MC-MBRT with mixed integer programming to choose collimator amongst varying ctc distances

**Summary of MC-pMBRT (literature review):**

The work in [1] proposes MC-pMBRT modeling approach which utilizes a set of generic and premade collimators for general-purpose pMBRT treatment planning where general-purpose collimators of varying Dctc are used for each field to achieve a desirable OAR-specific *dPVDR*, while optimizing target dose uniformity using multiple fields. Here, Dctc is the center-to-center distance for multi-slit collimator (MSC), and *dPVDR* is the depth at which peak-to-valley dose ratio is calculated. They study two models: (a) Dose Only (DO) model that optimizes the dose, and (b) Joint Dose-PVDR Optimization (JDPO) model that optimizes the dose as well as the PVDR at specific BEV dose planes.

**Our goal:**

The DO and JDPO models in [1] use carefully but randomly chosen combination of multi-slit collimators with different ctc’s for each field. Note that there are an exponential number of possible combinations of MSCs. For example, if 3 collimators with ctc’s equal to 3mm, 4mm, 5mm are available at the following fields (45­ º, 135­ º, 225­ º, 315­º), then there are 81 possible combinations for various collimators.

In our work, we use a mixed integer modeling approach, which when combined with MC-pMBRT model provides a method to choose the best possible combination without enumerating over all possible combinations. This significantly reduces the computational time. Enumerating over different combinations will lead to exponential increase in the computational time, while the mixed integer model requires a small amount of additional computational time to find the nearly optimal combination.

**Work done so far:**

1. We define a mixed integer (MI) model to choose the optimal combination of MSCs for the DO and JDPO methods from [1]. The output of this model is the best possible choice of the combination of collimators in the fields.
2. We use the ADMM method to solve the (MI) model. Once the combination of MSCs is chosen, we define the DO and JDPO model which use the MSCs that are chosen to be optimal by the MI model. Finally, we use ADMM method (from [1]) to solve the models.
3. To show that the combination chosen by the MI model is indeed optimal, we define the DO and JDPO models using other possible combinations of MSCs and compare the performance of DO and JDPO models that use optimal combination with models that use other possible combinations.
4. We use the following output metrics to compare the performance: (a) Conformal Index (CI), (b) max dose received by target as a fraction of the prescription dose (Dmax), (c) percentage of mean doses received by body and OARs (Dmean).
5. We compare the performance for three test cases: (a) head and neck, (b) lung, (c) abdomen.
6. We observe that, for each test case, the optimal combination chosen by the MI model outperforms other combinations in terms of most of the performance metrics. This leads to the conclusion that while multi-collimator approach of [1] is superior to the conventional method as shown in [1], we still need to have an efficient method to choose the combination of collimators to get the best possible performance. The experimental setup and parameters, as well as the detailed comparison of results is provided in Section 2.

**Next steps:**

1. Currently, we are in the process of implementing the ADMM method for the MIP-JDPO model. We should have the results for the three test cases in 2-3 days.
2. We will also calculate other statistics such as PVDR to compare the performance of the various models (since one of the primary goals of JDPO model is to maximize PVDR).

# Introduction and methodology

The DO model is the classical model given as follows:

Here, *x* represents the proton spot weights to be optimized, *A* the dose influence matrix, *d* the 3D dose distribution, *G* the minimum-monitor-unit (MMU) threshold, and is the dose objective function. The dose objective consists of target/OAR-specific planning objectives from dose-volume-histogram (DVH) based planning constraints [2,3]; the MMU constraint is enforced for plan deliverability, i.e., the weights of deliverable spots must be at least *G* [4]. The JDPO model proposed by [1] is

where denotes the number of beam angles (fields), denotes the dose in the BEV dose plan at angle , and is the interpolation matrix that extracts the values dose values in the plane. The function is the regularization term used to optimize PVDR in the each of planes. We use the definition of the function as proposed by [1], which is

The matrix is the total variation (TV) linear operator that is defined to calculate the difference in the dose received by adjacent voxels in the 2D dose plane (see [1] for the exact definition of . The function acts as a regularizer for peak-valley dose modulation.

## 1.1 Mixed Integer-Dose Only (MI-DO) Model

In this work, we propose the mixed integer programming approach that seeks to find the best possible choices of collimators for each field for both DO and JDPO models. In this approach, we modify the existing DO and JDPO models, by introducing additional binary variables and by replacing the influence matrix in the problem definition as given in the following proposed formulation:

The formulation (MI-DO) is a mixed integer model adapted from the DO model. Here, is the number of available collimators (of different ctc’s) for each field, and is the number of fields. The optimal solution of the model provides the optimal choice of collimators for each field such that if , then collimator is the optimal choice for field . Note, however, that due to large size of the problem and the computational complexity of the MI nonconvex programming problem, it is not ideal to solve the problem above to optimality. Thus, we propose the following methodology to obtain a nearly optimal combination of collimators using the (MI-DO) model.

**Methodology:**

1. Solve the relaxed problem using ADMM method:
2. Project the solution of the relaxed problem onto the binary constraint such that the constraints in the (MI) model are satisfied. The resulting solution provides a set of collimators that are used in each field.
3. Solve the (DO) model with the corresponding collimators using the ADMM method.

## Mixed Integer -JDPO (MI-JDPO) model

Similar to the (MI-DO) model , we next propose (MI-JDPO) model which is obtained by modifying the JDPO model. The modifications include addition of binary variable and change to the definition of the influence matrix in the problem formulation. Our proposed model formulation is given below.

To find the near optimal collimator combination using (MI-JDPO), we use the methodology similar to the one proposed for the (MI-DO) model. Thus, we (a) relax the binary constraint in the (MI-JDPO) problem, (b) solve the relaxed problem using ADMM method, (c) project the solution of the relaxed problem onto the binary constraint, (d) solve the (JDPO) model with collimators chosen according to the projected binary variable. In the next section, we provide the experimental setup and the comparison of the performance of the four models (DO, JDPO, MI-DO, and MI-JDPO) for three different test cases.

# Numerical results

In this section, we provide the numerical results for the following cases: (a) head and neck case, where the fields available are (45­ º, 135­ º, 225­ º, 315­º), and collimators with ctc 3mm, 4mm, and 5mm are available, (b) lung case , where the fields available are (45­ º, 135­ º, 225­ º, 315­º), and collimators with ctc 3mm, 4mm, and 5mm are available, and (c) abdomen case where (45­ º, 135­ º, 225­ º, 315­º) fields, and collimators with ctc 3mm, 4mm, and 5mm are available. The output of MI-DO and MI-JDPO models provide a nearly optimal combination of collimators to use. We compare the performance of this combination with the performance of DO and JDPO models that use the combinations suggested in [1]. The output metrics used to compare the performance are (1) Conformal Index (CI), (2) Dmax - maximum dose received by target voxel, (3) Dmean – mean dose received by OARs and body.

**Head and neck case.** For the head and neck case, the solution of (MI-DO) as well as (MI-JDPO) model provides the following combination of collimators as optimal: (3mm, 4mm, 4mm, 3mm) ctc respectively for fields at (45­ º, 135­ º, 225­ º, 315­º). In the rest of the paper, we use (3443) as a simpler notation to indicate the (3mm, 4mm, 4mm, 3mm) combination. We compare the performance of (MI-JDPO) and (MI-DO) (i.e., the combination (3443)) with combination (3553) for (DO) and (JDPO) models. The combination (3553) was used in [1] during the experiments. Table 1 provides the comparison. During our experiments, we have compared the performance of (3443) combination with several other possible combinations. However, for brevity, we provide the comparison with a single combination in the paper. It should be noted that none of the possible combinations outperform the combination (3mm, 4mm, 4mm, 3mm) which was chosen by (MI-DO) and (MI-JDPO) models.

**Lung case.** In this case, the solution of (MI-DO) model provides the following combination of collimators as optimal: (3mm, 3mm, 5mm) ctc respectively for fields at (0­ º, 120 º, 240 º). While the (MI-JDPO) model provides the following optimal combination (3mm, 5mm, 5mm) for joint dose-PVDR optimization. We compare the performance of these two models with (357) combination for the (DO) and (JDPO) models as implemented in [1] (see Table 2). In this case, we observe that there are other combinations that have similar performance. However, no other model significantly outperforms the (3mm, 3mm, 5mm) combination (the output of (MI-DO) model) for dose only optimization. Similarly, we observe that (355) combination of (MI-JDPO) model outperforms any other combination for the (JDPO) model.

**Abdomen case.** In this case, the solution of (MI-DO) as well as (MI-JDPO) model provides the following combination of collimators as optimal: (3mm, 3mm, 3mm) ctc respectively for fields at (0­ º, 120 º, 240 º). We compare the performance of this combination with the (337) combination for (DO) and (JDPO) models. The (337) combination was used to perform experiments in [1]. While Table 3 provides this comparison, we have also checked the performance of other combinations. However, none of the other combinations perform as well as (333) or (337) combination. From Table 1, we observe that while the (337) combination has mean dose values for OARs and body similar to the optimal combination (3mm, 3mm, 3mm) during the dose only optimization as well as the dose-PVDR optimization, the optimal combination outperforms (333) combination in terms of the conformal index.

Table : Comparison of output of (MI) model with randomly chosen combinations for head and neck case. PVDR for BEV dose planes for (45º, 135º, 225º, 315º) fields are calculated at the depth (2.5cm, 5cm, 5cm, 2.5cm) respectively.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | MI-DO (3443) | DO (3553) | MI-JDPO (3443) | JDPO (3553) |
| Obj Fn Val | 14.999 | 15.013 | 15.192 | 15.166 |
| Obj Fn Val g | - | - | 2.927 | 3.227 |
|  | - | - | 18.119 | 18.394 |
| CI | 0.613 | 0.578 | 0.703 | 0.615 |
| Dmax | 123.44% | 123.32% | 125.21% | 124.55% |
| Dmean (body) | 0.765% | 0.765% | 0.738% | 0.76% |
| Dmean (larynx) | 5.214% | 5.216% | 5.445% | 5.5% |
| Dmean (mandible) | 7.66% | 7.688% | 6.241% | 6.204% |
| Dmean (oral) | 5.448% | 5.444% | 4.728% | 4.645% |
| Dmean 45 º | 1.33 | 1.342 | 1.139 | 1.143 |
| PVDR 45 º | 12.737 | 12.821 | 11.692 | 11.687 |
| Dmean 135 º | 0.83 | 0.806 | 0.854 | 0.881 |
| PVDR 135 º | 7.948 | 9.034 | 7.686 | 10.66 |
| Dmean 225 º | 0.784 | 0.794 | 0.781 | 0.865 |
| PVDR 225 º | 6.773 | 8.438 | 6.749 | 9.267 |
| Dmean 315 º | 1.248 | 1.254 | 1.081 | 1.084 |
| PVDR 315 º | 12.823 | 13.013 | 13.357 | 13.436 |

Table : Comparison of output of (MI) model with randomly chosen combinations for lung case. PVDR for BEV dose planes for (0º, 120º, 240º) fields are calculated at the depth (3cm, 7cm, 12cm) respectively.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | MI-DO (335) | DO (357) | MI-JDPO (355) | JDPO (357) |
| Obj Fn Val | 6.111 | 6.198 | 6.38 | 6.321 |
| Obj Fn Val g | - | - | 4.251 | 4.369 |
|  | - | - | 10.631 | 10.69 |
| CI | 0.645 | 0.593 | 0.626 | 0.612 |
| Dmax | 134.17% | 135.44% | 136.62% | 135.53% |
| Dmean (body) | 3.932% | 3.905% | 3.832% | 3.902% |
| Dmean (lung) | 7.636% | 7.636% | 7.449% | 7.611% |
| Dmean (heart) | 2.02% | 2.027% | 2.101% | 2.138% |
| Dmean (eso) | 7.553% | 6.982% | 5.821% | 6.475% |
| Dmean 0 º | 5.547 | 5.783 | 5.111 | 5.034 |
| PVDR 0 º | 15.307 | 15.491 | 13.993 | 13.969 |
| Dmean 120 º | 4.059 | 3.883 | 3.995 | 3.908 |
| PVDR 120 º | 4.615 | 5.757 | 7.973 | 7.915 |
| Dmean 240 º | 2.529 | 2.426 | 2.287 | 2.545 |
| PVDR 240 º | 4.043 | 6.121 | 6.138 | 7.949 |

Table : Comparison of output of (MI) model with randomly chosen combinations for abdomen case. PVDR for BEV dose planes for (0º, 120º, 240º) fields are calculated at the depth (4cm, 2cm, 9cm) respectively.

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| --- | --- | --- | --- | --- |
|  | MI-DO (333) | DO (337) | MI-JDPO (333) | JDPO (337) |
| Obj Fn Val | 278.021 | 278.149 | 277.95 | 277.895 |
| Obj Fn Val g | - | - | 3.635 | 3.709 |
|  | - | - | 281.585 | 281.604 |
| CI | 0.833 | 0.802 | 0.837 | 0.782 |
| Dmax | 122.25% | 122.44% | 122.89% | 122.98% |
| Dmean (body) | 4.16% | 4.152% | 4.114% | 4.12% |
| Dmean (large bowel) | 18.045% | 18.054% | 18.305% | 18.281% |
| Dmean (spinal cord) | 5.859% | 5.692% | 5.904% | 6.003% |
| Dmean (left kidney) | 21.76% | 21.802% | 21.808% | 21.796% |
| Dmean 0 º | 20.083 | 20.052 | 18.137 | 18.01 |
| PVDR 0 º | 16.017 | 16.082 | 15.374 | 15.41 |
| Dmean 120 º | 14.806 | 14.822 | 15.154 | 15.075 |
| PVDR 120 º | 10.309 | 10.197 | 9.907 | 9.751 |
| Dmean 240 º | 4.172 | 4.092 | 4.392 | 4.507 |
| PVDR 240 º | 5.072 | 6.621 | 4.979 | 7.328 |

# References

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# Appendix A. Extra material

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| --- | --- | --- | --- | --- |
| HN | MI model (3443) | DO model (3553) | DO model (5335) | DO model (3333) |
| Obj Fn Val | 14.999 | 15.013 | 16.047 | 14.966 |
| CI | 0.613 | 0.578 | 0.504 | 0.503 |
| Dmax | 123.44% | 123.32% | 140.21% | 124.58% |
| Dmean (body) | 0.765% | 0.765% | 0.967% | 0.805% |
| Dmean (larynx) | 5.214% | 5.216% | 7.796% | 6.198% |
| Dmean (mandible) | 7.66% | 7.688% | 6.725% | 8.759% |
| Dmean (oral) | 5.448% | 5.444% | 8.538% | 7.052% |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Lung | MI model (335) | DO model (357) | DO model (337) | DO model (535) |
| Obj Fn Val | 6.111 | 6.198 | 6.153 | 6.298 |
| CI | 0.645 | 0.593 | 0.627 | 0.616 |
| Dmax | 134.17% | 135.44% | 134.92% | 138.69% |
| Dmean (body) | 3.932% | 3.905% | 3.905% | 4.28% |
| Dmean (lung) | 7.636% | 7.636% | 7.595% | 8.12% |
| Dmean (heart) | 2.02% | 2.027% | 2.024% | 2.2% |
| Dmean (eso) | 7.553% | 6.982% | 6.817% | 10.091% |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Abdomen | MI model (333) | DO model (337) | DO model (335) | DO model (357) |
| Obj Fn Val | 278.021 | 278.149 | 278.096 | 283.646 |
| CI | 0.833 | 0.802 | 0.827 | 0.618 |
| Dmax | 122.25% | 122.44% | 122.26% | 134.99% |
| Dmean (body) | 4.16% | 4.152% | 4.155% | 4.261% |
| Dmean (large bowel) | 18.045% | 18.054% | 18.045% | 15.63% |
| Dmean (spinal cord) | 5.859% | 5.692% | 5.757% | 6.586% |
| Dmean (left kidney) | 21.76% | 21.802% | 21.772% | 21.473% |