Laws of Propositional logic

1) Commutativity

$$P \vee Q = Q \vee P$$
$$P \wedge Q = Q \wedge P$$

2) Associativity

$$(P \vee Q) \vee R = P \vee (Q \vee R)$$
$$(P \wedge Q) \wedge R = P \wedge (Q \wedge R)$$

3) Distributing

$$(P \vee Q) \wedge R = (P \wedge R) \vee (Q \wedge R)$$
$$(P \wedge Q) \vee R = (P \vee R) \wedge (Q \vee R)$$

4) identity

$p \cup False = p$

$p \wedge True = p$

$p \wedge False = False$

$p \vee True = True$

5) Negation

$p \vee \sim p = True$

$p \wedge \sim p = False$

6) idempotence

$p \vee p = p$

$p \wedge p = p$

7) Absorption

$p \wedge (p \vee q) = p$

$p \vee (p \wedge q) = p$

8) Demorgan's law

$\sim (p \vee q) = \sim p \wedge \sim q$

$\sim (p \wedge q) = \sim p \vee \sim q$

9) Involution

$\sim (\sim p) = p$

			L.H.S.			R.H.S.
p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Inference in

→ Inference is a technique by which given set of facts or postulates or axioms or premises $f_1, f_2, \ldots f_n$ a goal G can be derived

→ For example the statement "where there is smoke there is fire"
"There is smoke on the hill"
Hence it can be reduce as hill

→ In propositional logic three rules are widely use for inferring facts.

1) modus ponens :- Given $p \Rightarrow q$, and p to be true, then q is true.

$$\left.\begin{array}{c} p \Rightarrow q \\ p \end{array}\right\} \text{Premises}$$

$$\overline{\quad q \quad} \; \} \text{ Goal}$$

2) modus tollen :- Given $p \Rightarrow q$ and $\neg q$ to be true then $\neg p$ is true

$P \Rightarrow Q$

$\sim Q$

$\sim P$

3) chain Rule : Given $P \Rightarrow Q$, $Q \Rightarrow R$ then $P \Rightarrow R$

$$\left.\begin{array}{l} P \Rightarrow Q \\ Q \Rightarrow R \end{array}\right\} \text{premises}$$

$P \Rightarrow R$ } Goal.

The chain rule is also referred as ~~lade~~ of ~~consitivity~~ transitivity.

Given

1) $C \vee D$

2) $\sim H \Rightarrow (A \wedge \sim B)$

3) $(C \vee D) \Rightarrow \sim H$

4) $(A \wedge \sim B) \Rightarrow (R \vee S)$

can $R \vee S$ be inferred

P (C∨D)

$C \vee D = \sim H$ Q

P Q

$C \vee D \Rightarrow \sim H$

$C \vee D$. by modus ponens

$\sim H \longrightarrow$ 5

(2) $\sim H \Rightarrow (A \wedge \sim B)$

~~$(A \wedge \sim B)$~~
$(A \wedge \sim B) \Rightarrow R \vee S$

$\sim H \Rightarrow R \vee S$ — (5)

$$\left.\begin{array}{l}\sim H \Rightarrow R \vee S \\ \underline{\sim H} \\ R \vee S\end{array}\right\} \text{modes ponens}$$

Hence $R \vee S$ is ~~equal~~ inferred.

* Predicate Logic

At the propositional logic at 5 other things,

constants | quantifier → For some, For all
variable | function
predicate

→ In propositional logic events are symbolized as proposition which acquire true or false value however there are real world situation where propositional logic falls short.

→ Hence, it need to be augmented with more tools to enhance logic capability. Hence, predicate logic comprise of the following apart from connective and proposition recognized by propositional logic

1) constant

→ constant represent object that do not change value.

→ ex. pencil, 100 degree centegrade

2) Variable

→ Variables are symbol which represent values aquired by the object as qualified by the quantifier

→ eg. x, y, z

3) Predicate

→ predicate are representative of association between objects that are constant or variables and aquire truth value, true or false

→ The predicate also carries a name that represent association between objects

eg. likes(Ram, tea)

4) Quantifiers

Quantifiers are symbol which indicate two type of quantification

1) ∃ for some

2) ∀ for all

$\forall$ is referred as universal quantifier $\exists$ is symbol existential quantifier

e.g. for all $\rightarrow$ man(x) : x is a man
mortal (x) : x is a mortal
$\forall x$ (man (x) $\Rightarrow$ mortal (x))

e.g. for existential
$\Rightarrow$ mushroom (x) : x is a mushroom
poisonous (x) : x is a poisonous
some mushroom is ~~posinous~~ poisonous

$\exists x$ (mushroom (x) $\wedge$ posinous (x)).

5) Function.
$\rightarrow$ It is similar to predicate in form of representation but unlike predicate they can acquire values other than truth values

eg plus (2, 3) = 5

Generate Predicate logic for following

1) Ram likes all kind of food

→ $\forall x \; food(x) \Rightarrow likes(Ram, x)$.

2) Sita likes everything that Ram likes

→ $\forall x \; (likes(Ram, x) \Rightarrow likes(Sita, x))$

3) Raja likes those, which sita and Ram both likes

→ $\forall x \; (likes(sita, x) \wedge likes(Ram, x)) \Rightarrow like(Raj, x)$

4) Ali likes some of which Ram likes

→ $\exists x \; (likes(Ram, x) \wedge likes(\cancel{Sita}\;Ali, x))$

Fuzzy Propositional logic

→ A fuzzy proposition is a statement which acquire fuzzy truth value thus, $\tilde{P}$, $T(\tilde{P})$ represents truth value attached to $\tilde{P}$.

→ The fuzzy membership value associated with fuzzy set $\tilde{A}$ for $\tilde{P}$ is treated as fuzzy truth value $T(\tilde{P}) = \mu_{\tilde{A}}(x)$

where $0 \leq \mu_{\tilde{A}}(x) \leq 1$

→ There are four fuzzy connectives indicated in table below:

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Consider $\tilde{P}$ and $\tilde{Q}$ as fuzzy proposition with $T(\vec{P})$ and $T(\vec{Q})$ as their truth value where fuzzy corrective table are define following.

Symbol	connective	usage	Description
$\sim$	~~$\sim\vec{P}$ or~~ $\bar{\vec{P}}$ Negation	$\sim\vec{P}$ or $\bar{\vec{P}}$	$1-T(\vec{P})$
$\vee$	Disjunction	$\vec{P}\vee\vec{Q}$	$\max(T(\vec{P}), T(\vec{Q}))$
$\wedge$	Conjunction	$\vec{P}\wedge\vec{Q}$	$\min(T(\vec{P}), T(\vec{Q}))$
$\Rightarrow$	Implication	$\vec{P}\Rightarrow\vec{Q}$	$\sim\vec{P}\vee\vec{Q}$ $\max((1-T(\vec{P}), T(\vec{Q}))$

$\vec{P}$: mary is efficient; $T(\vec{P}) = 0.8$

$\vec{Q}$: Rum is efficient; $T(\vec{Q}) = 0.65$

1) mary is not efficient

$\sim\vec{P}$,

$$T(\sim\vec{P}) = 1 - T(\vec{P})$$

$$= 1 - 0.8 = 0.2$$

2) mary is efficient and so is Rum

$\vec{P}\wedge\vec{Q}$ $\quad T(\vec{P}\wedge\vec{Q}) = \min(T(\vec{P}), T(\vec{Q}))$

$$= \min(0.8, 0.65)$$

$$= 0.65$$

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3) Either mary or Ram is Efficient

$\tilde{P} \vee \tilde{Q}$ $\quad T(\tilde{P} \vee \tilde{Q}) = \max(T(\tilde{P}), T(\tilde{Q}))$

$= \max(0.8, 0.65)$

$= 0.65$

4) If mary is efficient, then so is Ram

$\vec{P} \Rightarrow \vec{Q}$ $\quad T(\vec{P} \Rightarrow \vec{Q}) = \cancel{\max} \; T(\sim\vec{P} \vee \vec{Q})$

$= \max((1-T(\vec{P})), T(\vec{Q}))$

$= \max((1-0.8), 0.65))$

$= \max(0.2, 0.65)$

$= 0.65$

Fuzzy quantifier