A Level Mathematics
C1

JOSEPH CHAMBERLAIN
SIXTH FORM COLLEGE

Contents

1 Algebra

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Algebra Assessed Homework

1. Solve each equation
   (a) $2x-3=3(7-2x)$ **[2]**
   (b) $\frac{7x-1}{5}=\frac{3(5+2x)}{2}$ **[3]**
2. Solve the simultaneous equations: **[4]**

$$x-7y=-11$$
$$3x+4y=-8$$

3. Solve the inequalities:
   (a) $4(x-5)>2x-9$ **[2]**
   (b) $\frac{3x-1}{5}-\frac{x+1}{2}>3$ **[3]**
4. (a) For the function $f(x)=3x^2-x-2$, find the values of: **[4]**
   i. f(0) ii. f(1) iii. f(-1) iv. f(-3)
   (b) Find and simplify an expression for $f(z+1)$ **[3]**
5. Work out the answer, giving your solution in simplest form: **[6]**
   (a) $\frac{3}{8}+\frac{1}{6}$ (c) $\frac{3}{8}\times\frac{1}{2}$ (e) $\frac{3}{8}\times 2$
   (b) $\frac{3}{8}-\frac{1}{2}$ (d) $\frac{3}{8}\div\frac{1}{2}$ (f) $\frac{3}{8}\div 2$
6. Expand and simplify:
   (a) $(x-3)(x+4)$ **[1]**
   (b) $(2-x)(x+8)$ **[1]**
   (c) $(x-6)^2$ **[1]**
7. Solve by factorising:
   (a) $x^2+6x+5=0$ **[2]**
   (b) $2x^2+5x-3=0$ **[2]**
   (c) $6x^2-x=1$ **[3]**
8. Simplify as much as possible the fraction **[4]**

$$\frac{2x^2-8}{4x^3-4x^2-24x}$$

Total marks: [41]

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Algebra Assessment

Total marks: [0]

2 Surds

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Surds Assessed Homework

1. Express each of the following in the form $a\sqrt{b}$ where $\sqrt{b}$ can be simplified no further.

   (a) $\sqrt{45}$ **[1]**

   (b) $\sqrt{8}$ **[1]**

   (c) $\sqrt{128}$ **[1]**

   (d) $\sqrt{256}$ **[1]**

2. Express each of the following in the form $a+b\sqrt{2}$ where a and b are rational:

   (a) $\left(3+\sqrt{2}\right)^2$ **[2]**

   (b) $\left(3+5\sqrt{2}\right)\left(4-3\sqrt{2}\right)$ **[2]**

   (c) $\frac{10+\sqrt{8}}{4}$ **[3]**

3. Show that **[2]**

$$\frac{2\left(1-2\sqrt{2}\right)-2}{\sqrt{8}}$$

   can be written as an integer.

4. The function $f(x)$ is defined by

$$f(x)=x^2-3x+2$$

   Expressing your answers in the form $m+n\sqrt{3}$, where m and n are integers, find the value of $f(x)$ when:

   (a) $x=\sqrt{3}$ **[2]**

   (b) $x=2\sqrt{3}$ **[2]**

   (c) $x=\sqrt{3}+1$ **[3]**

5. Express the following in the form $a+b\sqrt{2}$ where a and b are integers:

   (a) **[3]**

$$\frac{3-2\sqrt{2}}{3+2\sqrt{2}}$$

   (b) **[3]**

$$\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

6. Solve the inequality **[4]**

$$\sqrt{5}\left(\sqrt{5}-x\right)>5\sqrt{5}-x$$

   giving your solution in the simplest form possible.

Total marks: [30]

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Surds Assessment

Total marks: [0]

3 Geometry

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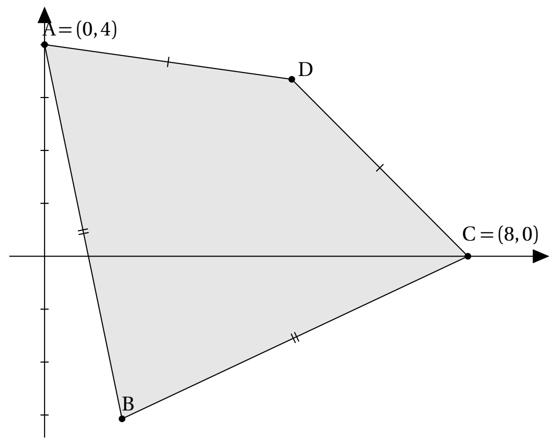
Geometry Assessed Homework

1. The points P and Q have coordinates $(-2,1)$ and $(3,-5)$ respectively. Find the following:
   (a) The midpoint of PQ **[1]**
   (b) The gradient of PQ **[1]**
   (c) The length of PQ **[2]**
2. The line L_1 has equation $y+3x=6$.
   The line L_2 is perpendicular to L_1 and passes through the point $(3,1)$.
   (a) Draw a sketch of L_1 and L_2, indicating the coordinates of the point where L_1 crosses the coordinate axes. **[2]**
   (b) Find an equation for the line L_2 **[3]**
3. Find and equation for the line joining the points $(3,2)$ and $(5,12)$. **[3]**
   Give your answer in the form $y=ax+b$.
4. The line L_1 has equation $3x+y=1$ and the line L_2 has equation $y=7x-2$. **[4]**
   Find the coordinates of the point of intersection of L_1 and L_2.
5. The points A and B have coordinates $(5,3)$ and $(9,9)$ respectively.
   The line AC has equation $2x-3y=1$.
   (a) Find the gradient of AB **[2]**
   (b) Determine whether the lines AB and AC are perpendicular **[3]**
6. The points $A=(2,5), B=(k,1)$ and $C=(-3,t+1)$ lie in the plane. It is given that $t>0$.
   (a) Given that the gradient of AB is equal to 4, find the value of k. **[2]**
   (b) Given that the length of BC is $6\sqrt{2}$, find the value of t, giving your answer in the form $n\sqrt{p_1}\sqrt{p_2}$ where p_1 and p_2 are prime numbers. **[3]**
7. The points P and Q have the coordinates $(3,-5)$ and $(1,1)$ respectively.
   (a) Find the equation of the line PQ in the form $ax+by+c=0$ **[3]**
   (b) Find the equation of the perpendicular bisector of PQ **[4]**

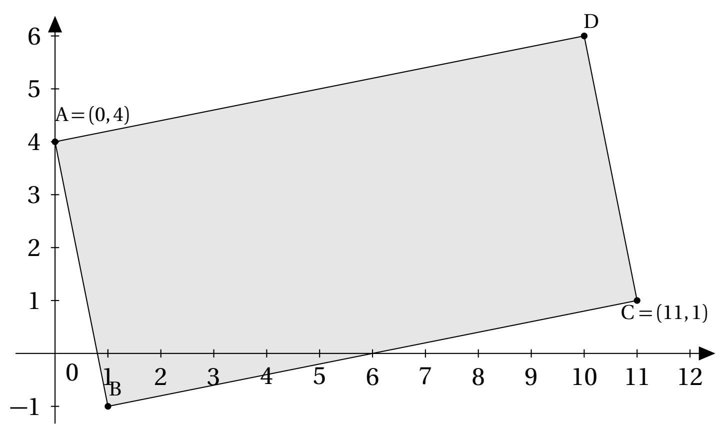
[Questions continued overleaf.]

8. The polygon ABCD is a kite. The point A has coordinates $(0, 4)$ and the point C has coordinates $(8, 0)$. **[4]**



Find the equation of the diagonal BD.

9. The polygon ABCD is a rectangle. The point A has coordinates $(0, 4)$ and the point C has coordinates $(11, 1)$. The line segment AB has gradient -5.



(a) Find the equations of the sides AB and BC **[3]**

(b) Prove that the coordinates of B are $(1, -1)$ **[3]**

(c) Find the area of the rectangle **[4]**

Total marks: [47]

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Total marks: [0]

4 Quadratics 1

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Quadratics1 Assessed Homework

1. (a) Factorise the expression $6x^2 - x - 5$ **[2]**

   (b) Hence draw a sketch of the curve $y = 6x^2 - x - 5$, stating the coordinates of the points where the curve crosses the x and y axes. **[4]**

2. (a) Express $x^2 + 12x + 5$ in the form $(x + a)^2 + b$, stating the values of a and b **[2]**

   (b) Hence state the minimum value of $x^2 + 12x + 5$ and the value x at which it occurs. **[1]**

   (c) Draw a sketch of the graph of $y = x^2 + 12x + 5$ stating the coordinates of the point where it crosses the y axis. Label the coordinates of the vertex of the parabola on your graph. **[3]**

3. State with a reason the number of real solutions to each equation:

   (a) $x^2 - 3x + 1 = 0$ **[2]**

   (b) $-x^2 + 6x - 9 = 0$ **[2]**

   (c) $-3x^2 + 5x - 3 = 0$ **[2]**

4. Determine the values of k for which the equation **[4]**

$$x^2 + 3(k-2)x + (k+5) = 0$$

   has equal roots.

5. (a) Express $2x^2 - 8x + 5$ in the form $p(x+q)^2 + r$, stating the values of p, q and r. **[3]**

   (b) Hence write down the coordinates of the vertex of the curve given by $y = 2x^2 - 8x + 5$. **[1]**

   (c) Describe fully the single transformation from which the curve $y = 2x^2 - 8x + 5$ may be obtained from the curve $y = x^2$. **[3]**

6. Sketch the curve $y = -4x^2 + 4x + 3$ indicating the coordinates of all points where the curve crosses the coordinate axes and the coordinates of the vertex of the curve. **[5]**

7. Using the quadratic formula, find both values of p which satisfy the equation **[4]**

$$2p^2 + 2p - 5 = 0$$

   Show that your solutions can be given in the form

$$p = \frac{-1 \pm \sqrt{k}}{2}$$

   where k is a prime number, stating the value of k.

Total marks: [38]

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Quadratics1 Assessment

Total marks: [0]

5 Quadratics 2

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Quadratics2 Assessed Homework

1. Find the coordinates of the points of intersection between the curve **[4]**

$$y = x^2 - 2$$

and the line

$$y = -2x - 2$$

2. (a) State, with a reason, the number of points of intersection between the curve

$$y = -x^2 - x + 1$$

and the lines:

i. $y = x - 1$ **[2]**

ii. $y = x + 4$ **[2]**

(b) Find the value of k for which the line **[4]**

$$y = x + k$$

is a tangent to the curve with equation

$$y = -x^2 - x + 1$$

3. (a) Solve the equation **[3]**

$$3x^2 + 4x - 5 = 0$$

giving your answers in surd form.

(b) Hence, solve the quadratic inequality **[3]**

$$3x^2 + 4x - 5 > 0$$

4. Find the range of possible values of k for which the equation **[4]**

$$2x^2 + (3 - k)x + (k + 3) = 0$$

has no real solutions.

5. It is given that the curve with equation

$$y = x^2(z - 3) + x + 2$$

intersects with the line

$$y = 3x + (z + 3)$$

in two distinct points (z is an unknown constant).

(a) By considering the discriminant, show that z must satisfy the inequality **[4]**

$$z^2 - 2z - 2 > 0$$

(b) By solving this inequality, find the range of possible values of z. **[4]**

Total marks: [30]

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Quadratics2 Assessment

Total marks: [0]

6 Polynomials

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Polynomials Assessed Homework

1. The polynomial $p(x)$ is given by $p(x) = x^3 + 4x^2 + x - 6$.
   (a) Use the factor theorem to show that $(x-1)$ is a factor of $p(x)$. **[2]**
   (b) By dividing $p(x)$ by $(x-1)$, or otherwise, find the quadratic factor of $p(x)$. **[2]**
   (c) Write $p(x)$ as a product of three linear factors. **[2]**
   (d) Sketch the curve $y = p(x)$, indicating the coordinates of the points where is crosses both the x and the y axis. **[3]**
2. A polynomial $f(x)$ is given by $f(x) = x^3 + ax^2 - 10x - 24$, where a is a constant.
   (a) Given that $(x+2)$ is a factor of $f(x)$, find the value of a. **[3]**
   (b) Factorise $f(x)$ completely. **[3]**
   (c) Solve the equation $f(x) = 0$. **[2]**
3. Find the remainder when the polynomial $q(x) = x^3 - 3x^2 + 5x - 7$ is divided by:
   (a) $(x-1)$ **[1]**
   (b) $(x+2)$ **[1]**
   (c) x **[1]**
   (d) $(x-3)$ **[1]**
4. When the polynomial $r(x) = x^3 + 5x^2 - 3x + k$ is divided by $(x-2)$, the remainder is 5. Find the value of k. **[3]**
5. When $x^3 + ax^2 + bx + 1$ is divided by $(x+1)$, the remainder is 7; when divided by $(x-2)$ the remainder is 19. Find the values of a and b. **[4]**
6. The polynomial $p(x)$ is given by $(x+1)(x^2 - 4x + 5)$
   (a) Find the remainder when $p(x)$ is divided by $(x-2)$. **[2]**
   (b) Express $p(x)$ in the form $x^3 + mx^2 + nx + 5$, stating the values of m and n. **[2]**
   (c) Show that the equation $x^2 - 4x + 5 = 0$ has no real solutions. **[2]**
   (d) Find the coordinates where the curve $y = p(x)$ meets the coordinate axes. **[3]**

Total marks: [37]

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Polynomials Assessment

Total marks: [0]

7 Circles

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Circles Assessed Homework

1. Find the coordinates of the radius and the centre of the circle with equation:

   (a) $(x-3)^2+(y+4)^2=25$ **[2]**

   (b) $x^2+(y-4)^2=16$ **[2]**

   (c) $x^2+y^2-4x+6y+9=0$ **[3]**

2. Write down the equation of the circle with the following features:

   (a) centre $=(0,0)$, radius $=5$ **[2]**

   (b) centre $=(2,3)$, radius $=\sqrt{5}$ **[2]**

   (c) centre $=(-4,3)$, radius $=6$. **[2]**

3. The circle C is given by the equation $x^2+y^2-6x+8y+9=0$.

   (a) Find the coordinates of the centre and the radius of C. **[2]**

   (b) Find the exact coordinates of the points where the C crosses both axes. **[3]**

   (c) Find, in terms of π, the area and circumference of C. **[1]**

4. Describe the geometrical transformation which has been applied to the circle with equation $x^2+y^2=25$ to obtain the circle with equation $x^2+y^2-2x+4y-20=0$. **[3]**

5. A and B are the coordinates of the end points of the diameter AB of a circle, where $A=(7,-3)$ and $B=(1,-11)$. Find the equation of the circle. **[4]**

6. A triangle PQR has vertices $P(-1,5)$, $Q(7,1)$ and $R(-5,-3)$.

   (a) Prove that triangle PQR is right-angled. **[2]**

   (b) Find the equation of the circle which passes through the points P,Q and R. **[4]**

7. The circle C is given by the equation $x^2+y^2-4x+10y+4=0$.

   (a) Find the coordinates of the centre and the radius of the circle. **[2]**

   (b) Hence sketch the circle. **[4]**

   (c) Show that $P(6,-2)$ lies on the circle. **[1]**

   (d) Find the equation of the tangent to the circle at the point P, giving your answer in the form $ax+by+c=0$ where a,b,c are integers. **[3]**

8. The equation of a circle is given by $x^2+y^2-4x+2y+2=0$.

   (a) Find the coordinates of the centre and the radius of the circle. **[2]**

   (b) P is a point outside the circle whose coordinates are $(5,-5)$. Find the exact value of the lengths of the tangents drawn from P to the circle. **[3]**

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Circles Assessment

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8 Differentiation

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Differentiation Assessed Homework

1. (a) Find the equation of the tangent to the curve $y = 2x^2 - 3$ at the point $(2, 5)$. **[3]**

   (b) Find the coordinates of the point where the tangent meets the x-axis. **[2]**

2. The normal to the curve $y = x^2 - 4x$ at the point $(3, -3)$ cuts the x-axis at A and the y-axis at B.

   (a) Find the equation of the normal. **[4]**

   (b) Find the coordinates of A and B. **[2]**

   (c) Find the area of the triangle AOB, where O is the origin. **[2]**

3. The curve $y = ax^3 - 2x^2 - x + 7$ has a gradient of 3 at the point where $x = 2$. Determine the value of a. **[2]**

4. Let $f(x) = 2x^3 + 3x^2 + 1$.

   (a) Find $f'(x)$, factorising your answer. **[2]**

   (b) Hence find the coordinates of the stationary points on the curve and determine their nature. **[5]**

   (c) Find the range of values of x for which $f(x)$ is an increasing function. **[3]**

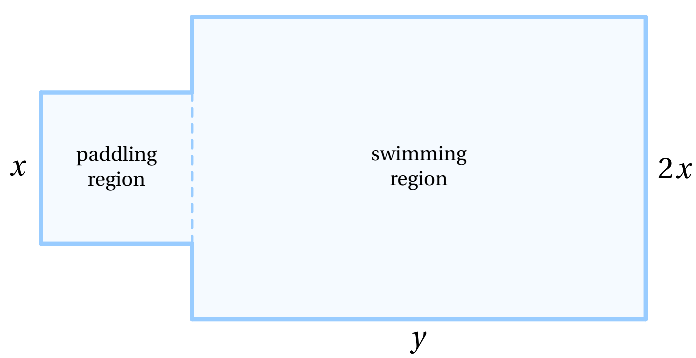
5. A curve has equation $y = x^3 + x^2 - 8x + 1$.

   (a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ **[4]**

   (b) Find the coordinates of the turning points and determine their nature. **[6]**

   (c) Draw a sketch of the curve, labelling *only* the turning points and y-intercept. **[3]**

6. The figure shows the plan of a bathing pool in which the width, x, of the paddling region is half the width of the swimming region. The paddling region is a square.



   (a) Show that the perimeter, P, and the area, A, are given by the formulas **[3]**

$$P = 6x + 2y$$
$$A = x^2 + 2xy$$

(b) You are given that the perimeter is 200m. By substituting an expression for y, show that **[2]**

$$A = 200x - 5x^2$$

(c) Find $\frac{dA}{dx}$ and determine for which value of x the area A has a stationary value. **[3]**

(d) Determine whether the stationary value found in part (c) is a maximum or minimum. **[3]**

(e) Find the maximum possible area of the pool. **[2]**

Total marks: [51]

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Differentiation Assessment

Total marks: [0]

9 Integration

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Integration Assessed Homework

1. Find:
   (a) $\int 12x\,dx$ **[2]**
   (b) $\int \left(x^3+x\right)dx$ **[2]**
   (c) $\int x(x+1)\,dx$ **[3]**
   (d) $\int (x+6)(x-4)\,dx$ **[3]**
   (e) $\int \frac{x^5+3x^2}{x}\,dx$ **[3]**
2. The gradient of a curve for each value of x is given by $6x$. The curve passes through the point $(1,4)$. Use this information to find the equation of the curve. **[4]**
3. You are given that $f'(x)=3x^2-2$ and that $f(-2)=6$. Find $f(x)$. **[4]**
4. Evaluate the following definite integrals:
   (a) $\int_1^5 2x\,dx$ **[2]**
   (b) $\int_0^2 3x^2\,dx$ **[2]**
   (c) $\int_{-1}^4 (6-2x)\,dx$ **[2]**
   (d) $\int_{-2}^{-1} (x^2+2x-1)\,dx$ **[2]**
   (e) $\int_2^3 (1+2x-3x^2)\,dx$ **[2]**
5. (a) For each of the following curves, find the coordinates of the point where the curve crosses the coordinate axes and draw a sketch.
      i. $y=4+3x-x^2$ **[3]**
      ii. $y=x^2-4x-5$ **[3]**
   (b) For each curve, find the area contained between the curve and the x axis. [Hint: for the limits of integration (a and b), use the x intercepts of the curve.] **[6]**
6. The curve $y=-3x^2+6x$ and the line $y=-3x+6$ intersect in two points.
   (a) Find the coordinates of the points of intersection of the line and the curve. **[3]**
   (b) Find the area of the region enclosed by the line and the curve. [Hint: Draw a sketch to help.] **[4]**

Total marks: [50]

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Integration Assessment

Total marks: [0]