Goal: Understand properties of tail inequalities for randomized algorithms

1) Coupon collector. Recall that we found that $\mathbb{E}[T(n)] = nH_n \leq n(\log n + 1)$. Use Markov's inequality to find how many coupons are required to ensure 90% probability of collecting all.

Markov: $\Pr[Z \geq z] \leq \mathbb{E}[Z]/z$ or $\Pr[X \geq (1+\delta)\mathbb{E}[X]] \leq \dfrac{1}{1+\delta}$.

Solution:

Let $Z = T(n)$ be our random variable of interest. We want to find the $z$ that guarantees $Z < z$ with probability of failure less than 10%. In other words,

$$\Pr[Z \geq z] \leq 10\%$$

Using Markov,

$$\Pr[Z \geq z] \leq \frac{\mathbb{E}[Z]}{z} \leq n(\log n + 1)/z \leq 10\%$$

This implies that $z \geq 10n(\log n + 1)$ is a sufficient number of coupons.
For example, if I plug in $n = 50$ this gives 2,456 coupons.