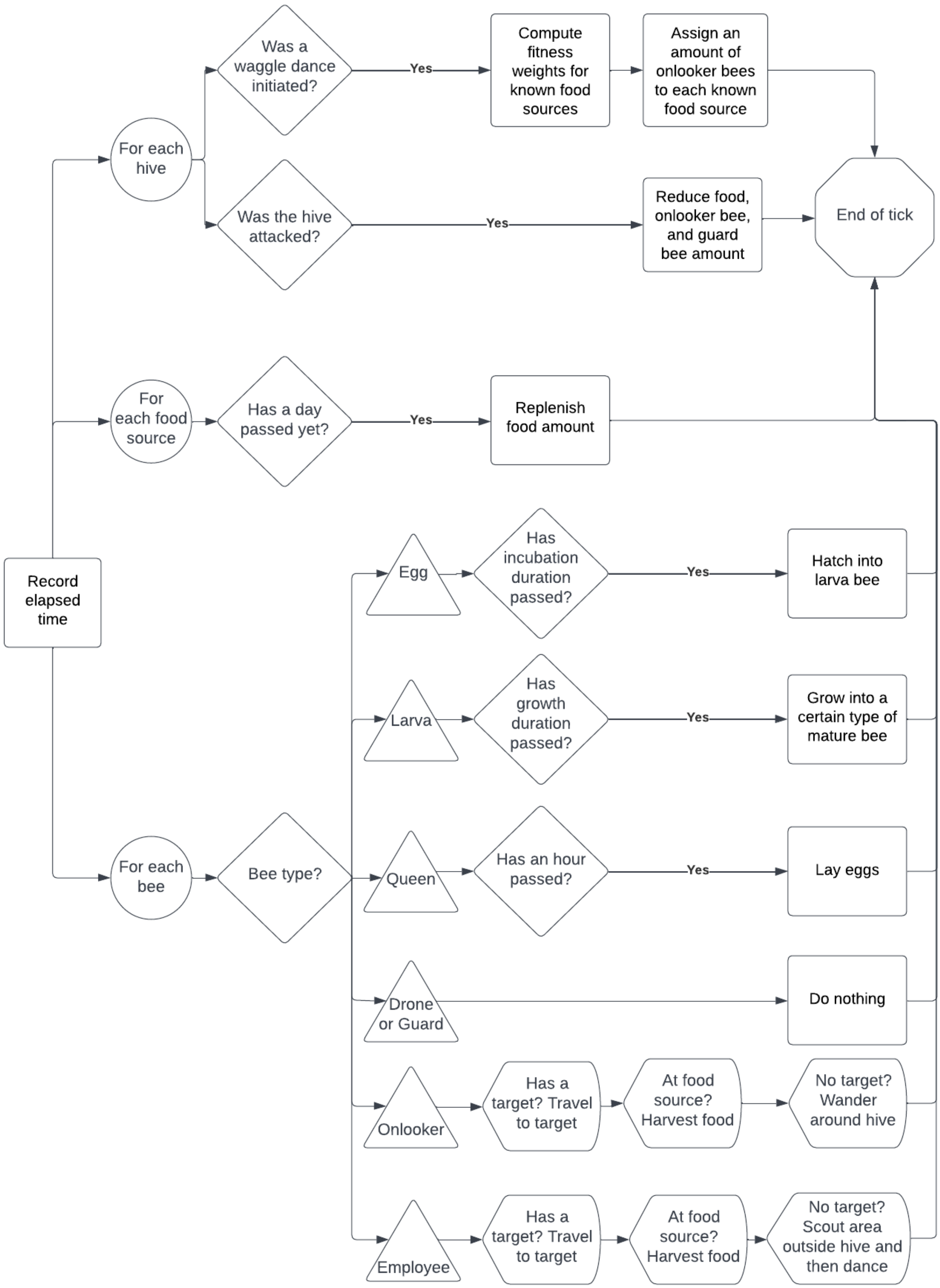


Figure 4. Flowchart for the loop occurring for each tick in the honey bee population model.

Task 2: Population Model Sensitivity Analysis

We utilize the different factors accounted for by our model from Task 1 to determine which of them are most influential on the population of the beehive. More specifically, Task 2 performs a sensitivity analysis on our developed time-series stochastic population model.

Overview

We ran several iterations of the simulation on six different factors tested, namely: **(a)** a control group, **(b)** changing abundance of food/flowering region for the bees (ρ_{flower}), **(c)** changing the rate that the queen lays eggs based on the population (δP_{egg}), **(d)** changing the lifespan of the bees (L), **(e)** changing the fraction of eggs that are not fertilized (η_{fert}), and **(f)** changing the intensity/probability of four negative externalities ($p_{weather}$, $p_{pesticide}$, $p_{predator}$, p_{viral}). Test (a) will act as a baseline for the numerical analysis of Tests (b)-(e), which account for internal variables to the colony, and the several subtests of Test (f), which account for external variables in the colony's environment.

Control Test

Test (a) models the time-series pattern of a single honey bee colony when there are normal internal and external conditions. Essentially, the results from this test can be interpreted as an "average hive conditions" and "perfect environmental conditions" setup. The default values we have used for the different variables are presented in Table 6 and Table 7.

Table 6. Variables with constant values

Variable	Value	Unit
η_{fert}	0.15	dimensionless
η_{hatch}	1.00	dimensionless
K_{flower}	50	[µL]
p_{weather}	0	dimensionless
$p_{\text{pesticide}}$	0	dimensionless
p_{predator}	0	dimensionless
p_{viral}	0	dimensionless

Table 7. Variables with normal distributions

Variable	Mean	Standard Deviation	Unit
L	Onlooker: 1008 Employee: 1008 Guard: 1008 Drone: 2160 Queen: 17820	168	[h]
δP_{egg}	1500	375	[day^{-1}]
C_{cap}	10	4	[µL]
C_{extract}	1.95	0.85	[µL]
u	100	10	[m^{-1}]
$E_{\max}$	22	5	[µL]
m_{work}	24	1	[h]
R_E	1	0.5	[µL h^{-1}]
T_{harvest}	0.1	0.05	[h]
T_{work}	16	1	[h]
T_{rest}	$T_{\text{rest}} = 24 - T_{\text{harvest}} - T_{\text{work}}$		[h]

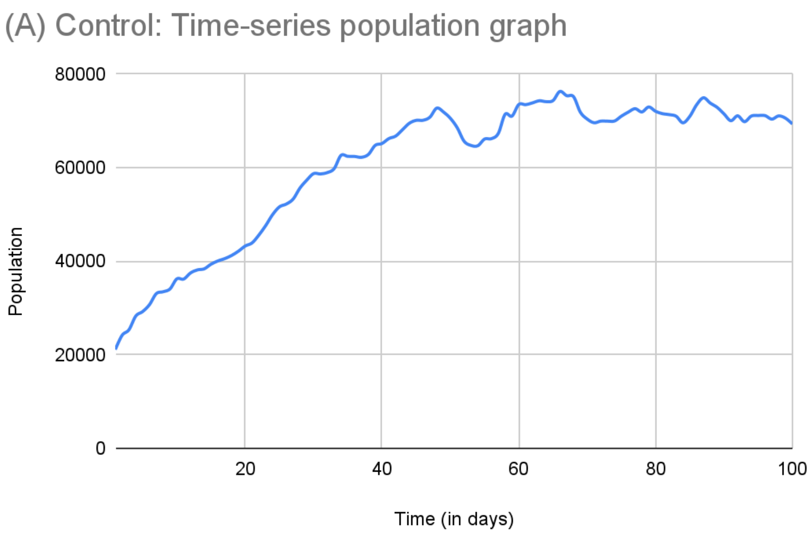


Figure 5. Population vs time graph over control test

From the graph in Figure 6, the data presented illustrate that the population of the colony increases during the first 60 days and becomes approximately constant during 60 days to 100 days in the simulation. We can conclude that this behavior is approximately equivalent to that of logistic growth, which is the accepted model for population growth[9]. Fitting a horizontal line from day 91 to day 100, we observed that the population stabilizes at approximately ~73,863 bees. Hence, we can verify our model to be valid since the population of the colony is within the acceptable range of 20,000 and 80,000.

Comparing Stabilized Population Graphs of Experimental Tests

We performed Tests (b) to (f) by changing only their corresponding experimental variable and maintaining the rest of the values from Tables 6 and 7. For each test, we selected five values for its experimental variable. Then, for every time-series population data produced, we determined their "stabilized population" similar to the method from Test (a). Figures 6 to 13 plots the stabilized population versus the value of the variable changed for every test. For raw time-series data, kindly consult Appendix B.

Several insights can be gained from these graphs. First, the trend of the best-fit linear show whether our model predicts that the population of a honey bee colony would increase (c, d), decrease (e, f(1), f(2), f(3), f(4) - values are below control), or stay relatively constant (b) if that test's variable is increased.

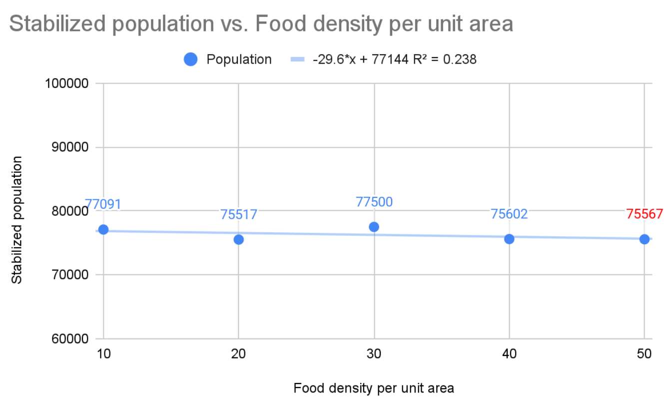


Figure 6. Test (b) stabilized population vs abundance of flowering region (ρ_{flower}). The selected values of ρ_{flower} are 10, 20, 30, 40, 50. The control value is marked in red.

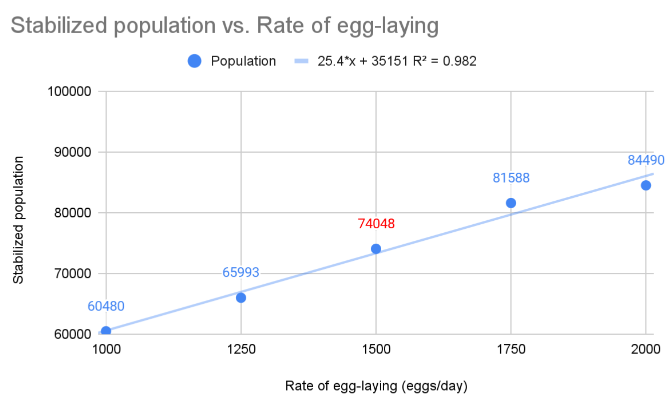


Figure 7. Test (c) stabilized population vs rate of egg-laying egg (δP_{egg}). The selected values of δP_{egg} are 1000, 1250, 1500, 1750, 2000. The control value is marked in red.

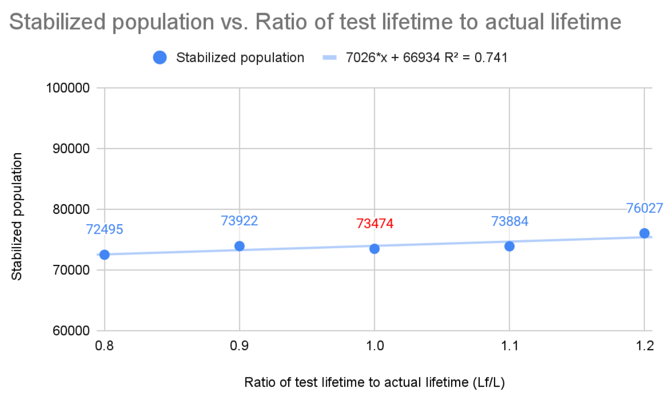


Figure 8. Test (d) stabilized population vs ratio of test lifetime to control lifetime (***L_f/L***). The values of ***L_f/L*** are changed to 0.8, 0.9, 1.0, 1.1, 1.2. The control value is marked in red.

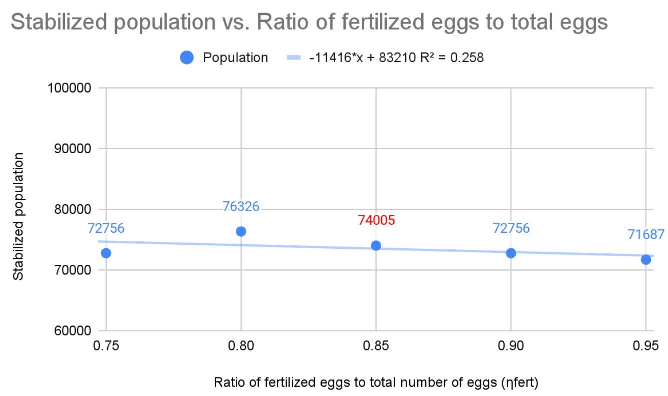


Figure 9. Test (e) stabilized population vs ratio of fertilized eggs to total number of eggs (η_{fert}). The values of η_{fert} are changed to 0.75, 0.80, 0.85, 0.90, 0.95. The control value is marked in red.

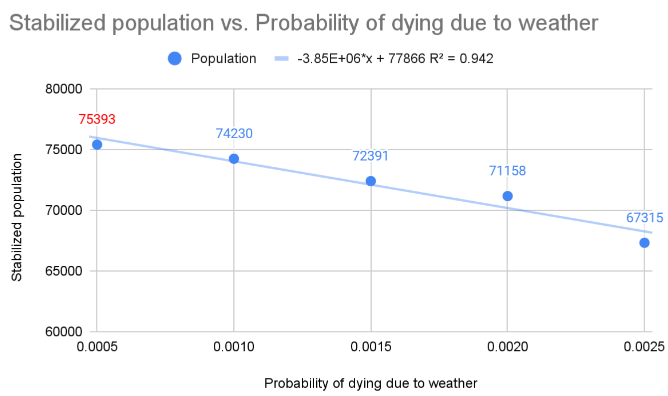


Figure 10. Test (f(1)) stabilized population vs probability of dying due to weather ($p_{weather}$). The values of $p_{weather}$ are changed to 0.0005, 0.0010, 0.0015, 0.0020, 0.0025. The control value is marked in red.

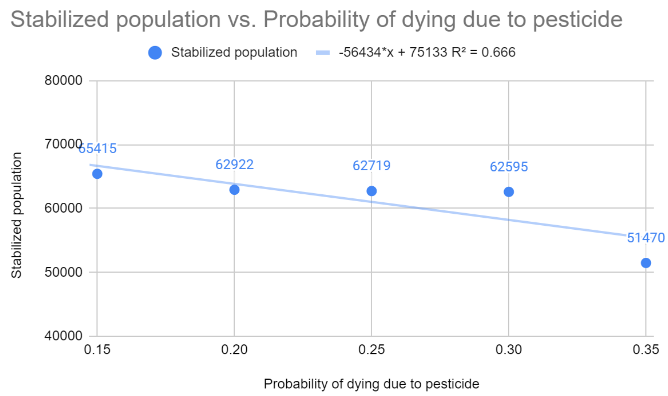


Figure 11. Test (f(2)) stabilized population vs probability of dying due to pesticide ($p_{pesticide}$). The values of $p_{pesticide}$ are changed to 0.15, 0.20, 0.25, 0.30, 0.35. The control value is $p_{pesticide}$ = 0.

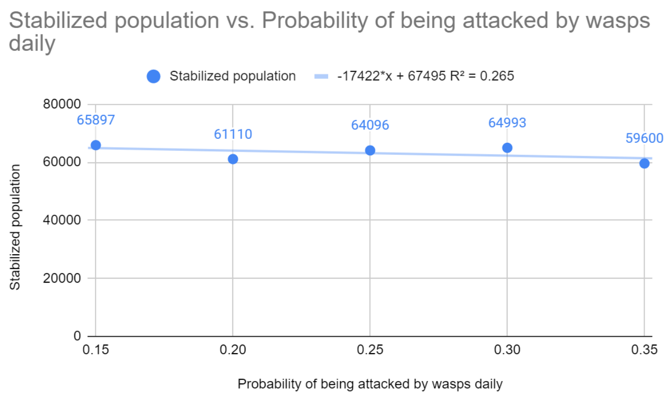


Figure 12. Test (f(3)) stabilized population vs probability of dying due to wasps ($p_{predator}$). The values of $p_{predator}$ are changed to 0.15, 0.20, 0.25, 0.30, 0.35. The control group is $p_{predator}$ = 0.

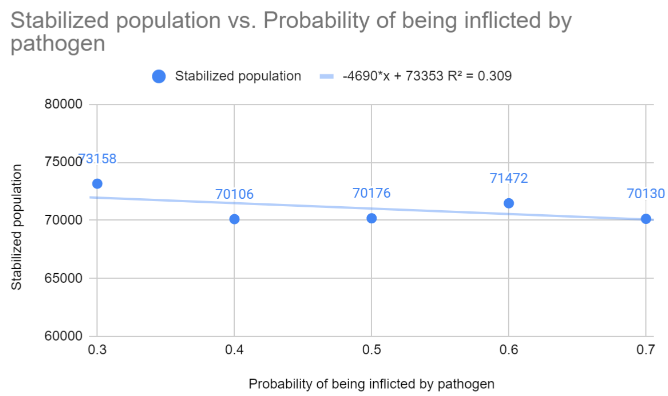


Figure 13. Test (f(4)) stabilized population vs probability of dying due to wasps (p_{viral}). The values of p_{viral} are 0.3, 0.4, 0.5, 0.6, and 0.7. The control group is p_{viral} = 0.

Percent Sensitivity of Factors

We measured the sensitivity of each factor to be the percentage change in output (the change in stabilized population) over the percent change in input (the change in the variable). These percentage changes will be compared to the control variables. However, since there are four variables, we will calculate the average sensitivity. This equation is shown in Equation 4 below:

$$\text{Average sensitivity} = \frac{1}{n-1} \cdot \sum \frac{\%\text{change in output}}{\%\text{change in input}} \qquad \text{(Eq. 4)}$$

A higher sensitivity implies that changing that independent variable is more significant than changing other independent variables.

Table 8. Average sensitivity metric of the tested experimental variables

Experimental Variable	Average Sensitivity
K_{flower}	0.0231
δP_{egg}	0.5591
L	0.0098
η_{fert}	0.0244
$p_{weather}$	0.0830
$p_{pesticide}$	0.3008
$p_{predator}$	0.2453
p_{viral}	0.0231

From the results in Table 8, we conclude that the three most influential variables on the population of a single honey bee colony are (1) the rate at which queens produce eggs (δP_{egg}) with a sensitivity of 0.5591, (2) the probability of the pesticides killing the bees ($p_{pesticide}$) with a sensitivity of 0.3008, and (3) the probability of the predators attacking the bee every single day ($p_{predator}$) with a sensitivity of 0.2453. These three significant variables that our Task 1 model predicted are also accurate for several reasons. First, we can observe that the population for each of these parameters change significantly whenever we change the value even slightly, which indicates that these parameters significantly affect the population. Second, these accurately model real-world data. For the environmental externalities pertaining to pesticides and predators, we observe that real-world pesticides truly kill a large percentage of bee populations, sometimes wiping out 72% of the population[10], while predators naturally kill off a lot of bees.

As a real-world interpretation, the appearance of the internal variable δP_{egg} in our three influential factors suggests that investing in technology to increase the rate of queen bee egg production would contribute to increasing the population of the bees. Not only does this have improvements in the commercial value of bees by potentially producing more honey, but it would also initiate more ecological movement. Second, we should monitor the field where beehives belong so that no predators or pesticides are used around the field. Our model shows that these negative externalities drastically reduce the population of bees. Aside from this, it may also harm the humans that consume products coming from these bees. Moreover, even though the sensitivity is not as large, the weather is also an important contributor to CCD. While these variables do not show it yet, increasing the probability of bees dying due to the weather increases through climate change, which happens a lot at the moment. Therefore, our model is justifiably accurate.

Strengths and Limitations

A primary strength of this model is that, despite being stochastic in nature, it reflects the **natural behavior of logistical growth** as proven by the fact that we eventually reach a final stabilized population or a state of equilibrium within the model such as that shown in the various tests (Figure 5-13). Therefore, this model justifiably determines how the honey bee population changes over time and over changes in variable values.

However, the primary limitation of this model is the **absence of a statistical test** that determines the change in values across a specific time interval. The group initially attempted to use an F-test, but realize that it is an insufficient test as the F-test compares the variance of the colony populations without regard to the order by which these populations are presented (i.e., it does not pair up the population of the colony with the time it occurs). Moreover, the model is easily susceptible to the randomization of the properties of the bees as mentioned. For our data in Figure 9, 11, and 13, we had to run our stochastic model several more times in order to get averaged data with a well-fit curve..

Task 3: Prediction for Fields

In this task, we were asked to develop a model for determining the number of hives needed to pollinate an 81,000-square meter piece of land. As pollination is an important ecological function that bees play a role in, it is imperative that we create a model that helps us understand how to optimize it via strategic hive positioning.

Assumptions and Justifications

Similar to Task 1, we made several assumptions to simplify our modeling process as real-world pollination is also very stochastic in nature.

Table 9. Assumptions and justifications for Task 3

#	Assumption	Justification
1)	Worker bees are homogenous	Worker bees are the main agents behind pollination. In contrast to Task 1, accounting for stochastic properties in bees wouldn't serve much purpose in the modeling of pollination.
2)	Honey bee hives are homogenous in population.	We have determined in Task 2 that there are many factors that can impact the population of a single honey bee colony. Accounting for these factors in our pollination model is highly unnecessary as we are only concerned with the strategic positioning of bee hives.
3)	Bee pollination is positively correlated with bee foraging patterns.	As a worker honey bee travels and forages nectar from one flower to another, it transports flower pollen that attaches to its body. This shows that pollination is mechanized by bee foraging patterns in the real world.
4)	The 81,000-square meter environment is homogenous and completely filled with crops (i.e., food sources for bees) benefitting from pollination.	In a real-world crop field, there would be a high density of plants in order to maximize the limited space. Additionally, accounting for different kinds of "crop regions" would be unnecessary as it would only complicate our modeling.
5)	The field is a square of side length $\sqrt{81000} = 284.605$ m.	This simplifies the computation. However, as would be described later, one could find the answer for a general field with area 81000 m^2 by overlaying a triangular lattice with a certain side length and counting the number of points that lie in the field

Model Development

For this task, we decided to use a geometric approach in minimizing the number of bee hives needed to pollinate the given field, and in the process, optimizing the spatial configuration of these bee hives.

Definition of Complete Pollination

In Task 1, we modeled how a region containing flowering plants (i.e., a flowering region) holds a certain amount of food K_{flower} = **50 μL** for the bees. Since pollination is positively correlated with foraging (Assumption 3), we can define the state of "complete pollination" for a 1 meter by 1 meter unit region when all of its available food has been foraged by the honey bees. Since every bee can only extract C_{cap} = **10 μL**, a single unit region will require $K_{pollination}$ = **50/10 = 5** honey bees foraging on it to be considered completely pollinated.

Determination of Pollination Gradient

In Task 1, we have also developed a weighing formula (Eq. 2) wherein the "priority" of a unit region to be visited is inversely proportional to its distance from a bee hive. Consequently, that determines the number of bees proceeding to a food source. Therefore, we ran our model in Task 1 to determine the relationship between the exact relationship between the number of bees foraging from a food source and its distance from the hive. Our model ran with a constant population of 20,000 bees in the first trial, 50,000 bees in the second trial, and 80,000 bees in the third trial, where 85% of the bees are foragers. The graphs can be found in Figures 12 to 14 for the graph of N vs d while the graphs for N vs d^2 can be found in Figures 17 to 19.

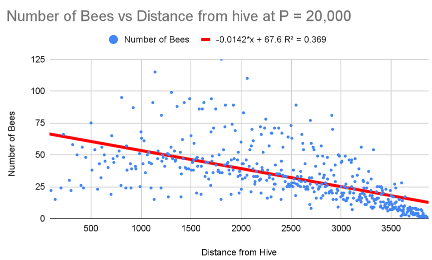


Figure 14. Number of bees (N) vs distance from hive (d) at initial population P = 20,000. The equation is $N = -0.0142d + 67.6$, $R^2 = 0.369$.

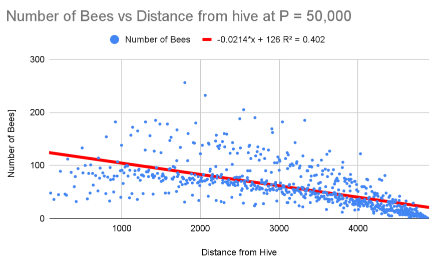


Figure 15. Number of bees (N) vs distance from hive (d) at initial population P = 50,000. The equation is $N = -0.0214d + 126$, $R^2 = 0.402$.

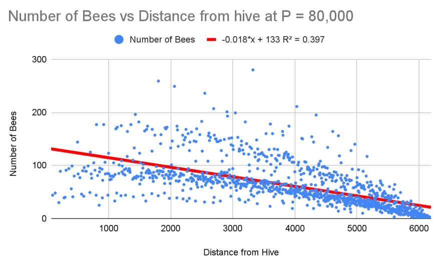


Figure 16. Number of bees (N) vs distance from hive (d) at initial population P = 20,000. The equation is $N = -0.0142d + 67.6$, $R^2 = 0.369$.

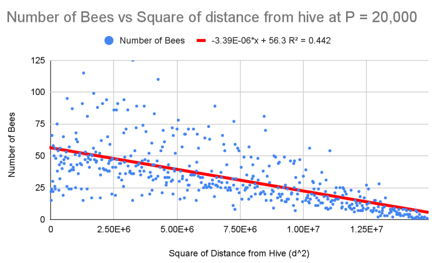


Figure 17. Number of bees (N) vs square of distance from hive (d^2) at initial population P = 20,000. The equation is $N = -3.39\times10^{-6}d^2 + 56.3$, $R^2 = 0.442$.

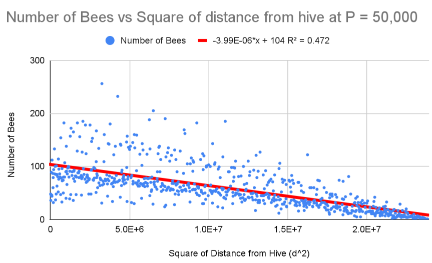


Figure 18. Number of bees (N) vs square of distance from hive (d^2) at initial population P = 50,000. The equation is $N = -3.99\times10^{-6}d^2 + 104$, $R^2 = 0.472$.

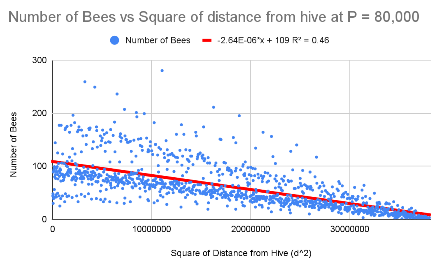


Figure 19. Number of bees (N) vs square of distance from hive (d^2) at initial population P = 80,000. The equation is $N = -2.64\times10^{-6}d^2 + 109$, $R^2 = 0.46$.

Since the data fits better as shown by the higher R^2 values in Figures 17 to 19, we chose to use these equations for developing our Task 3 model. The variation in the model is accounted for by the random nature of the model. However, the best fit line performs excellently in passing through the densest data points.

Geometric Determination of the Number of Hives

It is well-known that equilateral triangles (theoretically with side length l) tile the plane, as well as the fact that selecting an equilateral triangle with a vertex of the equilateral triangle as the center and radius s covers 6 other vertices. Hence, we will utilize this as motivation in order to pack the hives correctly. Observing the fact that we require at least 5 honey bees in order to pollinate a region, a minimum of 2 circles intersect at a vertex (when the center of the circle lies on the edge). Thus, at least 5 / 2 = 2.5 bees have to pollinate at that distance. Hence, in order to determine the side length s, we determine the distance $s = d$ satisfying $N = ms^2 + b$, where $N = 3$, where $N = ms^2 + b$ is the linear regression equation used in Figures 17 to 19. Calculating the values of s for each population, $s_{P=20000} = 39.651$ m, $s_{P=50000} = 50.312$ m, $s_{P=50000} = 63.365$ m **… (*)**.

We present two methods for calculating the bounds for the number of hives that we require for the hive: one that is theoretically optimal but impractical and one that is unoptimal but is practical.The first method is the most optimal packing method. In this method, we place a hive in all of the points in a triangular lattice with side length l, as shown in Figure 20**.** It is easy to observe that $l = s\sqrt{3}$ by trigonometry. Moreover, we can observe that along the width, we want to have at least $w/l = w/(s\sqrt{3})$ hives but at most $w/l + 1 = w/(s\sqrt{3}) + 1$ hives across the width; and along the breadth of the field, at least $h/(l\sqrt{3}/2) = h/(3s/2)$ hives but at most $h/(l\sqrt{3}/2) + 1 = h/(3s/2) + 1$ hives. Hence, our bound for the number of hives for this method can be seen in Equation 5. An example tiling can be shown in Figure 21.

$$\left\lceil \left(\frac{\sqrt{A}}{\frac{3}{2}s}\right)\left(\frac{\sqrt{A}}{\sqrt{3}s}\right)\right\rceil \leq N \leq \left\lceil \left(\frac{\sqrt{A}}{\frac{3}{2}s}+1\right)\left(\frac{\sqrt{A}}{\sqrt{3}s}+1\right)\right\rceil \qquad \text{(Eq. 5)}$$

The second method is almost similar, except we place a hive at every single point in the triangular lattice with side length s, where s is calculated in Figures 17 to 19. By the same argument on counting the number of hives to be placed across the width and the breadth, we can calculate our bound for the number of hives for this method in Equation 6. An example tiling can be found in Figure 22.

$$\left\lceil \left(\frac{\sqrt{A}}{s}\right)\left(\frac{\sqrt{A}}{\frac{\sqrt{3}}{2}s}\right)\right\rceil \leq N \leq \left\lceil \left(\frac{\sqrt{A}}{s}+1\right)\left(\frac{\sqrt{A}}{\frac{\sqrt{3}}{2}s}+1\right)\right\rceil \qquad \text{(Eq. 6)}$$

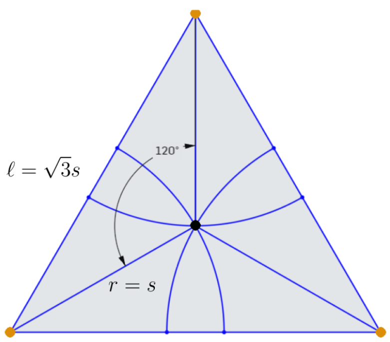


Figure 20. Calculation of length of side of triangular lattice for most optimal packing, with the value of ***s*** calculated from the model. Hives will be placed on the yellow points.

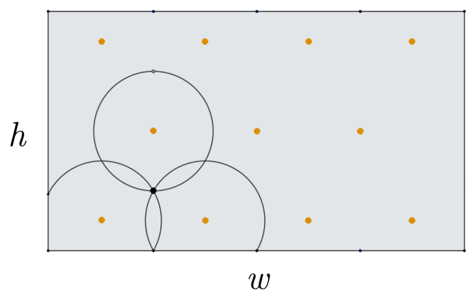


Figure 21. Example tiling for the amount of hives required for the first method to work. Hives will be placed on the yellow points. Circles demonstrate the range of the hive. We assumed $h = w = \sqrt{A}$, but the figure is a rectangle for the sake of brevity.

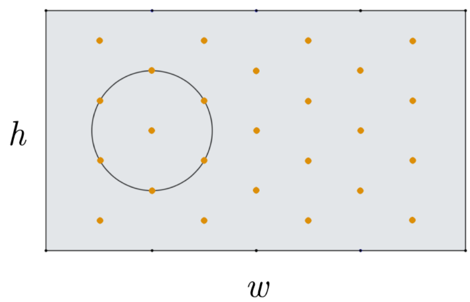


Figure 22. Example tiling for the amount of hives required for the first method to work. Hives will be placed on the yellow points. Circle demonstrates the range of the hive. We assumed $h = w = \sqrt{A}$, but the figure is a rectangle for the sake of brevity.

Results and Analysis

After using the values of s calculated in **(*)** above, we can already calculate the amount of hives required at a certain population of a bee colony, which can be shown in Table 10.

Table 10. Required number of bee colonies using specific methods

Population	Method 1 (Equation 5)		Method 2 (Equation 6)	
	Lower Bound	Upper Bound	Lower Bound	Upper Bound
20000	20	30	60	76
50000	12	20	36	50
80000	8	15	23	34

Two patterns are apparent right away from our demonstration in Table 10. First, we can observe that as the population of the hives increases, the number of hives decreases regardless of the method used to pack the hives. Therefore, an optimal strategy for maximizing the population of the bee colony is to invest in increasing the population of the hives before placing the hives across the field. Although the model assumes that the number of queens remains constant throughout the model, in reality, the number of queens could potentially increase upon sufficient nourishment. Hence, beekeepers could contribute to the well-being of the field of these crops. Second, the amount of hives required for the second method is significantly higher than that used in the first method. However, the way that we pack the values in the second method will remain more reliable as the bees could decide to take up a smaller distance from the hive (as apparent in the data from Figures 12 to 17).

In order for the model to be more useful, we can similarly for a given population. Eventually, we can create a regression model that determines the confidence intervals on how many hives are necessary in order to pollinate the entire field.

Strengths and Limitations

The first primary strength of this model is the **use of previous models** (i.e., the population model in Task 1 and Task 2) in order to produce the data in Task 3. Not only does this minimize computational intensity by no longer calculating new data, but this also **reduces the effort required to reimplement** this model in other areas. We can see that the values used in the previous model (Task 1 and Task 2) are also used in this problem; hence, this is also flexible to changes in real-world data which could be applied to this model.

The second primary strength of this model is by assigning the **most optimal locations** in the field. The model uses the triangular lattice, a known compact infinite tiling for planes. This reduces the time required in identifying the most optimal locations for placing hives and would be useful to beekeepers in making sure that they keep the bees safe while **keeping commercial and ecological benefits**. This would also be **flexible to other field shapes** as we can just overlay the existing triangular lattice with side lengths s or l, and as such, we can determine the most optimal locations.

However, this model is limited by its **inability to model competition between colonies**, since it was assumed in Task 1 that bee colonies will not interact with each other and in Task 3 that competition between bee colonies will not be modeled. In the real world, opposing bee colonies would fight for resources especially when they are in the same area. When this happens, this can be mitigated; however, by either introducing that to the model or using the first method of tiling in order to guarantee that common areas between colonies are as small as possible.

Conclusion

This model summarizes the population and pollination patterns of *Apis mellifera*, the European honey bee, which is calculated through various methods. This is performed through three tasks: (1) determining the population of a honey bee colony, (2) determining the most significant factors that affect the population, and (3) determining the number of hives that initiate pollination in an 81,000-m^2 field.

Task 1 describes a time-series stochastic model that models the population through several steps: mapping the environment, analysis of the roles of the bees, and implementation of the mechanisms of population variation (i.e., scouting and foraging of food sources, energy penalties, and negative externalities). Our model is able to justifiably use real-world data to accurately predict correct behavior and patterns of the population in response to the variables in the problem. The object-oriented model allows for changes in the model. Although we are limited by long runtimes and computational intensity for visualization and the fact that no internal factors are accounted for, the model remains accurate due to the strengths of the model.

Task 2 determines the most significant factors that affect the bee population. This was performed by changing several variables in Task 1 in order to produce changes in Task 2. The final population was compared to each other by determining the sensitivity, a ratio between the percent change in output and the percent change in input. A larger sensitivity implies a larger change. We found that there are three factors with a significantly large sensitivity compared to others: (1) the rate at which queens produce eggs, (2) the probability of the pesticides killing the bees, and (3) the probability of the predators attacking the bee every single day. This suggests investing in technology that increases the rate of production of eggs by the queens and by monitoring the field that the bees belong in so that no predators or pesticides are used around the field.

Task 3 aims to determine the necessary number of hives needed for complete pollination to occur. This is performed by defining the state of complete pollination when honey bees have consumed all of the food within a region. Two geometric packing methods were described: one that is optimal yet impractical, and one that is non-optimal yet impractical. We concluded that 23–34 hives are required in order to completely pollinate an 81,000-m^2 field with the maximum possible population.

The results from our models can help beekeepers in keeping track of factors that may negatively impact the growth of honey bee populations, and how to strategically position honey bee hives in order to pollinate a given crop area. The primary suggestions that were developed in this model are through the investment in technology that improves the production of eggs and by separating the predators and pesticides from bees. Our model produces accurate data with justifiable assumptions and is beneficial to conservationists. Through this, we can protect the bees and protect human life as we know it.

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Appendix A: Code Snippets

Code Snippet 1. Driver code for the simulator.

```
#include "pch.h"

const float TIME_SCALING = 1;
const float RUN_DURATION = 24 * 30 * 4 / TIME_SCALING;

const std::string inputPath  = "../Utilities/World.txt";
const std::string outputPath = "../Utilities/Result.txt";

int main() {
    std::ofstream output(outputPath);
    World::get()->generateFrom(inputPath);

    sf::Clock duration; duration.restart();
    sf::Clock simulate; simulate.restart();
    do {
        double time = simulate.restart().asSeconds();
        Hives::get()->update(time);
        Food::get()->update(time);
        Bees::get()->update(time);

        int hiveNumber = 1;
        for (auto i = begin(Hives::get()->list); i != end(Hives::get()->list); i++) {
            auto hive = *i;
            output << hiveNumber
                << '\t' << hive->count[AllBees]
                << '\t' << hive->count[Foragers]
                << '\t' << hive->count[Drones]
                << '\t' << hive->count[Guards]
                << '\t' << hive->count[Larvas]
                << '\t' << hive->count[Eggs] << '\n';

            hiveNumber++;
        }
    } while (duration.getElapsedTime().asSeconds() <= RUN_DURATION);
}
```

Code Snippet 2. Completion code for any waggle dance initiated within a hive.

```
void Hive::completeWaggleDance() {
    assert(waggleDancing);

    float minFood = foodData[0]->foodLeft;
    float maxFood = foodData[0]->foodLeft;
    float minDist = foodData[0]->distanceFrom(this);
    float maxDist = foodData[0]->distanceFrom(this);
    for (int i = 1; i < foodData.size(); i++) {
        minFood = std::min(minFood, foodData[i]->foodLeft);
        maxFood = std::max(maxFood, foodData[i]->foodLeft);
        minDist = std::min(minDist, foodData[i]->distanceFrom(this));
```

```
            maxDist = std::max(maxDist, foodData[i]->distanceFrom(this));
        }

        std::vector<std::pair<Food*, float>> weights;
        for (int i = 0; i < foodData.size(); i++) {
            float weight = compute(foodData[i], minFood, maxFood, minDist, maxDist);
            weights.push_back({foodData[i], weight});
        }

        std::sort(begin(weights), end(weights), decreasingOrder);

        std::vector<int> percentages;
        for (int i = 0; i < weights.size(); i++) {
            percentages.push_back(i + 1);
        }

        std::discrete_distribution<int> generator(begin(percentages), end(percentages));
        for (int i = 0; i < idleForagerBees.size(); i++) {
            int index = generator(engine);
            idleForagerBees[i]->setTarget(weights[index].first);
        }

        waggleDancing = false;
    }
```

Code Snippet 3. Bee traveling to target position code.

```
void Bee::travel(double deltaTime) {
    float rotationRadians = atan2(goal.y - position.y, goal.x - position.x);
    Point newPosition = position;
    newPosition.x += cos(rotationRadians) * speed * deltaTime;
    newPosition.y += sin(rotationRadians) * speed * deltaTime;
    if (distance(newPosition, position) > distance(target, position)) {
        setPosition(target, rotationRadians);
    } else {
        setPosition(newPosition, rotationRadians);
    }
}
```

Code Snippet 4. Bee abstraction implementation.

```
class Bee {
public:
    Bee(Point position, Hive hive, BeeType type);

    void update(double deltaTime);
    void setPosition(Point position, float rotationRadians);
    void setTarget(Food* foodsource);
    void setTarget(Point position);
    void harvest(float amount);
    void deposit(float amount);
    bool near(Structure* structure);
    bool at(Structure* structure);
    bool hungry();

    BeeState state;
private:
    Hive hive;
    Food* targetFoodsource;
    Point targetPosition;
    float harvestingDuration;
    float workDuration;
```

```
    float restDuration;
    float carryingFood;
    float lifespan;
    float maxEnergy;
    float energy;
    float fatigueEnergyPenalty;
    float speed;
};
```

Appendix B: Raw Population vs Time Graphs

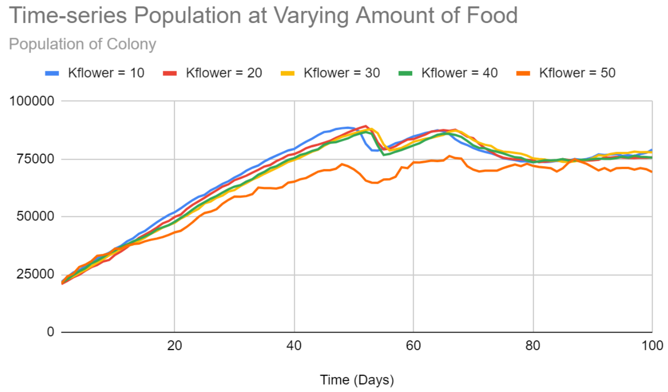

Figure B1. Time-series population graph with varying ($\boldsymbol{\rho_{flower}}$). The values of $\boldsymbol{\rho_{flower}}$ are changed to 10, 20, 30, 40, 50. The control group is marked in red.

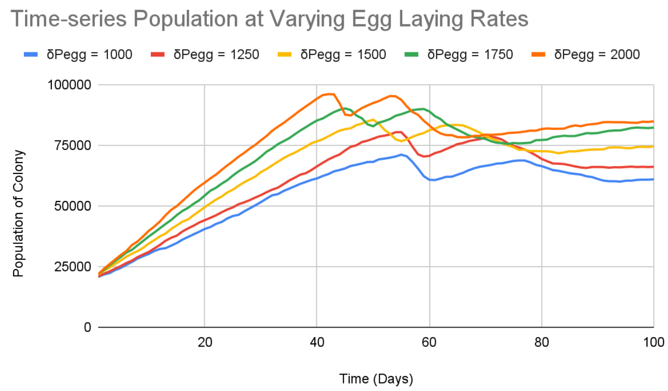

Figure B2. Time-series population graph with varying rates of egg-laying egg ($\boldsymbol{\delta P_{egg}}$). The values of $\boldsymbol{\delta P_{egg}}$ are 1000, 1250, 1500, 1750, 2000.

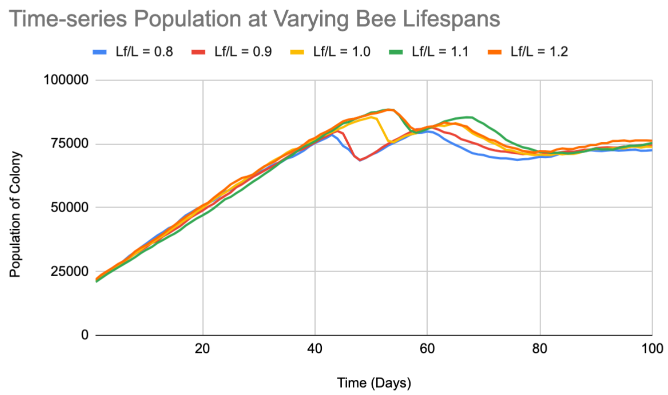

Figure B3. Time-series population graph with varying ratio of test lifetime to actual lifetime ($\boldsymbol{L_f / L}$). The values of $\boldsymbol{L_f / L}$ are 0.8, 0.9, 1 (control), 1.1, 1.2.

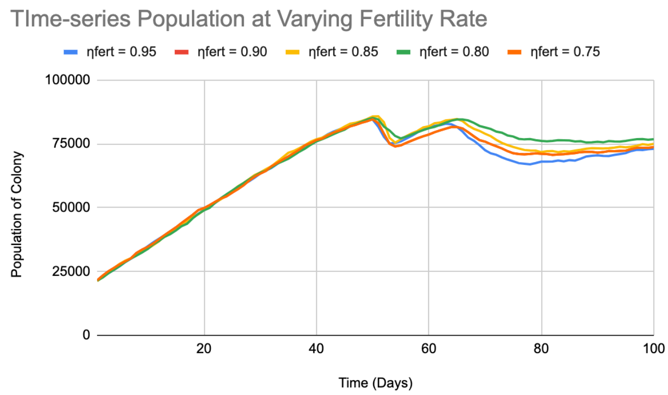

Figure B4. Time-series population graph with varying ratio of fertilized eggs to total number of eggs ($\boldsymbol{\eta_{fert}}$). The values of $\boldsymbol{\eta_{fert}}$ are 0.75, 0.80, 0.85 (control), 0.90, 0.95.

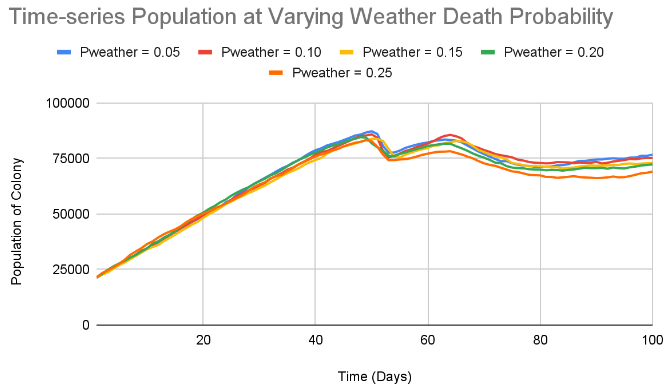

Figure B5. Time-series population graph with varying probability of dying due to weather ($p_{weather}$). The values of $p_{weather}$ are 0.05, 0.10, 0.15, 0.20, 0.25.

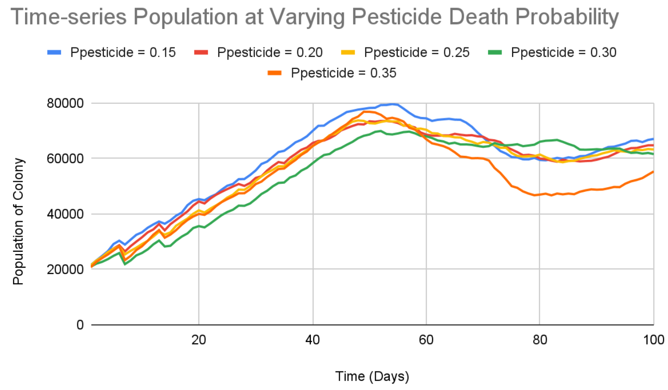

Figure B6. Time-series population graph with varying probability of dying due to pesticides ($p_{pesticide}$). The values of $p_{pesticide}$ are 0.15, 0.20, 0.25, 0.30, 0.35.

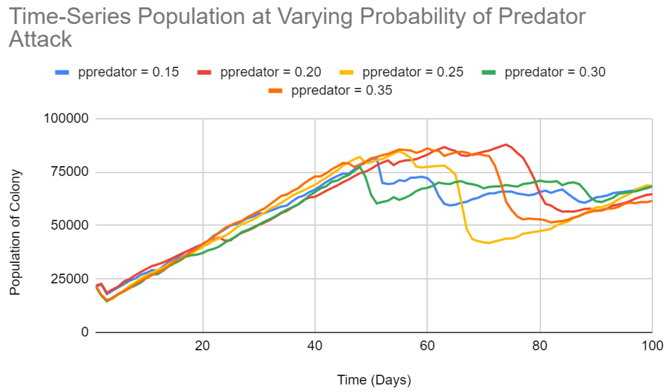

Figure B7. Time-series population graph with varying probability of dying due to pesticides ($p_{predator}$). The values of $p_{predator}$ are 0.15, 0.20, 0.25, 0.30, 0.35.

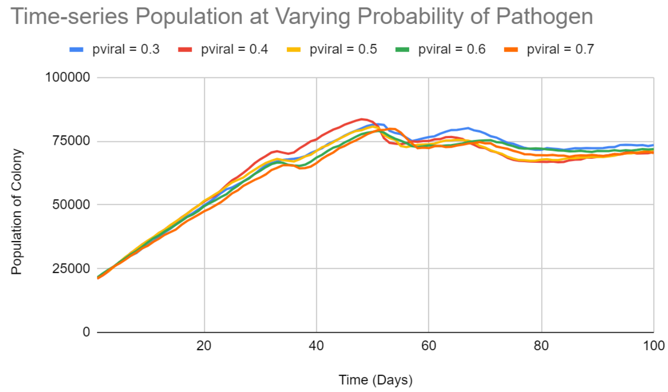

Figure B8. Time-series population graph with varying probability of dying due to pesticides (p_{viral}). The values of p_{viral} are 0.3, 0.4, 0.5, 0.6, and 0.7.