Homework 1

1.1(a) $P := \{x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mid 2x_1 - 3x_2 + 4x_3 = 0\}$

$l := \{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \mid t \in \mathbb{R}\}$

If l intersects P, then $\exists p \in l$ s.t.

$\begin{bmatrix} 1+2t \\ 1+1t \\ 1-1t \end{bmatrix} \in P$

Check if perpendicular:

Normal of plane $\underline{n} := \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$
Direction of line $\underline{v} := \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$

If $\underline{n} \cdot \underline{v} \neq 0$, then $l \not\perp P$.

$(2)(2) + (-3)(1) + (4)(-1) = 4 - 3 - 4 = -3$
$\rightarrow l \not\perp P$

Check if valid point on P: $\begin{bmatrix} 1+2t \\ 1+t \\ 1-t \end{bmatrix}\begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix} = 0$

$2(1+2t) - 3(1+t) + 4(1-t) = 2 + 4t - 3 - 3t + 4 - 4t$

$= -3t = -3 \leftrightarrow 3t = 3 \leftrightarrow t = 1$
P.O.I. $= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

(b) $\underline{n} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$

(c) No. since $\underline{n} \perp P$ and $l \not\perp \underline{n}$.

(d) $\underline{n} = \begin{bmatrix} 2 \\ -3 \\ 4 \end{bmatrix}$. Let $\underline{u} := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ for some point $\in P$

$2(2) - 3(0) + 4(1) = 0$ ✓ $\quad 2(0) - 3(4) + 4(3) = 0$ ✓

$\underline{u} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

Pick $\underline{v}$ s.t. $\underline{u} \not\perp \underline{v}$. Let $\underline{v} = \begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix}$.