4.4 CIRCUIT MODEL OF TRANSFORMER

Both primary and secondary of a transformer have winding resistances. Apart from this the two windings have leakage flux; ϕ_{l1} linking only the primary and ϕ_{l2} linking only the secondary (see Fig. 4.3). These leakage fluxes do not contribute in the process of energy transfer, which takes place via the mutual flux ϕ_m, but these cause the primary and secondary windings to possess leakage inductances and, therefore, leakage reactances at steady sinusoidal operation. The winding resistances and leakage reactances can be lumped in series with the ideal windings (resistance and leakage-less) in a circuit model. The ideal primary and secondary windings along with the core (which now carries only the mutual flux ϕ_m) indeed constitute the ideal transformer. Let windings resistances be r_1, r_2 and winding reactances (inductive) be x_1 and x_2.

The complete circuit model (commonly called *equivalent circuit*) is drawn in Fig. 4.4. It comprises the following circuit elements.

1. Magnetising shunt branch—B_m and G_i in parallel
2. Primary resistance r_1 and leakage reactance x_1 in series
3. Ideal transformer (turn ratio $N_1/N_2 = a$)
4. Secondary resistance r_2 and leakage reackage x_2 in series

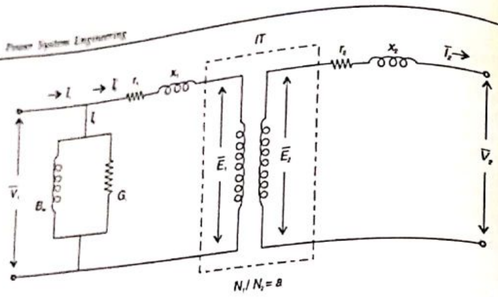

Fig. 4.4

By the technique of impedance transformation, these can be transferred to one side of the transformer say the primary. Then equivalent series resistance and reactance of the transformer referred to the primary side are

Equivalent resistance $R = r_1 + r_2' = r_1 + a^2 r_2$ (4.2)

Equivalent resistance $X = x_1 + x_2' = x_1 + a^2 x_2$ (4.3)

The transformer circuit model (equivalent circuit) of Fig. 4.4 with secondary resistance and reactance, referred to the primary side, gets modified to the form shown in Fig. 4.5 where

$$\bar{I}'_2 = \bar{I}_2/a \quad (4.4)$$

$$\bar{V}'_2 = a\bar{V}_2 \quad (4.5)$$

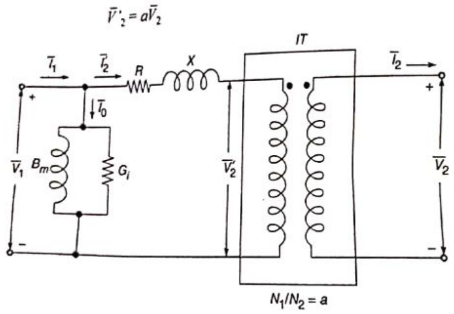

Fig. 4.5 Circuit model of transformer referred to the primary side

In the circuit model of a transformer, it is not necessary to carry the ideal transformer as these voltage and current conversions (Eqs (4.4) and (4.5)) can always be carried out computationally. The transformer circuit model with ideal transformer left out is drawn in Fig. 4.6.

The magnetising shunt branches in the circuit model of Fig. 4.6 do not affect voltage computation and may therefore be ignored. Further, since R is much smaller in a transformer than X, R may also be ignored. These two steps lead to the simplified circuit models of Fig. 4.7. It is also unnecessary to carry the superscript 'dash' on current and voltage.

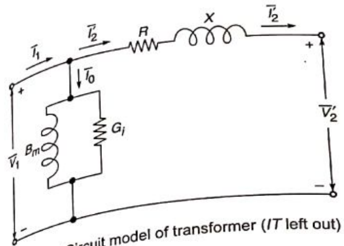

Fig. 4.6 Circuit model of transformer (IT left out)

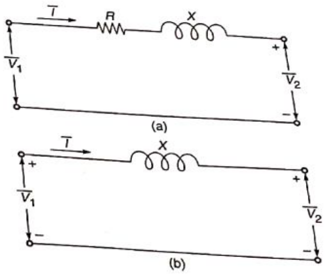

Fig. 4.7 Simplified circuit model of transformer

4.5 DETERMINATION OF PARAMETERS OF CIRCUIT MODEL OF TRANSFORMER

It is not practical to test a transformer for its voltage drop characteristic and its efficiency by a direct loading test. Such a test would suffer from three disadvantages, viz.

1. Loss of energy during testing
2. It may not be practical to arrange for load except for small size transformers
3. Losses as administered by direct loading would be in serious error as these are found by the difference of the input and output power readings which are close to each other, the losses being very small. It is well known that errors in meter readings add up and become much larger per cent of the difference. It is therefore standard practice in transformer testing to determine the transformer losses and the parameters of the circuit model by means of *nonloading* tests. The transformer performance is then computed from the circuit model.

Transformer parameter determination necessitates two tests, viz. open-circuit test and short-circuit test.

Open-Circuit (OC) or No-Load Test

The transformer is excited at rated voltage (and frequency) from one side while the other side is kept open-circuited as shown in Fig. 4.8(a). It is usually convenient to conduct such a test from the LV side. The circuit model under open-circuit is drawn in Fig. 4.8(b); it follows from Fig. 4.6 by setting $I'_2 = 0$.

Let the meter readings be

voltage (V) = V_1

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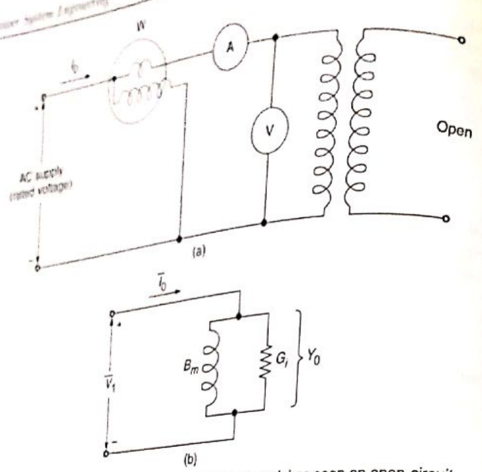


Fig. 4.8 (a) Circuit diagram for OC test (b) Circuit model as seen on open-circuit

current (A) = I_0 (4.6)

and power (W) = P_0 = core loss (P_i)

It then follows that

$$Y_0 = \frac{I_0}{V_1} \tag{4.7}$$

$$G_i = \frac{P_0}{V_1^2} \tag{4.8}$$

and

$$B_m = \sqrt{Y_0^2 - G_i^2} \tag{4.9}$$

By connecting a voltmeter on the secondary side, the OC test also yields the voltage ratio of the transformer, which is practically its turn ratio *a*.

The values of G_i and B_m as computed can be transferred to the other side of the transformer, if so desired.

It is seen that the OC test yields (i) core loss and (ii) parameters of the shunt branch of the transformer model.

The OC test is usually conducted from the LV side as low voltage small current supply is needed for the test.

Example 4.1 A 50 kVA, 2200/110 V transformer is connected to 110 V supply with metering. With 2200V side open circuited, the motor readings are 110V, 10 A, 400 W. Compute the parameters of the shunt branch of the equivalent circuit as seen from the LV and HV sides. Also compute the pu values. LOD E

Note: Difficulty Level E — Easy; M — Medium; D — Difficult

Solution

OC test on LV side—shunt branch praameters:

$$Y_0 = \frac{10}{110} = 0.091\ \mho$$

$$G_i = \frac{400}{(110)^2} = 0.033\ \mho$$

$$B_m = (Y_0^2 - G_i^2)^{1/2} = 0.085\ \mho$$

As seen on HV side:

$$G_i\,(HV) = 0.033 \times \left(\frac{110}{2200}\right)^2 = 8.25 \times 10^{-3}\ \mho$$

$$B_m\,(HV) = 0.085 \times \left(\frac{110}{2200}\right) = 21.25 \times 10^{-3}\ \mho$$

PU value :

$$(VA)_B = 50 \times 10^3$$

$$V_B\,(LV\text{ side}) = 110$$

Then

$$G_i\,(\text{pu}) = 0.091 \times \frac{(VA)_B}{V_B^2} = 0.033 \times \frac{50 \times 10^3}{(110)^2}$$

$$= 0.136\ \mho\ (\text{pu})$$

$$B_m\,(\text{pu}) = 0.085 \times \frac{50 \times 10^3}{(110)^2}$$

$$= 0.351\ \mho\ (\text{pu})$$

Short-Circuit (SC) Test

This test determines the series parameters of the transformer circuit model. The transformer is shorted on one side and is excited from a reduced voltage (rated frequency) source from the other side as shown in Fig. 4.8. The transformer circuit model under short-circuit conditions is drawn in Fig. 4.10(a). As the primary current is limited only by the resistance and leakage reactance of the transformer, V_{SC} needed to circulate full-load current is only of the order of 5–8 per cent of the rated voltage. At this reduced voltage, the exciting I_0 which is 2 to 5 per cent of the rated current gets reduced to 5 per cent of 2 per cent which is 0.1 per cent to 8 per cent of 5 per cent = 0.4 per cent of the rated current. The magnetising shunt branch of the circuit model can therefore be conveniently dropped resulting in the circuit of Fig. 4.10(b).

For convenience of the supply voltage and current needed, the SC test is usually conducted from the HV side and the LV side is short circuited.

In conducting the SC test, as in Fig. 4.9, the source voltage is gradually raised till the transformer draws *full-load* current. The meter readings under these conditions are

$$\text{voltage (V)} = V_{SC}$$

$$\text{current (A)} = I_{SC}$$

$$\text{power input (W)} = P_{SC} = I^2R\text{ loss* or copper loss } P_c \tag{4.10a}$$

(total in the two windings)

* As the transformer is excited at 5–8 per cent of rated voltage, the core flux gets reduced by the same percentage and the core losses being proportional to square of core flux are reduced to 0.25–0.64 per cent of that at rated voltage and are hence negligible. The power drawn by the transformer under SC condition is therefore wholly I^2R loss for all practical purposes.

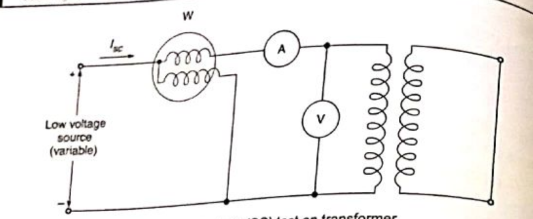


Fig. 4.9 Short-circuit (SC) test on transformer

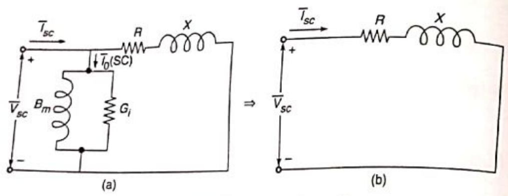


Fig. 4.10 Circuit model under SC conditions

From the circuit model of Fig. 4.10(b)

$$Z = \frac{V_{SC}}{I_{SC}} = \sqrt{R^2 + X^2} \tag{4.10b}$$

Equivalent resistance $R = \dfrac{P_{SC}}{(I_{SC})^2}$ (4.11)

Equivalent reactance $X = \sqrt{Z^2 - R^2}$ (4.12)

It is thus seen that the SC test yields information about (i) full-load copper loss and (ii) equivalent resistance and reactance of the transformer.

Together OC and SC tests determine all the four parameters of the transformer circuit model of Fig. 4.6—two shunt parameters (G_i, B_m) and two series parameters (R, X).

Example 4.2 A 25 kVA, 2200/220 V, 50 Hz single-phase transformer is found to have the following parameters: LOD M

$r_1 = 2\ \Omega$ $\quad r_2 = 0.025\ \Omega$
$x_1 = 7\ \Omega$ $\quad x_2 = 0.07\ \Omega$
$X_m = 16000\ \Omega$

Find the following:

(a) No-load current, its *pf* and power when excited from the LV side.
(b) With the LV side shorted, the HV side voltage needed to circulate full-load current. What is exciting current compared to full-load current?
(c) What is the power factor under part (b)?
(d) Convert the given parameter values of the transformer to *pu*.

Solution

(a) No-load, LV excited at 220 V:
Only magnetising current will be drawn. The core loss current is zero (negligible).

$$I_m = \frac{220}{16000} \times 10^3 = 13.75 \text{ mA, } 90° \text{ lagging}$$

$$pf = 0,\ P_0 = 0$$

Note: There be a small amount of core loss power draw which is being ignored here.

(b) Turn ratio, $a = \dfrac{2200}{220} = 10$

Referred to HV side:

$$R = 2 + (10)^2 + 0.025 = 4.5\ \Omega$$
$$X = 7 + (10)^2 + 0.07 = 14\ \Omega$$
$$Z = \sqrt{R^2 + X^2} = \sqrt{(4.5)^2 + (14)^2} = 14.7\ \Omega$$

Full-load current, $I\,(fl) = \dfrac{25 \times 10^3}{2200} = 11.36$ A

$$V_{sc} = Z\,I_{sc} = 14.7 \times 11.36 = 167 \text{ V}$$

It is $\dfrac{167}{2200} \times 100 = 7.59\%$ of rated voltage.

At this voltage, magnetising current $= \dfrac{167}{16000} = 10.44$ mA

It is $\dfrac{10.44 \times 10^{-3}}{11.36} \times 100 = 0.09$ per cent. So magnetising current can be neglected in the SC test.

(c) SC $pf = \cos\left(\tan^{-1}\left(\dfrac{X}{R}\right)\right) = \cos\left(\tan^{-1}\dfrac{14}{2.5}\right)$
$= 0.31$

(d) Parameters in pu
$(\text{kVA})_B = 25$ $(\text{kV})_B$ (HV) = 2.2

$$Z_B = \frac{1000 \times (2.2)^2}{25} = 193.6$$

$$r_1 = \frac{2}{193.6} = 0.0103, \qquad r_2 = \frac{(10)^2 \times 0.025}{193.6} = 0.0129$$

$$x_1 = \frac{7}{193.6} = 0.0362, \qquad x_2 = \frac{(10)^2 \times 0.025}{193.6} = 0.0362$$

$$R = r_1 + r_2 = 0.0103 + 0.0129 = 0.0232$$
$$X = x_1 + x_2 = 0.0362 + 0.0362 = 0.0724$$

$$X_m = \frac{16000}{193.6} = 82.64\ \Omega$$

4.6 AUTOTRANSFORMER

A two-winding transformer when electrically connected as shown in Fig. 4.11 is known as an autotransformer. Unlike a two-winding transformer, the two windings of an autotransformer are not electrically isolated.

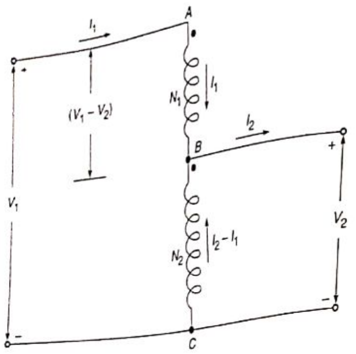


Fig. 4.11 Autotransformer

Let the two-winding transformer connected as an autotransformer be regarded as ideal. With this assumption, in Fig. 4.11 all voltages will be in phase and so will be all currents. The two-winding voltage ratio is

$$a = \frac{V_1 - V_2}{V_2} = \frac{N_1}{N_2} \tag{4.13}$$

The autotransformer voltage ratio is

$$a' = \frac{V_1}{V_2} = \frac{(V_1 - V_2) + V_2}{V_2}$$

or

$$a' = 1 + a \tag{4.14}$$

Now

$$(\text{VA})_{\text{TW}} = (V_1 - V_2)I_1 = (I_2 - I_1)V_2 \tag{4.15}$$

$$(\text{VA})_{\text{Auto}} = V_1 I_1 = V_2 I_2$$

But

$$\frac{I_2 - I_1}{I_1} = \frac{N_1}{N_2} = a$$

or

$$\frac{I_1}{I_2} = \frac{1}{1 + a} \tag{4.16}$$

Substituting Eq. (4.16) in Eq. (4.15),

$$(\text{VA})_{\text{TW}} = \left(1 - \frac{1}{1+a}\right) V_2 I_2$$

$$= \left(1 - \frac{1}{a'}\right)(\text{VA})_{\text{Auto}}$$

or

$$(\text{VA})_{\text{Auto}} = \left(1 - \frac{1}{1 - 1/a}\right)(\text{VA})_{\text{TW}} \tag{4.17}$$

or

$$(\text{VA})_{\text{Auto}} > (\text{VA})_{\text{TW}} \tag{4.18}$$

It is easily seen from Eq. (4.17) that the nearer a' is to unity, the larger is $(\text{VA})_{\text{Auto}}$ compared to $(\text{VA})_{\text{TW}}$. *An autotransformer is therefore applied for voltage ratios close to unity.*

The explanation of Eq. (4.18), lies in the fact that in an autotransformer, part of VA is conducted electrically, whereas in a two-winding transformer, all VA is transferred magnetically.

Example 4.3 A 2500/250 V, 25 kVA transformer has a core loss of 130 W and full-load copper loss of 320 W. Calculate its efficiency at full load, 0.8 pf. **LOD M**

The transformer is now connected as an autotransformer to give 2500/2750 V. Calculate its kVA rating and efficiency at full load, 0.8 pf. Compare with the two-winding kVA rating and efficiency.

Solution

(i) Two-winding Transformer:

Power output $= 25 \times 0.8 = 20$ kW

Losses $= 130 + 320 = 450$ W

$$\eta = \frac{20}{20.45} \times 100 = 97.8\%$$

(ii) Autotransformer: With reference to Fig. 4.12.

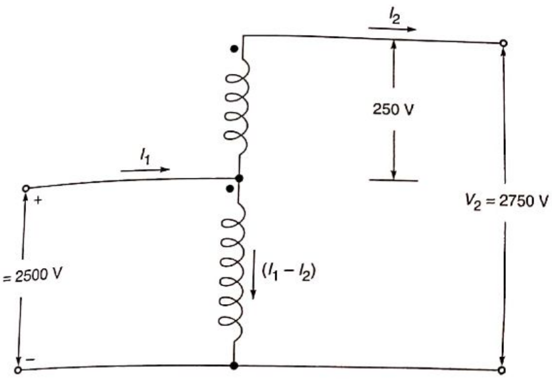


Fig. 4.12

$$I_2 = \frac{25 \times 1000}{250} = 100 \text{ A}$$

$$I_1 - I_2 = \frac{25 \times 1000}{2500} = 10 \text{ A}$$

$$I_1 = 110 \text{ A}$$

$$\therefore \quad \text{kVA rating} = \frac{2500 \times 110}{1000} = \frac{2750 \times 100}{1000} = 275$$

$$\text{Power output} = 2.75 \times 0.8 = 220 \text{ kW}$$

$$\eta = \frac{220}{220 + 0.45} = 99.8\%$$

It is seen that when a two-winding transformer is connected as an autotransformer, its rating goes up from 25 kVA to 275 kVA and its efficiency from 97.8% to 99.8%. This is possible because a large part of its kVA is transported conductively.

4.7 THREE-PHASE TRANSFORMERS

Three identical single-phase transformers can be connected to form a 3-phase bank. Primary and secondary sides of the bank can be connected in star/delta with various possible arrangements as

- star/star
- delta/delta
- star/delta or delta/star

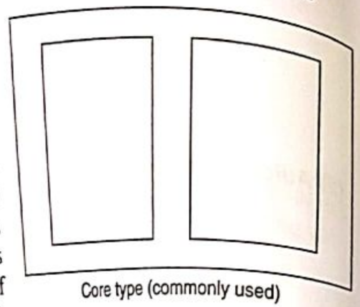


Fig. 4.13 Three-phase transformer core

Instead of three single-phase transformers, it costs about 15 per cent less to have a single 3-limb core as shown in Fig. 4.13 with primary and secondary of a phase wound on each limb. Like the sum of the currents in 3-phase in zero, the sum of the fluxes in the three limbs at any instant is zero providing for csontinuous flux paths. For reasons of economy, this arrangement (3-limb core) is popularly used. Of course, if one phase is out, the complete trans-former must be replaced.

In finding voltages and currents in a 3-phase transformer along with the ratio of transformation between the coupled windings, one must employ the line and phase relationship of star/delta connections with the assumption that the transformer is feeding a balanced load. Figure 4.14 shows a 3-phase transformer connected in delta on the primary side and star on the secondary side. In this figure, the coupled windings are drawn parallel to each other for ease of identification. Various line and phase voltages and currents are indicated on the figure (these follow easily). For a phase-to-phase transformation ratio of $a : 1$ (delta/star)

$$\frac{V_{\text{line}}(\text{star})}{V_{\text{line}}(\text{delta})} = \frac{\sqrt{3}V/a}{V} = \frac{\sqrt{3}}{a}$$

and

$$\frac{I_{\text{line}}(\text{star})}{I_{\text{line}}(\text{delta})} = \frac{aI/\sqrt{3}}{I} = \frac{a}{\sqrt{3}}$$

Phase Shift

In star/star and delta/delta connection, the line voltages and currents are in phase on the primary and secondary sides. However, in a delta/star connection the line voltages and currents undergo a shift in phase which can be ±30° or ± 90° depending upon the connections.

The delta/star connection of Fig. 4.14 with polarity marks indicated is a commonly used connection. The phasor diagram for voltages is shown in Fig. 4.15. The phase sequence is assumed to be *abc*/*ABC*.

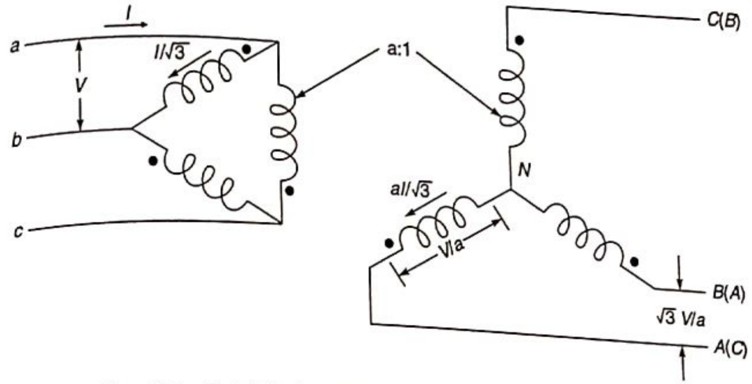


Fig. 4.14 Delta/star transformer connection (phase shift + 30°)

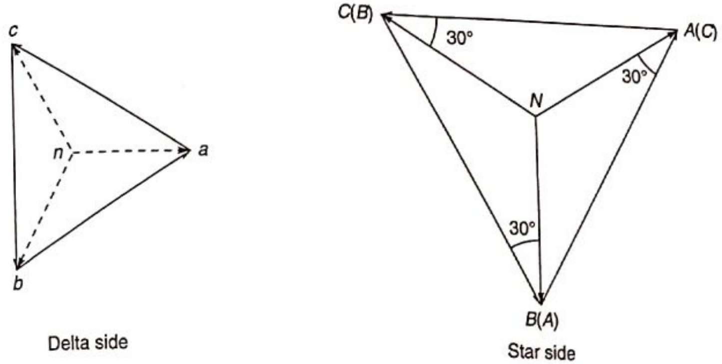


Fig. 4.15 Voltage phasor diagram of delta/star connection of Fig. 4.15

It is observed from above that the line voltages on star side lead the line voltages on delta side by 30°, viz. V_{ab} by 30°. The phase shift would become –30° by changing the phase sequence to *acb*/ACB, viz. V_{AC} by 30°. Relabeling the terminals on the star side, as shown within brackets, would make the phase shift –90° viz. V_{AB} lags V_{ab} by 90°. The reader may relabel to make the phase shift +90°. The line currents would undergo the same phase shift as line voltages in balanced 3-phase loading.

In power system applications of transformers, it is standard practice to connect the transformer (Δ/Y) such that the phase shifts by + 30° in going from LV side to HV side.

Star/Delta Connection

It is the most commonly used connection as the delta side provides a low impedance path for third harmonic current to flow, thereby reducing third harmonic voltage on the lines. At transmission level the low voltage side is connected delta and the high voltage side is connected. This provides for neutral grounding connection for high voltage transmission. However, at distribution level, the delta-star with star connection on low voltage side is employed to provide a neutral wire for feeding 3-phase and single-phase loads.

Example 4.4 A 3-phase transformer consisting of three 1-phase transformers with turn ratio of 10 : 1 (primary : secondary) is used to supply a 3-phase load of 120 kVA at 400 V on the secondary side. Calculate the primary line current and voltage if the transformer is connected (a) Δ/Y (b) Y/Δ. What is the line-to-line transformation ratio in each case? LOD M

Solution

(a) Δ/Y-connection (Fig. 4.16(a)):

$$I = \frac{120 \times 1000}{\sqrt{3} \times 400} = 173.2 \text{ A}$$

$$\text{Primary line-to-line voltage} = \frac{aV}{\sqrt{3}} = 10 \times \frac{400}{\sqrt{3}} = 2309 \text{ V}$$

$$\text{Primary line current} = \frac{\sqrt{3}I}{a} = 1.732 \times 173.2 \times \frac{1}{10} = 30 \text{ A}$$

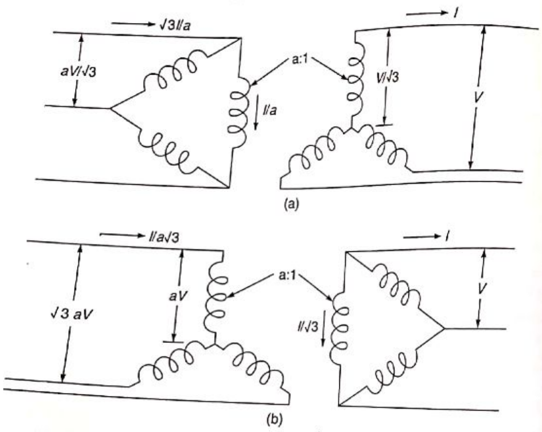


Fig. 4.16

Line-to-line transformation ratio (primary/secondary):

$$= \frac{aV/\sqrt{3}}{V} = \frac{a}{\sqrt{3}} = \frac{10}{\sqrt{3}}$$

(b) Y/Δ-connection (Fig. 4.14(b))

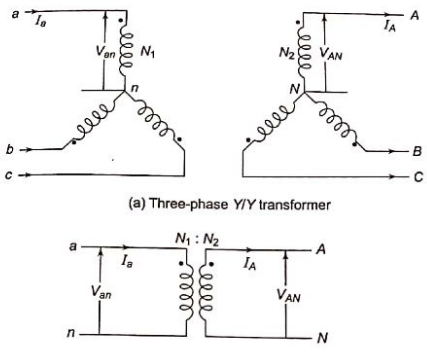
$$I = 173.2 \text{ A}$$

$$\text{Primary line-to-line voltage} = \sqrt{3}\, aV = \sqrt{3} \times 10 \times 400 = 6928 \text{ V}$$

$$\text{Primary line current} = \frac{1}{a\sqrt{3}} = \frac{173.2}{10 \times 1.732} = 10 \text{ A}$$

$$\text{Line-to-line transformation ratio} = \frac{\sqrt{3}aV}{V} = \sqrt{3}a = 10\sqrt{3}$$

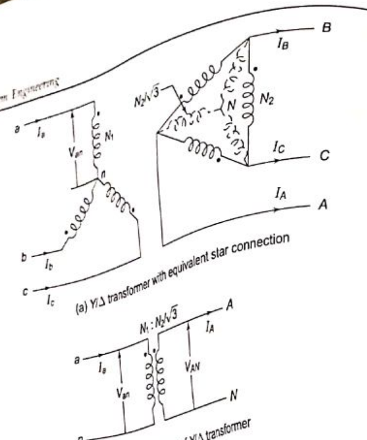
Consider now the case where a three-phase transformer forms part of a three-phase system. If the transformer is Y/Y connected as shown in Fig. 4.17(a), in the single-phase equivalent of the three-phase circuit it can be obviously represented by a single-phase transformer (as in Fig. 4.17(b)) with primary and secondary pertaining to phase a of the three-phase transformer.



(b) Single-phase representation of three-phase Y/Y transformer

Fig. 4.17

If the transformer is Y/Δ connected as in Fig. 4.18(a), the delta side has to be replaced by an equivalent star connection as shown dotted so as to obtain the single-phase equivalent of Fig. 4.18(b). An important fact has, however, to be observed here. On the delta side the voltage to neutral V_{AN} and line current I_A have a certain phase angle shift from the star side values V_{an} and I_a (90° for the phase labelling shown). In the single-phase equivalent (V_{AN}, I_A) are respectively in phase with (V_{an}, I_a). Since both phase voltage



(a) Y/Δ transformer with equivalent star connection

(b) Single-phase equivalent of Y/Δ transformer

Fig. 4.18

and line current shift through the same phase angle from star to delta side, the transformer per phase impedance and power flow are preserved in the single-phase equivalent. In most analytical studies, we are merely interested in the magnitude of voltages and currents so that the single-phase equivalent of Fig. 4.18(b) is an acceptable proposition. Wherever proper phase angles of currents and voltages are needed, correction can be easily applied after obtaining the solution through a single-phase trans-former equivalent.

It may be noted here that irrespective of the type of connection, the transformation ratio of the single-phase equivalent of a three-phase transformer is the same as the line-to-line transformation ratio.

4.8 One-Line Diagram and the Impedance or Reactance Diagram

A one-line diagram of a power system shows the main connections and arrangements of components. Any particular component may or may not be shown depending on the information required in a system study, e.g. circuit breakers need not be shown in a load flow study but are a must for a protection study. Power system networks are represented by one-line diagrams using suitable symbols for generators, motors, transformers and loads. It is a convenient practical way of network representation rather than drawing the actual three-phase diagram which may indeed be quite cumbersome and confusing for a practical size power network. Generator and transformer connections—star, delta, and neutral grounding are indicated by symbols drawn by the side of the representation of these elements. Circuit breakers are represented as rectangular blocks. Figure 4.19 shows the one-line diagram of a simple power system. The reactance data of the elements are given below the diagram.

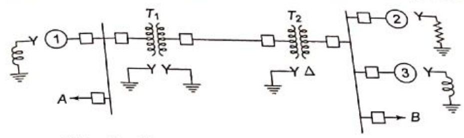


Fig. 4.19 One-line representation of a simple power system

Generator No. 1	30 MVA,	10.5 kV,	$X'' = 1.6$ ohms
Generator No. 2	15 MVA,	6.6 kV,	$X'' = 1.2$ ohms
Generator No. 3	25 MVA,	6.6 kV,	$X'' = 0.56$ ohms
Transformer T_1 (3 phase)	15 MVA,	33/11 kV,	$X = 15.2$ ohms per phase on high tension side
Transformer T_2 (3 phase)	15 MVA,	33/6.2 kV,	$X = 16$ ohms per phase on high tension side
Transmission line	20.5 ohms/phase		
Load A	40 MW,	11 kV,	0.9 lagging power factor
Load B	40 MW,	6.6 kV,	0.85 lagging power factor

Note: Generators are specified in three-phase MVA, line-to-line voltage and per phase reactance (equivalent star). Transformers are specified in three-phase MVA, line-to-line transformation ratio, and per phase (equivalent star) impedance on one side. Loads are specified in three-phase MW, line-to-line voltage and power factor.

The impedance diagram on single-phase basis for use under balanced operating conditions can be easily drawn from the one-line diagram. For the system of Fig. 4.19 the impedance diagram is drawn in Fig. 4.20. Single-phase transformer equivalents are shown as ideal transformers with transformer impedances indicated on the appropriate side. Magnetising reactances of the transformers have been neglected. This is a fairly good approximation for most power system studies. The generators are represented as voltage sources with series resistance and inductive reactance (synchronous machine model will be discussed in Sec. 4.11). The transmission line is represented by a π-model (to be discussed in Ch. 11). Loads are assumed to be passive (not involving rotating machines) and are represented by resistance and inductive reactance in series. Neutral grounding impedances do not appear in the diagram as balanced conditions are assumed.

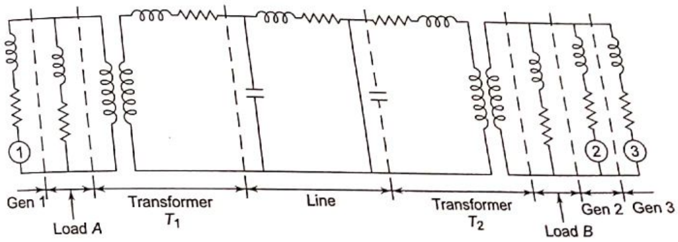


Fig. 4.20 Impedance diagram of the power system of Fig. 4.19

Three voltage levels (6.6, 11 and 33 kV) are present in this system. The analysis would proceed by transforming all voltages and impedances to any selected voltage level, say that of the transmission line (33 kV). The voltages of generators are transformed in the ratio of transformation and all impedances by the square of ratio of transformation. This is a very cumbersome procedure for a large network with several voltage levels. The per unit method discussed in the next section is the most convenient for power system analysis and will be used throughout this book.

4.9 Per Unit (PU) System

It is usual to express voltage, current, voltamperes and impedance of an electrical circuit in per unit (or percentage) of base or reference values of these quantities. The per unit* value of any quantity is defined as:

$$\frac{\text{the actual value in any units}}{\text{the base or reference value in the same units}}$$

The per unit method is particularly convenient in power systems as the various sections of a power system are connected through transformers and have different voltage levels.

Consider first a single-phase system. Let

Base apparent power in voltamperes, $S_B = (\text{VA})_B$

Base voltage $= V_B$ V

Then

$$\text{Base current, } I_B = \frac{S_B}{V_B} = \frac{(\text{VA})_B}{V_B} \tag{4.19a}$$

$$\text{Base impedance, } Z_B = \frac{V_B}{I_B} = \frac{V_B^2}{S_B} = \frac{V_B^2}{(\text{VA})_B} \tag{4.19b}$$

If the actual impedance is Z (ohms), its per unit value is given by

$$Z(\text{pu}) = \frac{Z}{Z_B} = \frac{Z \times S_B}{V_B^2} = \frac{Z \times (\text{VA})_B}{V_B^2} \tag{4.20}$$

For a power system, practical choice of base values are:

Base apparent power in megavoltamperes, $S_B = (\text{MVA})_B$

or

Base apparent power in kilovoltamperes, $S_B = (\text{kVA})_B$

Base voltage in kilovolts, $V_B = (\text{kV})_B$

$$\text{Base current, } I_B = \frac{S_B}{V_B} = \frac{1000 \times (\text{MVA})_B}{(\text{kV})_B} = \frac{(\text{kVA})_B}{(\text{kV})_B}\ \text{A} \tag{4.21}$$

$$\text{Base impedance, } Z_B = \frac{V_B}{I_B} = \frac{V_B^2}{S_B} = \frac{(\text{kV})_B^2}{(\text{MVA})_B} = \frac{1000 \times (\text{kV})_B^2}{(\text{kVA})_B}\ \Omega \tag{4.22}$$

* Per cent value = per unit value × 100
Per cent value is not convenient for use as the factor of 100 has to be carried in computations.

$$\text{Per unit impedance } Z(\text{pu}) = \frac{Z}{Z_B} = \frac{Z \times S_B}{V_B^2} = \frac{Z \times (\text{MVA})_B}{(\text{kV})_B^2} = \frac{Z \times (\text{MVA})_B}{(\text{kV})_B^2 \times 1000} \tag{4.23}$$

In a three-phase system rather than obtaining the per unit values using per phase base quantities, the per unit values can be obtained directly by using three-phase base quantities. Let

Three-phase base apparent power in megavoltamperes, $S_B = (\text{MVA})_B$

Line-to-line base voltage in kilovolts, $V_B = (\text{kV})_B$

Assuming star connection (equivalent star can always be found),

$$\text{Base current } I_B = \frac{1000 \times (\text{MVA})_B}{\sqrt{3}(\text{kV})_B}\ \text{A} \tag{4.24}$$

$$\text{Base impedance } Z_B = \frac{1000 \times (\text{kV})_B}{\sqrt{3}\, I_B}\ \text{ohms}$$

$$= \frac{(\text{kV})_B^2}{(\text{MVA})_B} = \frac{1000 \times (\text{kV})_B^2}{(\text{kVA})_B}\ \text{ohms} \tag{4.25}$$

$$\text{Per unit impedance } Z(\text{pu}) = \frac{Z(\text{ohms}) \times (\text{MVA})_B}{(\text{kV})_B^2} \tag{4.26}$$

$$= \frac{Z(\text{ohms}) \times (\text{kVA})_B}{(\text{kV})_B^2 \times 1{,}000}$$

When MVA base is changed from $(\text{MVA})_{B,\,\text{old}}$ to $(\text{MVA})_{B,\,\text{new}}$, and kV base is changed from $(\text{kV})_{B,\,\text{old}}$, to $(\text{kV})_{B,\,\text{new}}$, the new per unit impedance from Eq. (4.26) is given by

$$Z(\text{pu})_{\text{new}} = Z(\text{pu})_{\text{old}} \times \frac{(\text{MVA})_{B,\,\text{new}}}{(\text{MVA})_{B,\,\text{old}}} \times \frac{(\text{kV})_{B,\text{old}}^2}{(\text{kV})_{B,\,\text{new}}^2} \tag{4.27}$$

Per Unit Representation of a Transformer

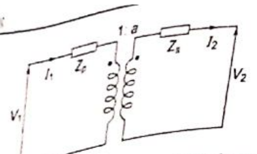
It has been said in Sec. 4.2 that a three-phase transformer forming part of a three-phase system can be represented by a single-phase transformer in obtaining per phase solution of the system. The delta connected winding of the transformer is replaced by an equivalent star so that the transformation ratio of the equivalent single-phase transformer is always the line-to-line voltage ratio of the three-phase transformer.

Figure 4.21(a) represents a single-phase transformer in terms of primary and secondary leakage reactances Z_p and Z_s and an ideal transformer of ratio $1{:}a$. The magnetising impedance is neglected. Let us choose a voltampere base of $(\text{VA})_B$ and voltage bases on the two sides of the transformer in the ratio of transformation, i.e.

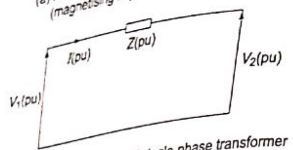
$$\frac{V_{1B}}{V_{2B}} = \frac{1}{a} \tag{4.28a}$$

Therefore,

$$\frac{I_{1B}}{I_{2B}} = a \text{ (as (VA)}_B \text{ is common)} \tag{4.28b}$$



(a) Representation of single-phase transformer (magnetising impedance neglected)



(b) Per unit equivalent circuit of single-phase transformer

Fig. 4.21

$$Z_{1B} = \frac{V_{1B}}{I_{1B}},\ Z_{2B} = \frac{V_{2B}}{I_{2B}} \tag{4.28c}$$

From Fig. 4.7(a), we can write

$$V_2 = (V_1 - I_1 Z_p)\,a - I_2 Z_s \tag{4.29}$$

We shall convert Eq. (4.29) into per unit form

$$V_2(\text{pu})V_{2B} = [V_1(\text{pu})V_{1B} - I_1(\text{pu})I_{1B}Z_p(\text{pu})Z_{1B}]$$
$$a - I_2(\text{pu})I_{2B}Z_s(\text{pu})Z_{2B}$$

Dividing by V_{2B} throughout and using base relations (4.28a, b, c), we get

$$V_2(\text{pu}) = V_1(\text{pu}) - I_1(\text{pu})Z_p(\text{pu}) - I_2(\text{pu})Z_s(\text{pu}) \tag{4.30}$$

Now
$$\frac{I_1}{I_2} = \frac{I_{1B}}{I_{2B}} = a$$

or
$$\frac{I_1}{I_{1B}} = \frac{I_2}{I_{2B}}$$

∴
$$I_1(\text{pu}) = I_2(\text{pu}) = I(\text{pu})$$

Equation (4.30) can therefore be written as

$$V_2(\text{pu}) = V_1(\text{pu}) - I(\text{pu})Z(\text{pu}) \tag{4.31}$$

where

$$Z(\text{pu}) = Z_p(\text{pu}) + Z_s(\text{pu})$$

Equation (4.31) can be represented by the simple equivalent circuit of Fig. 4.21(b) which does not require an ideal transformer. Considerable simplification has, therefore, been achieved by the per unit method with a common voltampere base and voltage bases on the two sides in the ratio of transformation.

Z(pu) can be determined directly from the equivalent impedance on primary or secondary side of a transformer by using the appropriate impedance base.

On primary side:

$$Z_1 = Z_p + Z_s/a^2$$

$$Z_1(\text{pu}) = \frac{Z_1}{Z_{1B}} = \frac{Z_p}{Z_{1B}} + \frac{Z_s}{Z_{1B}} \times \frac{1}{a^2}$$

But
$$a^2 Z_{1B} = Z_{2B}$$

∴
$$Z_1(\text{pu}) = Z_p(\text{pu}) + Z_s(\text{pu}) = Z(\text{pu}) \tag{4.32}$$

On secondary side:

$$Z_2 = Z_s + a^2 Z_p$$

$$Z_2(\text{pu}) = \frac{Z_2}{Z_{2B}} = \frac{Z_s}{Z_{2B}} + a^2\frac{Z_p}{Z_{2B}}$$

or
$$Z_2(\text{pu}) = Z_s(\text{pu}) + Z_p(\text{pu}) = Z(\text{pu}) \tag{4.33}$$

Thus, the per unit impedance of a transformer is the same whether computed from primary or secondary side so long as the voltage bases on the two sides are in the ratio of transformation (equivalent per phase ratio of a three-phase transformer which is the same as the ratio of line-to-line voltage rating).

The pu transformer impedance of a three-phase transformer is conveniently obtained by direct use of three-phase MVA base and line-to-line kV base in relation (Eq. 4.26). Any other impedance on either side of a transformer is converted to pu value just like Z_p or Z_s.

Per Unit Impedance Diagram of a Power System

From a one-line diagram of a power system we can directly draw the impedance diagram by following the steps given below:

1. Choose an appropriate common MVA (or kVA) base for the system.
2. Consider the system to be divided into a number of sections by the transformers. Choose an appropriate kV base in one of the sections. Calculate kV bases of other sections in the ratio of transformation.
3. Calculate per unit values of voltages and impedances in each section and connect them up as per the topology of the one-line diagram. The result is the single-phase per unit impedance diagram.

The above steps are illustrated by the following examples.

Example 4.5 Obtain the per unit impedance (reactance) diagram of the power system of Fig. 4.19. LOD E

Solution

The per phase impedance diagram of the power system of Fig. 4.19 has been drawn in Fig. 4.20. We shall make some further simplifying assumptions.

1. Line capacitance and resistance are neglected so that it is represented as a series reactance only.
2. We shall assume that the impedance diagram is meant for short circuit studies. Current drawn by static loads under short circuit conditions can be neglected. Loads A and B are therefore ignored.

Let us convert all reactances to per unit form. Choose a common three-phase MVA base of 30 and a voltage base of 33 kV line-to-line on the transmission line. Then the voltage base in the circuit of generator 1 is 11 kV line-to-line and that in the circuits of generators 2 and 3 is 6.2 kV.

The per unit reactances of various components are calculated below:

Transmission line: $\frac{20.5 \times 30}{(33)^2} = 0.564$

Transformer T_1: $\frac{15.2 \times 30}{(33)^2} = 0.418$

Transformer T_2: $\frac{16 \times 30}{(33)^2} = 0.44$

Generator 1: $\frac{1.6 \times 30}{(11)^2} = 0.396$

Generator 2: $\frac{1.2 \times 30}{(6.2)^2} = 0.936$

Generator 3: $\frac{0.56 \times 30}{(6.2)^2} = 0.437$

The reactance diagram of the system is shown in Fig. 4.22.

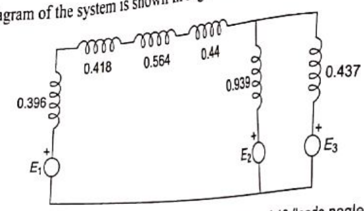


Fig. 4.22 Reactance diagram of the system of Fig. 4.19 (loads neglected)

E_1, E_2 and E_3 are per unit values of voltages to which the generators are excited. Quite often in a short circuit study, these will be taken as $1\angle 0°$ pu (no load condition).

Example 4.6 The reactance data of generators and transformers is usually specified in pu (or per cent) values, based on equipment ratings rather than in actual ohmic values as given in Example 4.5; while the transmission line impedances may be given in actual values. Let us resolve Example 4.5 by assuming the following pu values of reactances **LOD M**

Transformer T_1:	0.209
Transformer T_2:	0.220
Generator G_1:	0.435
Generator G_2:	0.413
Generator G_3:	0.3214

Solution

With a base MVA of 30, base voltage of 11 kV in the circuit of generator 1 and base voltage of 6.2 kV in the circuit of generators 2 and 3 as used in Example 4.5, we now calculate the pu values of the reactances of transformers and generators as per relation (Eq. 4.27):

Transformer T_1: $0.209 \times \frac{30}{15} = 0.418$

Transformer T_2: $0.22 \times \frac{30}{15} = 0.44$

Generator 1: $0.435 \times \frac{(10.5)^2}{(11)^2} = 0.396$

Generator 2: $0.413 \times \frac{30}{15} \times \frac{(6.6)^2}{(6.2)^2} = 0.936$

Generator 3: $0.3214 \times \frac{30}{25} \times \frac{(6.6)^2}{(6.2)^2} = 0.437$

Obviously these values are the same as obtained already in Example 4.5.

4.10 Complex Power

Consider a single-phase load fed from a source as in Fig. 4.23. Let

$$V = |V| \angle \delta$$

$$I = |I| \angle (\delta - \theta)$$

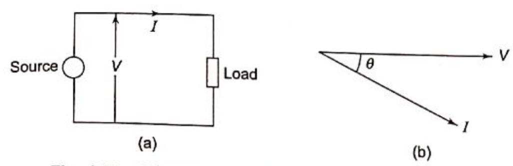


Fig. 4.23 Complex power flow in a single-phase load

When θ is positive, the current lags behind voltage. This is a convenient choice of sign of θ in power systems where loads have mostly lagging power factors.

Complex power flow in the direction of current indicated is given by

$$S = VI^*$$

$$= |V|\,|I| \angle \theta$$

$$= |V|\,|I| \cos\theta + j|V|\,|I| \sin\theta = P + jQ \tag{4.34}$$

or

$$|S| = (P^2 + Q^2)^{1/2}$$

Here

S = complex power (VA, kVA, MVA)

$|S|$ = apparent power (VA, kVA, MVA); it signifies rating of equipment (generators, transformers)

$P = |V|\,|I| \cos\theta$ = real (active) power (watts, kW, MW)

$Q = |V|\,|I| \sin\theta$ = reactive power

= voltamperes reactive (VAR)

= kilovoltamperes reactive (kVAR)

= megavoltamperes reactive (MVAR)

It immediately follows from Eq. (4.34) that Q, the reactive power, is positive for lagging current (lagging power factor load) and negative for leading current (leading power factor load). With the direction of current indicated in Fig. 4.23, $S = P + jQ$ is supplied by the source and is absorbed by the load.

Equation (4.34) can be represented by the phasor diagram of Fig. 4.24 where

$$\theta = \tan^{-1}\frac{Q}{P} = \text{positive for lagging current} \tag{4.35}$$

= negative for leading current

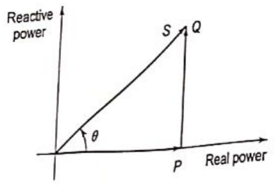


Fig. 4.24 Phasor representation of complex power (lagging pf load)

[I]f two (or more) loads are in parallel as in Fig. 4.25

$$S = VI^* = V(I_1^* + I_2^*)$$

$$= VI_1^* + VI_2^*$$

$$= S_1 + S_2 = (P_1 + P_2) + j(Q_1 + Q_2) \tag{4.36}$$

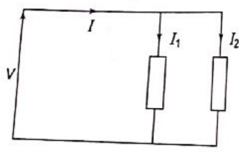


Fig. 4.25 Two loads in parallel

As per Eq. (4.36), Kirchhoff's current law applies to complex power (also applies separately to real and reactive powers).

In a series RL load carrying current I,

$$V = I(R + jX_L)$$

$$S = VI^* = I^2R + jI^2X_L$$

$$\therefore \quad P = I^2R = \text{active power absorbed by load}$$

$$Q = I^2X_L = \text{reactive power absorbed by load}$$

In case of a series RC load carrying current I,

$$P = I^2R$$

$$Q = -I^2X_C \text{ (reactive power absorbed is negative)}$$

Consider now a balanced three-phase load represented in the form of an equivalent star as shown in Fig. 4.26. The three-phase complex power fed into load is given by

$$S = 3V_PI_L^* = 3\,|V_P|\,\angle\delta_P I_L^* = \sqrt{3}\,|V_L|\,\angle\delta_P I_L^* \tag{4.37}$$

if

$$I_L = |I_L| \angle(\delta_P - \theta)$$

then

$$S = \sqrt{3}\,|V_L|\,|I_L| \angle\theta \tag{4.38}$$

$$= \sqrt{3}\,|V_L|\,|I_L| \cos\theta + j\sqrt{3}\,|V_L|\,|I_L| \sin\theta = P + jQ$$

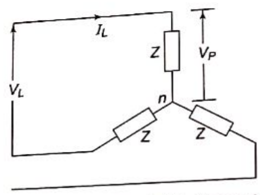


Fig. 4.26 Complex power fed to three-phase load

Here

$$|S| = \sqrt{3}\,|V_L|\,|I_L|$$

$$P = \sqrt{3}\,|V_L|\,|I_L| \cos\theta$$

$$Q = \sqrt{3}\,|V_L|\,|I_L| \sin\theta$$

where

θ = power factor angle

If V_L, the line voltage, is expressed in kV; and I_L, the line current in amperes, S is in kVA; and if the line current is in kiloamperes, S is in MVA.

In terms of load impedance Z,

$$I_L = \frac{V_P}{Z} = \frac{|V_L|\angle\delta_P}{\sqrt{3}Z}$$

Substituting for I_t in Eq. (4.37)

$$S = \frac{|V_t|^2}{Z^*} \tag{4.39a}$$

If V_t is in kV, S is now given in MVA. Load impedance Z if required can be calculated from

$$Z = \frac{|V_t|^2}{S^*} = \frac{|V_t|^2}{P - jQ} \tag{4.39b}$$

4.11 Steady State Model of Synchronous Machine

The synchronous machine is the most important element of a power system. It converts mechanical power into electrical form and feeds it into the power network or, in the case of a motor, it draws electrical power from the network and converts it into the mechanical form. The machine excitation which is controllable determines the flow of VARs into or out of the machine. Books on electrical machines [1, 2, 4] may be consulted for a detailed account of the synchronous machine. We shall present here a simplified circuit model of the machine which with suitable modifications wherever necessary (under transient conditions) will be adopted throughout this book.

Figure 4.27 shows the schematic cross-sectional diagram of a three-phase synchronous generator (alternator) having a two pole structure. The stator has a balanced three-phase winding—aa', bb' and cc'. The winding shown is a concentrated one, while the winding in an actual machine is distributed across the stator periphery. The rotor shown is a cylindrical* one (round rotor or non-salient pole rotor) with rotor winding excited by the DC source. The rotor winding is so arranged on rotor periphery that the field excitation produces nearly sinusoidally distributed flux/pole (ϕ_f) in the air gap. As the rotor rotates, three-phase emfs are produced in stator winding. Since the machine is a balanced one and balanced loading will be considered, it can be modelled on per phase basis for the reference phase a.

In a machine with more than two poles, the above defined structure repeats electrically for every pair of poles. The frequency of induced emf is given by

$$f = \frac{NP}{120} \text{ Hz}$$

where

N = rotor speed (synchronous speed) in rpm
P = number of poles

On no load the voltage E_f induced in the reference phase a lags 90° behind ϕ_f which produces it and is proportional to ϕ_f if the magnetic circuit is assumed to be unsaturated. This phasor relationship is indicated in Fig. 4.28. Obviously the terminal voltage $V_t = E_f$.

As balanced steady load is drawn from the three-phase stator winding, the stator currents produce synchronously rotating flux ϕ_a/pole (in the direction of rotation of the rotor). This flux, called *armature reaction* flux, is therefore stationary with respect to field flux ϕ_f. It intuitively follows that ϕ_a is in phase with phase a current I_a which causes it. Since the magnetic circuit has been assumed to be unsaturated, the superposition principle is applicable so that the resultant air gap flux is given by the phasor sum

$$\phi_r = \phi_f + \phi_a \tag{4.40}$$

* High-speed turbo-generators have cylindrical rotors and low-spped hydro-generators have salient pole rotors.

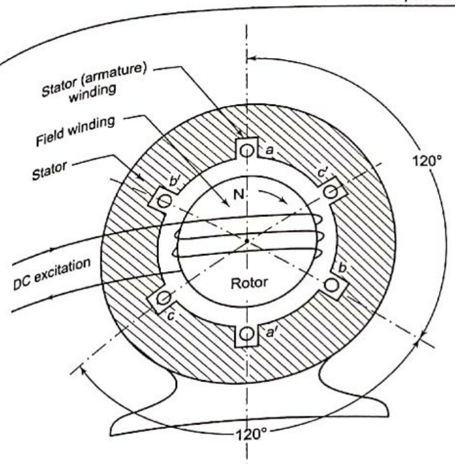


Fig. 4.27 Schematic diagram of a round rotor synchronous generator

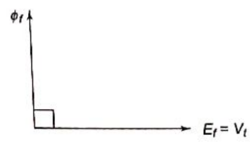


Fig. 4.28 Phasor relationship between ϕ_f and E_f

Further assuming that the armature leakage reactance and resistance are negligible, ϕ_r induces the armature emf which equals the terminal voltage V_t. Phasor diagram under loaded (balanced) conditions showing fluxes, currents and voltages as phasors is drawn in Fig. 4.29.

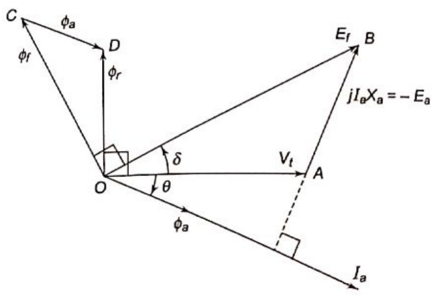


Fig. 4.29 Phasor diagram of synchronous generator

Here

θ = power factor angle
δ = angle by which E_f leads V_t called *load angle* or *torque angle*

We shall see in Sec. 5.10 that δ mainly determines the power delivered by the generator and the magnitude of E_f (i.e. excitation) determines the VARs delivered by it.

Because of the assumed linearity of the magnetic circuit, voltage phasors E_f, E_a and V_r are proportional to flux phasors ϕ_f, ϕ_a and ϕ_r respectively; further, voltage phasors lag 90° behind flux phasors. It therefore easily follows from Fig. 4.29 that phasor $AB = -E_a$ is proportional to ϕ_a (and therefore I_a) and is 90° leading ϕ_a (or I_a). With the direction of phasor AB indicated on the diagram

$$AB = jI_aX_a$$

where X_a is the constant of proportionality.

In terms of the above definition of X_a, we can directly write the following expression for voltages without the need of invoking flux phasors.

$$V_t = E_f - jI_aX_a \tag{4.41}$$

where

E_f = voltage induced by field flux ϕ_f alone

= no load emf

The circuit model of Eq. (4.41) is drawn in Fig. 4.30 wherein X_a is interpreted as inductive reactance which accounts for the effect of armature reaction thereby avoiding the need of resorting to addition of fluxes [Eq. (4.40)].

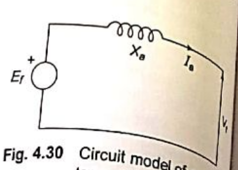


Fig. 4.30 Circuit model of round rotor synchronous generator (resistance and leakage reactance neglected)

The circuit of Fig. 4.30 can be easily modified to include the effect of armature leakage reactance and resistance (these are series effects) to give the complete circuit model of the synchronous generator as in Fig. 4.31. The total reactance $(X_a + X_l) = X_s$ is called the *synchronous reactance* of the machine. Equation (4.41) now becomes

$$V_t = E_f - jI_aX_s - I_aR_a \tag{4.42}$$

This model of the synchronous machine can be further modified to account for the effect of magnetic saturation where the principle of superposition does not hold.

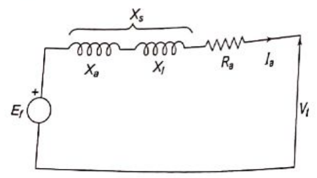


Fig. 4.31 Circuit model of round rotor synchronous generator

Armature resistance R_a is invariably neglected in power system studies. Therefore, in the place of the circuit model of Fig. 4.31, the simplified circuit model of Fig. 4.32 will be used throughout this book. The corresponding phasor diagram is given in Fig. 4.33. The field induced emf E_f leads the terminal voltage by the torque (load) angle δ. This, in fact, is the condition for active power to flow out of the generator. The magnitude of power delivered depends

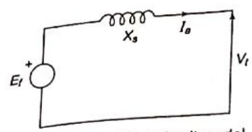


Fig. 4.32 Simplified circuit model of round rotor synchronous generator

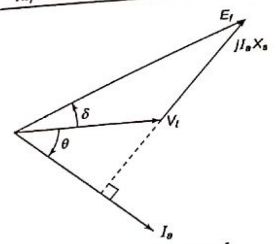


Fig. 4.33 Phasor diagram of synchronous generator

In the motoring operation of a synchronous machine, the current I_a reverses as shown in Fig. 4.34, so that Eq. (4.42) modifies to

$$E_f = V_t - jI_aX_s \tag{4.43}$$

which is represented by the phasor diagram of Fig. 4.35. It may be noted that V_t now leads E_f by δ. This in fact is the condition for power to flow into motor terminals.

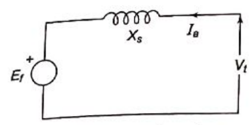


Fig. 4.34 Motoring operation of synchronous machine

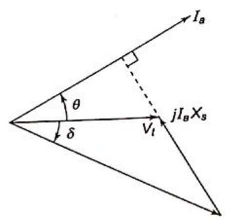


Fig. 4.35 Phasor diagram of motoring operation

The flow of reactive power and terminal voltage of a synchronous machine is mainly controlled by means of its excitation. This is discussed in detail in Sec. 5.10. Voltage and reactive power flow are often automatically regulated by voltage regulators acting on the field circuits of generators and by automatic tap changing devices on transformers.

Normally, a synchronous generator operates in parallel with other generators connected to the power system. For simplicity of operation we shall consider a generator connected to an *infinite bus* as shown in Fig. 4.36. As infinite bus means a large system whose voltage and frequency remain constant independent of the power exchange between the synchronous machine and the bus, and independent of the excitation of the synchronous machine.

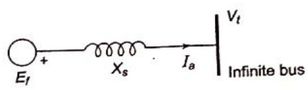


Fig. 4.36 Synchronous machine connected to infinite bus

Consider now a synchronous generator feeding constant active power into an infinite bus bar. As the machine excitation is varied, armature current I_a and its angle θ, i.e. power factor, change in such a manner as to keep

$$|V_t|\,|I_a|\cos\theta = \text{constant} = \text{active power output}$$

It means that since $|V_t|$ is fixed, the projection $|I_a|\cos\theta$ of the phasor I_a on V_t remains constant, while the excitation is varied. Phasor diagrams corresponding to high, medium and low excitations are presented in Fig. 4.37. The phasor diagram of Fig. 4.37(b) corresponds to the unity power factor case. It is obvious from the phasor diagram that for this excitation

$$|E_f|\cos\delta = |V_t|$$

This is defined as *normal excitation*. For the *overexcited* case (Fig. 4.37 (a)), i.e. $|E_f|\cos\delta > |V_t|$, I_a lags behind V_t so that the generator feeds positive reactive power into the bus (or draws negative reactive power from the bus). For the *underexcited* case (Fig. 4.37(c)), i.e. $|E_f|\cos\delta > |V_t|$, I_a leads V_t so that the generator feeds negative reactive power into the bus (or draws positive reactive power from the bus).

Figure 4.38 shows the overexcited and underexcited cases of synchronous motor (connected to infinite bus) with constant power drawn from the infinite bus. In the overexcited case, I_a leads V_t, i.e. the motor draws negative reactive power (or supplies positive reactive power); while in the underexcited case I_a lags V_t, i.e. the motor draws positive reactive power (or supplies negative reactive power).

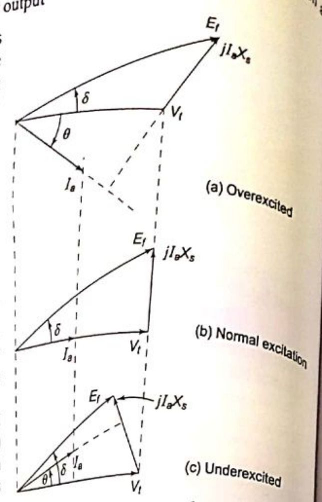


Fig. 4.37 Phasor diagrams of synchronous generator feeding constant power as excitation is varied

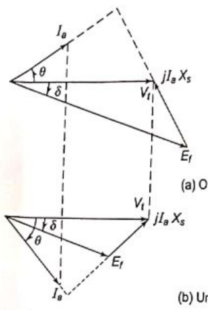


Fig. 4.38 Phasor diagrams of synchronous motor drawing constant power as excitation is varied

From the above discussion we can draw the general conclusion that a synchronous machine (generating or motoring) while operating at constant power supplies positive reactive power into the bus bar (or draws negative reactive power from the bus bar) when overexcited. An underexcited machine, on the other hand, feeds negative reactive power into the bus bar (or draws positive reactive power from the bus bar).

Consider now the power delivered by a synchronous generator to an infinite bus. From Fig. 4.33 this power is

$$P = |V_t|\,|I_a|\cos\theta$$

The above expression can be written in a more useful form from the phasor geometry. From Fig. 4.33

$$\frac{|E_f|}{\sin(90° + \theta)} = \frac{|I_a|\,X_s}{\sin\delta}$$

or

$$|I_a|\cos\theta = \frac{|E_f|}{X_s}\sin\delta \tag{4.44}$$

$$\therefore \quad P = \frac{|E_f||V_t|}{X_s}\sin\delta \tag{4.45}$$

The plot of P versus δ, shown in Fig. 4.39, is called the *power angle curve*. The maximum power that can be delivered occurs at $\delta = 90°$ and is given by

$$P_{\max} = \frac{|E_f||V_t|}{X_s} \tag{4.46}$$

For $P > P_{\max}$ or for $\delta > 90°$ the generator will have stability problem. This problem (the stability) will be discussed at length in Ch. 13.

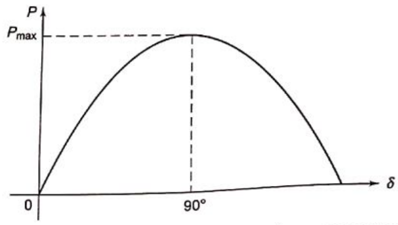


Fig. 4.39 Power angle curve of a synchronous generator

Power Factor and Power Control

While Figs 4.37 and 4.38 illustrate how a synchronous machine power factor changes with excitation for fixed power exchange, these do not give us a clue regarding the quantitative values of $|I_a|$ and δ. This can easily be accomplished by recognising from Eq. (4.44) that

$$|E_f|\sin\delta = |I_a|X_s\cos\theta$$

$$= \frac{PX_s}{|V_t|} = \text{constant (for constant exchange of power to infinite bus bar)} \tag{4.47}$$

Figure 4.40 shows the phasor diagram for a generator delivering constant power to infinite bus but with varying excitation. As $|E_f|\sin\delta$ remains constant, the tip of phasor E_f moves along a line parallel to V_t as excitation is varied. The direction of phasor I_a is always 90° lagging jI_aX_s and its magnitude is obtained from $(|I_a|X_s)/X_s$. Figure 4.41 shows the case of limiting excitation with $\delta = 90°$. For excitation lower than this value the generator becomes unstable.

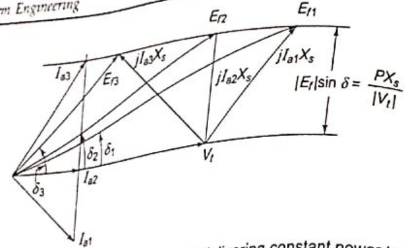


Fig. 4.40 Effect of varying excitation of generator delivering constant power to infinite bus bar

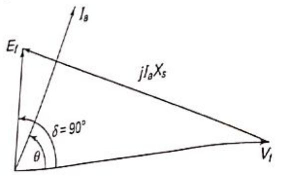


Fig. 4.41 Case of limiting excitation of generator delivering constant power to infinite bus bar

Similar phasor diagrams can be drawn for synchronous motor as well for constant input power (or constant load if copper and iron losses are neglected and mechanical loss is combined with load).

Another important operating condition is variable power and fixed excitation. In this case $|V_t|$ and $|E_f|$ are fixed, while δ and active power vary in accordance with Eq. (4.45). The corresponding phasor diagram for two values of δ is shown in Fig. 4.42. It is seen from this diagram that as δ increases, current magnitude increases and power factor improves. It will be shown in Sec. 5.10 that as δ changes, there is no significant change in the flow of reactive power.

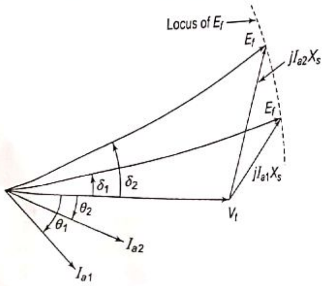


Fig. 4.42 Operation of synchronous generator with variable power and fixed excitation

Salient Pole Synchronous Generator

A salient pole synchronous machine, as shown in Fig. 4.43, is distinguished from a round rotor machine by constructional features of field poles which project with a large interpolar air gap. This type of construction is commonly employed in machines coupled to hydroelectric turbines which are inherently slow-speed ones so that the synchronous machine has multiple pole pairs as different from machines coupled to high-speed steam turbines (3,000/1,500 rpm) which have a two- or four-pole structure. Salient pole machine analysis is made through the *two-reaction theory* outlined below.

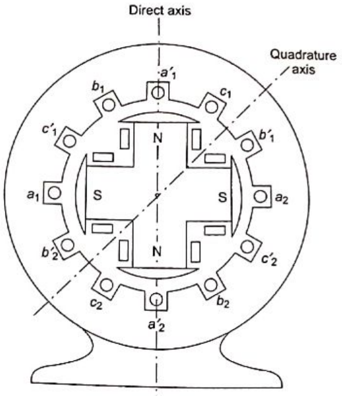


Fig. 4.43 Salient pole synchronous machine (4-pole structure)

In a round rotor machine, armature current in phase with field induced emf E_f or in quadrature (at 90°) to E_f, produces the same flux linkages per ampere as the air gap is uniform so that the armature reaction reactance offered to in-phase or quadrature current is the same ($X_a + X_l = X_s$). In a salient pole machine, air gap is non-uniform along rotor periphery. It is the least along the axis of main poles (called *direct axis*) and is the largest along the axis of the interpolar region (called *quadrature axis*). Armature current in quadrature with E_f produces flux along the direct axis and the reluctance of flux path being low (because of small air gap), it produces larger flux linkages per ampere and hence the machine presents larger armature reaction reactance X_d (called direct axis reactance) to the flow of quadrature component I_d of armature current I_a.

On the other hand, armature current in phase with E_f produces flux along the quadrature axis and the reluctance of the flux path being high (because of large interpolar air gap), it produces smaller flux linkages per ampere and hence the machine presents smaller armature reaction reactance X_q (quadrature axis reactance $< X_d$) to the flow of in-phase component I_q of armature current I_a.

Since a salient pole machine offers different reactances to the flow of I_d and I_q components of armature current I_a, a circuit model cannot be drawn. The phasor diagram of a salient pole generator is shown in Fig. 4.44. It can be easily drawn by following the steps given below:

1. Draw V_t and I_a at angle θ.
2. Draw I_aR_a. Draw $CQ = jI_aX_d$($\perp$ to I_a).
3. Make $|CP| = |I_a|\,X_q$ and draw the line OP which gives the direction of E_f phasor.
4. Draw a $\perp$ from Q to the extended line OP such that $OA = E_f$.

It can be shown by the above theory that the power output of a salient pole generator is given by

$$P = \frac{|V_t||E_f|}{X_d}\sin\delta + \frac{|V_t|^2(X_d - X_q)}{2X_dX_q}\sin 2\delta \tag{4.48}$$

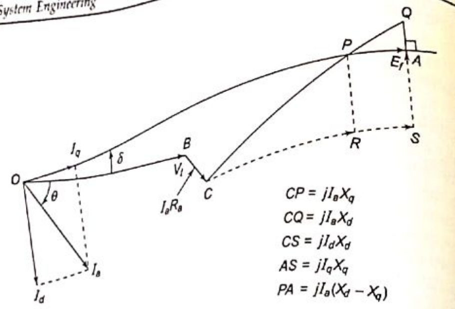


Fig. 4.44 Phasor diagram of salient pole synchronous generator

The first term is the same as for a round rotor machine with $X_s = X_d$ and constitutes the major part in power transfer. The second term is quite small (about 10–20 per cent) compared to the first term and is known as *reluctance power.*

P versus δ is plotted in Fig. 4.45. It is noticed that the maximum power output occurs at $\delta < 90°$ (about 70°). Further $\frac{dP}{d\delta}$ (change in power per unit change in power angle for small changes in power angle), called the *synchronising power coefficient*, in the operating region ($\delta < 70°$) is larger in a salient pole machine than in a round rotor machine.

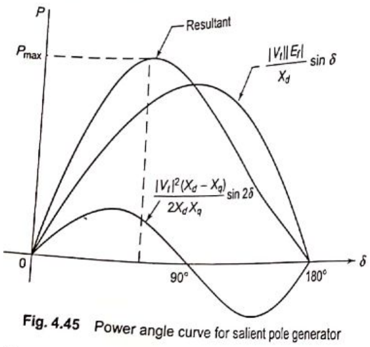


Fig. 4.45 Power angle curve for salient pole generator

In this book, we shall neglect the effect of saliency and take

$$X_s = X_d$$

in all types of power system studies considered.

During a machine transient, the direct axis reactance changes with time acquiring the following distinct values during the complete transient.

X''_d = subtransient direct axis reactance
X'_d = transient direct axis reactance
X_d = steady state direct axis reactance

The significance and use of these three values of direct axis reactance will be elaborated in Ch. 10.

Operating Chart of a Synchronous Generator

While selecting a large generator, besides rated MVA and power factor, the greatest allowable stator and rotor currents must also be considered as they influence mechanical stresses and temperature rise. Such limiting parameters in the operation are brought out by means of an *operating chart* or *performance chart.*

For simplicity of analysis, the saturation effects, saliency, and resistance are ignored and an unsaturated value of synchronous reactance is considered. Consider Fig. 4.46, the phasor diagram of a cylindrical rotor machine. The locus of constant $|I_a|X_s, |I_a|$ and hence MVA is a circle centered at M. The locus of constant $|E_f|$ (excitation) is also a circle centered at O. As MP is proportional to MVA, QP is proportional to MVAR and MQ to MW, all to the same scale which is obtained as follows.

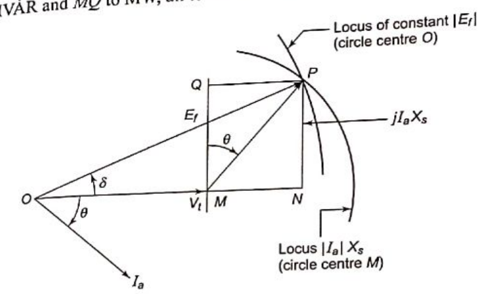


Fig. 4.46 Phasor diagram of synchronous generator

For zero excitation, i.e. $|E_f| = 0$

$$-jI_aX_s = V_t$$

or

$$I_a = jV_t/X_s$$

i.e. $|I_a| = |V_t|/X_s$ leading at 90° to OM which corresponds to VARs/phase.

Consider now the chart shown in Fig. 4.47 which is drawn for a synchronous machine having $X_s = 1.43$ pu. For zero excitation, the current is 1.0/1.43 = 0.7 pu, so that the length MO corresponds to reactive power of 0.7 pu, fixing both active and reactive power scales.

With centre at O, a number of semicircles are drawn with radii equal to different pu MVA loadings. Circles of per unit excitation are drawn from centre M with 1.0 pu excitation corresponding to the fixed terminal voltage OM. Lines may also be drawn from O corresponding to various power factors but for clarity only 0.85 pf lagging line is shown. The operational limits are fixed as follows.

Taking 1.0 per unit active power as the maximum allowable power, a horizontal limit-line *abc* is drawn through *b* at 1.0 pu. It is assumed that the machine is rated to give 1.0 pu unit active power at power factor 0.85 lagging and this fixes point *c*. Limitation of the stator current to the corresponding value requires the limit-line to become a circular arc *cd* about centre O. At point *d* the rotor heating becomes more important and the arc *de* is fixed by the maximum excitation current allowable, in this case assumed to be $|E_f| = 2.40$ pu (i.e. 2.4 times $|V_t|$). The remaining limit is decided by loss of synchronism at leading power factors. The theoretical limit is the line perpendicular to MO at M (i.e. $\delta = 90°$), but in practice a safety margin is brought into permit a further small increase in load before instability. In Fig. 4.47, a 0.1 pu margin is employed and is shown by the curve *afg* which is drawn in the following way.

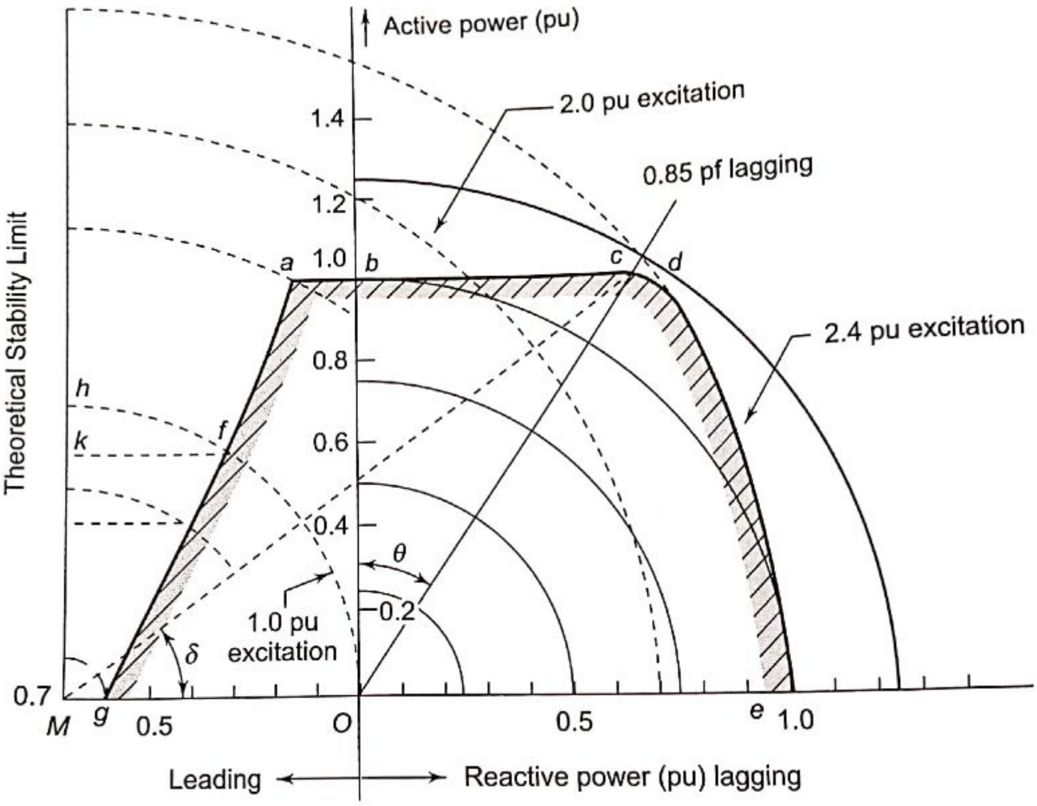


Fig. 4.47 Operating chart for large synchronous generator

Consider a point *h* on the theoretical limit on the $|E_f| = 1.0$ pu excitations arc, the power *Mh* is reduced by 0.1 pu to *Mk*; the operating point must, however, still be on the same $|E_f|$ arc and *k* is projected to *f* which is the required point on the desired limiting curve. This is repeated for other excitations giving the curve *afg*. The complete working area, shown shaded, is *gfabcde*. A working point placed within this area at once defines the MVA, MW, MVAR, current, power factor and excitation. The load angle δ can be measured as shown in the figure.

4.1 What is per unit system? How are these quantities selected? LOD E
4.2 What are the advantages of per unit system? LOD E
4.3 What is the procedure to draw the one-line diagram? LOD E
4.4 What is the difference between one-line impedance diagram and reactance diagram. LOD E
4.5 Obtain the steady state characteristics of round rotor synchronous generator. LOD M
4.6 Find the power output relation from the salient pole synchronous generator. LOD M
4.7 What is an infinite bus? LOD E
4.8 Express the per-unit admittance of a power system in terms of base voltage and base voltampere. LOD E

Problems

4.1 Figure P-4.1 shows the schematic diagram of a radial transmission system. The ratings and reactances of the various components are shown therein. A load of 60 MW at 0.9 power factor lagging is tapped from the 66 kV substation which is to be maintained at 60 kV. Calculate the terminal voltage of the synchronous machine. Represent the transmission line and the transformers by series reactances only. LOD M

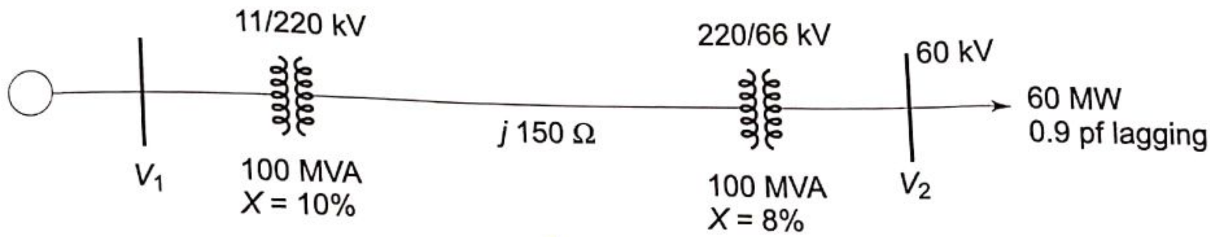


Fig. P-4.1

4.2 Draw the pu impedance diagram for the power system shown in Fig. P-4.2. Neglect resistance, and use a base of 100 MVA, 220 kV in 50 Ω line. The ratings of the generator motor and transformers are LOD M

Generator	40 MVA,	25 kV,	$X'' = 20\%$
Motor	50 MVA,	11 kV,	$X'' = 30\%$
Y–Y transformer,	40 MVA,	33 Y–220 Y V	$X = 15\%$
Y–Δ transformer,	30 MVA,	11 Δ–220 Y kV,	$X = 15\%$

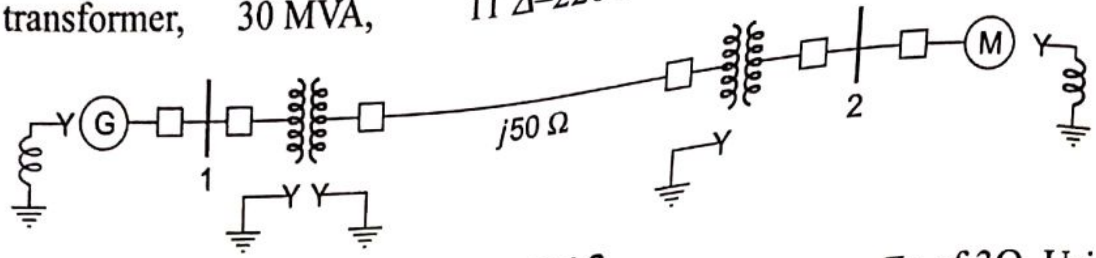


Fig. P-4.2

4.3 A single phase, 20kVA, 220-V generator has an internal impedance Z_g of 3Ω. Using the ratings of the generator as base values, determine the generated per-unit voltage that is required to produce full-load current under short-circuit condition. LOD E

4.4 A 7.5kVA, 400/200V transformer is approximately represented by a 2.5Ω reactance referred to the low-voltage side. Considering the rated values as base quantities, express the transformer reactance as a per-unit quantity. Also express the transformer reactance as a per-unit quantity in-term of high voltage side. LOD E

4.5 A portion of a power system consists of two generators in parallel, connected to a step-up transformer that links them with a 230-kVA transmission line. The ratings of these components are; LOD M

Generator, G1: 10MVA, 12% reactance

Generator G2: 5MVA, 8% reactance

Transformer: 12.5 MVA, 6% reactance

Transmission line: (4+j66)Ω, 230kV

where the % reactances are computed based on the individual component ratings. Express the reactance and the impedance in percentage with 25MVA as the base value.

4.6 Draw an impedance diagram for the system shown in Fig.Q4.6, expressing all values as per-unit values. LOD M

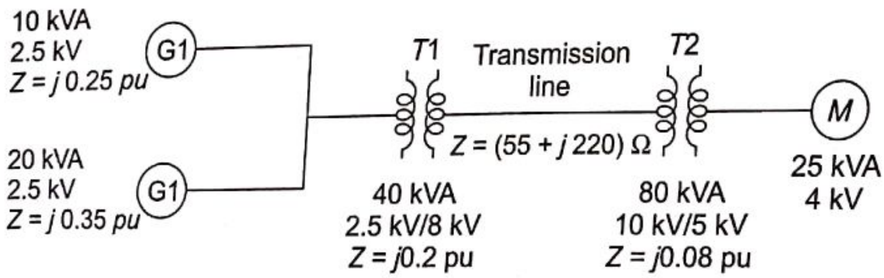
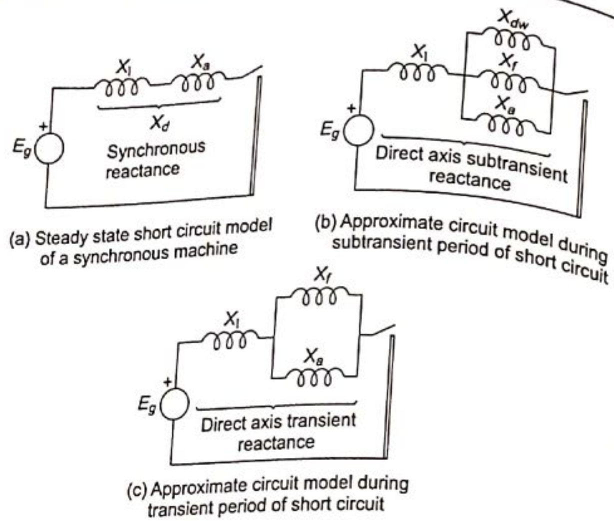


Fig. P-4.6 Single line diagram of power system

10.3 Short Circuit of a Synchronous Machine (On No Load)

Under steady state short circuit conditions, the armature reaction of a synchronous generator produces a demagnetising flux. In terms of a circuit, this effect is modelled as a reactance X_a in series with the induced emf. This reactance when combined with the leakage reactance X_l of the machine is called *synchronous reactance* X_d (direct axis synchronous reactance in the case of salient pole machines). Armature resistance being small can be neglected. The steady state short circuit model of a synchronous machine is shown in Fig. 10.3(a) on per phase basis.

Consider now the sudden short circuit (three-phase) of a synchronous generator initially operating under open circuit conditions. The machine undergoes a transient in all the three phases finally ending up in steady state conditions described above. The circuit breaker must, of course, interrupt the current much before steady conditions are reached. Immediately upon short circuit, the DC off-set currents appear in all the three phases, each with a different magnitude since the point on the voltage wave at which short circuit occurs is different for each phase. These DC off-set currents are accounted for separately on an empirical basis and, therefore, for short circuit studies, we need to concentrate our attention on *symmetrical (sinusoidal) short circuit current only*. Immediately in the event of a short circuit, the symmetrical short circuit current is limited only by the leakage reactance of the machine.



(a) Steady state short circuit model of a synchronous machine

(b) Approximate circuit model during subtransient period of short circuit

(c) Approximate circuit model during transient period of short circuit

Fig. 10.3

Since the air gap flux cannot change instantaneously (*theorem of constant flux linkages*), to counter the demagnetisation of the armature short circuit current, currents appear in the field winding as well as in the damper winding in a direction to help the main flux. These currents decay in accordance with the winding time constants. The time constant of the damper winding which has low leakage inductance is much less than that of the field winding, which has high leakage inductance. Thus, during the initial part of the short circuit, the damper and field windings have transformer currents induced in them so that in the circuit model their reactances—X_f of field winding and X_{dw} of damper winding—appear in parallel* with X_a as shown in Fig. 10.3(b). As the damper winding currents are first to die out, X_{dw} effectively becomes open-circuited and at a later stage X_f becomes open-circuited. The machine reactance, thus, changes from the parallel combination of X_a, X_f and X_{dw} during the initial period of the short circuit to X_a and X_f in parallel (Fig. 10.3(c)) in the middle period of the short circuit, and finally to X_a in steady state (Fig. 10.3(a)). The reactance presented by the machine in the initial period of the short circuit, i.e.

$$X_l + \frac{1}{(1/X_a + 1/X_f + 1/X_{dw})} = X_d'' \tag{10.5}$$

is called the *subtransient reactance* of the machine, while the reactance effective after the damper winding currents have died out, i.e.

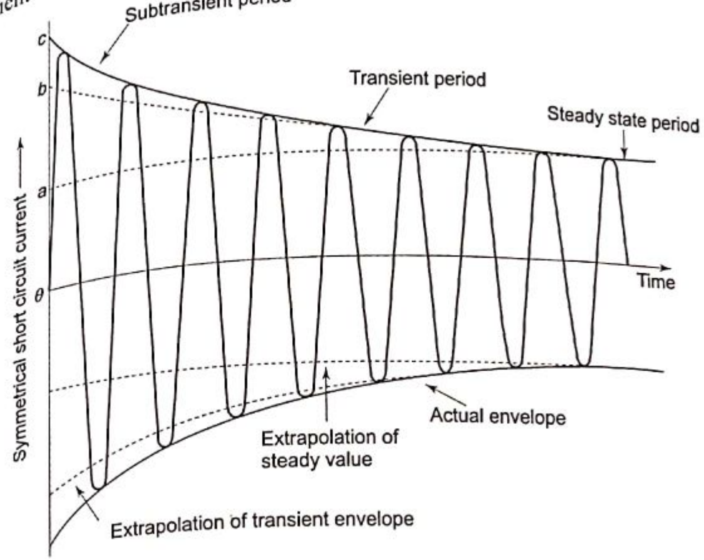
$$X_d' = X_l + (X_a \parallel X_f) \tag{10.6}$$

is called the *transient reactance* of the machine. Of course, the reactance under steady conditions is the *synchronous reactance* of the machine. Obviously, $X_d'' < X_d' < X_d$. The machine, thus, offers a time-varying reactance which changes from X_d'' to X_d' and finally to X_d.

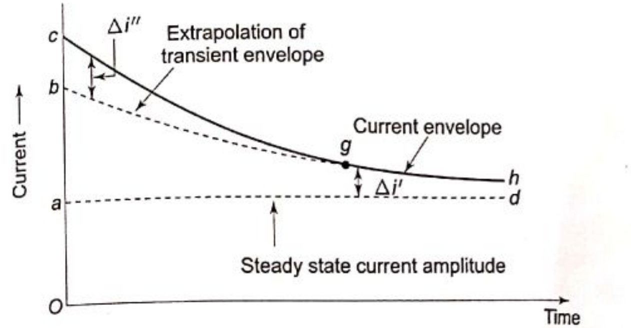
* Unity turn ratio is assumed here.

If we examine the oscillogram of the short circuit current of a synchronous machine after the DC off-set currents have been removed from it, we will find the current wave shape as given in Fig. 10.4(a). The envelope of the current wave shape is plotted in Fig. 10.4(b). The short circuit current can be divided into three periods—initial subtransient period when the current is large as the machine offers subtransient reactance, the middle transient period where the machine offers transient reactance, and finally the steady state period when the machine offers synchronous reactance.

If the transient envelope is extrapolated backwards in time, the difference between the transient and subtransient envelopes is the current, $\Delta i''$ (corresponding to the damper winding current) which decays fast according to the damper winding time constant. Similarly, the difference $\Delta i'$ between the steady state and transient envelopes decays in accordance with the field time constant.



(a) Symmetrical short circuit armature current in synchronous machine



(b) Envelope of synchronous machine symmetrical short circuit current

Fig. 10.4

In terms of the oscillogram, the currents and reactances discussed above, we can write

$$|I| = \frac{Oa}{\sqrt{2}} = \frac{|E_g|}{X_d} \tag{10.7a}$$

$$|I'| = \frac{Ob}{\sqrt{2}} = \frac{|E_g|}{X'_d} \tag{10.7b}$$

$$|I''| = \frac{Oc}{\sqrt{2}} = \frac{|E_g|}{X''_d} \tag{10.7c}$$

where

$|I|$ = steady state current (rms)
$|I'|$ = transient current (rms) excluding DC component
$|I''|$ = subtransient current (rms) excluding DC component
X_d = direct axis synchronous reactance
X'_d = direct axis transient reactance
X''_d = direct axis subtransient reactance
$|E_g|$ = per phase no load voltage (rms)
Oa, Ob, Oc = intercepts shown in Figs 10.4(a) and (b).

The intercept Ob for finding transient reactance can be determined accurately by means of a logarithmic plot. Both $\Delta i''$ and $\Delta i'$ decay exponentially as

$$\Delta i'' = \Delta i''_0 \exp(-t/\tau_{dw})$$

$$\Delta i' = \Delta i'_0 \exp(-t/\tau_f)$$

where τ_{dw} and τ_f are respectively damper, and field winding time constants with $\tau_{dw} \ll \tau_f$. At time $t \gg \tau_{dw}$, $\Delta i''$ practically dies out and we can write

$$\log(\Delta i'' + \Delta i')|_{t \gg \tau_{dw}} \gg \log \Delta i'_0 - t/\tau_f = -\Delta i'_0(-t/\tau_f)$$

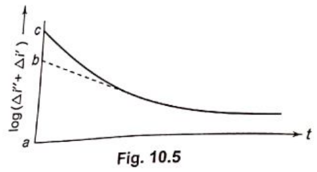


Fig. 10.5

The plot of $\log(\Delta i'' + \Delta i')$ versus time for $t \gg \tau_{dw}$ therefore, becomes a straight line with a slope of $(-1/\tau_f)$ as shown in Fig. 10.5. As the straight line portion of the plot is extrapolated (straight line extrapolation is much more accurate than the exponential extrapolation of Fig. 10.4), taking inverse log of intercept corresponding to $t = 0$ is

$$\log^{-1}(ab) = \Delta i'|_{t=0} = \Delta i_0' \exp(-t/\tau_f)|_{t=0} = \Delta i'_0 = ab$$

Though the machine reactances are dependent upon magnetic saturation (corresponding to excitation), the values of reactances normally lie within certain predictable limits for different types of machines. Table 10.1 gives typical values of machine reactances which can be used in fault calculations and in stability studies.

Table 10.1 Typical values of synchronous machine reactances (All values expressed in pu of rated MVA)

Type of machine	Turbo-alternator (Turbine generator)	Salient pole (Hydroelectric)	Synchronous compensator (Condenser/capacitor)	Synchronous motors*
X_s (or X_d)	1.00–2.0	0.6–1.5	1.5–2.5	0.8–1.10
X_q	0.9–1.5	0.4–1.0	0.95–1.5	0.65–0.8
X'_d	0.12–0.35	0.2–0.5	0.3–0.6	0.3–0.35
X''_d	0.1–0.25	0.13–0.35	0.18–0.38	0.18–0.2
X_2	$= X''_d$	$= X''_d$	0.17–0.37	0.19–0.35
X_0	0.04–0.14	0.02–0.2	0.025–0.16	0.05–0.07
r_a	0.003–0.008	0.003–0.015	0.004–0.01	0.003–0.012

r_a = AC resistance of the armature winding per phase
*High-speed units tend to have low reactance and low-speed units high reactance.

Normally both generator and motor subtransient reactances are used to determine the momentary current flowing on occurrence of a short circuit. To decide the interrupting capacity of circuit breakers, except those which open instantaneously, subtransient reactance is used for generators and transient reactance for synchronous motors. As we shall see later, the transient reactances are used for stability studies.

The machine model to be employed when the short circuit takes place from loaded conditions will be explained in Sec. 10.4.

The method of computing short circuit current is illustrated through examples given below.

Example 10.1 For the radial network shown in Fig. 10.6, a three-phase fault occurs at F. Determine the fault current and the line voltage at 11 kV bus under fault conditions. LOD M

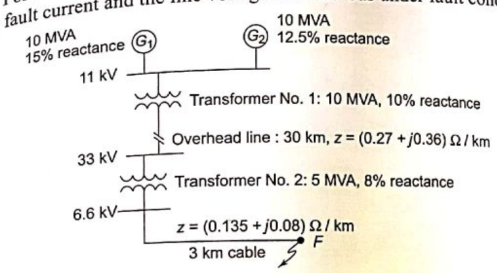


Fig. 10.6 Radial network for Example 10.1

Note: Difficulty Level E — Easy; M — Medium; D — Difficult

Solution

Select a system base of 100 MVA.
Voltage bases are: 11 kV in generators, 33 kV for overhead line and 6.6 kV for cable.

$$\text{Reactance of } G_1 = j\frac{0.15\times 100}{10} = j1.5 \text{ pu}$$

$$\text{Reactance of } G_2 = j\frac{0.125\times 100}{10} = j1.25 \text{ pu}$$

$$\text{Reactance of } T_1 = j\frac{0.1\times 100}{10} = j1.0 \text{ pu}$$

$$\text{Reactance of } T_2 = j\frac{0.08\times 100}{5} = j1.6 \text{ pu}$$

$$\text{Overhead line impedance} = \frac{Z\text{ (in ohms)}\times \text{MVA}_{Base}}{(\text{kV}_{Base})^2}$$

$$= \frac{30\times(0.27+j0.36)\times 100}{(33)^2}$$

$$= (0.744 + j0.99) \text{ pu}$$

$$\text{Cable impedance} = \frac{3(0.135+j0.08)\times 100}{(6.6)^2}$$

$$= (0.93 + j0.55) \text{ pu}$$

Circuit model of the system for fault calculations is shown in Fig. 10.7. Since the system is on no load prior to occurrence of the fault, the voltages of the two generators are identical (in phase and magnitude) and are equal to 1 pu. The generator circuit can, thus, be replaced by a single voltage source in series with the parallel combination of generator reactances as shown.

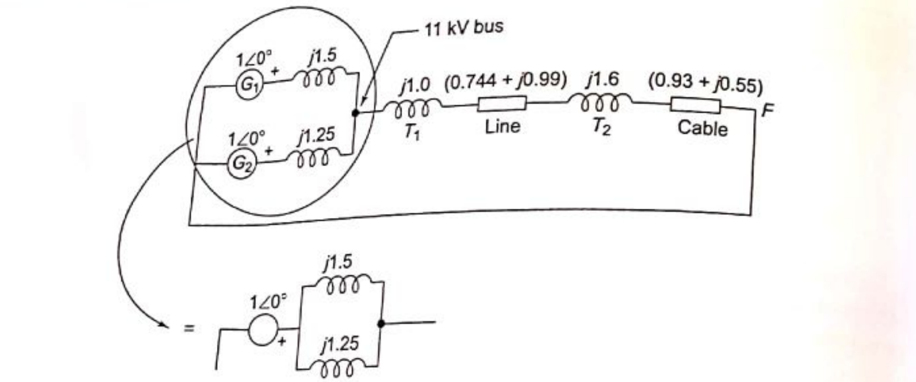


Fig. 10.7

$$\text{Total impedance} = (j1.5 \,||\, j1.25) + (j1.0) + (0.744 + j0.99) + (j1.6) + (0.93 + j0.55)$$

$$= 1.674 + j4.82 = 5.1 \angle 70.8^\circ \text{ pu}$$

$$I_{SC} = \frac{1\angle 0}{5.1\angle 70.8^\circ} = 0.196 \angle -70.8^\circ \text{ pu}$$

$$I_{Base} = \frac{100\times 10^3}{\sqrt{3}\times 6.6} = 8750 \text{ A}$$

$$I_{SC} = 0.196\times 8750 = 1715 \text{ A}$$

Total impedance between F and 11 kV bus

$$= (0.93 + j055) + (j1.6) + (0.744 + j0.99) + (j1.0)$$

$$= 1.674 + j4.14 = 4.43 \angle 76.8^\circ \text{ pu}$$

$$\text{Voltage at 11 kV bus} = 4.43 \angle 67.8^\circ \times 0.196 \angle -70.8^\circ$$

$$= 0.88 \angle -3^\circ \text{ pu} = 0.88\times 11 = 9.68 \text{ kV}$$

Example 10.2 A 25 MVA, 11 kV generator with $X_d'' = 20\%$ is connected through a transformer, line and a transformer to a bus that supplies three identical motors as shown in Fig. 10.8. Each motor has $X_d'' = 25\%$ and $X_d' = 30\%$ on a base of 5 MVA, 6.6 kV. The three-phase rating of the step-up transformer is 25 MVA, 11/66 kV with a leakage reactance of 10 per cent and that of the step-down transformer is 25 MVA, 66/6.6 kV with a leakage reactance of 10 per cent. The bus voltage at the motors is 6.6 kV when a three-phase fault occurs at the point F. For the specified fault, calculate **LOD M**

(a) the subtransient current in the fault
(b) the subtransient current in the breaker B
(c) the momentary current in breaker B
(d) the current to be interrupted by breaker B in five cycles

Given: Reactance of the transmission line = 15% on a base of 25 MVA, 66 kV.
Assume that the system is operating on no load when the fault occurs.

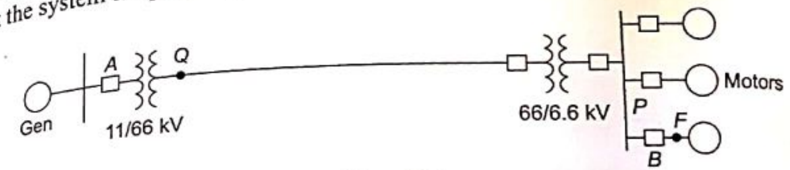


Fig. 10.8

Solution

Choose a system base of 25 MVA.
For a generator voltage base of 11 kV, line voltage base is 66 kV and motor voltage base is 6.6 kV.
(a) For each motor

$$X''_{dm} = j0.25 \times \frac{25}{5} = j1.25 \text{ pu}$$

Line, transformers and generator reactances are already given on proper base values.

The circuit model of the system for fault calculations is given in Fig. 10.9(a). The system being initially on no load, the generator and motor induced emfs are identical. The circuit can, therefore, be reduced to that of Fig. 10.9(b) and then to Fig. 10.9(c). Now

$$I_{SC} = 3 \times \frac{1}{j1.25} + \frac{1}{j0.55} = -j4.22 \text{ pu}$$

$$\text{Base current in 6.6 kV circuit} = \frac{25 \times 1000}{\sqrt{3} \times 6.6} = 2187 \text{ A}$$

$$\therefore \quad I_{SC} = 4.22 \times 2187 = 9229 \text{ A}$$

(b) From Fig. 10.9(c), current through circuit breaker B is

$$I_{SC}(B) = 2 \times \frac{1}{j1.25} + \frac{1}{j0.55} = -j3.42$$

$$= 3.42 \times 2187 = 7479.5 \text{ A}$$

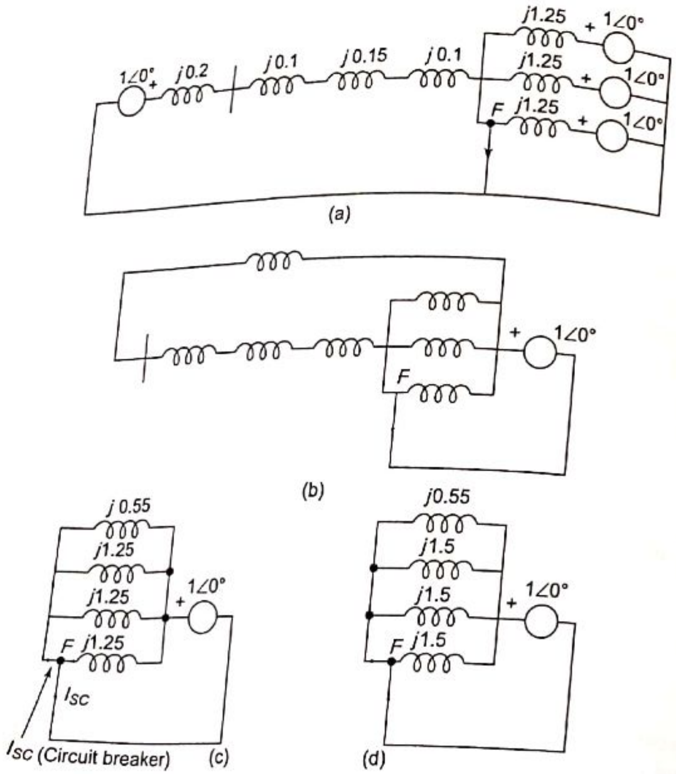


Fig. 10.9

(c) For finding momentary current through the breaker, we must add the DC off-set current to the symmetrical subtransient current obtained in part (b). Rather than calculating the DC off-set current, allowance is made for it on an empirical basis. As explained in Sec. 10.5,

$$\text{Momentary current through breaker } B = 1.6 \times 7479.5$$
$$= 11967 \text{ A}$$

(d) To compute the current to be interrupted by the breaker, motor subtransient reactance ($X_d'' = j0.25$) is now replaced by transient reactance ($X_d' = j0.30$).

$$X_d'(\text{motor}) = j0.3 \times \frac{25}{5} = j1.5 \text{ pu}$$

The reactances of the circuit of Fig. 10.9(c) now modify to that of Fig. 10.9(d). Current (symmetrical) to be interrupted by the breaker (as shown by arrow) is given as

$$= 2 \times \frac{1}{j1.5} + \frac{1}{j0.55} = 3.1515 \text{ pu}$$

Allowance is made for the DC off-set value by multiplying with a factor of 1.1 (Sec. 10.5). Therefore, the current to be interrupted is

$$1.1 \times 3.1515 \times 2187 = 7581 \text{ A}$$

10.4 Short Circuit of a Loaded Synchronous Machine

In the previous article on the short circuit of a synchronous machine, it was assumed that the machine was operating at no load prior to the occurrence of short circuit. The analysis of short circuit on a loaded synchronous machine is complicated and is beyond the scope of this book. We shall, however, present here the methods of computing short circuit current when short circuit occurs under loaded conditions.

Figure 10.10 shows the circuit model of a synchronous generator operating under steady conditions supplying a load current I^0 to the bus at a terminal voltage of V^0, E_g is the induced emf under loaded condition and X_d is the direct axis synchronous reactance of the machine. When short circuit occurs at the terminals of this machine, the circuit model to be used for computing short circuit current is given in Fig. 10.11(a) for subtransient current, and in Fig. 10.11(b) for transient current. The induced emfs to be used in these models are given by

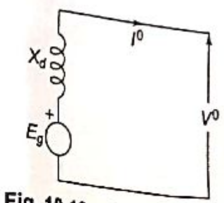


Fig. 10.10 Circuit model of a loaded machine

$$E_g'' = V^0 + jI^0X_d'' \tag{10.8}$$

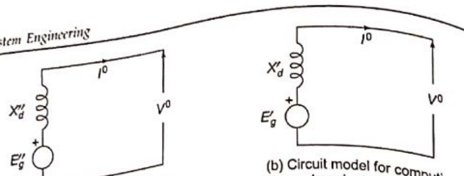
$$E_g' = V^0 + jI^0X_d' \tag{10.9}$$

The voltage E_g'' is known as the *voltage behind the subtransient reactance* and the voltage E_g' is known as the *voltage behind the transient reactance*. In fact, if I^0 is zero (no load case), $E_g'' = E_g' = E_g$, the no load voltage, in which case the circuit model reduces to that discussed in Sec. 10.3.

Synchronous motors have internal emfs and reactances similar to that of a generator except that the current direction is reversed. During short circuit conditions, these can be replaced by similar circuit models except that the voltage behind subtransient/transient reactance is given by

$$E_m'' = V^0 - jI^0X_d'' \tag{10.10}$$

$$E_m' = V^0 - jI^0X_d' \tag{10.11}$$



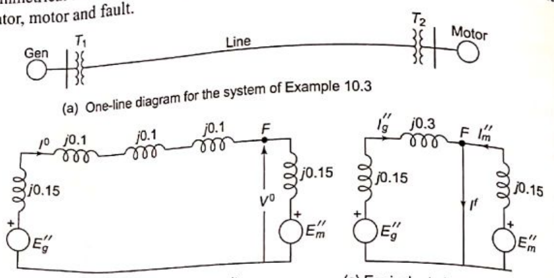
(a) Circuit model for computing subtransient current

(b) Circuit model for computing transient current

Fig. 10.11

Whenever we are dealing with short circuit of an interconnected system, the synchronous machines (generators and motors) are replaced by their corresponding circuit models having voltage behind subtransient (transient) reactance in series with subtransient (transient) reactance. The rest of the network being passive remains unchanged.

Example 10.3 A synchronous generator and a synchronous motor each rated 25 MVA, 11 kV having 15 per cent subtransient reactance are connected through transformers and a line as shown in Fig. 10.12(a). The transformers are rated 25 MVA, 11/66 kV and 66/11 kV with leakage reactance of 10 per cent each. The line has a reactance of 10 per cent on a base of 25 MVA, 66 kV. The motor is drawing 15 MW at 0.8 power factor leading and a terminal voltage of 10.6 kV when a symmetrical three-phase fault occurs at the motor terminals. Find the subtransient current in the generator, motor and fault. LOD M



(a) One-line diagram for the system of Example 10.3

(b) Prefault equivalent circuit

(c) Equivalent circuit during fault

Fig. 10.12

Solution

All reactances are given on a base of 25 MVA and appropriate voltages.

$$\text{Prefault voltage } V^0 = \frac{10.6}{11} = 0.9636 \angle 0^\circ \text{ pu}$$

$$\text{Load} = 15 \text{ MW, 0.8 pf leading}$$

$$= \frac{15}{25} = 0.6 \text{ pu, 0.8 pf leading}$$

$$\text{Prefault current } I^0 = \frac{0.6}{0.9636 \times 0.8} \angle 36.9^\circ$$

$$= 0.7783 \angle 36.9^\circ \text{ pu}$$

Voltage behind subtransient reactance (generator)

$$E_g'' = 0.9636 \angle 0^\circ + j0.45 \times 0.7783 \angle 36.9^\circ$$

$$= 0.7536 + j0.28 \text{ pu}$$

Voltage behind subtransient reactance (motor)

$$E_m'' = 0.9636 \angle 0^\circ - j0.15 \times 0.7783 \angle 36.9^\circ$$

$$= 1.0336 - j0.0933 \text{ pu}$$

The prefault equivalent circuit is shown in Fig. 10.12(b). Under faulted condition (Fig. 10.12(c))

$$I_g'' = \frac{0.7536 + j0.2800}{j0.45} = 0.6226 - j1.6746 \text{ pu}$$

$$I_m'' = \frac{1.0336 - j0.0933}{j0.15} = -0.6226 - j6.8906 \text{ pu}$$

Current in fault

$$I^f = I_g'' + I_m'' = -j8.5653 \text{ pu}$$

$$\text{Base current (gen/motor)} = \frac{25 \times 10^3}{\sqrt{3} \times 11} = 1{,}312.2 \text{ A}$$

Now

$$I_g'' = 1{,}312.2\,(0.6226 - j1.6746) = (816.4 - j2{,}197.4) \text{ A}$$

$$I_m'' = 1{,}312.2\,(-0.6226 - j6.8906) = (-816.2 - j9{,}041.8) \text{ A}$$

$$I^f = -j11{,}239 \text{ A}$$

Short Circuit (SC) Current Computation through the Thevenin Theorem

An alternate method of computing short circuit currents is through the application of the Thevenin theorem. This method is faster and easily adopted to systematic computation for large networks. While the method is perfectly general, it is illustrated here through a simple example.

Consider a synchronous generator feeding a synchronous motor over a line. Figure 10.13(a) shows the circuit model of the system under conditions of steady load. Fault computations are to be made for a fault at F, at the motor terminals. As a first step the circuit model is replaced by the one shown in Fig. 10.13(b), wherein the synchronous machines are represented by their transient reactances (or subtransient reactances if subtransient currents are of interest) in series with voltages behind transient reactances. This change does not disturb the prefault current I^0 and prefault voltage V^0 (at F).

As seen from FG, the Thevenin equivalent circuit of Fig. 10.13(b) is drawn in Fig. 10.13(c). It comprises prefault voltage V^0 in series with the passive Thevenin impedance network. It is noticed that the prefault current I^0 does not appear in the passive Thevenin impedance network. It is, therefore, to be remembered that this current must be accounted for by superposition after the SC solution is obtained through use of the Thevenin equivalent.