

General Relativity Assignment

Meenal Punia, Kusum Meena, Humayun Akbar
Shashwat Kashyap, Nimesh Khandelwal

November 25, 2019

1 Important Symbols and Tensors

1.1 Metric

$$g_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sin^2(x_1) & 0 \\ 0 & 0 & 0 & \sin^2(x_1) \sin^2(x_2) \end{bmatrix}$$

1.2 Christoffel Symbols

$$\Gamma^1_{22} = -\frac{\sin(2x_1)}{2}$$

$$\Gamma^1_{33} = -\sin(x_1) \sin^2(x_2) \cos(x_1)$$

$$\Gamma^2_{12} = \frac{1}{\tan(x_1)}$$

$$\Gamma^2_{21} = \frac{1}{\tan(x_1)}$$

$$\Gamma^2_{33} = -\frac{\sin(2x_2)}{2}$$

$$\Gamma^3_{13} = \frac{1}{\tan(x_1)}$$

$$\Gamma^3_{23} = \frac{1}{\tan(x_2)}$$

$$\Gamma^3_{31} = \frac{1}{\tan(x_1)}$$

$$\Gamma^3_{32} = \frac{1}{\tan(x_2)}$$

1.3 Ricci Tensor

$$R_{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 \sin^2(x_1) & 0 \\ 0 & 0 & 0 & 2 \sin^2(x_1) \sin^2(x_2) \end{bmatrix}$$

1.4 Curvature scalar

$$R = 6$$

1.5 Einstein Tensor

$$G_{\mu\nu} = R_{\mu\nu} - R g_{\mu\nu} = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -4 \sin^2(x_1) & 0 \\ 0 & 0 & 0 & -4 \sin^2(x_1) \sin^2(x_2) \end{bmatrix}$$