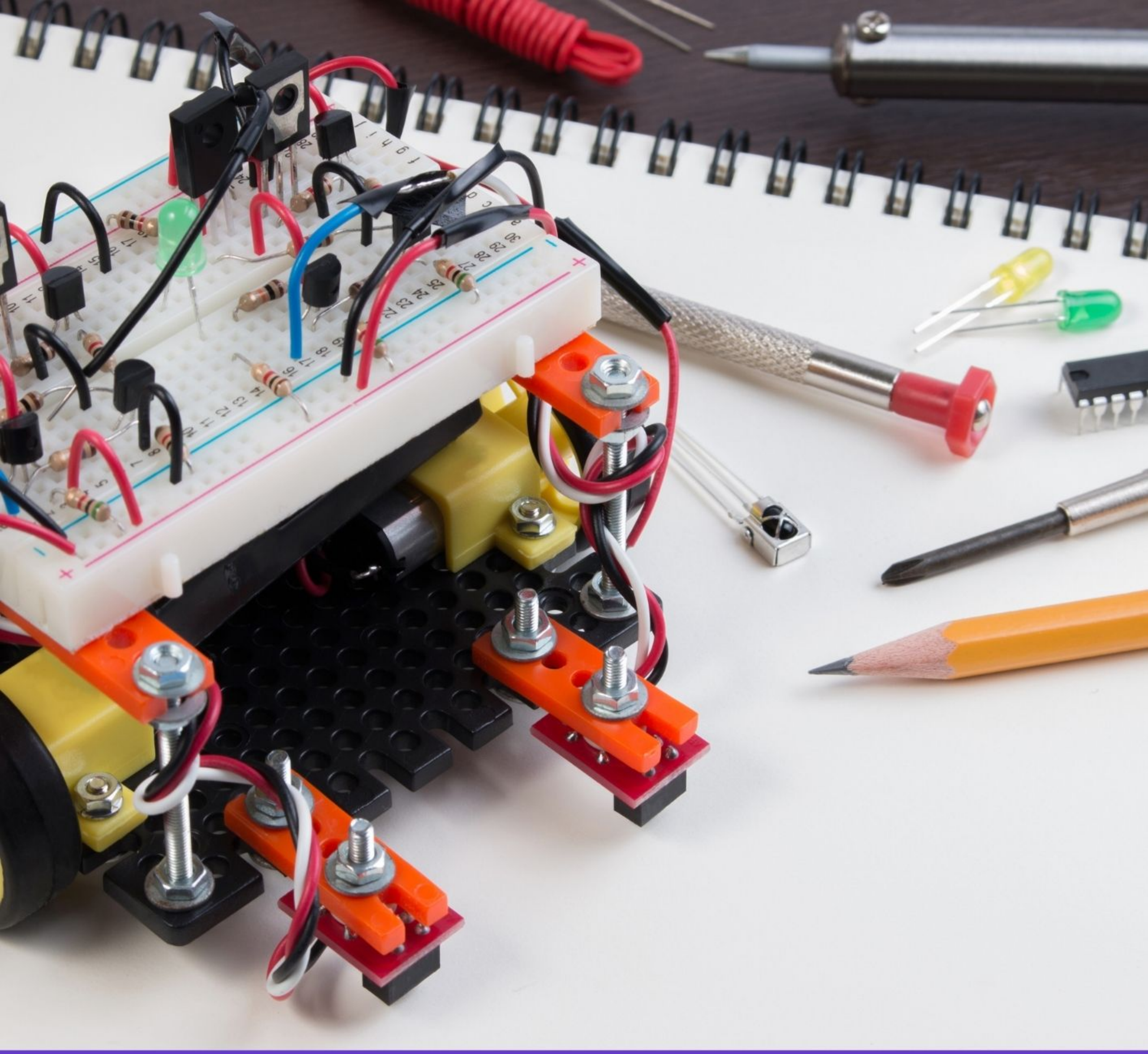

ROBOTICS

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INDEX

Robotics
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Lecture - 01
Introduction to Robot and Robotics

Let us start with the course on Robotics. The first topic is on Introduction to Robots and Robotics.

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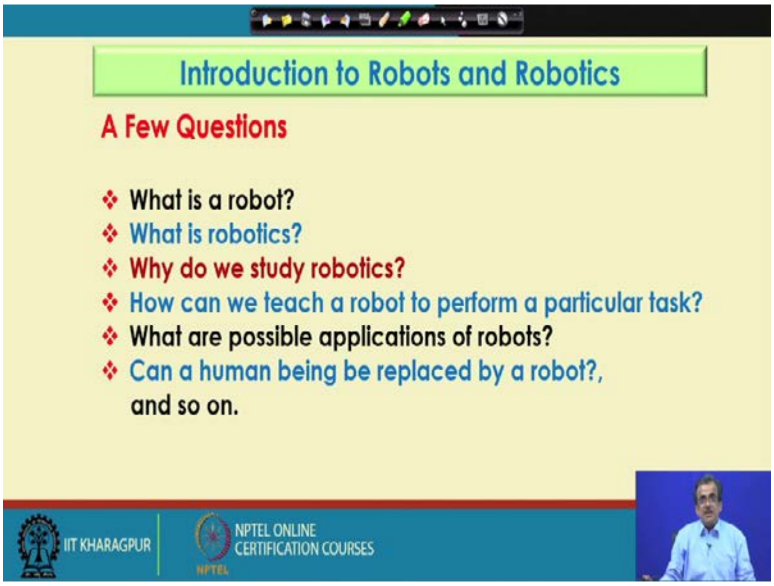

Now, before we start learning robotics, a few questions may come to our mind, these are as follows: What is a robot? What is robotics? Why should we study robotics? What is motivation behind robotics? How can we give instruction to a robot that you perform this particular task? What are the different types of robots, we generally use? What are the possible applications of robots? Can a human being be replaced by a robot?, and so on.

Similarly, there are many other questions. Now, here actually, what I am going to do, I am just going to give answer to the first few questions. But, the last one, that is can a human being be replaced by a robot?, that I will try to answer towards the end of this particular course. Now, let me start with the first one, that is, what is a robot?

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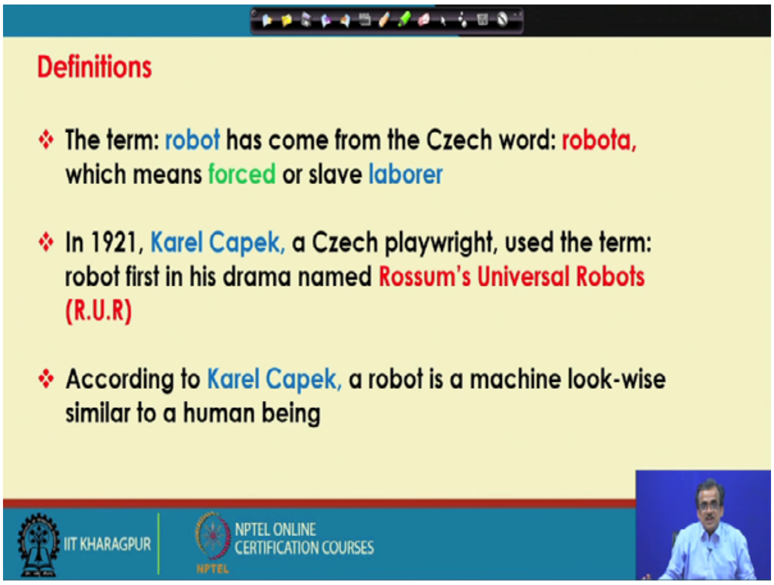

So, I am just going to define the term: robot. The term: robot has come from the Czech word: robota, which means the forced or the slave laborer. This is just like a servant, and we are going to give some tasks to the robot, and it is going to perform those tasks just like a servant.

Now, the term robot was introduced in the year 1921 by Karel Capek. Karel Capek was a Czech playwright, he wrote one drama and the name of the drama was: Rossum's Universal Robot (R.U.R). And, in that particular drama, he introduced a term: robota, that is, the robot. But, the way he described robot is as follows: the robot was look-wise similar to a human being. But, nowadays we use a few robots, which do not look like the human being. So, this is the way actually, the term robot was introduced in the year 1921. But, during that time, there was not even a single robot in the world.

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Now, if you see the literature, the term robot has been defined in a number of ways. For example say, according to the Oxford English Dictionary, robot is a machine capable of carrying out a complex series of actions automatically, especially one programmable by a computer, so this is nothing but an automatic machine. Then, according to ISO, that is, International Organization for Standardization, the robot has been defined as follows: the robot is an automatically controlled, reprogrammable, multifunctional manipulator, programmable in three or more axes, which can be either fixed in place or mobile for use in industrial automation applications.

Now, as I mentioned, that robot is nothing but an automatically controlled machine. And, it is reprogrammable that means the same robot can perform a variety of tasks, and to perform the variety of tasks, we will have to change its program. And, it is multifunctional, that means, the same robot, the same manipulator can perform the different types of machining operations. It can do some sort of peak and place type of operation, and so on.

Now, here actually, we are using the term: manipulator. By manipulator, we mean that it is a robot with fixed base. Now, this manipulator could be either serial manipulator or parallel manipulator. So, these things, I will be discussing in details after some time.

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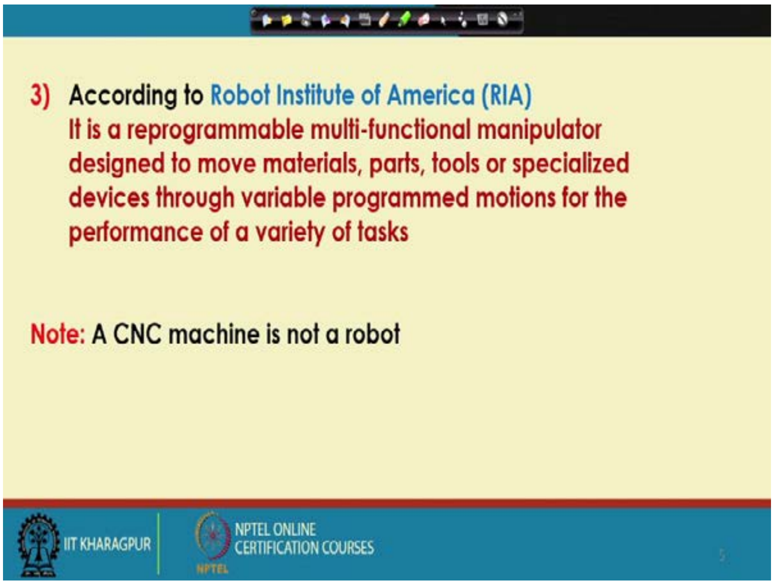

Now, another very popular definition is given by RIA, that is, Robot Institute of America. Now, they defined robot as follows: it is a reprogrammable multi-functional manipulator designed to move materials, parts, tools or specialized devices through variable programmed motions for the performance of a variety of tasks.

Now, these terms, I have already defined. For example, by manipulator we mean robot with fixed base, and that is nothing but a mechanical hand; that means, the human hand we are going to model, design and develop in the form of an artificial hand, and that is nothing but the manipulator, and it is reprogrammable and multifunctional. Now, in terms of re-programmability, if we compare a robot with one NC, CNC machine; now in CNC machine like computerized numerical control machine, we can perform a variety of tasks by changing the program.

Similarly, in robots, the same robot I can use to serve a variety of purposes, simply by changing the program. But, here, there is a basic difference between the level of re-programmability, which can be achieved by a robot, and that can be achieved by a CNC machine. Now, it is important to note, that the level of re-programmability, which can be achieved by a robot is more compared to that of the CNC machine. And, that is why, a CNC machine is not a robot. I have put one note here, that CNC machine is actually not a robot.

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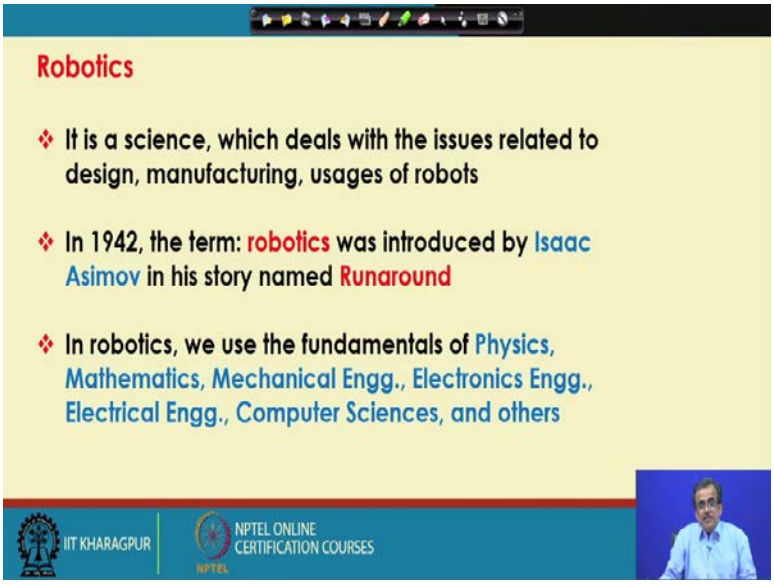

Now, next I am just going to define, what do we mean by the robotics. Now, the robotics is a science, which deals with the issues related to design, development, applications of robots to perform a variety of tasks. The term: robotics actually, it was coined by Isaac Asimov in the year 1942. Isaac Asimov, wrote one story, the name of this story was Runaround. And in that particular story, he used the term robotics first, but once again let me mention that during that time, that is, during 1942, there was not even a single robot in this world.

Now, here in robotics, we use the fundamentals of different subjects, for example, physics, mathematics, mechanical engineering, electrical and electronics engineering, computer science. And, that is why, it is bit difficult to become a true roboticist, because if we want to become an expert, a true expert of robotics, we will have to know the fundamentals of all these basic subjects, and a robotics is actually a multi-disciplinary subject.

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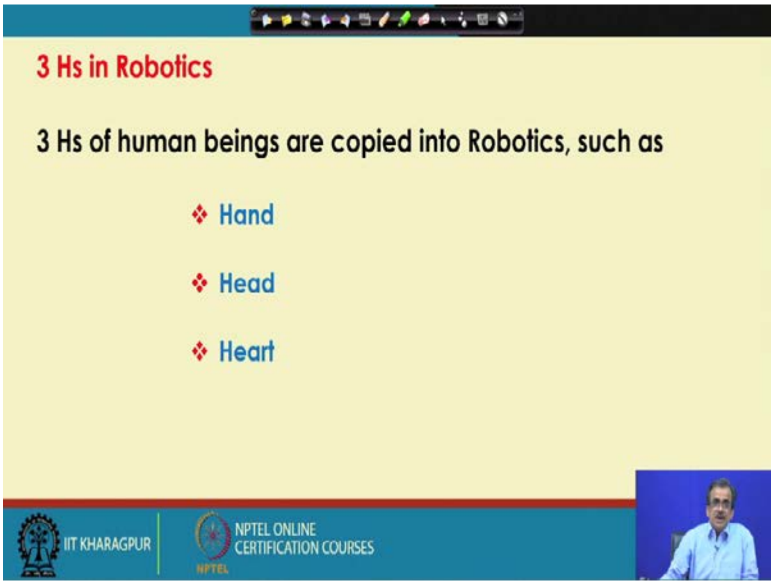

Now, I am just going to define one concept, which I have already mentioned a little bit, like in robotics, we try to copy 3 Hs. Now, these 3 Hs are nothing but, the Hand, Head, and Heart, that means, we try to copy the hand of a human being in the artificial way, in the form of one manipulator, that is, the mechanical hand. We try to copy the head of a human-being, that is, nothing but the intelligence.

And, we also try to copy the heart of a human being, but not the mechanical heart, but the emotion of a human-being. And, that is why, in future, the robot will be intelligent and at the same time emotional too. Now, if we consider the human-beings, we are intelligent, we are emotional, and in robotics, we try to copy everything from the human being. So, in future, we are trying to design and develop intelligent and emotional robots.

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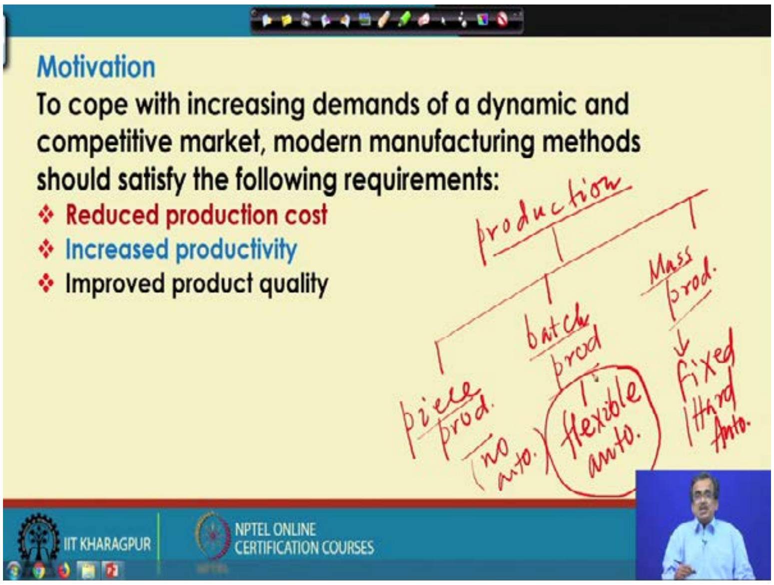

Now, the next is, what is the motivation behind robotics?, why should we study robotics?, what is the reason?. Now, if you see today's market, it is dynamic and competitive. And, if you want to be in competition, and if you want to be in business, what you will have to do is. You will have to fulfill at least three requirements. Now, these requirements are as follows: like you will have to produce good at low cost; and at the same time the productivity has to be high; and the quality of the product has to be good. Now, you see the three objectives, like reduced production cost, increased productivity, and improved product quality. Now, it is bit difficult to achieve all these three things at a time, and some of them are actually conflicting.

Now, if you want to achieve all three, there is only one solution, and that is nothing but automation. So, you will have to go for automation, if you want to achieve all three requirements. Now, if I proceed further, let me tell you something regarding, the different types of products and methods, which we generally use. Now, if you see the production methods, the purpose of production is actually to convert the raw materials into the finished product. Now, this production could be of three types. For example, we can have the piece production, then there could be batch production, then there could be mass production.

Now, for piece production, we have got several designs and each design, we will have to manufacture small in number. Now, for batch production, we have got a few designs;

and each design, we produce a few in numbers. Now, in mass production, we have got only one design, and that particular product is to be produced a large in number. Now, we can automate this particular batch production, mass production and of course for piece production, automation is not possible; so there is no automation for this particular piece production.

But, for batch production, we can go for automation. And, for mass production, we go for automation. For mass production, we generally go for the fixed automation or hard automation. For this particular batch production, we generally go for the flexible automation.

Now, robotics is an example of this flexible automation. And, that is why, for batch production, particularly in the manufacturing unit, we will have to go for the robots, if you want to survive in this competitive market and that is why, the robotics and the robots have become so much popular in manufacturing units. But, nowadays, not only in manufacturing units, the robots are used in different areas. For example, robots are nowadays used in space science, used in medical science, robots are also used for sea-bed mining, in agriculture, fire-fighting, and so on. So, there are various applications of robots, nowadays.

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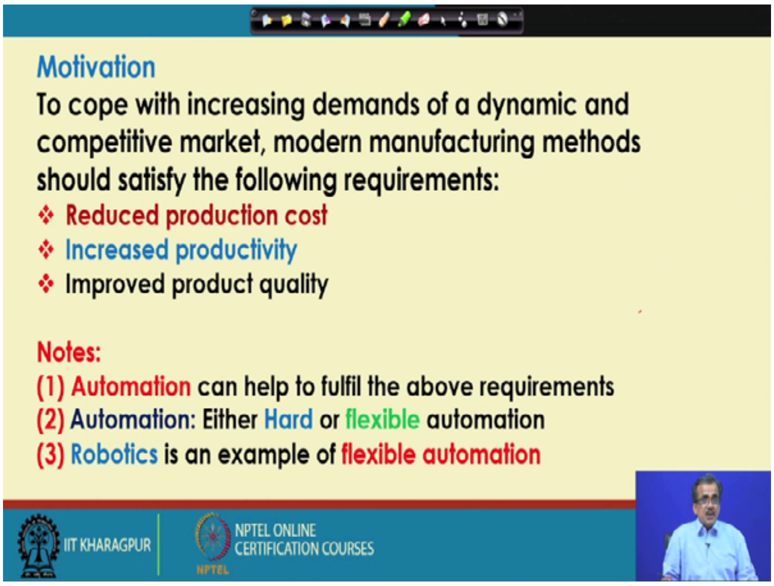

Now, here all such things I have noted. Automation can help to fulfill the requirements of the above requirements. And, robotics is an example of the flexible automation, and that is why, we should study robotics.

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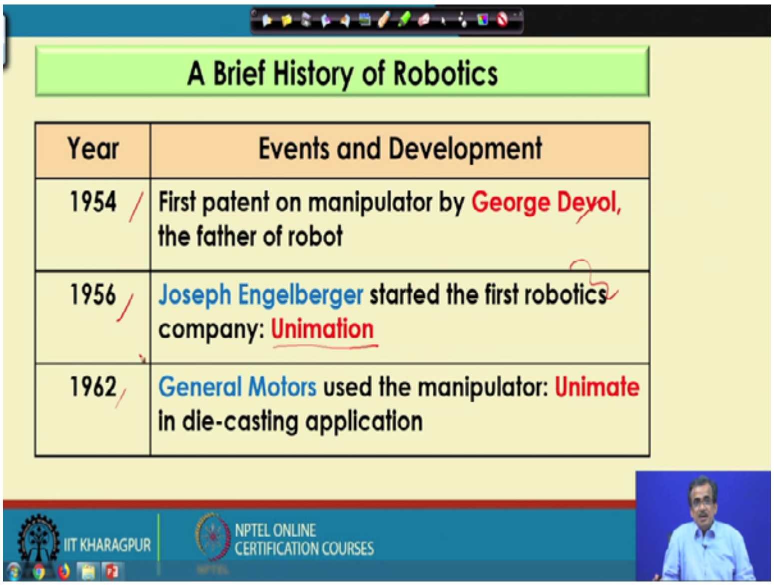

Now, I am just going to concentrate on a brief history of robotics. Now, if you see the NC machine, that is, the numerical controlled machine; that was developed first in the year 1950, but robot came after that. So, the first robot, which was developed, that was developed in the year: 1954. In 1954, the first patent on the manipulator was filed by George Devol, and he is known as the father of robot. In 1956, Joseph Engelberger started the first robotics company, and the name of the company is Unimation. So, Unimation is the first robotics company, which was started in the year 1956. Then, in the year 1962, General Motors used the manipulator, the name of the manipulator is Unimate, and this particular robot was used in die-casting application.

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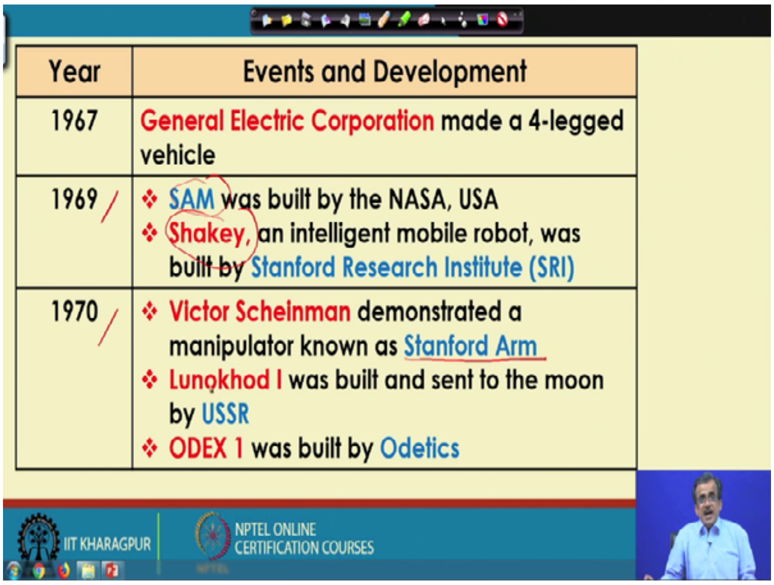

Now, next, in the year 1967, General Electric Operation made one 4-legged robot, and this is a 4-legged vehicle, and they demonstrated and it worked well. Then, in the year 1969, SAM was built by the NASA, USA. SAM was the name of that particular robot, which was built by the NASA, then Shakey, an intelligent robot, was actually manufactured by Stanford Research Institute SRI. In fact, Shakey is the first intelligent mobile robot that was developed in the year 1969. In 1970, Victor Scheinman, demonstrated a manipulator known as Stanford Arm, and then, Lunokhod 1 was another robot, that was sent to the moon by USSR, then ODEX 1, another robot, was built by Odetics, in the year 1970.

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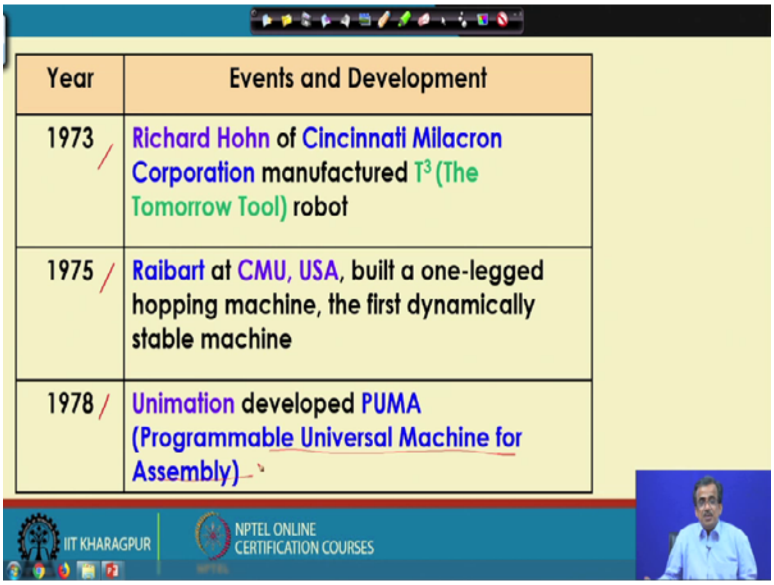

Then, in the year 1973, Richard Hohn of Cincinnati Milacron Corporation manufactured one robot, the name of the robot was T^3, The Tomorrow Tool. Then, in the year 1975, Raibart at Carnegie Mellon University, USA, built one one-legged hopping machine, and that is the first dynamically stable machine. Raibart, in fact, is known as the father of multi-legged robots.

In the year 1978, Unimation, the first robotics company, could develop the PUMA, that is, Programmable Universal Machine for Assembly. And, this is actually a manipulator, whose current version is having 6 degrees of freedom, and it is very frequently used in various industries.

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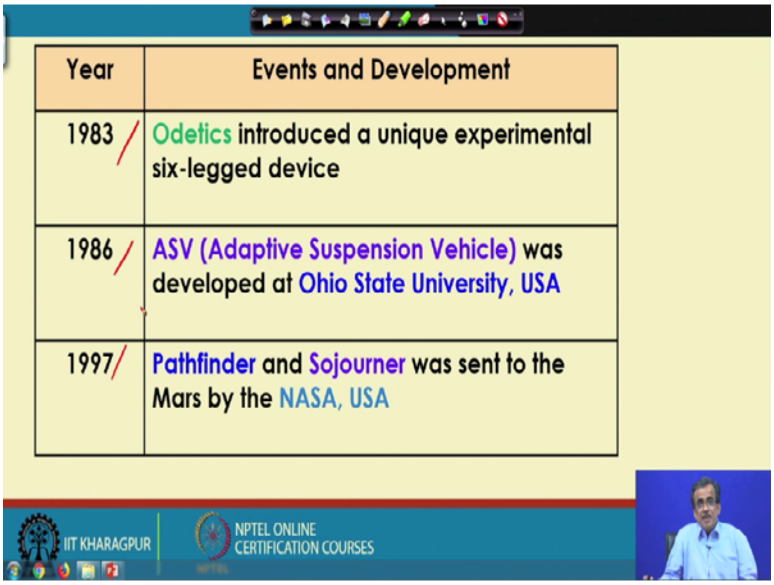

Then, in the year 1983, Odetics, a robotics company, introduced a unique experimental six-legged device. In the year 1986, Adaptive Suspension Vehicle, in short, ASV was developed by Ohio State University, USA. In 1997, NASA, USA, developed the intelligent robots like Pathfinder and Sojourner, and they sent them to the Mars, but that particular mission was a failure. And, that particular failure was due to some sorts of mismatch of the specifications.

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Next, in the year 2000, Honda could develop one Humanoid robot, Asimo robot. So, Asimo Humanoid robot was developed by Honda, in 2000. Then, comes in 2004, the surface of the Mars was explored by Spirit, and Opportunity, and this particular mission was successful. And, you might be knowing, what happened in 2012, the Curiosity, one intelligent autonomous robot, was sent to the Mars by the NASA, USA, and this particular mission was successful.

Then, all of you might be knowing, what happened in the year 2015, Sophia, that is one intelligent and a little bit emotional humanoid robot, was built by Hanson Robotics, Hong Kong, and this is actually, as on today, the most sophisticated intelligent humanoid robot. And, a few weeks ago, this particular robot was brought to IIT, Bombay, and there she could talk, she could communicate with other people, and some of you might have seen in paper or a television. So, that particular very sophisticated intelligent humanoid robot is Sophia.

So, these are in sort the brief history of the robotics. Now, the purpose behind giving this brief history of this robotics is just to tell you that we started a bit late in India. The study on robotics, we started around 1979, 80. So, we started a little bit late, although the first manipulator, the first patent was filed in the year 1954.

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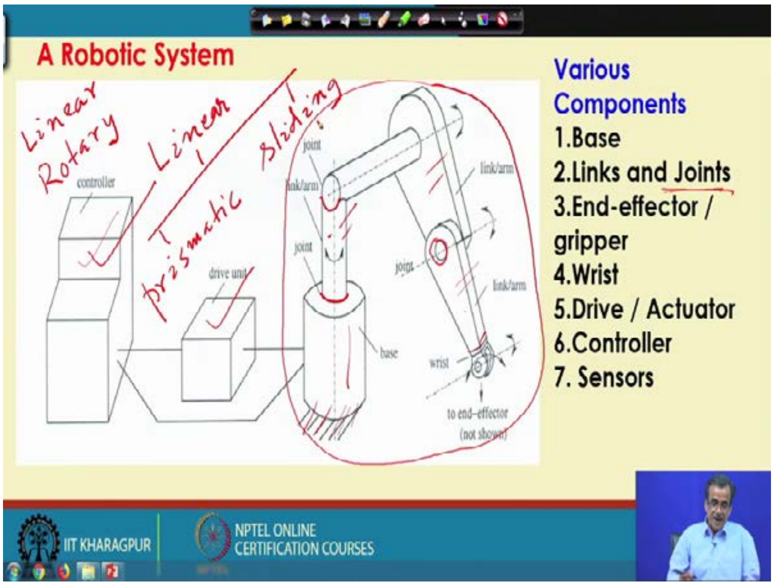

Now, I am just going to concentrate on a particular robotic system. So, what are the different components of a typical robotic system? Now, here, in this particular schematic view, you can see that that this particular thing is nothing but a robot. So, this is actually the robot, and this is the manipulator, this is a serial manipulator. And, this is the drive unit for this serial manipulator. And, this is the controller or the director for this particular manipulator.

Now, as I told that this is a serial manipulator, and by manipulator, we mean a robot with fixed base. So, here, the base of this particular robot is fixed. So, it is a fixed base, we have got one link here, another link here, another link here, and these links are used just to transmit the mechanical power. And, in between the two links, we have got the joints, so we have got a few joints. For example, say if I consider that this is the base of this particular manipulator, and this is the next link, so in between these two, you have got a joint here.

Similarly, in between this link, and that particular link, we have got a joint here. Similarly, here in between this link and that link, we have got a joint here, between these and these we have got another joint here. So, in between the two links, so we have got a particular the joint. Now, if you see the robotic joint, the robotic joint could be basically of two types, it could be either the linear joint, or there could be rotary joints. So, the linear joint, it could be either prismatic joint or sliding joint. Now, here I am just going to draw a rough sketch for these prismatic and sliding joints.

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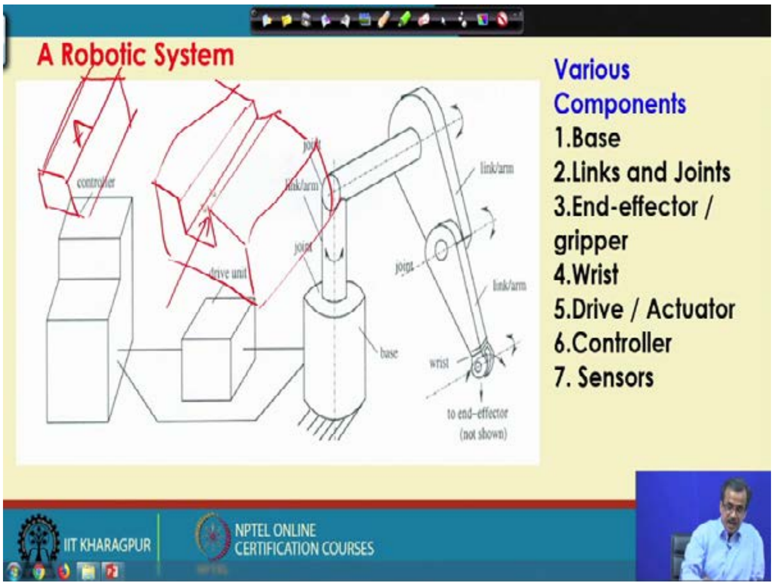

Now, if I just draw this particular prismatic joint, supposing that I have got a block like this. So, if I consider a block like this. Now, here, I can insert one this type of key. Now, if I insert this particular key here. So, this particular joint will be nothing but a prismatic joint, and this is a linear joint. So, this particular part, say part A can be just moved in the linear direction here, and this is an example of the prismatic joint.

Now, similarly, I am just going to take the example of one sliding joint, now supposing that I have got a block like this. Say, I have got a block like this. And, here, I will have to insert one pin, that pin could be something like this. Say, I will have to insert a pin something like this here, and this particular pin can be inserted here and there will be only the linear movement, and this is the example of one sliding joint. So, these are all linear joints.

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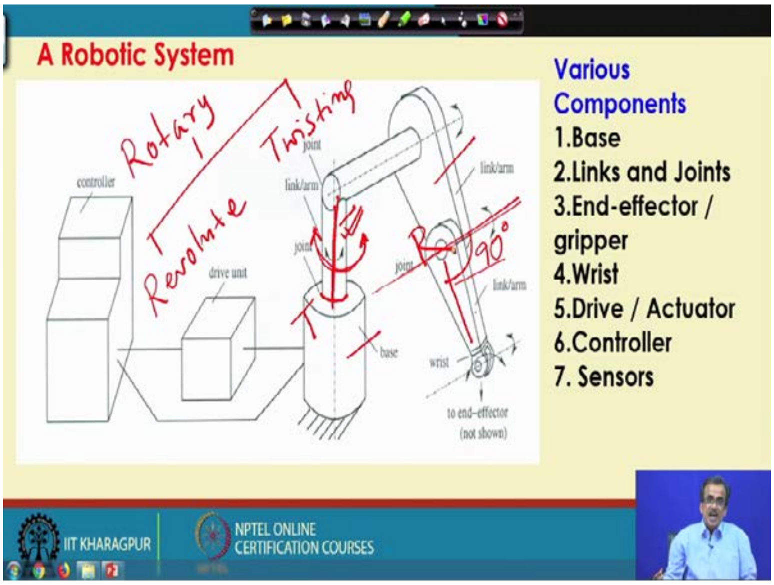

Now, next come to the rotary joint. Now, here so, if you see the rotary joint, it could be of two types basically, we could have the revolute joint, and there could be twisting joint. Now, both are the rotary joints, but basically there is a difference between these revolute joint, and twisting joint. Now, to find out the difference between the revolute joint, and twisting joint; I am just going to take one example here.

Now, let me take one example. So, this is the fixed base, and this is the link, and in between I have got a joint here. Now, with the help of this particular joint, so this particular link can be rotated something like this. So, it can be rotated something like this. Now, if this is the output link and this side is input. The axis of the output link is nothing but this about which I am taking the rotation. And, this particular axis is coinciding with the axis of the output link. This is the output link. This is the axis of the output link, and I am taking this rotation about this particular axis.

So, this particular rotary joint is nothing but, the twisting joint denoted by T. Now, let me take another example, say this is one link and this is another link. So, here, I am just going to take the rotation, the rotation about this particular axis. Now, here if this is the input side, that is the output side. So, the axis of the output link is something like this, and the axis about which I am taking the rotation is this, and they are at 90 degrees.

So, if this is output, the axis of the output link, and the axis about which I am taking the rotation, they are at 90 degrees. So, that type of rotary joint is known as the revolute joint, so this is nothing but a revolute joint denoted by R. So, basically once again let me repeat that we use two types of joints, namely linear joint, and rotary joint. And, once again, there are two types of linear joint, the prismatic joint and sliding joint, and two types of rotary joint we use, one is the revolute joint, another is the twisting joint.

Thank you.

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Lecture - 02
Introduction to Robot and Robotics (Contd.)

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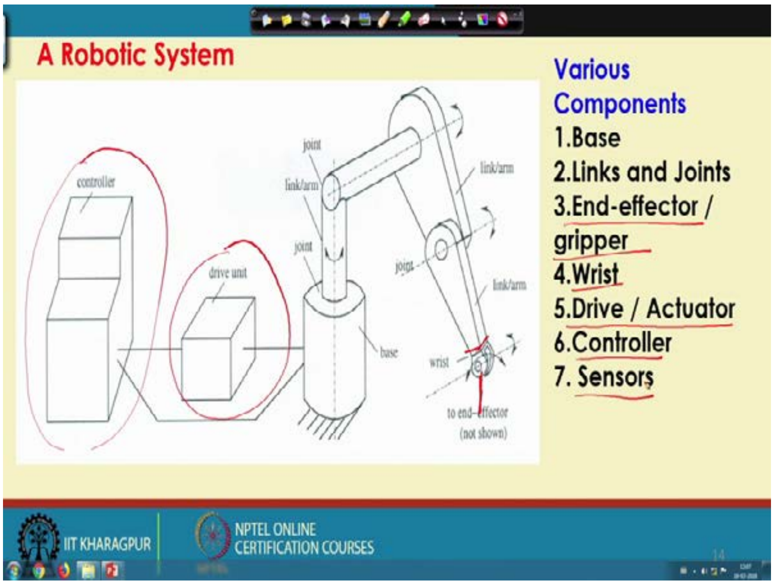

So, we are discussing the different components of a Robotic System. And, we have seen that, the robot is having base, links and joints. And, the next is, we have got some sort of end-effector or the gripper. Now, the purpose of using the end-effector is to grip that particular object, which I am going to manipulate. And, this particular end-effector or the gripper will be connected here, but is not shown here in this particular sketch, so the gripper will be connected here.

The next is this gripper or the end-effector, I will be discussing in much more details after some time. The wrist joint, so this particular joint is nothing but the wrist joint, where I am just going to grip, so this particular end-effector with the last link of the robot. Next is the drive system or the actuator. Regarding this drive system or the actuator, now let me see, what type of drive system we have in our body, like we human -beings are dependent on both the mechanical drive systems, that is with the help of your muscles. And, we also take the help of hydraulic system, that is with the help of blood, and the blood is pumped with the help of heart.

So, now all such things have been copied in robotics also. And, in robotics, we have got the pure mechanical drive. Now, mechanical drive means, in the form of the gears, gear and pinions, in the form of chain drives, belt drive and so on. Now, supposing that the load requirement is more or the power requirement is more, so what we will have to do is, we will have to take the help of some sort of hydraulic drive. We also take the help of pneumatic drive, using some sort of compressed air, we use electrical drive also. And, sometimes we combine, that means, use electro-hydraulic, electron-pneumatic drives. So, different types of drive unit we generally use here, in the robots.

The next is actually the controller. Now, this controller is nothing but the brain of this robot. So, just like our head, this controller contains this particular brain or the intelligence. And, here in the controller, there will be software, and hardware also. Now, these actually constitute one robotic system all such components. Now, here if I want to make it intelligent, the robot will have to collect information of the environment with the help of some sensors.

And, that is why, we use some sensors along with the robot, just to collect information of the environment and operate the drive units, so that we can make these particular robots intelligent. This is once again has been copied from human-being, because we have got a few sensors like we have got eyes, ears, nose, skin and all such things. We collect information with the help of these senses, take the decision on our head, the same thing is done by an intelligent robot. The information collected with the help of these particular sensors, will be processed in the controller, then the decision will be taken, and that particular decision will be executed. This is the way, actually, one intelligent robot will be working. All such things will be discussed in much more details after some time.

So, the different types of sensors, we generally used in the robots, these things will be a discussed in details after some time. In fact, in robots, we use both internal as well as external sensors. Internal sensors are used to operate the drive units. For example, say we have got the position sensor, velocity sensor, acceleration sensor, forces or the moment sensor, and so on. On the other hand, we have got a few external sensors, which are used to collect information of the environment. For example, we have got some sort of range sensor, proximity sensor, and all such things will be discussed after some time in much more details.

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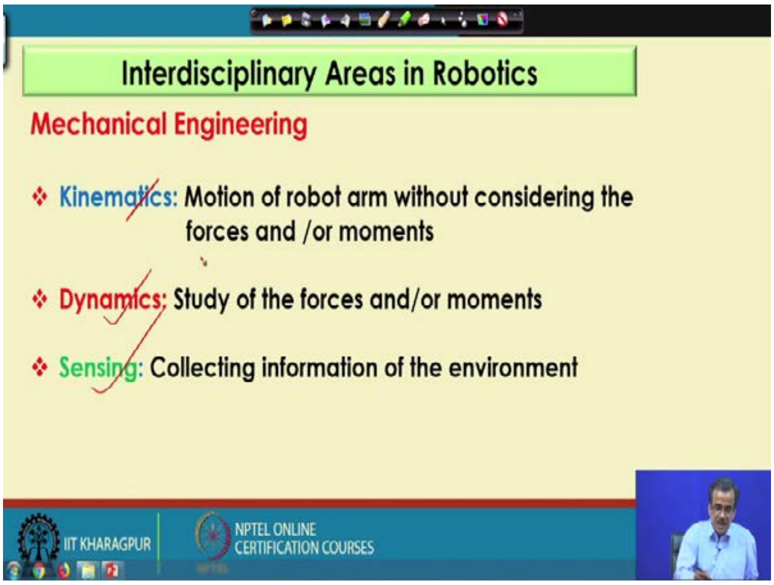

Now, I am just going to see; what are the different areas in robotics. As I told that in robotics, there are four distinct modules. Like these modules are coming under the purview of different disciplines. For example, say we have got the kinematics, dynamics, and sensing, which are coming under the umbrella of this mechanical engineering.

Now, in kinematics actually what we do is, we try to consider the motion, the relative motion of the different joints, different links ok, but we generally do not try to find out the reason behind this particular movement or this relative movement, and that is actually done in dynamics. So, in dynamics, we try to find out, how much is the force required?, if it is a linear joint, and how much is that moment or torque required, if it is a rotary joint. So, all such things are mathematically determined in dynamics. And, in sensing, we try to collect information of the environment with the help of sensors. So, all such things are coming under the umbrella of mechanical engineering.

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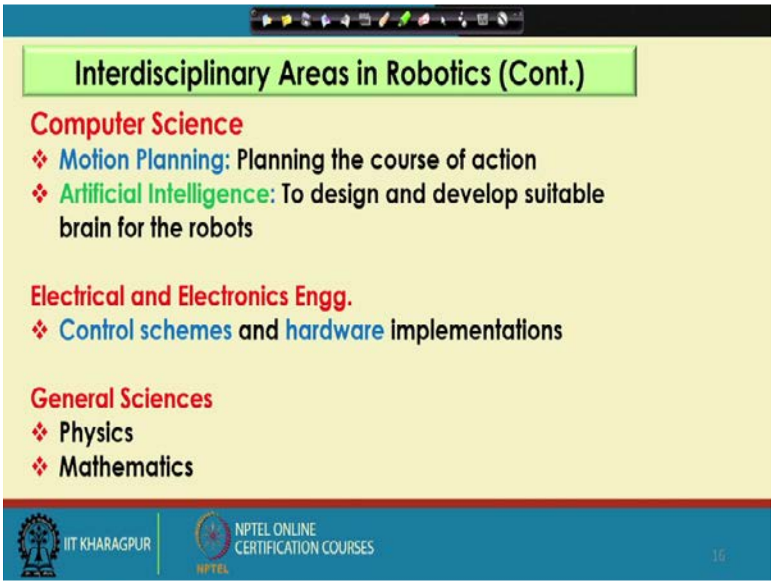

And, then, comes the motion planning. So, what we do is, if you want to make these particular robots intelligent, what you will have to do is, you will have to make some planning. We will have to plan the course of action, and we take the help of a few motion planning algorithms.

Now, if you see the literature, a huge literature is available on robot motion planning using both traditional as well as soft computing-based approaches. So, here, in this course, basically I will concentrate only on the traditional approaches of motion planning, and these things will be discussed in much more details after some time. But, the purpose of using motion planning is to decide the course of action, depending on the input situation. So, what should be the output, how to decide that, that is the purpose of the motion planning.

Now, then comes the artificial intelligence. So, as we told that, we try to copy the human brain in the artificial way, using the principle of artificial intelligence. Now, to design and develop the suitable brain for the robots, we will have to model the human brain, the human intelligence in the artificial way, using the principle of artificial intelligence. And, once again, this artificial intelligence is a very big area of research. And, distinctly there are two groups of algorithms, one is called the traditional AI techniques, and we have got the non-traditional AI techniques, that is, called the computational intelligence, that is the artificial intelligence using the principle of soft computing.

So, using this principle actually, we can make a plan depending on the requirement. So, that the robot can be made intelligent, and it can take the decision and execute that particular task depending on the requirement. Now, all such things are coming under the umbrella of computer science.

Now, next comes the control scheme. Now, supposing that to perform a particular task. So, my motion planning algorithm has given some decision, which has to be executed. So, how to execute? So, what you do is at each of the robotic joint, we use some motor. Generally, we use DC motor, and to control these motors actually there should be controller. So, definitely the robot should have one control architecture, and one control scheme has to be used to control this particular robot in a very efficient way. And, these control schemes, and its hardware implementations are coming under the purview of your electrical engineering, and the electronics engineering.

Now, here, actually to develop the robots, we will have to have very good knowledge of the general science like physics, mathematics, because we will have to use the principle of physics and mathematics very frequently to design and develop the robots. Particularly, if I want to design from kinematic point of view, dynamics point of view, so a lot of mathematics, a lot of physics I will have to use. Similarly, if I want to design and develop suitable sensors for these particular robots, we will have to use the basic principles of physics, all such things I will be discussing after some time in much more details.

Now, that means, if somebody wants to become a true expert of robotics, he or she should have at least some basic fundamental knowledge of all these pillars. And, that is why, the robotics is little bit difficult, and it is bit difficult to become a true roboticist.

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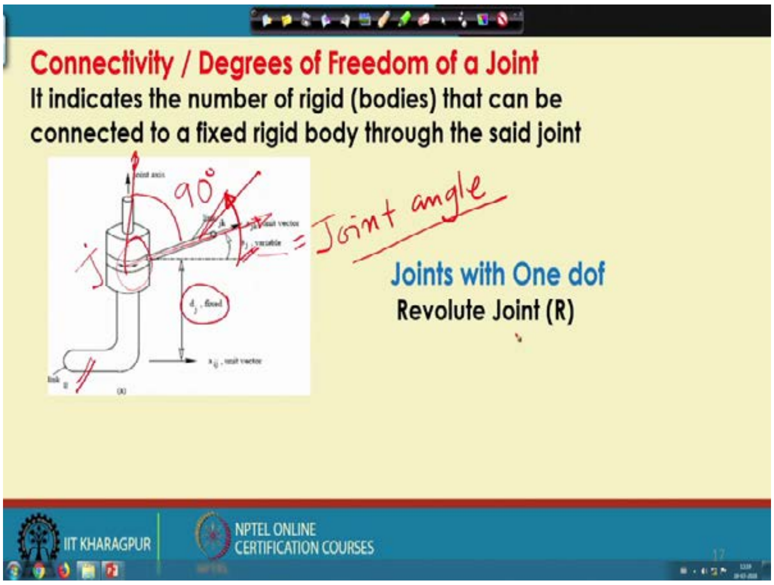

Now, I am just going to concentrate on once again the different types of joints, which is generally used in robots. I have already mentioned that basically we use two types of joints, one is called the linear joint, and another is called the rotary joints. Once again, the linear joint we have got basically of two types, one is called the sliding joint, another is called the prismatic joint. Similarly, the rotary joints, we have got, may be either the revolute joint, or twisting joint. Now, each of these particular joints is having the degree of freedom or connectivity. So, by connectivity we mean, how many rigid link can be connected to one fixed link through that particular joint.

Supposing that this is the input link, and I have got one output link here, now if I want to join this output link with the input link, I will have to put one joint. Now, if I can join only one output link to the fixed input link with the help of that particular joint, this particular joint is having one connectivity or one degree of freedom one.

Now, here, I am just going to discuss in details, the joint like the revolute joint. Now, this revolute joint as I told, has got only one connectivity, and one degree of freedom, because I can connect only one output link with the input with the help of this particular joint.

Now, one very simple example of this type of joint, that is, revolute joint is this particular joint. Here, you can see, that if this is the input side, that is the output link, this particular output link can be connected to the input link with the help of this particular joint. So, this joint is nothing but similar to a revolute joint.

Now, here, in this particular sketch if you see, this is the input link, denoted by i j. And this is the output link denoted by j k. And, in between the input link i j, and the output link j k, we have got a joint, and that is nothing but the j-th joint. So, this is nothing but the j-th joint.

Now, here, if you see, this is the axis about which I am taking this rotation. And, the axis of the output link is nothing but this, and they are at 90 degrees. That is why, this is a typical example of the revolute joint. Now, let us see, how does it work, to explain its working principle, this is the input link and that is the output link.

Here, I have got a fixed offset that is denoted by d_j fixed, so this is the fixed quantity. And, so this particular output link can only rotate with respect to the input in this particular direction. OK? This rotation is given by θ_j. This θ_j is nothing but a variable. So, here, θ_j is known as the joint angle, so this is known as the joint angle. And, this joint angle is nothing but a variable here for this particular revolute joint.

As I told that this particular joint is a revolute joint. So, this particular angle is going to vary, and this angle is nothing but the joint angle. So, this is the variable for this revolute joint.

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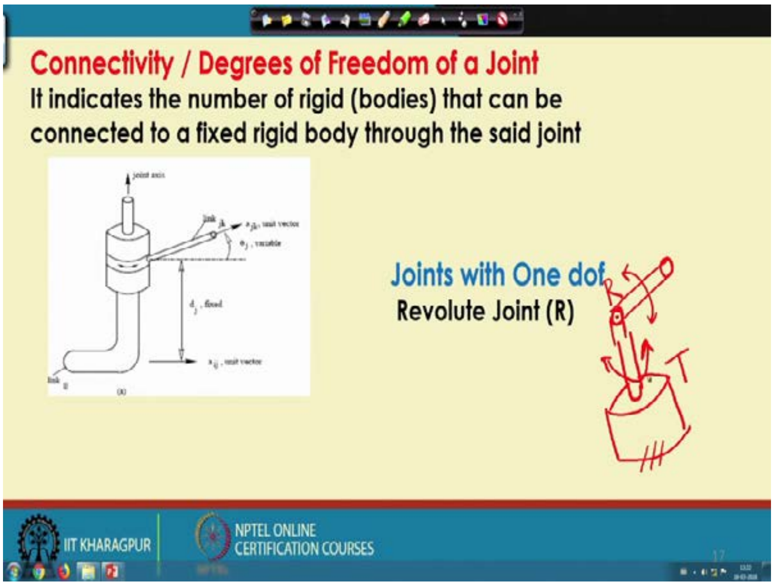

Now, then, comes the prismatic joint. Now, before I go once again for this particular prismatic joint, let me just tell you one more thing. Now, here I did not discuss the twisting joint that is another type of the rotary joint. Now, I am just going to show you one very practical example, with the help of which you can find out the difference between, the revolute joint, and the twisting joint.

Let us concentrate on the joint which you have at this particular neck, say our neck. Now, with the help of this particular joint, I can rotate my head in two different ways. For example, say this is nothing but my fixed link. And, supposing that, I am just going to rotate the head, so this is actually the axis about which I am rotating my head, and this is something like this. So, I am just rotating my head something like this. And, here in this particular rotation, this is the axis about which I am rotating my head, OK, so this is one rotation.

Now, I am just going to rotate my head in another way. So, this indicates the axis about which I am taking the rotation. So, here, in this particular rotation, so I am able to rotate my head like this. So, if I rotate my head like this, my output axis of the head is here, and this is the axis about which I am taking the rotation, so this angle is nothing but 90 degrees. So, the moment I am rotating my head like this, this particular joint is nothing but the revolute joint. But, the moment I am rotating my head like this, this particular joint will act like the twisting joint, so this is the practical example, just to find out the difference between the revolute joint and the twisting joint.

Now, here if I just draw one revolute joint and the twisting joint on the same manipulator, I can draw very easily. Say, this is one manipulator with one twisting joint, and one revolute joint. So, let me try to prepare one sketch. So, this is actually a robot with fixed base. This is a very simple robot. So, with respect to the fixed base, I have got a joint here, and it can rotate something like this, so this joint is nothing but a twisting joint; it is a rotary joint. But, here, I have got another joint with the help of which I can rotate, and this is nothing but a revolute joint. So, this is the difference between the twisting joint, and revolute joint.

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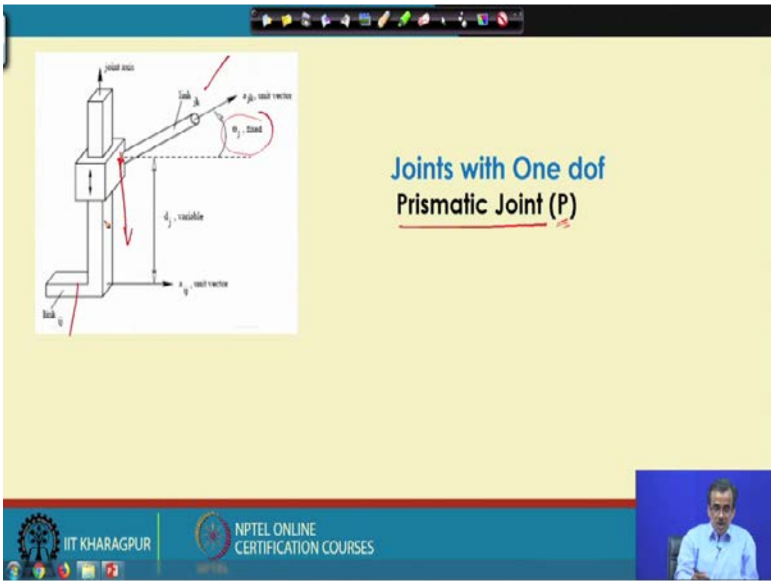

Now, I am just going to start with the prismatic joint, which is nothing but a linear joint. And, this particular prismatic joint is having only one connectivity or one degree of freedom; now, here on this particular sketch. So, this is the input link i j. So, this is actually the input link i j. And, the output link is nothing but the link j k. And, here, we have got the j-th joint. Now, you can see that, so this particular θ_j, that is, the joint angle is kept fixed.

Now, this block can move up and down, so it can slide, it will have only the linear movement, only one connectivity, one only one degree of freedom, and there could be only linear movement. So, this particular prismatic joint, which is denoted by P, will have only one connectivity or one degree of freedom. So, this prismatic joint is having one connectivity or one degree of freedom.

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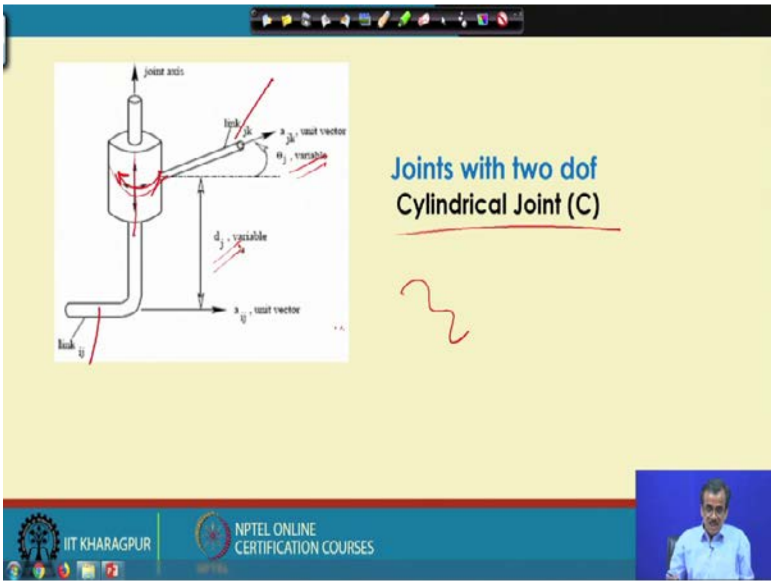

Now, then comes the cylindrical joint. This particular cylindrical joint is having, in fact, two degrees of freedom. So, cylindrical joint has got two degrees of freedom; this is the input link i j, the output link is j k. Now, this particular block, can slide up and down. And at the same time, it can also rotate something like this. This is called the cylindrical joint.

So, here, we have got one linear joint, and another rotary and this is a combination, and it has got two degrees of freedom. And, that is why, both θ_j as well as this particular d_j are kept as the variables. So, this is a typical example of this particular cylindrical joint, which has got two degrees of freedom or two connectivities.

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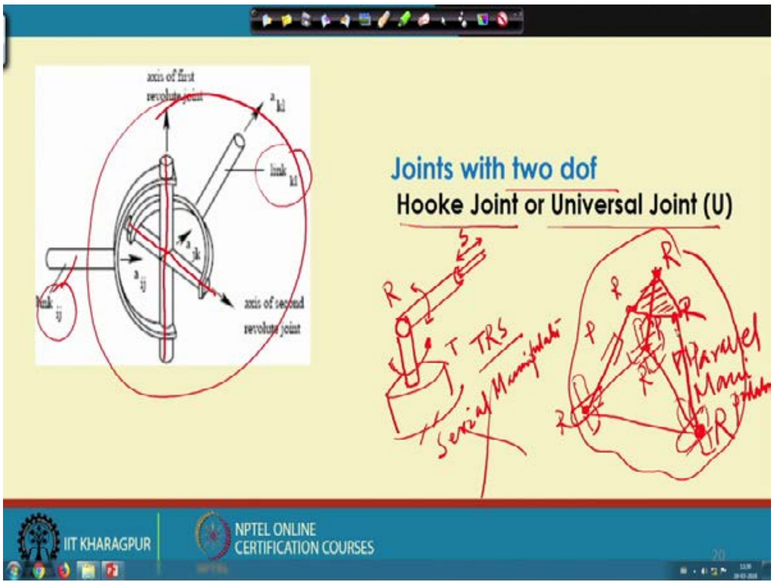

Now, then comes the concept of Hooke joint or the Universal Joint. So, this Hooke joint or the Universal Joint, this is actually a combination of two rotary joints. In fact, here we are going to combine, two such revolute joints. And, this particular Hooke joint or the universal joint has got two degrees of freedom.

Now, let us try to understand the principle of this Hooke joint. Now, here, the input link is i j, and the output link is nothing but k l. Now, here you can see that we have got one revolute joint here. So, this is the axis for the first revolute joint. And, this is the axis for the second revolute joint, OK? Now, I am just going to show you the physical concept of this Hooke joint or the universal joint.

Now, let me consider that this is a joint this is a revolute joint. So, this is the axis about which I am taking the rotation, and with respect to this particular axis. So, the joint can be rotated something like this. So, it has got some theta variation here. Similarly, with respect to this. So, I can have another revolute joint like this. So, here with respect to this the axis, I can I can rotate something like this. So, this is another revolute joint.

So, these two revolute joints if I connect, then it will form the Hooke joint or the universal joint. Supposing that this is actually my input side, and this is my output side. So, with respect to the input, the output will have two degrees of freedom. And, this type of joint is generally not used in serial manipulator, but this is used in parallel manipulator. Now, the concept of serial manipulator and parallel manipulator, I am not yet discussed. But, I am just going to give a very rough sketch for this serial manipulator and a very rough sketch for the parallel manipulator, just to find out their differences.

Now, if you see the manipulator, which I just drew a few minutes ago, that is nothing but a serial manipulator. Now, in serial manipulator, actually all the links, all the joints are in series. For example, say the same picture I can consider for the serial manipulator, so this is nothing but a serial manipulator. So, this is another joint. And here, I can put one linear joint also. So, let me put one linear joint here. So, there is a twisting joint here, denoted by T, there is a revolute joint here, denoted by R, and there is a sliding joint here, say denoted by S, that is a linear joint. This is actually known as, TRS manipulator, this is nothing but a serial manipulator. So, this is a serial manipulator.

And, now, I am just going to draw a rough sketch for one parallel manipulator. A very simple parallel manipulator, if I just draw. So, a very simple sketch, very simple design, I am just going to make. So, this is the parallel manipulator, this is the top plate for the parallel manipulator. And here, we have got a few joints.

For example, say I can put one revolute joint here, I can put one revolute joint here, I can put one linear joint here, say one prismatic joint I can put. Similarly, at each of these particular legs; I have got one revolute joint here, I have got one linear joint here, this is a revolute joint, OK?. Similarly, I have got a revolute joint here, I have got another revolute joint here, I have got one prismatic joint or the linear joint here, so this is nothing but a parallel manipulator. So, this is actually a parallel manipulator.

Now, the reason why I am just trying to find out the difference between the serial and parallel manipulators is as follows. So, these types of joint are generally not used in serial manipulator. But, in some of the complicated parallel manipulator, we use this type of Hooke joint. For example, in place of this rotary joint, I can put one Hooke joint here, I can put one Hooke joint here, I can put one Hooke joint here, I can put one Hooke joint here.

So, these type of joints are generally used in parallel manipulator, but not in serial manipulator. Those things I will be discussing after sometime in much more details.

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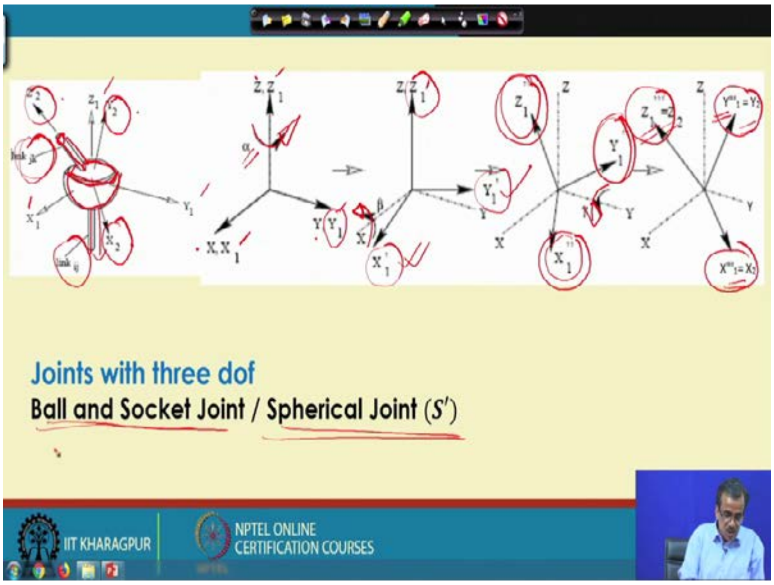

Now, then comes the concept of the ball and socket joint or the spherical joint. Now, this ball and socket joint or the spherical joint is having actually three degrees of freedom, and all three are rotations. Now, here actually what we do is, the input link and output link are connected, so that one understands this arrangement. So, the input link, that is, link ij is having the coordinates X_1, Y_1 and Z_1, and input link is connected here, so this is actually connected to the input link, OK? And, the output link is jk, whose coordinates are nothing but X_2, Y_2 and Z_2, that is connected to this part, and this is nothing but the output link, and this is a link j k.

Now, starting from input link ij, if I want to go to this link jk, how many rotations are required. Now, if I can start from X_1, Y_1 and Z_1, and if I can reach X_2, Y_2 and Z_2, through some minimum number of rotations, that will be the degrees of freedom or connectivity of this type of ball and socket joint or the spherical joint. Now, let us try to find out what should be the degrees of freedom of this ball and socket joint or the spherical joint. And, before I do that, let me once again mention that this type of joint is generally used only in the parallel manipulator, but not in serial manipulator.

Now, let us try to understand, why does it have three degrees of freedom. Now, this X_1, Y_1 and Z_1 are initially coinciding with the universal coordinate system X, Y and Z. And here, I am just going to give some rotation about Z by an angle α in the anti-clockwise sense. Now, if I give rotation about Z, my Z will remain same as Z. So, this will become

Z_1'. So, this will become Z_1'. But, X_1 will become X_1' that will be different from X_1. And, Y_1 will become Y_1'. And, Y_1' will be different from Y_1, but Z_1' will remain same as Z_1.

Now, I am just going to give some rotation about X by an angle β in the anti-clockwise sense. So, if I give rotation about X the original X or the universal X by an angle β, I will be getting change in all X, Y and Z, that means your X_1'' will be different from X_1'; Y_1'' will be different from Y_1'; and Z_1'' will be different from your Z_1'.

And now, I am just going to give rotation about the universal Y by an angle γ in the anti-clockwise sense. So, all three rotations will come that means your X_1''', will be different from your X_1''; then Y_1''' will be different from Y_1''; and Z_1''' will be different from Z_1''. And now, X_1''' is nothing but X_2, Y_1''' is nothing but Y_2 and Z_1''' is nothing but Z_2.

Now, here, this X_2 is nothing but this particular X_2. This Y_2 is nothing but this particular Y_2. And, this particular Z_2 is nothing but this particular Z_2, that means, starting from the input link, that is, X_1, Y_1 and Z_1, I am able to reach the output link, that is, X_2, Y_2 and Z_2. And, to reach that, I need to take the help of three rotations. All three rotations are taken with respect to the universal coordinate system, that means I need three rotations. Thus, this ball and socket type of joint or the spherical joint is having 3 degrees of freedom or the mobility level of 3.

Thank you.

Robotics
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Lecture – 03
Introduction to Robot and Robotics (Contd.)

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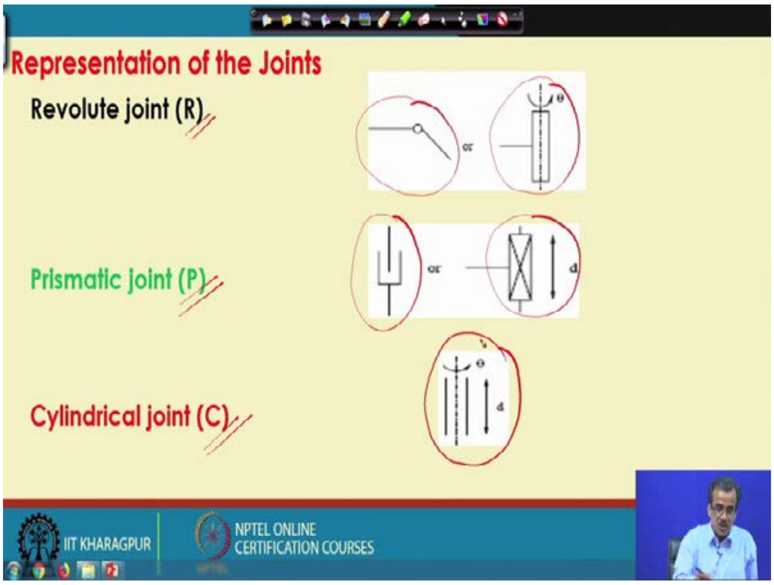

Now, let us see how to represent the different types of joints used in robot with the help of a few symbols, so that we can represent the whole manipulator with the help of these symbols. This revolute joint, we have already discussed, that is denoted by R. And, this particular symbol is also used to represent the revolute joint. This is another symbol, which is also used to represent the revolute joint.

Now, then comes the prismatic joint, that is denoted by P, we use either this particular symbol to represent the prismatic joint or that particular symbol to represent the prismatic joint. Now, then comes the cylindrical joint, that is denoted by C. And here, we can use this particular symbol to represent the cylindrical joint.

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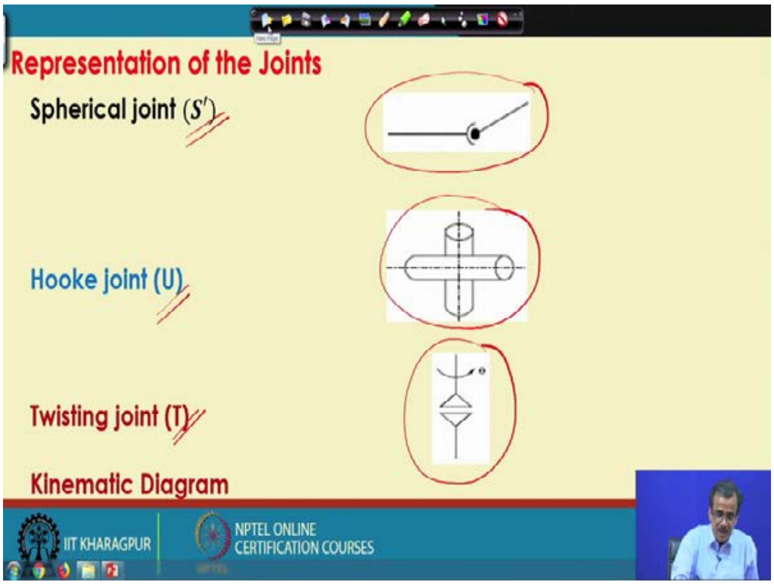

Then comes the spherical joint, which is having 3 degrees of freedom, is represented using this particular symbol S prime, and we can also use this particular symbol to represent the spherical joint. The Hooke joint having 2 degrees of freedom is denoted by U. And, this particular symbol is also used to represent the Hooke joint.

Now, then comes the twisting joint, this is also a rotary joint, that is denoted by T. And, this particular symbol is also used to represent the twisting joint. Now, with the help of these symbols, we can represent the manipulator. So, before we start doing kinematic analysis, what we do is, we try to represent the whole robot or the whole manipulator with the help of some symbols used for the joints.

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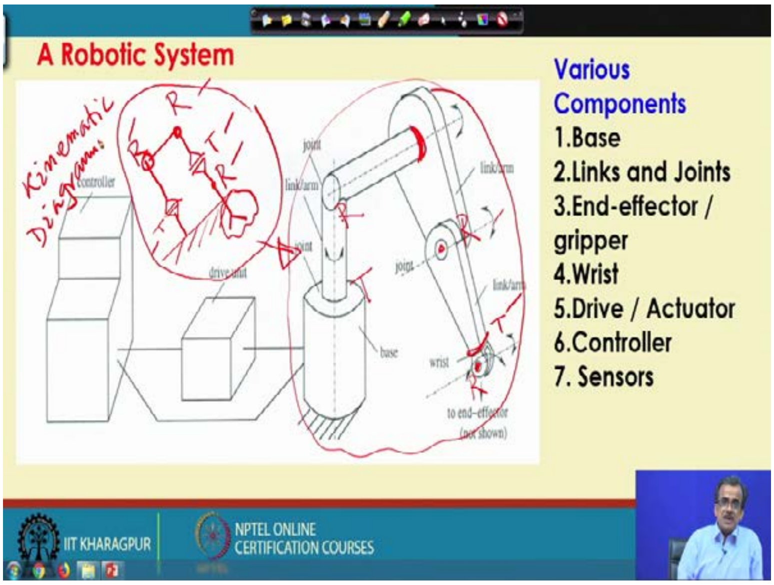

Now, let me take one example here, and to take the example actually, I will have to go back to the previous slides, where we consider a robotic system. Now, here, in this particular robotic system is one serial manipulator. And, the same serial manipulator I just want to represent with the help of the symbols that means, I want to prepare the kinematic diagram of this manipulator or the robot.

Now, let us see how to prepare the kinematic diagram. To prepare the kinematic diagram, what we do is, we start from the base. So, here, I have got a fixed base, so this is denoted by fixed base. And I have got one twisting joint here, so here I have got a twisting joint. So, let me draw one twisting joint with the help of symbol, so this is the symbol for the twisting joint said T.

Next joint is here, that is a revolute joint. Now, remember here, there is no joint actually and this is rigidly connected. Here, there is no joint, the rotary joint is here, that is a revolute joint. So, to represent the revolute joint we take the help of this type of symbol, this is the symbol for the revolute joint. Next joint is here, so this is another revolute joint. So, I am just going to use the symbol for the revolute joint.

The next is the twisting joint, and this twisting joint is the wrist joint, so this is a twisting joint. So, I will have to draw one twisting joint here, so this is a twisting joint. And after that, actually I am just going to connect one end-effector or the gripper here. And, this particular joint will be nothing but a revolute joint. So, there will be a revolute joint here, and after that there will be the end-effector. So, this is actually the symbol for the end-effector.

So, this is the last one is nothing but a revolute joint. So, let me repeat. So, this is the twisting joint, a revolute joint, revolute joint, revolute joint, then we have got the twisting joint. And here, so we have got the revolute joint with the help of which I connect the end-effector. This is what, we mean by the kinematic diagram of this particular robot, OK? So, this is known as the kinematic diagram for this particular robot.

In robotics actually as we mentioned little bit, that there are four modules. And all such modules are actually explained one after another, and these are all dependent also. So, gradually I will be discussing all such things. But, the starting point is the kinematic diagram, based on the kinematic diagram.

So, I am just going to carry out the kinematic analysis, that is, kinematics, based on kinematics; I will be discussing dynamics, based on dynamics; I will be discussing the control. And, once that particular robot is made ready, after that I will try to incorporate intelligence to make it intelligent and autonomous. So, all such things actually, I am just going to discuss one after another.

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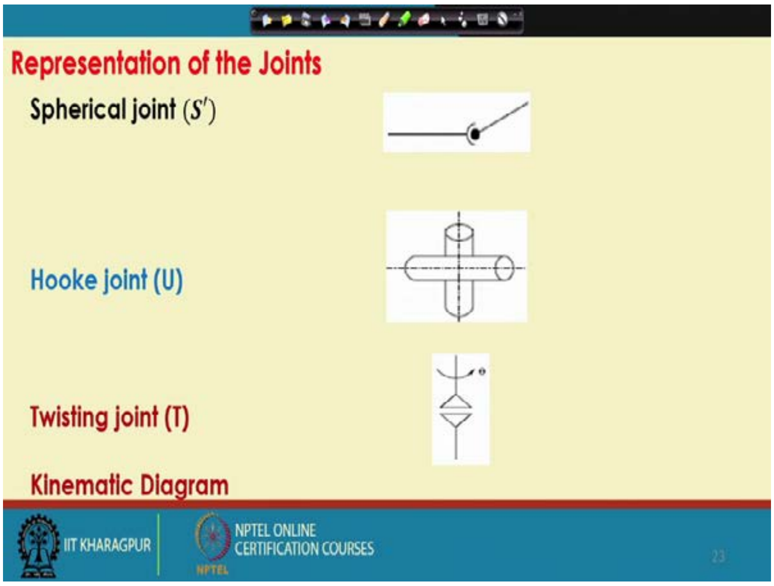

So, let us try to come back to the original discussion, where we stopped. In fact, this is the place. So, we have seen how to prepare the kinematic diagram, and the purpose of kinematic diagram, as I told, that just to represent (with the help of a few symbols), that complicated robotic system. So, this is the purpose of making kinematic diagram.

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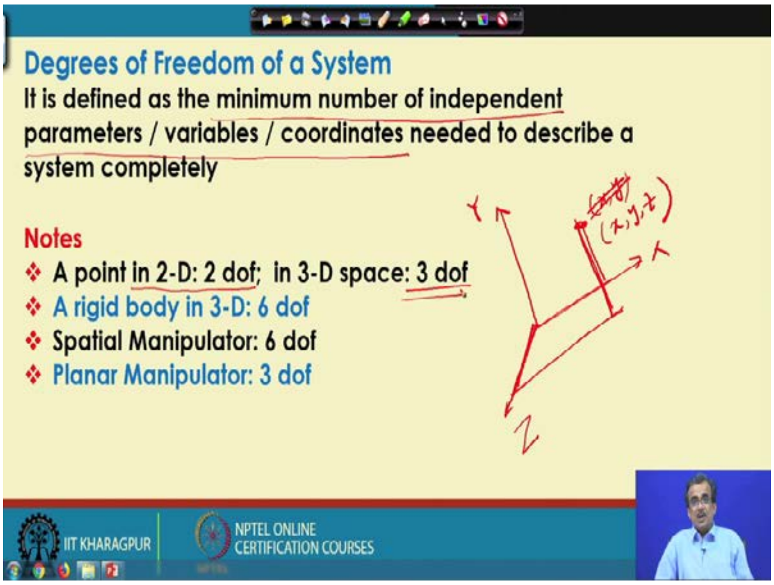

Now, once you have studied the degrees of freedom or connectivity of the different types of robotic joints, I am in a position to discuss about the degrees of freedom of a robotic system. Now, the degrees of freedom of robotic system is defined as the minimum number of independent parameters, variables, or coordinates needed to describe a robotic system completely, and that is nothing but the degrees of freedom of a robotic system.

Now, before I discuss, the degrees of freedom of a robotic system a few preliminaries, which all of you know, I am just going to recapitulate. For example, say a point in 2-D plane has got 2 degrees of freedom. For example, say I have got a 2-D plane like this, say x and y. And if I want to represent a particular point, I need only two information, one is this x information, another is y information. And, supposing that it is having the coordinate (x, y), so I need only two information. So, a point on 2-D has got only 2 degrees of freedom.

Similarly, if I consider a point in 3-D, for example, if I add one more dimension here, say z, that is, x, y and z, so what I need is, the z information also to represent. So, x, y and z information, so all 3 information actually I will have to find out, so, this is one information, this is another information, this is another information.

So, in place of x y, now I need x, y and z. If I consider the 3 dimensions, that means, a point in 3-D space has got 3 degrees of freedom. So, I think this is clear to all of you. Now, a rigid body in 3-D space has got 6 degrees of freedom. So, how to define that, how to how to explain that a rigid body in 3-D space has got 6 degrees of freedom.

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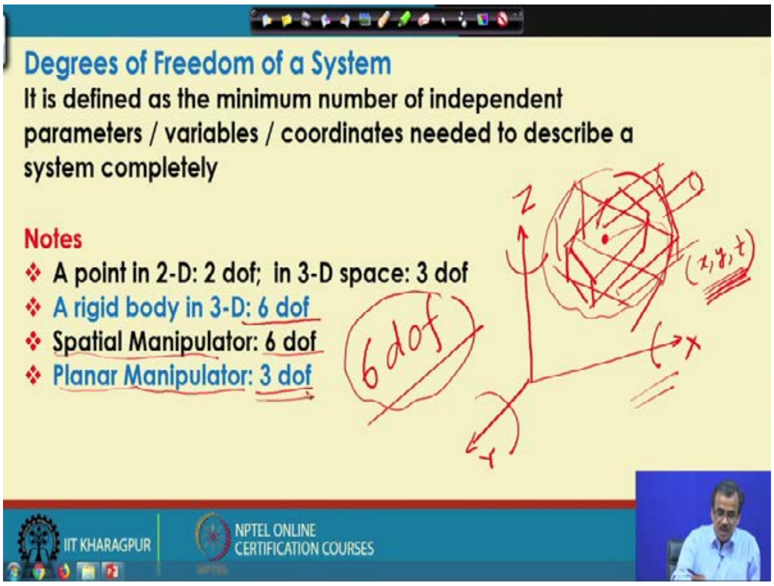

Let me take a very simple example. Supposing that I am once again considering X, Y and Z, so X, Y and Z, in the 3-D space. And, I have got one 3-D object, very simple 3-D object like this. So, this is the 3-D object, which I have. Now, if I want to represent, so this particular 3-D object in this 3-D space, how to represent. To represent this 3-D body in 3-D space, actually what we do is, we first try to find out the mass-center. Now, supposing that the mass-center of this particular 3-D object is this, and it is having the coordinate, say x, y and z.

So, to represent the position of this particular mass-center, I need three information: x, y and z. And, now, this particular 3-D object can have different orientations also, so this is one orientation. Similarly, there could be some other orientations also, this could be another orientation. Now, to represent the orientation, once again I need to take the help of rotation about X, rotation about Z, rotation about Y. So, I need three more information. So, three information for position, and three information for orientation or the rotation, that is why, a 3-D object in 3-D space has got 6 degrees of freedom, OK?.

Now, if I want to manipulate this particular 3-D object in 3-D space. For example, say one serial manipulator is going to come here, just to grip this particular object. Supposing that, it is going to grip it like this. Say, I have got a gripper here, and with the help of this gripper, say I am just going to grip it. Now, with the help of this particular gripper, if I want to grip this particular object, what I will have to do is: this particular gripper should be able to grip this particular 3-D object in different orientations, and different positions, that means, if I want to grip with the help of a serial manipulator.

So, this serial manipulator should have ideally 6 degrees of freedom. And, that is why, most of the industrial robot are having 6 degrees of freedom. Ideally, one industrial spatial manipulator should have 6 degrees of freedom. For example, if I take the example of PUMA, Programmable Universal Machine for Assembly, it should have 6 degrees of freedom, ideally speaking.

And, that is why, actually I have mentioned here, for an ideal spatial manipulator, there should be 6 degrees of freedom. For a planar manipulator, which is working on 2-D plane, it should have ideally 3 degrees of freedom. So, by definition, a spatial ideal manipulator should have 6 degree of degrees of freedom, and a planar manipulator should have 3 degrees of freedom.

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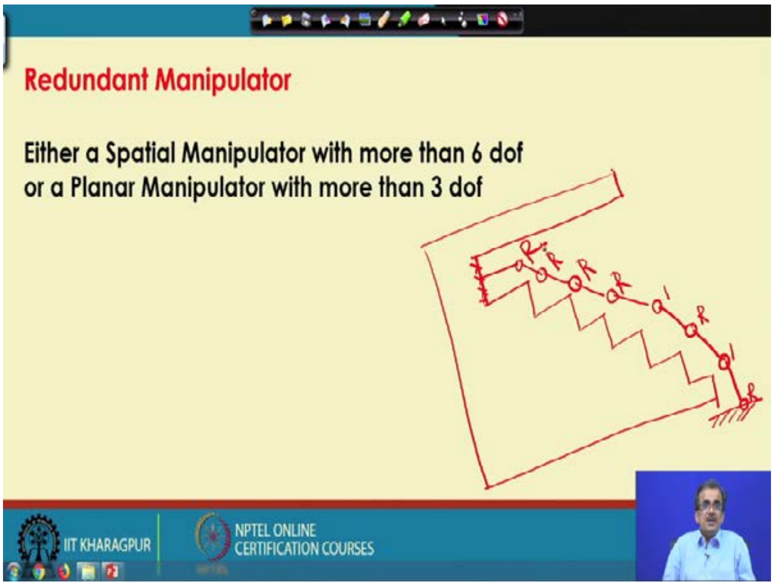

Now, comes the concept of redundant manipulator. Now, remember sometimes to serve a specific purpose, we need to use some sort of redundant manipulator. And, this redundant manipulator, if it is a spatial one, it should have more than 6 degrees of freedom, like 7 degrees of freedom, 8 degrees of freedom. If it is a planar manipulator, it should have more than 3 degrees of freedom; say 4 degrees of freedom, 5 degrees of freedom, and so on. And, as I told, these types of redundant manipulators are used just to serve some specific purposes.

Let me take one very simple example. This is a very practical example. Supposing that, say I am just going to do some sort of welding with the help of a serial manipulator at a place, which is very difficult to reach. Let me take a very hypothetical example. Say this is the place, where I will have to do this particular welding, and this place is so remote, that it is not so easy to reach that particular place. And, supposing that, this is the geometry, and it is so much constrained scenario. And, at this particular position, say I will have to do this particular welding with the help of a serial manipulator. The base of the serial manipulated is here, OK?.

Now, if I want to do the welding here, with the help of a serial manipulator, the welding torch has to be gripped by the end-effector of this particular serial manipulator. And, to reach that particular point the base is here. So, I need to use a number of links, number of joints, so might be one joint, one link another joint, another joint, another joint, another joint, another joint, another joint, and another joint and might be then only I will be able to reach this particular position.

Now, if I use this type of serial manipulator, (which is a closed, sorry) which is an open loop chain. So, how many revolute joints we are using: one revolute, another revolute, another revolute, 4th revolute, 5th revolute, 6th revolute 7th revolute 8th revolute joints here. So, I am using 8 revolute joint here, and that means, this particular manipulator should have more than 6. Now, how to determine that degrees of freedom, I am just going to discuss after some time but, here actually we need to note that the number of the degrees of freedom is more than 6. This is a typical example of the redundant manipulator.

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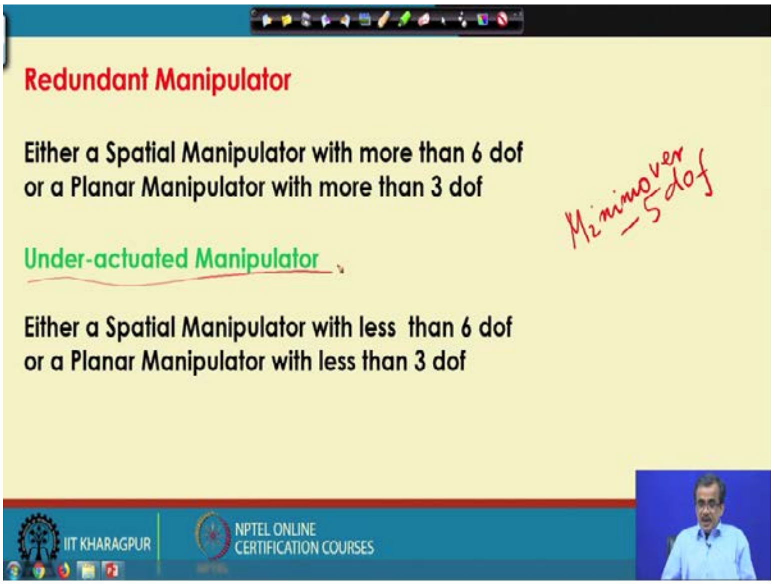

Now, similarly, sometimes we use actually some sort of manipulator, which is under-actuated. Now, by under-actuated manipulator, we mean that this is either a spatial manipulator with less than 6 degrees of freedom or a planar manipulator with less than 3 degrees of freedom. Now, here, if I use a special manipulator with less than 6 degrees of freedom or a planar manipulator less than 3 degrees of freedom, that is called the under-actuated manipulator.

Let me take one example. Supposing that, one manipulator is working in 3-D space, and it is doing some sort of pick and place type of operation. So much accuracy is not actually required. And, here, we can even use one manipulator, having say 5 degrees of freedom. For example, say we have got one manipulator, whose name is Minimover. So, Minimover is actually a manipulator having 5 degrees of freedom, and that is a spatial manipulator, so that is nothing but under-actuated manipulator, OK?

Now, I am just going to take another very practical example, just to find out the difference between the redundant manipulator, and this under-actuated manipulator.

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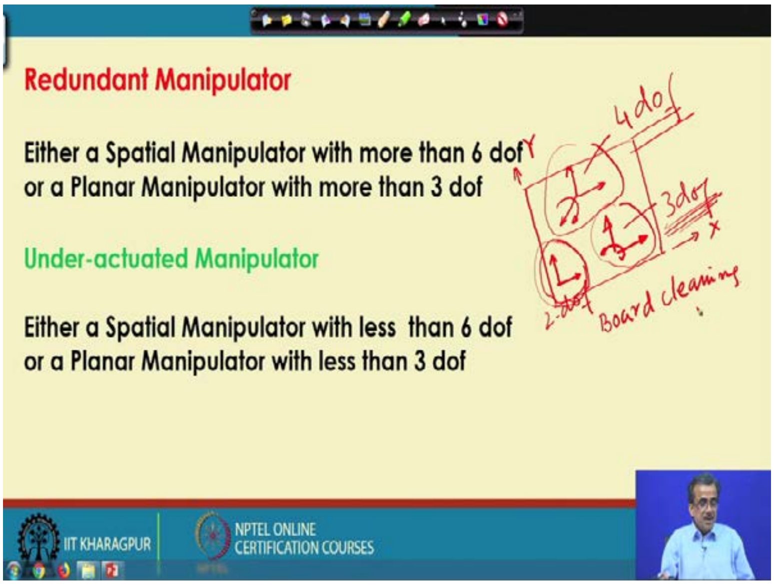

Let me take one task, very simple task. Supposing that, I have got one the board, the white board. Now, on this particular white board, say I have written something, I want to clean it with the help of a duster. Now, what are the different ways, I can clean this particular the board. Now, this particular board is in 2-D. So, this is say the X direction, this is your Y direction, and Z is perpendicular to the board.

Now, if I want to clean this particular board, I can use the duster in different ways, let me take one possibility. For example, say I can use one duster in this particular direction, and this particular direction, only in two directions. So, I will move duster along X, I will move duster along Y, I can clean the board, so this is one way of cleaning the board. Another way of cleaning the board should be as follows: I can move along X, I can move along Y, and I can also rotate about this particular Z, Z is perpendicular to the board, so this is another way of cleaning the board.

Now, I am just going to show another method to clean the board. So, I will move along X, I will move along Y, I will move along this particular Z direction, opposite to the Z, OK, and at the same time, I will just rotate about Z. Are you getting my point? So, for the same task of board cleaning, so what I can do is: I can use three types of serial manipulator.

Now, if I use this particular manipulator, it is having 2 degrees of freedom. If I use this particular manipulator, it is having 3 degrees of freedom. If I use this particular manipulator, it is having 4 degrees of freedom, OK?

Now, this is the 2-D plane. So, ideally speaking, if it is the ideal one, it should have 3 degrees of freedom. So, if I use this manipulator with 3 degrees of freedom, that is an ideal. Planar manipulator for cleaning this particular board, but if I use this particular manipulator, this will be an under-actuated planar manipulator for cleaning the board. And, if I use this particular board having 4 degrees of freedom that will be one redundant planar manipulator used for cleaning the board. I hope, the difference is clear between the ideal manipulator, redundant manipulator, and under-actuated manipulator.

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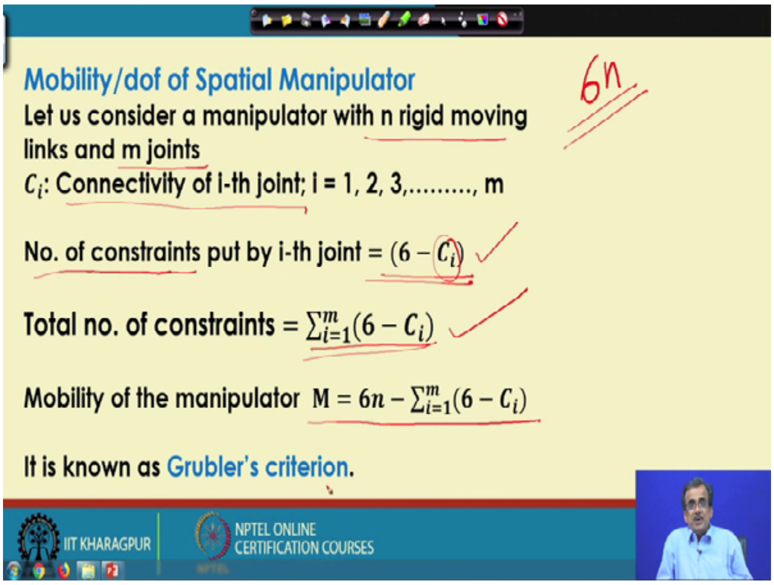

Now, I am just going to discuss the mobility or the degrees of freedom. So, how to do mathematically, calculate the mobility or degrees of freedom of a spatial manipulator. So, I am just going to start with the spatial manipulator, which is walking in 3-D space. Now, let us consider a manipulator with n rigid moving links and m joints. So, there are small n number of rigid links, and I have got small m joints. Now, as I discuss that each rigid body in 3-D space has got 6 degrees of freedom. So, I have got n such rigid links. So, I have got 6 n total degrees of freedom.

Now, C_i is the connectivity of i-th joint. Connectivity of the joint, I have already discussed, i varies from 1 to up to m. Now, a particular joint, say i-th joint, if it is having the connectivity C_i, it is going to put constraint, that is nothing but $(6-C_i)$, once again. So, C_i is the connectivity of the i-th joint. And, this particular i-th joint is going to put constraint, that is nothing but $(6-C_i)$. Similarly, we have got how many joints, small m number of joints. So, each joint is going to put $(6-C_i)$ constraints. So, the total number of constraint will be $\sum(6-C_i)$, where i varies from 1 to m. So, this is the total number of constraints. And, this is the total number of availability.

So, this particular difference is nothing but the mobility of the manipulator denoted by M, and that is nothing but $(6n-\sum 6-C_i)$, where i varies to 1 to m, and this particular formula is very well-known Grubler's criterion. And, by using this particular Grubler's criterion, very easily we can find out, what should be the degrees of freedom of a particular robotic system.

Now, the same thing we can also do it for the planar system to determine mobility or degrees of freedom of a planar manipulator. Now, I am going to consider a planar manipulator, which is working on 2-D plane. And, here, the same n number of moving links and small m number of joints have been considered and connectivity is C_i. The number of constraints put by i-th joint is $(3- C_i)$. And, the total number of constraint is $\sum(3-C_i)$, where i varies from 1 to m. And, the mobility of the manipulator is given by $(3n-\sum 3-C_i)$, where i varies from 1 to m. This is once again the well-known Grubler's criterion.

Now, here, I just want to mention one thing very purposefully, particularly in the previous slide. Let me go to the previous slide. I am using a particular term, that is, the mobility, ok?. So, in place of these degrees of freedom I am using this term: mobility. Now, here I have something to say regarding the concept of mobility and the degrees of freedom.

Now, here, on principle as I told by definition one serial manipulator should have 6 degrees of freedom and one spatial manipulators should have 6 degrees of freedom. Now, supposing that, one redundant manipulator is having certain degrees of freedom, truly speaking, we should not call, it is having 10 degrees of freedom. Instead we should say that, it has got the mobility levels of 10.

By definition, the maximum degrees of freedom can be equal to 6, and that is why, if it is more than 6, we generally use the term: mobility. We say that this particular manipulator is having the mobility level of 10, instead of saying that this serial manipulator is having 10 degrees of freedom. So, I think, it is clear.

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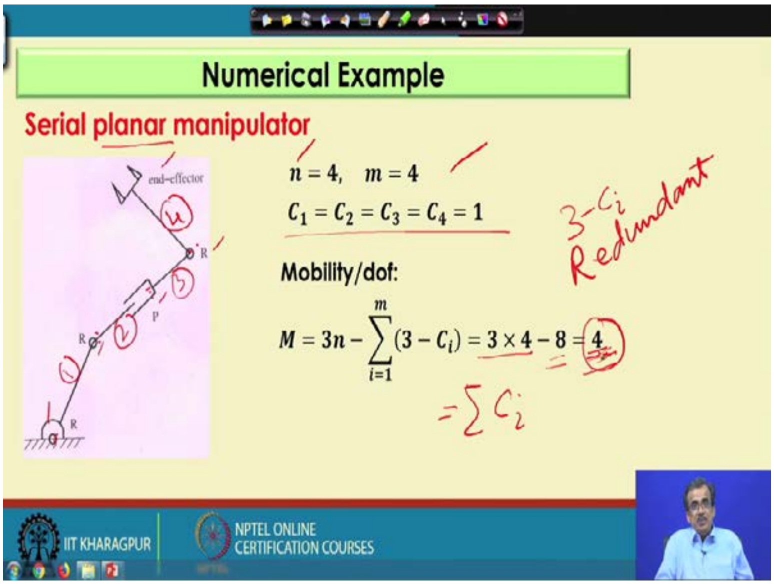

Now, I am just going to solve some numerical examples, just to show you, how to determine the degrees of freedom using the Grubler's criterion for some of the manipulators. Now, this is one serial manipulator, you can see, and here all the links are in series, ok?. So, this is the fixed base first revolute joint, second revolute joint, the linear joint (prismatic joint), the revolute joint, and this is the end-effector.

So, let us try to calculate its degrees of freedom or mobility. Now, here small n is nothing but the number of moving links. For example, 1, 2, 3, 4, so there are four number of moving links. The number of joints small m is equal to 4; 1, 2, 3, 4. Connectivity for each of these particular joints, this is, the revolute joint, prismatic joint is equal to 1, each of the joints is having connectivity of 1.

Now, supposing that, so this particular joint is having one connectivity, then how many constraints it put, this is a planar one. So, the number of constraints it is going to put is nothing but ($3-C_i$), C_i equals to 1. So, it is going to put two constraints. Each of the joint is having one connectivity. So, each of the joint is going to put two constraints, so 2 plus 2, 4 plus 2, 6 plus 2, 8. So, the degrees of freedom or the mobility M is nothing but ($3n-\sum(3-C_i)$), i varies from 1 to m, becomes equal to 4. So, 3 n is nothing but 3 multiplied by 4, and total number of constraint is 8. So, I am getting 4.

Although this is a planar manipulator, it is having 4 degrees of freedom, that means, this is one redundant serial planar manipulator, so this is nothing but the redundant planar serial manipulator. And, another observation we should take. Here, for this serial manipulator the degrees of freedom or the mobility is nothing but 4 and that is nothing but the sum of all C_i values. Like each of these C_i is equal to 1 and if you sum them up you will be getting 4, and this particular condition is true only for the serial manipulator, but not for the parallel manipulator.

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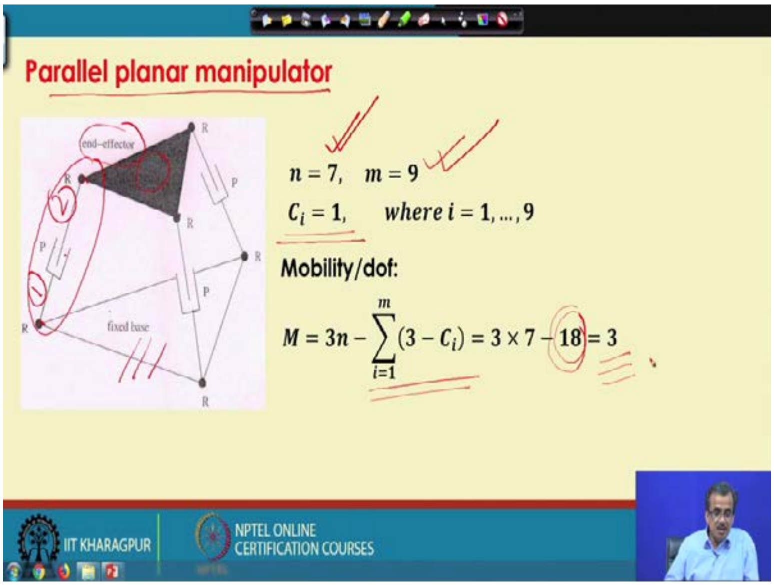

Then, comes your parallel planar manipulator. Now, here this is very simple. So, this is the fixed base and the revolute joint I have. So, there are 3 links and here, at the top, we have got a top plate, and that is nothing but the end-effector. And at each link we have got a revolute joint, one prismatic joint, and one revolute joint. Similarly, we have got one revolute joint, prismatic joint, one revolute joint. And, each link is having how many constraints, how many joints that we will have to count.

Now, here how many links we have, on each leg I have got 1, 2. So, 2 plus 2, 4 plus 2, 6 and this particular end-effector will be considered as one link. So, I have got a total of 6 plus 1, that is, 7 links. And, how many joints we have, on one leg, we have got 1, 2, 3. So, 3 multiplied by 3, so I have got 9 such joints, ok? These are all, the revolute joint, and prismatic joint, and each is having connectivity of 1, that means, each of the joints is going to put, how many constraints 3 minus 1, that is, 2 constraints.

So, each leg is putting how many constraints, 2 constraint here, 2 constraints here, 2 constraints here, 2 plus 2 plus 2. So, one leg is going to give 6 constraints and here also 6, here also 6. So, we have got 18 constraints. So, $\sum(3-C_i)$, i varies from 1 to m, becomes equal to 18. 3 n is 3 multiplied by 7, that is, 21; so, the mobility is coming to be equal to 3. So, this is an ideal parallel planar manipulator. This is the way actually, we can find out the degrees of freedom or the mobility of different types of manipulators.

Thank you.

Robotics
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Lecture – 04
Introduction to Robot and Robotics (Contd.)

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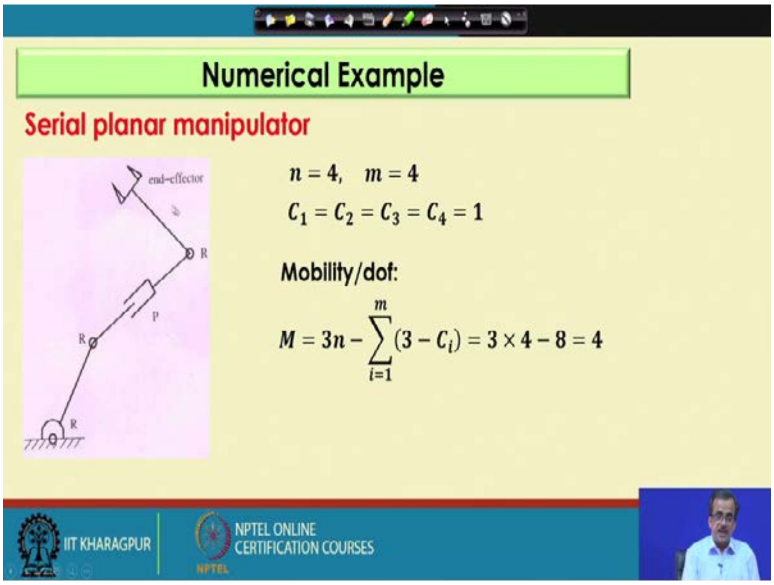

We are discussing how to use the principle of Grubler's criterion, to determine the degrees of freedom of different types of manipulator. Now, we have already seen for the serial planar manipulator. So, we got the degrees of freedom as 4. And here, this is a serial manipulator, because all the links are in series.

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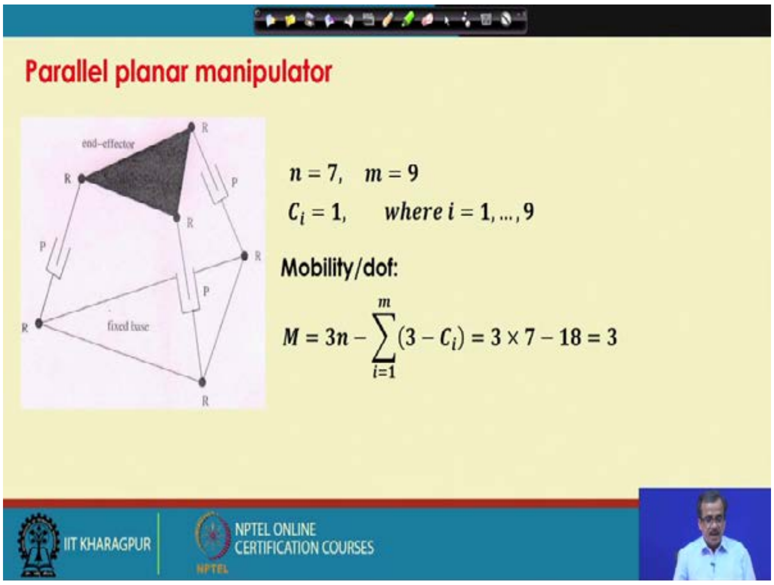

Now, for one very simple parallel manipulator, we also determined what should be the degrees of freedom. And, we got for this particular parallel manipulator, the degrees of freedom as 3.

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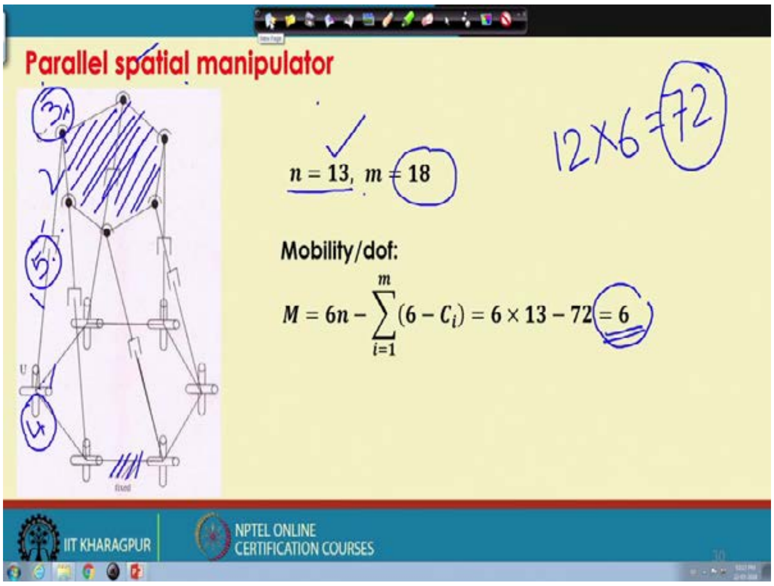

Now, I am just going to take the example of another more complicated parallel manipulator. And, I am just trying to find out, what should be the degrees of freedom for this parallel manipulator using Grubler's criterion. Now, this is nothing but a spatial parallel manipulator. Now, here, we have got 6 legs, and each leg consists of one universal joint or Hooke joint, and one prismatic joint, and we have got one spherical joint. Similarly, we have got 6 such legs. And the top plate is nothing but this, so this is nothing but the top plate for this particular manipulator and the base plate is kept fixed to the ground.

Now, we will have to find out the degrees of freedom of this manipulator. So, what we do is, for each leg, we try to find out, how many constraints it is going to put. Now, before that, let us try to find out, how many joints are there. In one leg, we have got 1, 2, 3. And, similarly we have got 6 such legs. So, 6 multiplied by 3, we have got 18. So, we have got 18 such joints.

And, the number of links we should try to find out, on one leg we have got 1, 2. So, similarly, we have got 6 legs. So, 6 multiplied by 12, and the top plate we will also be considered as one of the links, so you have got the number of links, that is, moving links equals to 12 plus 1, that is, 13. So, we have got small n that is the number of moving links is equal to 13 and the number of joint, that is, m is kept equal to 18.

Now, here, we will have to find out, how many constraints are put by one leg. So, this is the universal joint. Now, this universal joint has got 2 degrees of freedom, and this is a spatial manipulator, so this joint is going to put 6 minus 2, that is, 4 constraints. Similarly, so this particular joint is a prismatic joint is having only one degree of freedom, and it is going to put 6 minus 1, that is, 5 constraints. And, this particular joint is a spherical joint, which is having 3 degrees of freedom, and it is going to put 6 minus 3 that is nothing but 3 constraints.

So, the total number of constraints put by one leg, that is nothing but 4 plus 5 that is 9 plus 3, that is, 12. So, one leg is going to put 12 constraints. And, similarly, we have got 6 such legs, so it is going to put 12 multiplied by 6, that is 72 constraints. Now, we have already discussed, that we have got 13 number of moving links. So, the mobility or the degrees of freedom is nothing but $6n-\sum(6-C_i)$, where I varies from 1 to m, and C_i is nothing but the connectivity. So, this will become equal to 6 multiplied by 13 that is 78 minus 72 and, that is, equals to 6.

So, this parallel manipulator is having 6 degrees of freedom, and this is a spatial manipulator. So, this is nothing but an ideal parallel spatial manipulator having 6 degrees of freedom, that means, the top plate can have 3 translations, and there could be 3 rotation also with respect to the fixed ground. So, this is popularly known as the Stewart platform, which is generally used for the training purpose of the trainee pilot in aircraft. So, it has got other practical applications also. So, this is the way, actually we can use the Grubler's criterion to find out the degrees of freedom or the mobility of the robotic system.

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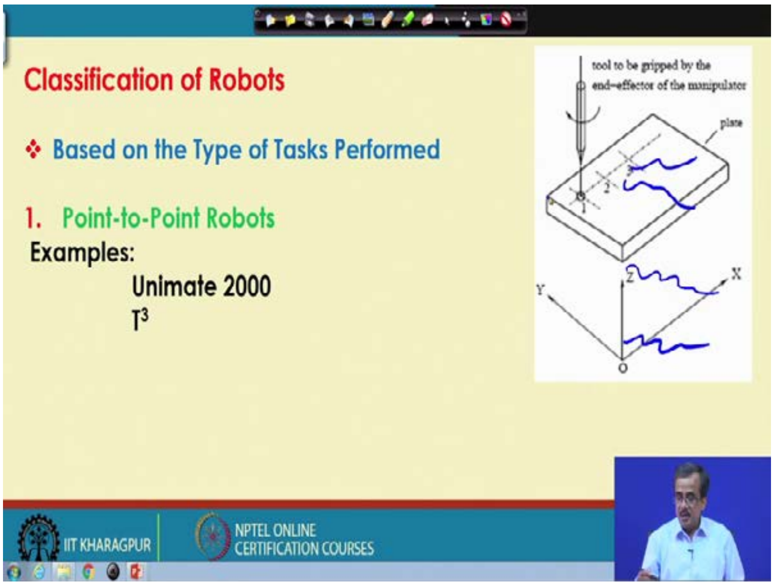

Now, I am just going to discuss, the classification of robots. Now, the robots are classified in a number of ways, if you see the literature, we have got different types of robots. Now, based on the type of task it performs, the robots are classified into two groups, one is called the point-to-point robot, and we have got the continuous path robot. Now, I am just going to discuss, the working principle of this particular the point-to-point robot.

Now, let me take one example, very simple example. Supposing that, I have got a steel plate, and on this particular steel plate, so I will have to make some drilled holes at some pre-specified locations; say location 1, location 2, and location 3 with the help of say one cutter, that is the twisted drill bit. Now, this particular twisted drill bit will be gripped by the end-effector or the gripper of the manipulator.

Now, if I want to make the drilled hole at location 1, the tip of this twisted drill bit should be able to coincide with the center of this particular hole. And, once it is made coincident, now we can be rotate this particular cutting tool, say either in clockwise or anticlockwise. Supposing that, I am rotating it clockwise, so it is going to generate that particular drilled hole. And, once that particular hole has been drilled, so what we do is, we rotate the cutter in the reverse direction, and the tool will be withdrawn from this particular the job.

Now, once the hole has been drilled at location 1, now we go to location 2, and repeat the process. And, the same process we also repeat for the point 3. Now, here after this particular the hole has been drill at location 1, the tool is withdrawn from the job, and then we move to location 2, as the tool is withdrawn from the job, so the tool is not in touch with the job continuously, this is known as the point-to-point task.

So, for this point-to-point task, we use a special type of robot, and that is called the point-to-point robot. Now, this Unimate 2000, then T^3, that is, The Tomorrow Tool, these are the typical examples of this type of point-to-point robots.

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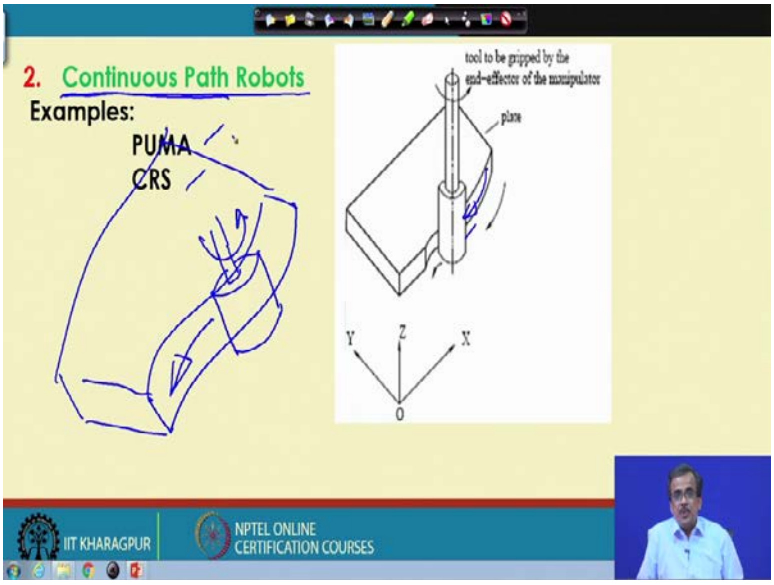

Now, let us try to explain the working principle of the continuous path robot. Now, here the tool will be in touch with the job continuously, and supposing that, I am just going to cut a complicated profile on say one side of a steel plate. Now, if I just draw this particular thing in a slightly different way, supposing that, I have got a profile, which is to be cut on one side of a steel plate. So, this is the steel plate say and here, so I will have to cut this particular profile. The way we cut is, we use one milling cutter and this milling cutter, so this we will be gripped by the end-effector of this manipulator.

Now, this milling cutter will have to rotate, and at the same time, it should trace this particular complicated profile. And, here, we can use this type of milling cutter. Now, here what we can do is, this particular end, we grip with the help of a gripper or the end-effector, and we generate the required motions, that is, the rotation and it will be able to trace this complicated profile while cutting.

Now, during this machining operation, so this particular tool is in touch with the job continuously, and that is why, this is known as continuous path task. And the robots, which are typically designed for this type of task is known as the continuous path robot. Now, the typical example for continuous path robot is your PUMA, CRS, so these are all continuous path robot. Now, here I just want to make one comment. Now, supposing that, I have got one continuous path robot, the same continuous path robot can also be used as a point-to-point robot, but the reverse is not true. And, so we have got both point-to-point robot as well as the continuous path robot.

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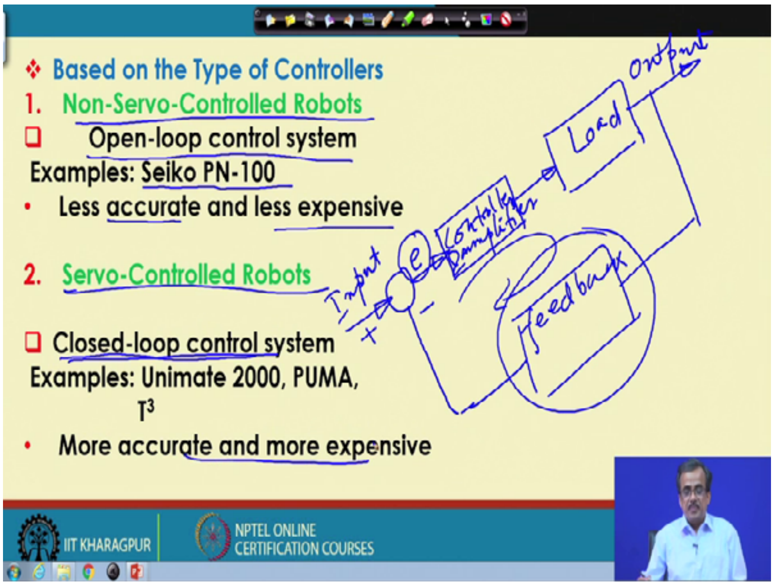

Now, another classification like based on the type of controller, which is generally used. So, we divide these robots into two groups; one is called the non-servo-controlled robots and another is known as servo-controlled robots. So, we have got the non-servo-controlled robots, and servo-controlled robots. So, in non-servo-control robots, we use open loop control system. Now, in open-loop control system, we generally do not measure the output for the purpose of comparison, and just to find out the error. Now, this error is neither measured nor compared and fedback for the purpose of compensation of this error.

On the other hand, in case of servo-control robots, we use some feedback device, and we use closed-loop control system. Now, here, in closed-loop control system, actually what we do is, we measure the output for the purpose of comparison. We try to find out the error, and this particular error is fed back, and we try to minimize this particular error, and that is the principle of the closed-loop control system.

Now, here in robots, if you want to perform some very precise tasks, we will have to go for the servo-controlled robot, and it has got the closed-loop control system. Now, regarding the closed-loop control system, the way it works is as follows. Supposing that, I have got the controller; say this is nothing but the controller, the block diagram for this controller. And here, I have got the mechanical load, which I am just going to handle.

Now, what you do is, we try to give some sort of input here, through some error junction. So, here, we have got the input and based on this particular input, so we will be getting some output here. Now, this particular output will be measured, and output will be fed back with the help of one feedback circuit, so this is nothing but the feedback. And, this is compared here, and we try to find out the error here, that is, e. And, in this particular summing junction, we try to compare, and try to find out how much is this particular error.

So, this error will pass through the controller and amplifier. So, generally we use amplifier also, and once again it will pass to the load, and we will be getting some output. And, this particular cycle will go on and go on, and we will be getting very accurate movement at the end. This is what we follow in closed-loop control system. But, in open-loop control system, this feedback circuit is absent.

So, we have got two types of robots, non-servo- controlled robots, where we use open-loop control system, the examples of which are Seiko PN-100. As there is no error compensation, it is less accurate and less expensive, because there is no such feedback device. On the other hand, the servo- controlled robot like Unimate 2000, PUMA, then T^3 are more accurate and more expensive.

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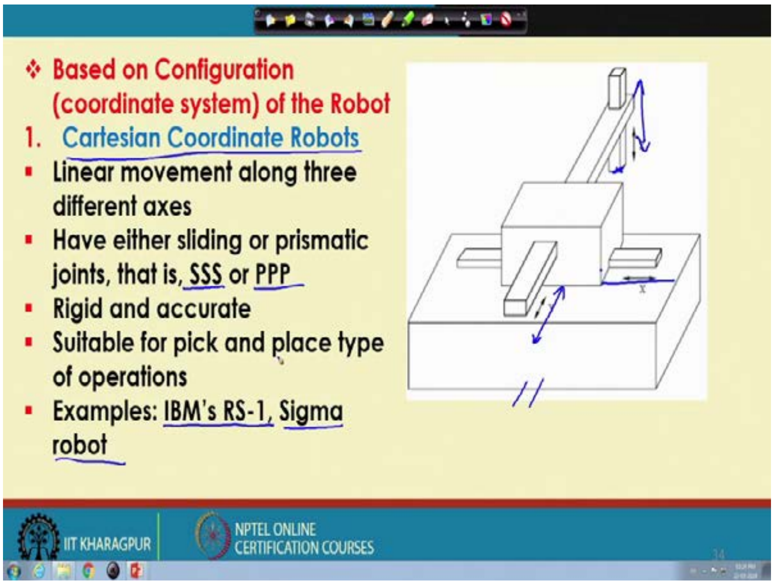

Now, the next classification is based on actually the type of the coordinate system, which is generally used. Now, based on the coordinate system used in robots, the robots are classified into four sub-groups; one is called the Cartesian coordinate robot, then we have got the cylindrical coordinate robot, then polar coordinate robot, and we have got the revolute coordinate robot.

Now, all such robots, I am just going to discuss one after another. So, based on the coordinate system as I told, there are four types of robots, the first one, that is, your Cartesian coordinate robot. Now, here, this schematic view shows a Cartesian coordinate robot. Now, for this type of robot, we have got the linear movement along the X direction, the linear movement along the Y direction, and we have got the linear movement along the Z direction. So, along X, Y and Z, we have got the three linear movements, and they are independent. And, this type of robot is known as the Cartesian coordinate robot.

Now, this particular linear joint it could be either prismatic or sliding. Now, if I use all three are prismatic, so this is called PPP robot and if I use sliding joint, so that it is called SSS robot. Sometimes we use a combination of P and S also. Now, here, as in this particular robot, we use prismatic joint, this robot is very rigid and very accurate. So, this is the end-effector and this is the fixed base. So, if we want very accurate movement, we can use this type of robot. And, this is suitable for pick and place type of operation.

Now, a typical example for this type of Cartesian coordinate robot is IBM's RS-1, then Sigma robot from Olivetti. Olivetti is actually the name of the robot manufacturer. They manufacture one Cartesian coordinate robot, which is known as the Sigma robot. So, here in Cartesian coordinate system or Cartesian coordinate robot, we get three linear movement. And, this robot as I mentioned is very rigid and accurate. We can use this robot in the shop-floor. So, if something is lying on the floor, with the help of this, we will be able to pick that particular object.

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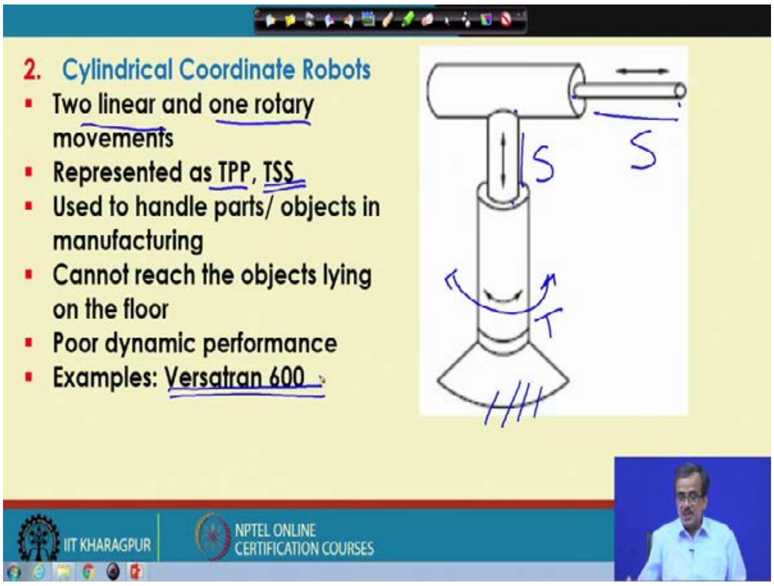

The next is the cylindrical coordinate robot. Now, here, we have got two linear, and one rotary joints. So, here we can see that we have got one linear joint, another linear joint, and we have got one rotary joint. Now, this rotary joint with respect to the fixed base is nothing but a twisting joint. And, this is actually the linear joint, say it is a sliding joint. And, this is another linear joint, say this is the sliding joint. So, this particular robot is known as TSS robot. Now, in place of sliding joint, I can also use the prismatic joint. Here also, I can use the prismatic joint, then it will be called TPP.

Now, if you see this particular robot, there is actually one problem, the way it works. So, with the help of this joint, I will be getting the maximum horizontal reach, and the minimum horizontal reach. Similarly, here, I will be getting the maximum vertical reach, and the minimum vertical reach. And, for this type of robot, if something is lying on the shop-floor, so it will not be able to pick that particular object.

And, it has got another problem, that problem is related to this rotary joint. Due to the presence of this particular rotary joint, the dynamic performance of this particular robot is poor, compared to Cartesian coordinate robot. Now, here, this Versatran 600 is a very typical example for this type of cylindrical coordinate robot. So, this is the way actually, this particular cylindrical coordinate robot is working.

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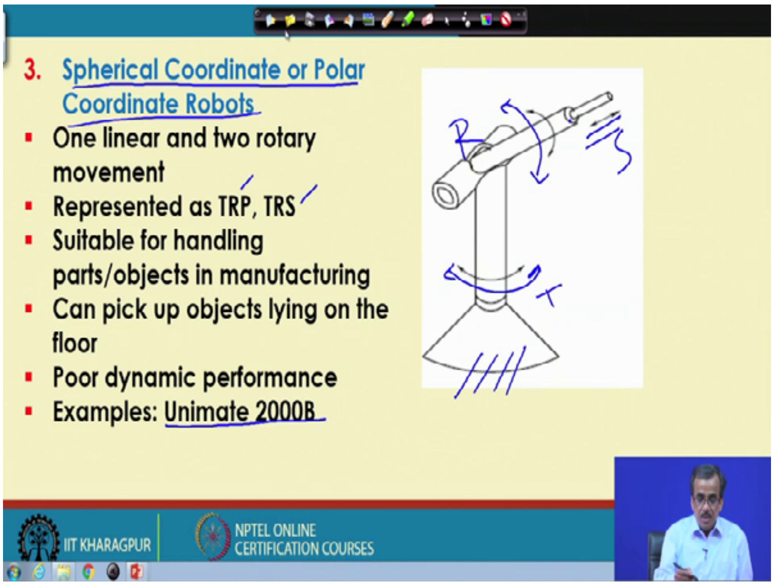

Now, then comes the spherical coordinate robot or polar coordinate robot. Now, here, we have got one linear joint. So, here we have got a linear joint and we have got two rotary joints, so, this is nothing but a twisting joint. And, we have got one revolute joint here with the help of which it can rotate something like this. Now, with the help of this linear joint, so I can represent what should be the maximum horizontal reach, and what should be the minimum horizontal reach.

Similarly, with the help of this revolute joint, I can find out what should be the maximum vertical reach, and what should be the minimum vertical reach. And here, with the help of this particular twisting joint, I can rotate with respect to the fixed base. Now, in this robot, if I use T here, R here, and say S here, this is known as TRS robot. And, if I use prismatic joint, this is known as TRP joint. Now, this robot is suitable for picking some objects, which are lying on the subfloor. And, but here, we have got another problem, the same problem like your poor dynamic performance due to this rotary joint. Now, a typical example for this type of spherical coordinate robot is Unimate 2000B.

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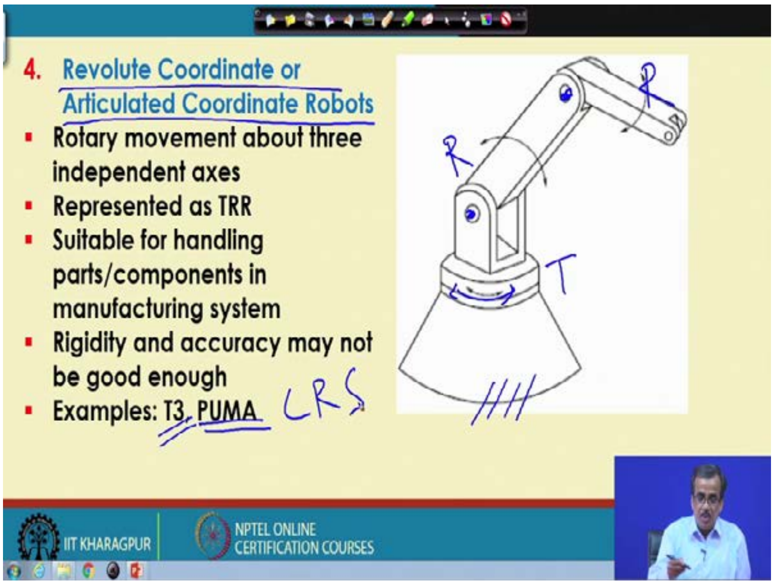

Now, I am just going to discuss another robot, which is also very frequently used, and that is called the revolute coordinate robot or articulated robot. So, this revolute coordinate robot, we have got three rotary joints. For example, say this is the schematic view of the revolute coordinate robot, now this is the fixed base. So, with respect to the fixed base we have got a twisting joint here, similarly we have got a revolute joint here, and we have got another revolute joint here, so we have got a revolute here, revolute here, and we have got the twisting here, and this is known as actually TRR robot.

And, this type of robot is actually very much used in industries. Nowadays to solve a variety of problems like pick and place type of operation or if we want to do some sort of drilling, milling that types of operation, this type of robot along with some more attachments are very frequently used. But, here once again, due to the presence of this rotary joint, its dynamic performance may not be so good.

Now, a typical example of this type of robot could be your PUMA, that is, Programmable Universal Machine Power Assembly, then T^3, The Tomorrow Tool, then comes CRS is another example of this type of revolute coordinate robot. Now, this particular robot, as I told, is very frequently used in industries.

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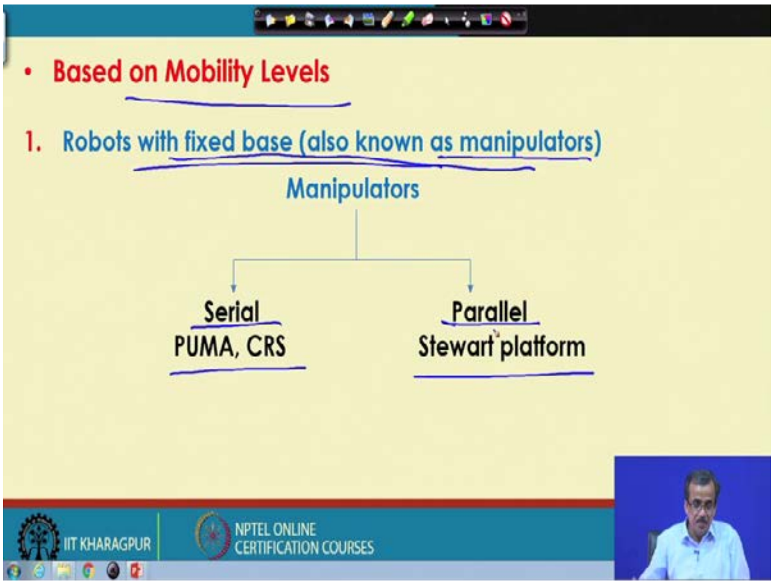

Now, another classification I am just going to discuss, and this is based on actually the mobility levels. Now, based on the mobility levels, the robots are classified into two groups, robot with fixed base, and the robot with moving base. Now, the robot with moving base, I am just going to discuss after some time.

Now, let me just concentrate on the robot with fixed base, and as I told these are also known as the manipulators. Now, these manipulators could be either serial manipulator like PUMA, CRS or it could be parallel manipulator like the Stewart platform. Now, both the things I have already discussed a little bit. For the serial manipulator, the links should be in series, the joints are in series.

On the other hand, for this parallel manipulator, the links will be in parallel and I can compare the load carrying capacity of this serial manipulator, and the parallel manipulator. The load carrying capacity of the parallel manipulator will be more compared to that of the the serial manipulator.

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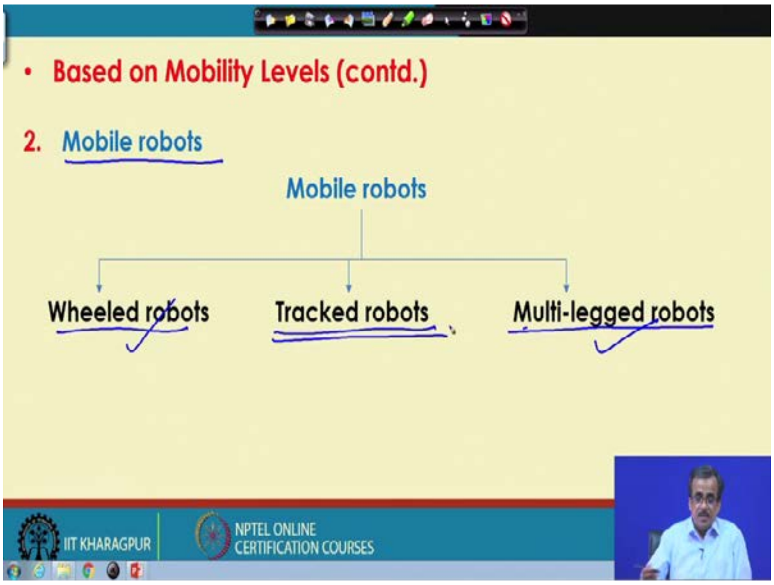

Now, I am just going to discuss the robot with moving base. Now, the robot with moving base, these are very popularly known as the mobile robots. Now, the mobile robots could be either the wheeled robots, there could be tracked robots (tracked vehicles), or there could be multi-legged robots. Supposing that the terrain is perfectly smooth, now for the smooth terrain, we can go for the wheeled robot. Now, if it is perfectly rough, there are many such ups and downs, staircases, so it is better to go for the multi-legged robots like 4-legged robot, 6-legged robot and so on. And, if the terrain is in between, that is, neither very smooth nor very rough, we can go for some sort of tracked vehicle.

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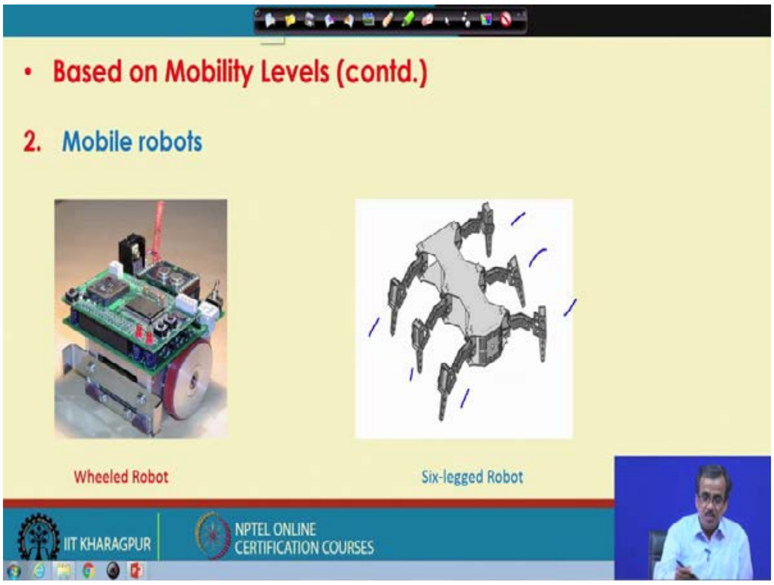

Now, here I am just going to take the example of one wheeled robot. So, this is one actually two wheeled one caster robot, it is a very simple wheeled robot. So, one wheel we can we can see here, the other wheel is on other side, and below that, there is one caster also, that is nothing but the support, so this is a typical example of wheeled robot.

Now, similarly, here I am just going to show the schematic view of a six-legged robot. Now, here you can see, it has got six-legs, and each of these particular legs are generally having 3 degrees of freedom, and this is actually the trunk for this particular the six-legged robot.

Now, depending on this particular duty factor, sometimes like 4 out of 6 will be on ground; sometimes 3 out of 6 will be on ground, and so on. So, these are actually you are the mobile robots. Similarly, we have got some other type of mobile robots also.

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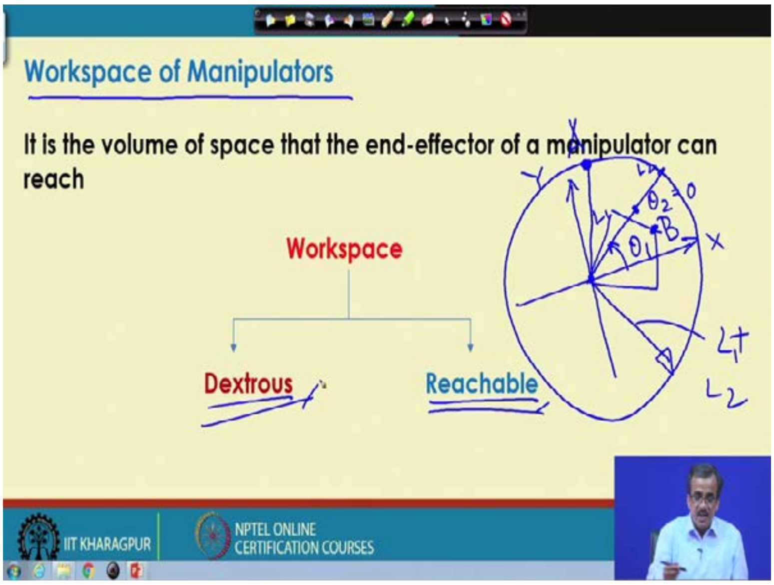

So, we have seen the classification of the robots. Now, if you see the literature, we have got different types of robots. And, as I discussed that these particular robots are classified in different ways, and we get different types of robots. Now, I am just going to discuss with another topic, which is very important, and it is little bit difficult to understand also to imagine also. Now, let us start with that, and let us try to see, how to determine the workspace of a manipulator. So, by a workspace we mean the volume of space that the end-effector of a manipulator can reach.

Now, let me take a very simple example. Now, if I consider that my hand is nothing but a serial manipulator, and this is nothing but the end-effector of the serial manipulator, so this is the fixed base. So, with respect to the fixed base, I can move this particular end-effector like this, so I can go to the top; I can go to this side; I can come to this side; I can go to bottom; I can go to up. So, this particular end-effector is having some locus, and it is going to maintain one volume of space. Now, this particular space has got a volume, and that is actually the workspace of this manipulator, ok?.

Now, if you see this particular workspace, workspace could be either dextrous workspace, or it could be the reachable workspace. Now, to define these two terms like the dextrous workspace and reachable workspace. Let me take one very practical example.

Now, let me take the example of a very simple manipulator. For example, say I have got one serial manipulator having say 2 degrees of freedom. So, this is X direction, this is Y direction in Cartesian. And, supposing that, the joints are such that, this is my the first links say L_1, say L_1 is the length of the first link, and L_2 is the length of the second link, say, this is, L_2.

Now, let us consider here, the angle between L_1 and L_2 has been assumed to be equal to 0. So, as if this will act as a manipulator having only one degree of freedom, that means, there will be only one joint angle theta and supposing that, so this theta can vary through say 360. Now, if I vary through this particular θ_1 through 360 degree, and of course, I have got θ_2, but θ_2 has been taken to be equal to 0.

So, if I just rotate, then there is a possibility that I will be getting the work plane, which is a circle with the radius nothing but $L_1 + L_2$. So, this $L_1 + L_2$ will be the radius of this particular circle, and this is nothing but the work plane, because this is in 2-D plane for this manipulator. Now, I am just going to take two points. Now, supposing that, I am just going to consider a point here, lying on the boundary of the circle and that particular point is a denoted by A. And, I am just going to take another point say point B, and that is lying inside the circle.

Now, if I want to reach this particular point A, I need a particular configuration, that is your theta 1 and theta 2, will have some values. On the other hand, to reach the point B, there could be two different configurations, one is this configuration, another could be your this particular configuration. And, now, I say that the point A is a point, which is lying on the reachable workspace for this particular manipulator. And point B is a point, which is lying on the dextrous workspace of this particular manipulator. Now, let us try to see the definitions of dextrous workspace and reachable workspace.

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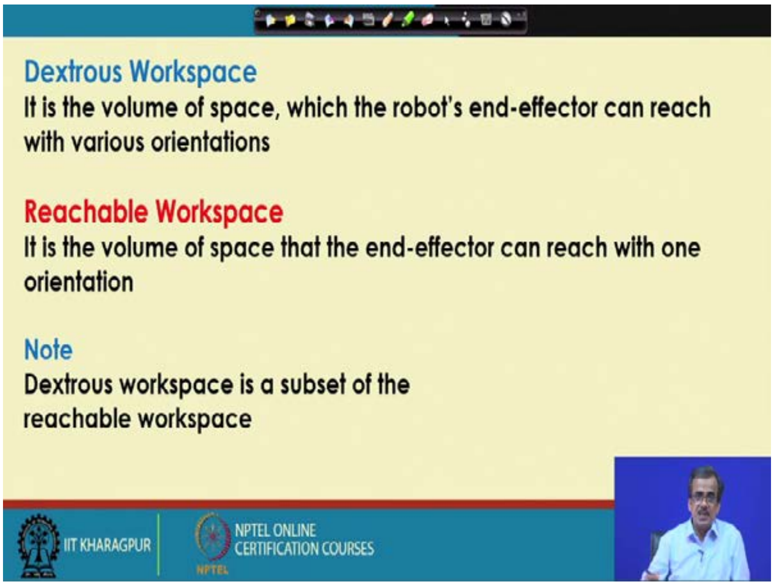

Now, if you see, the dextrous workspace is nothing but the volume of space that the robot's end-effector can reach with different combinations of the joint angles. On the other hand, the reachable workspace is that volume of space that the end-effector can reach with one orientation. And, this particular reachable point is lying on the boundary of the circle.

On the other hand, the dextrous point is lying inside that particular circle. And here, I have put one note, that dextrous workspace is a subset of the reachable workspace. So, the reachable workspace is the larger workspace, bigger workspace. And, dextrous workspace is nothing but a smaller workspace, so dextrous workspace is nothing but the subset of a reachable workspace.

Thank you.

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Lecture - 05
Introduction to Robot and Robotics (Contd.)

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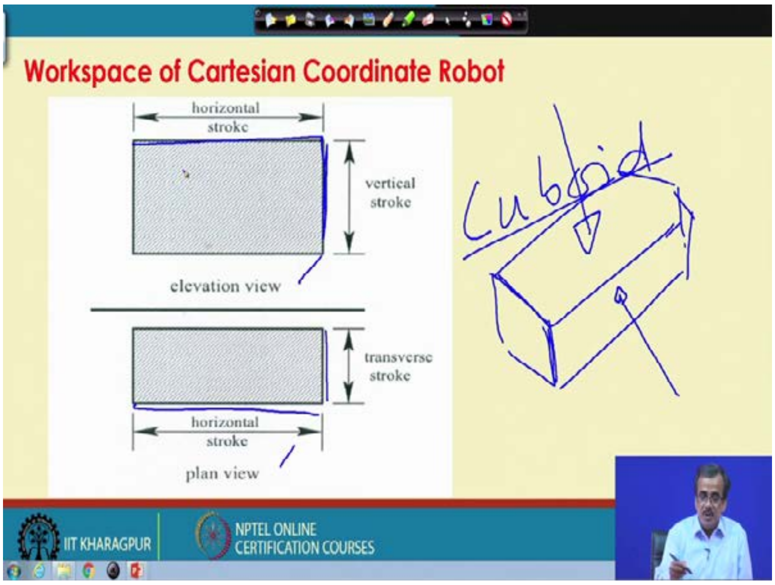

Now, I am just going to find out the workspace for different types of robots. We have already discussed that based on the coordinate system, the robots are classified into four groups. So, we have got the Cartesian Coordinate Robot, then comes Cylindrical Coordinate Robot, Spherical Coordinate Robot and Revolute Coordinate Robot.

Now, I am just going to spend some time to find out what should be their workspace. Let us first try to concentrate on the Cartesian coordinate robot. Now, let us see the picture once again, of the Cartesian Coordinate Robot. So, this is actually the picture for the Cartesian coordinate robot and I am going to find out its workspace.

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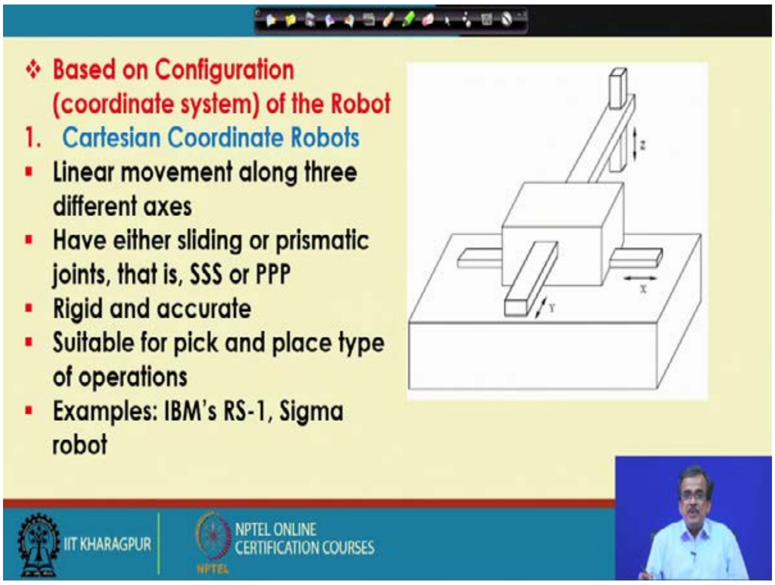

Now, here, there are three linear joints. So, very easily we can imagine that the workspace for this robot will be nothing but a cuboid. So, the workspace for this particular robot will be a cuboid. And, this particular cuboid, I can draw very easily. So, this is nothing but the cuboid and that will be the workspace for this manipulator

Now, here for some of the robots, you will be getting the workspace that is so much complicated that it becomes bit difficult to visualize its three dimensional view. And, that is why, actually what we do is, we take the help of at least two views like elevation view and plan view; just to identify or just to imagine, what should be the workspace for a particular manipulator

Now, for this Cartesian coordinate system, as I mentioned that the workspace is nothing but a cuboid. Now, if I take the elevation view, that means, I am just going to take the view in this particular direction. So, I will be getting this horizontal stroke. So, this is nothing but the horizontal stroke and this is the vertical stroke. So, I will be getting the vertical stroke. And, on the plan view, that means, if I take the view from the top, I will be getting this as the horizontal stroke and this transverse stroke will be nothing but this.

So, the same 3D view, I am just going to draw here with the help of this elevation view and plan view. And, I am just going to take the help of this just to indicate and imagine the workspace for the complicated manipulator. So, for this particular Cartesian coordinate system, it is very simple the workspace is nothing but the cuboid

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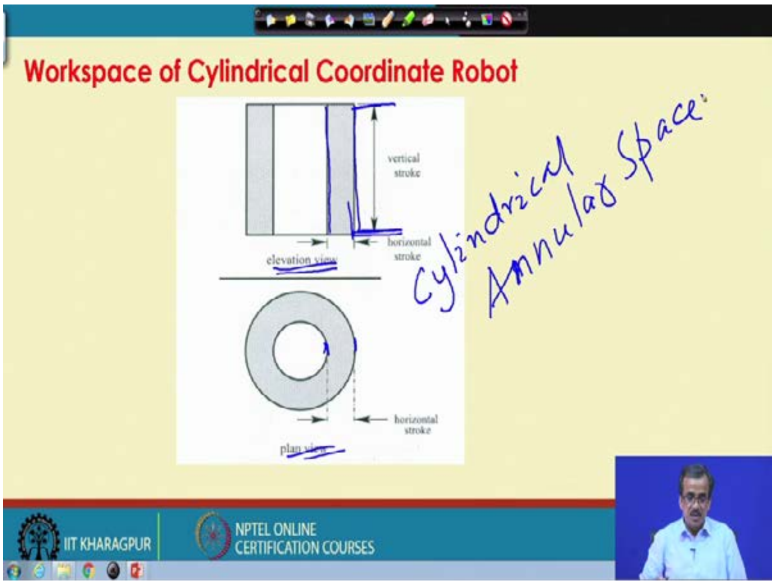

Now, I am going to consider the workspace for the Cylindrical coordinate robot. Now, if you remember the cylindrical coordinate robot, it was something like this. Let us try to see the cylindrical coordinate robot once again.

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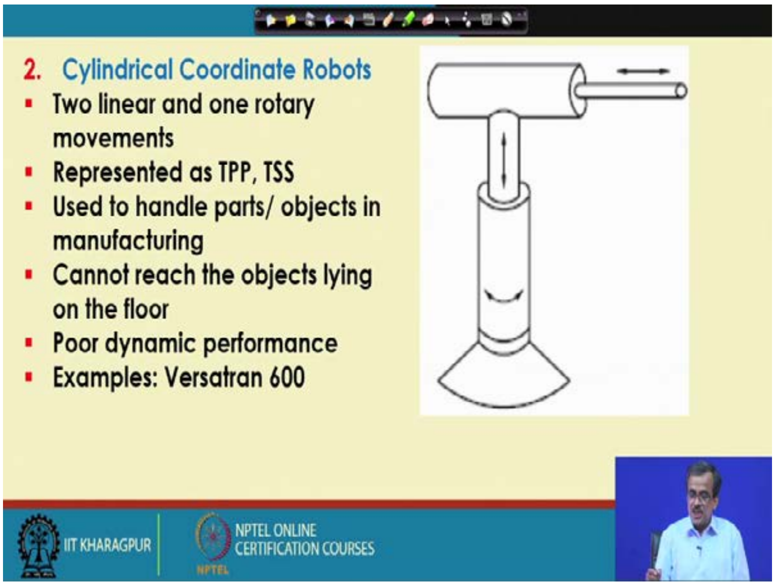

So, this is the cylindrical coordinate robot. As I told, this is going to show us the maximum horizontal reach and the minimum horizontal reach, and this will give the vertical reach and here, there will be some rotation. So, corresponding to this particular cylindrical coordinate robot, I am just going to imagine the workspace.

Now, as I told that the maximum horizontal reach will be identified. So, this is actually the maximum horizontal reach. This is actually the minimum horizontal reach. This is the elevation view and this is the plan view. So, here, I am going to consider 360 degrees rotation with respect to the fixed base and that is why, we are getting this type of circle here, ok?. So, this is the plan view. And, on the plan view also, once again, we will be getting the maximum horizontal reach and the minimum horizontal reach. In the elevation, I will be getting the maximum vertical reach and the minimum vertical reach and the way it is working.

So, it has got two linear and actually one rotary, that is, the twisting and this particular workspace, which will be getting is nothing but a cylindrical annular space. So, this workspace is nothing but the cylindrical annular space

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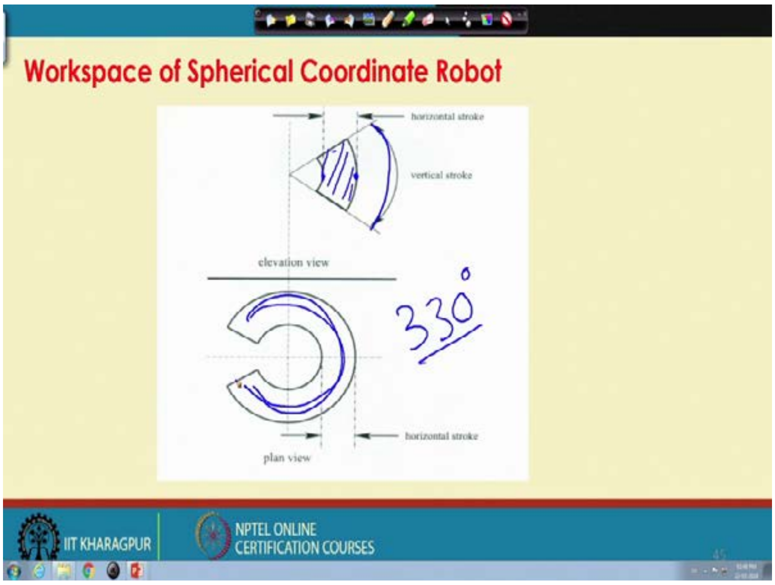

Now, next we try to find out the workspace for some other robot like the Spherical coordinate robot. Now, let us try to see the picture of the spherical coordinate robot once again. So, this is actually the spherical coordinate robot. So, here I have got only one linear joint.

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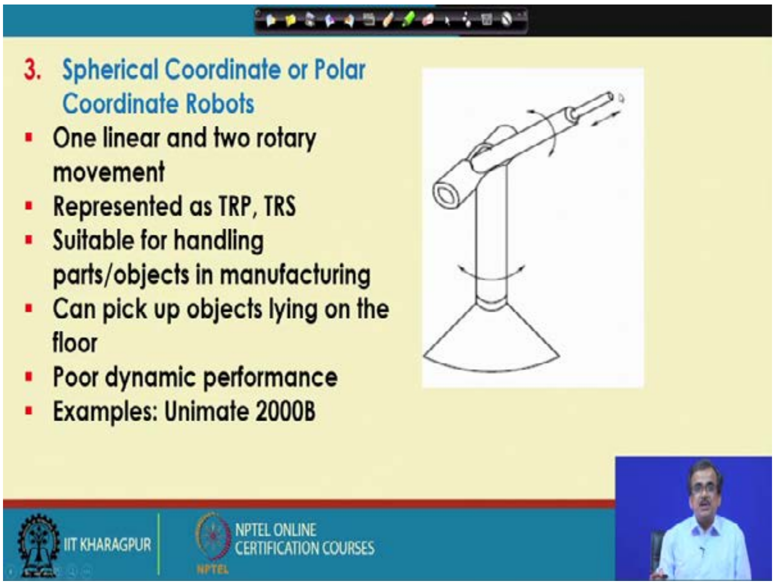

This is going to give the maximum horizontal reach and the minimum horizontal reach. So, here, we have got one revolute joint and here, we have got one twisting joint ok. So, I am trying to find out the workspace for this manipulator.

Now, as I told that with the help of this revolute joint; so, I will be getting the maximum vertical reach and the minimum vertical reach. So, let us try to see the workspace. Now, here, if you see on the elevation view, I will be getting the maximum horizontal reach that is something like this. And, this is the minimum horizontal reach and the vertical reach. So, this is will be the vertical reach and this vertical reach will be obtained with the help of that revolute joint. And, here, with the help of twisting joint, I am considering more or less say 330 degrees rotation, but not 360 and that is why, this type of plan view, I will be getting.

Now, if I want to imagine the 3D view of this particular workspace, what I will have to do is: this particular elevation view, whatever I am getting, the whole thing I will have to rotate through this particular 330 degrees and then, I can imagine like what should be the workspace in 3D for this particular spherical coordinate robot.

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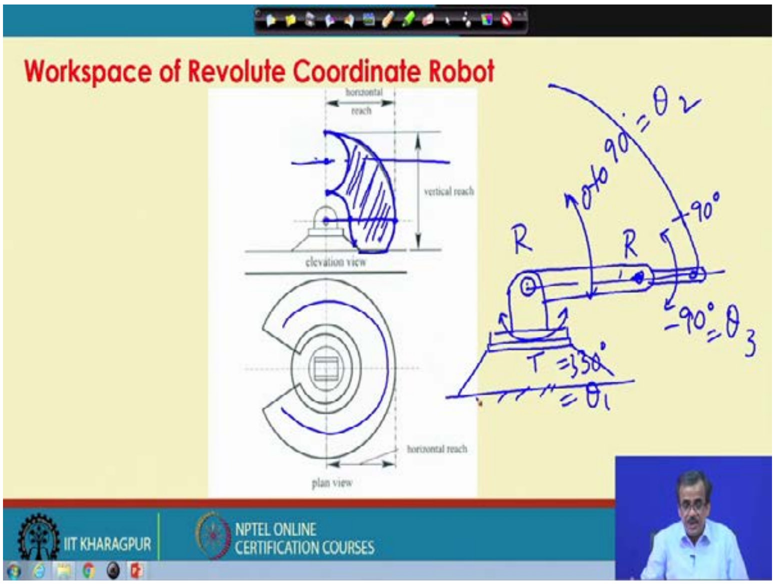

So, I hope you got some idea like how to imagine or how to determine the workspace for a particular manipulator

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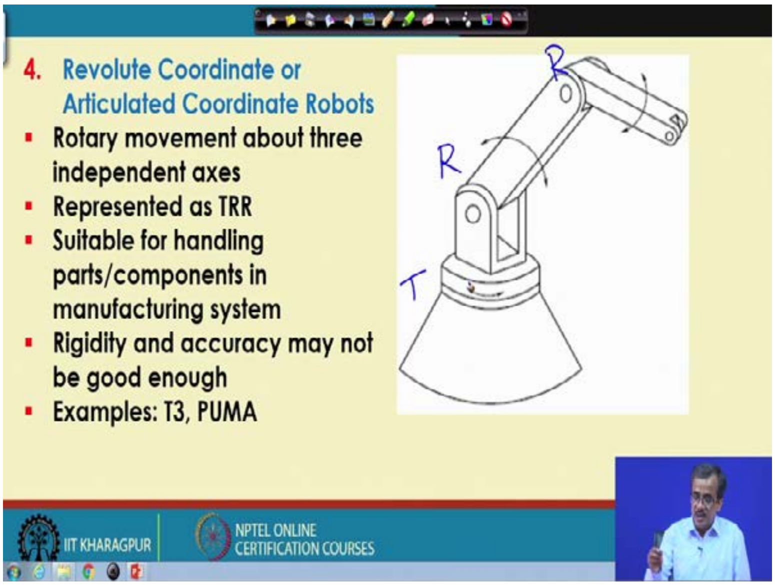

Now, I am just going to concentrate on another robot, that is, Revolute coordinate robot. Now, here, if I want to imagine the workspace, it is a bit difficult thing. And as I told, we have got one twisting joint here. We have got one revolute joint here and we have got another revolute joint here.

Now, if I want to imagine its workspace; it is a bit difficult because I have got 3 rotary joints. Now for a rotary joints in 3D, we will be getting actually the partial sphere. We will be getting the partial spheres and there should be intersection of partial spheres. So, it is a bit difficult to imagine the workspace for this manipulator

Now, to make it simple; actually I am just going to prepare one small sketch here; so, that I can imagine the workspace for this particular manipulator. Now, let me do one thing. So, that particular sketch I am just going to prepare in that slide, where I am just going to show you that particular workspace. So, I am just going to draw that particular picture there only.

So, let me just draw that simplified version of that revolute coordinate robot. Now, if I draw the revolute coordinate robot in its simplified version, it will look like this. So, this type of the simpler version will be getting. Now here, as I told that we have got the twisting joint here. So, with the help of this twisting joint; supposing that I am rotating by says not 360.

So, let me consider, it is rotated by say 330 degrees. And, I have got a revolute joint here. So, with the help of this revolute joint, let me assume that I am going to rotate by 90 degrees, ok?, say 0 to 90 degrees, that means, starting from here. So, I am just going to rotate by 90 degrees. And here, I have got another revolute joint and supposing that I am just going to rotate say from minus 90 degrees to plus 90 degrees, ok?. And, supposing that this is equal to θ_1, this is your θ_2 and this particular angle is nothing but θ_3.

So, this particular joint angle is say going to vary from 0 to 330, θ_2 is going to vary from 0. So, this corresponds to 0. So, zero to 90 degree in the anticlockwise sense and supposing that θ_3 is going to rotate from minus 90 to plus 90, say clockwise to anticlockwise 90 to 90; that means, total 180 degree. The moment we consider; so this type of configuration, then we will be getting starting from here. So, my position is here only. So, I am here.

Now, what I am going to do? I am just going to rotate by 90 degrees. So, if I rotate by 90 degrees. So, I will be getting this particular point, ok?. Now, you see, here, I have got a joint with the help of this particular joint. So, that joint could be here, ok?. Now reference if I draw, the parallel to this will be its reference. So, I can also rotate anticlockwise, I can rotate clockwise. So, this particular tip is going to be rotated. So, with the help of this particular joint; that means, this particular joint, this tip actually I can rotate something like this, ok?.

So, I have reached this particular point; the momentum I am here, that is, the folded-back situation. Now, I can rotate by 90 degrees. So, I can come over here. So, here is the tip. Now, if we just release it, the tip is going to fall. So, it is going to be obstructed here. Similar is the situation for the stretched condition. So, I am here. So, if I release this particular joint. So, it is also going to fall and it is going to be obstructed here. So, this will be your the elevation view of this particular manipulator corresponding to this rotation and here, you can see that.

So, with the help of the first joint, it can be rotated by 330 degrees. So, if I want to imagine the workspace of this particular manipulator, this particular shaded portion will have to be rotated by 330 degrees and then only, I will be able to imagine the workspace for this type of complicated manipulator.

Now, whenever you are going to give any task to the manipulator, its workspace analysis is very important to carry out and without this workspace analysis, actually, we should not proceed to solve a particular task. And, we will have to find out whether the tip of the manipulator, that is, the end-effector is able to reach that particular workspace in order to perform that particular the task

So, this workspace analysis is very important. And, as we know that we human-beings can visualize only up to three dimensions. We take the help of this plan view, elevation view, elevation and plan view and tried to imagine that particular the 3D, but supposing that I have got a manipulator having says 6 degrees of freedom like say PUMA. And PUMA has got 6 rotary joints: 3 twisting joints and 3 revolute joints and it is a bit difficult to imagine the workspace of this PUMA. So, there will be a lot of confusion, if you want to imagine the workspace for this particular industrial robot like PUMA having 6 degrees of freedom. So, the concept of workspace analysis is very important

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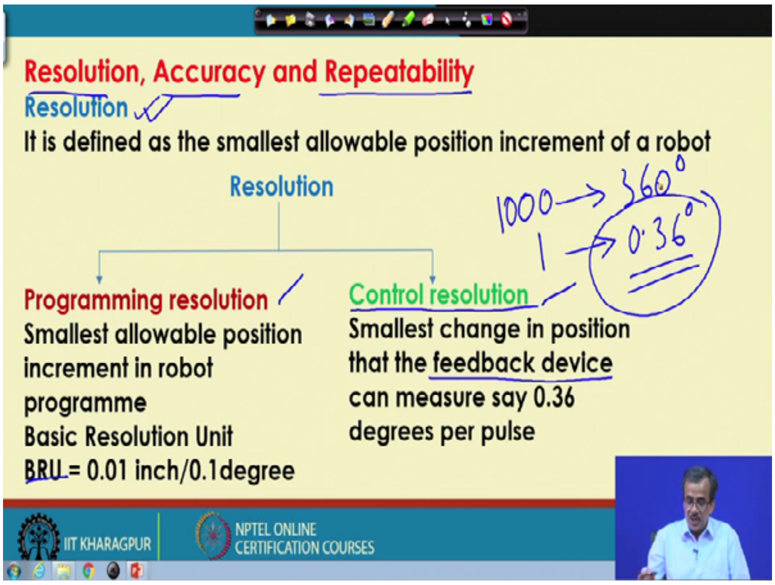

Now, I am just going to define 3 terms actually, which are very important and very frequently used, but truly speaking these 3 terms are not the same and there is a difference between among these 3 terms. So, I am just going to define these 3 terms like your resolution, accuracy and repeatability of a particular manipulator.

Now, this information of resolution, accuracy and repeatability is required, if you want to design the specification of a robot, which you are going to purchase. So, you will have to clearly mention, how much resolution you want, how much accuracy you want and what is the repeatability you want. And, let us try to understand the difference between or the difference among these three terms like resolution, accuracy and repeatability

Now, before I start, let me mention that these three terms are not the same. There is difference among resolution, accuracy and repeatability. Now, let us start with the one, that is, resolution. Now, the resolution is nothing but the smallest allowable position increment that the robot can measure and this is almost similar to the concept of the least count for a particular measuring device. For example, say if we use a screw gauge or if use slide calipers, there will be least count. So, that particular least count is nothing but the resolution of that particular measuring device.

Now, this resolution could be of two types. There could be programming resolution or there could be control resolution. Now whenever we write down the computer program, just to teach a particular point, which I have not yet discussed (I will be discussing after some time) I should know like 1 basic unit corresponds to how much displacement. So, this programming resolution is nothing but the smallest allowable position increment allowed in computer programming corresponding to 1 unit.

Now here, this particular programming resolution is expressed in basic resolution unit. Now, in short, this is known as BRU. So, 1 BRU equal to 0.01 inch or in millimeter, this could be 0.001 millimeter (nowadays it is also available). And, for the rotary movement, this particular BRU could be of 0.1 degree. So, whenever we are going to purchase a robot, we will have to mention how much is the programming resolution we want and what is the programming least count for this manipulator.

Now, then comes the concept of the control resolution. Now, as I mentioned that if we want to use the servo-controlled robot like the closed loop control system. We generally use some feedback device. And, in robots, very frequently, we use different types of sensors to measure the position. And, out of all the position measuring sensors, the optical encoder is very popular one.

Now, I can consider one optical encoder to measure the angular displacement of a particular rotary shaft. Now, let me take one example, supposing that, this is the output shaft of 1 electric motor. Now here, I want to measure the rotation of this particular output shaft of the electric motor. So, how to measure? So, what I do is: here, we put one optical encoder and optical encoder is nothing but a collection of a number of concentric rings placed one after another and on these particular concentric rings, there will be a dark region and clear region.

Now, if there is opaque region, the light will not be able to pass, but if there is clear region or the light region, the light will be able to pass. So, here, we have got this particular optical encoder. Now, on one side, we have got the light source and on the other side, we have got the photo detector. So, the shaft is rotating and the whole optical encoder is also rotating and on the left side, we have got the light source and on the other side, we have got the photo-detector. And, corresponding to this particular rotation, there will be some number, some binary number generated.

Now, by decoding this particular binary number, we can find out what should be the angular displacement of this rotary shaft and this particular angular displacement is compared with the target value and we try to find out the error and this particular error is compensated.

Now, the working principle of this particular optical encoder, I will be discussing after some time, in details. Now, for the time being, let me consider that we are using one optical encoder as a feedback device and supposing that so, this particular feedback device is going to give one complete revolution, that is, 360 degrees and corresponding to these 360 degrees, there is say 1000 electrical pulses. So, 1000 electrical pulse correspond to 360 degrees rotation of this particular shaft. So, one electrical pulse corresponds to 0.36 degrees and we cannot think of the faction of one electrical pulse; that means, the control resolution or the resolution of the feedback device will be 0.36 degrees.

So, this will be the control resolution for this feedback device. So, this is the way, actually, we define this resolution of a particular robot.

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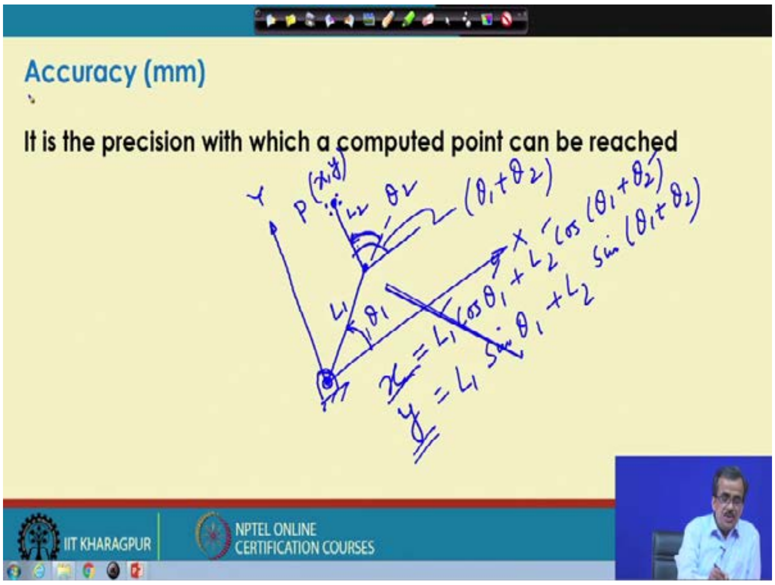

Now, I am just going to define what you mean by accuracy. Now, this accuracy is nothing but the precision with which a computed point can be reached. Now, let me just try to take one a very simple practical example, and for simplicity, let me take the example of a 2 degrees of freedom serial manipulator. So, let me draw one 2 degrees of freedom serial manipulator. So, this is X, this is Y in Cartesian coordinate system. So, this is the base and I have got one link like this and another link is something like this, ok?. The length of the first link is L_1, the length of the second link is say L_2. The joint angles; so, this particular joint angle is θ_1 and this particular joint angle is say θ_2. And supposing that the tip of this particular manipulator is denoted by P and it is having the coordinate like (x,y).

Now, here, at this particular joint, I have got a motor here, I have got another motor here. So, with the help of this motor, I am just going to generate this joint angle. Now, supposing that this particular joint angle is θ_1 and with respect to this L_1, the joint angle is θ_2. So, with respect to X; X axis, the total angle will be θ_1 plus θ_2. So, this is $\theta_1+ \theta_2$. So, this particular angle is with respect to X, that is, $\theta_1+ \theta_2$. So, this is nothing but θ_1.

So, if this is the situation, very easily, we can write down the general expression for this particular x and y. For example, say from trigonometry, you can write down x is nothing but $L_1\cos \theta_1+L_2\cos(\theta_1+ \theta_2)$. Similarly, y is nothing but $L_1\sin \theta_1+L_2\sin(\theta_1+ \theta_2)$.

If I know the values for this particular $\theta_1+ \theta_2$, can I not calculate x and y? We can. So, we can calculate what should be the numerical value for this particular x and y. Now I am just going to give that command to this particular robot, which is having the length of the links like L_1 and L_2 and I am just going to generate θ_1 and θ_2 and, supposing that the robot has started working. So, starting from a position corresponding to this L_1 and L_2, supposing that it is going to reach this particular point.

Now, what is the guarantee that it will be able to reach exactly the same point? There is no guarantee. There could be a little bit of error, while reaching this particular point, the point it is going to reach could be here or it could be here, or it could be here, or it could be here. So, there could be some small deviation from the computed point or the calculated point.

Now, this particular deviation from the computed point and the point which has been reached is known as the accuracy of this robot and, that is expressed in terms of millimeter, ok?. Now, it could be either positive or negative. It depends on whether that point is exceeding that or not. So, I can find out in both the ways. So, accuracy it could be either the positive or negative.

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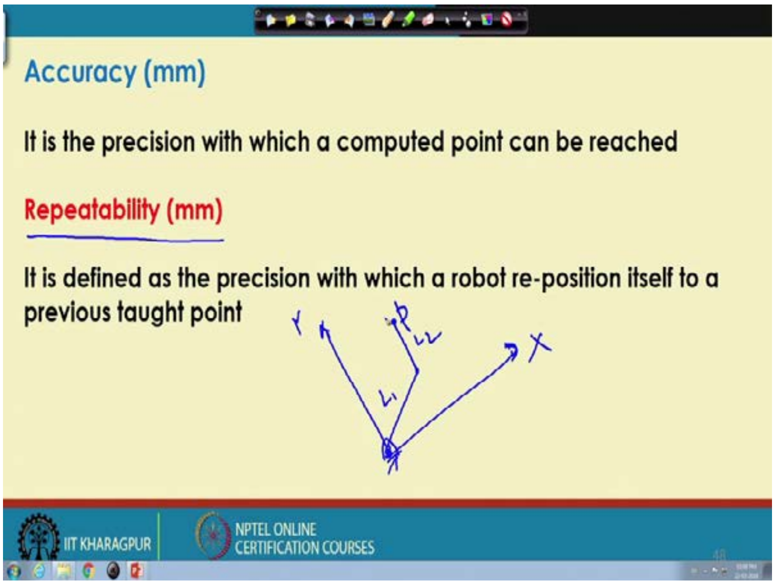

Now, I am just going to define another term that is called the repeatability. By repeatability, we mean supposing that we have taught a robot to reach a particular point, and there are several teaching methods, which I will be discussing after some time. So, with the help of that particular teaching method, say I have taught the robot to reach a particular point, say the point A. Let me take the same example of say 2 degrees of freedom serial manipulator. So, this is nothing but the manipulator this is L_1 and this is L_1.

So, I have taught it to reach this particular point. This is L_1 and this is L_2, ok?. Now, once I have taught, and if I just run this particular the robot say for 20 times. So, at each for at each of the 20 times, there is no guarantee that it is going to reach exactly the point, which I have taught; there could be some deviation. Now this particular deviation is known as the repeatability of this manipulator, ok?

So, this is the way actually, we define the repeatability of this particular manipulator and once again let me repeat like, if we want to prepare the specification of a robot, we will have to mention, what is the resolution, accuracy and repeatability we want and based on our requirements, the manufacturer is going to manufacture that particular robot to supply it to us.

Thank you.

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Lecture - 06
Introduction to Robots and Robotics (Contd.)

Now, we have already discussed that we use robots in manufacturing unit. Now, today, I am just going to discuss, the various applications of robots, and we know a little bit that the robots are used in manufacturing units, nowadays and there is specific requirement also that we have already discussed. Now, if I use robots in manufacturing unit, we will be getting a few advantages. So, I am just going to see these advantages first. For example, say the robot can work in dirty environment and hazardous environment like the nuclear power plant.

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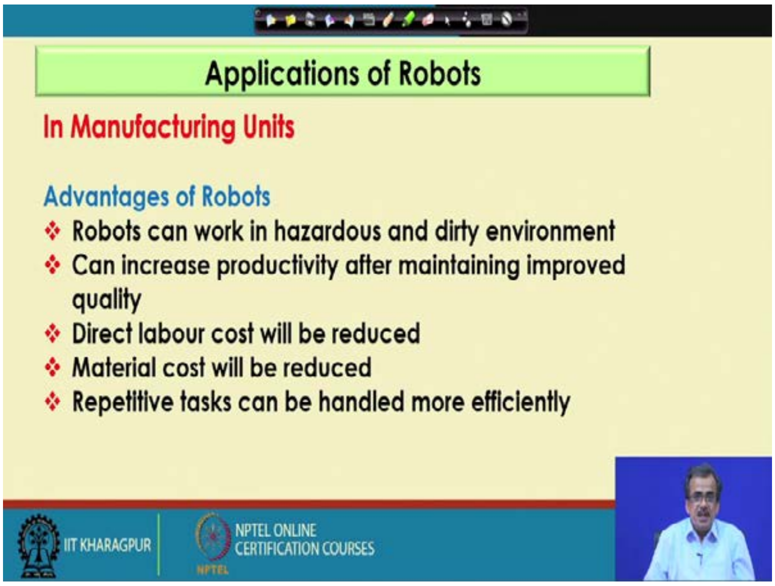

Now, if I use robot, there is a possibility that we will be able to produce goods with good quality with less error and the productivity will be high. And, if we use robots to replace the human labor, there is a possibility that one robot can replace a large number of workers and by doing that, there will be saving of labour cost. Now, if we just do some sort of repetitive jobs with the help of some operators, the manual operators.

So, what will happen is, those human-beings may not like that repetitive task and there will be a lot of mistakes, there will be a lot of wastage of that particular products. Now, if we can give this type of repetitive tasks particularly to the robot, there is a possibility that the chance of rejection will be less and due to that, there will be some sort of saving of material cost. So, if we use robots actually, these are some advantages we will be getting and moreover as I have already mentioned that for the repetitive task, it is better to use the robots because human operator may get bored to perform the repetitive task. And, that is why, the robots have become very popular nowadays in modern manufacturing units. And, nowadays, actually the robots are used in manufacturing units to perform a variety of tasks.

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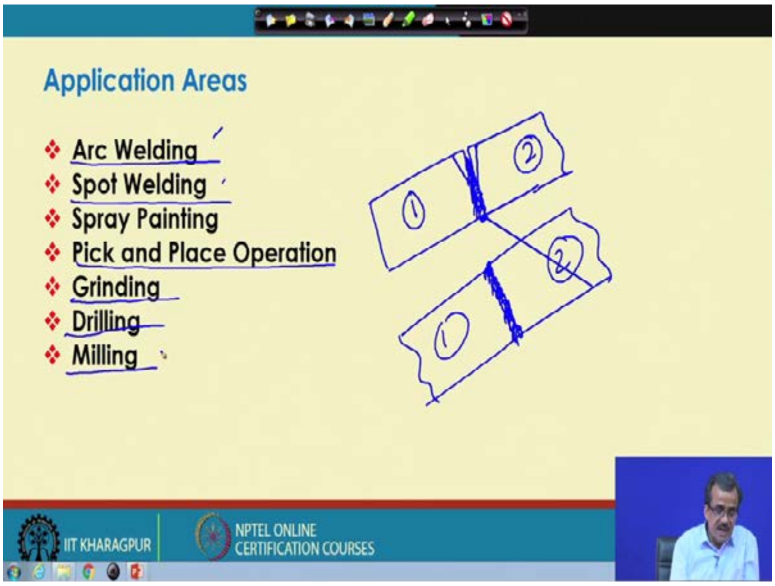

For example, say it can do arc welding, spot welding. It can do some sort of spray painting. For example, nowadays the spray painting on the car body, particularly Maruti uses that the manipulator to do this type of task of spray painting, but we will have to be careful while doing this particular spray painting. There should not be any such discontinuity.

Now, regarding this arc welding and the spot welding, the way the robot can help is as follows. Now, let me prepare a very simple sketch. We will understand, supposing that I have got two plates, two steel plates, which I am just going to join by welding. Say, I am just going for some sort of continuous arc welding and spot welding. So, this is plate 1 and this is plate 2, and these two plates I am going to join.

Now, what I will do is, I will start from here and I will just go on doing this particular continuous arc welding with the help of one robot, say PUMA. So, I am using PUMA to carry out this type of arc welding. Now, if I start from here; there is a possibility, that there will be some distortion and due to this distortion, the plate may take the position something like this. So, if I start the welding here; the other side may be distorted and here, I will not be able to carry out this type of arc welding, the continuous arc welding. Then, how to overcome this particular problem?.

To overcome this particular problem actually what we do is: before we carry out this type of arc welding with the help of the manipulator. The first we do is, so this is plate 1 and plate 2. So, before we start with this particular arc welding, we go for the spot welding. And, what we do with the help of the manipulator; we just do one spot welding here, another spot welding here, another spot welding here and then, we start the continuous arc welding here. So, this particular spot welding is going to arrest that particular distortion due to this welding and you will be getting continuous welding. So, this type of task you can give it to the manipulator. The same manipulator will be able to perform the spot welding and then, the continuous arc welding. So, this is one very good application of manipulator in industry.

Then, comes Pick and Place type of operation. In the machine shop say, before the assembly, the few components are to be transported from one place to another. So, we can use the robots. The robot can pick that particular object and place it to another place. Then, comes the Grinding, we can carry out the grinding operation and this particular grinding wheel will be attached to the end-effector of the manipulator. We can do drilling, which I have already discussed. We can make some drilled hole on some steel plates.

We can also do milling that also I have discussed to cut some complicated profile on one side of a steel plate. So, what you can do is, we can take the help of this type of milling cutter and this milling cutter will be gripped by the gripper or the end-effector of the manipulator.

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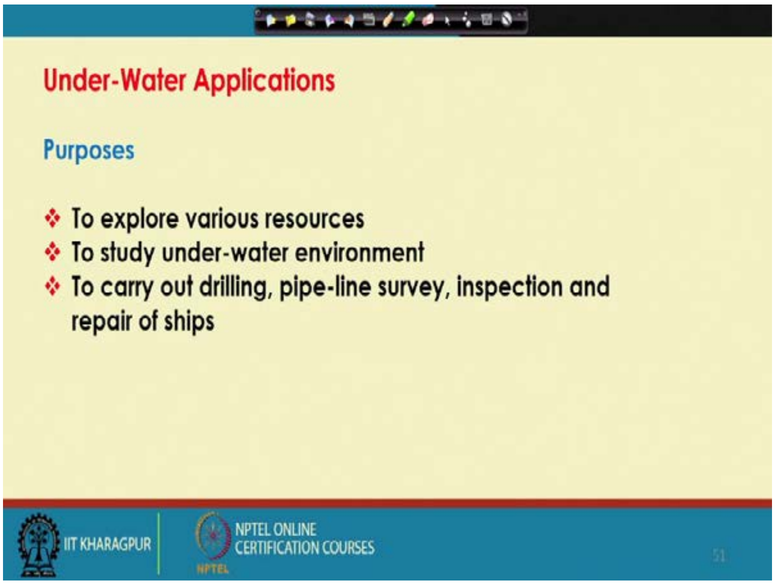

So, in manufacturing unit, nowadays, the robots are used actually to solve a variety of purposes, a variety of tasks, ok?. But, besides this manufacturing unit, nowadays, the robots are used for some other types of applications. For example, say, robots are nowadays used for underwater applications. For example, say inside the sea. So, we send robot just to search for the valuable stones or the gems. So, it will try to find out the possible location of the valuable stones or the gems.

Now, if we want to carry out some study of the underwater environment. So, inside water, we have some living creatures. So, if you want to study their lives or if you want to study that underwater environment; we can take the help this type of underwater robots.

Now, then comes the crude petroleum. Our seabed could be a very good source for the crude petroleum and you will have to drill it out. Now, for this drilling purpose, we can use robots. This particular crude petroleum is to be transported. There must be some pipelines through which the crude oil will pass and it will be transported to another place.

So, we can use the underwater robots to carry out some sort of inspection for this particular pipeline and if required, this underwater robot can also do a little bit of repair job, maintenance job. It can do a little bit of welding also, that underwater welding, nowadays, is also possible. So, by creating the vacuum there and with the help of the robots, this type of welding can be carried out. So, these are the various applications of underwater robots.

Now, these underwater robots actually are designed and developed in different ways. Different designs are available. Now, we can get some sort of multi-legged robot, as the underwater the robots; sometimes, we use tracked vehicle as the underwater robot, but generally, we do not use wheeled robots as the underwater robots, because the seabed may not be very smooth. But, we may have the provision that depending on the requirement, sometimes we can use it as the multi-legged robot or we can use it as the tracked vehicle.

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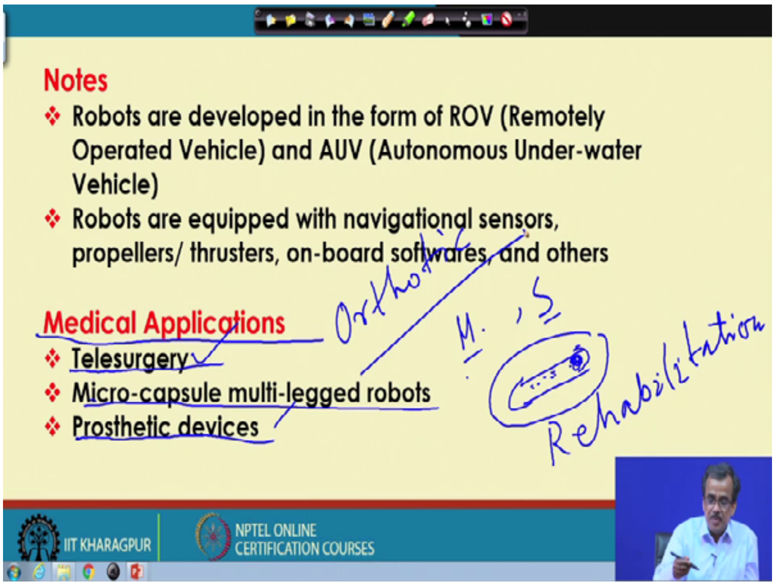

Now, if you see these underwater robots, these are actually developed in two different forms. Now, one is called the ROV, that is, the Remotely Operated Vehicle and another is AUV, that is called Autonomous Underwater Vehicle. Now, there is a basic difference between ROV and AUV.

So, ROV has actually the centralized control. So, there will be a central computer, which is going to control the movement of this particular robot and that is a remotely operated vehicle. But, AUV is nothing but the autonomous underwater vehicle. So, there actually the control system, which we use is decentralized. So, each of these particular robots is intelligent and they can take their own decisions and they are decentralized. So, this particular underwater robots are actually developed both in the forms of ROV as well as

AUV, and as usual, the robot should be equipped with some sensors just to collect information of the environment. And of course, there will be some propellers and thrusters; so just to control the movement of these underwater robots.

Now, next is, the medical applications. Nowadays, actually, the robots are extensively used in medical science and there are several applications of robots in medical science. I am just going to discuss a few. For example, say in Telesurgery, we extensively use the robots. We generally use two robots, one is called the master robot and another is called the slave robot.

Now, the slave robot is actually going to carry out the operation on the patient and with the help of the master robot, the doctor is going to give the instructions. Now, here, there is a physical distance between the doctor and the patient, and may be the patient is, say 5 kilometer away from the doctor. And, with the help of these two robots, the doctor is going to carry out that particular operation. Now, here, the slave robot is equipped with the surgical instruments like there will be knives, scissors and all such things. And, along with these knives and scissors, there will be force and the torque sensors mounted and on the other side, the doctor is having the master robot and there will be one control panel.

So, the moment, the slave robot is going to carry out some operations, with the help of that surgical instruments it will have to put some force. If it is linear or it will have to put some force or sometimes you will have to create some moment. And, all such forces, torques, moments, these things will be determined with the help of that force or the torque sensor. And, with the help of this wireless connection, the required torque, required force moment and all such things will come to the doctor, who is sitting at a place, may be around 5 kilometer away from that particular patient.

Now, after seeing that particular information on the computer screen; so, the doctor is going to give instruction to the slave robot with the help of this particular the master robot. So, this is the way actually we carry out the telesurgery with the help of this particular robot. So, we use two robots to carry out, this type of telesurgery.

Now, next come is the Micro-capsule Multi-legged robot. Now, this is another a very good design for the robots in medical science. Now, here the robot is actually a multi-legged robot, a very small robot like safe and size-wise could be equivalent to one capsule sort of thing; very small capsule sort of thing. And, inside this particular capsule, actually we have got that multi legged robot. And here this multi legged robot is equipped with the high speed camera and this particular robot can be used just to identify whether there is any such tumor, say inside the digestion system of the patient.

So, this is, as I told, just like a tablet or a capsule. The way we take capsule, the patient is going to swallow it. So, this particular capsule robot with the help of say water will go inside the digestion system and there, it will start walking. Now, remember here actually, we do not use any motor for this type of small robot. So, instead what we do is, we control the movement of this particular small robot from outside the body of the patient with the help of one permanent magnet; that means, here we have got one magnetic material inside and from outside the body of the patient, we move this particular permanent magnet, there is a possibility we can control the movement of this particular small robot.

Now, the reason why we do not use any motor is the size will become larger or the bigger, if I use some motor. So, generally, we do not use any such motor for this type of robot, but it has got camera. Now, the camera needs power and for that sometimes a very small a lithium battery sort of battery, we generally use along with the camera. So, whenever this particular robot is working inside the digestion system, at a regular interval, it will take the snap and it will send this particular information to the doctor.

So, the doctor on the screen will be able to see the picture of what is there inside that the stomach and whether there is any such tumor. So, the possible location of the tumor will be identified with the help of this type of micro-capsule multi-legged robots. So, it has got some applications.

Then, comes the Prosthetic device. Now, nowadays one field of robotic research has become very prominent, that is called the rehabilitation robotics. Now, in this rehabilitation robotics, actually what we do is, we try to take the help of different types of robots. For example, say in rehabilitation robotics, we design and develop some prosthetic device. Sometimes, we generally go for some sort of orthotic devices. So, these prosthetic devices and orthotic device are going to help the old people, the weaker people during walking. And, both these prosthetic and orthotic devices, nowadays have become very popular and these are nothing but once again some special types of robots, some intelligent robots. So, these are some of the applications of robots in medical science.

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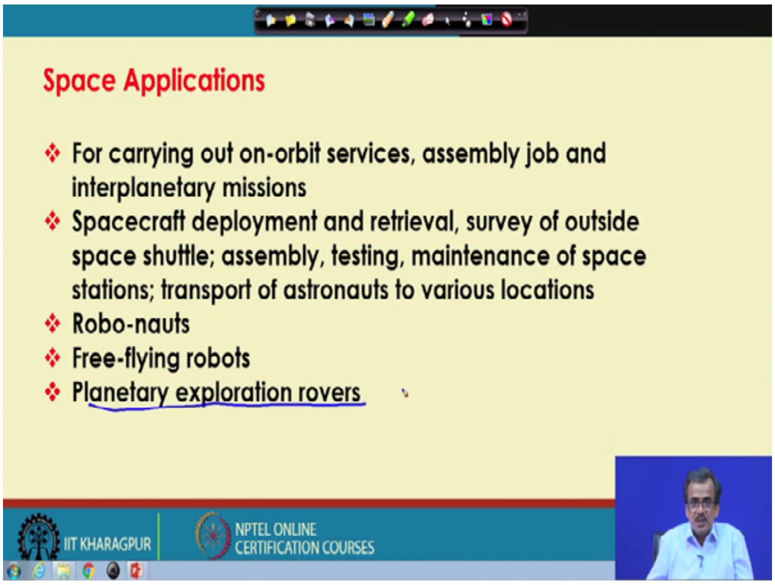

Then, comes some space applications. Now, nowadays, the robots are used in space very frequently. You might have heard about your the planetary exploration robots. So, we know about curiosity, and then spirit and opportunity. These are nothing but the planetary exploration rovers. So, with the help of these planetary exploration rovers, we can collect information of the planet. We can collect information of the Mars and these robots are all intelligent robots.

For example, if you talk about the curiosity, which is an intelligent robot sent to the Mars, and it has the capability to use be as a multi-legged robot or it can also be used as the tracked vehicle. Now, there are some other applications like in space station, we can use robot just to do some sort of inspections, survey maintenance job. We can carry the astronauts with the help of robots and in future, in fact, the astronauts will be replaced by the Robo-nauts.

Now, in the space station, we have got a few spacecraft. Now, what you can do is, we can use robots for the deployment and retrieval of this particular spacecraft. And, nowadays, we use some small robots, very small robots and these are known as the free-flying robots. Now, these free-flying robots are very small in size and we can also send it to the Mars, to collect information, and this type of free-flying robots are just like a fly, very small in size and this particular robots are in design and development stage. And, this will be once again, a little bit intelligent also. We can send it to the space to collect information of the space with the help of this robot.

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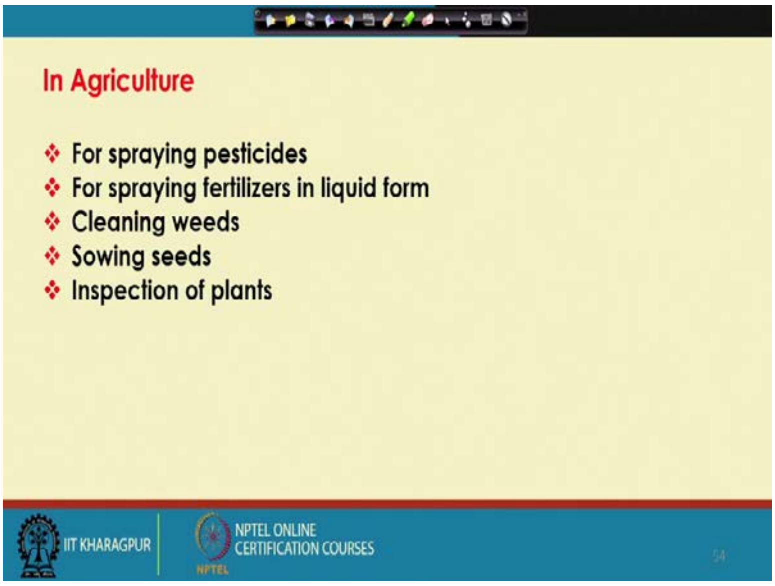

So, these are all some of the applications, which we have in the space science. Now, in agriculture, nowadays actually we are planning to use robots, different types of robots and in fact, we can use robots just for spraying pesticides in the field. We can use robots for spraying fertilizers in the form of liquid form, ok? So, the fertilizers, we can mix it with water to make it liquid and then, we can use some sort of robot in the field just to spray that particular fertilizer.

We can use robots for cleaning weeds. For example, there may be some unwanted small plants sort of thing in the field. Those are called weeds. Now, for cleaning the weeds, nowadays we are thinking like how to use the robots. We can use a robot for sowing seeds in the field. Now, while sowing seeds, we follow a certain pattern and accordingly, we can take the help of the robots just to actually sow the seeds in the field. We can use robot for inspection of the plants; the health inspection, the quality inspection of the food grain. So, we can use this type of robots.

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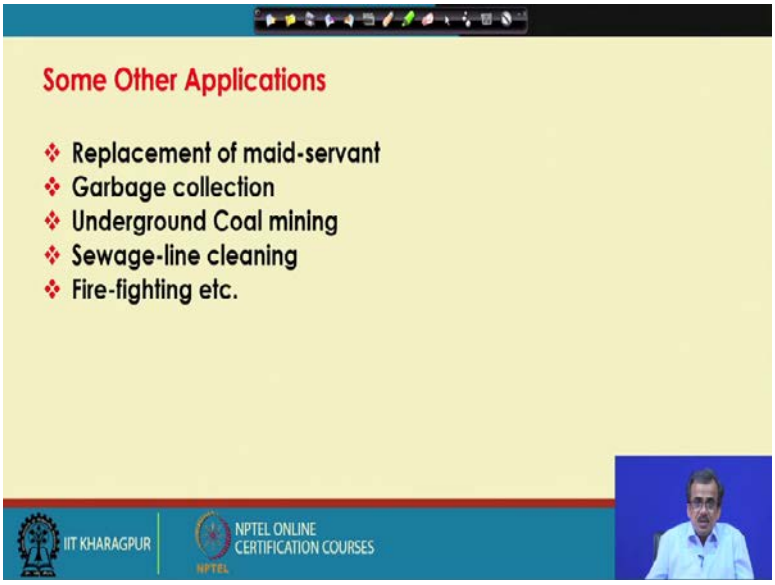

Now, here, I am just going to mention a few other applications. Nowadays, in the house actually, as a replacement of the maidservant, people are thinking whether we can use some robots. Then, for garbage collection, whether the robots can be used, then underground coal mining. There are already a few applications, where robots are used for coal mining. Then, comes the sewage line cleaning. This is already in use.

Now, just to clean the sewage line with the help of robots, some special type of robots can be designed and developed. And, these are nowadays, in fact, used just to clean the sewage line; then for firefighting, nowadays, the robots are also used, and so on. So, there are many applications of the robots and in future, the robots will be used to serve a variety of purposes in more fields. The robots will be used because the robots will become more intelligent and autonomous in future and we will be able to use this robot to serve a variety of purposes.

Thank you.

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Lecture - 07
Introduction to Robots and Robotics (Contd.)

Now, I am going to start with the end-effector, which is generally used in the Robots.

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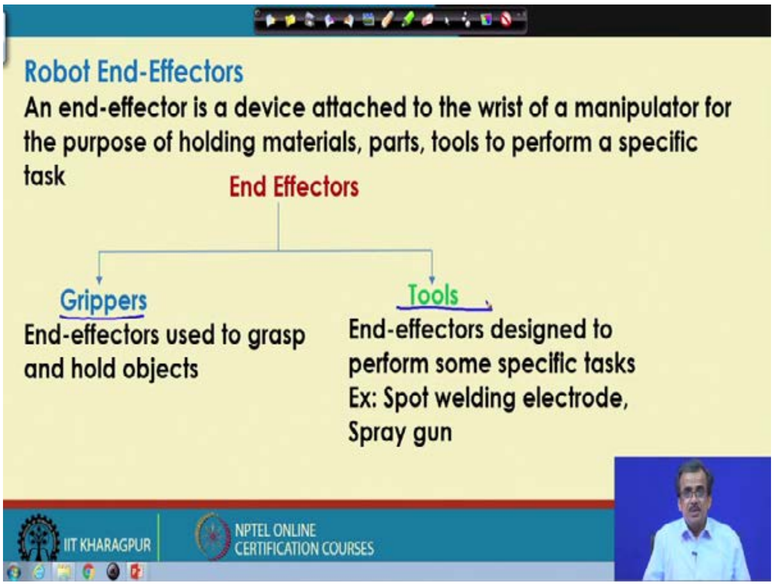

Now, if we see the robot's end-effector, we use generally two types of end-effectors, which I am going to discuss, but the basic purpose of using end-effector is to grip or hold some parts, materials, tools, just to perform some specific tasks. And, this particular end-effector is generally attached to the wrist joint, that is, the last joint. So, there, we attach this particular end-effector.

Now, if you see the end-effector, this end-effector could be of two types. Now, one is called the gripper and another is called some specific tools or the specialized tools. Now, this grippers are known, for example, say, we will have to grip some tools, like we will have to do some machinings, for example, drilling or say grinding or milling. So, we will have to use some special type of gripper, so that we can grasp that particular object. Now, sometimes, as we mention, the robots are used to perform some specific tasks, for example, say spray painting or welding.

Now, if we want to do the spray painting, the spray gun has to be attached to the wrist joint. Similarly, if we to want to do some sort of welding, then the welding electrode has to be attached to the wrist joint and that is, also an end-effector, but this particular end-effector is in the form of tools. So, the end-effectors could be grippers or it could be some specialized tools.

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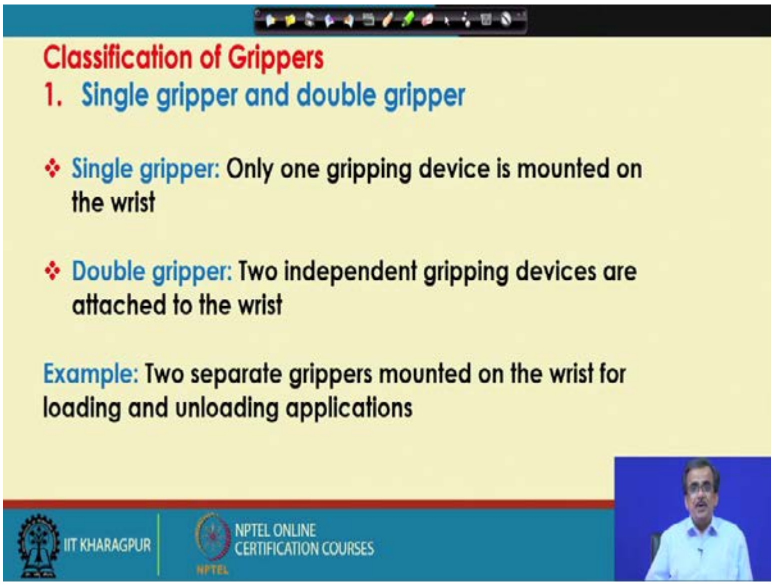

Now, I am just going to see the different types of grippers, which we generally use. So, I am just going to classify the grippers. Now, the first classification is: single gripper and double gripper. Now, as I mentioned that if this is the wrist joint. So here, in this particular wrist joint, we connect that particular gripper. Now, here, I can grip, I can connect only one gripper just to serve a specific purpose or depending on the requirement, I can just attach two independent grippers.

So, if I use two independent grippers, that will become the double gripper, and if I use only a one such gripper gripping device, that is called the single gripper. So, this is the difference between single gripper and double gripper.

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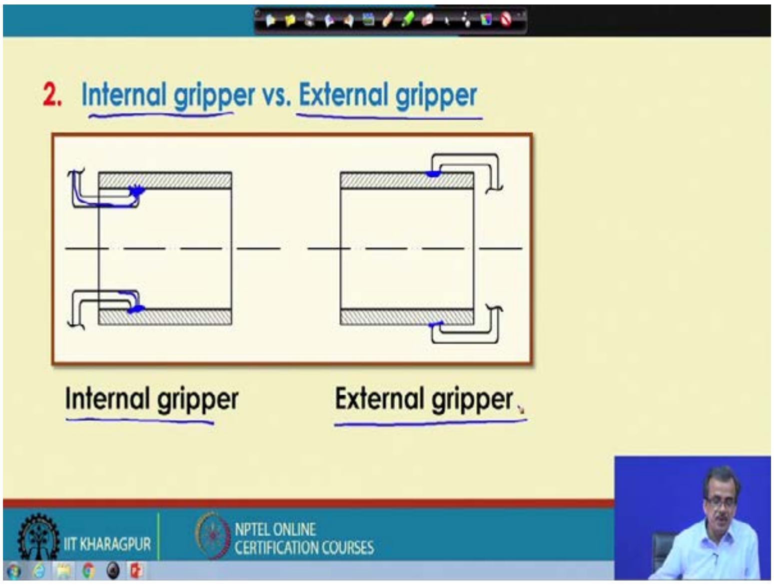

Now, then, comes the concept of the internal gripper and external gripper. Now, let me take one example. Supposing that I have got a steel pipe for example, say water pipe or oil pipe, ok? Now, this particular pipe, I want to grip.

Now, there are two possible ways with the help of which I can grip this particular pipe. So, I can follow this type of gripper, this type of gripping pad, or gripper, so I can use one gripper like this, I can use another gripper. So, one gripper is here, and another gripper is here and this is a hollow pipe, so I can grip with the help of these two fingers or there is another possibility, the same pipe I can grip here, externally. So, I can put a one gripping pad here and another gripping pad here and I can grip this particular hollow pipe.

So, if I use this type of gripper that is called the internal gripper. If I use this type of two fingers, that is called the external gripper. So, this is the difference between the internal gripper and external gripper.

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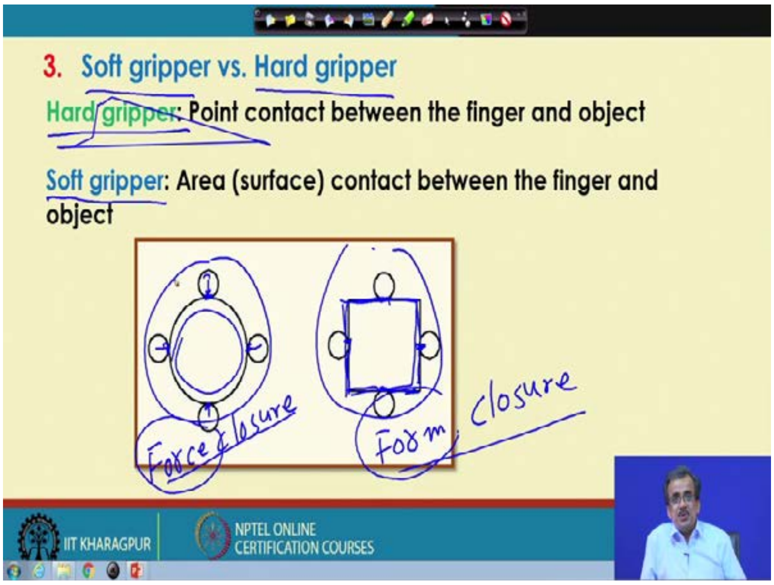

The next classification is the soft gripper versus the hard gripper, soft gripper and hard gripper. Now, let me take one example. Supposing that, this particular marker, say, if I just grip it like this. So, there is a possibility that I can grip it with the help of these two fingers, although this is not the perfect point contact, so might be it is almost similar to the point contact and with the help of these two fingers, I can grip it.

And, there is another way of gripping, I can also grip it like this. Now, if I grip it like this, this is nothing but the perfect area gripper, ok? So, if I use the point gripper like this, that is nothing but the hard gripper, that is actually the hard gripper and if I use this type of the area contact and that is called the soft gripper. So, for the hard gripper, we maintain the point contact just to grip that particular object and for the soft gripper, we consider the area contact just to grip that particular object.

Now, here, in this particular sketch, in fact, I am just going to give information of another concept. Now, that concept is also very important from the gripper design point of view and that is nothing but the concept between or the difference between the force closure and form closure. So, this is actually the force closure and this is nothing but the form closure form closure. So, the concept of these force closure and form closure is a very important.

Now, let me take one very simple example. Supposing that say I am just going to write something on the board with the help of a white chalk. So, the chalk is having circular cross-section. Now, to grip that particular chalk, so that I can write in a very nice way, so what I will have to do is, with the help of my finger, I will have to put some force, here I will have to put some force here, then only I can grip this particular chalk, because the chalk is having the circular cross-section and if I do not grip it properly, if I do not put force and there is a possibility that I will not be able to write or draw the picture which I am planning to do.

The same is true, if I do not grip it properly, so I will not be able to write anything here. Are you getting my point? Now, this is another example. Supposing that the object is having the cross-section, which is nothing but a square in place of a circular in place of circular. Now, this object is having the square cross-section. Now, if it is a square and if I just try to grip it. Here, gripping will be much easier compared to this particular object because here, so I have got a one finger here, another finger here, another finger here, another finger here. And, this particular geometry or the corner this geometry or the corner is going to help us in gripping, which is a absent here.

So, supposing that both the chalks are having the same mass, same weight and I have got chalk having circular cross-section, I am having chock having the square cross-section. So, handling the chalk having square cross-section will be much easier, and that is why, in fact, in some of the universities, they use the square chalk a particularly in foreign universities, they take the help of square chalks.

But, here, actually we take the help of this type of circular cross-section chalk. So, the difference between the concept of force closure and form closure is a as follows. Here, to grip this particular object, we will have to take the help of some force and here, this particular geometry is going to help us in gripping this particular object. So, this is the concept of the form closure and this is the concept of the force closure.

So, if we want to design and develop some sort of grippers, we will have to see the nature of the object, the cross-section of the object, which I am going to grip. And, accordingly, depending on the requirement, I will have to design that particular gripper. So, this is the concept of the force closure and form closure.

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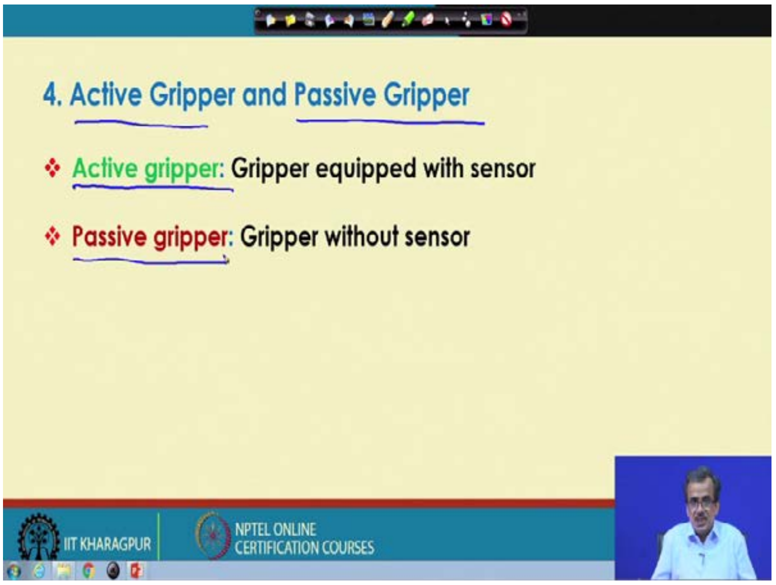

Now, another classification is actually the active gripper versus the passive gripper. Now, by active gripper, we mean the gripper having some sensor and by passive gripper, we mean it is a gripper without sensor.

Now, let me take the example of our gripper. So, if I just consider our gripper. So, for example, say with the help of this finger, I am just going to grip it. Now, for gripping definitely, I will have to put some force, but at the same time, this particular skin, which I have on the finger, is going to help us a little bit and this particular sensor, the skin is a touch sensor, which I will be discussing, in details, after sometime. So, this particular skin, that is, the touch sensor is going to help me in gripping also, besides that particular force, which I am putting, and that is nothing but an example of active gripper. On the other hand, we have got the concept of passive gripper, where we do not use any such sensor.

Now, I will be discussing the working principle of passive gripper in much more details, after some time.

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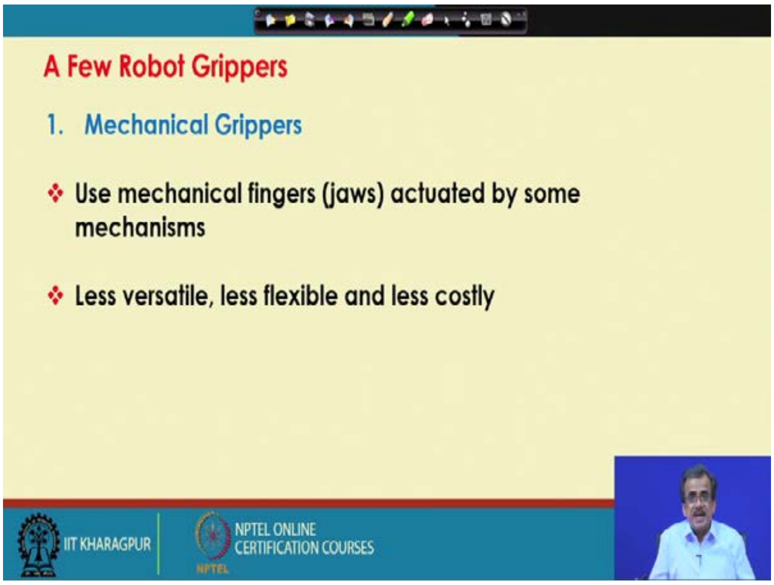

Now, I am just going to discuss the working principles of a few very simple mechanical gripper, which are very easy to understand. So, in mechanical gripper actually, what we do is, we try to design some fingers and we operate these fingers with the help of some mechanisms. And, these are purely mechanical, so I am just going to design and these particular grippers are very simple to design and these are less costly and less versatile. So, I am just going to discuss a few available very simple mechanical designs for this particular gripper.

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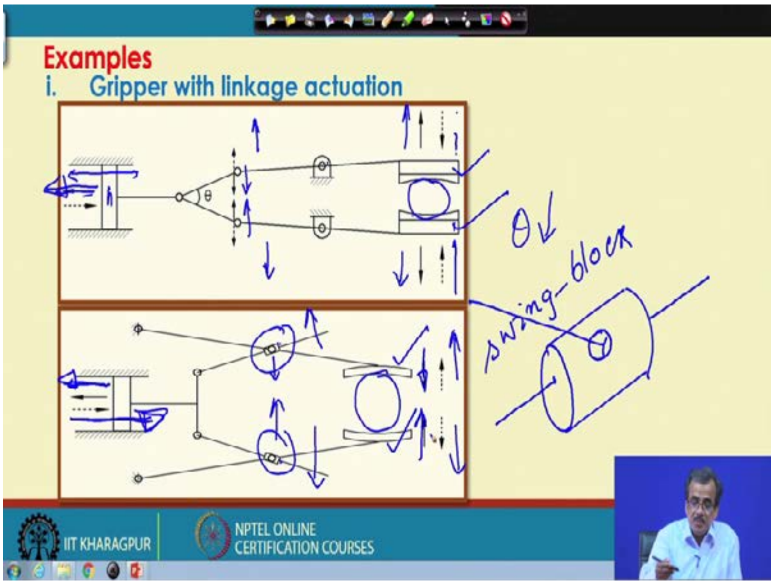

For example, say, the first one is a gripper with linkage actuation. So, let us try to understand the working principle of this particular very simple gripper.

So, here we have got two gripping pads. So, these two are the gripping pad or the gripping jaws. So, with the help of this gripping pad, I am just going to grip some objects here. So, I am just going to grip with the help of these two gripping jaws or gripping pads. Now, I will have to grip, I will have to un-grip also, how to do it? Now, the mechanism is very simple. So, what we do is here we have got one piston cylinder arrangement, so this particular piston can slide inside this particular the cylinder.

Now, the moment this particular piston moves towards this solid arrow; that means, in this particular direction, ok? So, here I have got a joint, the rotary joint. So, what will happen is, the moment it is sliding in this particular direction, this angle: θ is going to be reduced, and as θ decreases, these two points are going to come closure to each other and here, we have got the support and due to that, these two grippers are going to move away from each other, and it is going to un-grip.

On the other hand, if this particular piston is moving towards this particular arrow, this dotted one, this dotted arrow, what will happen is, θ is going to increase and these two points are going to move away from each other and consequently, these two gripping jaws are going to come closure to each other, and it is going to grip this particular object. So, this is a simple gripper designed with the help of the linkage or the mechanism, ok?

Now, I am just going to discuss the working principle of another very simple gripper. Now, here once again, this is the gripping pad or the gripping jaw with the help of which I am just going to grip this particular object, the object is here, say. Now, what we do is, once again we use the principle of the cylinder piston mechanism, so, this particular piston can slide.

Now, supposing that it is sliding towards this particular direction, shown as the solid arrow. Now, if it moves towards that. So, here, we have got one mechanism, that is called the swing-block mechanism. So, swing-block mechanism it is a very simple, supposing that I have got one this type of cylinder sort of thing. So, this is the cylinder and here, you will find that. So, this is connected here, one link and here another link is connected and here, you will be finding one circular groove here, and through this particular groove another link will pass. So, this particular link will pass through this, ok?

Now, here, the moment this particular piston slides towards this in this, particular direction. So, what will happen is, these two swing-blocks, these two swing-blocks will try to come closure to each other. So, if they come closure to each other, what will happen? So, it is going to grip this particular object. And, a reverse is the situation, if it is sliding towards this particular direction. So, if it is sliding towards this particular direction, these two swing-blocks are going to move away from each other and consequently, there will be gripping of this particular object, shown by this particular, the dotted one, ok? So, this is the way, actually, we can grip and un-grip with the help of this type of very simple mechanical gripper.

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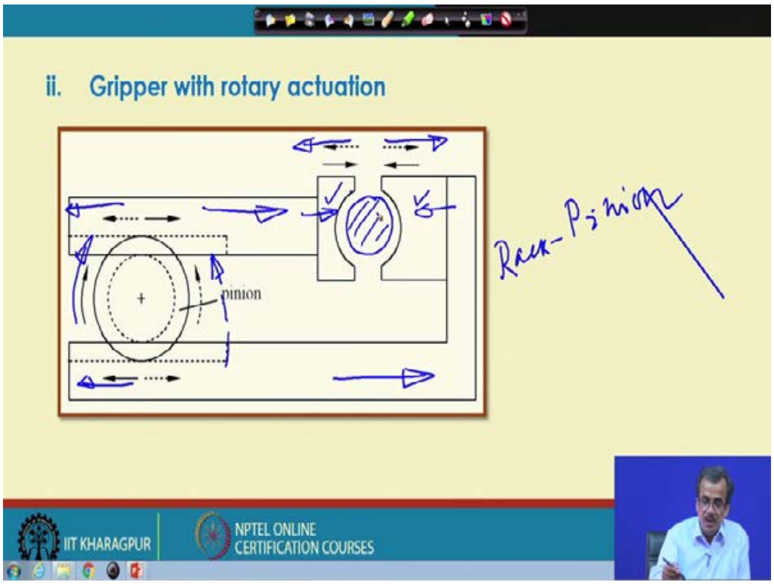

Now, I am just going to discuss the working principle of another very simple mechanical gripper. Now, here, this is nothing but one gripping pad, this is another gripping pad and I am just going to grip this particular objects, ok? Now, here, we have got one pinion, pinion is nothing but a small gear. So, I have got a pinion here, and I have got a rack here. So, this is nothing but the rack and pinion mechanism, rack-pinion mechanism.

So, a rack-pinion mechanism, we have. Now, here, so this particular pinion can rotate either clockwise or anticlockwise, and here, this upper part is connected to this gripping pad and the lower part is connected to the gripping pad in this particular fashion, ok?

Now, supposing that, so this particular pinion that is connected to the motor, say, is rotating in the clockwise sense. So, if it rotates in clockwise sense, the upper part is going to slide something like this and the lower part is going to slide something like this; that means, it is going to move towards that and that is going to come like this and it is going to grip this particular object, ok?

And, reversing the situation, this particular pinion is rotating in the anticlockwise sense. So, if it is rotating in the anticlockwise sense, this upper part is going to slide along this particular direction and this lower part is going to slide in this particular direction and consequently, this is going to slide towards this, this is going to slide towards this, and it is going to un-grip that particular object. So, this is the way, actually, we can grip and un-grip this particular object with the help of this type of very simple mechanical gripper.

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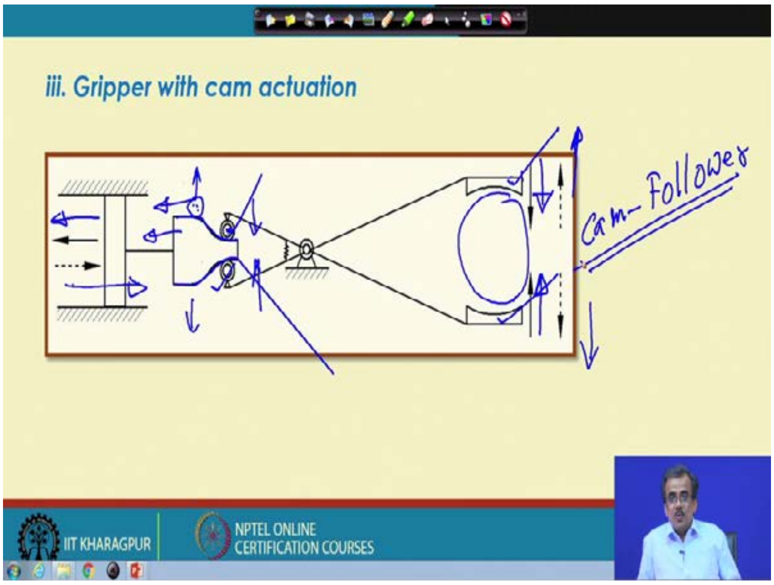

Now, then, comes the working principle of another very popular mechanical gripper, that is nothing but using the mechanism of cam and follower. So, by using the mechanism of cam and follower, we can design this particular gripper. Now, let us see how.

Once again, we have got the piston and cylinder arrangement and this is a one gripping pad, this is another gripping pad, ok? and here, we have got that particular cam profile, very complicated cam profile. So, we have got this cam profile and we have got the roller follower. So, this is the roller follower. So, cam and follower profile, ok?

Now, supposing that this particular piston is sliding towards this particular direction shown by the solid arrow. So, what will happen is: this is going to slide towards this. So, might be initially, the roller was here, are you getting my point? Now, this particular thing will slide in this particular direction. So, ultimately, the roller has come here, it has come up to this, ok? So, initially, the roller was here, but after sometime due to this sliding movement, the roller will come here; that means, these two rollers will come very close to each other.

Now, if they come closer to each other, then what will happen is: if it comes closer to each other. So, this is going to come, these two gripping pads are going to come closer to each other and it is going to grip this particular object and for reversing the situation, it slides towards this dotted direction. So, if it slides along this, what will happen is, initially, the roller was here, but after sometime, the roller could be here that means, the distance between the roller is going to be increased that means, these two rollers are separated out and due to that, what will happen, if it is going to be separated out, ok? So, this is going to be separated out. So, these two gripping pads are also going to be separated out and it is going to un-grip that particular the object, ok?

So, this is the way, this cam actuated gripper is working. These are all very simple mechanical designs of the grippers.

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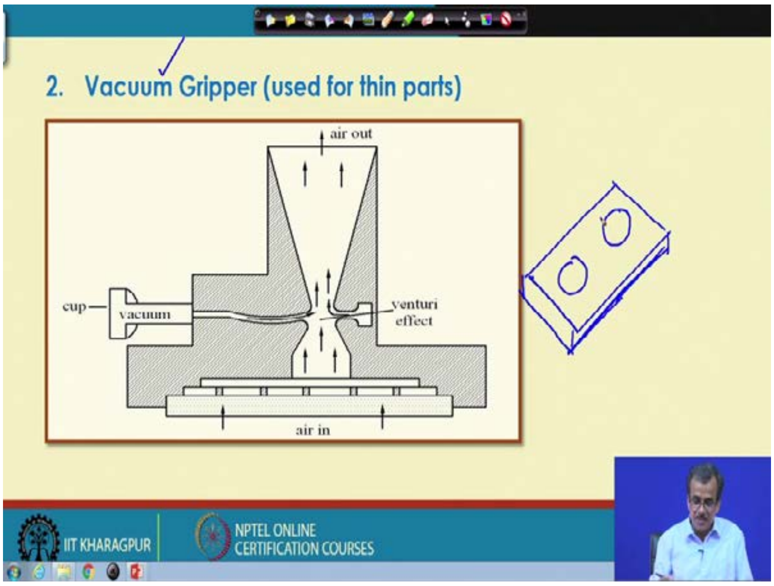

Then, comes the vacuum gripper, which is very frequently used nowadays, particularly for a handling some flat plate sort of thing. Now, let me take a very simple example. Supposing that the robot is going to do some sort of pick and place type of operation and it is going to pick one steel plate and it is going to place it to some other place, ok? Now, how to grip it?

Now, supposing that I have got a steel plate here, say, this type of steel plate. Say, might be the thickness is of 20 millimeter steel and the length could be like 1 meter, the breadth could be of 0.5 meter something like this, ok? So, this type of steel plate, the robot is going to grip and place it to another. How to do it?

So, what we do is: we take the help of this type of vacuum gripper, ok? Now, here we put one vacuum gripper, we put another vacuum gripper here and we will be able to grip this particular steel plate. Now, let us see the working principle of this type of gripper. Now, here, actually what we do is now, let us first see that particular picture, the way it is used and then, once again I will be coming back.

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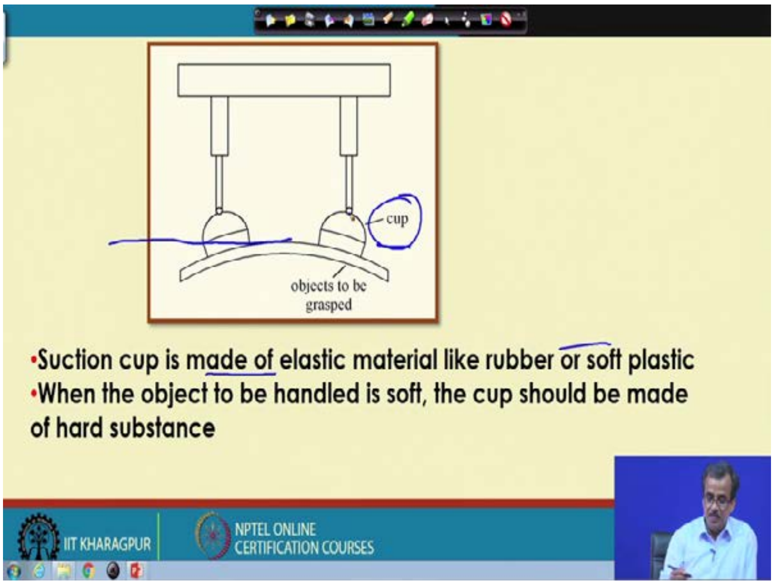

So, this is the way, for example, this is the object to be gripped, I am using these two vacuum grippers and this is connected to the robotic the wrist, ok?

So, this particular cup is nothing but the vacuum gripper. Now, let us see, how does it work. Now, to explain the working principle actually, I will have to go back to this particular design, ok? Now, here, the flowing fluid is air, you can see that air is actually coming in and going out, ok? Now, we know that the air is a compressible fluid ok?, but for simplicity, let me assume that air is incompressible fluid, that is, the density ρ is constant. So, this is an assumption. This is an assumption because, we know that air is nothing but a compressible fluid ok?, but for the purpose of analysis, let me assume that the ρ is kept constant, that is, nothing but the compressible fluid.

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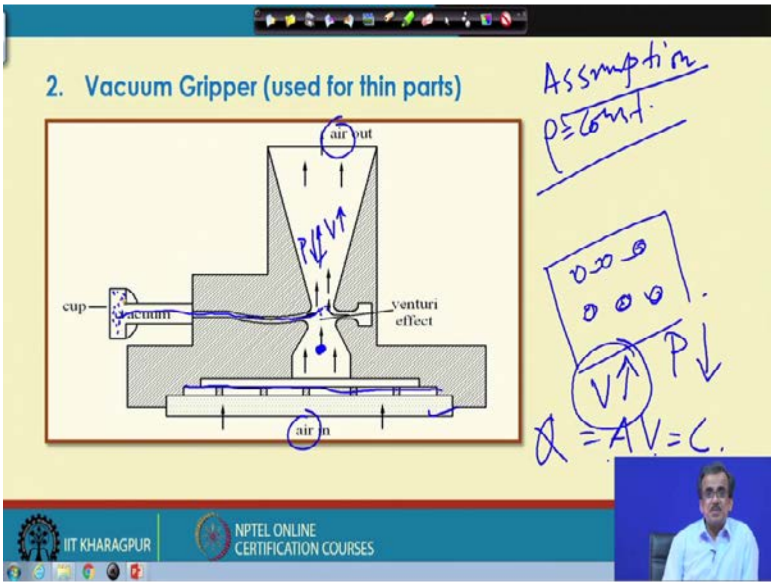

Now, here, the way it works is as follows: here, we have got a strainer. So, this is the strainer. It is just going to separate out the dirt particles. Now, inside, we have got one plate, so this is the plate and on this particular plate, actually I have got some small drilled holes, ok?, some small opening, we have and this is known as the orifice plate, ok?

So, this is actually the orifice plate and on this orifice plate, we have got some small openings, ok? Now, the air will be forced to pass through this orifice plate and the moment it passes through this small area, what will happen to its velocity? The velocity is going to increase based on the continuity equation because according to the continuity equation the volume rate of flow, that is, A×V, area multiplied by velocity should remain constant.

Now, here, the area is decreasing, so the velocity is bound to increase. Now, if velocity increases, what will happen to the pressure? According to Bernoulli’s equation, the pressure is going to be reduced, ok? So, as velocity increases, the pressure is going to be reduced. So, here, the velocity will be more, but pressure is going to be reduced, and then, this air will pass through the venturi. Now, in venture, once again, there will be some change in the cross-sectional area and due to this sudden change of the cross-sectional area, what will happen? Once again, the velocity is going to increase, but pressure will be further decreased and this particular region is connected to this part.

So, here, the pressure becomes below atmospheric. So, definitely, here, the pressure will be below atmospheric, ok? So, inside the cup, the pressure is below atmospheric, ok? Now, if you see this application, the way we are using it.

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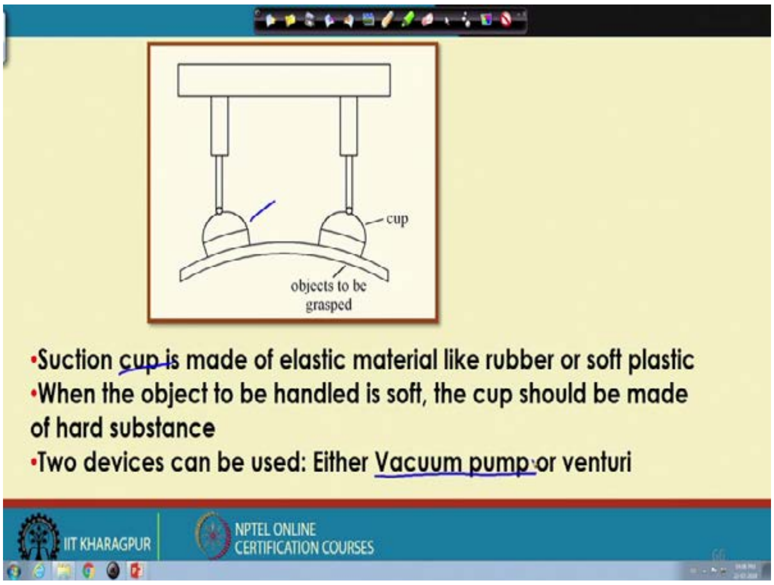

So, here, inside this, we have got the pressure below atmospheric and outside, we have got atmospheric pressure and due to this pressure difference, this particular elastic cup or the vacuum gripper is going to grip the object, ok?. So, this is the way, due to this pressure difference, it is able to grip that particular steel plate.

Now, how to un-grip? To un-grip it, actually what we will have to do is: we will have to stop the air flow. Now, if you stop the air flow what will happen? So, this particular area, is connected to air atmospheric air, inside the cup, the pressure will be atmospheric pressure, and outside is also atmospheric pressure, there is no pressure difference and then, due to the self-weight, this particular steel plate is going to be un-gripped. So, this is the way, actually, we grip and un-grip, if we use some sort of vacuum gripper.

Now, here, this vacuum gripper is developed in the form of some elastic cup. So, this is actually the elastic cup. So, this is the elastic cup. Now, this elastic cup is made of elastic material like rubber or the soft plastic. Now, if the object is hard, we generally use the soft elastic material and if the object is soft, we use some sort of hard material, as the vacuum gripper.

Now, to generate the vacuum, we can use a venturi and orifice, the way I discussed. We can also use some sort of vacuum pump. So, by using the vacuum pump also, we can create that particular vacuum inside the vacuum gripper or the elastic cup and using this we can grip and un-grip. So, this particular object, part will be connected to the robot and the robot is going to grip and un-grip that type of object like steel plates.

Thank you.

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Lecture - 08
Introduction to Robots and Robotics (Contd.)

We are discussing the working principles of different types of end-effectors used in robots.

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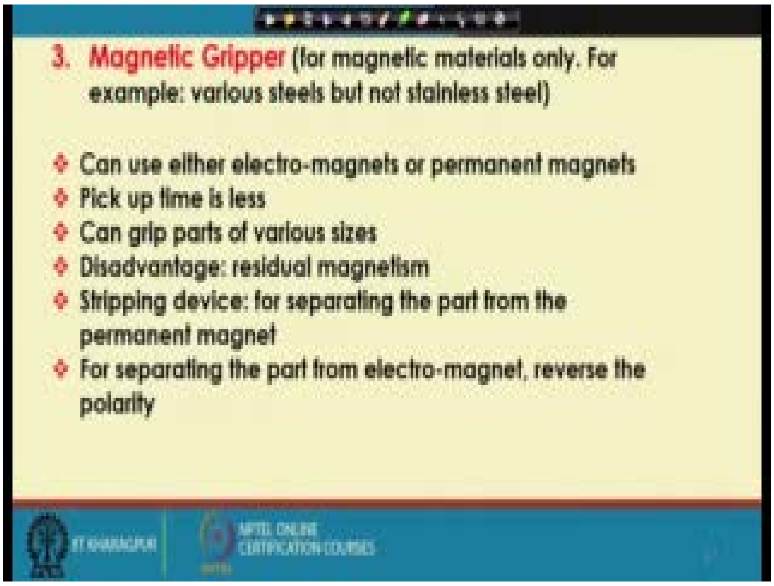

Now, I am going to start with the working principle of a magnetic gripper. Now, this magnetic gripper is suitable for the magnetic materials. For example, say, if I consider a component made of steel. So, this particular magnetic gripper is going to work, but it will not work for the stainless steel, because stainless steel is not magnetic.

Now, here, we can use both permanent magnet as well as electro-magnet. Now, if I use permanent magnet, the mechanism will be as follows:

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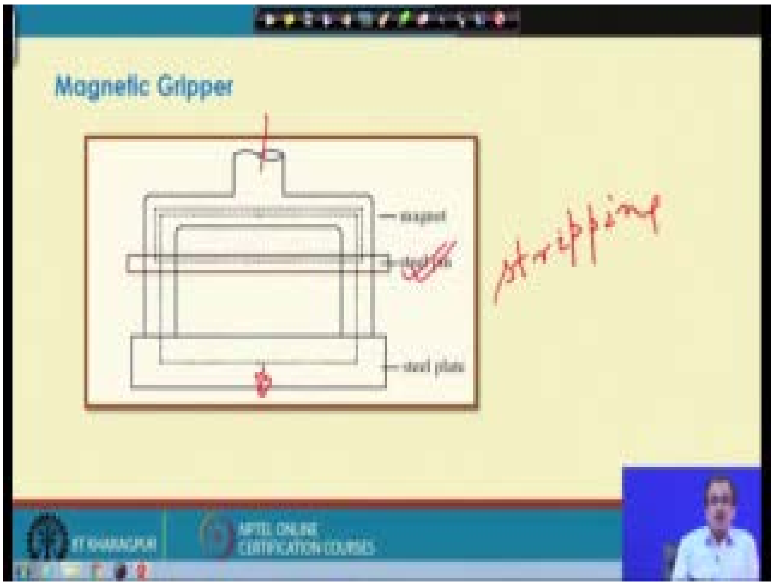

So, for example, say this is nothing but the permanent magnet and this permanent magnet will be connected to the robotic end-effector. Now, if I see this particular magnetic gripper, if we use magnetic gripper, we have got a few advantages, for example, it can grip objects of various sizes and moreover the pickup time will be less. On the other hand, it has got drawback like it is subjected to residual magnetism.

Now, supposing that I am using a permanent magnet now of this type. So, this is actually the permanent magnet, which I am going to use and this will be connected through the wrist end of the manipulator. Now, this is the steel plate which I am going to grip. The moment I put this permanent magnet very close to the steel plate, the magnetic lines of forces are going to pass through this particular steel plate, and due to this, this steel plate will be gripped by the permanent magnet.

Now, if I want to un-grip or if I want to remove this particular steel plate from this magnetic gripper, I will have to use one stripping device, that is nothing but a steel pin. So, this particular steel pin can be used as the stripping device. The way it works is as follows: here on this particular permanent magnet, I have got one circular hole here and I have got another circular hole. If I want to un-grip, this particular steel pin will be inserted through these two circular holes. The moment we put this particular steel pin, some of the magnetic lines of forces will pass through this particular steel pin and consequently, the strength of the magnetic field passing through the steel plate will be weaker and due to this weakness in strength of this particular magnetic field and the self-weight of this steel plate, the steel plate will be separated out from this particular permanent magnet. Now, this is the way actually, we can un-grip so, this particular steel plate from the permanent magnet.

Now, in place of permanent magnet, if I use the electro-magnet, and if I want to grip it, it is ok, but if I want to un-grip, what I will have to do is: I will have to reverse the polarity. So, if I reverse the polarity of the electro-magnet, I am just going to un-grip this particular steel plate. Now, this is the way actually, one magnetic gripper works and its working principle is very simple and this is very frequently used for the magnetic material, but this will not work for the nonmagnetic material.

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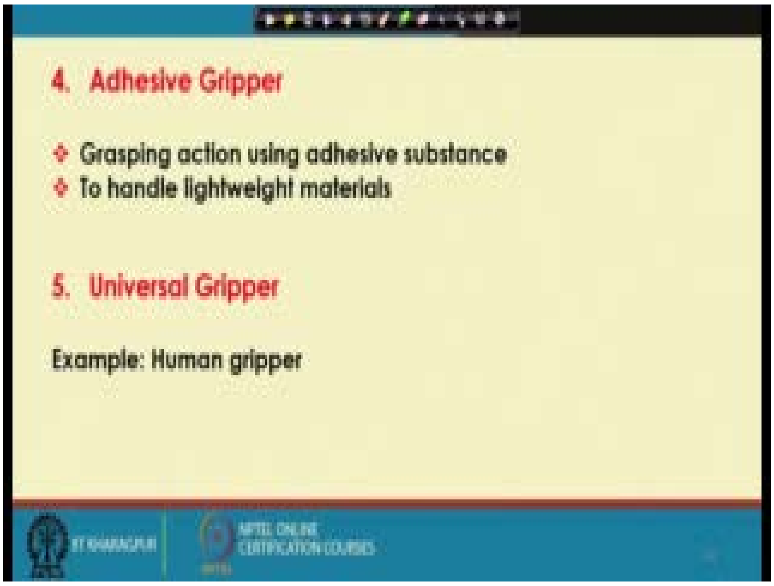

The next is the adhesive gripper. Now, this adhesive gripper is suitable only for the light object like the small weight objects and here, we use some sort of adhesive material. Just to grip that particular object, we take the help of adhesive material. Now, this is almost similar to the way one frog catches its prey. So, on the tongue actually it puts some sort of adhesive material and that particular tongue will be thrown towards that insect and the insect will be caught with the help of this particular the adhesive material. So, this particular adhesive gripper, as I told you, is suitable only for the very light material.

Now, then, comes the universal gripper. Now, our hand is actually a true example of this particular universal gripper, because with the help of our hand we can grip different types of object and our gripper is robust and it is flexible and it can grip a number of objects of different shapes and sizes. And, that is why, this is a very sophisticated one and our gripper is known as the universal gripper.

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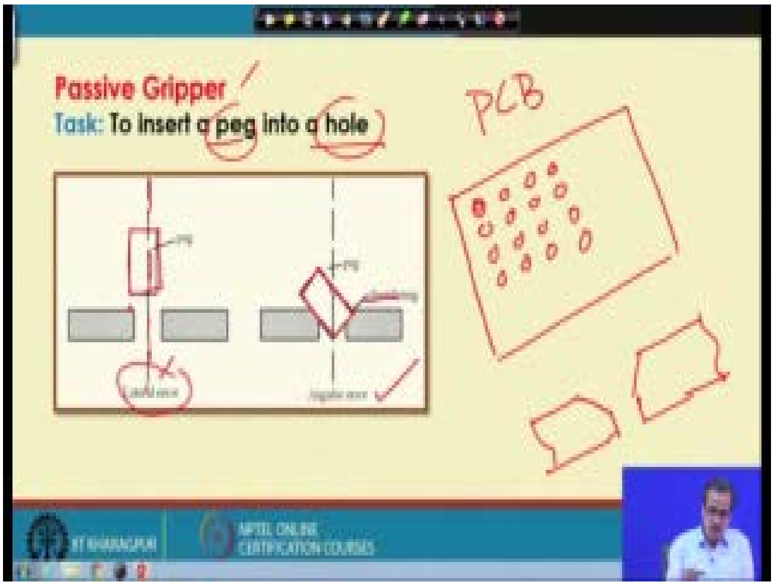

So, now, I am just going to start with the working principle of the passive gripper. Now, this passive gripper is used, whenever there is no such sensor. I have already mention that by passive gripper, we mean those grippers, where we do not use any such sensor. Now, before I proceed with the working principle of this particular gripper. Now, let us try to understand, why do we go for this type of gripper. Now, let me take one very simple example. Now, supposing that I want to develop one printed circuit board (PCB) and on the printed circuit board, there are some small small circular holes. And, what we will have to do is: depending on the requirement of this particular electrical or the electronic circuits, I will have to insert some sort of small elements like register, capacitor, and so on, just to design and develop a particular printed circuit board.

Now, this particular task, if I give it to the manipulator, if I give it to the robot, at the end -effector actually, we will have to put a special type of gripper, which is nothing but the passive gripper, if I want to insert some small items like register, capacitor into this particular hole. Now, here, the problem which we are going to face is like, it is a bit difficult to insert a peg into a hole. Now, this particular problem in robotics is the actually very popular and so, how to insert a particular peg into a hole, here. So, this particular schematic view shows that I have got a steel plate and on the steel plate, we have got one circular hole.

Now, on this circular hole like this, actually we will have to insert this particular peg. Now, supposing that, this is the central line for this hole and I have got this particular peg, which I will have to insert now, this peg is actually gripped with help of the gripper and now, that robot is going to put this particular peg into the hole. Now, if we to want to put this particular peg into the hole, there is a possibility that this part of the peg is going to collide here and due to this, it will not be able to insert. So, in this particular peg into the hole and this is what is known as the lateral error. So, the robot will not be able to insert this particular peg into the hole and it will be obstructed here, this is called the lateral error.

Now, to remove this lateral error actually, what we do is: we put some chamfering, that means, I have got this type of plate and I put this particular chamfering sort of thing. So, here, I am just going to put the chamfering. Now, if I put this particular chamfering and try to insert this peg with the help of the robot, there is a possibility that this particular lateral error will be solved. But, it is going to create another problem, might be this particular peg is going to take the position something like this and it is going to create another problem, another error, that is called the angular error. So, by inserting this particular chamfering, there is a possibility we can solve this lateral error, but we are going to create another problem that is called the angular error. So, how to solve both the errors, so that I can insert this particular peg into the hole.

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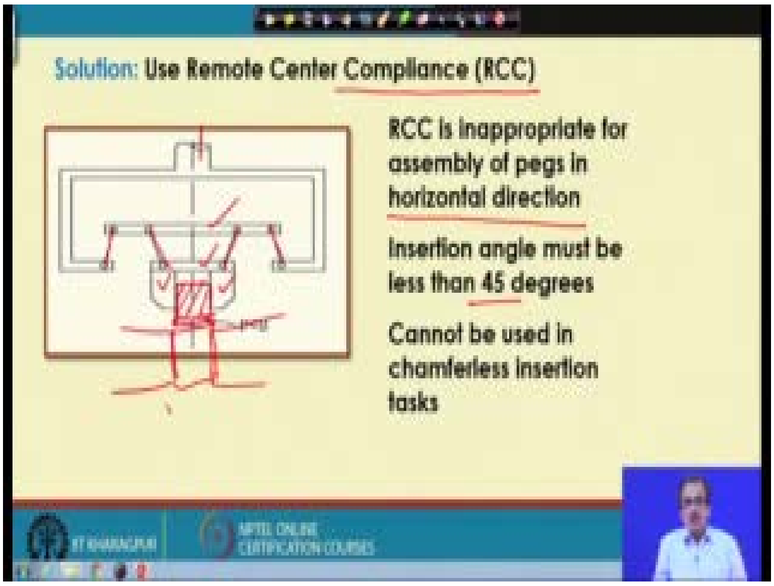

Now, to do that actually, we take the help of one passive gripper, which is very popularly known as remote center compliance and that is nothing but RCC (Remote Center Compliance). Now, the construction-wise it is very simple. For example, say, this part this is connected to the wrist end of the robot and here, I have got one steel frame sort of thing, I have got another frame here, steel frame sort of thing, I have got another steel plate sort of thing of small thickness and this particular plate and that particular plate are connected with the help of four such links like this and here we have got two fingers. So, this is finger 1, finger 2 and with the help of this two fingers, we try to grip this particular peg. So, this is actually nothing but the peg. So, this is the peg, which I will have to insert into that particular hole.

Now, this is connected to the wrist end, as I told. Now, this particular peg will be brought very near to the hole and supposing that the hole could be here. So, might be the hole could be here and here actually what we do is: this particular peg will have some sort of oscillatory movement like this and due to this particular oscillation and due to this error and trial, this peg will be inserted into this particular hole with the help of this RCC, which is nothing but a passive gripper.

Now, this RCC will work, provided we put some chamfering at this particular plate, otherwise, it may not work and this angle of chamfering has to be less than 45 degrees, otherwise there could be some sort of angular error and moreover so, this RCC can work in vertical direction, but it will not be working in the horizontal direction, but this particular gripper is very popular just to solve, how to insert small, small electronic items into that particular printed circuit board. So, this is the way, actually, this particular passive gripper works.

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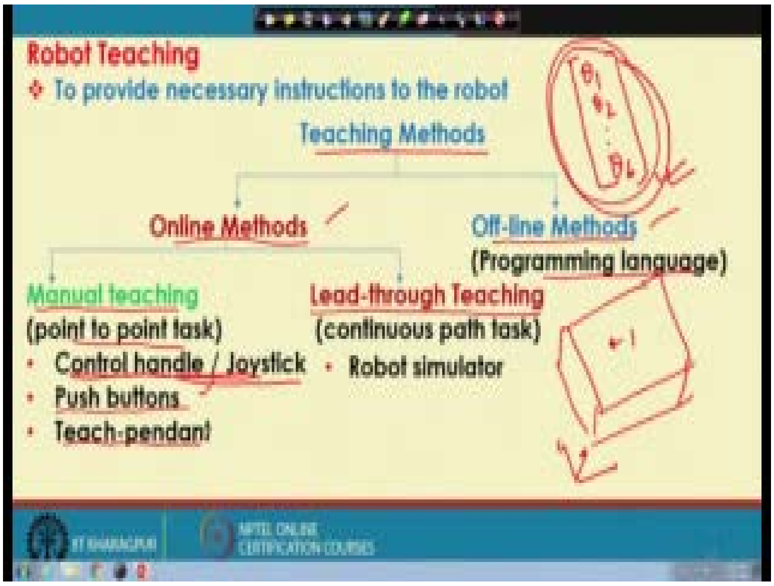

So, this is all about the end-effectors, the different types of end-effectors, which is generally used in robots. Now, here, I just want to mention that depending on the requirement, depending on the task, we will have to design the special type of gripper, special type of end-effector. So, the working principles of a few grippers, a few end-effectors, I discussed are actually very simple, very simple design, but depending on the complicated task, depending on the task, the nature of the task, we will have to design the most suitable gripper, and that is why, we see the task and try to design the end-effector or the gripper.

Now, I am going to start with the teaching methods like how to give instruction to a robot. Now, supposing that, say I have got one robot, say, one serial manipulator and I just want to give the instruction that you start from a particular point, say, the tip of this particular marker and you reach this particular point, the tip of this particular finger through a number of intermediate points.

Now, how to give this type of command, how to give this type of instruction to this particular robot; and now here, actually the purpose of teaching as I told, to provide necessary instruction to the robot. Now, these teaching methods are broadly classified into two groups. We have got online methods and we have got offline methods. Now, by online methods, we mean those methods, where while giving instruction, we use this particular robot. That means, we are going to teach a robot, but while giving instruction or while teaching it, we will have to use that particular robot. So, that particular method is known as the online method.

On the other hand, if I do not use the robot while teaching, that particular method is known as the offline method, and here in offline method, we will have to take the help of some sort of programming language. Now, let me first concentrate on these particular online methods. Now, these online methods are once again classified into two subgroups one is called the manual teaching, another is called the lead-through teaching. Now, let me try to discuss the working principle of this particular manual teaching first.

Now, supposing that I am just going to use one serial manipulator having, say 6 degrees of freedom like PUMA and I am just going to do some sort of drilling operation on a steel plate. So, what I will have to do is supposing that this is the plate and on this particular plate I want to just do some sort of drilling here, at location – 1. So, what I will have to do is as follows: this twisted drill bit has to be gripped by the gripper of this particular manipulator and the center of this particular hole and the tip of this particular the twisted drill bit should coincide.

Now, this is in 3D, for example, it has got like x y and z axis. So, this particular object is in 3D. So, how to reach this particular point, a 3D point in 3D space, with the help of that particular manipulator having 6 degrees of freedom. Now, to reach this particular point in 3D space with the help of a manipulator having 6 degrees of freedom, there could be several combination of the θ values, for example, say there could be several combination of θ_1, θ_2 ……, θ_6 values with help of which, I can reach this particular point, say point 1 and out of all the possible combination of the θ values, if I know at least one, my purpose will be served.

Now, how to collect this particular information, to collect the information, what I can do is so, I can take the help of manual teaching which is suitable for point-to-point task and this is nothing but a point-to-point task. So, there are several methods for this manual teaching, for example, say we can take the help of control handle or joystick. So, with the help of this control handle or joystick through some error and trial, the tip of this particular twisted drill bit will be able to reach the center of this hole.

The moment it reaches the center of this particular hole, we store all the θ values with the help of optical encoder, which are mounted at each of the robotic joints. And, we measure all such θ values and once, we have measured all such θ values corresponding to this particular hole which is to be drilled on this plate. So, what I do is: we replace this plate by a second one, and we make this particular drilled hole exactly at the same location, then once it is done on the second plate, we go for the third plate and solve; so for a large number of plates exactly at the same locations, I can make these types of drilled holes.

Now, to collect this particular information of θ_1, θ_2 ……, θ_6, we take the help of this control handle, which is nothing but a manual teaching. Now, then comes the push buttons. Now, we have already discussed, that for each of this particular robotic manipulator, there is a director or a controller. Now, on the body of the director or the controller; there will be a few push buttons and with the help of these push buttons, actually, we can control the movement of the tip of the manipulator either in Cartesian coordinate system like x, y and z or in joint space like in terms of θ_1, θ_2 ……, θ_6.

So, we can increase the values of θ_1, θ_2 ……, θ_6, we can increase and decrease the numerical values for these x movement y movement and z movement, then through some trial and error the movement it reaches this particular point, what I do is, we store all such θ values with the help of optical encoder. So, this is how to use the push button.

Now, next, I am going to discuss how to use a teach-pendant for the manual teaching. Now, this teach-pendant is nothing but one remote controller for this robot. So, just like the remote controller used in TV, it is look-wise almost similar, but slightly larger in size. Now, this teach-pedant can be operated either in the Cartesian coordinate system or the world coordinate system, that is, in x, y and z. It can also be operated in the joint scheme, that is, in terms of θ_1, θ_2 ……, θ_6. It can be operated in tool coordinate system also, and so on.

So, we will have to select a particular operating system or the coordinate system and then by using this particular teach-pendant manually, we can control the movement of the different joints. The moment, it reaches the tip of this particular the cutting tool reaches the center of the hole, so, what I do is: we store all such θ values with the help of optical encoder and the same set of θ values, we use for a large number of plates. Now, this is the method of how to use the teach-pendant to incorporate the manual teaching.

Now, I am just going to discuss the principle of another online method that is called the lead through teaching. Now, for this lead through teaching, actually, this is suitable for some sort of continuous path task, which I have already discussed and for this particular continuous path task, the tool should be in touch with the job continuously.

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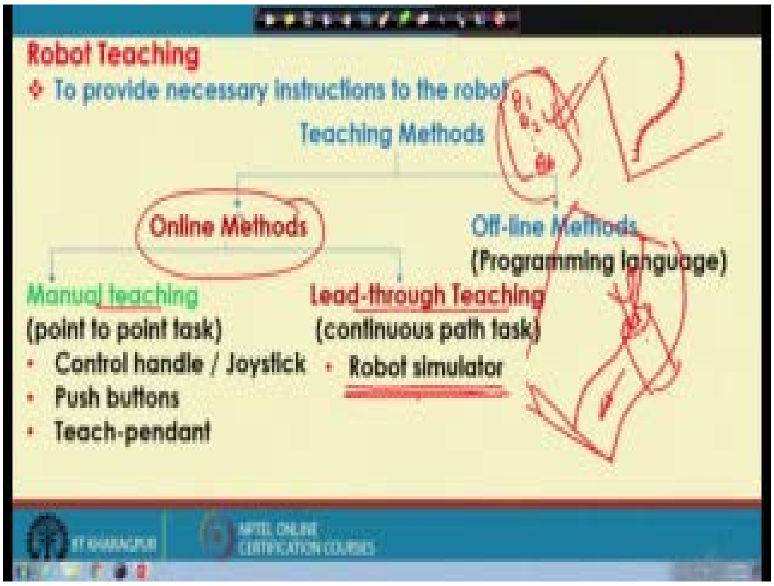

Now, let me take the same example, which I took for this particular continuous path task, supposing that this is actually a profile, which I will have to cut on one side of a steel plate. The way, it has to be done is as follows: actually, we use some sort of milling cutter. So, this type of milling cutter, we use and so, this milling cutter should rotate and it should be able to trace this particular complicated profile.

Now, this is in 3D. Now, if I consider the 2D view, supposing that x and y, so, this type of profile, I will have to cut. How to cut this type of profile? To cut this type of profile, actually, what we do is: we divide this profile into a large number of small segments and the more the number of segments, the better will be the precision. Supposing that, we are going to divide it into say one thousand segments now, for each of these particular one thousand one points, we cannot find out, so easily, the sets of θ values like θ_1, θ_2 ……, θ_6 and moreover, once we have got it somehow, we will have to store these particular sets of theta values. So, it requires a huge amount of memory, but the more difficult thing is actually, how to determine like one thousand one sets of such theta values.

Now, mathematically, it becomes very difficult to determine theses one thousand one sets of theta values and that is why, we will have to use some other practical methods like how to collect this particular information. Now, one method could be like, if this is the cutter, this particular cutter is gripped by the gripper or the end-effector. Now, here, this particular cutting tool is going to trace this complicated profile, Now, we can try, if this is the cutter, I can just grip it and try to trace this complicated profile, which I am just going to cut.

Now, if I want to trace this complicated profile manually, it becomes very difficult because at each of the robotic joint, there are some motors, there are some brakes, there could be chain drive, gear drive, belt drive, and so on. So, it becomes very difficult like if I just grip it and try to move according to my choice. So, it becomes very difficult to move manually. Then, how to trace? How to trace this particular complicated profile and if I can trace this complicated profile, which I am going to cut, and while tracing at regular interval so, if I can store the θ values with the help of optical encoder, I will be able to collect all sets of θ values.

And, once, we have collected those sets of theta values, we try to fit some smooth curve, which I will be discussing after some time, just to ensure the smooth variation for θ_1, θ_2 ……, θ_6; and once we have got that smooth variation, now, I can operate and I can run that particular robot, but here, the problem is that we will not be able to trace this complicated profile. Now, actually one method has been suggested that we are going to use one manipulator, a second manipulator, that is called the robot simulator.

Now, this robot simulator is actually not a simulation package. So, this is actually a physical robot and this particular robot is kinematically equivalent to the main robot, which I am going to teach, and by kinematic equivalence, we mean that both the main robot and this particular robotic simulator are having the similar type of joints, similar type of links, but this robot simulator could be in 1:1 scale with the main robot or it could be scaled up version or scaled out version.

Now, in this robotic robot simulator, actually, there is no such motor, there is no drive unit, but at each of the joint, we have got the optical encoder. Now, if you have the optical encoder at each of the joint, but there is no such drive unit, there is no such gear, no breaks nothing, now, I can just grip this particular end-effector or the this particular cutter, which is connected to the end-effector, and I can trace the complicated profile, which I am going to cut and while tracing at regular interval with the help of optical encoder I am just going to store the θ values.

Now, this is actually known as the lead-through teaching. Now, this robot simulator is actually the master robot and the main robot, which I am going to control is called the slave robot and this is, in fact, the working principle of master and slave robot. Now, so, this is actually the lead-through teaching. So, both lead-through teaching and manual teaching are coming under the umbrella of the online methods.

Now, I am just going to concentrate on the offline method and here, in offline method actually, we will have to use some sort of programming language.

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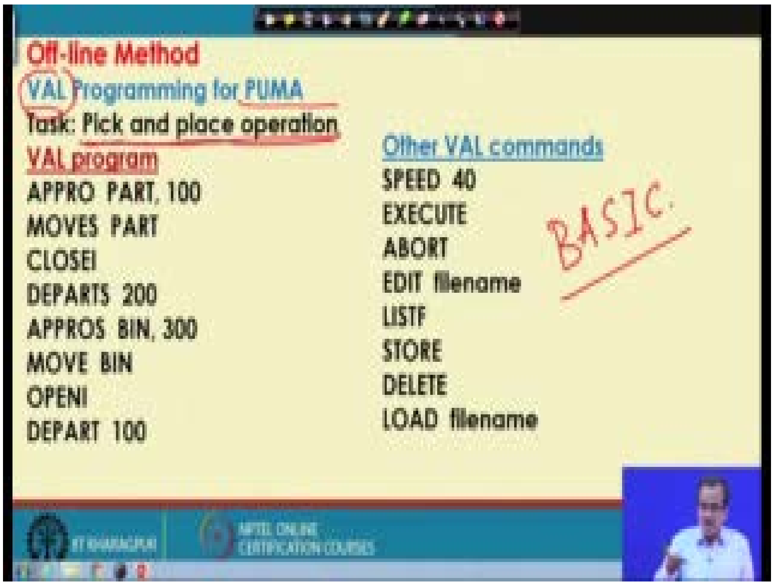

Now, if you see the offline method. So, we will have to use some programming language just like your computer program. Now, here, actually, we can use a language like VAL programming for the PUMA series robot. So, this particular example, I am just going to take for the PUMA series robot using the VAL, that is, versatile assembly language or variable assembly language, and this is suitable only for PUMA, that is, programmable universal machine for assembly.

Now, here, in this VAL programming, actually, we take the help of a few commands from the BASIC language, that is, your beginners all purposes symbolic instruction code. Now, here, we will see that the some of the codes are exactly the same, but we add a few extra the commands also, in this particular VAL.

Now, before I just can write one program with the help of this VAL programming, I just want to define the task, which I am going to give to the robot and that particular task is nothing but the pick and place type of operation. Now, let me discuss a little bit this pick and place type of operation first, then I will be discussing like how to write down that particular VAL program to solve this the problem.

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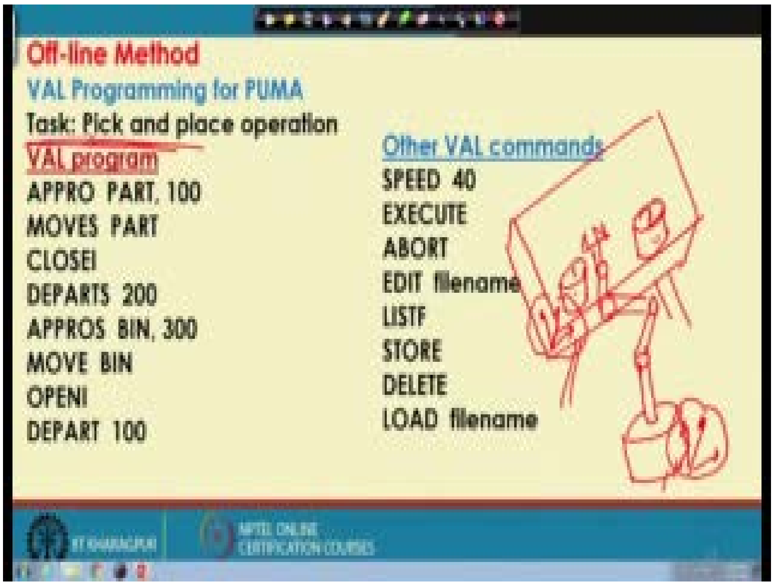

Now, let us try to define the problem, which I am going to solve. Supposing that I have got a table sort of thing. So, this type of table I have and on this particular table, I have got two bins, say I have got say one bin or the bucket here. So, this is one bin and this is another bin or bucket here. So, this is bin – 1 and this is bin – 2 lying on the top of the table so, this is the top of the table, say.

Now, here, actually I do is: I have got a manipulator, say, serial manipulator sort of thing, for example, say I have got a manipulator like this, very simple manipulator. I am just going to consider here that I have got a this type of manipulator, and here, we have got this particular gripper or the end-effector.

Now, this manipulator is having one coordinate system, one base coordinate system, like x, y and z coordinate system and this particular table is having another coordinate system, here, like x, y and z here, ok? So, if I want to give instruction to the robot that you just go to the bin – 1 and collect a particular job and place it to the bin – 2, their particular coordinate systems are to be known to one another. Supposing that, on this particular bin, I have got an object, a 3D object and we know how to represent the position and orientation of this particular 3D object. So, here, to represent the position and orientation, we need actually six information, three for the position and three for the orientation.

Now, let me take a very simple example supposing that this is the 3D object. Now, if I want to represent its position and orientation, I need three information for the position and three for the rotation, that is, the orientation. So, I need six information. Supposing that the position and orientation of the object lying on this particular bin – 1 are known and the position and orientation of this particular bin, that is, the bin – 2 are also known, and all such information I have stored at the top of this particular program. So, here, at the top of the program, I will have to write down the position and orientation of the bin – 1, position and orientation of the bin or the bucket – 2, and the position and orientation of this particular item, the 3D object.

Now, once I know all such things, let us see how to write down, this particular the VAL commands. Now, we are going to discuss, how to teach a robot practically. Now, here, the robot which we are going to consider is the PUMA, that is, programmable universal machine for assembly. Now, this PUMA is nothing but a serial manipulator having 6 degrees of freedom, there are six joints. All six joints are rotary joints and out of six, we have got three revolute joints and three twisting joints.

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Now, here so, this is actually the PUMA. Now, this is a robot with fixed base.

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So, this is the fixed base, the first joint, that is, nothing but the twisting joint. The second joint is the revolute joint.

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The third joint is another revolute joint.

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The fourth joint is a twisting joint. The fifth joint is another revolute joint and here we have got one twisting joint. So, we have got six joints, each joint is having one degree of freedom. So, this serial manipulator is having 6 degrees of freedom.

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Now, if I see the different components. So, this is the body of the robot. This is the controller or the director of this particular robot. Now, this is equipped with one display, but that display has become out of order.

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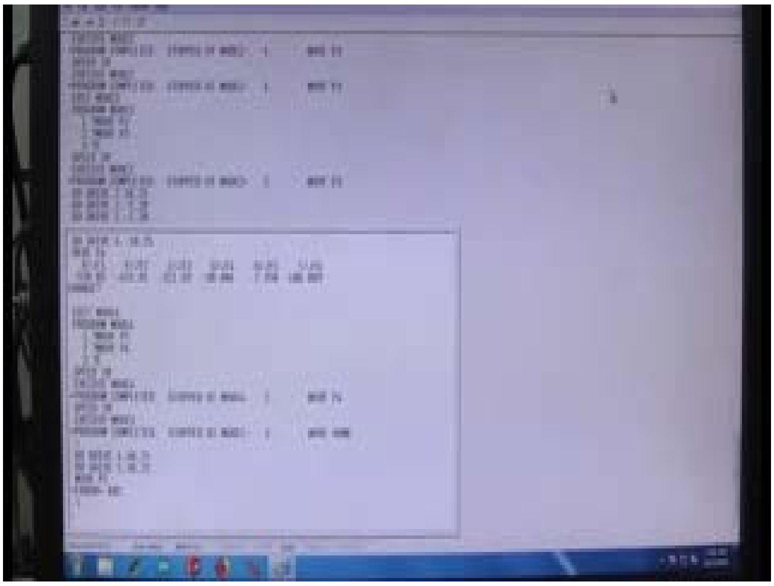

That is why, this particular display is used as the display for the controller or the director of this robot. As we have already discussed that the robot can be taught using either online or offline method. Now, out of these online methods, we have got the manual teaching and lead-through teaching. Now, here, I am just going to show, how to control or how to teach this particular robot using a manual teaching method, that is, with the help of one teach-pendant.

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Now, this particular teach-pendant is nothing but a remote controller for this particular robot. Now, with the help of this particular teach-pendant, the robot can be controlled either in world coordinate system or the Cartesian coordinate system or we can control it in joint space like in terms of the θ space or in tool coordinate system.

Now, these teach-pendant is used for the manual teaching. Now, regarding this offline teaching method, as I have already mentioned that we use some programming language. Now, for this particular PUMA series robot, the programming language, which we are using to teach this particular robot is VAL, that is, versatile assembly language or variable assembly language. Now, this versatile assembly language we can use to write down the program to solve some practical problems. Now, here, I am just going to discuss two practical problems and to solve these two practical problems, we are going to write down the VAL commands and we are going to control this particular manipulator.

Now, let us first concentrate on the first task. Now, the task is, we will give command with the help of the VAL programming like the first the robot will got to its HOME position, then from the HOME position so, it will be directly reach a particular point, a predefined point, say point A, and after that, from point A, it will once again go back to the home. Now, to solve this particular problem, we are going to show you the VAL programming first and then, with the help of this VAL programming, we are going to teach this particular the robot.

Now, here actually, this shows the VAL commands like with the help of this VAL commands actually what we can do is: the HOME is already defined and we are going to defined a particular point, say, either point 1, point 2 or point 3 or point 4 and from this particular point by using this VAL command, we can give the command that it goes to the HOME, then, from HOME we just go to the point A and once again, it comes back to the HOME. Now, we are going to execute this particular program to control the robot.

The second task is related to the pick and place type of operation. So, at location 1, there is an object. Now, the task of the robot is to pick that particular object and it is going to carry it to another location and it will place it there and this is very popularly known as pick and place type of operation.

Now, we are going to show you like how to use the VAL programming, so that the robot can perform this particular task. Now, this is the VAL programming. So, what we have done it here, we have to find the different points. So, the coordinates of the different points, we have already saved in the program and then, we are going to just give the command like: MOVE to that particular point, MOVE to that particular point, another point. So, this is the way actually we can write down this particular VAL programming to solve the pick and place type of operation.

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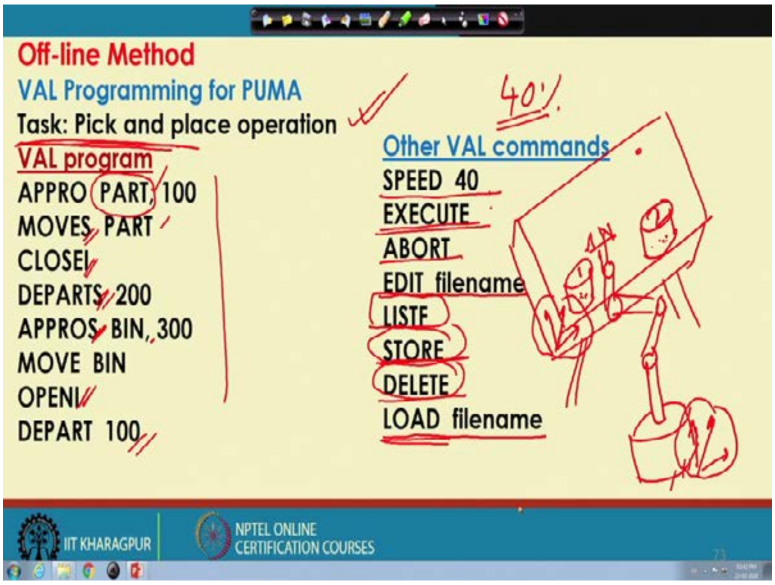

Now, I am just going to write down the VAL programming to solve the task, that is, pick and place type of operation, and I have already defined the position and orientation of this particular 3 D object, and that is nothing, but the PART. PART is actually that particular name of the 3 D object, and this particular part is here on this particular you are the bucket number 1 or the bin number 1, the tasks of the robot will be as follows:

So, this end-effector or the gripper will come to bin 1, it will grip that particular object and it will carry it to this particular bin 2 and it is going to place it there, and for that I am just going to write down this VAL programming.

So, the first command is APPRO PART comma 100, PART is actually the name of that particular 3 D object. So, if I write down APPRO PART comma 100; that means, this PART is defined its position and orientation are defined.

So, this particular end-effector will come to a position, which is 100 millimeter above this particular part in the z direction and it will stop there, the next command is your

MOVES PART, “S” means straight path. So, from here, it will move to the PART by following a straight path and it is going to reach that particular item, that is, PART then CLOSEI. So, with the help of this particular end-effector or the finger, so it is going to grip that object and “I” means, there will be a short delay.

So, “I” indicates a short delay, so now, this particular end-effector has already gripped that particular object, whose name is PART. The next command is DEPARTS 200, “S” is once again stands for straight. So, from here, so depart by 200 millimeter along this particular the z direction. So, by default, this particular movement is along the z direction, then APPROS BIN comma 300, “S” means the straight path. So, from here, you approach a particular point by following a straight line to a point and that point is actually 300 millimeter above this BIN, that is, your BIN_2.

So, here, I am here, so I am actually at the top of that particular BIN, then move to BIN. So, this particular end-effector will move to this particular BIN and once it is moved to this particular BIN, next is your OPENI; that means, you un-grip that particular object, you release the object and “I” means that there will be a short delay. And, once it has been un-gripped, it will depart in the z direction by 100 and that is nothing, but DEPART 100.

So, that completes actually the VAL programming to solve this particular task, that is, your pick and place type of operation. So, this is the way, actually, we can write down the program, the VAL programming just to teach the robot. Now, here, I am just going to tell regarding some other VAL commands, which are also used along with this. For example, say, we can mention this speed. Regarding the speed of movement now, if I write down speed 40, it means the speed will be 40 percent of the rated speed or the maximum speed.

So, actually, at each of the joint, we have got the motor and the motor is having the maximum rated speed. So, speed 40 means, it will take 40 percent of the maximum rated speed. Now, EXECUTE is actually the command, if you want to run this particular program. ABORT is another, if you want to stop running that particular robot, now EDIT filename. So, if we write down EDIT then blank filename, so that particular ready-made program will be opened, which is the already stored there. So, that will come on the display and now, you can add a few lines, you can delete a few lines, you can save it, that is, by using this particular command, that is, STORE.

So, by STORE, we can save that particular program and then LISTF is the listing of the file, supposing that there are a few files, which are stored in the program and if you want to see the listing of that particular files. So, we can use this particular command, the next is your DELETE. So, we can DELETE a line, we can DELETE program with the help of this particular command DELETE, next is your LOAD blank filename. So, if I write down LOAD blank filename, so with the help of this particular command actually we can load the program on the display and there, we can add a few new lines, we can delete a few lines and after that, we can save it, we can execute and we can run this particular robot.

Now, here, I just want to mention one thing that I have discussed the different robot teaching methods some specific commands to solve a particular task but the purpose is not to make it intelligent. So, by teaching, we cannot make a robot intelligent and to make it intelligent actually, we will have to do something else, which I am going to discuss at the end of this course, and once again let me repeat the purpose of teaching is just to give instructions, but not to make it intelligent. Now, actually, I am just going to concentrate on how to prepare the specification of a particular robot.

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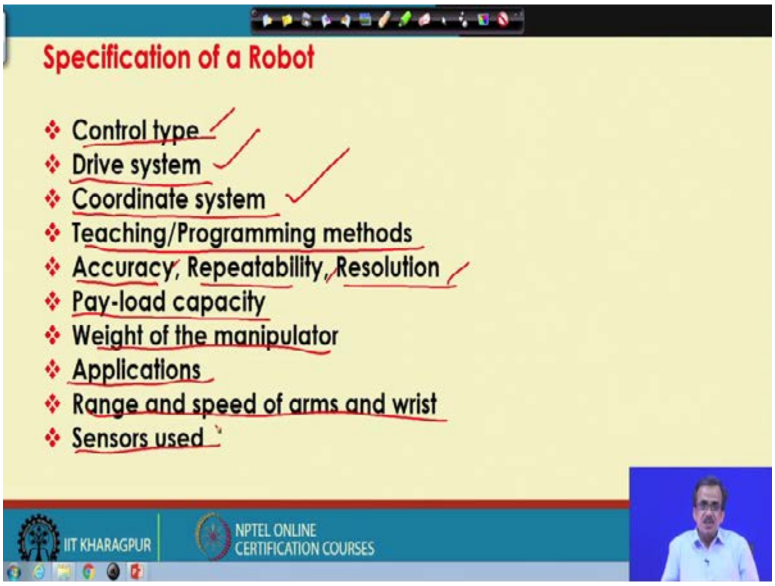

Supposing that I am just going to purchase a robot to solve some specific purposes, now, if I want to purchase a robot, what are the different information which I will have to give, which I have to specify while preparing that particular specifications. So, these things, I am just going to mention one after another, for example, say.

So, I will have to mention about the control type, that means, I should go for the servo-controlled robot or non-servo controlled robot. I have already discussed, non-servo controlled robot means you will not be getting very accurate movement, and if we want to get very accurate movement, we will have to go for actually, the servo-controlled robot. So, the control type, we will have to mention.

The next is the drive system, we will have to mention that whether we are going for the pure mechanical drive like the gear drive or chain drive or bell drive or we are going for some sort of hydraulic drive, pneumatic drive, electro-hydraulic drive, electro-pneumatic drive. So, that particular drive system, we will have to clearly mentioned, the next is the coordinate system like whether it is Cartesian coordinate robot or cylindrical coordinate robot or spherical coordinate robot or a revolute coordinate robot that we will have to clearly mentioned in the specifications.

The next is the teaching or the programming method, how can I teach a robot? which method I am going to use? So, that we will have to mention. The next are the accuracy, repeatability and resolution, we know the meaning of these particular terms. If we want to purchase a robot, we will have to mention how much resolution you want, what is the accuracy and repeatability you want, the next is actually the pay load capacity.

Now, depending on the pay load capacity, actually, we will have to find out how much should be the joint torque, how much is the capacity of the motor, and others and these things, I will be discussing after some time. So, this pay load capacity is actually the maximum amount of load that can be carried at the end-effector of this particular robot, we will have to clearly mention. Then, it comes the weight of the manipulator. So, we will have to mention, what should be the weight and accordingly, actually we will have to think about the foundation of this particular the robot, also.

Next is the application, the purpose for which we are going to use this particular manipulator, the range and speed of arms and wrist. So, we will have to specify, what should be the ranges of movement of the different joints and what should be the speed of this particular movement and what should be the workspace, these things actually, we will have to find out, beforehand.

Next is, if I want to make it intelligent, we will have to use some sensors, which I have not yet discussed and that particular sensor we will have to select, how to select what are the different types of sensors to be used. So, these things, I am going to discuss after some time. So, this is actually how to prepare that the specifications, if I want to purchase that particular robot.

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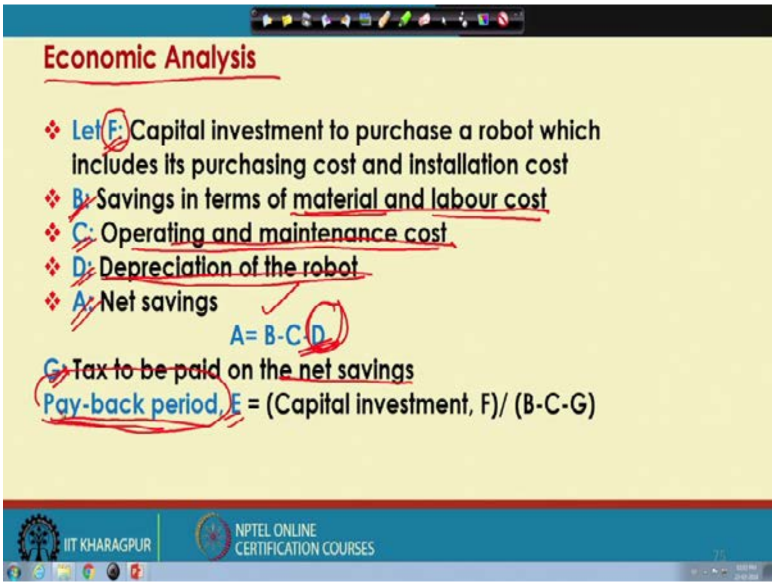

Now, I am just going to carry out one economic analysis now, before we just go for this particular economic analysis. Let me tell the purpose behind going for this particular economic analysis. Now, we have understood that modern manufacturing unit should keep some robots and moreover, the robots are having some other types of applications. Now, a particular manufacturing unit, if we want to purchase a robot, robot is costly, like if we just go for today's very sophisticated serial manipulator having say 6 degrees of freedom, the cost will be Indian rupees around 25 lakhs 20 lakhs something like that, so it is costly.

So, this manufacturing unit is going to take one decision, whether it should take a loan from the bank and purchase this particular robot or not. So, to take this particular decision, that whether I should take loan from the bank to purchase a robot for my own manufacturing unit. So, I will have to carry out one analysis and that is the purpose of this particular economic analysis. Now, here, actually I am just going to define a few terms, and to take the decision at the end, whether I should take loan from the bank to purchase a particular robot for my own manufacturing unit.

Now, here, the symbol: F indicates the capital investment to purchase a robot, which includes the purchasing cost and installation cost. So, if I want to purchase a robot and install that particular robot, I will have to spend F amount of money, now, this B indicates the savings in terms of material and labour cost. Now, we have already discussed that if we can replace the human operator. So, there will be some saving in terms of labour cost and as the chance of rejection will be less, there will be some sort of saving in terms of material cost.

So, this particular B indicates the savings in terms of the material and labour costs, the next is C, that is nothing, but the operating or the maintenance cost for this particular robot, the next is D, which indicates the depreciation of this particular robot. So, by depreciation, actually, we mean the falling value of an asset, now let me take one example, supposing that I purchase one car today by spending say rupees 5 lakhs. And, if I want to sell it after 10 days, I may not get rupees 10 lakhs, in return, provided the cost of that particular brand of the car remains the same.

So, I may get a slightly less, might be say 50000 or say 1 lakh less than the 5 lakhs, now this particular difference of 50000 rupees or 1 lakh that will be the depreciation value of this particular the car in 10 days. Now, actually, whenever we purchase the new machine, our maintenance cost is less and that is why, we consider that, initially, there will be more depreciation and with the age of this particular car or the machine, the maintenance cost is going to increase and this particular depreciation is going to decrease, so that the sum of the depreciation and maintenance costs remains more or less the constant.

But, here, actually what I am going to consider for simplicity that the rate of depreciation is not varying, but generally, we consider the varying rate for this particular depreciation, initially, the rate of depreciation will be more and with time, the rate of this depreciation will be less, but here, for simplicity, I am just going to consider the constant depreciation.

Now, next is A, A indicates the net savings. Now, if I purchase the robot, there will be some saving in terms of material and labour costs. So, $A = B - C - D$, where C is the operating or the maintenance cost and D is nothing but depreciation, that I am going to subtract here and this particular depreciation value is going to help us in, actually, the tax calculation.

Now, actually, what we do is, whenever we calculate tax, we have some standard deduction, now that particular standard deduction is nothing, but some sort of depreciation. So, we calculate the net saving by considering or by subtracting the depreciation value and here, G is nothing, but the tax to be paid on the net saving. So, based on our saving, certain percent of our saving or our income, we pay as tax say 30 percent or 35 percent or something like that.

So, G is actually the total amount of tax to be paid on this particular net saving. Now, pay-back period, I am just going to define in this particular term. So, this payback period is denoted by E and this is nothing, but the minimum amount of time required or the number of years required to get the money back for example, say while purchasing I have spent some money on the robot. So, how much time how much minimum time it will take in years to give me the money back, which I have spent. So, that particular time is nothing, but the payback period for a particular machine or the payback period for this robot.

And, this payback period E is defined as $E = \frac{F}{B - C - G}$, where G is nothing, but the amount of tax to be paid. So, this is the way actually we calculate this particular the payback period; that means, the number of years required to get that particular money back, which I have spent.

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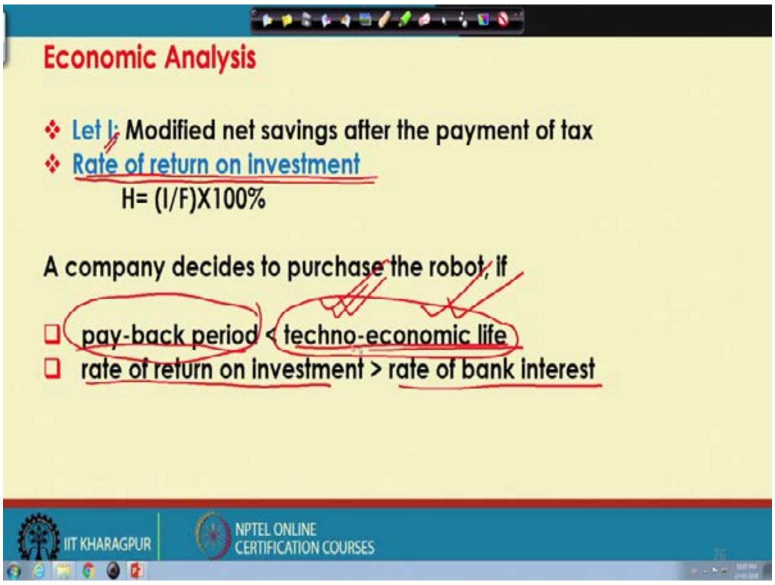

Now, once they have got this particular payback period, next, we try to find out what should be the rate of return on investment, and now to determine the rate of return on investment, what we do is: supposing that, based on my net savings, my income, say 30 percent or say 35 percent I have spent as tax, I have given as tax to the government. So, I am having the remaining 65 percent. So, that particular remaining 65 percent of my income is nothing but "I".

So, let "I" be the modified net saving after the payment of the tax, now if "I" is known. So, I can find out the rate of return on investment (H) as $H = \frac{I}{F} \times 100\%$ and we will be getting some numerical value. And, that is known as actually the rate of return on investment, now, we compare the payback period with the techno-economic life of the robot and this particular the rate of return on investment with the rate of bank interest.

Now, here, the first comparison is your payback period with techno-economic life of the robot, payback period I have already defined. Now, I am just going to define, what do you mean by this techno-economic life of a robot. Now, this techno-economic life of a robot is actually the intersection of the technical life and the economic life. Now, by intersection we mean the minimum supposing that the technical life of a robot is a 10 years and an economic life is a 6 years. So, the techno-economic life will be your 6 years not 10 years.

Now, by technical life, we mean the period up to which the robot can manufacture or the robot can produce goods within the technical specification, within the tolerance limit of this particular product, that is known as the technical life and by economic life, we mean the number of years during which the robot is going to manufacture within the profit zone. So, by using this particular robot in my manufacturing unit, so long as I am getting the profit, this particular robot is in economic life zone, so this is the way we try to find out the technical life of a robot, the economic life of a robot, and we try to find out the intersection and that is nothing, but the techno-economic life.

And, here, the rate of return on investment, I have already discussed and that has to be compared with the rate of bank interest, now, if I take loan from the bank. So, at the end of each year, I will have to give EMI to the bank.

So, this particular rate of return on investment that has to be greater than the rate of bank interest and this particular payback period has to be less than the techno-economic life, then only we should go for purchasing that particular robot by taking a loan from the bank. So, this is the way actually, we will have to take the decision that whether we should purchase a robot by taking a loan from the bank.

Thank you.

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Lecture – 10
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Now I am going to start with one numerical example based on this economic analysis, which I have already discussed.

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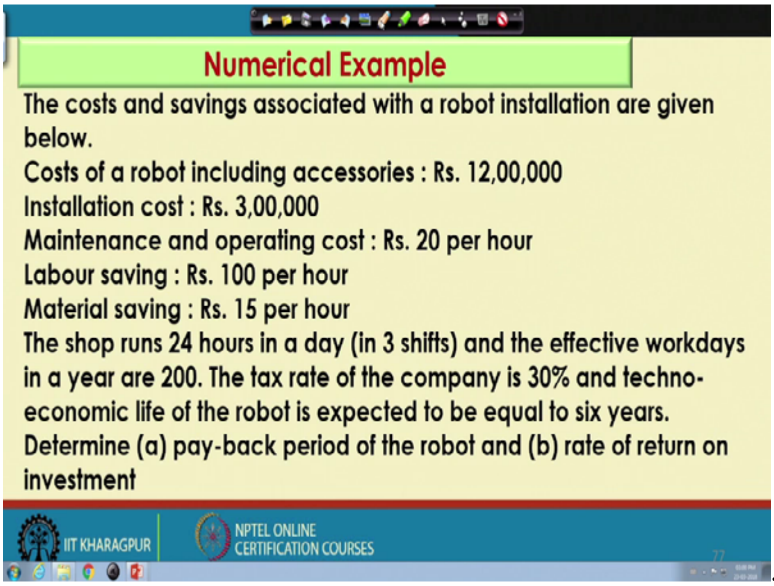

Now, here, I am just going to solve one case study sort of thing, the problem is as follows: supposing that the constant savings associated with a robot installation are given below, for example, the cost of a robot including accessories is say rupees 12 lakhs, the installation cost is rupees 3 lakhs.

The maintenance and the operating cost is say rupees 20 per hour, then, the labor saving is say rupees 100 per hour, then, material cost is rupees 15 per hour, and supposing that the shop is running for 24 hours in a day; that means, there will be three shifts, each shift is equivalent to 8 hours and the effective work days in a year is, say 200, the tax rate of the company is 30 percent and techno-economic life of the robot is expected to be equal to 6 years.

Now, we will have to determine the payback period of this particular robot and the rate of return on investment, and ultimately, we will have to take the decision, whether we should go for purchasing this particular robot by taking loan from the bank, so, that decision I am just going to take.

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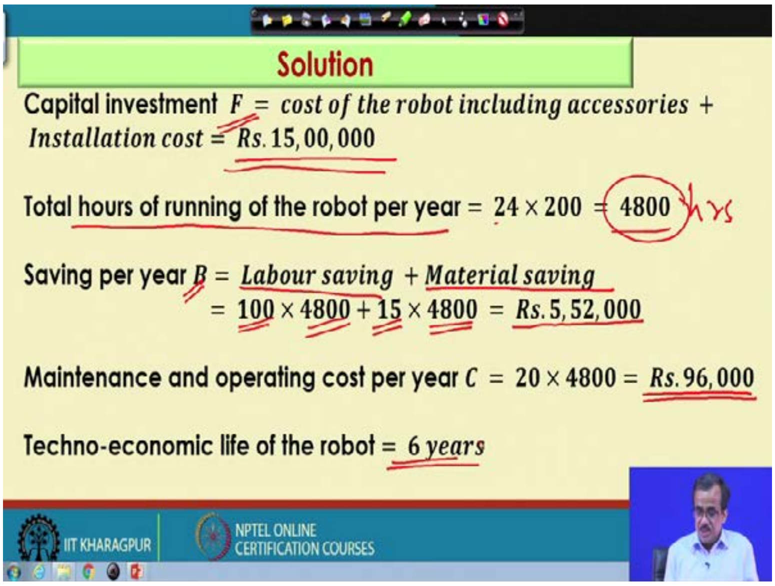

Now, here, the capital investment is denoted by F and this is nothing, but the cost of the robot including accessories plus installation cost. So, rupees 12 lakhs and 3 lakhs, the total is rupees 15 lakhs. So, this is nothing, but the capital investment denoted by F.

Now, here, the shop is running 3 shifts; that means, 24 hours in a day; that means, the whole day, it is running, and 200 days are the number of working days in a year. So, the total hours of running of the robot per year is nothing, but 24 multiplied by 200, that is, 4800 hours. So, this is the total hours of running.

Now, saving per year, say denoted by B, that is, the labor saving and the material saving. Now, the labor saving is rupees 100 per hour. So, rupees 100 multiplied by 4800 (total number of hours) plus the material saving (rupees 15 per hour multiplied by total number of hours 4800), and if we add them up, we will be getting rupees 5, 52000. So, this is nothing, but the total saving per year by using this particular robot.

Then, maintenance and operating cost per year and, that is, nothing, but rupees 20 per hour. So, rupees 20 multiplied by 4800, so this is nothing, but the maintenance and the operating cost and this is coming to be equal to your 9600 and techno-economic life of the robot is given as 6 years.

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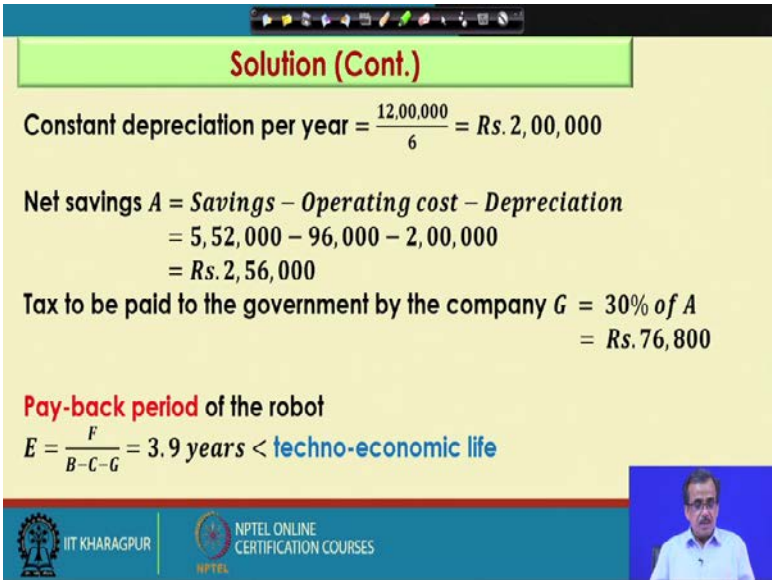

Now, capital sorry the depreciation of this robot per year and that is nothing, but the constant depreciation, which I have considered. So, the constant depreciation, for simplicity, we have considered and now, while calculating this particular depreciation, we should not consider the installation cost. So, actually, by definition, depreciation is the falling value of an asset.

So, we will have to consider the cost of the robot with accessories only, but not that installation cost and that is why, for calculating the depreciation, we consider rupees 12 lakhs divided by 6, but not rupees 15 lakhs divided by 6. So, the constant depreciation per year is coming to be equal to your rupees 2 lakhs and this is, as I told, almost similar to the standard deduction, whenever we calculate our income tax.

Now, the net saving is denoted by nothing, but the saving, which I have already calculated as rupees 5,52,000 minus the operating cost, and operating cost is coming to be equal to 96000, and depreciation is nothing, but rupees 2 lakh and if you calculate the net saving this will become equal to rupees 2,56000.

Now, a certain percentage of the net saving will be paid as tax, and here, the tax rate is 30 percent. So, the tax to be paid to the government and that is denoted by G is nothing, but 30 percent of the net saving, that is coming to be equal to rupees 76800. So, this is the tax to be paid to the government.

Now, this payback period, E is defined as $E = \dfrac{F}{B - C - G}$. So, we know the numerical values of all B, C, G, and F, and if we just insert the values here and calculate, we will be getting 3.9 years, and which is found to be less than the techno-economic life, that is nothing, but 6 years.

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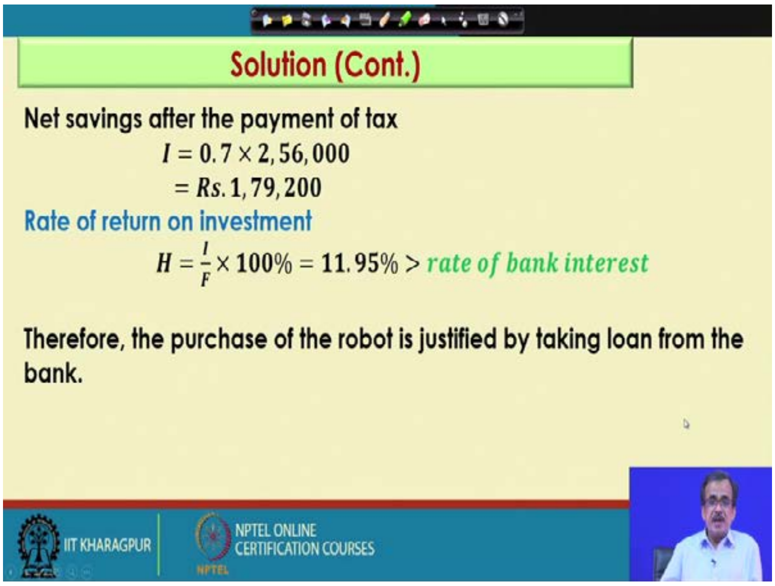

Now, next we try to find out what should be the rate of return on investment. Now, as I told that thirty percent of the net saving, we have paid as tax, the remaining amount, the net saving after the payment of tax is denoted by “I”. So, “I” is nothing, but 70 percent of rupees 2,56,000, that is nothing, but rupees 1, 79, 200.

Now, rate of return on investment is denoted by H and that is nothing, but I divided by F. So, F is the capital investment and “I” is the net saving after the payment of tax multiplied by 100 percent and this is nothing, but 11.95 percent, and the rate of bank interest is around 10 percent, 10 point something.

So, this particular rate of return on investment is more than the rate of bank interest and moreover, the payback period is less than the techno-economic life. So, both are favorable. So, we should purchase this particular robot by taking loan from the bank. So, this is the decision, that we can purchase the robot by taking loan from the bank through this economic analysis. So, this is the way actually, it helps to take the decision that whether we should take loan from the bank to purchase a particular robot.

Now, similar type of analysis, we can carry out for other machines also, for example, other conventional machines for your own manufacturing unit. If you want to purchase some machines, the similar type of analysis we can carry out. So, this is the way, we actually carry out the economic analysis for the robots.

Thank you.

Robotics
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Lecture - 11
Robot Kinematics

We are going to start with the second topic, and this is on Robot Kinematics. Now, here, the purpose of kinematics is to study the motion of the robotic link, but we do not try to find out the reason behind this particular motion. For example, say if it is a linear movement, there must be some force acting, if it is a rotary movement, there must be some torque acting, but here, in kinematics, we do not try to find out - what should be the amount of force, what should be the amount of torque. But, we study only the motion of the different links, the relative motion of the different links, and so on.

So, let us see how to carry out this particular kinematic analysis.

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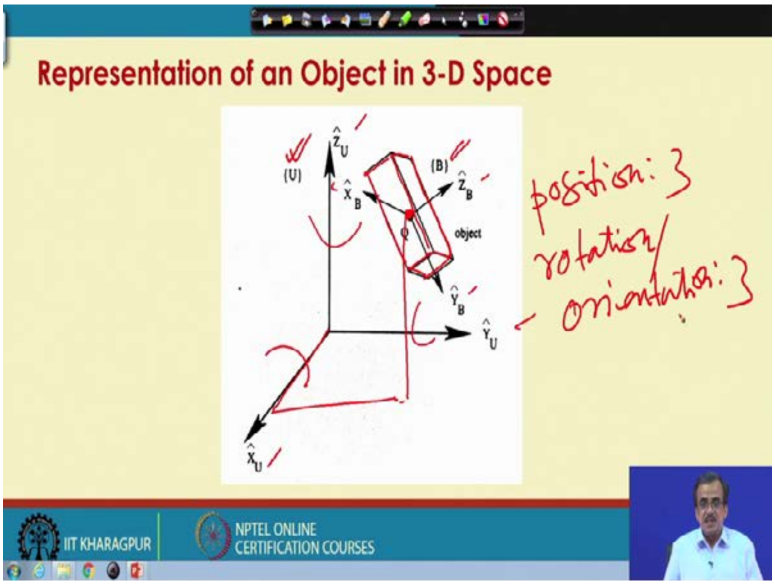

Now, before I start with this kinematic analysis, let me start with the very scratch like the very beginning I should say, supposing that I have got a 3-D object. So, this is nothing but a 3-D object, now this particular 3-D object, I will have to represent. So, this is actually the 3-D object, which I will have to represent; that means, its position and orientation, I will have to represent in 3-D space.

Now, here, U indicates the universal coordinate system and it has got the axis like X_U, Y_U and Z_U and they are independent, they are mutually perpendicular. Now, we will have to represent, this particular 3-D object in this 3-D space. So, how to represent, how to represent its position, how to represent its orientation? Now, here, to represent this particular position actually, what we do is: we try to find out the mass center of this particular 3-D object. So, this is the mass center. So, at this mass center, we just draw one coordinate system, that is nothing but the B coordinate system and it is having X_B, Y_B and Z_B.

Now, here, if I want to determine the position of this particular mass center. So, what I will have to do is, I will have to move along X, I will have to move along Y and I will have to move along Z just to find out this particular position. So, I need three information, and if I want to represent the orientation of this particular 3-D object in this 3-D space, I will have to consider the orientation or the rotations, rotation about X, rotation about Y, rotation about Z. So, I need three more information.

So, for position we need three information, that is, X, Y and Z ; and for this particular rotation or this orientation, we need actually the three more information. So, we need a total of six information. So, this is the way, actually, we can represent the position and orientation of a 3-D object in 3-D space.

Now, let us see like how to represent the position or what do you need to represent the position, only?

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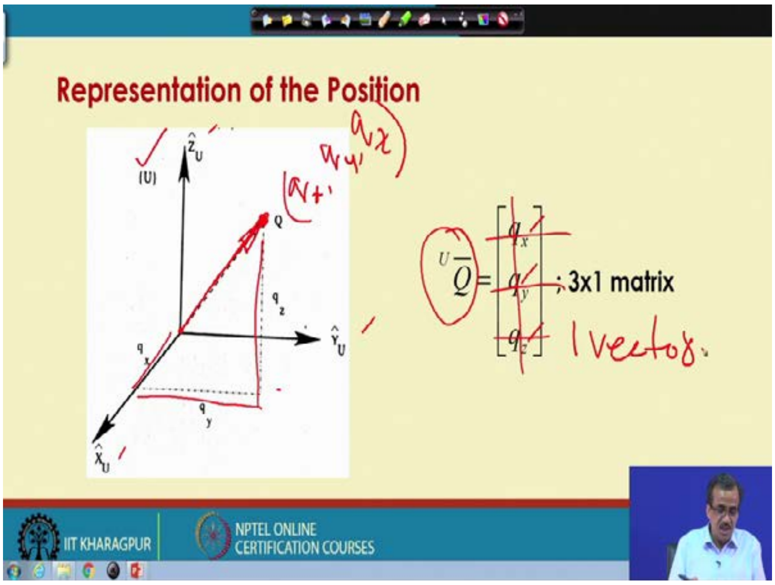

So, representation of the position, that means, the position of the mass center of the 3-D object, let me draw once again the same thing. So, I have got the universal coordinate system. So, this is X_U, Y_U and Z_U and Q is actually a point, whose position is to be determined. So, what I will do is: starting from the origin O, I will move along X, then I will move along Y, then I will move along Z. So, this particular point will be having the coordinate like (q_x, q_y, q_z). So, this is nothing but the coordinate.

And, here, so this particular point can be represented as a vector, as a position vector. So, this is nothing but the position vector and that is denoted by Q with respect to U. So, this particular point is actually Q with respect to U, that is, the universal coordinate system and to represent that particular position vector, I need the elements like (q_x, q_y, q_z). So, this is a vector and in matrix form, this is nothing but a 3×1 matrix. There are 3 rows and 1 column and this is nothing but a 3×1 matrix. So, this is nothing but a 3×1 matrix or a vector. So, to represent the position we need one vector or one 3×1 matrix. So, this is how to represent the position.

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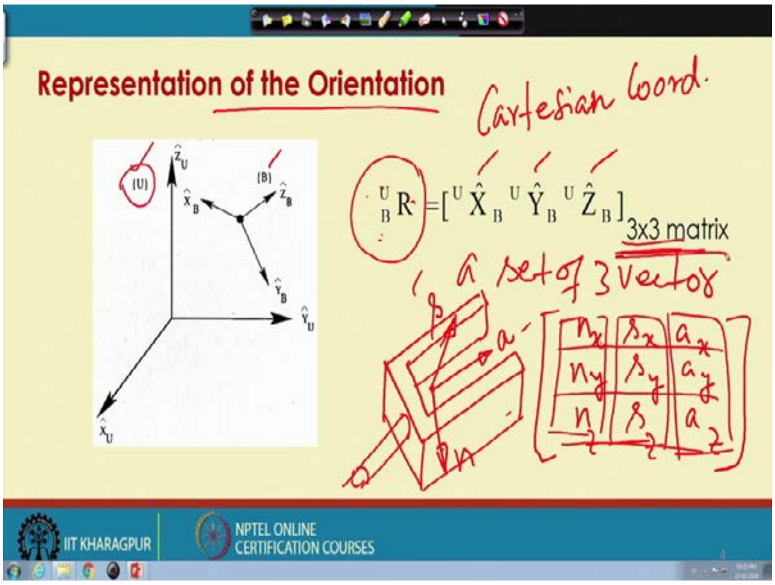

Now, let us see, how to represent the orientation. To represent the orientation, what I do is so, this is once again my U coordinate system, the universal coordinate system and this particular B is the body coordinate system, which is attached to the mass center of the body, whose orientation I am just going to represent. Now, this particular B, you can see that there have been some amounts of change in orientation. For example, say X_B is not parallel to X_U, Y_B is not parallel to Y_U. Similarly, Z_B is not parallel to Z_U, that means, there has been some rotation. So, orientation of this particular B has been changed with respect to this particular U.

Now, here, the way we represent this particular orientation or rotation is like this. So, ${}_B^U R$ is nothing but the rotation of B with respect to U, that is, the rotation of body coordinate system with respect to the universal coordinate system is nothing but R_B with respect to U. Now, to represent this ${}_B^U R$, we take the help of, in fact, three vectors. So, this is one vector, this is another vector, this is another vector. So, we need in fact, a set of three vectors, we need now one set of vector is nothing but a 3 cross 1 matrix and this will become a 3 × 3 matrix.

Now, how can we visualize this particular vector? So, I am just going to prepare one rough sketch just to understand like what are these the three vectors. Now, let me just prepare one very rough sketch for a robotic hand or a robotic gripper sort of thing. So, this is one actually one two finger, say, a very simple gripper. So, this is actually the gripper. So, I have got two fingers here, finger 1 and finger 2. Now, here, with the help of this particular gripper, I am just going to grip that particular 3-D object. Now, if I want to grip the 3-D object it may have different orientations and depending on the orientation of the 3-D object, I will have to change the orientation of this particular gripper or the finger, then only I can grip it and here, to represent the orientation, actually, we take the help of three vectors and this is, actually, one is called the normal vector, that is, denoted by n; another is called the sliding vector, that is, denoted by s; another is called the approach vector, that is, denoted by a, ok?

So, we have got actually normal vector, sliding vector and approach vector. So, if I just write down in the form of matrix n, s, a, this is, along x, x, x; n, s, a; along y, y, y; n, s, a; along z, z, z; n, s, a. So, in the matrix form this particular orientation will be represented like this and this is in Cartesian coordinate system. So, I am now, discussing here the Cartesian coordinate system, how to represent that particular orientation.

Now, here, this n_x, n_y, n_z are the elements of this particular the normal vector. So, these are the elements of the normal vector, these are the elements of the sliding vector and these are the elements of the approach vector. So, I need a set of three vectors and this is nothing but actually the 3 × 3 matrix. In the matrix form so, we have got 3 rows and 3 columns. So, this is nothing but a 3 × 3 matrix. So, to represent the orientation, we need a set of three vectors or we need a matrix and that is nothing but actually a set of 3 × 3 matrix.

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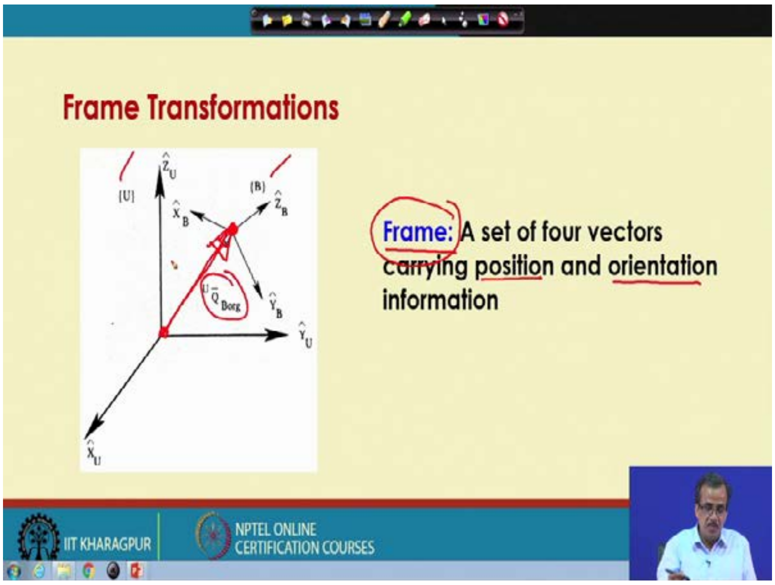

Now, I am just going to define the frame. Now, frame is actually a set of four vectors, which carry information of the position and orientation. We know that to represent the position, we need only one vector and to represent the orientation we need, in fact, three vectors, a set of three vectors; that means, to represent both position as well as orientation we need a set of four vectors. Now, this set of four vectors is known as the frame. Actually, what we do is, we try to assign frame at each of the joints.

Now, here, in this schematic view, if you see, this is nothing but the universal coordinate system and once again B is actually the body coordinate frame, which is attached to the mass center of the body. Now, this particular coordinate system X_B, Y_B, Z_B is having some translation with respect to U. So, this is the amount of translation, so this particular origin has been shifted to this particular point and this particular translation is nothing but Q_Borigin with respect to U. So, in the vector form, I can represent like this. And, here, there are some rotation and that is why, X_B is not parallel to X_U, Y_B is not parallel to Y_U and Z_B is not parallel to Z_U, and this is the way, actually, we can represent the position and orientation with the help of actually the four vectors.

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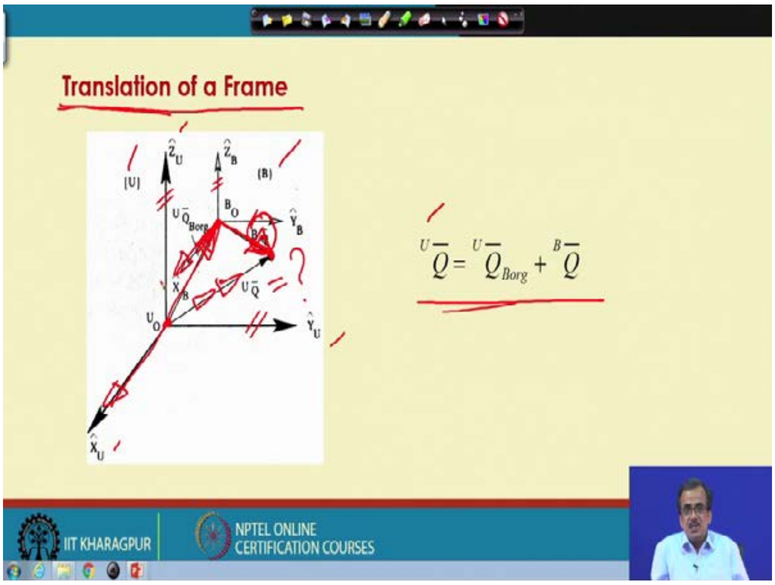

Now, I am just going to concentrate on the frame transformation. Now, frame transformation means it includes frame translation and frame rotation. So, here, actually, I am just going to concentrate on the translation of a frame first. So, only translation, the pure translation, I am just going to consider.

Now, once again, this is the universal coordinate system: X_U, Y_U and Z_U are the universal coordinate system, B is the body attached coordinate system and here, I am just going to consider only translation. So, there is no rotation of B with respect to X_U and that is why, you can see that this particular X_B is parallel to your X_U, then comes this Y_B is parallel to Y_U and Z_B is parallel to Z_U, but there has been a shifting of the origin from here. So, previously the origin was here, now, the origin has been shifted to this and here I am just going to show it with the help of one position vector.

So, this is the situation. So, the frame B or the coordinate system B has been translated only with respect to the universal coordinate system, but there is no such rotation. Now, supposing that, the position of a particular point with respect to the B is known, supposing that this particular vector is known, ok? So, if this vector is known and this particular translation is also known, then, how to find out this Q with respect to U, that is our aim.

So, our aim is to determine the position of the same point, which is lying on this particular body B with respect to the universal coordinate system, that means, I am trying to find out the position vector and very easily, I can find out, that is, Q with respect to U, that is, ${}^{U}Q = {}^{U}Q_{Borg} + {}^{B}Q$. So, this is your Q with respect to B. So, I will be getting, that is, Q with respect to U.

That means, if I know the position with respect to the body coordinate frame very easily you can find out the position information with respect to the universal coordinate system and here, I am just going to consider only the translation.

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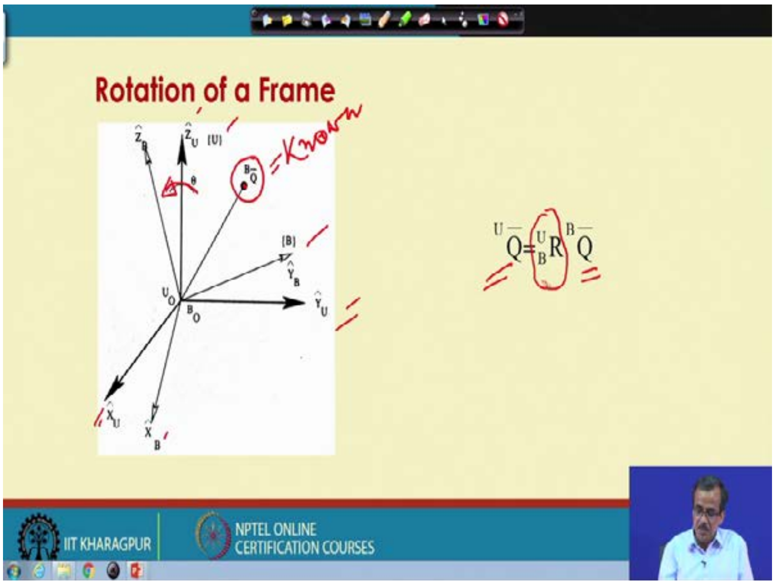

Now, I am just going to consider only rotation, that is, pure rotation. Now, once again let me consider that universal coordinate system U having X_U, Y_U and Z_U and there is pure rotation with respect to the universal coordinate system by some angle, say θ in the anticlockwise sense. Now, if I rotate by an angle θ in the anticlockwise sense, my Z_B, that is, the Z axis of the body coordinate system will be different from Z_U, although initially they were coinciding. Now, Z_B will be different from Z_U, X_B will be different from X_U and Y_B will be different from Y_U, ok?

And, we will be getting this particular rotated frames and supposing that the position of a particular point with respect to the rotated frame B is known to us. So, this is known. So, once again, let me repeat that the body coordinate system has been rotated with respect to the universal coordinate system, and I know the position, that is, Q with respect to B and our aim is to find out Q with respect to U, the position with respect to the universal coordinate system.

Now, this Q with respect to U is nothing but the rotation of B with respect to U multiplied by Q with respect to B, so this particular Q with respect to B is known and if I can find out, this particular rotation matrix, that is, ${}^{U}_{B}R$, very easily I can find out ${}^{U}Q$; now, how to determine that ${}^{U}_{B}R$? That I am going to discuss after some time.

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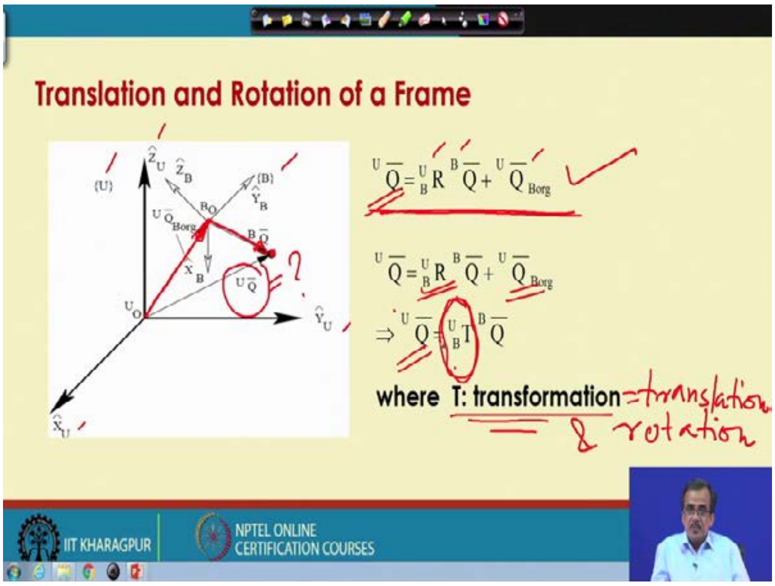

Now, I am just going to consider a more complicated situation, where I am just going to consider both translation as well as rotation, and once again let me repeat that, this is the universal coordinate system: X_U, Y_U and Z_U and B is the body coordinate system, there has been some translation, that means, the origin has been shifted. So, from here, to this particular point and this is actually the position vector, that is, ${}^{U}Q_{Borg}$. And so, this particular B, the B coordinate system has got some rotation with respect to this particular universal coordinate system U and that is why, X_B is not parallel to X_U, Y_B is not parallel to Y_U and Z_B is not parallel to Z_U. So, this is the situation, where there has been both translation as well as rotation.

Now, supposing that, this particular point, that is, Q with respect to B is known. So, this particular point is known and what is our aim? Our aim is to find out, say Q with respect to U. So, that is our aim. This is known, that is, ${}^{U}Q_{Borg}$. So, this particular vector is known, this particular position vector is known, moreover, the rotation matrix of B with respect to U, say, that is also known, then, how to find out this Q with respect to U.

Now, to find out this Q with respect to U, I will have to find out like this. So, I will have to find out the rotation of B with respect U multiplied by Q with respect to B. So, both the things are known plus ${}^{U}Q_{Borg}$. So, this is also known, so, very easily we can find out Q with respect to U. Now, this Q with respect to U is nothing but this expression. Now, here inside the expression there are two things; one is actually the translation of the origin of B, another is actually the rotation of B with respect to U. So, there are two things one is the translation, another is the rotation.

So, now, what I am going to do is: I am just going to use a particular term, which includes both the translation as well as rotation, and that particular term is nothing but transformation. So, this particular transformation, it includes both translation and rotation. So, translation and rotation are taken together inside this particular transformation.

Now, here so, this particular expression I am just going to write down in terms of the transformation matrix, that is, Q with respect to U is nothing but the transformation of B with respect to U. So, in place of this rotation and this position now, I am using this particular transformation; so transformation of B with respect to U multiplied by Q with respect to B. Now, if I do this transformation of B with respect to U now, I can multiply. So, this is Q with respect to B and you will be getting this Q with respect to U. So, this is the way actually we can find out if we have both translation as well as rotation.

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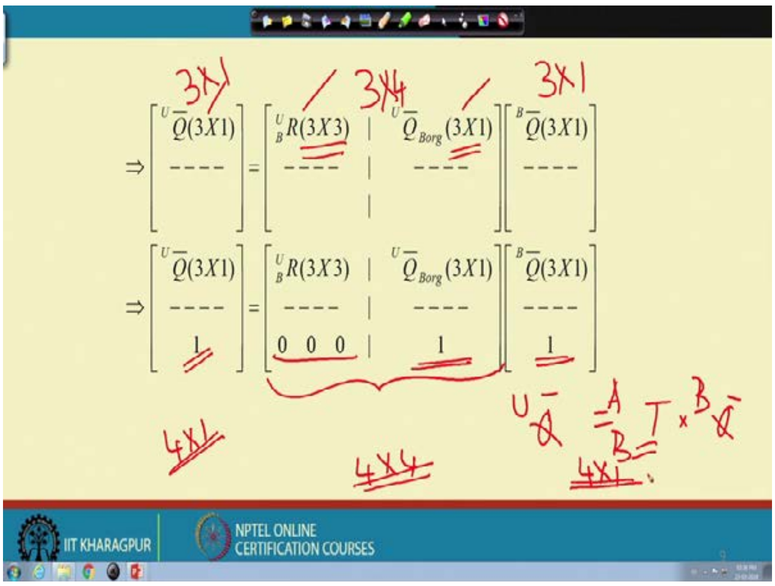

Now, let us try to check the dimension matching of this particular matrix. Now, if you see in the last slide, we wrote the equation, that is, Q with respect to U is nothing but the transformation of B with respect to U multiplied by actually Q with respect to B. Now, here, this Q with respect to U, if you see, in terms of matrix, this is nothing but a 3×1 matrix. Now, this particular transformation matrix has got two things; one is called the rotation matrix and we have got the position vector. Now, this rotation matrix is a 3×3 matrix and position vector is nothing but a 3×1 matrix.

So, taken both the things together, that is, rotation as well as translation will be actually 3×3 and 3×1, so, this will become your 3×4 matrix. So, there will be 3 rows and 4 columns and this is nothing but 3×1 and this particular Q with respect to B is nothing but your 3×1.

So, that means, I will have to multiply one 3×1 matrix by one 3×4 matrix, just to get a 3×1 matrix, which is not possible. Now, to solve this particular problem to make it possible actually, what I do is, here we do some modification sort of thing. So, here on this position term Q with respect to U we add 1, and here, just below the rotation matrix on the fourth row, we use 0 0 0 (three zeroes) and here, we add 1 and now, this particular transformation matrix will have the dimension, that is, 4×4. So, this was 3×4 previously.

Now, I have added one more row here. So, this will become your 4×4 and this particular thing will become 4×1 matrix and this will also become 4×1 matrix. Now, if I just multiply. So, this 4×1 with 4×4 so, I will be getting this 4×1 matrix. So, it is matching. Now, my question is like why do you put 1 here? why do you put 1 here? why 1 here and why do you put these three zeros? That I am going to discuss all such things.

Thank you.

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Lecture - 12
Robot Kinematics (Contd.)

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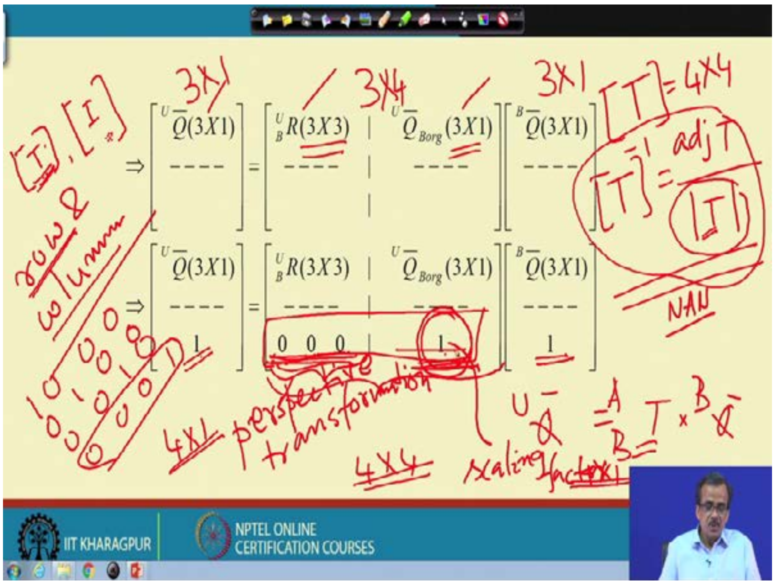

Now, I am going to discuss, why do you put three zeros here and one 1 here. Now, to discuss why do you put three zeros and one 1 here, I will have to see the definition of the inverse of a matrix, and how do you calculate the inverse of a matrix using the computer program?

Now, let me take a very simple example, supposing that I have got a matrix, say, this particular T matrix, and this T matrix is having the dimension, this is, 4 × 4 matrix, and this transformation matrix. I will have to find out its inverse, that means, this transformation matrix has to be invertible, that is, non-singular. Now, by definition, the inverse of this particular matrix, that is, nothing but your adjoint of T divided by the determinant of T, and we know how to find out the adjoint. Adjoint is nothing but the transpose of the co-factors. And, we will have to find out the determinant, this is, by definition, how to find out the inverse of this matrix.

Now, if you write down one computer program. So, in the denominator, there will be actually the determinant of this particular T matrix. Now, the determinant of the T matrix, it may also become equal to 0, sometimes. Now, if T becomes equal to 0, this will become equal to adjoint T divided by 0. So, something divided by 0 so, this is going to generate one NAN, not a number. So, your computer program is going to give a NAN and that is why, to determine the inverse of a particular matrix, this particular definition, we generally do not use in the computer program, instead we follow another method.

Supposing that I will have to find out the inverse of a particular matrix, say T, and T is nothing but a 4×4 matrix. So, what we do is: side by side, we just write down one identity matrix I, and that is also a 4×4 matrix, that is your 1 0 0 0, 0 1 0 0, 0 0 1 0, 0 0 0 1, something like this, ok? Now, we take the help of some row and column operations. So, we take the help of row and column operations. Now, our aim is to convert this particular T into one identity matrix, ok, we take the help of row and column operations to convert this particular T matrix to the identity matrix and the same set of operations, you will have to carry out here on the identity matrix. So, at the end of this particular operation, I will be getting one identity matrix in place of T and an inverse of T matrix in place of this particular identity matrix (I). So, whatever matrix I will be getting here, that will be the inverse of this particular T matrix. So, this is the way, actually, we try to find out the inverse of a matrix in computer program.

So, this particular T matrix actually has to be invertible, and if we can make it invertible, we will be in advantageous position, if we can keep the fourth row to at least contain 0 0 0 1, because a 4×4 identity matrix is nothing but this. So, very purposefully, I am generating this particular fourth row, so that this particular T becomes invertible.

So, I am just going to help that particular transformation matrix to become invertible just by putting 0 0 0 1 here, that is the reason, why do we put 0 0 0 and 1 there; there is another reason behind that. So, this particular 0 0 0 is known as perspective transformation, if you see the literature and this particular 1 is known as the scaling factor.

So, why do you call it a scaling factor. We call it a scaling factor because in place of 1, I can write down 5, I can write down 6, and so on. There is no problem, if I write 5 here. So, I can take 5 out of the matrix and I can make it 1, and that is why, this is known as the scaling factor. I hope, I am clear why do you put this particular the fourth row as 0 0

0 1 and now onwards so, we are going to consider that the transformation matrix is nothing but a 4 × 4 matrix and this is known as the homogeneous transformation matrix.

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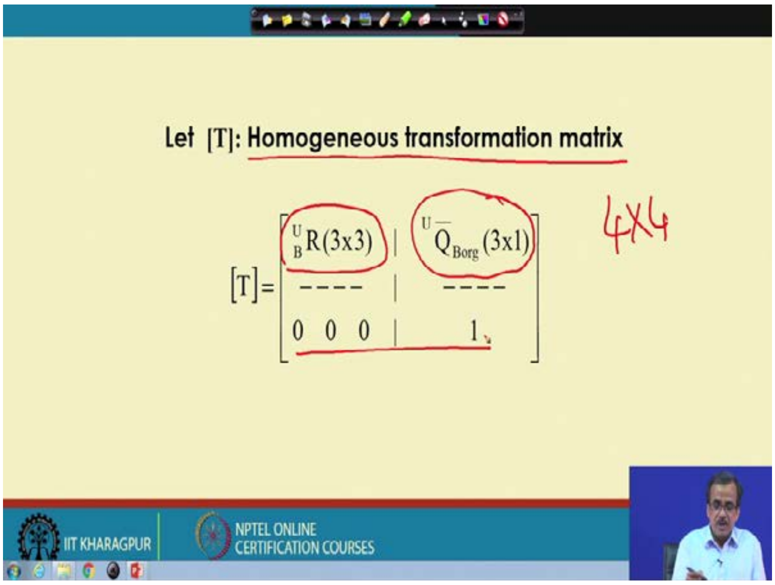

So, this is known as the homogeneous transformation matrix, and we will have to determine, to find out its inverse also, because if I just assign one matrix here, if I assign another matrix here, my aim is to represent this with respect to the previous one and the riverse, that means, my aim is to represent this with respect to that. So, this particular thing is possible, if and only if you have that particular invertible transformation matrix or that nonsingular transformation matrix and that is why, this particular transformation matrix has to be invertible.

Now, once again, if I concentrate this 4 × 4 homogeneous transformation matrix, out of these four, I have got 3 × 3 rotation matrix and I have got a position vector. So, this is nothing but 3 × 1 in matrix form, and I have got 0 0 0 1.

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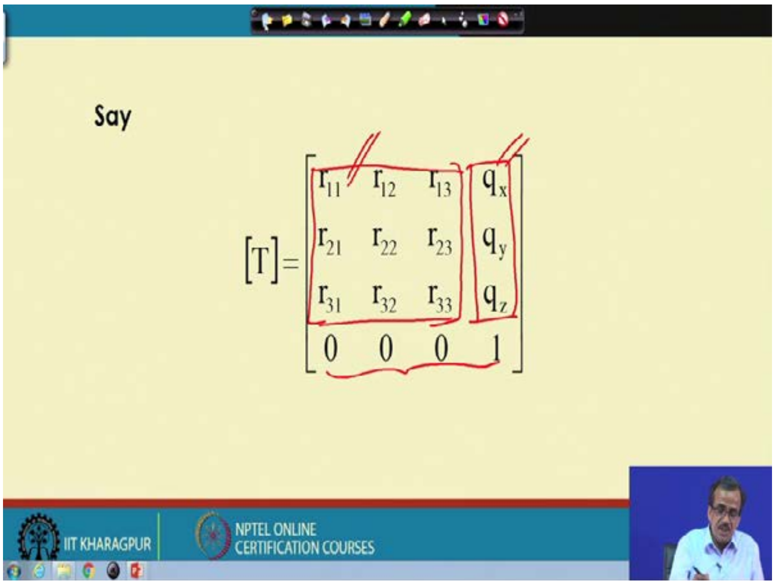

Now, if I take one typical example of one transformation matrix, it will look like this. For example, this particular this 3 × 3 matrix is going to carry information of this particular rotation. So, this is going to carry information of the rotation and this is going to carry information of the position and the fourth row is your 0 0 0 1. So, this is nothing but a 4 × 4 homogeneous transformation matrix carrying this particular rotation matrix and position vector. So, this is the way, actually, we represent the transformation matrix.

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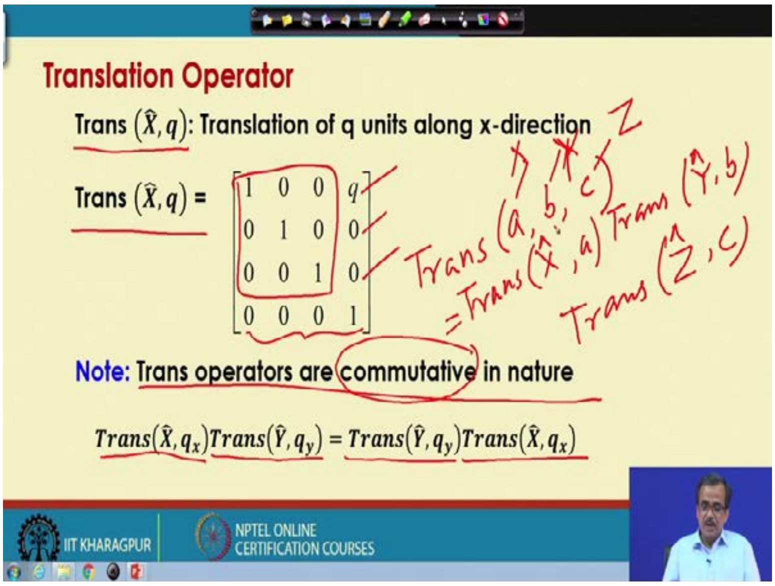

Now, I am just going to concentrate on the translation operator the properties of the translation operator. The translation operator, in short, is written as $Trans(X,q)$; that means, along the X direction the translation is only by q units and here, the rotation matrix is nothing but the identity matrix. You can see that, so this is nothing but the identity matrix, 3 × 3 identity matrix like 1 0 0, 0 1 0, 0 0 1, and along this X direction, I have got the position information, that is, q along y it is 0, along z it is 0 and as usual we have got 0 0 0 1. So, this is nothing but $Trans(X,q)$. So, this is the way we can write down this 4 × 4 matrix.

Now, here, I have put one note, the Trans operator is commutative in nature, that means, it does not depend on the sequence. For example, say I can write down $Trans(X,q_x)$; that means, along X there is a translation by q_x amount, along Y there is a translation by q_y amount. So, $Trans(X,q_x)Trans(Y,q_y)$ is nothing but $Trans(Y,q_y)Trans(X,q_x)$. So, it does not depend on the sequence and they are commutative in nature. So, Trans operators are commutative in nature.

Now, another thing, I just want to tell you in some of the literature, you will find one notation, that notation is something like this, $Trans(a,b,c)$. In some of the literature, we can find this type of notation. Now, it means that translation along X by a units, along Y by b units and along this particular Z by c units, and this is equivalent to your $Trans(X,a)$, $Trans(Y,b)$, $Trans(Z,c)$. So, they are equivalent.

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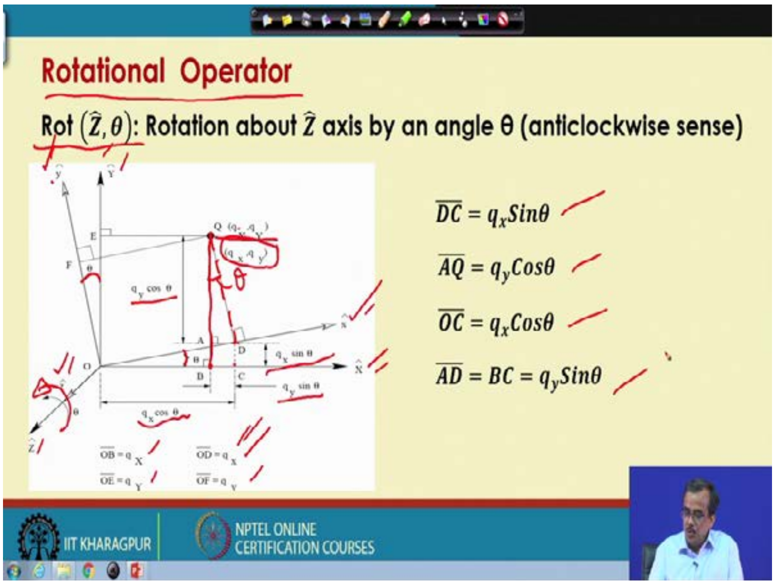

Now, I am just going to start with the rotation operator; that means how to determine that 3 × 3 rotation matrix. Now, here, I am just going to derive something, that is nothing but the 3 × 3 matrix corresponding to the rotation about Z by θ. So, I am just going to find out the rotation about Z by θ. Now, here, so this particular capital X, capital Y and capital Z represents the main coordinate system or the universal coordinate system, and small x then comes small y and small z represent the rotated coordinate system and here, the rotation is about Z by an angle theta.

So, here you can see that I am rotating about Z by angle θ in the anticlockwise sense. Now, initially this capital X, capital Y and capital Z and small x small y and small z, they were coinciding. Now, if I take the rotation about capital Z by an angle θ in the anticlockwise sense, my small z will remain same as capital Z, but small x will be different from capital X and this particular rotation will be by the angle θ and small y will be different from capital Y and this particular rotation will be θ, moreover, this particular angle will also become θ, ok?

Now, let us try to concentrate on the main coordinate system or the universal coordinate system first. Supposing that I have got a point, here, Q in the main coordinate system and its coordinates are (q_x, q_y), so, this is the coordinate and what about Z? Z is, here, actually perpendicular to the board and this particular thing, as if I am considering on the

2D, but Z is perpendicular to the board. In fact, and that is why, very purposefully, I have not written any Z value here on this 2D plane, x-y plane. In fact, Z is put equal to 0.

Now, with respect to the main coordinate system, if the coordinates are (q_X, q_Y), so, I can find out this OB; OB is nothing but q_X and this BQ is equal to OE is nothing but is your q_Y, ok? Now, I just concentrate on the rotated frame, that is, your small x, small y and small z, the same point q, its coordinate in the rotated frame is denoted by (q_X, q_Y). That means, if I just draw this particular, if I just draw this particular perpendicular here, ok, this particular OD will be q_X. So, OD is q_X and this particular DQ is equal to your q_Y, ok?

Now, if I know this particular DQ; DQ is how much? So, this particular DQ is nothing but q_Y, this angle is θ. So, I can find out that this particular AQ is your $q_y \cos\theta$, and similarly, this AD, which is equal to BC is nothing but $q_y \sin\theta$. Similarly, this OD, this OD is nothing but q_X; this angle is θ. So, it is cos component that is, OC will be your $q_x \cos\theta$ and your CD, this particular CD is nothing but $q_x \sin\theta$, ok? So, these things I have written here like DC, DC is nothing but $q_x \sin\theta$ then AQ; AQ is nothing but $q_y \cos\theta$, then OC; OC is nothing but $q_x \cos\theta$ and AD equals to BC, AD equals to BC is nothing but $q_y \sin\theta$. So, all such things, we can find out very easily.

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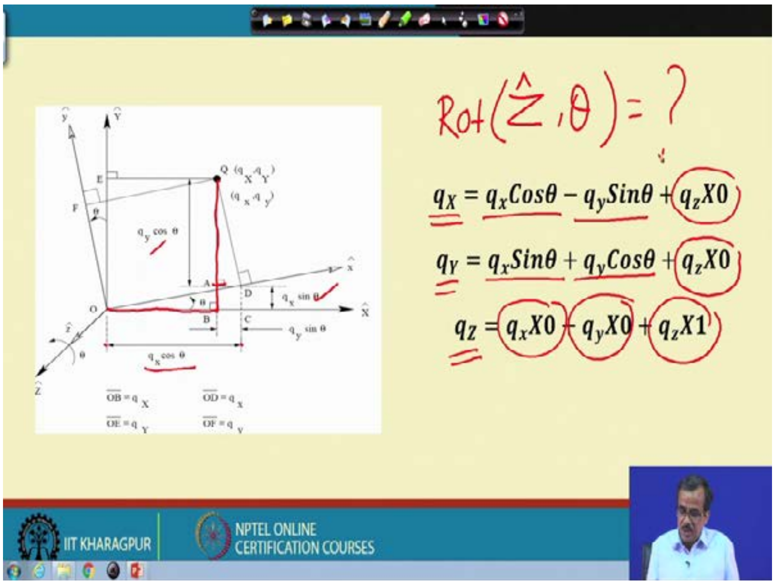

Now, I am just going to write down this particular q_X. This q_X is how much? So, up to this, is your q_X, OB. OB is nothing but is your q_X and that is nothing but $q_X \cos\theta$, $q_X \cos\theta$ is from here to here $-q_y \sin\theta$, and here, the Z component is 0, because this is on the 2D plane. So, I am just writing here, q_z multiplied 0.

The next is q_Y. So, what is q_Y? q_Y is nothing but is your this BQ, and this BQ is how much, that is, $q_x \sin\theta$, that is up to this. So, this is $q_x \sin\theta + q_y \cos\theta$ and after that I am adding q_z multiplied 0 and this particular q_Z because here Z is perpendicular to the board. So, its x component will be multiplied by 0, y component is multiplied by 0 and z is multiplied by 1, ok? So, I am just trying to find out the relationship between the original coordinate system and this particular rotated frame. That means, I am trying to find out the expression for rotation about Z by an angle theta in the anticlockwise set. So, this particular 3 cross 3 matrix, I am just going to find out.

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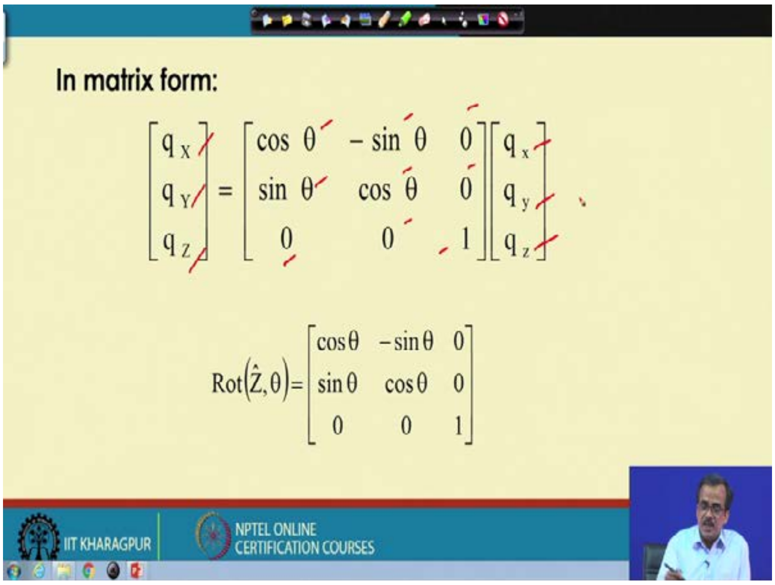

Now, this can be written, as I think, this can be written as in the matrix form q_X, q_Y, q_Z is nothing but $\cos\theta, -\sin\theta, 0; \sin\theta, \cos\theta, 0; 0, 0, 1. q_x, q_y, q_z$, if you just see the previous one, from there, I can write down this particular thing.

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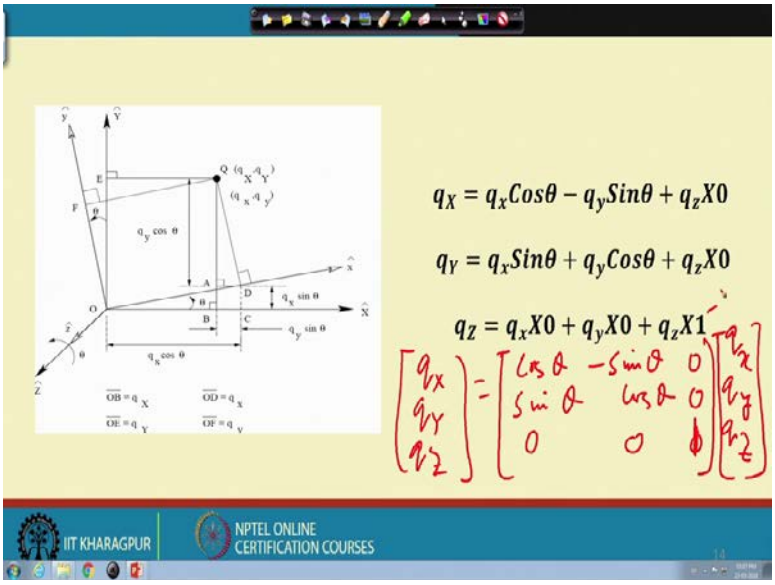

For example, say let me write it here, once again, so this q_x, q_y, q_z can be written as your $\cos\theta, -\sin\theta, 0$ then comes your $\sin\theta, \cos\theta, 0$; 0, 0, 1 and this is multiplied by your q_x, q_y, q_z something like this, ok? I am sorry, here there will be 1, so we have got 1 here.

So, this is the way, actually, we can write down this particular rotation term. So, this is actually this rotation term, that is, the rotation about Z by an angle θ. So, rotation about Z by an angle θ is $\cos\theta, -\sin\theta, 0; \sin\theta, \cos\theta, 0; 0, 0, 1$. So, this is nothing but the rotation about Z by an angle theta.

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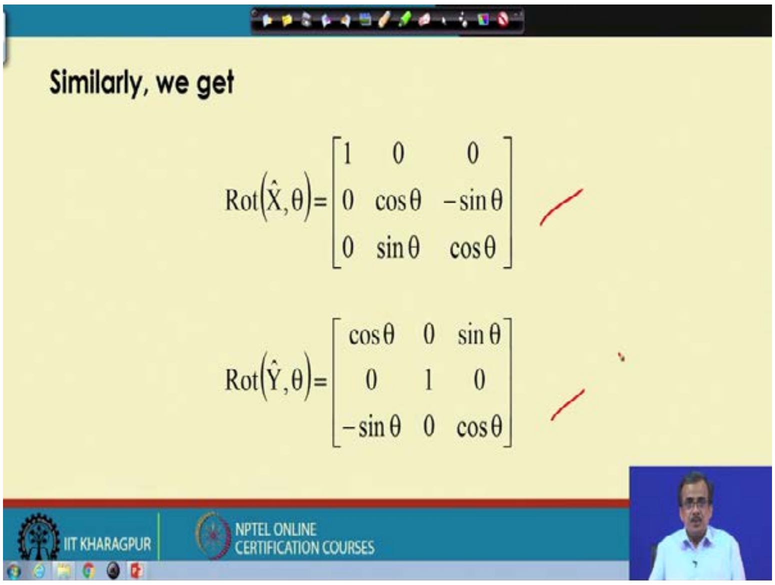

Now, by following the similar procedure, the similar procedure in fact, I can find out the rotation about X by an angle θ, that is nothing but $1, 0, 0; 0, \cos\theta, -\sin\theta; 0, \sin\theta, \cos\theta$. Similarly, I can also find out rotation about Y by an angle θ is $\cos\theta, 0, \sin\theta; 0, 1, 0; -\sin\theta, 0, \cos\theta$. So, using these, I can find out rotation about X by an angle θ, rotation about Y by an angle θ.

Thank you.

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Lecture - 13
Robot Kinematics (Contd.)

So, we have seen how to determine the 3 × 3 matrix corresponding to rotation about Z by an angle θ.

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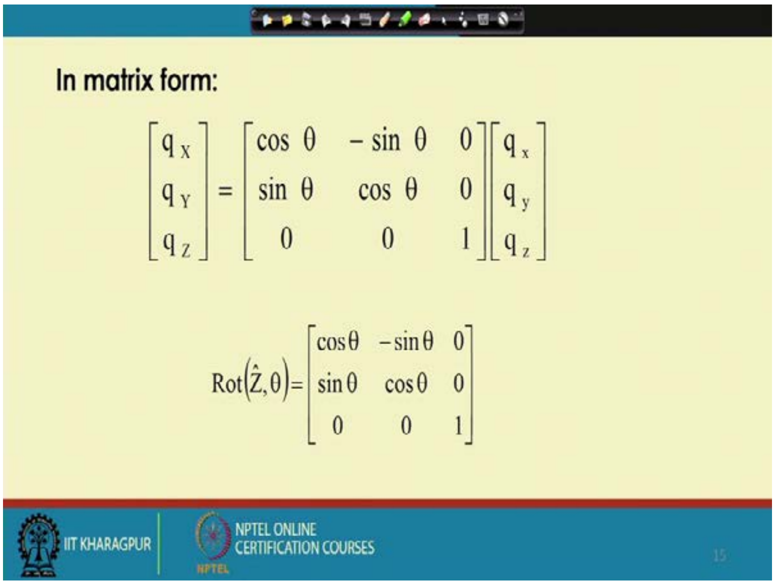

So, this is the matrix.

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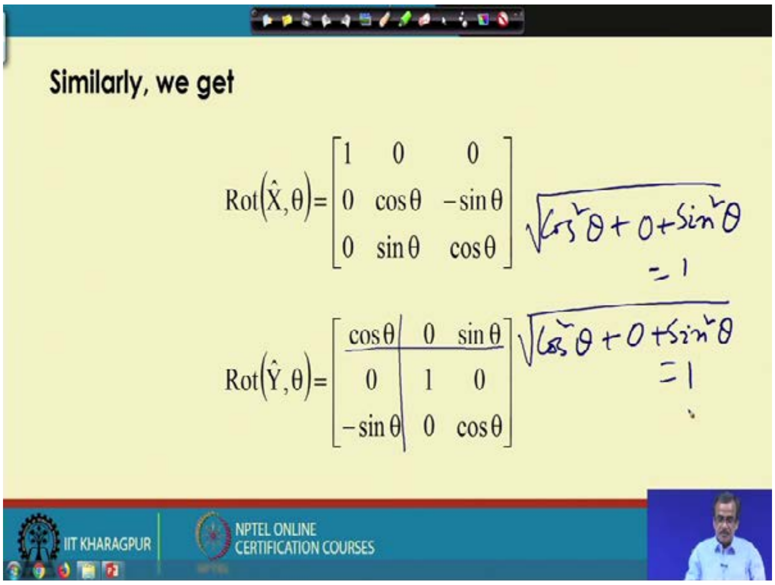

Similarly, we can find out the rotation about X by an angle θ and that is nothing but 1, 0, 0; 0, cos θ, minus sine θ; 0, sine θ, cos θ; then, rotation about Y by an angle θ in the anticlockwise sense is cos θ, 0, sine θ; 0, 1, 0; minus sine θ, 0, cos θ.

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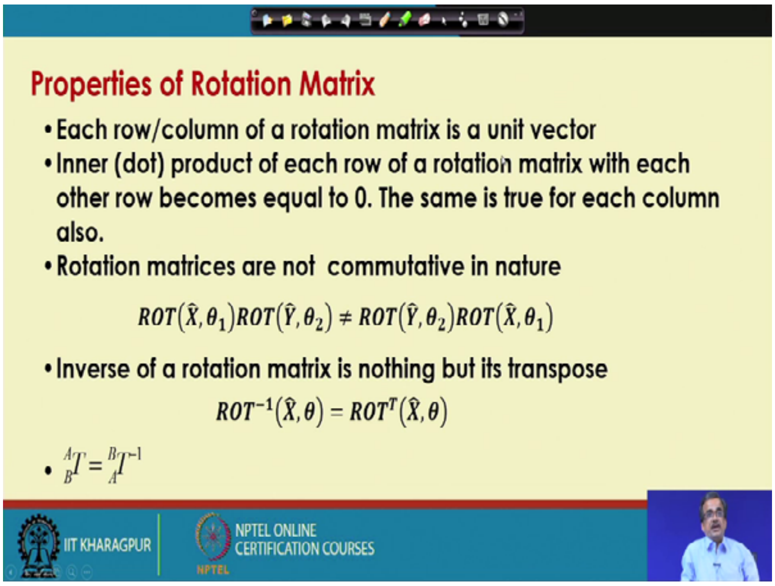

Now, I am just going to discuss the properties of rotation matrix. The first one: each row or column of a rotation matrix is a unit vector. That means, if I just concentrate on this particular rotation matrix, like each row. For example, if I consider the first row that is your cos θ, 0, sine θ. Now, this is the unit vector representation; that means your

$\sqrt{(\cos^2\theta + 0 + \sin^2\theta)} = 1$ Similarly, if I concentrate on a particular column then, cos θ, 0, minus sine θ and once again, $\sqrt{(\cos^2\theta + 0 + \sin^2\theta)} = 1$ the squared root of cos squared theta plus 0 plus sine squared theta, becomes equal to 1, this is what, we mean by the first property.

The next is the inner product or the dot product of each row of a rotation matrix will become equal to 0. So, if I concentrate on a particular row or if I concentrate on a particular the column: for example, say if I concentrate on this particular the row, that is, cos θ, 0, sine θ and the second row is 0, 1, 0.

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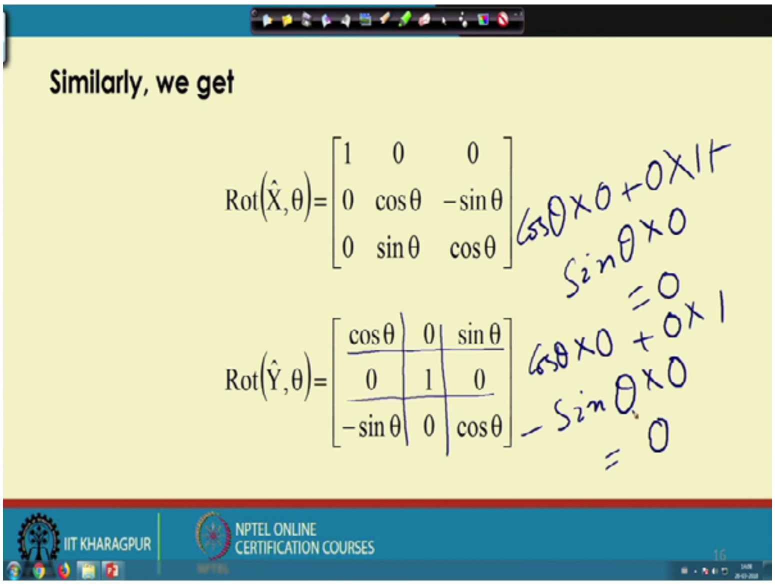

And, if I try to find out their inner product or their dot product, the inner product of the first row, and the second row it is nothing but $\cos\theta \times 0 + 0 \times 1 + \sin\theta \times 0 = 0$. Similarly, if I consider the two columns of the rotation matrix like the first column, that is, cos θ, 0, -sinθ and 0, 1, 0. And, if we try to find out the inner product this will become equal to $\cos\theta \times 0 + 0 \times 1 - \sin\theta \times 0 = 0$. So, this is the way actually we can check the second property of this particular rotation matrix, that is, an inner product of each row or each column of a rotation matrix with each other row becomes equal to 0.

Next property is the rotation matrices are not commutative in nature; that means, it depends on the sequence. So, sequence is very much important, while writing down the rotation matrix. For example, say $ROT(X,\theta_1)ROT(Y,\theta_2) \neq ROT(Y,\theta_2)ROT(X,\theta_1)$.

Now, if I calculate the left hand side and determine the right hand side separately and try to compare, we will see that they are not equal. That means, this particular rotation matrix depends on the sequence, thus, the way, the sequence along which we are writing down these rotation matrices and they are not commutative in nature. The next property is the inverse of a rotation matrix is nothing but it is transpose. So, if it is a pure rotation matrix, then if you want to find out the inverse of that particular rotation matrix, it is very easy. So, what you can do is: if we can find out the transpose of that particular rotation matrix, that will be equal to its inverse.

For example, say if I see that $ROT(X,\theta)^{-1} = ROT(X,\theta)^{T}$ but, this is true only when, this is a pure rotation matrix. Now, if there is a any such translation term, this particular condition will not hold good. Now, here, I have written another thing that is ${}^{A}_{B}T = {}^{B}_{A}T^{-1}$, ok?

Now, this is very much essential because say we want to find out a particular joint with respect to the previous and the reverse, that is, this particular joint with respect to the next and for that what you need is, this particular inverse we will have to find out and ${}^{A}_{B}T = {}^{B}_{A}T^{-1}$. So, this particular condition we can use.

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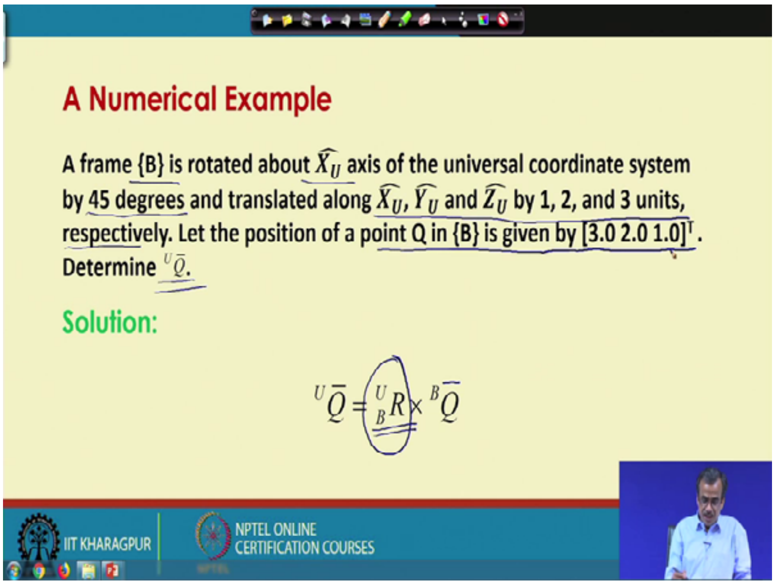

Now, I am just going to solve one numerical example based on the theory, which I have already discussed. Now, for this numerical example, the statement is as follows: a frame

B. So, B is nothing but a body coordinate frame is rotated about X_U, that is, the X axis of the universal coordinate system by 45 degrees. So, this is positive 45 degrees; that means, the anticlockwise direction and translated along X_U, Y_U and Z_U by 1, 2, and 3 units, respectively. Let the position of a point Q in B is given and this is given by 3, 2, 1 transpose, then how to determine the position of that same point with respect to the universal coordinate system, that is, Q with respect to U.

So, here, the way we will have to find out it, is very simple. So, this ${}^{U}Q = {}^{U}_{B}R \times {}^{B}Q$, and of course, this is a vector so, we will have to put this vector sign. So, if I can find out this rotation of B with respect to U and I know this Q with respect to B so, very easily, I can find out what is Q with respect to U. Now, how to determine this rotation matrix, that is, ${}^{U}_{B}R$.

So, once again, let me read that there is a rotation about X_U by an angle 45 degree in the anticlockwise sense and there are translation along X_U, Y_U and Z_U by 1, 2 and 3 units, respectively. So, by using that particular information, what I will have to do is so: I will have to find out what should be this particular ${}^{U}_{B}R$.

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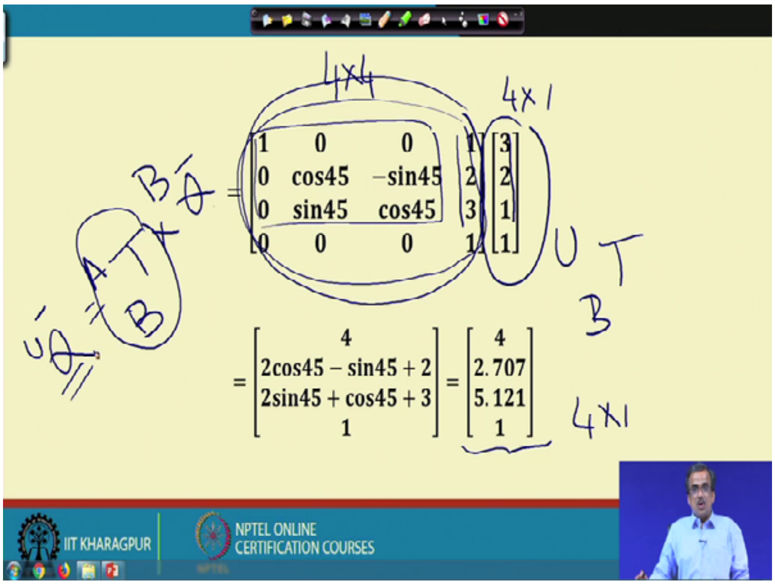

Now, this ${}^{U}_{B}R$ if you see the rotation term, the rotation terms is nothing but 1, 0, 0; 0, cos θ, -sinθ; 0, sinθ, cosθ; now, here, theta is nothing but 45 degrees. So, this is actually nothing but this particular rotation about X_U by an angle 45 degrees and these are nothing but the translation terms.

So, I will be getting the transformation matrix corresponding to that particular rotation and that is nothing but this 4 × 4. In fact, although I have written there it is ${}^{U}_{B}R$. Truly speaking, corresponding to that it should be your ${}^{U}_{B}T$. So, if I write ${}^{U}_{B}T$, then it will carry actually the 4 × 4 matrix and this is multiplied by 3 2 1 1 and these (3 2 1) is nothing but the position terms, that is, Q with respect to B.

So, Q with respect to B and if I know actually, we will have to find out Q with respect to U and truly speaking, this is nothing but ${}^{U}_{B}T$ multiplied by actually Q with respect to B. Now, Q with respect to B is this much, and this ${}^{U}_{B}T$ is nothing but this much and if I just multiply, this is a 4 × 4 matrix and this is a 4 × 1 matrix, and if I multiply then, I will be getting this particular the 4 × 1 matrix and this is nothing but Q with respect to U.

So, very easily, we can find out, what is this particular the Q with respect to U. So, Q with respect to U, you can find out.

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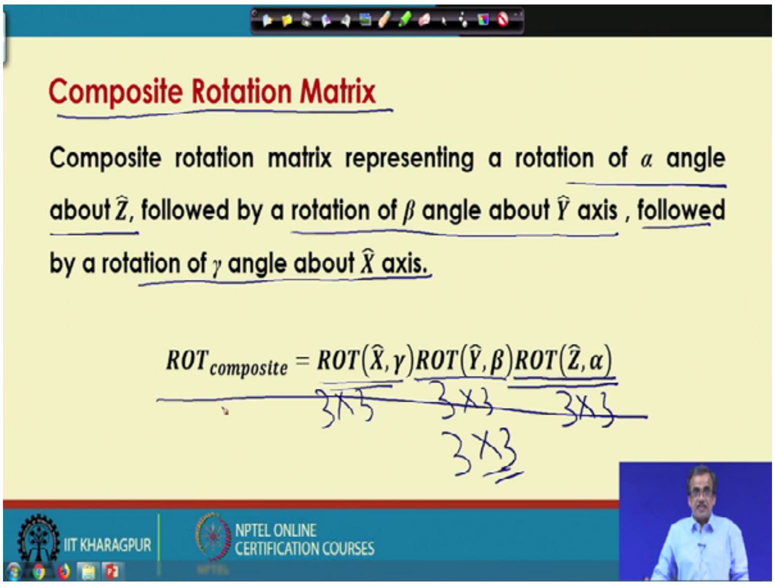

Now, let us try to concentrate on this particular rotation matrix, that is called the composite rotation matrix. Now, I have already discussed that while writing this particular rotation matrix, the sequence is very important and we will have to follow a particular sequence, otherwise altogether you will be getting the different the final results and that is why, to write down this particular rotation matrix all of us, who work on robotics, follow one rule that is called the composite rotation matrix rule.

Now, the rule is as follows: the composite rotation matrix representing a rotation of α angle about Z axis followed by a rotation of β angle about Y axis followed by a rotation of γ angle about X axis is written in this particular the format. The rule is as follows: whatever I state first; that means, the rotation about Z by an angle α, that particular thing will be written at the last, that is rotation about Z by an angle α will be written at the end followed by the rotation of β about Y axis; so rotation of β about Y axis is followed by the rotation of a γ angle about X axis. So, $ROT(X,\gamma)$ is written first.

Now, for each of these, actually we can find out like what should be the 3 × 3 matrix. So, this is a 3 × 3 matrix, this is also a 3 × 3 matrix and this is also a 3 × 3 matrix. Now, if I multiply then finally, we can find out one 3 × 3 matrix and that is nothing but the composite rotation matrix. So, we can find out the 3 × 3 matrix, that is nothing but ROT_composite, that is the composite rotation matrix. Now, this particular rule is actually followed by the whole robotics community, just to write down, whenever there is rotation term.

Now, here, I am just going to take some examples and I am going to show you that this particular rule is correct. So, indirectly, I am just going to prove that this particular rule that is the rule, for the composite rotation matrix is correct.

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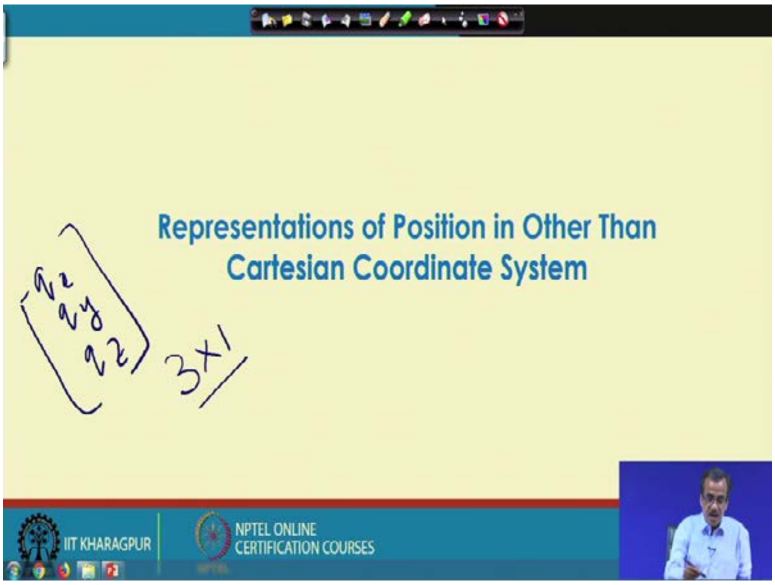

So, here, till now, whatever we have seen is: we have expressed position in terms of the Cartesian coordinate system and we have seen that, in terms of Cartesian coordinate system, the position can be expressed in terms of a vector and that is nothing but a 3 × 1 matrix. For example, say the position vector is nothing but say q_x, q_y, q_z and once again, let me repeat that this is nothing but a position vector in matrix form, this is nothing but a 3 × 1 matrix. Now, the same position can also be expressed in other coordinate system now that I am going to discuss.

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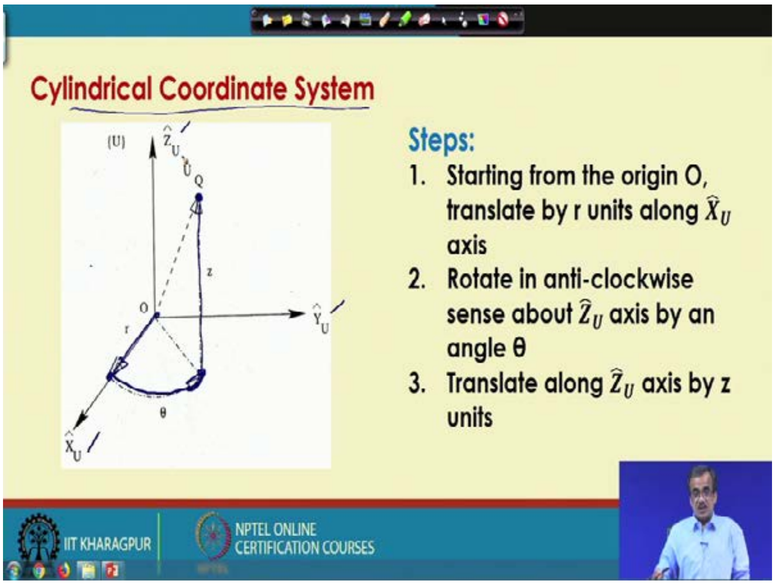

Now, here, I am just going to see how to represent the position in cylindrical coordinate system. Now, in cylindrical coordinate system, the position of a particular point in 3D can be expressed with the help of actually two translations and one rotation in a particular the sequence.

Now, here, supposing that the problem is as follows: so I have got the universal coordinate system and in this particular universal coordinate system, this is nothing but X_U, Y_U, and Z_U. Now, supposing that I have represented a particular point, that is nothing but Q with respect to U. So, in Cartesian coordinate system if I want to represent this, it is very easy. So, if I know the translation along X direction, if I know the translation along Y direction and if I know the translation along Z direction. So, very easily, I can find out this Q with respect to U in Cartesian coordinate system.

Now, here, my aim is to reach the same point using the cylindrical coordinate system; so how to reach, we take the help of these steps. So, in step one, we start from the origin O. So, this is the origin O and then, we translate by r units along X_U. So, from here, we translate by r unit along X_U. So, I am here the next is the rotation in a anticlockwise sense about Z_U axis by an angle θ. So, I am here now, I am taking rotation about this particular Z_U by an angle θ in the anticlockwise sense. So, I am here so, this is my position. Now, the next is translate along Z_U axis by Z unit. So, starting from here, we translate along this particular Z direction by this small z unit, and I am just going to reach the same point, that is nothing but Q with respect to U.

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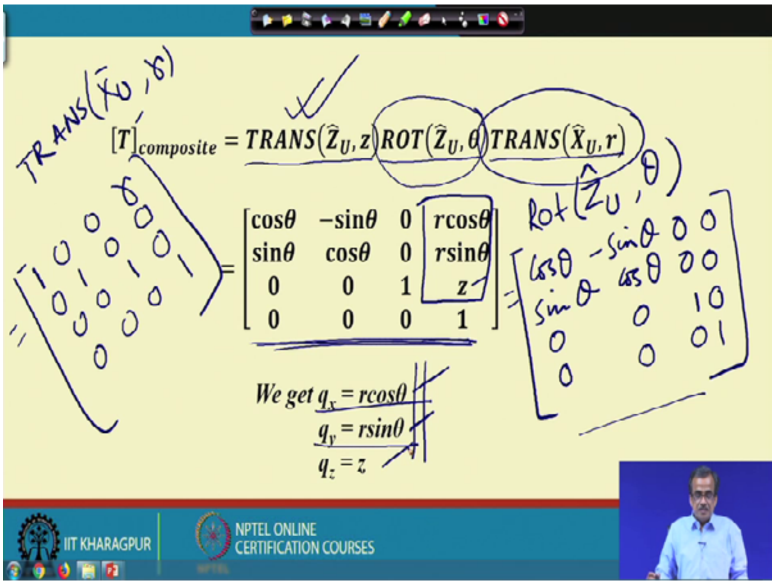

Now, this particular sequence, if I just take the help of the rule for the composite rotation matrix, I can find out the final transformation matrix or if I want, I can find out the corresponding rotation matrix also, but, here, I am interested mostly in position.

So, what I can do is: I can find out the transformation matrix. So, T_composite, that is, the transformation matrix corresponding to this particular composite, whatever I stated first that will go to the last, that is translation along X_U by r unit followed by rotation about Z_U by theta followed by translation along Z_U by z. Now, corresponding to each of these particular things, we can write down the 4 × 4 matrix. For example, say corresponding to this rotation about Z_U by an angle θ, I can write down very easily this particular transformation matrix, that is, 4 × 4.

For example, say, this will be the cos θ, - sinθ, 0; then comes sine θ, cosθ, 0; 0, 0, 1 and this is the pure rotation so, the translation terms will be 0. So, 0 0 0 and of course, I have got the fourth row, that is, 0 0 0 1. So, this is nothing but the rotation about Z_U by an angle theta in 4 × 1 matrix form. Similarly, I can also find out the translation along X_U by an angle r, sorry by r.

Now, this translation it is a pure translation. So, this rotation terms will be nothing but a 3 × 3 identity matrix. So, very easily, I can write down this particular thing, for example, say the $TRANS(X_U, r)$ so, this can be written in the matrix form in a 4 × 4 form as follows: So, this is 1 0 0; 0 1 0; 0 0 1. So, this is nothing but the rotation term, this is an identity matrix. Then, along X_U, I have got r translation, then Y it is 0, Z it is 0 and the fourth row is 0 0 0 1.

So, similarly, I can also write down the 4 × 4 matrix corresponding to this $TRANS(Z_U, z)$. Now, I am getting 3 matrices each having 4 × 4 dimensions and if I just multiply, then finally, I will be getting this particular 4 × 4 matrix. Now, in this 4 × 4 matrix, these particular terms are going to represent the position terms. For example, say in Cartesian whatever was q_x that is nothing but $r\cos\theta$, whatever was q_y that is nothing but $r\sin\theta$. Similarly, your q z is nothing but is equal to z. So, this particular relationship between the Cartesian coordinate system and this particular the cylindrical coordinate system, I can establish very easily using the rule of the composite rotation matrix.

Now, all of us, we know that the relationship between the Cartesian coordinate system and the cylindrical coordinate system is nothing but this, that is, q_x equals to $r\cos\theta$, q_y equals to $r\sin\theta$. So, these are actually very well-known relationships between the Cartesian coordinate system and the cylindrical coordinate system. So, the rule for composite rotation matrix, whatever we have used, that is correct. So, this is the way, actually, indirectly, we can prove the correctness of this particular rule for composite rotation matrix.

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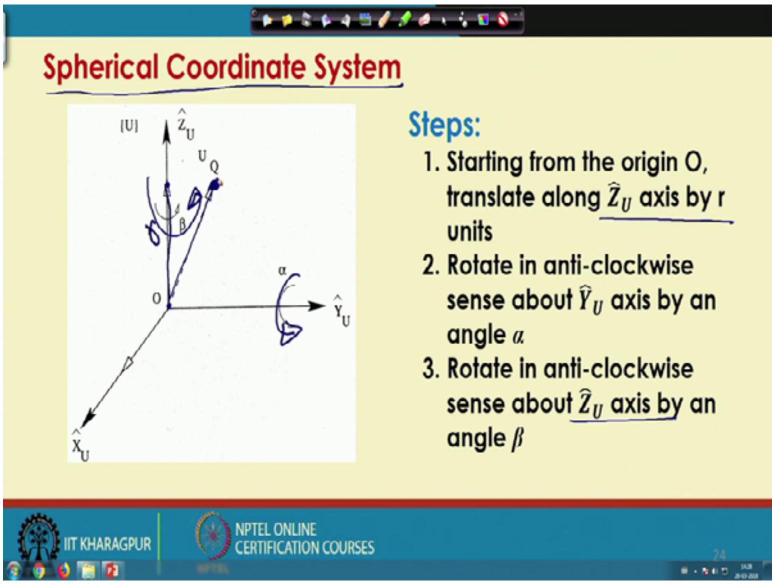

So, now, I am just going to discuss another coordinate system and that is called the spherical coordinate system. Now, here, our aim is to reach the same point, which I represented in Cartesian coordinate system, that is, this particular Q with respect to U. With the help of Cartesian coordinate system, that is, q_x, q_y and q_z, the same point, I want to reach with the help of this spherical coordinate system. So, in spherical coordinate system, in fact, there is one translation and there are two rotations in a particular the sequence. So, let us try to check that particular sequence.

So, in step – 1: starting from the origin O, we translate along Z_U axis by r. So, this is nothing but the origin of this particular coordinate system. So, starting from here, along this particular Z_U, we translate by r units, I am here, this is by r units. Now, the next is, rotate in anticlockwise sense about Y_U by an angle α. So, this is nothing but my Y_U axis so, about Y_U, I just rotate by α in the anticlockwise sense. So, whatever was here, this particular thing will be rotated something like this.

So, let me repeat, supposing that this is along this particular Z_U and here, say I have got, Y_U axis. So, I am just rotating about this particular Y_U. Now, it will be rotated something like this and after that, we take another rotation, that is, rotation in anticlockwise sense about Z_U by an angle β.

So, now, I am rotating about Z_U by an angle β so, there is a possibility that I am going to reach this particular point, that is, nothing but Q with respect to U. So, what we do, let me repeat again, I am just going to translate it, translate along the Z by r units, after that I am just going to rotate about Y_U by an angle α and after that we are going to rotate about Z_U. So, this is nothing but Z_U, so rotate by an angle β. And, then, I will be able to reach this particular point. So, with the help of one translation and two rotations I am just going to reach this particular point.

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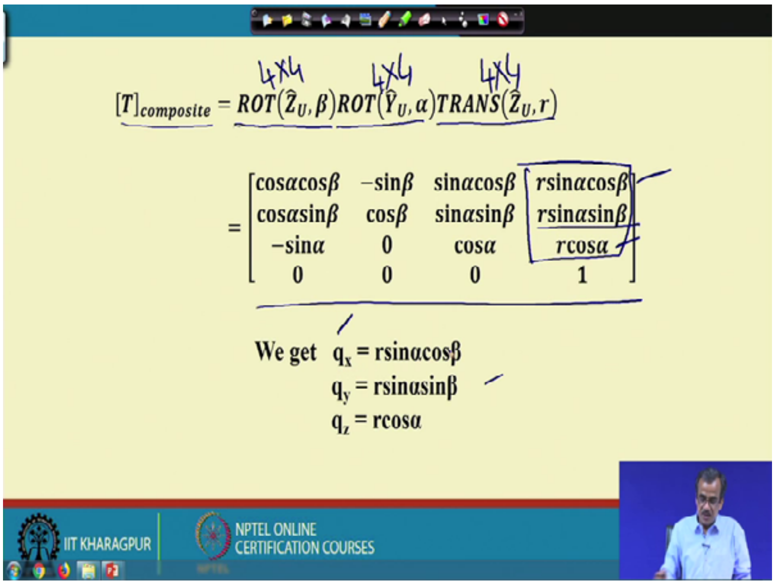

Now, all such translation and rotations, if I just write with the help of this composite rotation matrix, I will be getting this particular form, that is, your that transformation matrix corresponding to this composite. Now, whatever I stated first, will go to the last. So, translation along this particular Z_U by r unit, so I will have to write at the end, because that I stated first, followed by the rotation about Y_U by α, followed by rotation about Z_U by an angle β.

And, as I discussed this particular 4 × 4 matrix, I can write down, and all of us we know how to multiply like these two matrices and so, it is having the dimension of 4 × 4, this is once again like the 4 × 4. So, these 4 × 4 matrices we can multiply and then, you will be getting one 4 × 4 matrix, and that particular 4 × 4 matrix, we can multiply with this, then, I will be getting this final 4 × 4 matrix.

Now, in this particular 4 cross 4 matrix, actually, the position terms are denoted by these three, that means, in Cartesian, what is q_x that is nothing but $r\sin\alpha\cos\beta$ and q_y is nothing but your $r\sin\alpha\sin\beta$ and q_z is nothing but is your $r\cos\alpha$. Now, here, if I know, in this Cartesian coordinate system, the same point I can also represent in the spherical coordinate system; that means, this is the known relationship between the Cartesian and the spherical coordinate systems. So, once again, actually, I have a re-derived.

So, this particular relationship between your the Cartesian and the spherical coordinate systems is actually known to us and this is another indirect proof for this particular rule of composite rotation matrix. That means, the position of a point with respect to the universal coordinate system can be expressed in Cartesian coordinate system in cylindrical coordinate system, and in spherical coordinate system. And, that is why, the same robot can be actually controlled either in Cartesian coordinate system or cylindrical coordinate system or in spherical coordinate system.

Now, here, actually, I am just going to express and discuss how to represent the orientation other than the Cartesian coordinate system. So, the position we have seen that we can represent in other coordinate system. Now, I am just going to show like, how to represent the orientation with respect to the other coordinate system.

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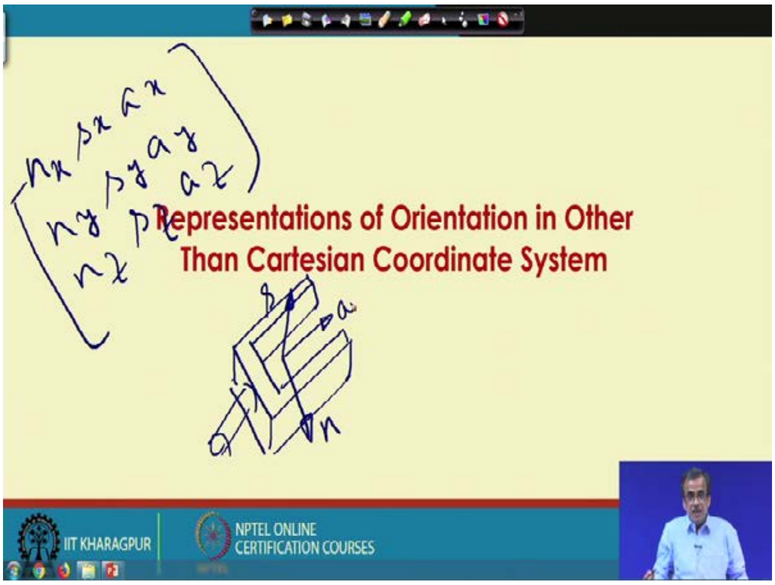

For example, in terms of Cartesian, we have already discussed that we take the help of like 3 × 3 matrix to represent the orientation. Now, if we remember, we have seen that with the help of this normal vector, sliding vector and approach vector actually we can represent this particular orientation, n_x, n_y, n_z; s_x, s_y, s_z; a_x, a_y, and a_z. So, this is nothing but the 3 × 3 rotation matrix in Cartesian to represent the orientation. These I have already discussed in details. Now, this n stands for the normal vector, s stands for the sliding vector and a stands for the approach vector.

Now, if I just draw once again the same picture like one end-effector with two such fingers for a particular manipulator, so very easily, actually, I can represent; these are the normal vector, sliding vector and approach vector. So, if this is the end-effector with two fingers, the normal vector is nothing but this, and the sliding vector is nothing but this, and the approach vector is nothing but this. This is how to represent the orientation in terms of your Cartesian coordinate system.

So, now, we will be discussing how to represent the same orientation in other coordinate system.

Thank you.

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Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 14
Robot Kinematics (Contd.)

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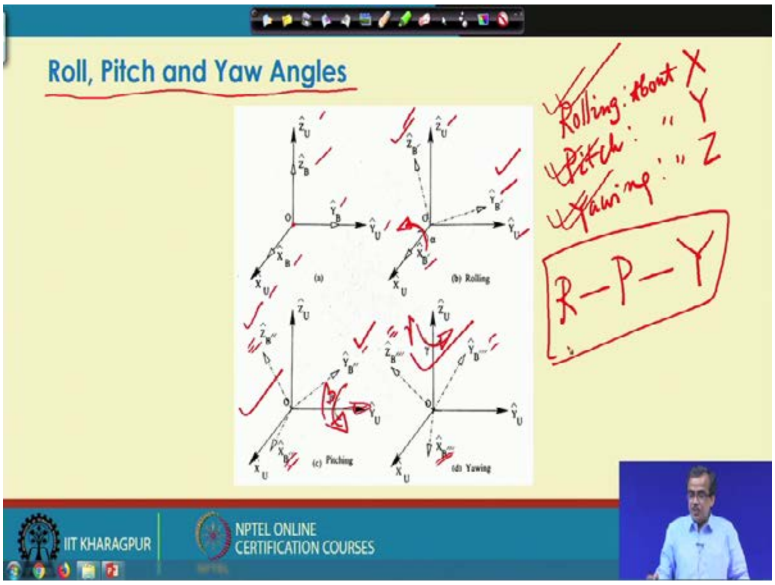

Now, I am going to discuss how to represent the orientation using the principle of this Roll, Pitch and Yaw angles. Now, the concept of this roll, pitch and yaw, actually, we have copied from the movement of a ship. Now, let me first define the rolling, pitching and yawing movement of a particular ship and let us see, how to copy these to represent the orientation with the help of this roll, pitch and yaw.

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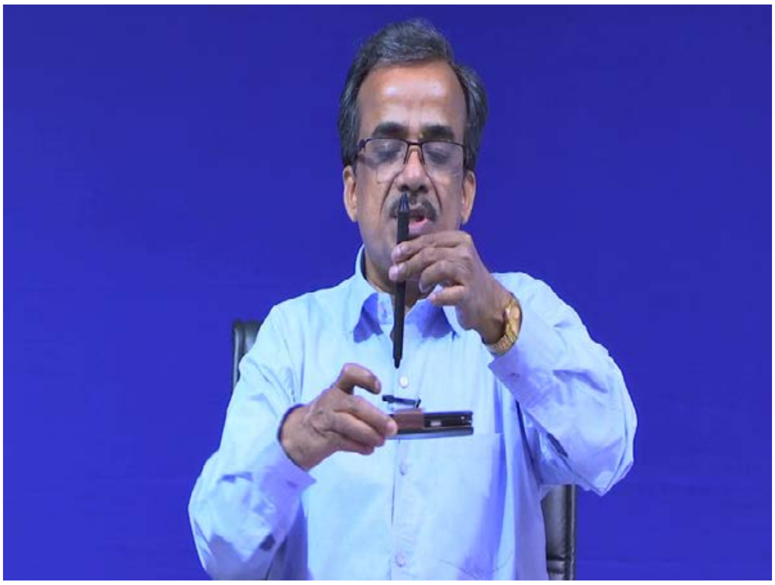

Now, supposing that this is nothing but a ship, now, this ship will have rolling movement, pitching movement and yawing movement. For example, say, if I consider this particular movement, this particular movement is nothing but the rolling movement. Similarly, this type of movement of the ship is nothing but the pitching movement, and this particular movement of the ship is nothing but the yawing movement; for this type of movement of the ship, as if, this is the axis about which I am taking the rotation.

Similarly, this is the type of movement of this particular ship that is your pitching, as if this is the axis about which I am taking the rotation. And, the moment, we consider like this type of movement of the ship that is called the yawing movement, and as if this is nothing but the axis about which I am taking the rotation.

Now, if I call the rotation about X is the rolling movement, then, the rotation about Y is the pitching movement, and then, rotation about Z is nothing but the yawing movement. So, rolling is nothing but the rotation about X, then the pitching movement is nothing but about Y, and this particular yawing movement is nothing but the rotation about Z, the same concept we are going to use it here.

Now, let us try to explain the way, we can copy here. Now, here, this particular universal coordinate system is denoted by, once again, X_U, Y_U and Z_U, and the body coordinate system, which is attached to the 3D body, whose rotation I am just going to represent, that particular body coordinate system is B, and it has got X_B, Y_B and Z_B and initially, they are coinciding and origin is exactly the same, that is, O for both the coordinate systems.

Now, what you do is: we take some rotation about the universal coordinate system and we try to rotate that particular B, for example, say we first take the rotation about this particular X_U. So, this is my X_U and take rotation by an angle α in the anticlockwise sense, that is, + α. Now, if I take rotation about X_U and initially X_U and X_B were coinciding. So, my X_B prime will remain same as X_U, because I have taken rotation about X_U. Now, this particular Y_B prime will be different from Y_U, similarly, this Z_B prime will be different from Z_U. So, this is what you mean by the rotation about X.

Now, I am just going to take the rotation about Y_U. So, this is nothing but your Y_U direction. So, I am taking the rotation about Y_U by an angle β in the anticlockwise sense. So, if I take the rotation about Y_U by an angle β so, what will happen to my X_B double prime? So, X_B double prime will be different from X_B prime, then Y_B double prime will be different from Y_B prime and Z_B double prime will be different from your Z_B prime. And, now, I am just going to take the rotation about this particular the Z_U. So, if I take the rotation about Z_U, by an angle γ and in the anticlockwise sense.

So, what will happen to my X_B triple prime? So, X_B triple prime will be different from X_B double prime, then Y_B triple prime will be different from Y_B double prime and Z_B triple prime will be different from Z_B double prime. So, the final coordinate system, the final frame, I will be getting: X_B triple prime, Y_B triple prime, Z_B triple prime after taking three rotations in a particular sequence.

Now, here, actually what you do is: the rotation about this particular X, we call it this is nothing but the rolling motion, then the rotation about Y we call it this is nothing but the pitching motion and the rotation about Z is nothing but the yawing motion, and these particular rotations, three rotations are in a particular sequence. The sequence is nothing but the roll, pitch and yaw, and this particular sequence, we are going to follow.

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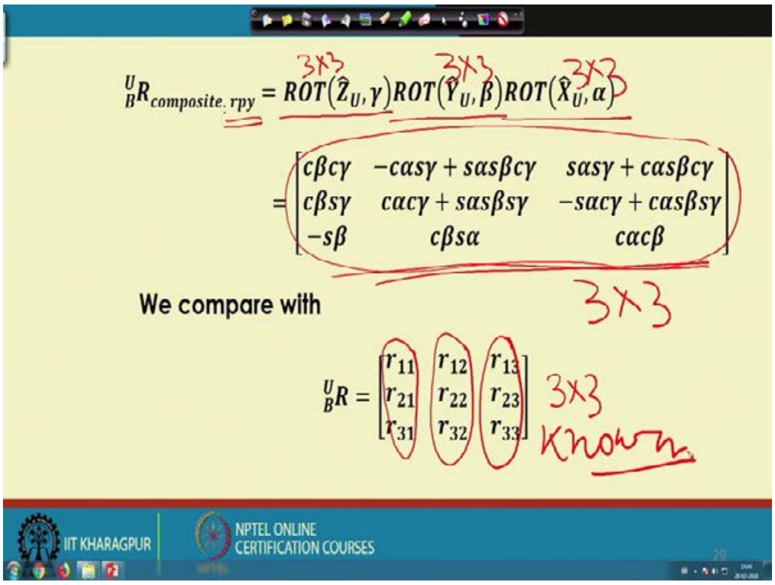

We are going to write down the composite rotation matrix as follows: like $^U_B R_{composite:rpy}$ and here, I am writing rpy, that is, roll, pitch and yaw, this is very important, because if you just change the sequence, you will be getting all together a different final matrix.

So, this particular sequence (rpy), I will have to write. So, rotation of B with respect to U, that means, I am just going to find out what is the orientation of that particular body with respect to the universal coordinate system. So, we will have to find out this R_B with respect to U composite comma rpy ($^U_B R_{composite:rpy}$), that is, roll-pitch-yaw and once again, the same rule is to be used. So, whatever I stated first that will go to the end. So, that is rotation about X_U by an angle α followed by the rotation about Y_U by an angle β, followed by the rotation about Z_U by an angle γ.

Now, each of these rotation matrices is nothing but 3 cross 3 matrix. So, this is a 3 cross 3 matrix. We know the expression of rotation about Z by an angle γ, similarly, the rotation about Y_U by an angle β is 3 cross 3 and this is rotation about X_U by an angle α is once again 3 cross 3 and if you multiply, then ultimately, I will be getting this as the final matrix and this is once again a 3 cross 3 matrix,. So, by using the concept of the roll, pitch and yaw, I will be getting that this is nothing but the final form of the rotation matrix.

Now, in Cartesian coordinate system, which I have already discussed the same rotation can be represented with the help of your 3 cross 3 matrix and this is nothing but r_11, r_12, r_13, these are the elements for the first row, then r_21, 22, 23, and then 31, 32, 33. Now, in vector form, if you see, this is corresponding to the normal vector, this is corresponding to the sliding vector and this is corresponding to the approach vector.

So, if this is known and this is the final expression for the rotation matrix using the concept of the roll, pitch and yaw and element-wise, if I just compare, I will be able to find out that corresponding to this known rotation, what should be the angles for the rolling, pitching and yawing, so that we can determine very easily.

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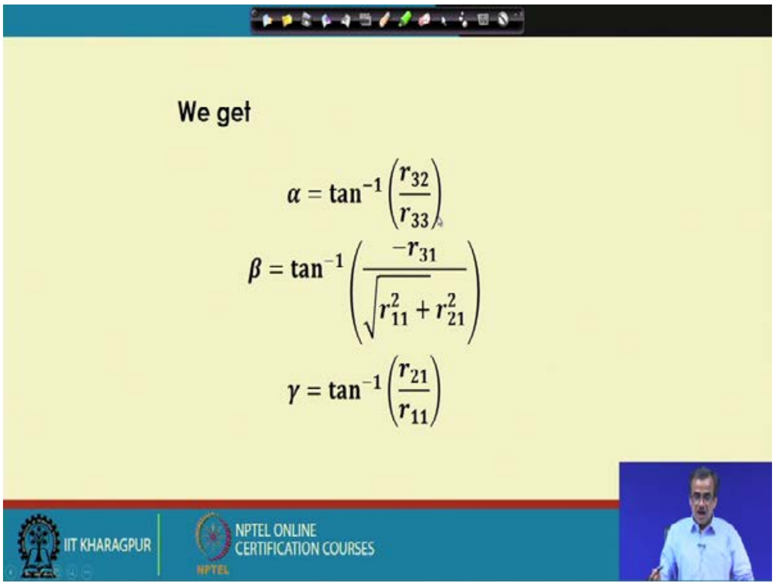

So, what we will have to do is: element-wise we will have to compare and then, we will have to find out. For example, the angle of rolling alpha is nothing but tan inverse of r_32 divided by r_33. So, r_32 is this, r_33 is this and if I compare, r_32 is nothing but this and r_33 is nothing but this.

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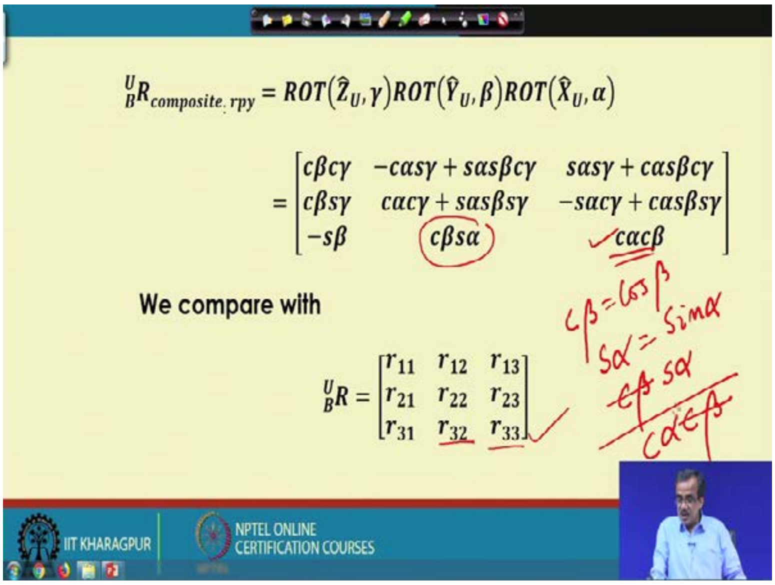

Here, $c\beta$ means $\cos\beta$, $s\alpha$ is nothing but $\sin\alpha$. So, $c\beta s\alpha$, that is, $\cos\beta \sin\alpha$ is divided by $\cos\alpha$ then comes your $\cos\beta$, this particular thing. Now, if I just see, $c\beta$, that is, cos beta, cos beta gets cancelled. So, I am getting tan α, that is, $\sin\alpha$ is divided by the $\cos\alpha$. So, this is nothing but tan α. So, very easily, I can find out that α is nothing but $\tan^{-1}\frac{r_{32}}{r_{33}}$.

Now, following the same principle, I can also find out the angle for this particular pitching. So, $\beta = \tan^{-1}(-r_{31} / \sqrt{r_{11}^2 + r_{21}^2})$, it is up to this, ok? So, we can find out this particular β. Similarly, the angle γ, that is nothing but the angle of yaw can be determined as $\gamma = \tan^{-1}(r_{21} / r_{11})$. So, if I know the orientation in Cartesian coordinate system, and to achieve the same orientation, I can also find out what should be the corresponding values for the angles of rolling, pitching and yawing. And, that is why, the orientation of the robot can also be actually expressed using the principle of roll, pitch and yaw.

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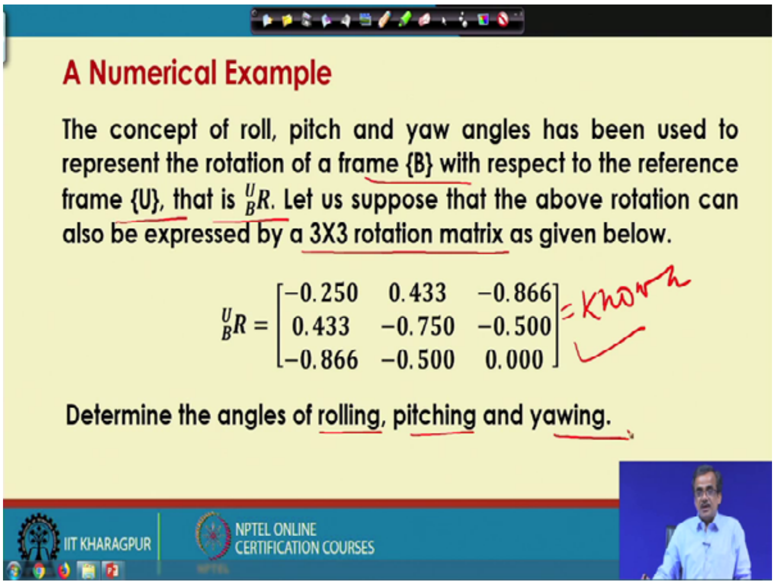

Now, here, just to explain it further, I am just going to take the help of one numerical example. So, very easily, we can find out the angles for the rolling, pitching and yawing. The statement of the problem is as follows: the concept of roll, pitch and yaw angels has been used to represent the rotation of B with respect to the reference frame U, that is your R B with respect to U (${}^{U}_{B}R$).

Let us suppose that the above rotation can also be expressed by a 3 × 3 rotation matrix, as given bellow. So, this particular rotation matrix is known. So, this is given, we will have to determine the angles of rolling, pitching and yawing, it is very simple.

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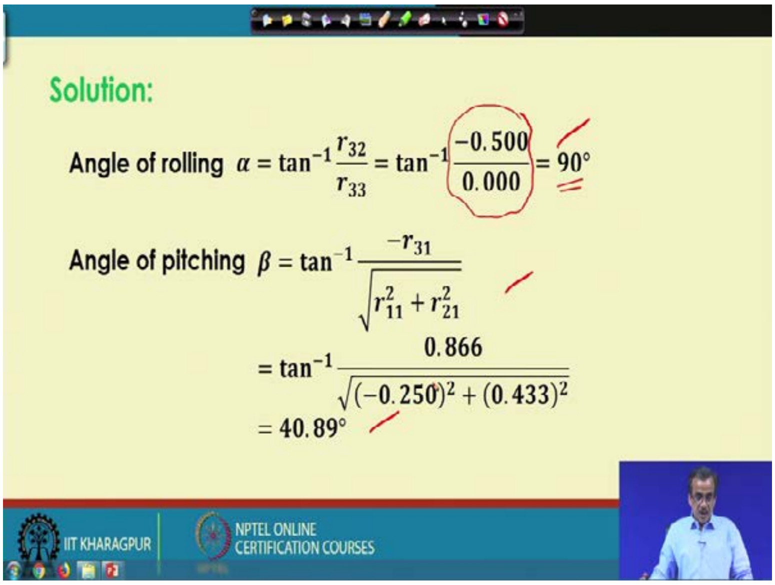

So, what I am we are going to do is: very easily, we are going to use those expressions that is, angle of rolling α is nothing but $\tan^{-1}\frac{r_{32}}{r_{33}}$. So, this is coming as tan inverse minus 0.5 divided by is your 0.000. So, this is nothing but actually the infinity and that is why, this α is equal to 90 degrees. Similarly, the angle for pitching, $\beta = \tan^{-1}(-r_{31} / \sqrt{r_{11}^2 + r_{21}^2})$ and if you just put the numerical value and calculate you will be getting 59.99 degrees (sorry for the calculation mistake in the .ppt file).

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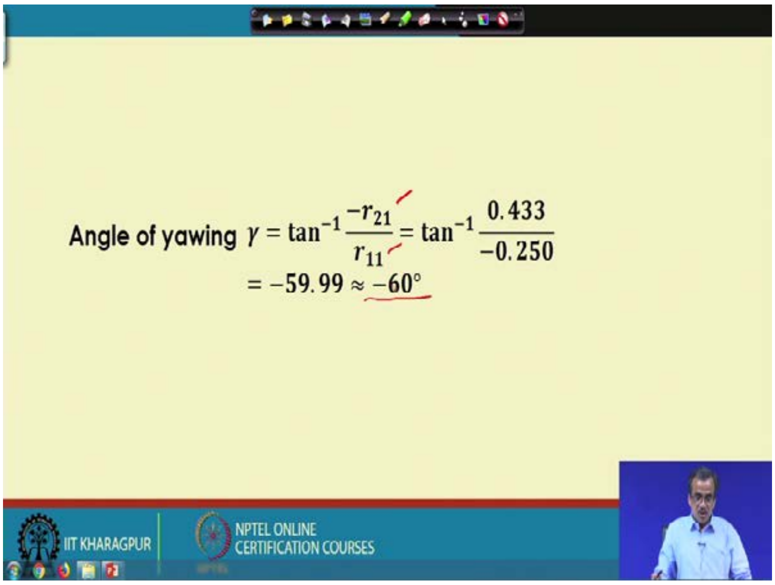

Now, similarly, we can also find out, actually, the angle for the yawing. So, this $\gamma = \tan^{-1}(r_{21}/r_{11})$ and if you put the numerical values, we will be getting this as equal to more or less, approximately equal to minus 60 degrees, ok? So, if we consider that positive is anticlockwise and your negative is clockwise, this is nothing but the clockwise. So, this is the way, actually we can find out these angles for the rolling, pitching and yawing.

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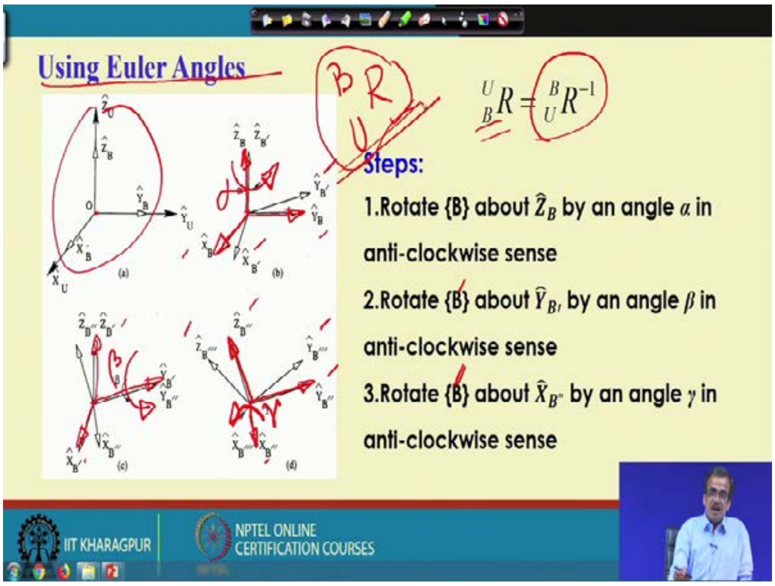

Now, I am just going to discuss another method, which is also very frequently used to represent the orientation of this particular object. Now, supposing that the initial coordinate systems are the same, for example, we are discussing this Euler angles, we have got this X_U, Y_U, Z_U as the universal coordinate system; X_B, Y_B and Z_B is a body coordinate system and the origin is at nothing but O.

Now, here, actually, what I do is: initially, they are coinciding, but after that, we are going to leave this particular universal coordinate system and we are going to rotate only the B coordinate system with respect to the rotated coordinate system, but not with respect to the universal coordinate system. But, what is our aim? Our aim is to determine; what is R B with respect to U, that is, rotation of B with respect to U, that we are trying to find out ok, but that will be found out by following some sort of indirect method.

So, what we will do is we will try to find out first, what is R U with respect to B and we will try to find out the inverse of that and that is nothing but R B with respect to U. Let me repeat, we are trying to find out first what is R U with respect to B and as I told that in this particular method, we are taking rotation of B with respect to B itself and we will be taking rotation with respect to the rotated frame itself.

Now, let us try to explain, so initially, you forget about U coordinate system. So, initially, this is actually the origin, this is my X_B, this is, my Y_B and this is my Z_B. So, what I do is: we rotate B about Z_B by an angle alpha in the anticlockwise sense. So, we are rotating with respect to this Z_B by an angle alpha. Now, if I do that what will happen to my Z_B prime, Z_B prime will remain same as Z_B, because we took the rotation about Z_B, but what will happen to my X_B prime.

So, this X_B prime will be different from X_B, then Y_B prime will be different from your Y_B and now, I am just going to take rotation. Rotate B prime about this Y_B prime by an angle beta in the anticlockwise sense. So, this is nothing but Y_B prime. So, whatever Y_B prime we have got, you draw this particular Y_B prime similarly, you draw this particular Z_B prime and you draw this X_B prime.

Now, I am just going to rotate about this Y_B prime by an angle beta in the anticlockwise sense. So, what will happen to my Y_B double prime? Y_B double prime will remain same as Y_B prime, but what will happen to X_B double prime? X_B double prime will be different from X_B prime. Similarly, your Z_B double prime will be different from your the Z_B prime and after that, actually, what we do is: we rotate B double prime with respect to X_B double prime. Now, here, let me draw this X_B double prime. So, this is nothing but X_B double prime, this is nothing but is your Y_B double prime and this is nothing but the Z_B double prime.

Now, we are going to rotate this B double prime with respect to X_B double prime. So, this is my X_B double prime. So, I am just going to rotate by the angle gamma in the anticlockwise sense. So, what will happen to my X_B triple prime? X_B triple prime will remain same as X_B double prime, but Y_B triple prime will be different from Y_B double prime and Z_B triple prime will be different from Z_B double prime and till now, all the rotations, we have taken are with respect to the B coordinate itself and I have not yet involved, I have not yet used this particular universal coordinate system.

Now, let us try to understand one thing. So, initially, this particular U coordinate system and B coordinate system were coinciding and after that, we took rotation of B with respect to B itself, that means, I am rotating the B coordinating system, but I am not doing anything with U. Now, can I not consider a similar situation that here, initially they are coinciding and U is kept constant and B is rotating, can I not find out one equivalent situation that as a B is kept constant and U is rotated by the same angle in the opposite direction? So, let me repeat again, initially the U coordinate system and B coordinate system were coinciding. Now, B is rotated by some angle say α in the anticlockwise direction, now can I not say that this is equivalent to the situation that B is kept constant and U is rotated by the same angle α in the opposite direction, that means, just like the velocity and relative velocity that particular concept sort of thing.

Now, the reason why I am going for this type of thing is as follows: my aim is to determine actually R_B with respect to U (${}^{U}_{B}R$), but before that, I will have to find out what is R_U with respect to B (${}^{B}_{U}R$). Now, if I want to find out what is R_U with respect to B and if I do not include U, I cannot find out this R U with respect to B. Just to include U with B I am taking in the help of that particular concept, that means by using that particular concept if I write down the composite rotation matrix. So, this is nothing but R_U with respect to B and Euler angles will have to write down Euler angles here.

So, that is nothing but whatever I consider first, but with the negative sign because we rotated B keeping U fixed. Now, we are considering, as if we are rotating U keeping B fixed in the opposite direction. So, we are considering the rotation about Z_B by an angle -α followed by the rotation about Y_B by an angle -β, the rotation about X_B by an angle -γ. Now, we know the expression of each of these rotation matrices.

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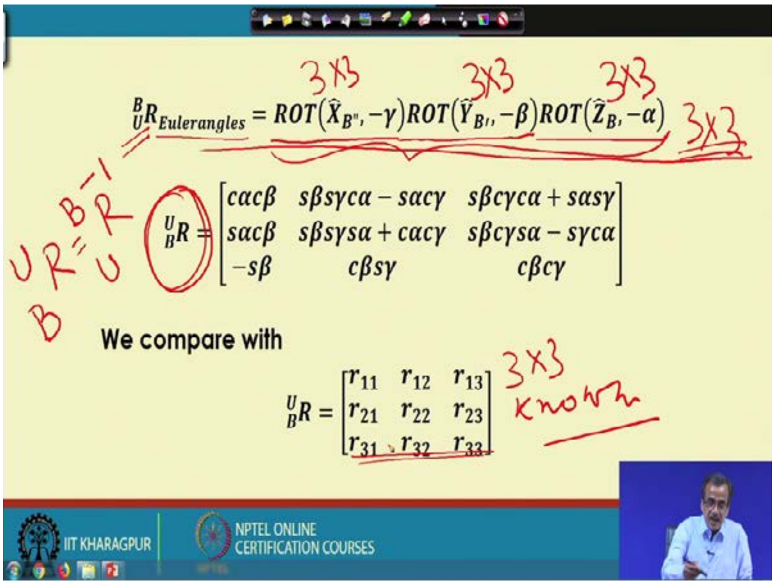

For example, rotation about X by - γ, I can write down the 3 × 3 matrix. Similarly, rotation about Y_B by an angle β, so, I can write down this particular 3 × 3 matrix then comes your rotation about Z_B by an angle α. So, I can write down the 3 × 3 matrix, and these three 3 × 3 matrices I can multiply. And finally, I will be getting one 3 × 3 matrix and that is nothing but is your R_U with respect to B, but what we need is just the reverse. So, what I need is R B with respect to U. So, this R B, that is, the rotation of B that is the body coordinate system, with respect to the universal coordinate system U and that is nothing but R U with respect to B inverse of that.

Now, if I can find out so, this R U with respect to B, this is nothing but a 3 × 3 matrix and this is a pure rotation matrix, and as it is a pure rotation matrix, very easily we can find out its inverse, because for a pure rotation matrix, the inverse is nothing but its transpose. So, that means, the row will become column and column will be row. So, whatever matrix we are getting here, the 3 × 3 matrix you try to find out the transpose of that, and that is nothing but the inverse. So, you will be getting the rotation of B with respect to U. So, this particular thing we can find out. So, by using this Euler angles, we can represent the orientation with respect to the universal coordination system.

Now, supposing that in Cartesian coordination system, this particular R B with respect to U, so, this particular 3 × 3 matrix is known to us. Now, if it is known, then element-wise I just compare. So, very easily, we can find out the numerical values for these particular α, β and γ, that is, the Euler angles. So, let us see how to find out the Euler angles values. So, α is nothing but $\tan^{-1}(r_{21}/r_{11})$. So, r_21 is nothing but this, and r_11 is nothing but this. So, r_21 is nothing but $\sin\alpha\cos\beta$.

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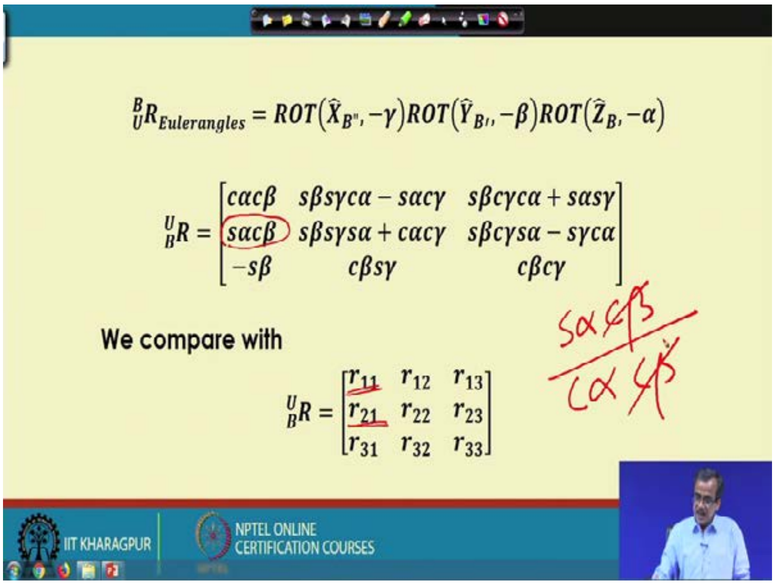

So, $\sin\alpha\cos\beta$ is $s\alpha\, c\beta$ and r_11 is nothing but is $c\alpha\, c\beta$, that is, c alpha c beta. So, $\cos\beta$, $\cos\beta$ gets cancelled and I will be getting $\tan\alpha$ and if I get $\tan\alpha$, very easily, I can find out, α is tan inverse this. Similarly, I can find out your α is this, β is nothing but this. So, minus r_31 divided by r_11 square plus r_21 square and it should come up to this, ok? Then, γ is nothing but $\tan^{-1}\frac{r_{32}}{r_{33}}$.

So, in this way, actually we can represent both the position as well as orientation in different coordinate system. And, if we can represent the position and orientation in different coordinate systems, the same robot you can control in different coordinate systems, and that is why, if you see the remote controller for the robot, for that is the teach-pendant (explained while discussing the robot teaching methods) can be used in different coordinate systems to control the manipulator.

So, once again, let me repeat that the position can be expressed either in Cartesian coordinate system or in cylindrical coordinate system or in spherical coordinate system.

Similarly, the orientation or the rotation can be represented either in Cartesian coordinate system or in roll, pitch and yaw or in Euler angles.

Thank you.

Robotics
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Lecture - 15
Robot Kinematics (Contd.)

Let us take the example of my own hand in the form of a serial manipulator. Say, this is the serial manipulator, this is the end-effector ok? Now, if I want to find out the position and orientation of this particular end-effector, with respect to the fixed coordinate system, then, what I will have to do is: I will have to assign the coordinate system at the different joints, and then, we will have to assign the frame and talk about the frame transformation.

Now, to make it possible, actually, the first thing we will have to do is: at each of this particular joint, we will have to assign the coordinate system. Now, to assign the coordinate system, we will have to follow certain rules, and these rules, actually, are nothing but the Denavit Hartenberg notation rules, and this particular concept was proposed in the year 1955 by Denavit and Hartenberg.

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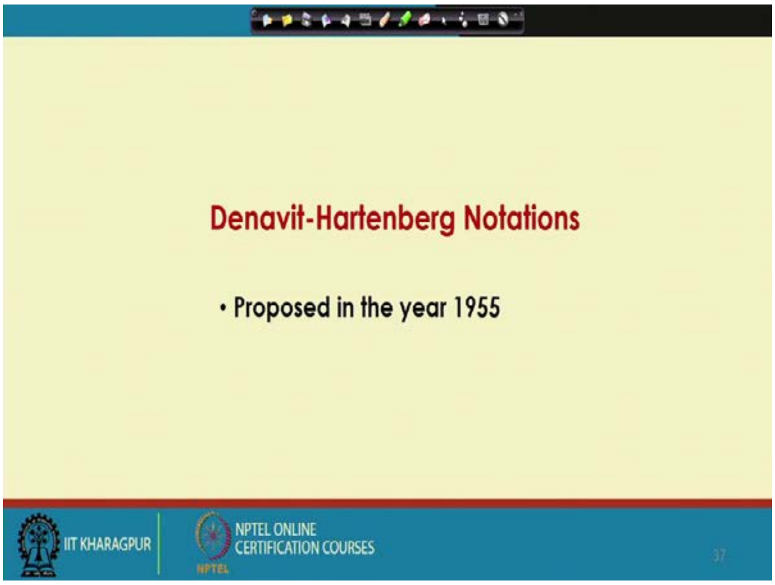

Now, according to this Denavit and Hartenberg rules, actually, we can assign coordinate systems at the different joints. Then, very easily, we will be able to find out the position and orientation of this particular end-effector with respect to the base coordinate frame and vice-versa. So, now, we are going to discuss the Denavit Hartenberg notation and Denavit Hartenberg rules to assign the coordinate system at the different joints.

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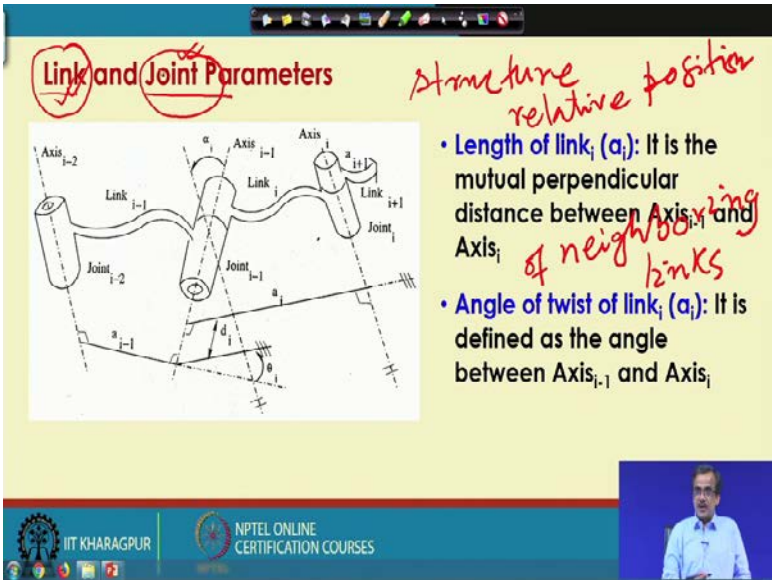

Now, before I go for that, what we will have to do is: we will have to define a few parameters. For example, say, we will have to define two link parameters and there are two joint parameters. Now, before I am just going for deriving or explaining the meaning of this link parameters and the joint parameters, let me tell you the purpose of using them. Now, the link parameters are used to represent the structure of a link.

So, if I want to represent the structure of a link, I will have to use the link parameter. Similarly, the joint parameters are used to represent relative position of the neighboring links. So, the relative position of the neighboring links, if you want to find out, we take the help of the joint parameters, ok?

Now, I am just going to define the link and joint parameters. Now, here, the first thing I am just going to define is the link parameter, that is, the length of a link. The length of a link that is denoted by a_i, and by definition it is the mutual perpendicular distance between the axis_i minus 1 and axis_i. Now, before that, let me tell you that in this particular sketch, supposing that, this is the joint_i, similarly, this is joint_i minus 1, this is joint_i minus 2. So, for this particular joint_i, this is the axis_i and for the joint_i minus 1, the axis is axis_i minus 1 and for joint_i minus 2, the axis is denoted by i minus 2.

Now, here, if you see this particular axis_i and axis_i minus 1. So, this upper part is actually in 3-D. So, if I consider this axis_i and axis_i minus 1, they may lie on the same plane, they may not lie on the same plane, too. For example, say if I consider say, this is one axis, say axis_i and this is another axis, the axis_i minus 1, they could be like this, they could be like this, they could be like this. So, they may not lie on the same plane or there is a possibility that both of them are lying on the same plane, same 2-D plane.

Now, supposing that, they are not lying on the same plane, they are lying on the two different planes. So, this is axis_i, this is axis i_minus 1, and they are lying on two different planes, if they are lying on the two different planes like this, I can find out the mutual perpendicular distance, and that particular mutual perpendicular distance is nothing but the length of link_i, that is, denoted by a_i.

Now, similarly, if they are lying on the same 2-D plane, the plane of the board now, here, their mutual perpendicular distance will be this. So, this will be the mutual perpendicular distance and they are parallel. So, if they are parallel and lying on the same 2-D plane. So, I can find out the mutual perpendicular distance. Once again, let me repeat that particular definition, the length of link, that is, a_i, it is the mutual perpendicular distance between axis_i minus 1 and axis_i.

So, this is nothing but axis_i, this is nothing but axis_i minus 1 and here, this particular is nothing but is your a_i, that is, the length of the link and this particular angle is 90 degrees. And, this axis_i and axis_i minus 1, according to this figure, might be, they are lying on two different planes, here. So, this particular angle is 90 degree and this is also 90 degree, and this is the mutual perpendicular distance and this a_i is nothing but the length of the link i.

Now, let me take one very special case, supposing that axis_i and axis_i minus 1. So, this is my axis_i minus 1 and this is your axis_i and they are going to intersect at this particular point, and supposing that, they are lying on the same 2-D plane. And, if they are going to intersect at this particular point, then what will be the value for this particular a_i? a_i will become equal to 0.

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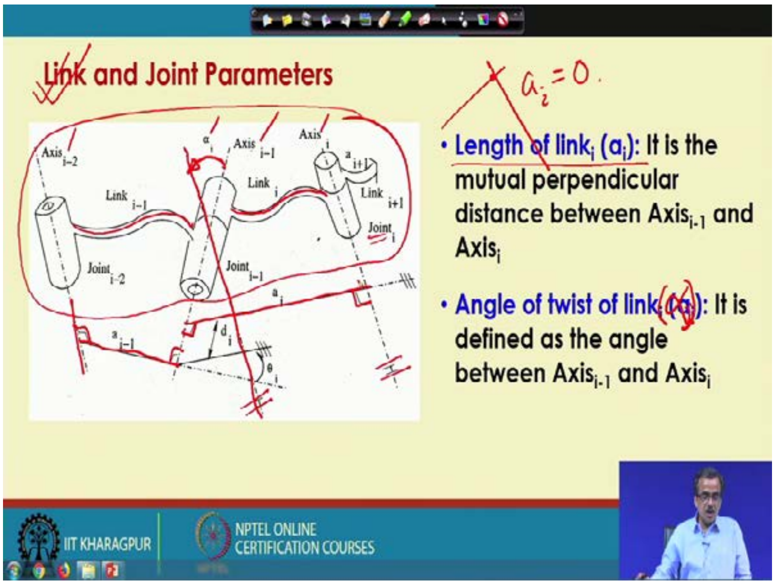

For this type of case, actually, a_i becomes equal to 0.

So, this particular, the length of the link could be 0 and it could be nonzero. So, this is what you mean by the length of link i. The next is the angle of twist of link, I think here, there is a typo-graphical error. So, this particular symbol, we generally use as α. So, this is α_i. So, in place of a_i, this is actually α_i, this is the α_i, ok? So, angle of twist of link, that is denoted by α_i, it is defined as the angle between the axis_i minus 1 and axis_i, ok?

Now, this is the axis_i, this is my axis_i minus 1. So, what you do is: this particular axis i, we draw it here, now, if I just draw it here, this particular axis_i. So, this particular line is parallel to this, ok? Now, here, I will be getting one angle, the angle between axis_i minus 1 and axis_i measured from axis_i minus 1. So, this angle is nothing but α, that is the angle of twist, α. So, this is actually the angle between the axis_i minus 1 and axis_i.

Now, remember, this particular angle alpha, that is the angle of twist, it could be either positive or negative or sometimes it may also become 0. If the two axes are parallel, then this particular angle will become equal to 0 here. So, this angle is measured from axis_i minus 1 to i and, here, it is anticlockwise. So, this is positive α, similarly, if it is found to be clockwise, α could be negative also, ok? So, these two things are actually nothing but the link parameters, and the link parameters are used to represent the structure of a particular link. For example, this is link_i, this is your link_i minus 1. So, this is a_i similarly, this is you're a_i minus one. So, this is the mutual perpendicular distance between axis_i minus 2 and axis_i minus 1. So, this particular is a_i minus 1. So, this is a_i minus 1 and this is your a_i.

So, till now, I have defined only two link parameters, now, I am just going to define the joint parameters.

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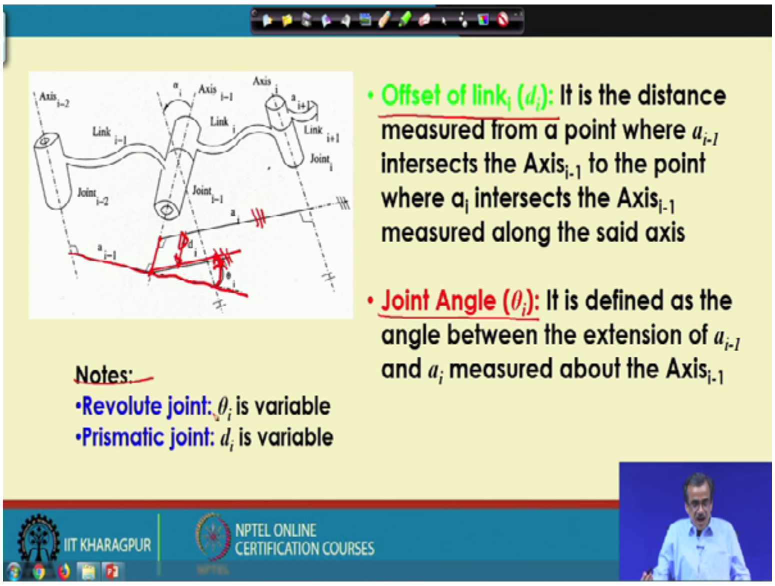

The purpose of using joint parameters, I have already told that just to represent the relative position of the neighboring links. Now, here, there are two joint parameters, one is called the offset of link, that is, denoted by d_i, the link offset. And, this particular link offset is nothing but the distance between the two points, the first point is a point where a_i minus 1 intersects the axis_i minus 1 and another point is, where a_i intersect the axis_i minus 1.

So, this is the distance measured from a point where a_i minus 1 intersects the axis_i minus 1 to the point, where a_i intersect the axis_i minus 1. So, this particular distance is actually nothing but the link-offset and that is denoted by d_i, ok? So, this is actually the d_i. Now, this particular d_i, link offset, could be 0, sometimes the link offset becomes equal to 0 and it could be the positive value also and in some special case due to the coordinate system, it may take a negative value, also.

Now, then comes the joint angle that is denoted by θ_i, now, this particular joint angle is defined as the angle between the extension of a_i minus 1 and a_i measured about axis_i minus 1. So, if I extend, this particular a_i minus 1. So, I will be getting this particular line and so, this line is actually parallel to this particular a_i, now, this angle between the extension of a_i minus 1 and a_i measured from a_i minus 1 is nothing but the joint angle, that is, θ_i and θ_i is actually measured from an extension of a_i minus 1 to a_i. Now, this θ_i it could be once again 0, it could be positive or it could be negative.

Now, here, I have put actually two notes: for a revolute joint, θ_i is the variable, it is very obvious, for example, if I take the example of a revolute joint like this, this particular joint is a revolute joint. So, if I take this is the axis about which I am taking the rotation and if you concentrate on this particular angle, this particular angle is the joint angle and that is actually a variable, ok?

So, for a revolute joint, θ_i is the variable and what about the other three parameters. Other three parameters, like a_i, α_i, d_i are kept constant, similarly, for a prismatic joint, d_i is link offset, which is the variable and the other three remaining parameters, for example, say a_i, α_i and θ_i, these things are kept constant. So, for revolute joint, θ_i is the variable, for prismatic joint, d_i is the variable.

Now, till now, actually, we have defined two link parameters, and two joint parameters, and with the help of these four terms, I am just going to now state the rules to be used to assign the coordinate system at a particular joint, how to assign the coordinate system at a particular joint.

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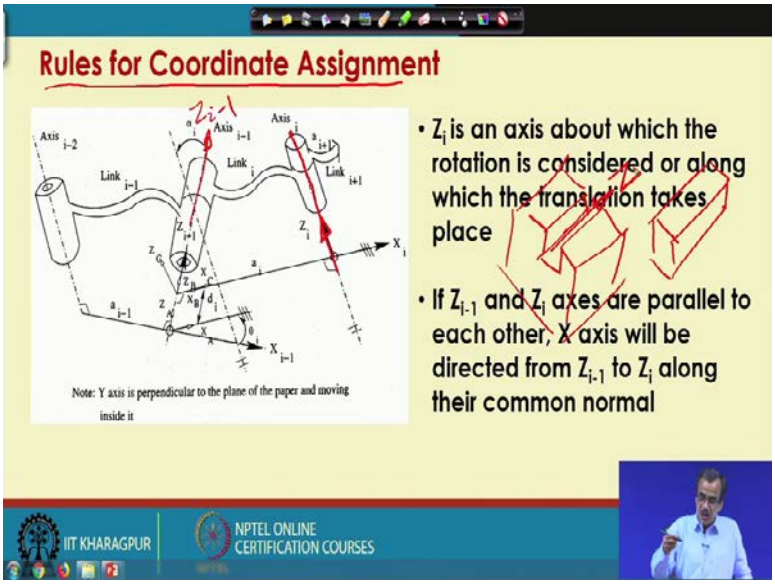

The rules for the coordinate assignment, we will have to find out. Now, remember one thing, the first thing we will have to do is: we will have to assign the Z coordinate system, next we try to find out the X coordinate system, and after that, we go for the Y coordinate system.

Now, let us see, how to represent or how to find out that Z_i axis first. The rule for determining the Z_i axis is as follows: Z_i is an axis about which the rotation is considered and along which the translation takes place, now let us try to understand. So, Z_i is an axis about which the rotation is considered, now, here according to this so, for example, for this particular joint. So, this is the axis about which I am taking the rotation. So, this is nothing but your Z_i.

Similarly, here, you can see that, this is the axis about which I am taking the rotation at this particular joint. So, this is actually the axis, that is, Z_i minus 1. So, this is Z_i and this is your Z_i minus 1, now let me repeat. Now let me take the example of the same revolute joint. Now, this is the axis about which I am taking the rotation. So, this particular thing is going to represent my Z axis, and if it is a linear joint, if there is a translation here, I have already taken the example of linear joint like say key and keyway for example, let me prepare a very simple sketch once again.

So, for example, say if I take this type of sketch for this linear joint, which I have already discussed, this type of linear joint if I take, and here, if I just insert one key sort of thing.

So, this particular key, I am just going to insert here, ok? So, this will be the Z direction. So, this is the direction of Z, actually, for the linear joint. So, Z is the axis along which, this particular translation takes place, and if it is a rotary joint, Z is the axis about which the rotation takes place, ok?

Now, I am just going to consider one case, if Z_i minus 1 and Z_i axes are parallel to each other, ok, then X axis will be directed from Z_i minus 1 to Z_i along their common normal. Now, as I mentioned that these particular things, that is Z_i and this particular Z_i minus 1, say, they may belong to two different planes, but they could be parallel or they are lying on the same plane and they could be parallel. So, if they are found to be parallel, this is the mutually perpendicular direction, that is, the way, we define this particular a_i. So, a_i is actually the direction of X.

So, X will be along the length of the link. Once again, let me repeat, if the two Z axes are parallel. So, X will be along their common normal and that means, you are here Z_i minus 1 and Z_i are parallel. So, this is a_i direction. So, this is nothing but X direction. So, this is your X_i direction. So, I will have already got this particular Z_i and X_i. So, this is your Z_i minus 1 and this is your X_i minus 1, this is X_i minus 1 and this is Z_i minus 1, ok?

Now, let us see, now, there could be some other cases, ok? So, I am just going to discuss these special cases.

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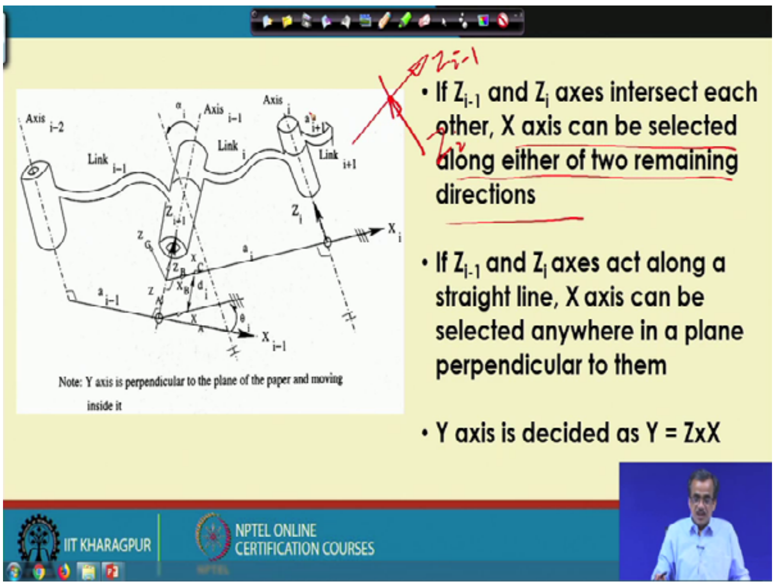

For example, say, if Z_i minus 1 and Z_i axes intersect each other. So, X axis can be selected in either of the two remaining directions. Now, if Z_i and Z_i minus 1axes intersect each other. So, this I have already discussed a little bit, supposing that, this is my Z_i minus 1 and this is the Z_i axes.

So, this Z_i minus 1 and Z_i are intersecting. If they are intersecting, then what will happen to the value of the length of the link a_i, that will become equal to 0. And, if they are intersecting, then X can be selected along either of the two remaining directions, so at a particular joint. So, Z has been selected. So, I have got two remaining direction and out of these two remaining directions, anyone can be selected as X. So, this is a very special case.

Now, similarly, there could be some other very special cases, like if Z_i minus 1 and Z_i axes act along a straight line; that means, they are collinear. So, Z_i minus 1 and Z_i they are collinear, then X axis can be selected any where in a plane perpendicular to them. So, if they are found to be collinear, it is a very special case, for example, say might be this is my Z_i minus 1 and this is my Z_i.

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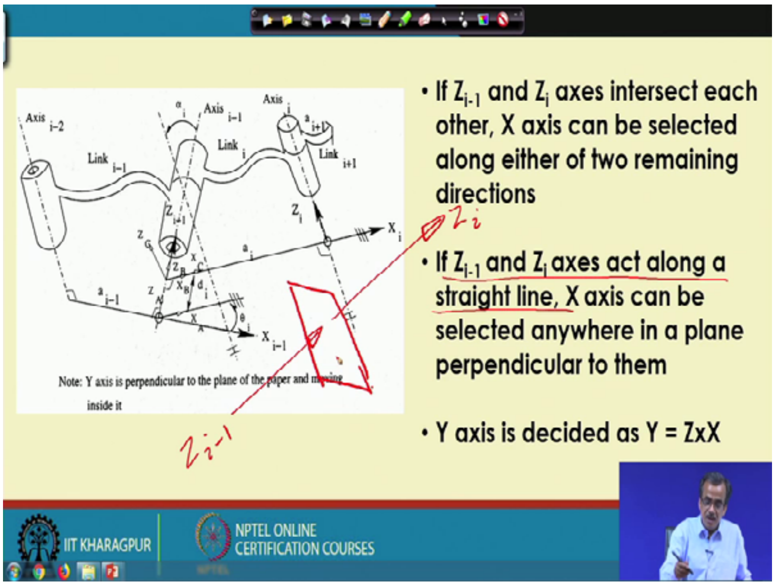

So, this is Z_i and this is Z_i minus 1, it is a very special case. So, what I will have to do is: I will have to consider a plane, which is perpendicular to them.

Now, a plane perpendicular to them could be of this type, for example, if I just draw it here, roughly one plane perpendicular to them could be something like this, which is perpendicular to this particular line. Now, X may lie on this particular plane, ok? So, this is actually one possibility, similarly, there could be another possibility like, this is Z_i minus 1 and this is Z_i. Now, I am just going to define a plane, which is perpendicular to both now that particular plane could be something like this. So, my X can lie here, also ok? So, both the possibilities are there. So, what you can do is: for these special cases, we will have to find out very carefully that particular X direction.

So, till now, we have discussed actually how to determine the Z axis and then, X axis and this particular sequence has to be maintained; that means, first, we will have to find out the Z axis, then we go for X axis and after that, we try to find out the Y axis. Now, Y axis is nothing but the Z × X. Now Z, X: these are all unit vectors. So, I can find out that the cross product of these particular Z and X, and Y will be nothing but Z × X.

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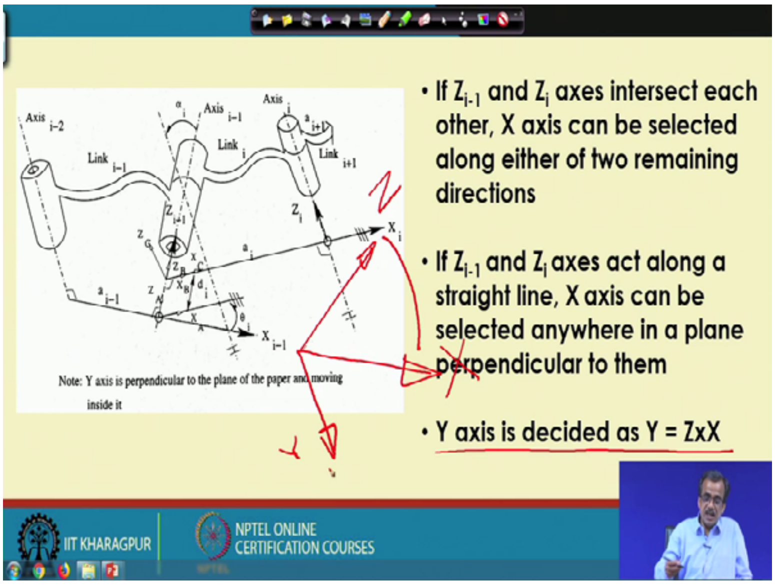

Now, let me take a very simple example supposing that this is my Z. So, I have already defined and say this is my X. So, that also I have got, now I will have to find out your Y direction, now the Z × X. So, according to the rule of cross product: Z × X will be something like this. So, this will be the direction of this particular Y. So, Z × X will be Y, are you getting the point?. So, this is the way, actually, we will have to find out the Y direction.

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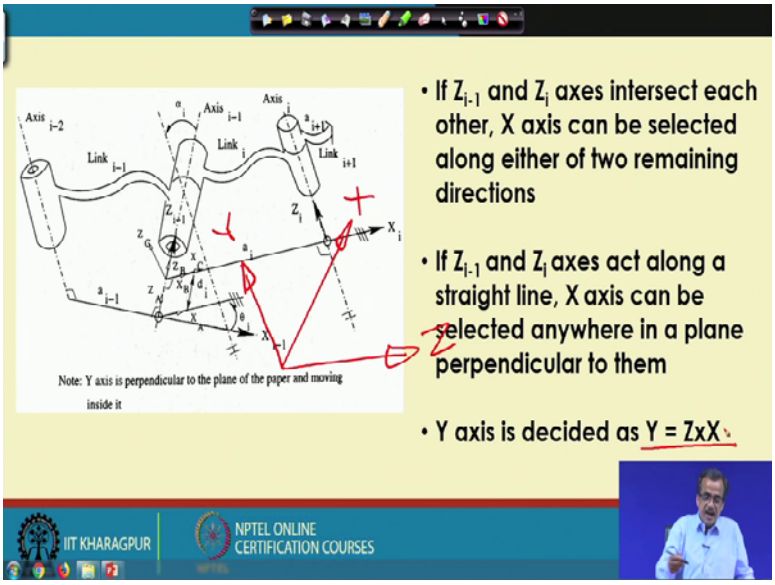

Now, say, this is my Z and this is my X, ok? Now, Z × X. So, Z × X will be something like this. So, this will be Y. So, Z × X will be something like Y. So, we will have to be very careful while determining, so this particular your Y direction. So, Y is nothing but is your Z × X and by following this particular rule at each of the joint, we can actually define the coordinate system like your X,Y and Z. And, once you have got this particular thing, now what you will have to do is:

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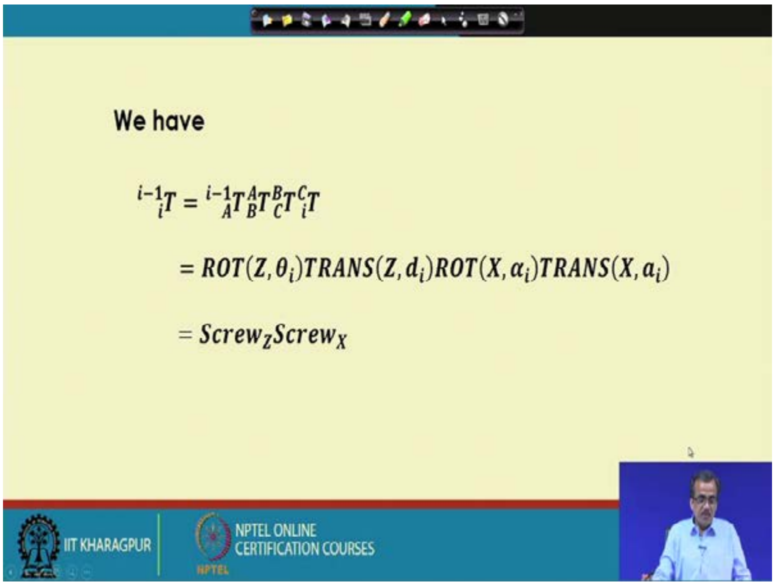

So, we will have to find out what is this transformation matrix, that is, T_i with respect to T_i minus 1.

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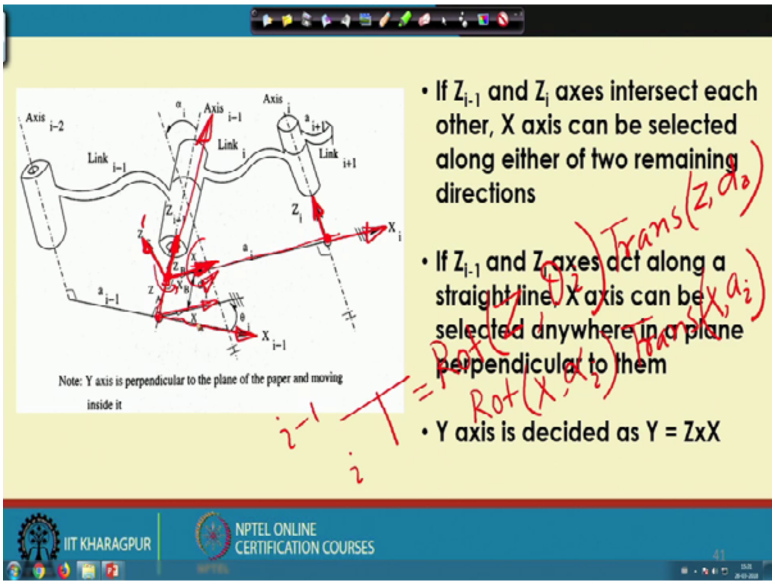

So, to determine this T_i with respect to i minus 1; so this is nothing but i, and this is nothing but i minus 1. So, my aim is to determine that T_i with respect to i minus 1. Let us see how to find out how to find out? So, my i minus 1 frame is here, and i-th frame is here. So, from here, so I will have to reach this particular, how to do it? Now, to do that actually, what we do is, I am just going to follow one sequence of rotations and translations.

So, let me try from X_i minus 1 and Z_i minus 1. As I told that this is nothing but is X_i minus 1 and this is nothing but is Z_i minus 1. So, what I will have to do is: I will have to take some rotation by an angle theta i about this particular Z axis, ok, then, only I will be able to move, here ok. So, I will be getting X_A and I am taking rotation about Z_A, so, Z_A will remain same as Z_i minus 1, ok. So, first, I will have to take some the rotation sort of thing that is nothing but I am just going to write down. So, rotation about Z by angle θ_i, the next from here I will have to reach this particular point, how to reach? I will have to translate along Z, so, I am just going to translate along Z by d_i. So, I will be getting actually this particular thing, that is, X_C and this is your Z_C, sorry, X_B and this is your Z_B.

And, after that, actually, I am just going to take some rotation about this particular X axis by an angle α. So, I am just going to take rotation about X by an angle α and then, I will be getting these as nothing but X_C because I have taken a rotation about X_C and Z_B will take the position like Z_C. So, Z_C will be something like this and once we have got this particular X_C and Z_C, now, I can translate along X direction.

So, TRANS along this particular X direction by this particular amount a_i. So, I will be able to reach this point and I will be getting these Z_i and X_i. So, with the help of these translations and rotations, actually, starting from here, I am just going to reach this particular point.

Thank you.

Robotics
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Lecture - 16
Robot Kinematics (Contd.)

Now, I am just going to discuss once again, like how to determine this particular T_i with respect to i minus 1, that is, the transformation matrix of i with respect to i minus 1.

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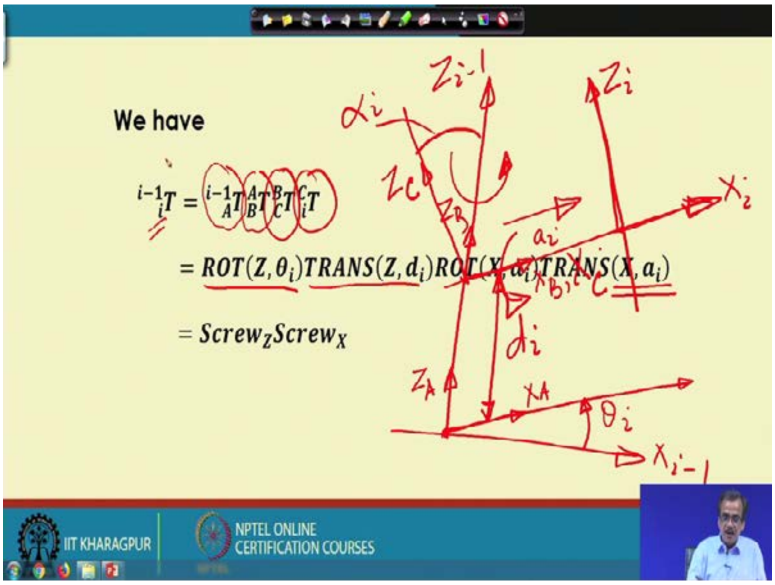

Now, if i draw the same thing once again, for example, say this is my Z axis, this is my X axis. So, this is Z_i, this is X_i and supposing that this is my Z_i minus 1 and let me consider this is X_i minus 1.

So, starting from here, actually, I will have to reach this, how to do it? So, what I do is, first we take the help of T_A with respect to i minus 1. So, what I do? So, we rotate about Z by angle θ. So, here, this is nothing but a_i. So, I am just going to draw one line parallel to that and this particular angle is nothing but θ_i.

So, we will start from this Z_i minus 1 and X_i minus 1 and the first is T_A with respect to i minus 1. So, what we do is. We draw this is nothing but X_A and this is nothing but is your Z_A and that is nothing but rotation about Z by θ_i, ok? So, as if I am taking rotation about Z by an angle θ_i; so this X_i minus 1 will take the position of X_A and this particular Z_A will remain the same along this particular Z_i minus 1.

Now, I am just going to do one thing, that is, T_B with respect to A. So, from here, actually, I will have to reach this X_B and this will become your Z_B and this distance is nothing but the offset, that is, d_i. Now, this is nothing but translation along Z by d_i. So, X_A will become X_B and your Z_A will become Z_B.

Now, I am just going to take this thing, like your C with respect to B. So, if I draw one parallel here. So, this particular angle is nothing but α. So, this is the α angle. So, what I will have to do is: I am going to take rotation about X by an angle α. So, what will happen to my X_C? X_C will remain same as X_B, but Z_C will be different from Z_B. So, this will become equal to Z_C and now, I am translating along this particular X, that is, a_i with respect to C. So, I am translating along X by a_i. So, from here, I am just going to reach this particular Z_i and X_i. This is the way, actually, I can find out what is T_i with respect to i minus 1, ok?

So, this is the way, I can reach this T_i with respect to i minus 1. Now, if you see this particular sequence, for example, say the rotation about Z by θ_i translation along Z by d_i, this is one thing, then rotation about X by α and translation along X by a_i is another.

Now, this rotation about Z followed by translation along Z by d_i is nothing but the screw Z. Now, this is very simple for example, say if this is the Z axis. So, I am rotating about Z, and if I have got a threaded part here, the screwed part is going to have some linear displacement along this particular Z direction. So, I am a rotating about Z and there is translation along Z. So, this is nothing but the screw principal and that is why, the rotation about Z and translation along Z is nothing but screw Z, then a rotation about X and translation along X is nothing but the screw X.

So, we will have to follow this particular screw rule. Now, you see, I know the expression for rotation about Z by θ_i, I know the expression of translation along Z by d_i, then rotation about X by α_i and translation along X by a_i.

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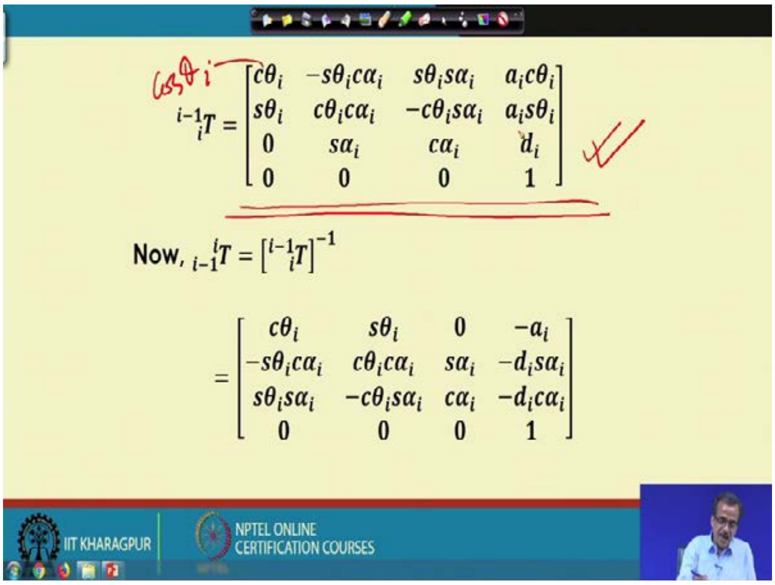

So, I will be getting some matrices and these mattresses if I multiply, then I will be getting this type of 4 × 4 matrix. So, this is the 4 × 4 matrix. Corresponding to each of these particular transformations, I can find out the 4 × 4 matrix. So, there are 4 such matrices, each having 4 × 4 dimensions and if I multiply, then I will be getting finally this particular 4 × 4 matrix.

Now, here, c theta_i means $\cos\theta_i$, this is the short form cos theta_i minus s theta_i is minus sine theta_i, c alpha_i is cos alpha_i and so on, and d_i is nothing but the offset. So, this is nothing but say T_i with respect to i minus 1. So, this is the final matrix, which we are going to get. So, I think up to this, it is clear to all of you, but here, I have got one query, that is the rule which you have followed to derive this particular expression, are we not violating the rule for the composite rotation matrix?

According to the rule for the composite rotation matrix, whatever we state fast that should go to the end, ok, but here, we stated first the rotation about Z by θ_i, that I have written at the beginning, but not at the end, ok? So, my question is, are we violating the rule for composite rotation matrix? The answer is, no. Now, the reason why that particular answer is “no” is as follows: if you follow the rule for the composite rotation matrix, actually, whatever we were doing is nothing but T_i minus 1 with respect to i.

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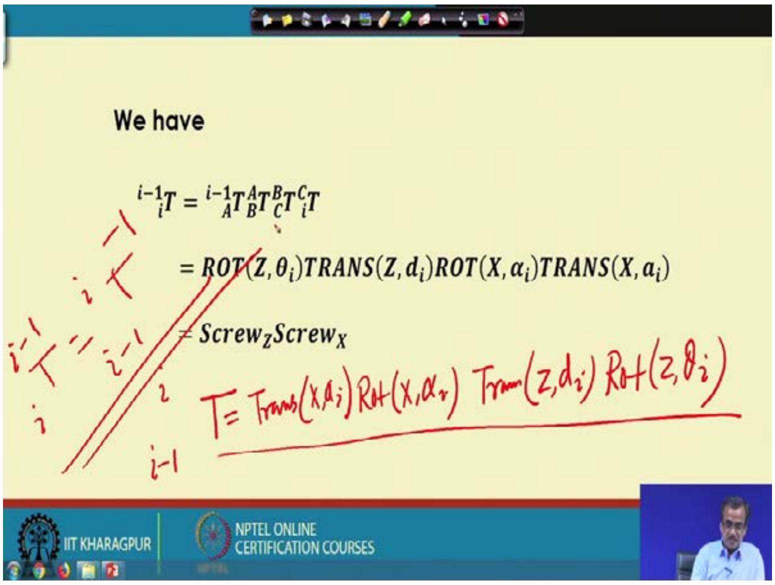

With respect to I, if you want to find out these, whatever I stated first that I will have to write at the end.

So, rotation about Z by θ_i, then comes translation along this Z_i, then comes your rotation about X by α_i, then comes your translation along X by a_i. So, according to the rule for composite rotation matrix this should be the sequence. Whatever I stated first should go to the end, followed by this, followed by this, followed by this and truly speaking, this is nothing but T_i minus 1 with respect to i, but what we are trying to find out is just the reverse, that is, T_i with respect to your i minus 1, and all of you know that this particular T_i with respect to i minus 1 is nothing but inverse of T_i minus 1 with respect to I, ok?

So, we are not violating the rule for the composite rotation matrix and according to that, T_i minus 1 with respect to I, as I told, it is the inverse of that, and if we just try to find out the inverse of this particular matrix, we will be getting. So, this is the way, actually, we can find out the final matrix by using that particular rule for the composite rotation matrix and using the Denavit-Hartenberg notation, we can find out the expression for this transformation matrix.

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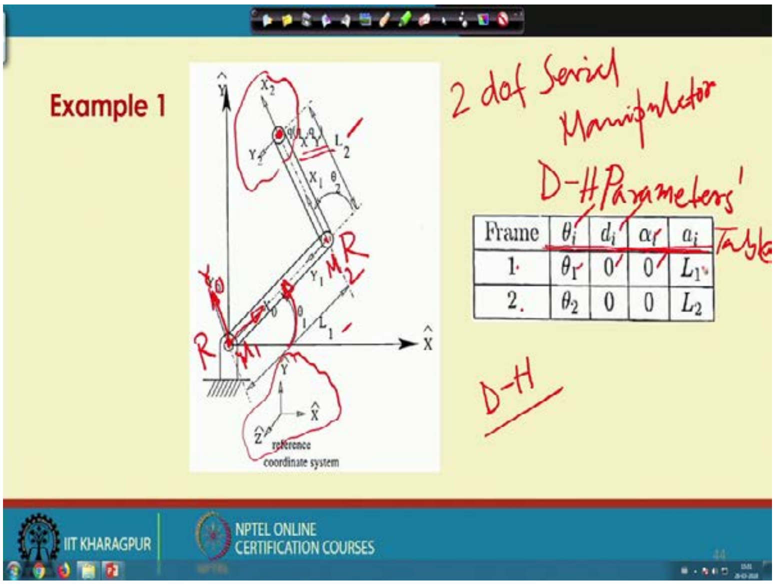

Now I am just going to solve one example, one very practical example by using the rules, which I have already discussed. So, how to assign the coordinate system and how to carry out the kinematic analysis? Now, here, for simplicity, I am just going to consider a very simple problem, a problem of 2 degrees of freedom serial manipulator.

So, in this 2 degrees of freedom serial manipulator, here we have got 2 joints, this is joint 1 and this is joint 2, the link 1 is having the length L_1, the link 2 is having the length L_2 and this is, say, the wrist joint or say, approximately this is the end-effector. And, here, the joint angles are nothing but θ_1 and here with respect to the previous. So, this is nothing but θ_2. So, θ_1, θ_2 are nothing but the joint angles, I have already defined the joint angles.

Now, let us try to assign the coordinate system first, according to the D-H parameters setting rule or the Denavit-Hartenberg notations, now, let us try to concentrate on the first joint. So, as I told that we have got two joints here, two motors here, one motor is here, another motor is here, here, there is no motor. Now, here, how to assign the coordinate system? The first thing we will have to see is, we will have to see the reference coordinate system. So, this is nothing but the reference coordinate system, we can see that the Z × X is nothing but Y and this is actually the reference coordinate system.

So, with respect to this, by following these, I will have to assign the coordinate systems according to the rule. So, this particular joint is a revolute joint and this is a rotary joint, this is also a revolute joint, this is a rotary joint. Now, this is in Cartesian X and Y and this particular end-effector is having the coordinate (q_X, q_Y) and you forget about Z, as this is on the two dimensions.

Now, here, actually what we do is, we will have to find out first the Z. So, Z is the axis about which I am taking the rotation. So, Z will be what? Z will be perpendicular to this board. So, Z will be perpendicular to the board and here, also the Z will be perpendicular to the board and X is what? The Z is here at the first joint and the Z is here at the second joint, they are parallel.

So, their mutually perpendicular direction is this direction. So, X should be along that particular the direction. So, Z is perpendicular to the board away from the board and X is in this particular direction and Z × X, so, this will be my Y_0 direction and Z is perpendicular to the board, which is not shown here and it is coming out of the board.

Similarly, here, the Z is perpendicular to the board coming out of the board, and this will be my X direction and this will be my Y direction and what we do is, whatever coordinate system we assign here, the same thing we copy at the last joint although here there is no motor. So, in place of your X_1 I will have to write X_2 in place of Y_1, I will have to write Y_2. So, this X_2 and Y_2 is nothing but we are copying this X_1 and Y_1. So, this is an extra coordinate system, we are adding at the end, although there is no such motor here.

There are two motors, as I told here, I have got motor_1 and here, I have got motor_2, just to create the rotary movement. So, this is how to assign the coordinate system, according to the D-H parameters setting rule. Now, once we have actually assigned this particular coordinate system, now I can prepare the D-H parameters table, this is called the D-H parameters' table. Now, here, what we write is your frame_1, frame_2. Whenever we write frame 1, what you will have to do is, one with respect to 0 and whenever we consider 2, that is 2 with respect to the previous, that is, 1. So, this particular sequence is very important, for example, first we consider θ_i next we consider d_i, if you remember the screw Z rule.

Now, screw Z, there is a rotation about Z by an angle θ_i, there is translation along Z by d_i. So, I am following this particular screw Z, then I am going to follow the screw X, that is the rotation about X and then translation along X. So, this is nothing but screw X. So, screw Z screw X.

So, that particular rule, we will have to follow, while writing down the link and joint parameters in the D-H parameters' table. In some of the textbooks, they do not follow this particular rule, they write in a slightly different fashion like they first write α, θ, then d_i, a_i something like that, but if you write in that particular fashion, whenever we are going to write down the forward kinematic equation. So, you will have to make the correction. But, if you follow this particular sequence like the screw Z and screw X, this particular sequence, you need not make any change and directly, you can write down the forward kinematic equation, that I am going to show.

Now, how to determine these numerical values or how to find out these variables; now, as I told 1 means what? 1 means 1 with respect to 0, what is θ_i? This is a rotary joint, this is a revolute joint and for this particular revolute joint for example, this type of revolute joint, sort of thing. So, this particular angle is the variable, ok? So, this θ_i has to be a variable. So, at this particular joint, this is my θ_i and that is variable, then what is d_i? By definition, if you remember, d is the distance between two X, measured along Z.

Now, this is X_0 direction and this is X_1. So, if I extend X_1 they are going to intersect. So, X_0 and X_1 are going to intersect, then what is the distance between X_0 and X_1 is 0. So, here, I have put 0. Now, then comes alpha, alpha is the angle between two Zs, if you remember. So, this is actually my Z_0, which is perpendicular to the board, Z_1 is perpendicular to the board and they are parallel. So, their included angle is 0.

Now, next is you're a_i. So, a_i is the distance between two Zs. So, here, I have got one Z, here I have got one Z and along X, I can find out the mutual perpendicular distance and that is nothing but the length of the link. So, a_i is nothing but the length of the link. Next, we can find out that frame_2, that is, 2 with respect to 1, that is 2 with respect to 1. So, once again, I have got a rotary joint here, revolute joint here. So, the variable is theta_i next is the d, that is the distance between two Xs. So, X_1 and X_2 are in the same line. So, the distance between them is 0, the next is your alpha.

Hypothetically, we have assumed that this is your Z_2 and this is Z_1 and they are parallel. So, the angle is 0, then comes the length of the link. So, this is your Z_1 and Z_2 they are parallel. So, this particular L_2 is nothing but the length of the link. So, we can find out, all the entries of the D-H parameters' table. And, once you have found out the entries for that, now, very easily can find out, what should be the kinematic equation.

So, what I am going to do is: I am trying to find out the kinematic equation; the purpose of kinematic equation is to represent the position, and orientation of end-effector with respect to the base coordinate system.

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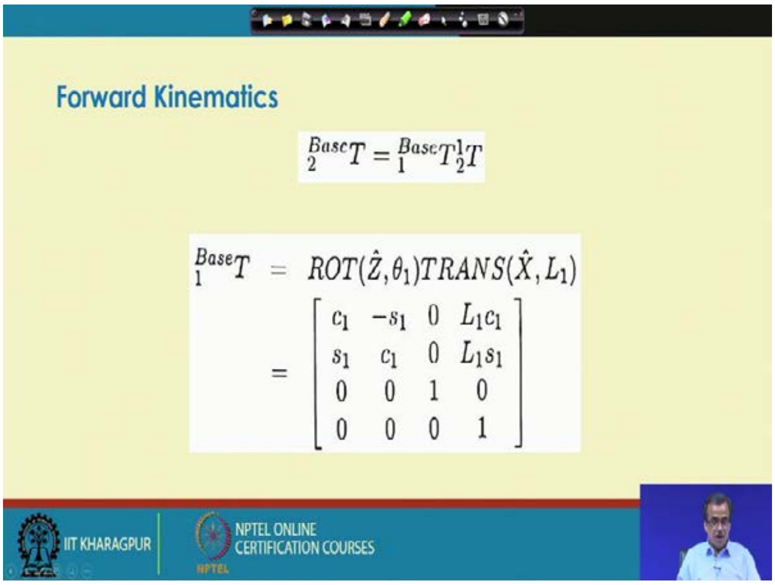

And, to get it actually, what I will have to do is: I will have to take the help of this type of transformation matrix, that is, T_2 is respect to base. So, this is T_1 with respect to the base. So, T_2 with respect to base is nothing but T_1 with respect to base multiplied by T_2 with respect to 1.

Now, how to find out T_1 with respect to base, now to write down T_1 with respect to base, you concentrate here.

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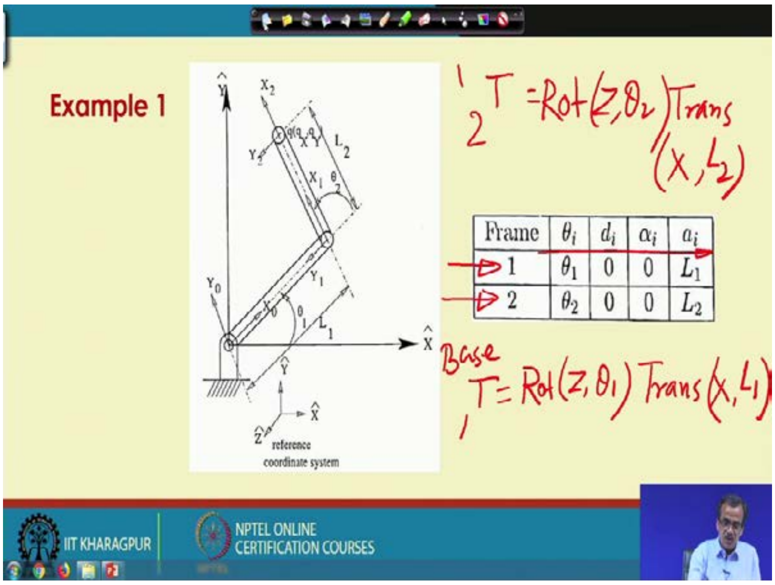

And, you just move along this particular direction, without making any change. So, T_1 with respect to base, the first is θ_1 that is your rotation about Z by θ_1. Next is 0, 0, I am not going to write anything and the in the last one is translation, Trans along X by L_1. So, this is what you mean by T_1 with respect to base.

Similarly, we can also write down, that is, T_2 with respect to 1. So, I will have to concentrate here, and this is nothing but rotation about Z by an angle θ_2, then comes your translation along X by L_2 and all such things, actually, I am just going to consider next. So, this T_1 with respect to base is nothing but is rotation about Z by θ_1, translation along X by L_1. So, I know the 4 × 4 matrix corresponding to this, I know the 4 × 4 matrix corresponding to this and if I just multiply then I will be getting this particular 4 × 4 matrix for this T_1 with respect to base.

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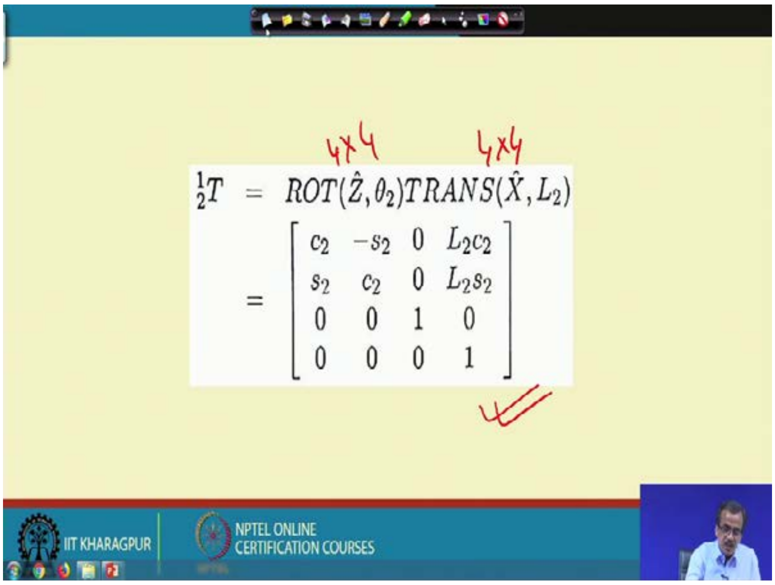

Now, similarly, actually what we can do is: we can find out this T_2 with respect to one, that is, your rotation about Z by an angle θ_2 and translation along X by L_2. So, I know the 4 × 4 matrix here, and I know the 4 × 4 matrix here, and I can multiply for getting the final matrix.

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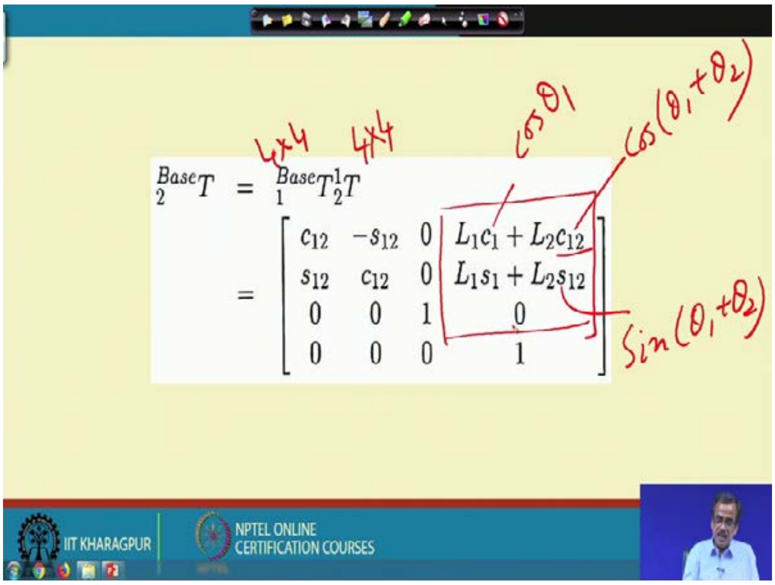

And, once we have got this particular final matrix now, I am in a position to find out, what is T_2 with respect to base, that is, nothing but T_1 with respect to base multiplied by T_2 with respect to 1.

Now, if I just multiply. So, this T_1 with respect to base that is nothing but 4 × 4 matrix, and this T_2 with respect to 1 is nothing but another 4 × 4 matrix, then I will be getting actually this final 4 × 4 matrix and here, actually this carries information of this position, ok?

So, the position information is given by this particular information and here, c_1 means cos theta_1 and c_12 means cos of theta_1 plus theta_2. Similarly, s_12 is nothing but sine of theta_1 plus theta_2, now if you just compare, whatever position information we are getting. So, if I just compare with our general information of trigonometry. So, we can find out that this particular expression is correct.

For example, say, if you see this particular 2 degrees of freedom serial manipulator.

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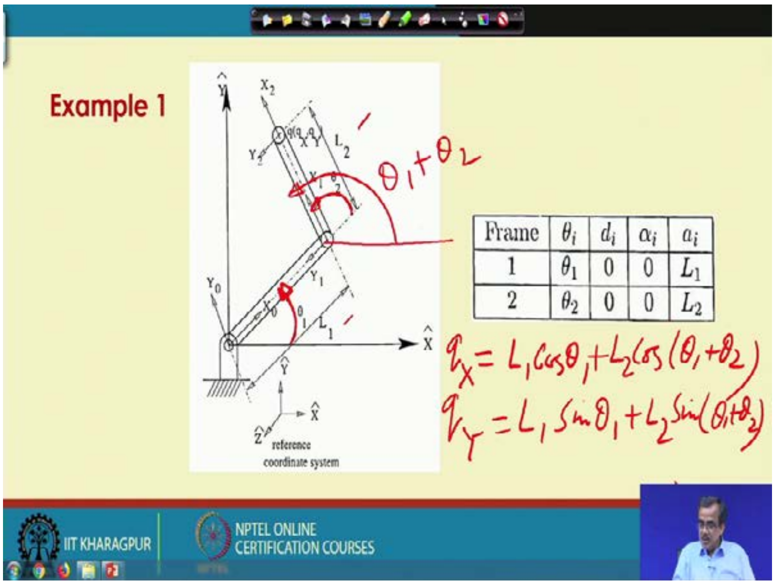

So, this is theta_1, the length of the link is your L_1 and the second link is L_2 and with respect to this particular, joint angle is theta_2.

Now, with respect to X, the total angle with respect to X is nothing but theta_1 plus theta_2. So, this is your theta_1 plus theta_2, now very easily, you can find out using the principle of trigonometry, we can find out the general expression for q_X is nothing but $L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$. Similarly, we can find out this q_Y is nothing but $L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$, this we can we can find out using the principle of trigonometry.

Now, the same thing, we are getting after carrying out this particular analysis. So, we get the same expression, that is your $L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$ and that is nothing but q_X.

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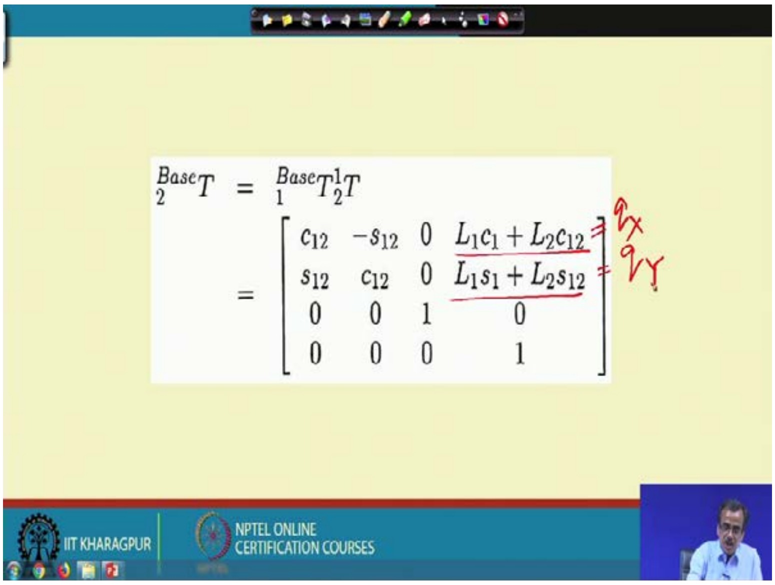

$L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$ is q_Y and q_Z is equal to 0.

The same expression we are getting. Now this particular problem is actually known as the forward kinematics problem.

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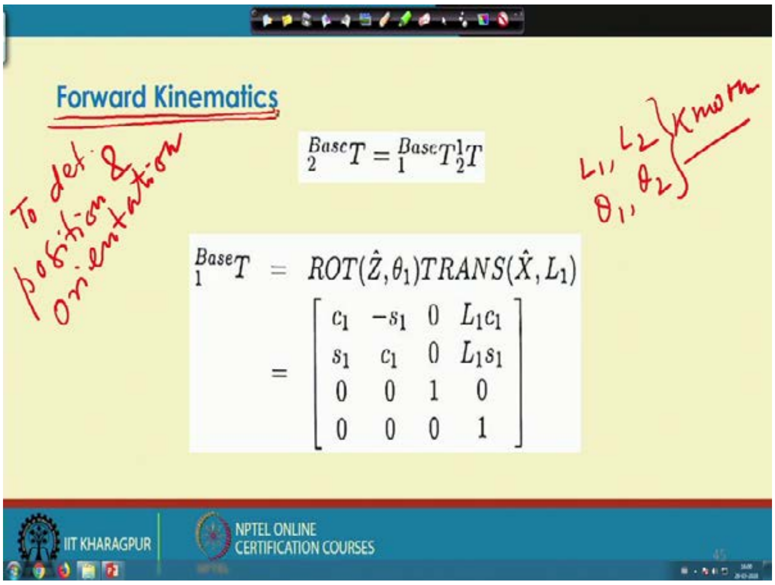

Now, in forward kinematics problem, actually what we do is: our aim is to determine the position and orientation of the end-effector of the robot, provided the length of the links are known and the joint angles are known. So, that is actually the problem of the forward kinematics.

Once again, let me repeat supposing that the length of the links say L_1 and L_2 are known, the joint angles θ_1, θ_2 are known. So, these are known and if these values are known, can I not find out the position and orientation of the end-effector with respect to the base coordinate system of the robot? If I take the physical example, if this is the end-effector and this is my base coordinate system, can I not find out the position and different orientation of this particular end-effector with respect to the base coordinate system?

So, this particular problem is the problem of the forward kinematics. So, the forward kinematics problem we can solve very easily using this principle of Denavit-Hartenberg notation and then, this frame transformation. So, very easily, we can find out, we can solve the problem, that is the forward kinematics problem.

Thank you.

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Lecture - 17
Robot Kinematics (Contd.)

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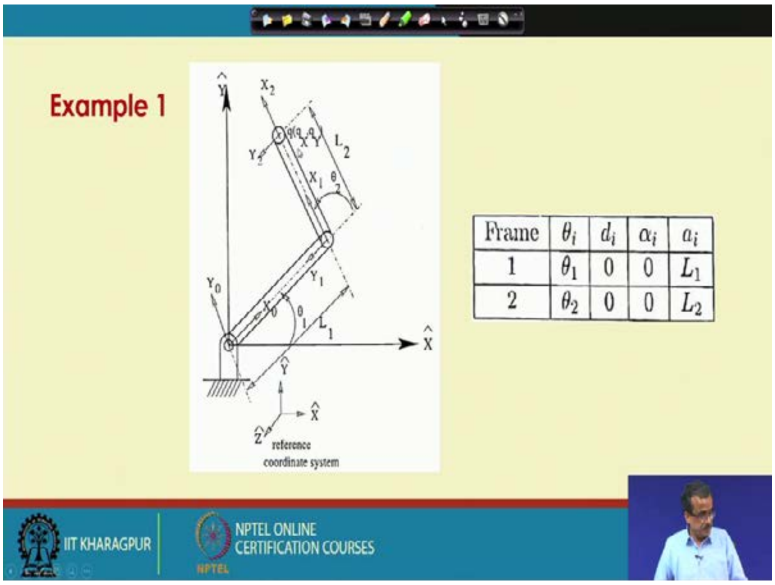

Now, we are going to discuss how to carry out the analysis related to inverse kinematics of this particular 2 degrees of freedom serial manipulator.

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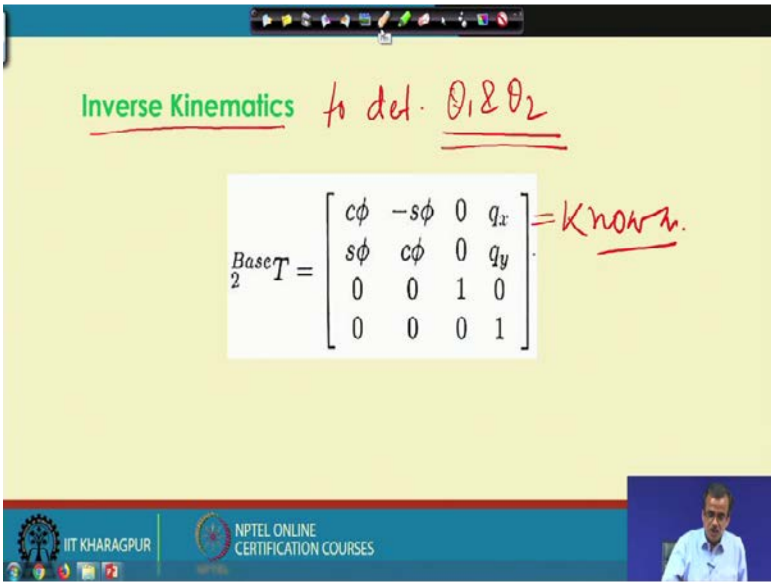

Now, in inverse kinematics, we try to find out the joint angles; that means, the purpose is to determine the joint angles θ_1 and θ_2, provided the position and orientation of the end-effector with respect to the base coordinate frame and length of the links are known.

Now, this particular matrix, that is, T_2 with respect to base carries information of the position and orientation of the end-effector and this particular matrix is known to us. And, our aim is to determine the values for these particular joint angles: θ_1 and θ_2, that is the purpose of inverse kinematics.

Now, let us see how to carry out this inverse kinematics the problem, how to find out the solution.

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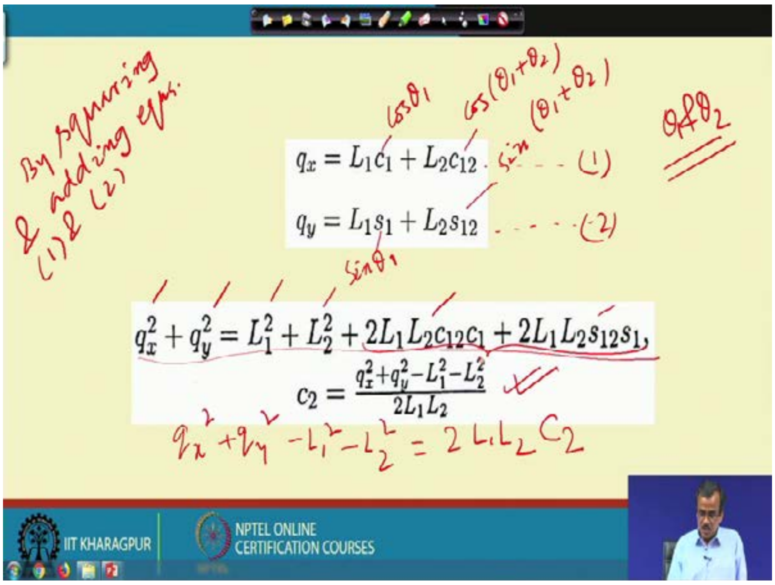

Now, here, if I just compare like this particular q_x and q_y, that is the coordinates of the position of the end-effector with the position terms, that is nothing but $q_x = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$; $q_y = L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2)$. So, we have got two equations. So, there are two equations and there are two unknowns, that is, θ_1 and θ_2. So, we can solve for these two unknowns, θ_1 and θ_2.

Now, let us see, how to solve it, now, this is the very simple set of equation and as I told, there are two unknowns and two equations. So, what we can do is: we can square and add equations (1) and (2). So, by squaring by squaring and adding equations (1) and (2), we get that $q_x^2 + q_y^2 = L_1^2 + L_2^2 + 2L_1L_2c_{12}c_1 + 2L_1L_2s_{12}s_1$.

So, this particular expression we will be getting here. So, this part of this expression can be further simplified and we can write that q_x square plus q_y square minus L_1 square minus L_2 square is nothing but $2L_1L_2$ (cos $(\theta_1+\theta_2)$ cos θ_1+ sin $(\theta_1+\theta_2)\sin\theta_1$). Now, $\cos(\theta_1+\theta_2-\theta_1)$. So, we will be getting cos of θ_2.

Now, from here, we will be getting this particular the expression, that is, $c_2 = (q_x^2 + q_y^2 - L_1^2 - L_2^2)/2L_1L_2$.

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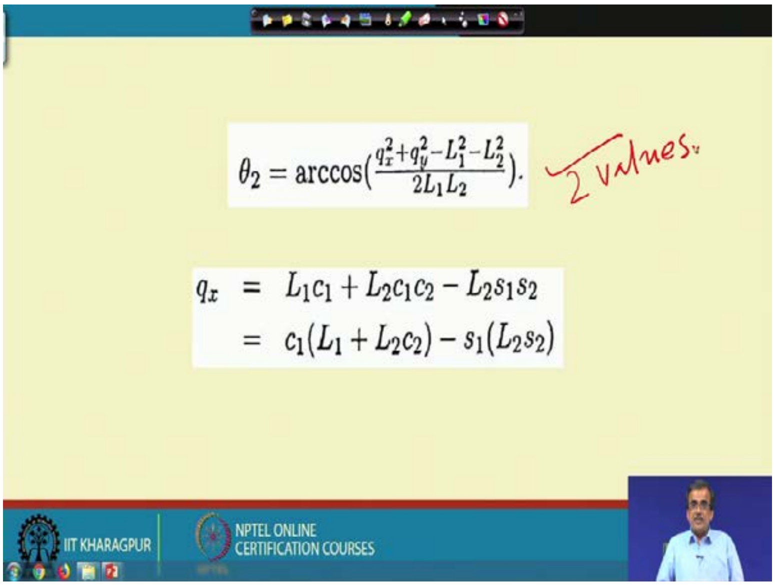

Now, once you have got this particular cos θ_2, very easily you can find out what is θ_2. So, θ_2 is nothing but $\cos^{-1}((q_x^2 + q_y^2 - L_1^2 - L_2^2)/2L_1L_2)$. So, we will be getting two values for this particular θ_2.

So, θ_2 is known, now will have to determine what is θ_1. Now, to determine this particular θ_1, what we do is: we try to concentrate on this equation, that is, equation (1). So, from equation (1) actually, we can find out, we can further write this $q_x = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$.

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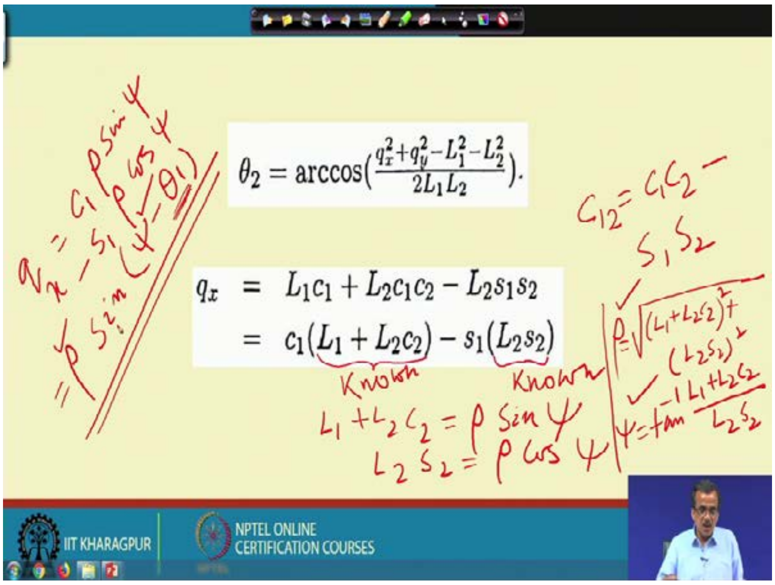

So, $\cos(\theta_1+\theta_2)$ can be written as $\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$; so this will become q_x becomes L_1 cos θ_1 + L_2 cos of θ_1 cos of θ_2 - L_2 sine of θ_1 sine of θ_2 because C_12 is nothing but cos θ_1 cos θ_2 - sine θ_1 sine θ_2.

Now, here, now this particular expression can be rearranged, we can take this cos theta_1 as common and within the first bracket we can write down L_1 plus L_2 C_2 and here I can take S_1 is common and within bracket I can write down L_2 S_2, now here, if you see, this L_1 plus L_2 C_2, C_2 we have already determined theta_2 is known. So, cos theta_2 is known, then L_1 and L_2 are the lengths of the links. So, L_1 and L_2 are known. So, this particular expression is actually known. So, this is known and your cos theta_1 is unknown.

Similarly, L_2 is known sine θ_2 is known. So, this part is known. Now, this known part actually, we can assume that L_1 + L_2 cos θ_2 is nothing but ρ sine ψ and we can take this L_2 cos theta sorry sine θ_2 is nothing but ρ cos ψ, ok?

So, if I just assume like that this known part is nothing but your ρ sinψ and L_2 S_2 is nothing but is your ρ cos ψ. So, very easily, we can write down that q_x is nothing but cos θ_1 multiplied by ρ sinψ, then comes your – sin θ_1 and this is nothing but is your ρ cos ψ, and from here, I can take rho constant and sine ψ cos θ_1 - cos ψ sin θ_1 is nothing but sin (ψ-θ_1).

Now, here, these part $q_x = \rho \sin(\psi - \theta_1)$. Now, from here, actually, we can also find out what is ρ? Now, ρ is nothing but is your squared root of L_1 plus L_2 C_2 square plus L_2 S_2 square. So, the squared root of that. So, this is nothing but ρ and we can also find out, what is ψ. ψ is nothing but tan inverse of L_1 plus L_2 cos θ_2 divided by L_2 sine θ_2, and that is nothing but is your ψ.

So, we can find out ρ, we can find out ψ. So, here rho is known, here ψ is known. So, only unknown is your θ_1. So, from here, directly you can find out by sine inverse or what we can do is we take the help of another equation, that is, your that q_y equation.

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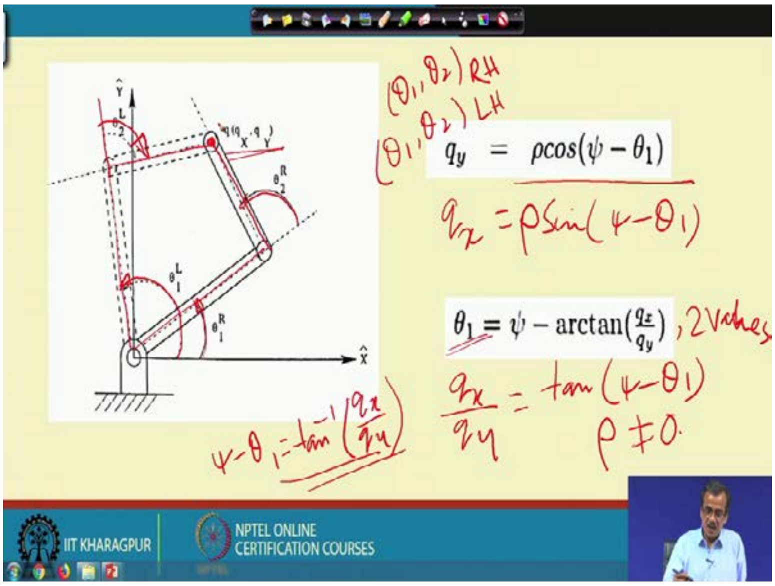

Now, this q_y by following the same procedure, can be written as ρ cos ψ - θ_1 and if we get q_x equals to say ρ sin ψ - θ_1, that we have already seen.

So, from here, we can find out that q_x by q_y is nothing but is your $\tan(\psi - \theta_1)$ and ρ is not equal to 0. So, from here, we can find out that ψ - θ_1 is nothing but tan inverse of q_x by q_y. So, from tan inverse q_x by q_y. We can find out the θ_1, this particular θ_1 is nothing but ψ - tan inverse of q_x by q_y. So, we can find out this particular the θ_1.

Now, once again, we will be getting two values for this particular θ_1; so for this problem of 2 degrees of freedom serial manipulator, there are two sets of theta values, we are getting: one is called θ_1, θ_2 right hand solution, another is called θ_1, θ_2 left hand solution.

Now, if we consider that this particular manipulator will have to reach this particular point, whose coordinates are q_x q_y, this is one configuration with the help of which, it can reach this particular point, and here, this is your θ_1, θ_2 right hand solution.

Now, another solution could be another configuration like this. So, this is L_1, this is L_2, where θ_1 is nothing but this. So, this is my θ_1 and θ_2 will be clockwise. So, θ_2 will be negative. So, we will be getting two sets of θ_1 and θ_2 values; that means for this particular serial manipulator having 2 degrees of freedom, there are two solutions for this inverse kinematics.

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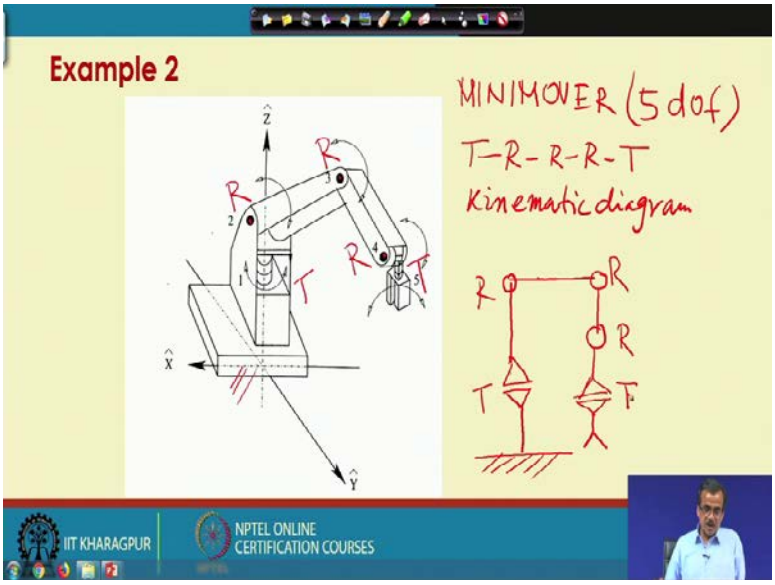

So, one is called the left hand solution, another is called the right hand solution. So, this is the way actually, we can carry out the inverse kinematics and forward kinematics for the serial manipulator. Now I am just going to take the example of a more complex manipulator and this particular manipulator is having actually 5 degrees of freedom and this is a spatial manipulator. So, a spatial manipulator with 5 degrees of freedom, this is nothing but an under-actuated manipulator, as we discussed.

So, this is a manipulator, the name of this particular manipulator is MINIMOVER and it is having 5 degrees of freedom. Now, here, if I just try to understand the nature of the joints, we can see that. So, this is nothing but the fixed base and with respect to the fixed base, here we have got one joint, now this particular joint is actually the twisting joint, then we have got another joint here. So, this is nothing but a revolute joint, the third joint is here this is once again a revolute joint, the fourth joint is here, this is once again a revolute joint and we have got another joint here, the twisting joint and this is nothing but the fifth joint.

So, this particular robot, as we discussed, is known as is T-R-R-R-T manipulator. So, each of these particular rotary joints is each having one degree of freedom and this is the serial manipulator, thus, it is having 5 degrees of freedom. Now, if you draw the kinematic diagram. So, let us try to draw the kinematic diagram of this particular manipulator. We start with the fixed base. So, this is the fixed base, the first joint is the twisting joint. So, this is the symbol for the twisting joint. So, let us draw this twisting joint.

The second joint is the revolute joint. So, this is actually the revolute joint, the third joint is once again a revolute joint, the fourth joint is once again a revolute joint and the fifth joint is nothing but a twisting joint. So, this is nothing but a twisting joint and this is the symbol for the gripper. So, we have got twisting joint, revolute joint, revolute joint, revolute joint and this is the twisting joint. So, this is nothing but the kinematic diagram of this serial manipulator having 5 degrees of freedom.

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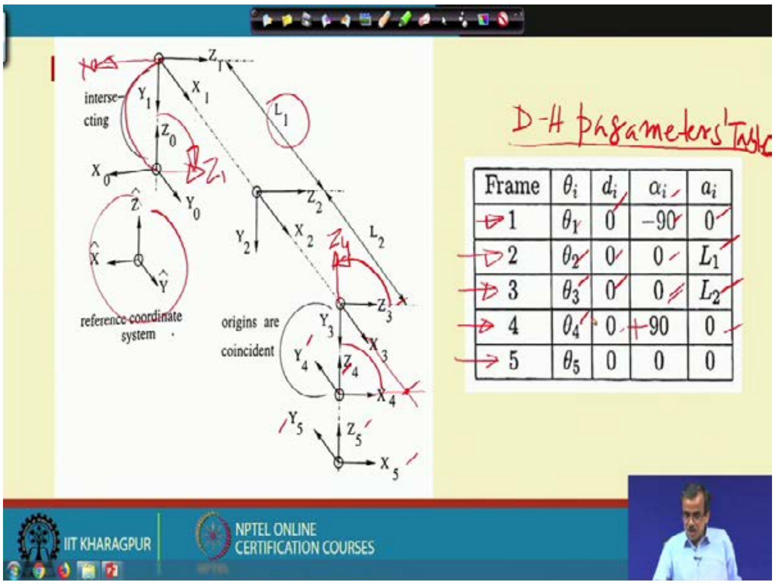

Now, here, actually what we do is, we try to assign the coordinate system at the different joints according to the D-H parameters' setting rule. Now, as I told that we will have to see the reference coordinate system, at first. So, if you see the reference coordinate system. So, this is X ,Y and Z and Z × X is nothing but Y.

Now, if you see the first joint, the first joint is nothing but a twisting joint and for these twisting joint, Z is what? So, this is my Z direction, and that is why, we have considered. So, this is your Z_0 and this X_0, I have consider along this particular direction, in the direction of this reference X reference coordinate X and Y_0 is in this particular the direction.

So, this indicates actually the coordinate system at the twisting joint and if you see the second joint is actually a revolute joint and this particular Z and that particular Z are going to intersect and that is why, actually, we have written. So, as if this particular and that particular Zs are intersecting, and this is the Z about which we take the rotation for this revolute joint.

Now, if this is Z and they are intersecting, now according to the D-H parameters' setting rule, X could be either this one or it could be this one; that means Z_1 is selected, now X could be either this direction or that particular direction. So, what we do is, we will have to select X in such a way, that we can show the length of the link because the next joint if you see is once again a revolute joint and the Z here and the Z here are parallel, and if they are parallel, their mutual perpendicular distance is going to be the direction of X and that is nothing but the length of the link and that is why, we have considered, this is your Z_1 and this we have selected as X_1.

Now, if this is selected as X_1. So, $Z \times X$ is nothing but Y_1. So, this is your Y_1, ok? The third joint is once again a revolute joint. So, its Z will be parallel to Z_1. So, this is your Z_2 and if you see this particular the manipulator; actually the fourth joint is once again a revolute joint. So, here, this particular Z and that particular Z should be parallel and that is why. So, if this Z and that particular Z are parallel. So, the mutual perpendicular distance should be the X and that is why, we have considered this, as X_2. So, these are Z_2 and X_2 and $Z \times X$ is nothing but Y_2, as I discussed. So, here, this is once again a revolute joint.

So, I have taken Z along this, X_3 along this, now $Z \times X$ is nothing but Y_3. So, Z_3 X_3 and Y_3 we can find out, now,if you see the next joint, the next joint is nothing but a twisting joint and the Z here for the fourth joint. So, the Z for this particular fifth joint, that is the twisting joint, they are going to intersect, now, if the two Zs are intersecting, I have got both the options. So, Z is selected because this is the twisting joint. So, Z_4 is selected here. So, this is my Z_4.

Now, regarding the X_4; as this particular Z_3 and Z_4 are intersecting. So, X_4 could be either this particular direction or this particular direction. So, anyone can be taken as X_4, and here, I have taken this as X_4 and if this is taken as X_4 then $Z \times X$. So, $Z \times X$ will be or Y_4. So, this is your Y_4. And, as I told that the same coordinate system will be copied at the last, that is my Z_5, this is my X_5 and this is Y_5.

Now, this completes actually the assignment of these particular the coordinate system. Now, here, one thing I just want to mention that this is not the only way of showing this type of coordinate systems at the different joints, in some of the textbooks, they follow a slightly different method that method is something like this. So, at each of the rotary joint there will be some rotations. So, what they want to do is they show the rotation even while showing that particular coordinate system.

For example, here, there will be some rotation, now if I show that particular rotation, this particular X_0, Y_0, Z_0 will be rotated and here, once again, there is some rotation and if I once again show that particular rotation, then it will be further rotated.

So, each of this particular coordinate systems, will be rotated on this particular figure, then it becomes difficult to visualize actually, the coordinate systems assigned at the different joints. That is why, actually, this is the better way of representing, where we show the coordinate system at each of these particular joints. So, this particular reference coordinate system is followed at each of these particular joint and wherever we have some rotation. So, that particular rotation we are going to show while preparing the D-H parameters' table.

So, in the D-H parameters' table, we show all such rotations, now let us see how to fill up; this table, the D-H parameters' table. Now, to fill up, actually, what we do is: first, we concentrate on the frame_1 and as I told that while preparing this particular table, we will have to follow the screw Z and screw X; that means, I will have to write θ_i first, then d_i, then α_i after that a_i.

So, what I do is: the first joint that is this particular twisting joint is a rotary joint. So, definitely the variable will be the joint variable and this particular joint variable, I have considered as θ_1. Now, here, I just want to say that here I can write down θ_1 or if you just draw this particular X_0 here, if I draw the X_0 here, let me just draw it X_0 here. So, if I draw the X_0 here, the angle between X_0 and X_1 is already 90 degree. So, what you can do is in place of θ_1 you can also write down 90 plus θ_1.

Now, if I write 90 plus θ_1, then this particular θ_1 could be the acute angle, but here for simplicity, whatever I am doing, I am writing the whole thing as θ_1; that means, this particular θ_1 will be more than 90 degrees. So, what in place of this 90 plus θ_1, for simplicity, I am writing this as θ_1 so, that θ_1 is actually your obtuse angle.

Now, next is your how to find out this particular d. According to the definition of d, d is the offset and that is nothing but the distance between two X, the distance between two X measured along Z. Now, these two are coinciding they are in fact, intersecting. So, if they are intersecting then the distance between X_0 and X_1 is nothing but 0, then we try to find out alpha.

So, by definition α is nothing but the angle between two Zs measured about X. So, if I draw this particular Z_1, this is my Z_1; so Z_0 to Z 1. So, I will have to move in the clockwise direction by 90 degrees, clockwise we have considered negative, so, this is nothing but negative 90 degrees, then comes a_i. a_i is the distance between two Zs, if two Zs are parallel, we try to find out the mutual perpendicular distance.

Now, they are intersecting. So, the distance between Z_0 and Z_1, that is nothing but 0. So, I can fill up actually all the entries corresponding to frame_1 now, we can concentrate on this particular 2. Now 2 means what? 2 means 2 with respect to 1, now here. So, once again the joint is nothing but a revolute joint. So, the variable is theta_2 and d is the distance between two Xs. So, X_1 and X_2 are on the same line. So, the distance is 0, then comes here, alpha Z_1 and Z_2 are parallel. So, the angle between them is 0, then comes here a. So, if these two Zs are parallel, Z_1 and Z_2. So, X is along the mutual perpendicular distance and this is actually L_1. So, L_1 is nothing but the length of the link. So, this is your L_1.

Now, 3 means 3 with respect to 2 now, here. So, this particular joint is a revolute joint. So, the variable is θ_3, then comes d, which is the distance between two Xs. So, X_2 and X_3 are on the same line. So, this is 0, then comes your alpha is the angle between two Zs, Z_2 and Z_3 are parallel. So, this is 0, now then comes here this "a", that is, Z_2 and Z_3 are parallel and this is the mutual perpendicular distance. So, the length of the link is L_2, then comes 4, that is, 4 with respect to 3. So, once again, this particular joint is a revolute joint. So, the variable is θ_4.

The next is d, d is the distance between the two Xs. Now, here, you can see if I extend this particular X_3, it is going to intersect the X_4. So, the distance between them is equal to 0. Next is the alpha, that is the angle between Z_3 and Z_4. Now, here, if I just draw this particular Z_4. So, this is my Z_4 and Z_3 to Z_4 if I want to move, I will have to move in the anticlockwise direction by 90 degrees. So, this is nothing but positive 90, then comes your "a'. "a" is the distance between two Zs and two Zs are actually intersecting and coinciding.

So, this particular distance is nothing but is your 0 and then, comes here, 5. So, 5 with respect to 4 and this is a twisting joint. So, the joint angle is nothing but your θ_5 and the other things will be equal to 0. So, this is the way, actually, we will have to prepare the D-H parameters' table, that is, Denavit-Hardenberg parameters' table. And, once you have got this particular table, carrying out the forward kinematics becomes very easy.

Thank you.

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Lecture - 18
Robot Kinematics (Contd.)

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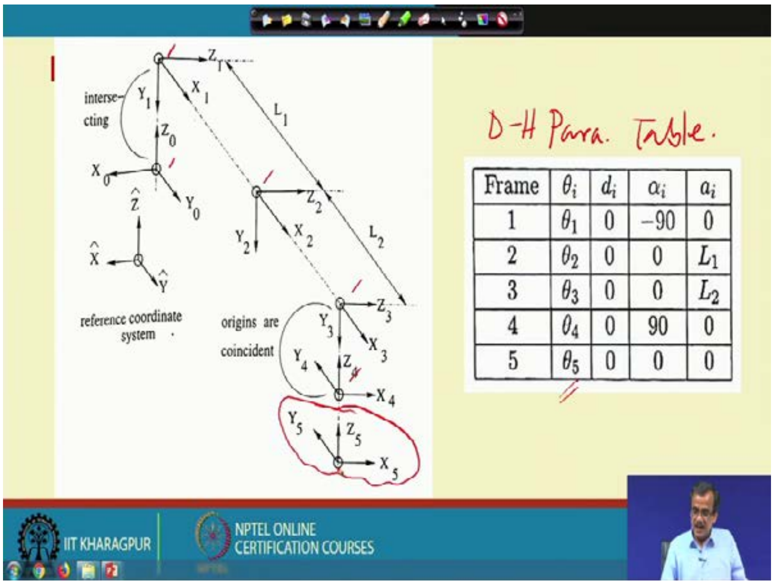

Now, this is the DH parameters' table, which we got for this MINIMOVER. Now as I mentioned earlier. So, we have got 5 joints here, the first joint, that is denoted by this, second joint is denoted by this, third joint, fourth joint and fifth joint and this particular joint is an extra, whatever coordinate system I am showing it here. So, this is hypothetical and what we do is, exactly whatever we have like your Z_4, X_4 and Y_4, the same thing I have just copied it here and this is hypothetical, and this is required just to define this particular joint angle.

In fact, this particular coordinate system has got no physical existence, this is an imaginary coordinate system, it is attached at the end, otherwise. So, first, second, third, fourth, fifth joints. So, all 5 joints I have assigned the coordinate system. So, this is the purpose of attaching this hypothetical extra coordinate system at the end.

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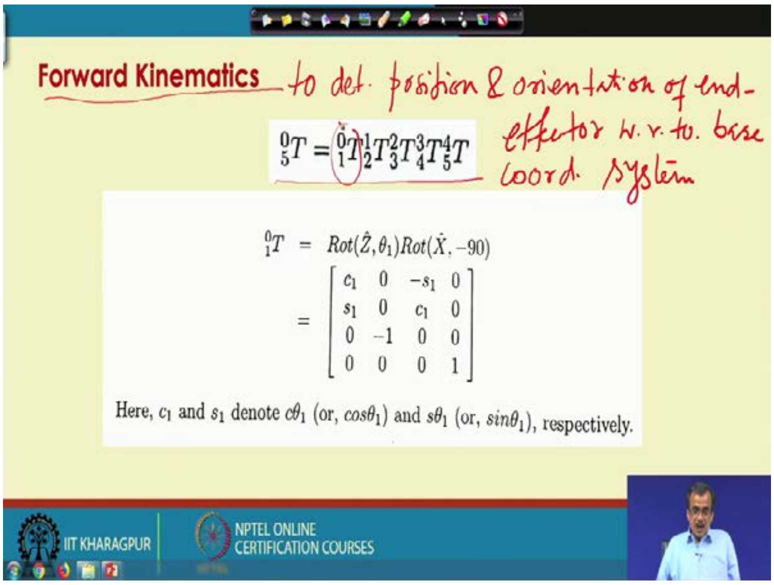

Now, I am just going to start with the forward kinematics of this particular the manipulator. Once again for the forward kinematics, the purpose is the same to determine the position and orientation of the end-effector with respect to the base coordinate system, provided the length of the links and the joint angles are given or they are known.

So, let me write here, the purpose is once again to determine the position position and orientation of end-effector; that is, end-effector with respect to the base coordinate system, provided the length of the links and the joint angles are known. So, this is the purpose of this particular the forward kinematics.

Now, let us see how to carry out this particular the forward kinematics, it is exactly the same way, we did for the 2 degrees of freedom serial manipulator. So, our aim is to determine T_5 with respect to 0, that is transformation matrix of 5 with respect to 0, that is an end-effector with respect to the base coordinate frame and this is nothing but T_1 with respect to 0 multiplied by T_2 with respect to 1 multiplied by T_3 with respect to 2 multiplied by T_4 with respect to 3 multiplied by T_5 with respect to 4.

Now, we will try to find out each of these particular the transformation matrix. Now what you will have to do is. So, you will have to go back to this particular the DH parameter table. Now, if we just go back to the DH parameter table. So, I am just going to concentrate here, now here, if you see we have got one rotation here and another rotation here. Now this particular rotation is rotation about Z by an angle θ_1 and this is the rotation about X by an angle minus 90. And, as I told that, you follow this particular sequence like your screw Z and screw X that particular rule and if I just follow that. So, very easily I can I can write down that, T_1 with respect to 0 is nothing but rotation about Z by θ_1 then rotation about X by minus 90. And, we know, this rotation matrix in 4 × 4 that is nothing but the rotation about Z by an angle θ_1.

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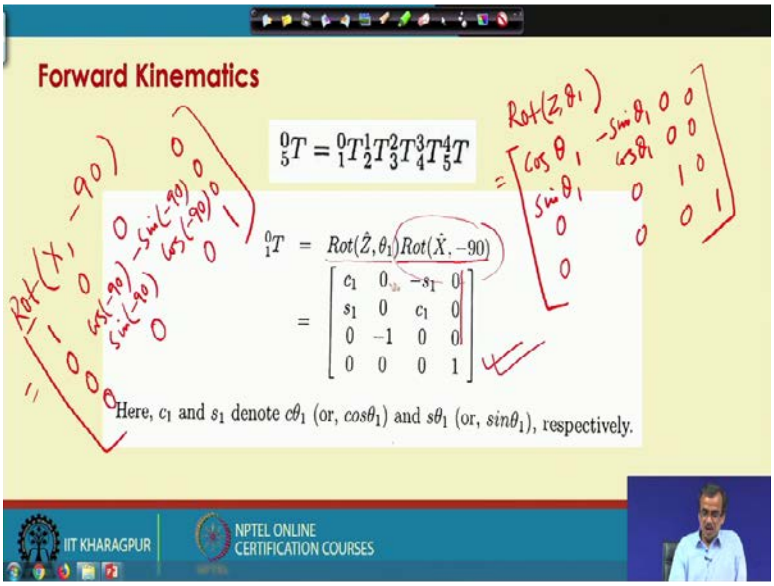

So, in terms of 4 × 4. So, we can write down that is nothing but $\cos\theta_1, -\sin\theta_1, 0, 0$; then comes your $\sin\theta_1, \cos\theta_1, 0, 0$; then comes 0, 0, 1, 0; 0, 0, 0, 1 and this is here there is no position term. So, these things are all 0. So, this is nothing but rotation about Z by an angle θ_1, similarly this rotation about X by an angle - 90. So, we can write down. So, rotation about X by - 90 is nothing but. So, rotation about X is 1, 0, 0, 0; then comes 0, cos of - 90 then comes $-\sin(-90)$ then comes 0; then comes 0, 0, 0, 1 and the position terms are all 0.

So, this is nothing but that 4 × 4 matrix, now these 4 × 4 matrix, we will have to multiply. Now, if I multiply this particular matrix with this. So, I will be getting this particular the 4 × 4 matrix, that is nothing but cos θ_1 denoted by c_1, 0, $-\sin\theta_1$, that is denoted by - s_1, 0; s_1, 0, c_1, 0; 0, -1, 0, 0; 0 0 0 1. So, this particular final matrix, you will be getting corresponding to this particular 4 sorry ${}^0_1T$.

Now, here, if you see: I have multiplied two rot matrices and finally, I am getting one pure rotation matrix and here the position terms are all zeros. That means, this is the pure rotation matrix and if I want to find out the inverse of this particular matrix very easily I can find out because inverse of this particular matrix is nothing but the transpose of this matrix. Now, if I try to find out the transpose of this particular the matrix and that is nothing but is your ${}^0_1T^{-1}$.

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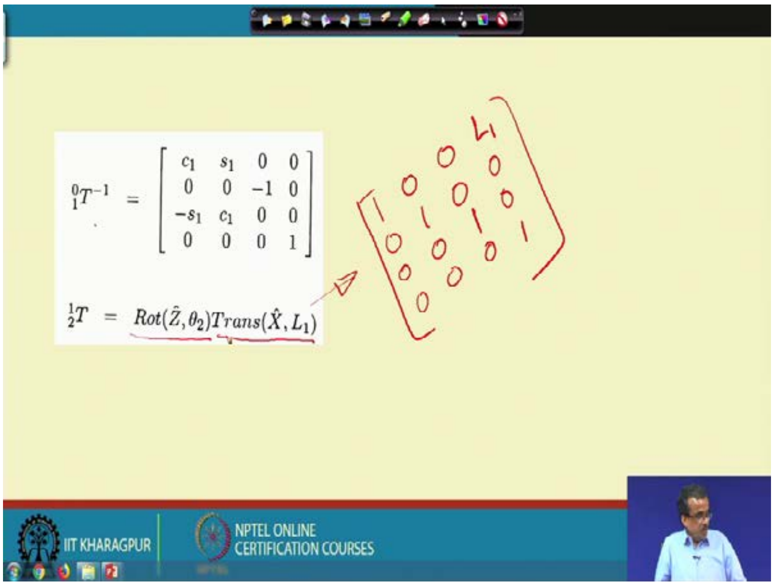

So, whatever was the row, so that became the column. So, first row became first column, second row became second column, and so on. So, we will be getting the inverse of this particular T_1 with respect to 0. So, this is nothing but the inverse. So, in future, we may need this particular the inverse, we will see that how to utilize this particular the inverse, now, the next is your T_2 with respect to 1. So, what I will have to do is once again I will have to go back to I will have to go back to the DH parameter table, and I will have to concentrate on this particular like 2 with respect to 1. So, I have got θ_2 here and I have got L_1 here.

So, this θ_2 is nothing but the rotation about Z and this particular L_1 is nothing but the translation along X ok. So, this thing I am just going to write it here just to find out ${}^1_2T$.

So, ${}^{1}_{2}T$ is nothing but rotation about Z by θ_2 then translation along X by L_1 and once again I know the expression and for this particular the translation along X by L_1, the 4 × 4 matrix is very simple and this will be your the rotation terms will be just identity matrix.

So, this is the rotation term, then 0 0 0 1 and this is translation along X is L_1 0 0 and we know the expression that is rotation about Z by θ_2 and those two 4 × 4 matrices, if I just multiply, then we will be getting your like this particular expression that is ${}^{1}_{2}T$, so, this is the expression, which we will be getting.

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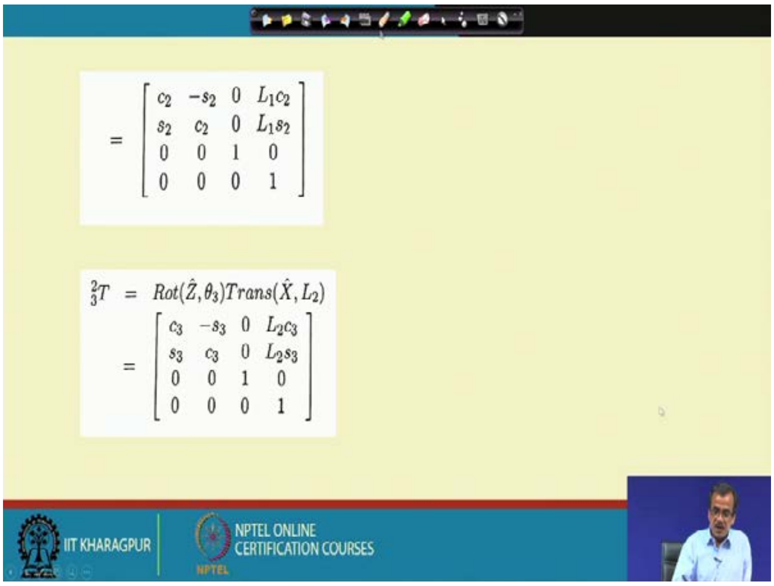

Following the same procedure; so, we can also find out what is this ${}^{2}_{3}T$ and if you see that DH parameters table. So, in the DH parameters table, let me once again go back. So, I am just going to find out T_3 with respect to 2. So, I will have to concentrate here. So, I have got theta_3, I have got L_2. And, now, corresponding to this what we can do is very easily we can find out the T_3 with respect to 2 is nothing but rotation about Z by theta_3 translation along X by L_2 and I will be getting these two 4 × 4 matrices.

And if you multiply you will be getting this particular the matrix that is T_3 with respect to 2.

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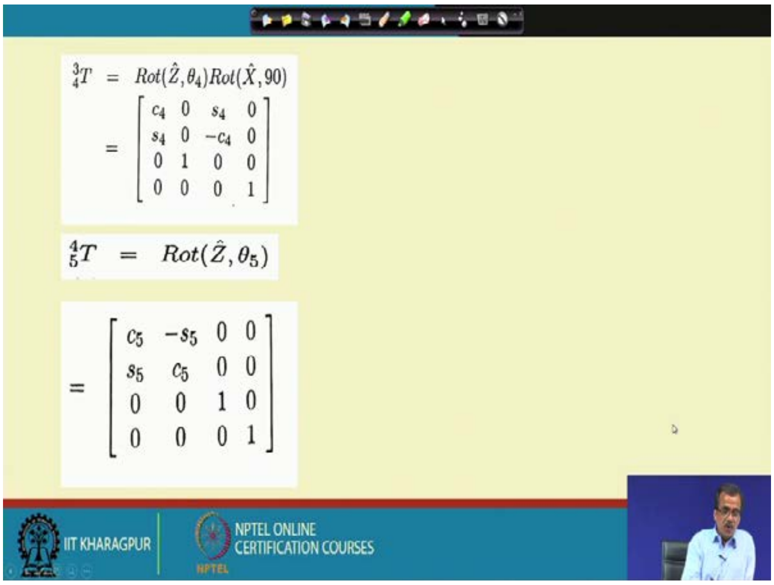

Now, is your T_4 with respect to 3, from the DH parameters' table, we can find out this is nothing but is your rotation about Z by θ_4 rotation about X by 90. So, we know the expression and if we multiply then we will be able to find out. So, this particular ^3_4T ; and by following the same method we can also find out ^4_5T and this is nothing but rotation about Z by θ_5 . So, this is the expression for is your ^4_5T .

Now, here, once you have got these particular all the expressions the individual expressions for transformation matrix now, you can multiply.

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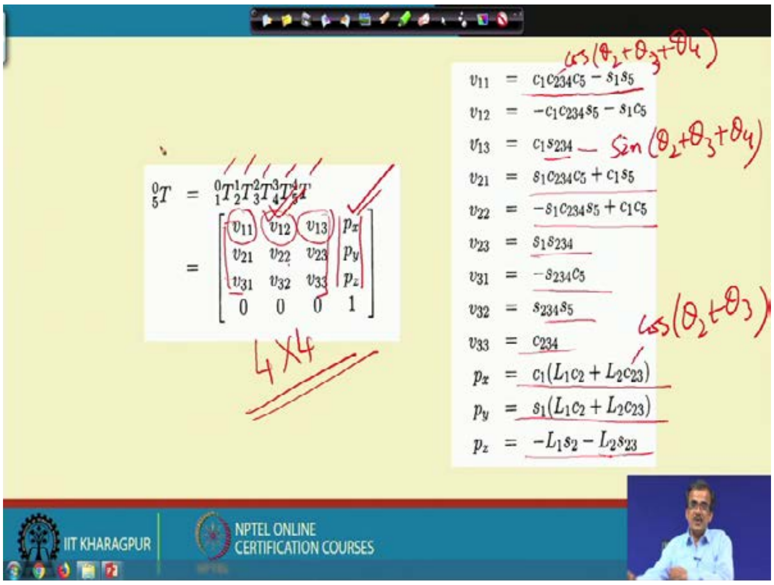

And if we multiply that is ^0_5T, if you want to find out that is nothing but is your ^0_1T then ^1_2T, ^2_3T, ^3_4T, ^4_5T and this particular the 5 transformation matrices if you multiply then ultimately I will be getting one 4 × 4 matrix like this.

Now, this particular matrix actually is going to carry information of the position and orientation of the end-effector with respect to the base coordinate pair now here. So, this particular p_x p_y and p_z. So, this is going to carry the position information and your orientation information that is denoted by a actually your. So, this particular the 3 × 3 matrix. So, this is these are actually the elements of the rotation terms. Now if you see the expression for each of these particular terms. So, we will be getting a very complex expression.

For example, say if you see, this particular v_1, that is, the first term the first element of the rotation matrix. So, you will be getting the expression like c_1 c_234 c_5 - s_1 s_5. Now, by this c_234 we mean $\cos(\theta_2 + \theta_3 + \theta_4)$ then c _5 is $\cos\theta_5$, c_1 is $\cos\theta_1$, s_1 is $\sin\theta_1$ and s_5 is $\sin\theta_5$.

Similarly, if you see this particular v_12; so here once again c_234 will come then v_13 we have got c_1 s_234 and s_234 is nothing but $\sin(\theta_2 + \theta_3 + \theta_4)$, then v_21 is this particular expression, then v_22 is this particular expression v_23 is this v_31 is this, v_32 is this, and v_33 is this.

Now, if you see the position term: p_x, p_x is nothing but $\cos\theta_1$ multiplied by L_1 $\cos\theta_2$ + L_2 $\cos(\theta_2+\theta_3)$. So, this is actually the expression for this particular the p_x and similarly we can find out. So, this particular p_y is $\sin\theta_1$ multiplied by L_1 $\cos\theta_2$ + L_2 $\cos(\theta_2+\theta_3)$. Then, p_z is nothing but - L_1 $\sin\theta_2$ - L_2 $\sin(\theta_2+\theta_3)$. So, such a big expression you will be getting for this particular the final matrix.

Now, you see. So, this particular manipulator is having only 5 degrees of freedom, now if I consider an ideal spatial manipulator like say PUMA, which is having 6 degrees of freedom. So, this particular expression will be even more complex and another term will come that is θ_6 and more complex expression will be getting for each of these particular elements of this 4 × 4 matrix, which is nothing but the position and orientation information of the end-effector with respect to the base coordinate frame.

And, this is actually the purpose of the forward kinematics. So, once again, let me repeat in forward kinematics, we try to represent the position and orientation of the end-effector of the manipulator with respect to the base coordinate frame provided we know, the length of the links, that is, L_1, L_2 and all such things we know and all the joint angles like $\theta_1, \theta_2, ..., \theta_5$ are known, then only we can find out the position and orientation information of this particular the end-effector with respect to base.

Now, here, once again let me just try to take one physical example, supposing that say this is my serial manipulator. So, this is the end-effector and supposing that this is the fixed. So, the fixed base; so I have got a number of joints, number of links and this is actually the serial manipulator. And, with the help of this manipulator, supposing that I am just going to manipulate this particular object. So, it is something I want to grip it, it is something like this I want to grip it, it is something like this I want to grip it.

So, I am just going to grip this particular object. So, I should know the position and orientation and accordingly, I will have to orient this particular end-effector. So, that I can grip this particular object and I can do this particular the manipulation task.

So, in forward kinematics, we try to find out the position and orientation of this particular end-effector with respect to base, but remember we can also think in the reverse direction like for example; say if I know this with respect to base, can I not find out the base with respect to this.

Now, let me take a very simple example, if I know this particular joint with respect to this, I should also be able to find out, so this particular thing with respect to this and vice-versa, that is with respect to this. So, we should know the information in both the direction. That means this particular transformation matrix has to be invertible.

Thank you.

Robotics
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Lecture – 19
Robot Kinematics (Contd.)

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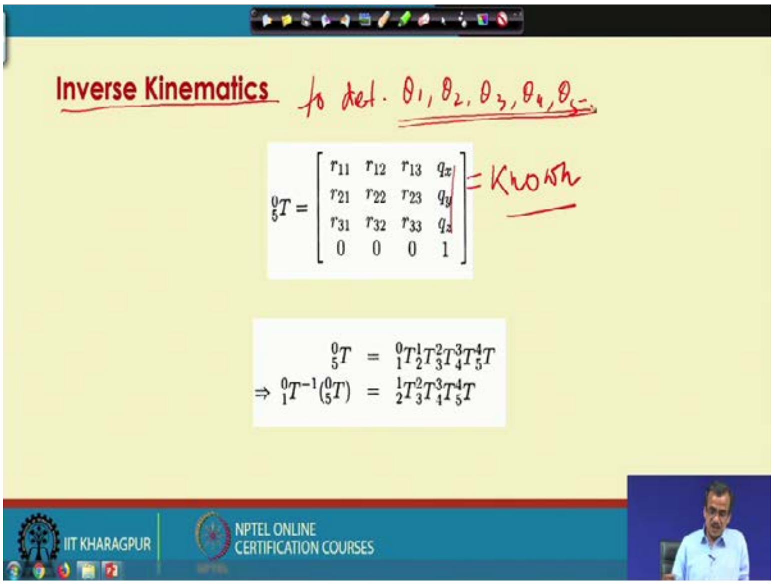

Now, let us start with the inverse kinematics of this particular manipulator, that is, the MINIMOVER. Once again, the purpose is to determine the values of the joint angles like your $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ provided the position and orientation of the end-effector with respect to the base coordinate system are known.

Now, here, this particular ${}^0_5T$, is nothing but r_11, r_12, r_13, so on. So, these are 3 × 3 the rotation or orientation information. And, q_x, q_y, q_z is nothing but the position information. Now, supposing that this particular matrix is given. So, this is known, ok? And, our aim is to determine these particular $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$. Now, here, actually what you can do is, in the forward kinematics calculation like whatever we have already discussed.

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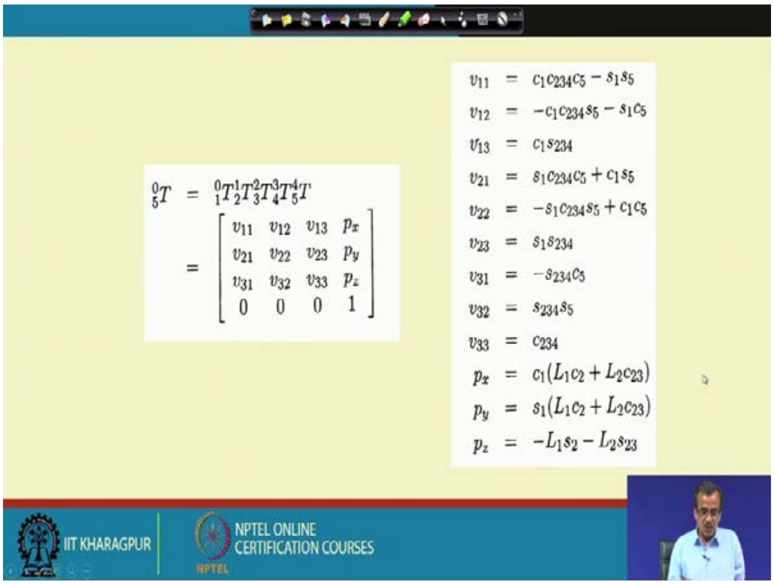

For example, so this particular we have already got the final matrix and each of these elements of this particular final matrix is having a very big expression. Now, here, this particular calculated matrix, that is, ${}^0_5T$.

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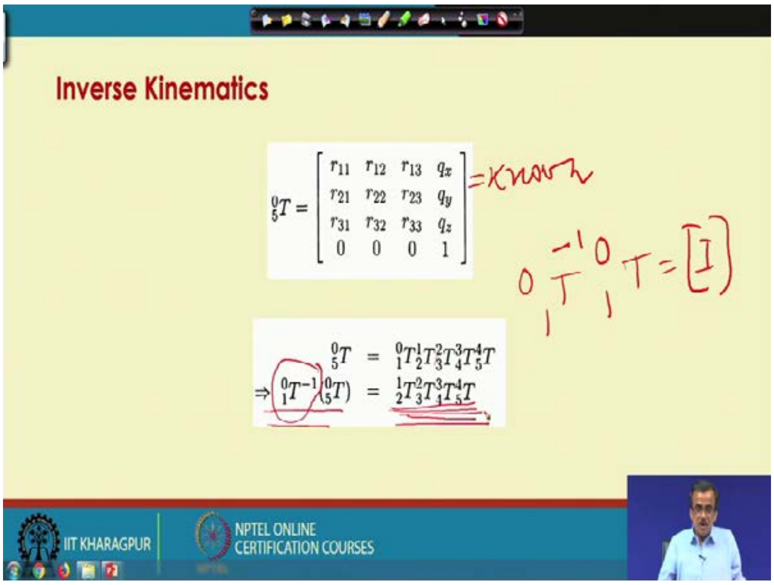

Now, if I equate it to the known matrix, so this particular known matrix, I will be able to find out some equations. And, we have got five unknowns. Now, for the five unknowns, if you want to solve we need at least five equations, but here, in fact, we will have to use more than five equations, otherwise, we will not be getting all five joint angle values.

Now, if I just equate element-wise these two matrices like the matrix which you have got through forward kinematics calculation and this particular known matrix. So, if I just equate element-wise, we will be getting some equation, but it could be difficult to solve those equations to find out the values of $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$. And, that is why, actually we are going to use a particular trick, a particular method like to simplify, so that we can get the equation in such a form that can be solved easily.

Now, here, actually what we do is. So, from forward kinematics, we know that ${}^0_5T$ is nothing but ${}^0_1T$ multiplied by ${}^1_2T$, ${}^2_3T$, ${}^3_4T$, ${}^4_5T$. Now, what I do is, both the sides we multiply by ${}^0_1T^{-1}$. Now, if you remember, we have already determined that particular matrix, that is, ${}^0_1T^{-1}$. So, both the sides, both the left hand side and the right hand side, what we do is, we try to we multiply by ${}^0_1T^{-1}$.

Now, if I multiply, here, like ${}^0_1T^{-1}$ with your ${}^0_1T$. So, I will be getting the identity matrix, that is, [I], ok? So, this particular identity matrix [I] will be getting and that is why, left hand on this particular side, we will have ${}^1_2T$, ${}^2_3T$, ${}^3_4T$, ${}^4_5T$, and we will be getting this particular expression.

Now, what I am going to do is, I am just going to find out, the final expression for this side and the final expression for that particular side. Let us see how to find out those expression, and then, we are going to equate just to find out like what should be the two sides of this particular equation, and then, element-wise, we are going to equate.

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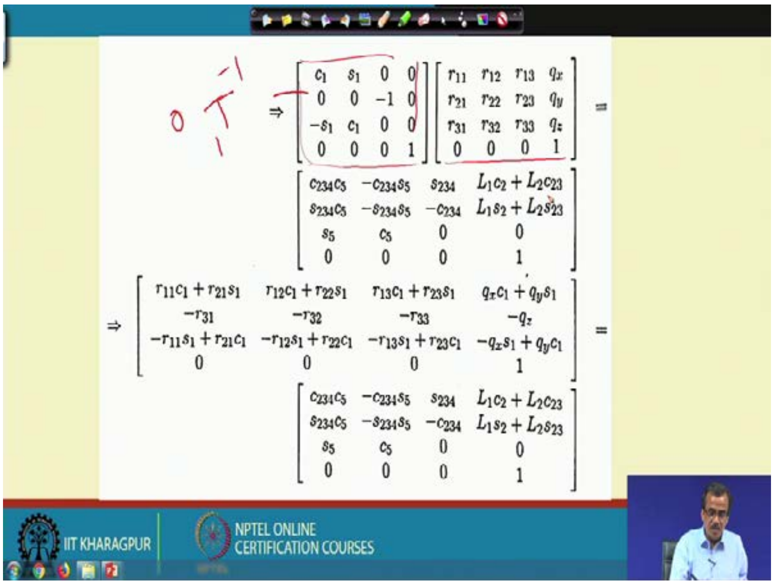

Now, left hand side, if you see, so what we will can see this is nothing but your ${}^{0}_{1}T^{-1}$. So, this particular matrix is ${}^{0}_{1}T^{-1}$, if you remember, this is actually the matrix. And, this is the known matrix, that is, r_11, r_12, r_13, q_x; ………….. So, this is nothing but the known matrix.

So, these two matrices, now, we are going to multiply. And, on the right hand side, what we are going to write is, on the right hand side, if you see, so we have got these ${}^{1}_{2}T$, ${}^{2}_{3}T$, and then, comes your ${}^{3}_{4}T$, ${}^{4}_{5}T$. And, each of these particular expressions, we have already seen, while carrying out that forward kinematics calculations.

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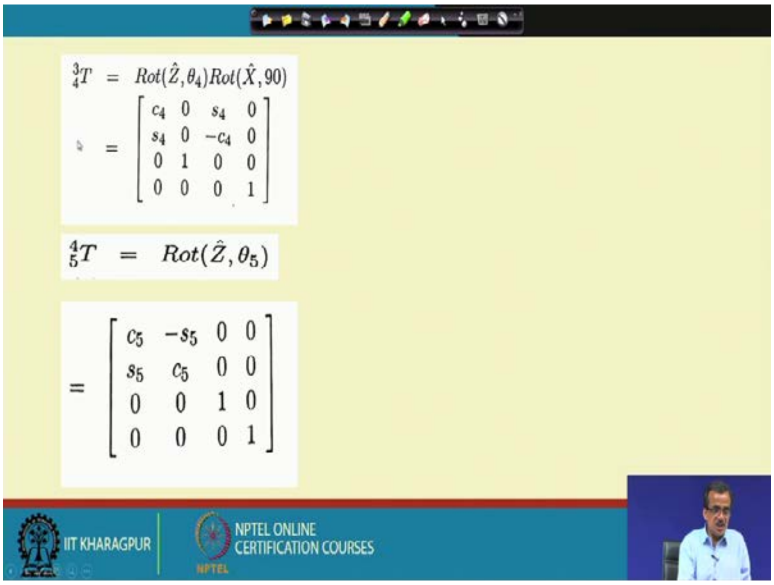

For example, if you want to have a look. So, this is my ${}^{4}_{5}T$ or say this is ${}^{3}_{4}T$, and so on. So, what you will have to do is here we have got four such matrices each having 4 × 4 dimensions and if you multiply them, then, I will be getting, finally, one 4 × 4 matrix.

Now, if I just go, so we will be getting such a big expression and that is nothing but this 4 × 4 matrix. So, this 4 × 4 matrix will be getting. And, on the other side, so we have got this 4 × 4 matrix. And, if you multiply, you will be getting this particular 4 × 4 matrix. And, on this side, we have got the same thing, as I have written it here. So, once again, let me repeat this particular matrix, if you multiply with this, so I will be getting, this particular matrix.

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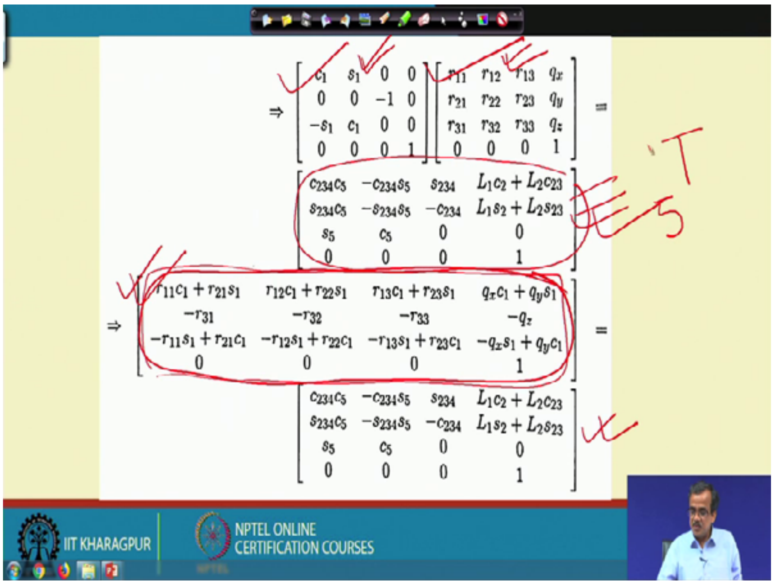

And, this is nothing but is your ${}^{1}_{5}T$, that is, your so this particular thing like your this is nothing but what ${}^{1}_{2}T$; ${}^{2}_{3}T$.

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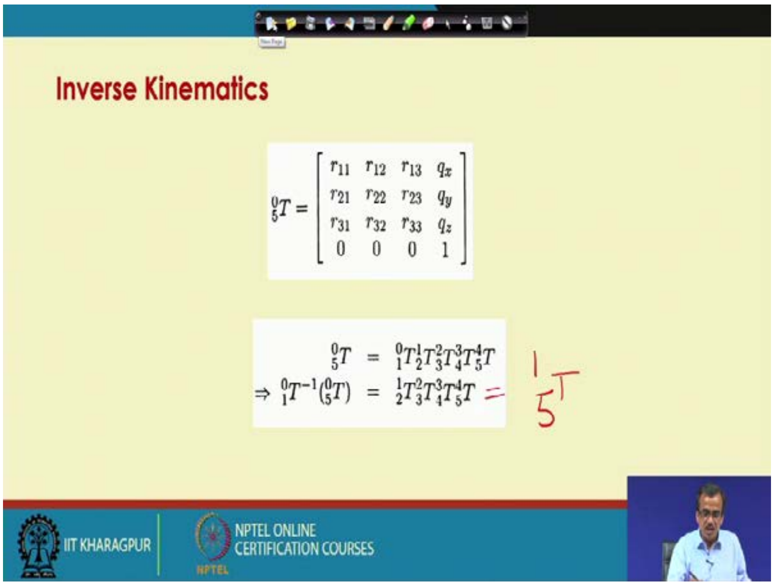

So, this can be written as your ${}^{1}_{5}T$. So, this is ${}^{1}_{5}T$. So, this ${}^{1}_{5}T$, I am just going to write it here. So, this is nothing but your ${}^{1}_{5}T$. So, this is ${}^{1}_{5}T$. So, the this side I am getting these two, so this particular matrix I am getting; and this side I am getting this particular matrix. Now, element-wise, I am just going to equate, ok? For example, first row first column, this particular element will be put equal to this, if it is required, if it is going to help us similarly. So, let us concentrate on these particular position terms. So, I can equate to this, I can equate to this, then, this I can equate to this, ok? So, by equating these particular elements, we will be able to find out the different terms, the different equations rather.

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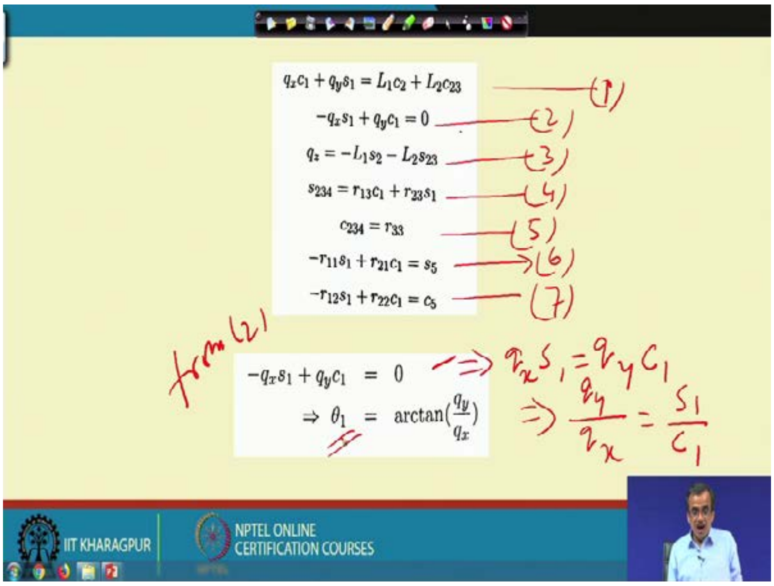

Now, if I just equate, I will be getting a set of equations. For example, say if you see the position term, $q_x c_1 + q_y s_1 = L_2 c_2 + L_2 c_{23}$ And, this is nothing but is your equation (1). Similarly, the second position term, if you see that is your $-q_x s_1 + q_y c_1 = 0$. So, you will be getting equation (2). Then, the third position term if we equate, we will be getting equation (3).

And, now that orientation term or the rotation term, we will have to equate. And, if we equate then we can find out s_234 equals to this and c_234 is equals to this. So, what I can do is I am just going to find out s_234 and c_234. So, this is nothing but is your s_234, and this is c_234. So, this has to be equated to s_234 should be equal to this; c_234 should be equal to r_33, ok? And, then we can also find out another expression $\sin\theta_5$ and $\cos\theta_5$. So, this particular $\sin\theta_5$ will be made equal to this, ok; and this $\cos\theta_5$ will be made equal to this, ok. And, position terms we have already equated.

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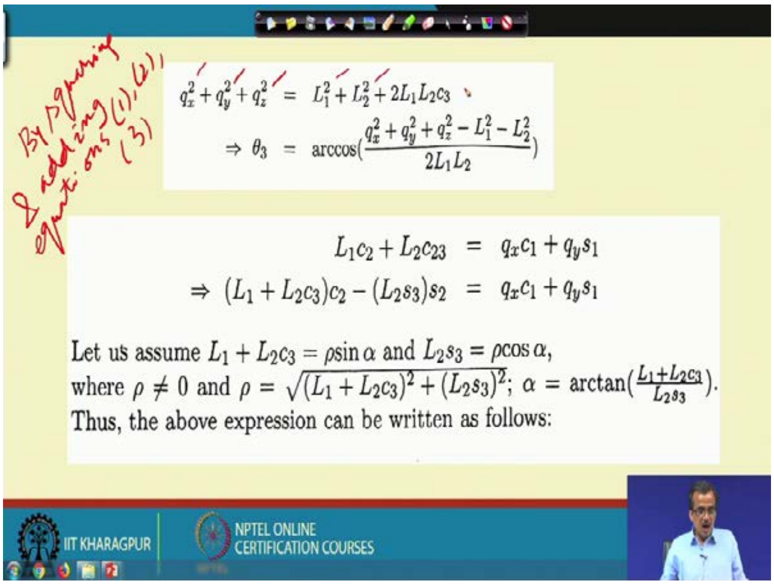

Now, if this is the situation, I will be getting all such equations like these, this is your equation (1); this is your equation (2); this is equation (3); equation (4); then we have got 5th equation; this is your 6th equation; and this is your 7th equation. Now, although we have got only 5 unknowns, we will have to take the help of these seven equations to find out the solutions. So, although the minimum number of equations required is 5, with the help of 5 equations, we will not be able to find out all five θ values.

Now, let us see how to determine the values of the theta like $\theta_1, \theta_2, \ldots\ldots, \theta_5$, using this set of equations. Now, first let us concentrate on equation (2). So, from equation (2), we can find out $-q_x s_1 + q_y c_1 = 0$. So, this can be written as your $q_x s_1 = q_y c_1$, then we can write down q_y divided by q_x is nothing but $\sin\theta_1$ divided by $\cos\theta_1$. So, this is nothing but $\tan\theta_1$. So, $\theta = \tan^{-1}(q_y / q_x)$. So, out of these five theta values, so one theta value, I am able to find out very easily and that is nothing but θ_1.

Now, let us see how to find out the other theta values. Now, to find out the other theta values actually, what I will have to do is, we will have to take the help of other equations. For example, say once again if you write down the equation number, so this is (1), this is (2), this is (3), this is (4), this is (5), this is (6) and this is (7). So, we will have to take the help of equations (1), (2) and (3). And, let us see by taking the help of 1, 2 and 3, how can you find out the other joint angle values.

So, what I do is, we use actually the same method like by squaring and adding equations (1), (2) and (3), we get $q_x^2 + q_y^2 + q_z^2 = L_1^2 + L_2^2 + 2L_1L_2c_3$, how to get it, let us try to see.

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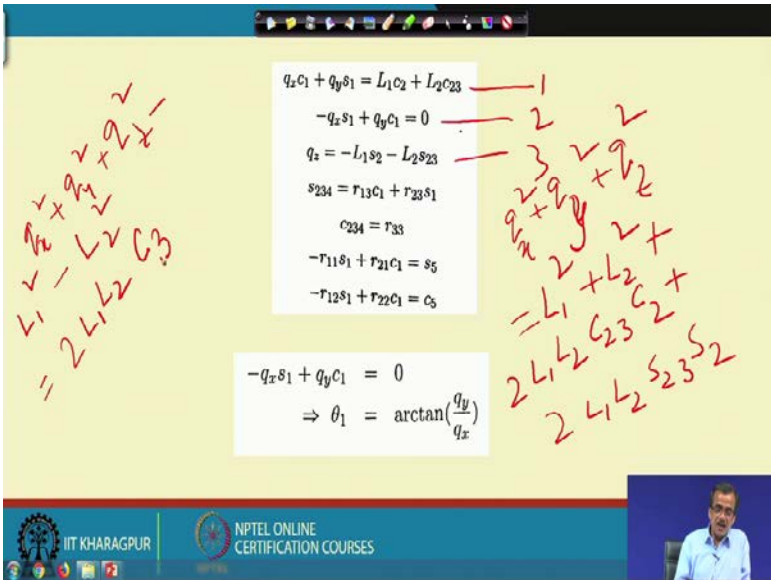

So, what I do is, this is my equation (1), this is (2), and this is (3). So, what I do is, by squaring and adding, you will be getting q_x square, then I will be getting q_y square, then I will be getting q_z square and that is nothing but is your L_1 square then comes your L_2 square plus 2 L_1 L_2 cos of theta_3. And, then here, I will be getting $2L_1L_2\cos\theta_3$.

Now, this part actually can be written as your $q_x^2 + q_y^2 + q_z^2 - L_1^2 - L_2^2 = 2L_1L_2\{\cos(\theta_2+\theta_3)\cos\theta_2 + \sin(\theta_2+\theta_3)\sin\theta_2\}$. So, this can be written as $\cos(\theta_2+\theta_3-\theta_2)$. So, this will become cos of theta_3, ok? And, from here, we can write down so that particular expression is theta_3 is nothing but cos inverse of this particular expression. So, theta_3 is known.

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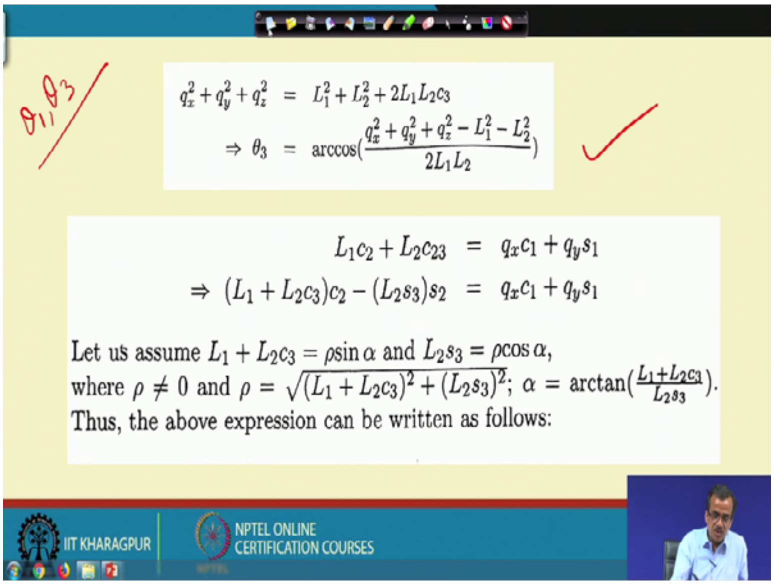

So, till now, we have determined theta_1, theta_1 we have already got and we have got now theta_3; out of five values, two values, we have already got. Now, we concentrate on the first equation, that is, your $q_xc_1+q_ys_1=L_1c_2+L_2c_{23}$.

Now, this is that particular equation. So, this particular equation $q_xc_1+q_ys_1=L_1c_2+L_2c_{23}$. Now, let us try to concentrate here, now here. So, this is from equation (1). So, from equation (1), this can be written as so $L_2\cos(\theta_2+\theta_3)=L_2\cos\theta_2\cos\theta_3-L_2\sin\theta_2\sin\theta_3$.

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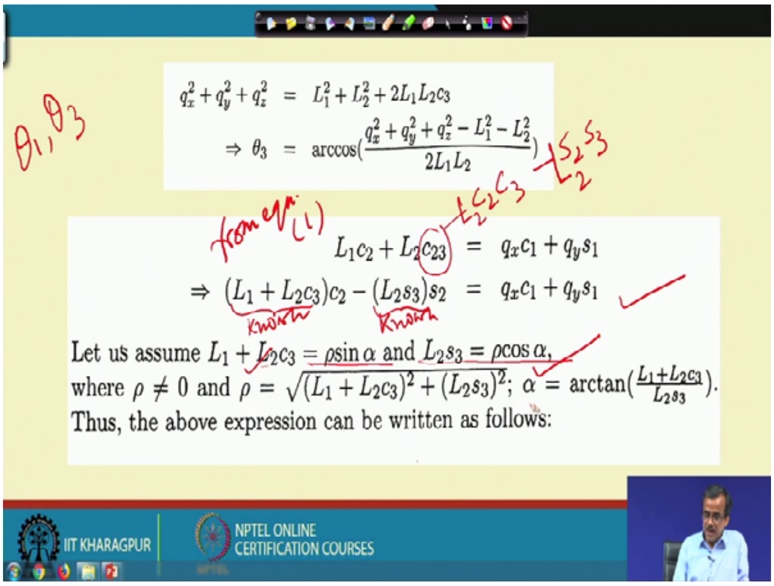

So, this particular thing can be written as cos of theta_2 cos of theta_3 minus sin of theta_2 sin of theta_3. So, if I just write here, I can take this particular c_2 common and within bracket I can write down L_1 plus L_2 cos theta_3 minus L_2 s_3 within a bracket then s_2. And, this is nothing but q_x cos theta_1 plus q_y sin theta_1, ok?

Now, if you see so till now as I mentioned that we have calculated theta 1, we have calculated theta 3; Now, here if you see so this L_1 plus L_2 cos theta_3, so this is the known quantity because I know that L_1 L_2, I know theta_3. So, this is the known quantity; similarly, this L_2 sin theta_3, so this is also the known quantity. And, here, so we can assume that the known quantity $L_1 + L_2 \cos\theta_3 = \rho \sin\alpha$; and $L_2 \sin\theta_3 = \rho\cos\alpha$.

And, here, $\rho = \sqrt{(L_1 + L_2 c_3)^2 + (L_2 s_3)^2}; \rho \neq 0$; and moreover $\alpha = \tan^{-1}(\frac{L_1 + L_2 c_3}{L_2 s_3})$. So, I can find out rho; I can find out this particular alpha. And, if I know this rho and alpha, ok? so I can write down, I can find out this expression as follows:

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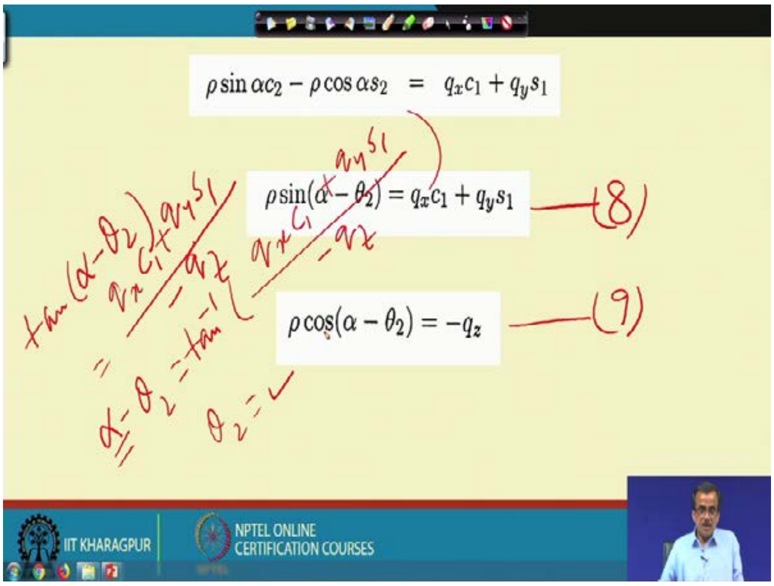

So, this can be written as $\rho \sin\alpha c_2 - \rho\cos\alpha s_2 = q_x c_1 + q_y s_1$. So, this is actually one equation, we will be getting. So, previously we got seven equation and these can be written as $\rho\sin(\alpha - \theta_2) = q_x c_1 + q_y s_1$. So, this is actually nothing but equation number (8). So, this is your equation number (8).

Now, by following the same principle actually, what you can do is, so we can concentrate on this particular equation, that is, your equation (3). So, we can concentrate on this particular equation, and we can follow the same method. And, if you follow the same method, then I can find out this particular expression that is rho cos alpha minus theta_2 is nothing but minus q_z. So, this is actually this is actually equation (8); this is equation (8), and this is your equation (9), ok?

Now, by solving these equations (8) and (9), we can find out actually $\tan(\alpha - \theta_2) = (q_x \cos\theta_1 + q_y \sin\theta_1)/(-q_z)$. So, this is nothing but tan of alpha minus theta_2. And, $\alpha - \theta_2 = \tan^{-1}(q_x \cos\theta_1 + q_y \sin\theta_1)/(-q_z)$. So, this is the expression of this particular alpha minus theta_2. And, alpha we have already determined, so we can find out this particular theta_2. So, theta_2 is also known, now.

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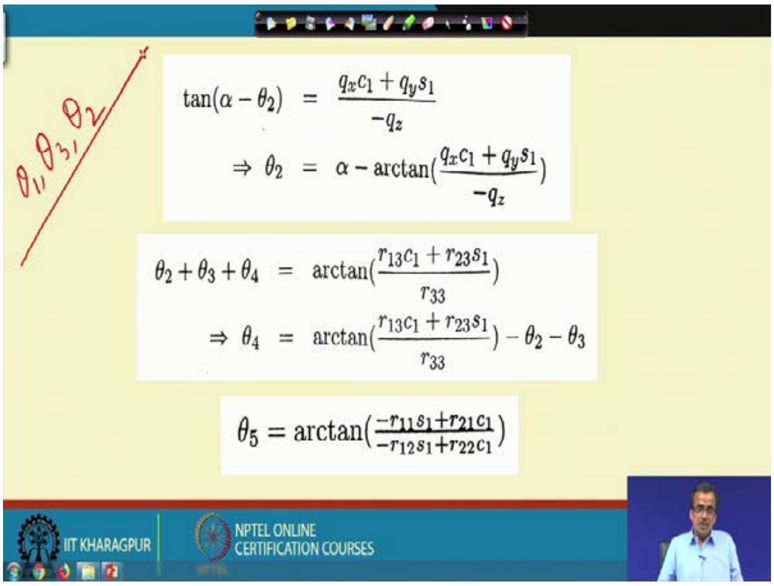

So, once you have determined this particular theta_2, now so till now, actually, we have got theta_1 then, we determine theta_3, then we determined actually theta_2 so, out of these five values, three values are known. Now, if we just go back to the set of equation, we have got some equations like $\sin(\theta_2 + \theta_3 + \theta_4)$, $\cos(\theta_2 + \theta_3 + \theta_4)$.

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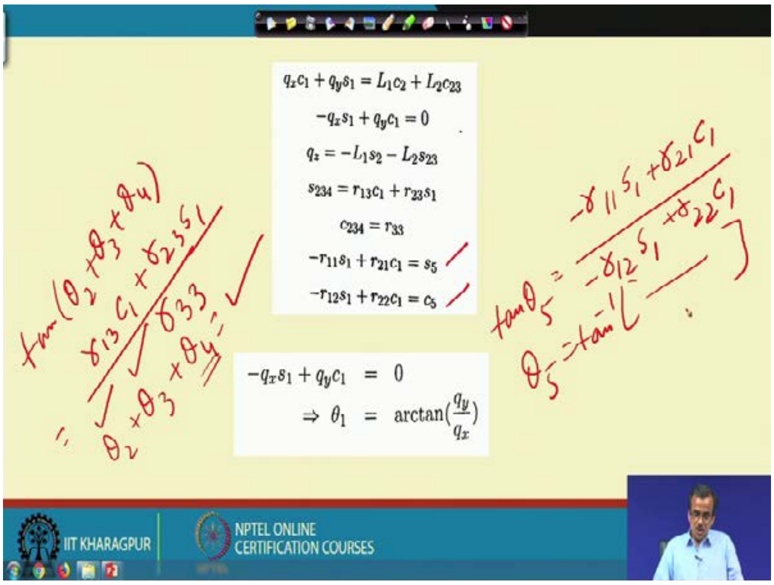

So, I can find out very easily, that is your $\tan(\theta_2+\theta_3+\theta_4) = \frac{r_{13}c_1 + r_{23}s_1}{r_{33}}$. So, this particular thing we can find out. So, this is known quantity, so I can find out, what is theta_2 plus theta_3 plus theta_4. Now, this is known, and theta_2 I have already calculated; theta_3, I have already calculated. So, I can find out what is theta_4.

And, once you have got this particular theta_4, then using these two equations very easily, we can find out $\theta_5 = \tan^{-1}(\frac{-r_{11}s_1 + r_{21}c_1}{-r_{12}s_1 + r_{22}c_1})$. So, from here, you can find out tan inverse of that, tan inverse of this particular expression, if you write down, you will be getting this particular theta_5. So, this is the way actually, we can find out all the values of the joint angles, that is, your theta_1, theta_2, theta_3, theta_4 and theta_5. And, we can solve this particular inverse kinematics in this way.

Now, remember one thing, here, we may not get and we will not be getting the unique solution for this particular inverse kinematics. Like let me take a very simple example, supposing that I have got a manipulator having five degrees of freedom or say 6 degrees of freedom. And if, I give the task to the manipulator that the tip of the manipulators touch touch this particular point.

Now, this point can be touched with the different configurations of the joints of the manipulator and that is why, to touch the same point, actually, there will be the different combinations of the theta values with the help of which, actually I can reach the same point. And, there will be multiple sets of solutions for this particular joint angles with the help of which, the tip of the manipulator or the end-effector will be able to reach a particular point in 3D space.

Thank you.

Robotics

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Lecture - 20
Robot Kinematics (Contd.)

So, we have seen how to solve the forward kinematics problem and inverse kinematics problem for serial manipulator. To gain confidence, like how to assign the coordinate system, how to carry out the forward kinematics, how to carry out the inverse kinematics; in fact, we need some more examples, more practice and that is why, actually, I am just going to take two more examples just to explain it again like how to assign this particular coordinate system, because this is very important to assign the coordinate system correctly.

Now, if you can correctly assign the coordinate system, then, we will be able to find out the kinematics equation correctly and once the kinematics equations are made correct, then only, we can find out the correct dynamic equation and control is based on the dynamic equation. So, the accuracy lies with actually, how to assign the coordinate system correctly and that is why, let us spend some more time on how to assign the coordinate system. And, let us try to spend some more time and let us try to solve some very simple examples.

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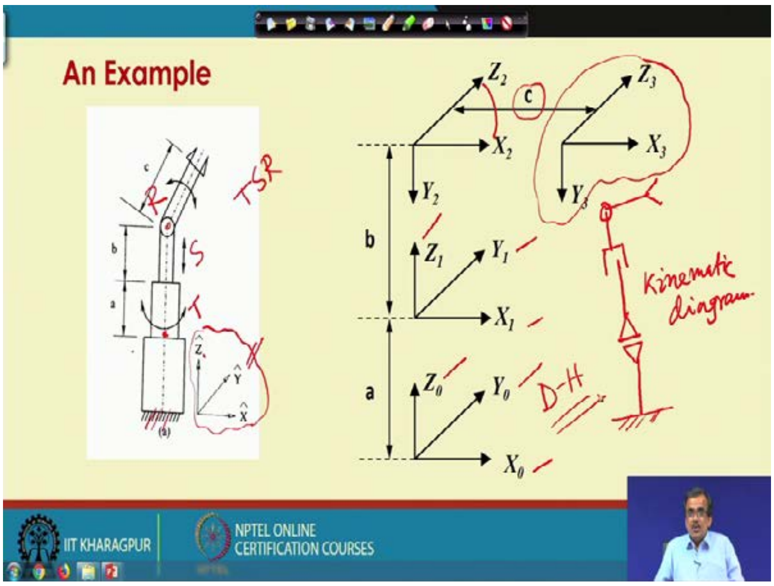

Now, this is one example; example of one manipulator, the serial manipulator having only 3 degrees of freedom; for example, say this is the fixed base. So, here, I have got one twisting joint, say T and I have got one sliding joint, that is, S and I have got a revolute joint, that is R. So, this is nothing but a T-S-R manipulator and its kinematic diagram can be very easily drawn. For example, say this is the fixed base. So, I have got the twisting joint here, and then, I have got one sliding joint.

Now, this particular sliding joint is denoted by this particular symbol, then we have got one revolute joint and after that there is one end-effector. So, this is nothing but the kinematic diagram for this particular manipulator, ok?. Now, let us try to assign the coordinate system, once again. Now, to assign the coordinate system, as I told, the first thing, we will have to do is: we will have to concentrate on the reference coordinate system. So, this is Z, X and Z × X is Y and the first joint, so this particular joint is the twisting joint; that means, this will be rotated and Z will be along this particular direction. So, you select this particular as Z_0 and this particular X_0, you try to take in the same direction of the reference. So, this is your X_0 and Y is nothing but Z × X. So, this is your Y_0, ok?

Now, you see here, we have got one sliding joint. So, with the help of the sliding joint, it can move up and down, and this is a linear joint and this particular linear joint, it will move along this. So, my Z_1 will remain same as in the direction of Z_0. So, this is my Z_1 direction and here, so, what you can see is, they are on the same line. So, Z_0 and Z _1, they are on the same line and along this I take this particular X_1.

Now, I take along this as X_1; so that I can show this particular dimension because they are on the same line and if I do not show X_1 here, so I will not be able to capture this particular dimension. So, let me take X_1 here and if it is X_1, then Y is Z × X. Then comes this revolute joint. Now, for this revolute joint, Z will be different from this particular Z. So, let me take Z along this particular direction; that means this is the Z_2 direction. So, this is the Z_2 direction, ok?

Now, here, this particular Z and that particular Z, they are intersecting. So, to select this particular X, I have got, in fact, both the options but I selected this particular as X_2; so that I can capture this particular dimension because this particular coordinate system I am just going to copy it here and so that the mutual perpendicular distance between the two Zs, that will be the length of the link. That means we will be able to capture this particular C. That is why, instead of this particular direction, I have selected X_2 along this.

Now, if this is Z_2, this is X_2, then Z × X, this will be Y_2 and as I discussed earlier, the same thing will be copied here and this is actually the hypothetical joint. So, Z_3 will be this, X_3 will be this, Y_3 will be this. So, this is the way actually, we will have to assign this particular coordinate system. Now, once you assign this particular coordinate system, according to the D-H parameters setting rule, now what you can do is.

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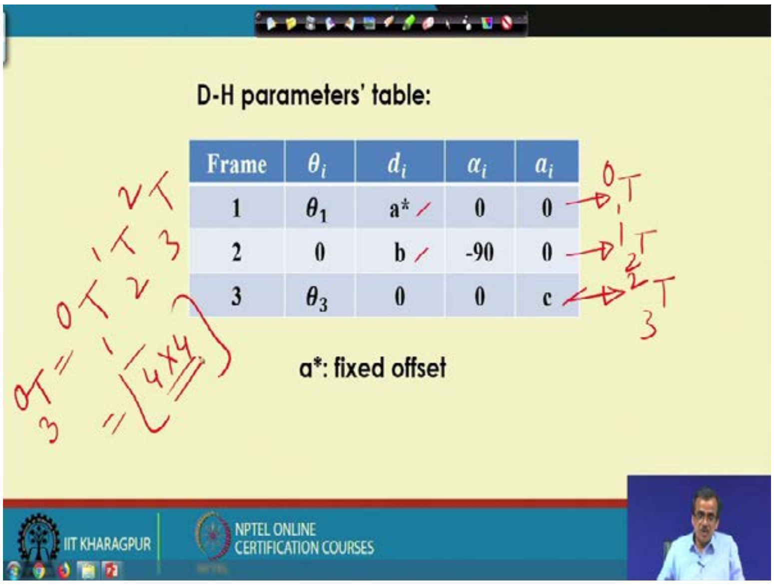

We will have to prepare the D-H parameters table and to prepare the D-H parameters table, what we can do is. So, once again in the D-H parameters table, follow the same sequence like $\theta_i, d_i, \alpha_i, a_i$. The first one, the first joint is actually a twisting joint. So, definitely the variable will be your theta_1, then comes here d.

Now, we see that d is the distance between two X. So, this is X_0, this is X_1. So, this is the distance between two X and it is measured along Z, ok? So, this particular a is nothing but an offset, but this offset is a fixed offset and that is why, this is shown as a star. So, this particular a^* is the fixed offset. This is not the variable offset, ok? The next is your alpha. So, alpha is the angle between two Zs. As the two Zs are lying on the same line, the angle between them is 0, then comes your a_i; a_i is the distance between two

Zs measured along X. The two Zs are on the same line. So, here, so this particular a_i is 0.

Now, the next is your 2; that is, 2 with respect to 1; so, 2 with respect to 1, ok? Now, here, this particular joint; for example, this particular joint is the sliding joint. So, the variable will be the offset. So, offset will be the variable and here, that is your angle that is your joint angle will become equal to 0, because this is a sliding joint; not a rotary one. So, here, this variable offset is nothing but b and that is the distance between X_1 and X_2 measured along Z_1. So, this will be the variable offset and theta is equal to 0. Now, if you see this particular table, so, in place of theta_i, I put 0; in place of d_i, I put b.

Now, this particular b is actually the variable offset. Then, comes your alpha. So here, this is Z_1 and this is Z_2, ok? Now, if I draw the Z_2 here, say this is Z_2, if I draw from Z_1 to Z_2, so from Z_1 to Z_2, this is clockwise. So, this will be your minus 90 and a_i is nothing but the distance between two Zs but they are intersecting. So, the distance between two Zs is 0. The next is your 3 with respect to 2. Now, here, this particular joint is actually a revolute joint. So, definitely the variable will be theta_3. Now, then comes here b; b is the distance between two Xs. The 2 X are lying on the same line. They are collinear. So, the distance will be 0, then comes your angle alpha and alpha is the angle between two Zs, they are parallel. So, we will be getting 0 here, and then comes a_i.

Now, this a_i is actually, the distance between two Zs measured along this particular X and here, we will be getting c. And, we can see that we are able to capture a, b and c, ok? And now, once you have got this particular thing, the way I discussed, very easily you can carry out the forward kinematics. Once you have got the forward kinematics equation for the known matrix, for the known position and orientation, you can carry out the inverse kinematics.

Now, as I told, it is very easy like how to find out the forward kinematics solution like your ${}^{0}_{3}T$. So, this is nothing but ${}^{0}_{1}T$, then comes ${}^{1}_{2}T$, ${}^{2}_{3}T$. Now, T_1 with respect to 0. So, this will give actually T_1 with respect to 0, this will give T_2 with respect to 0, this will give your T_3 with respect to 2. And, if you multiply, then we will be getting this particular final the 4 × 4 matrix, which will carry position and orientation information.

So, this is the forward kinematics and by following the method, actually you can carry out the kinematic analysis, the way I discussed in the last two problems. You can follow the similar type of approach to carry out the inverse kinematics. So, this is one problem. Now, just to gain more confidence, let us try to look into another similar type of problem but a slightly different.

Now, here, I am just going to consider another serial manipulator having 3 degrees of freedom. So, this is nothing but the 3 degrees of freedom, serial manipulator.

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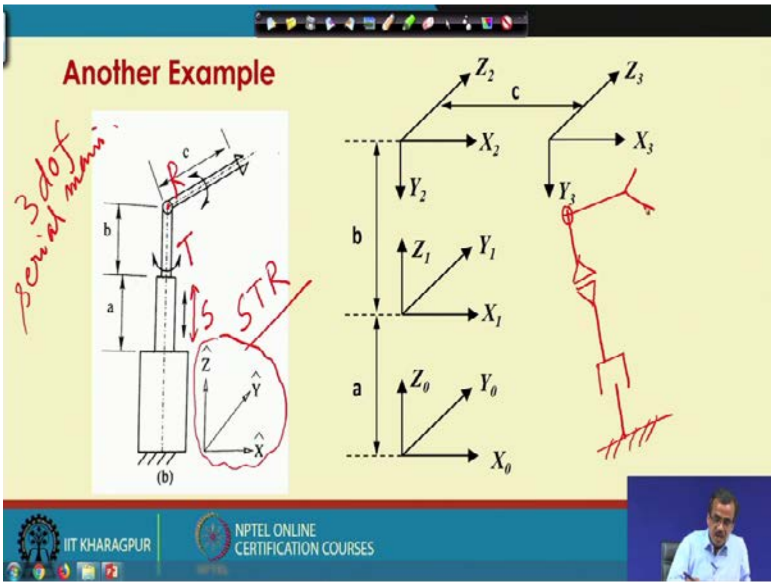

Now here, we have got the sliding joint first. So, this is S, then we have got the twisting joint that is T and here, we have got the revolute joint that is R, and this is nothing but actually S-T-R manipulator. By following the same method, first you look into that particular reference coordinate system and before that, we can also find out very easily the kinematic diagram; for example, say first, we have got the sliding joint. So, this is actually the sliding joint, then we have got the twisting joint and after that, we have got the revolute joint and then we have got the end-effector. So, this is nothing but the kinematic diagram.

And, once you have got this, you are in a position to, in fact, draw the coordinate system at the different joints. Now, this is the twisting joint, sorry, this is a sliding joint. So, Z will be along this. So, I have taken Z_0 along this particular direction; X_0 is along my base X, Y_0 is along my base Y. The next is your this particular twisting joint. Now, for this twisting joint, once again, so Z_1 will be in the same line of Z_0 and let me put this as X_1 and this as Y_1 so that I can capture; so this particular “a” value, this particular “a” dimension, ok?

The next is your, next joint is the revolute joint. So, this is nothing but Z_2 and once again, the way I discuss to capture this particular “c” information. Now here, this particular Z and that particular Z, they are intersecting. So, I have got both the options as X_2 but I am going to take this as X_2, so that I can capture this particular “c”. It is very simple, very similar almost but slightly different. So, using this actually, now, we can find the D-H parameters table. For the first one, the first joint is actually a sliding joint. So here, theta is equal to 0 and here, the variable is nothing but “a”. So, “a” is nothing but the variable. This is the variable offset. So, “a” is the variable offset, alpha is nothing but 0, alpha is the angle between two Zs and that is 0. Then comes a_i, so, a_i is the distance between your two Zs, they are on the same line. So, here, “a_i” is kept equal to 0. So, we can find out this.

The next is your 2, that is, 2 with respect to 1. Now, 2 with respect to 1; this is a twisting joint. So, theta_2 is the variable and then, there is another offset. So, this is X_1, this is X_2, the distance between X_1 and X_2 measured along Z_1. So, this particular “b” is the offset but this is actually the fixed offset that is why, we have used b^*. Then, comes your alpha; alpha is nothing but the angle between two Zs. So, what we do is, here, we draw this particular Z_2, the way I did earlier. Then, Z_1 to Z_2 is in clockwise sense, ok? So, very easily actually, we can find out. So, this is minus 90 and a_i is the distance between two Zs, they are intersecting. So, this is equal to 0.

Similarly, you can find out this particular thing also because this joint is the rotary joint that is a revolute joint. So, theta_3 will be the variable and d_i, the distance between two x, they are on the same line. So, this is 0 and alpha, they are parallel. So, alpha is 0. Then comes “a”, that is the distance between two Zs and that is nothing but is your c, so this is c. So, you can prepare the D-H parameters table and as I told, once you have got this particular D-H parameters table correctly, you can carry out the forward kinematics and if you know the final position and orientation, so very easily, you can find out the inverse kinematic solution but here, if we solve the inverse kinematics problem, we will have to solve for theta_2, theta_3 and this particular variable “a”, ok?

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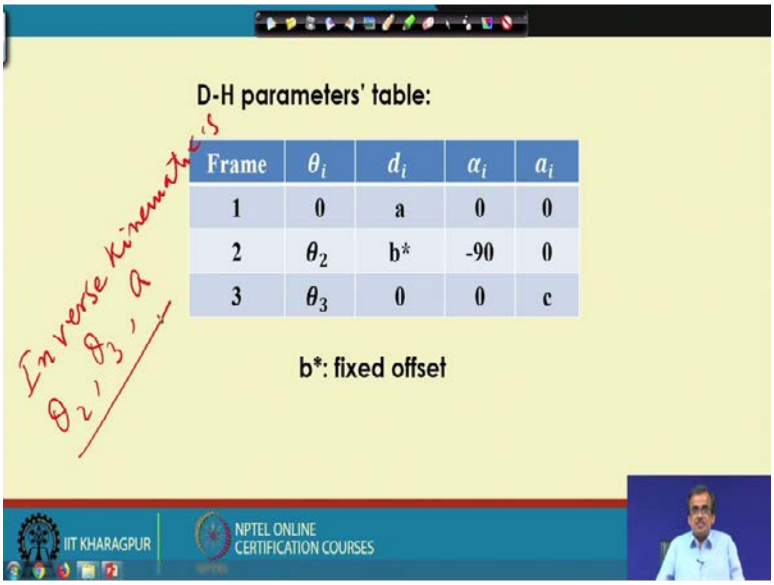

So, if I want to solve the inverse kinematics, the variables which we are going to find out are your theta_2, theta_3 and “a”. So, these are actually the variables, whose values are to be determined, ok? So, this is the way actually, we can carry out your the forward and inverse kinematics.

Now, once you have done this kinematics, we know the movement of the different links, the movement of the different joints, but till now we have not considered the reason behind this particular movement, that there could be a force, there could be moment. So, these things we have not considered in the kinematics, and these things are going to be considered in the dynamics. But, before I go for this particular dynamics, we will have to discuss another thing, another very fundamental thing that is called the trajectory planning that I am going to discuss next.

Thank you.

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Lecture - 21
Trajectory Planning

Now, I am going to start with a new topic that is topic 3, that is, Trajectory Planning. Now, before I start, let me tell you that this particular trajectory planning is not the same with the robot motion planning. The purpose of trajectory planning is to ensure the smooth variation in the robotic joint. On the other hand, the purpose of robot motion planning is to make that particular robot intelligent.

Now, here, I am just going to concentrate on the principle of the trajectory planning only, like how to make that particular trajectory planning, so that that the operation or the variation in a particular robotic joint becomes smooth. Now, before I proceed further, let me tell you the reason behind going for this particular trajectory planning. Now, till now, we have discussed the robot kinematics. The purpose of kinematics is to study the motion of the different robotic joints without considering the reason behind that particular motion, that is, the force or the torque.

Supposing that we have got a linear joint, so we will have to find out the force; if we have got the rotary joint, we will have to find out the torque. Now, supposing that I have got rotary joint, now in that particular rotary joint, so I will have to find out the joint torque and in a particular cycle time, the variation of joint torque has to be gradual and there should not be any such abrupt change in the value of that particular torque.

Now, if you see the expression of joint torque, which I am going to derive after some time, you will see that this particular joint torque has got a few components like the inertia component, Coriolis and centrifugal component, gravity terms, friction terms and, so on. And, if you see the expression, we have got the terms like joint angle, that is nothing but the angular displacement, angular velocity, angular acceleration. Now, as I told that we will have to ensure the smooth variation of this particular joint torque with time; that means, the joint angle should vary in a very smooth way with time and to ensure that, the first time derivative of this angular displacement, that is angular velocity, and the second order derivative, that is angular acceleration are to be continuous.

Now, to ensure that actually, we will have to study the trajectory planning before you start with the dynamics. And, that is why actually, we should study this trajectory planning before the robot dynamics.

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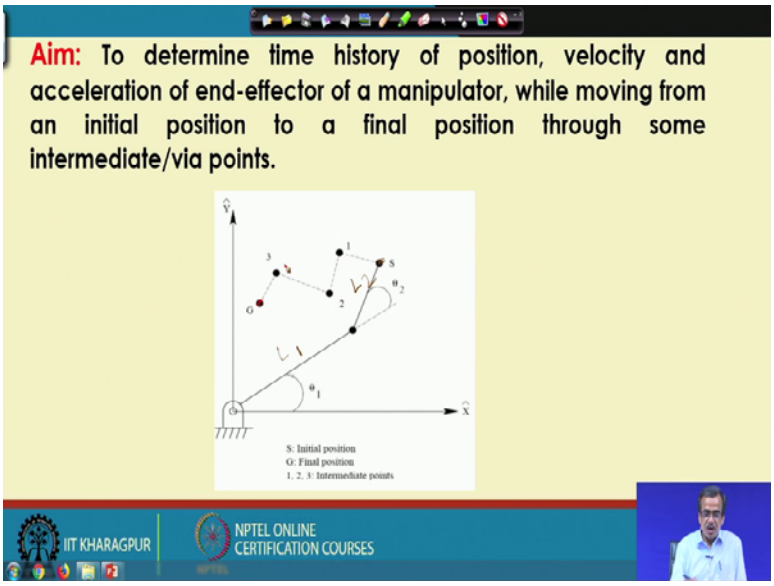

Now, supposing that we have got a serial manipulator like 2 degrees of freedom serial manipulator. So, let me concentrate, so this is nothing but your link_1. So, this is the link_1, L_1 and the link_2, the length is L_2. Now, here the aim is as follows: this is the starting point of this particular end-effector; the end-effector starting from here will reach this particular, the final point, that is, the goal point through a number of intermediate points like 1, 2, 3 and so on.

Now, the purpose of this particular trajectory planning is to determine the time history of position, velocity and acceleration of end-effector, while moving from the initial position S to the final position G through a number of intermediate points or the via points. Now, let us see, how to ensure the smooth variation.

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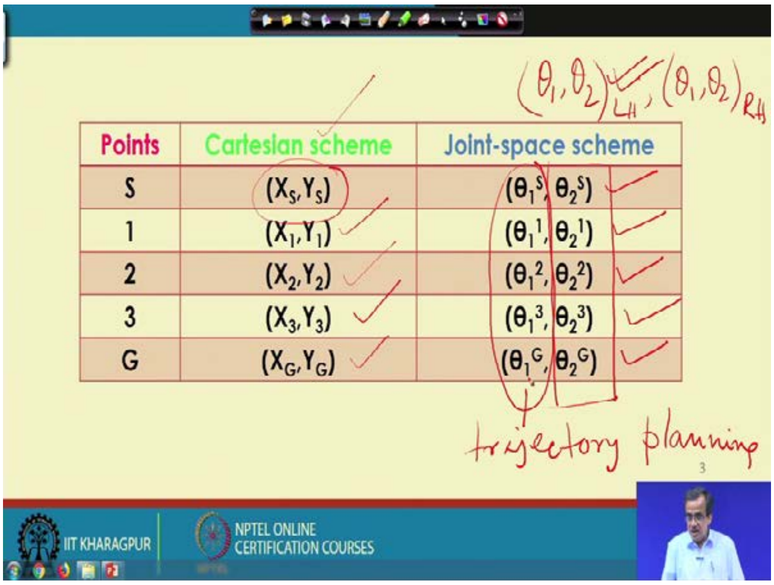

So, corresponding to this particular point S, in Cartesian coordinate system, we have got X_S, Y_S, this particular coordinate, and corresponding to this the first intermediate point, we have got X_1, Y_1.

Similarly, second intermediate point X_2, Y_2, third one X_3, Y_3 and the final one that is X_G, Y_G. Now, we have already discussed inverse kinematics. Now, supposing that in Cartesian coordinate system, we know the position of this end-effector and if we know the position of the end effector, we can solve the inverse kinematics and we can find out two sets of θ values like your θ_1, θ_2, the left hand solution and we have got the θ_1, θ_2, right hand solution.

Now, here, out of these left and right hand solutions, let me consider any one, supposing that I am concentrating on the left hand solution. So, corresponding to these X_S, Y_S, I will be getting θ_1^S, θ_2^S. Similarly, corresponding to X_1, Y_1 in Cartesian, in joint space scheme, I will be getting θ_1^1, θ_2^1. Corresponding to X_2, Y_2, I will be getting θ_1^2, θ_2^2, then corresponding to X_3, Y_3, I will be getting X_1 sorry, θ_1^3, θ_2^3. Then corresponding to X_G, Y_G, I have got θ_1^G, θ_2^G.

Now, as I know that at each of the robotic joints, we have got, the motor, the dc motor and this particular angular displacement will have to generate with the help of that particular the motor. Now, that is why, for the controlling of the motor, there is no way out but we will have to go for the joint space scheme. Now, if I want to ensure the smooth variation of theta during a particular cycle time, so what I will have to do is, I will have to feed a smooth curve passing through your $\theta_1^S, \theta_1^1, \theta_1^2, \theta_1^3, \theta_1^G$. Similarly, I will have to actually find out another smooth curve for the variation, the smooth variation of θ_2. This is what, we mean by the trajectory planning. So, in trajectory planning actually what we do is, we try to fit a smooth curve for this particular joint variable. Now, if it is a rotary joint, then of course, it will be the joint angle.

So, here let us see how to fit a smooth curve, so that I can ensure the smooth variation of θ_1 and θ_2, separately. So, this is actually the thing, which I am going to discuss in details.

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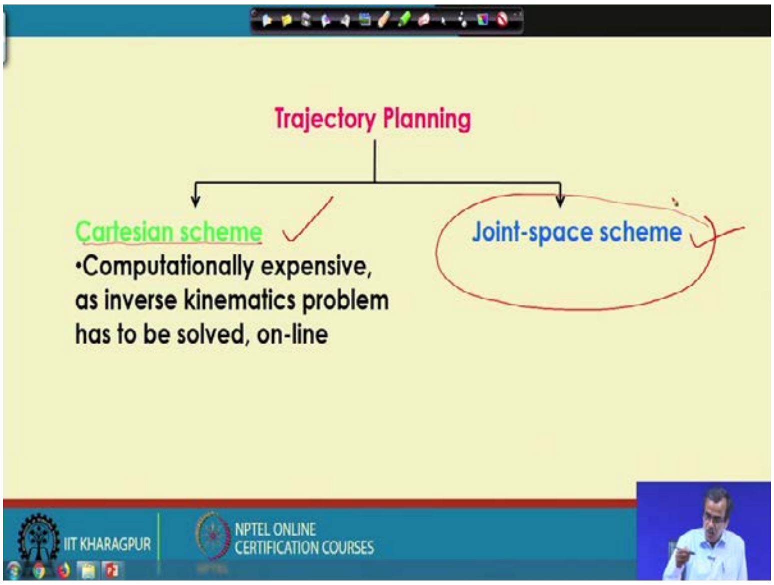

Now, if you see the trajectory planning, the trajectory planning can be done either in Cartesian coordinate system or in joint space scheme. Now, as the motor is controlled in joint space scheme, and if I want to follow the trajectory planning in Cartesian coordinate system, we will have to carry out the inverse kinematics and we have seen, how much complex this particular inverse kinematics is and it is a bit difficult to solve this inverse kinematics problem, online. And, that is why, the trajectory planning in Cartesian scheme is not so much important but we will have to concentrate more on trajectory planning in joint space scheme because the motor is controlled in the joint space.

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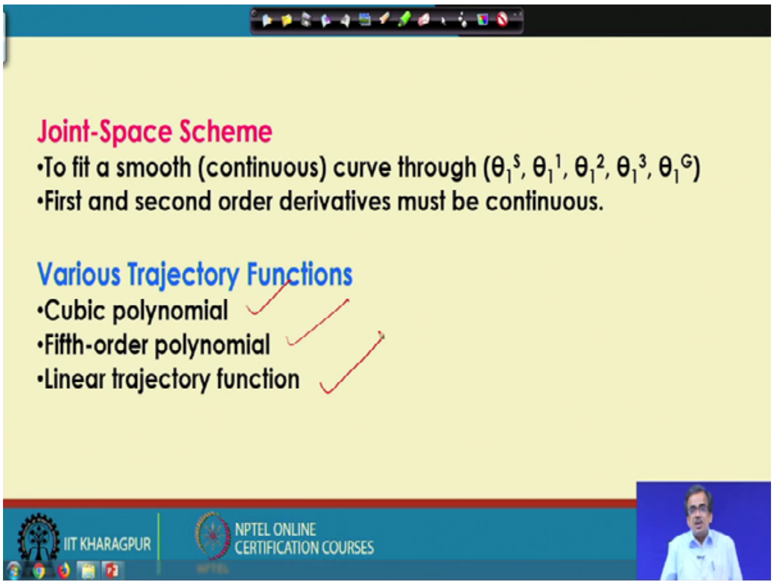

Now, let us see how to make this particular planning for the variation of joint angle in joint space scheme. Now, as I told, the purpose of this trajectory planning in joint space scheme is to fit a smooth curve passing through $\theta_1^S, \theta_1^1, \theta_1^2, \theta_1^3, \theta_1^G$ and to ensure actually, the first order derivative and the second order derivative must be continuous; that means, the first order derivative that is the angular velocity. So, we will have to find out this particular $\dot{\theta}$ and this $\ddot{\theta}$ is the actually the angular acceleration and they are to be continuous.

Now, here, actually what we do is we try to find out like how to fit different types of trajectory function. For example, say, we are going to concentrate on the cubic polynomial, then comes the fifth order polynomial and the linear trajectory function. Let us see, how to fit the cubic polynomial first, then I will move to the fifth order polynomial and we will also see how to fit one linear trajectory to ensure the smooth variation of the joint angle.

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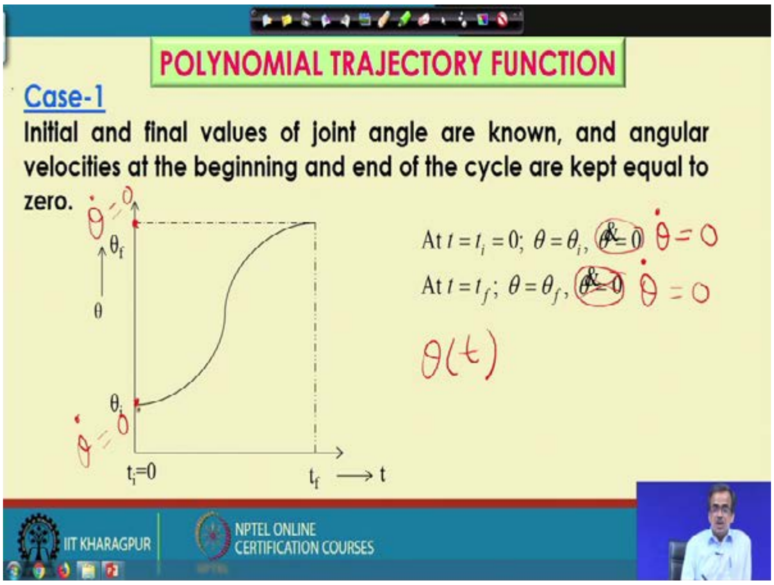

So, let me concentrate on the polynomial trajectory function, first. Now, case 1; this is a very typical problem, where this is the variation of theta, that is, the joint angle with time. Now, what you do is, we will have to find out a smooth curve of theta as a function of time. So, theta should be a function of time and we will have to find out a smooth curve. Now, here, the conditions are as follows: at time t equals to t_i equals to 0; that means, initially, theta equals to θ_i. So, θ is equal to θ_i and $\dot{\theta}$ that is actually the angular velocity is kept equal to 0. So, here, there is some typographical error. So, truly speaking, it should be $\dot{\theta}$ is equal to 0.

So, angular velocity here will be equal to 0, that is a time t equals to t_i, your theta^dot that is the angular velocity is kept equal to 0. Similarly, at time t equals to t_f, t_f indicates the finishing time, theta is nothing but θ_f. So, this is actually the θ_f and the angular velocity, that will be $\dot{\theta}$ and that is equal to 0. So, here once again, the $\dot{\theta}$ is equal to 0.

So, let me repeat at time t equals to t_i initially, theta equals to θ_i, angular displacement and $\dot{\theta}$, that is the angular velocity that is equal to 0; a time t equals to t_f, theta equals to θ_f, the angular displacement and velocity, that is $\dot{\theta}$ is equals to 0. So, there are four known conditions. So, for these four known conditions actually, I can fit one cubic polynomial in the form of this. So, let us consider the cubic polynomial like this.

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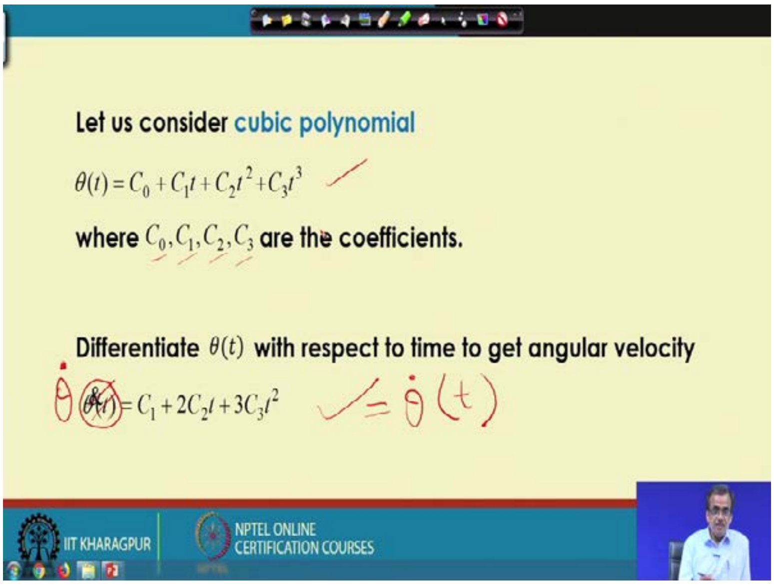

Now, this particular cubic polynomial $\theta(t)$ is nothing but $C_0 + C_1 t + C_2 t^2 + C_3 t^3$. Here, C_0, C_1, C_2, C_3 are the coefficients. Now, we have got four known conditions and those four known conditions will have to be utilized just to find out the expressions or the values for these coefficients: C_0, C_1, C_2, C_3.

Now here, actually, what we do is, we differentiate $\theta(t)$ with respect to time that is, we try to find out actually what is $\dot{\theta}$. Now, this particular $\dot{\theta}$ is nothing but your $\frac{d\theta}{dt}$. So, this is $\theta(t)$. So, if you find out the derivative. So, this here, this contribution will be 0. So, we will be getting $C_1 + 2C_2 t_f + 3C_3 t_f^2 = 0$. So, this is nothing but $\dot{\theta}$ or the angular velocity and truly speaking, so this is also a function of time, so, $\dot{\theta}(t)$.

So, here, we can find out this angular displacement and angular velocity and now, I am just going to put all four conditions, which I discussed a little bit early. So, what I am going to do is, I am just going to put these four conditions in these two equations.

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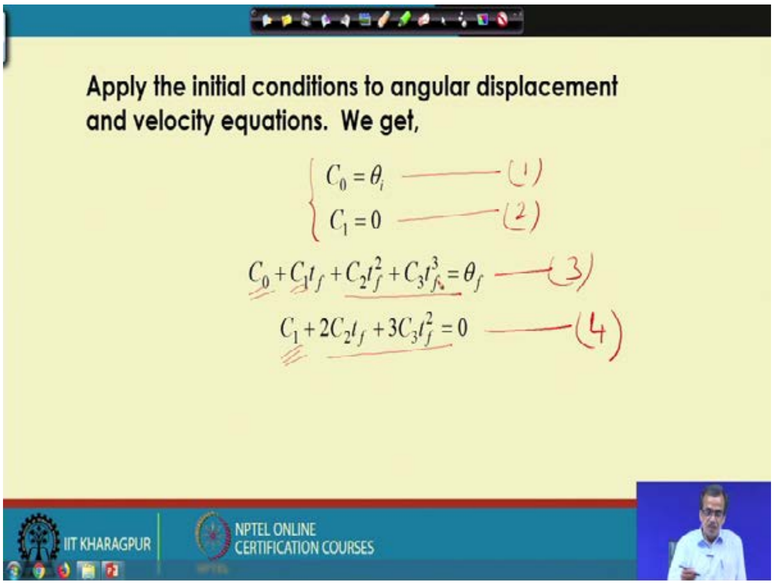

Now, if I just put those conditions, so we will be getting this particular the equations; for example, the first equation will be $C_0 = \theta_i$, then the second equation is $C_1 = 0$, then $C_0 + C_1 t_f + C_2 t_f^2 + C_3 t_f^3 = \theta_f$ that will be equation (3). Then, comes your $C_1 + 2C_2 t_f + 3C_3 t_f^2 = 0$. So, this is equation (4).

So, there are four equations and there are four unknowns and out of these four unknowns, two have already determined, that is C_0 and C_1. Now, if I substitute the values of C_0 and C_1 here, similarly, the value of C_1 here. So, I will be getting one equation in terms of C_2, C_3, I will be getting another equation in terms of C_2, C_3.

So, there are two equations and there are two unknowns C_2 and C_3 and those things can be very easily solved.

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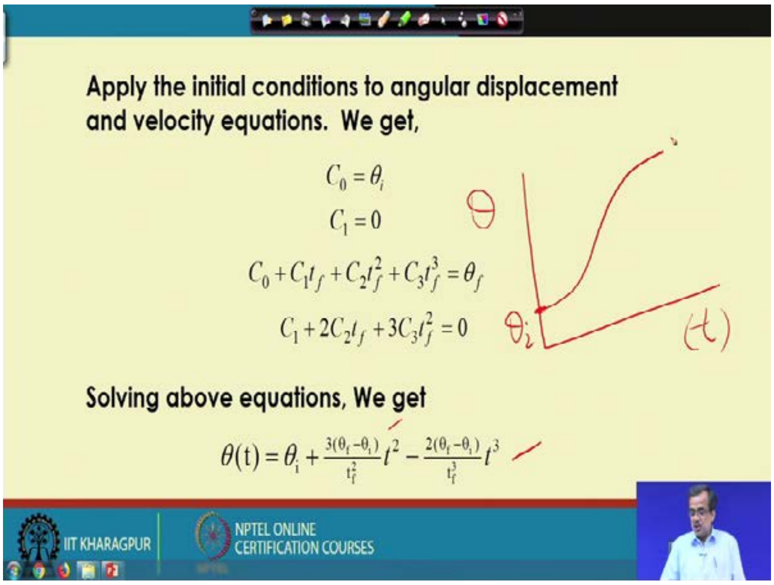

And, if I solve it, then I will be getting actually the final expression and this particular final expression is nothing but $\theta(t) = \theta_i + \frac{3(\theta_f - \theta_i)}{t_f^2} t^2 - \frac{2(\theta_f - \theta_i)}{t_f^3} t^3$. So, this particular theta is actually a function of time.

Now, if I just plot it, for example, now we can plot theta as a function of time and I can plot this particular curve and this is up to t cube. So, this will be a some sort of cubic polynomial and very easily, I can find out the suitable plot. Now, here, approximately, I am just putting the plot but that may not be the correct one; for example, at time t equals to 0. So, these will be θ_i and there is a possibility that it will be something like this. So, this type of plot will be getting. So, this is the approximate plot for this particular the function. Now, this could be your θ_i, the starting is θ_i at time t equals to 0, theta is nothing but θ_i.

So, I can find out approximately this type of the smooth curve further, the θ_i. So, this is the way actually, I can use cubic polynomial to find out what should be the trajectory function.

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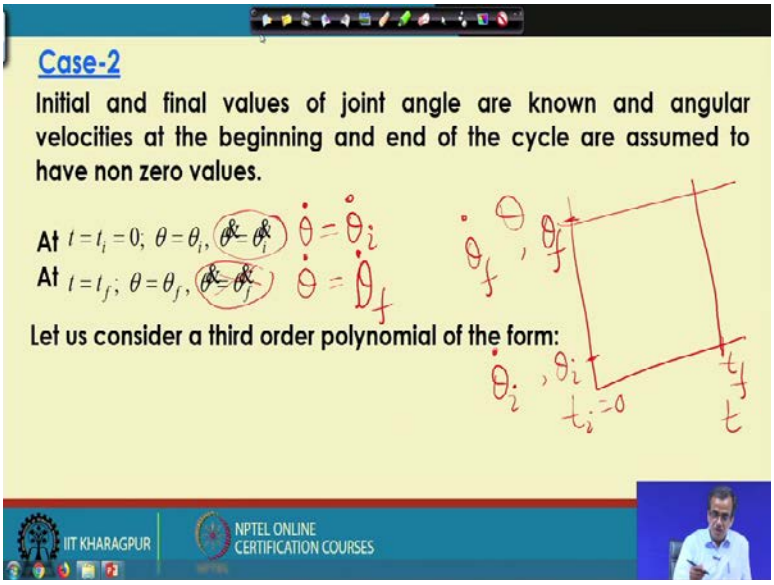

The expression for theta as a function of time, so that the variation of theta becomes smooth. Now, I am just going to concentrate on case 2. Now here, actually what I do is, this is slightly different from case 1. So, in case 1, at time t equals to t_i, the velocity was equal to 0 but here it will be non-zero.

So, at time t equals to t_i, that is, t_i equals to 0, so if I just plot it here, this is time and this is theta. So, at time t equals to t_i equals to 0, theta is equal to θ_i and here, there is typographical error. So, this particular $\dot{\theta}$, that is the angular velocity, is nothing but $\dot{\theta}_i$, it is non-zero. So here, the angular velocity is nothing but $\dot{\theta}_i$.

Similarly, at time t equals to t_f, θ equals to θ_f, so might be at time t equals to t_f, θ equals to θ_f. So, I have got θ_f here and once again there is typographical error; so, this particular $\dot{\theta}$, that is the angular velocity that will be equal to $\dot{\theta}_f$. So, this is non zero. So, here the angular velocity will be $\dot{\theta}_f$. So, using these four conditions, so once again, we can go for some sort of cubic polynomial. So, there are four known conditions. So, I can fit one cubic polynomial.

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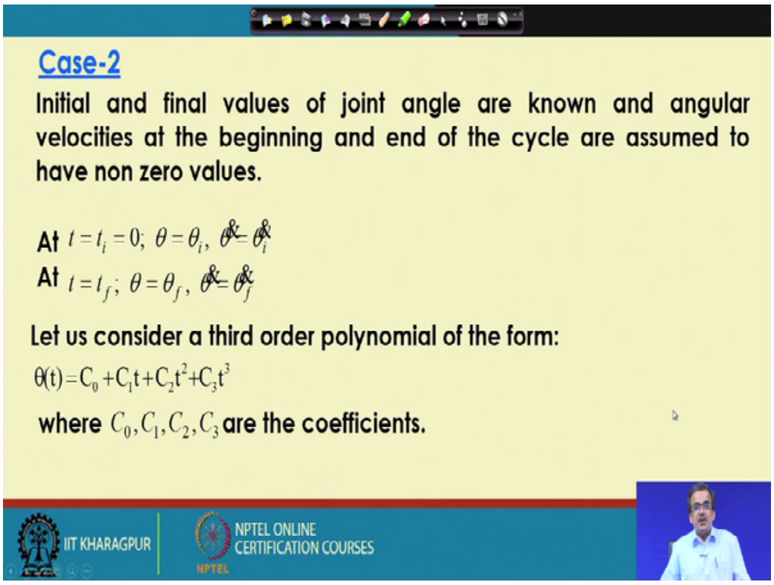

So, let us try to fit one cubic polynomial. Now, if I just try to fit the cubic polynomial, so this is nothing but the expression for the cubic polynomial. $\theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3$. Once again, there are four coefficient C_0, C_1, C_2 and C_3 and we will have to find out the values for this C_0, C_1, C_2 and C_3.

Now, by following the same method, actually what you can do is, there are four conditions and we can put the conditions on the equation and we can try to find out like, what should be the equations.

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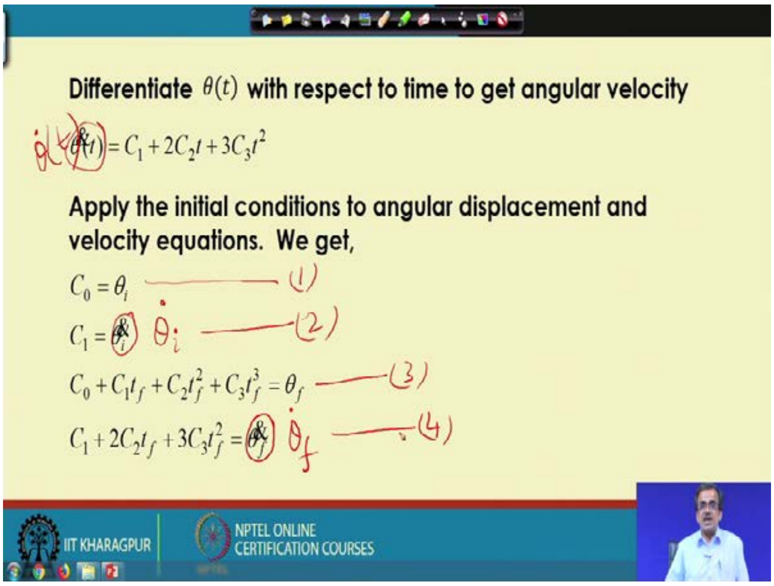

For example, say what you can do is; so, once again we can find out, we can differentiate this particular theta with respect to t and you will be getting $\dot{\theta}(t) = C_1 + 2C_2 t + 3C_3 t^2$ and we apply the four initial conditions just to find out the equations. The equations are as follows: $C_0 = \theta_i$. So, this is equation (1) then $C_1 = \dot{\theta}_i$, that is equation (2). Then, $C_0 + C_1 t_f + C_2 t_f^2 + C_3 t_f^3 = \theta_f$, that is your equation (3); then comes your $C_1 + 2C_2 t_f + 3C_3 t_f^2 = \dot{\theta}_f$.

So, this is your equation (4). Now, there are four equations and there are four unknowns. So, these four equations can be solved just to find out the values of the four coefficients. Now, if we just solve those equations.

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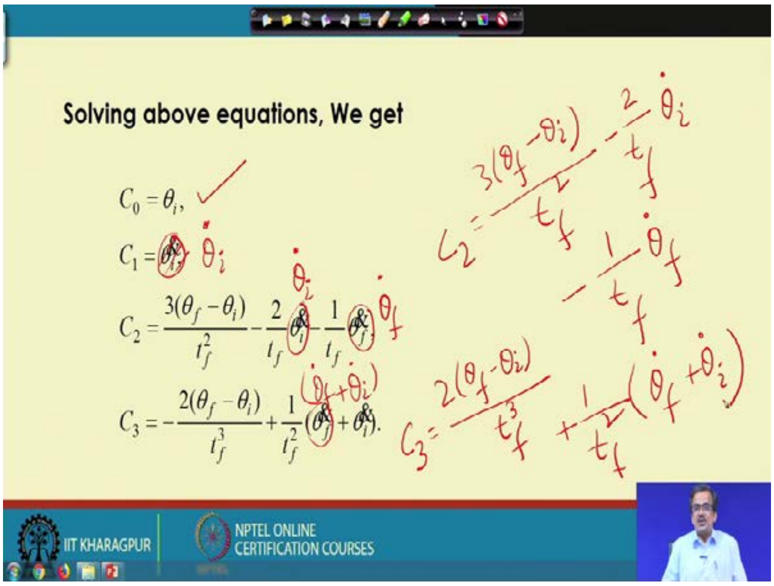

So, we will be getting the terms like this, the coefficients like this. For example, $C_0 = \theta_i$, then $C_1 = \dot{\theta}_i$. So, I am sorry for this type of graphical error, then $C_2 = \frac{3(\theta_f - \theta_i)}{t_f^2} - \frac{2}{t_f}\dot{\theta}_i - \frac{1}{t_f}\dot{\theta}_f$. Now, if I just a write down, let me write down once again; $C_2 = \frac{3(\theta_f - \theta_i)}{t_f^2} - \frac{2}{t_f}\dot{\theta}_i - \frac{1}{t_f}\dot{\theta}_f$. So, this is the final expression for C_2.

Now, similarly, the $C_3 = \frac{2(\theta_f - \theta_i)}{t_f^3} + \frac{1}{t_f^2}(\dot{\theta}_i + \dot{\theta}_f)$. Now, if I write down once again. So, this $C_3 = \frac{2(\theta_f - \theta_i)}{t_f^3} + \frac{1}{t_f^2}(\dot{\theta}_i + \dot{\theta}_f)$. So, this is the expression, which will be getting for C_3.

Now, I will request all of you just to carry out this particular derivation and check whether you are getting the same expression or not. So, this is how to find out the cubic polynomial.

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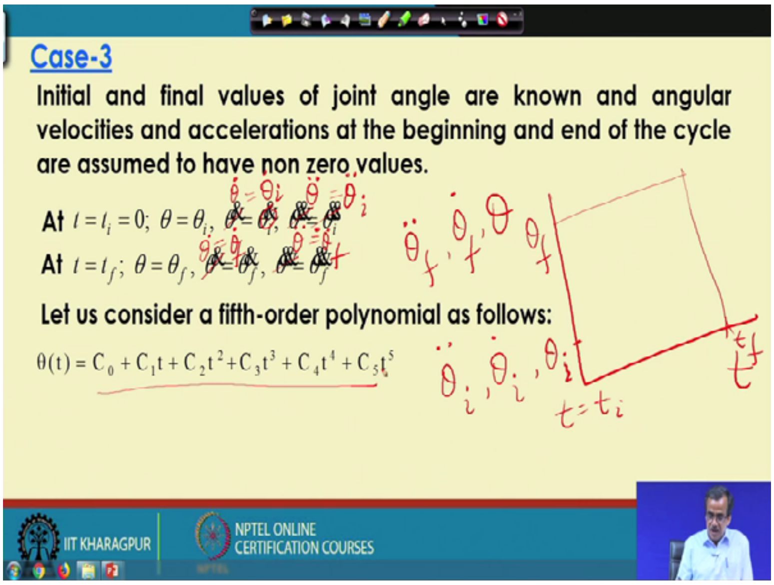

Now, I am just going for case 3. Now, the case 3 is more complicated. More complicated in the sense, like we have got more known conditions.

Now, once again the problem is as follows: say, I will have to fit one smooth curve for this particular theta. So, theta as a function of time, so at time t equals to t_i that is the initial condition and at time t equals to t_f it ends. So, at time t equals to t_i, I have got the initial displacement as θ_i. Then, comes the initial velocity that is nothing but $\dot{\theta}_i$ and initial acceleration that is nothing but $\ddot{\theta}_i$; that means, this will be your, this will be $\dot{\theta}$ is equals to $\dot{\theta}_i$. Then comes, this will be $\ddot{\theta}$ that is the angular acceleration is nothing but $\ddot{\theta}_i$.

Similarly, at time t equals to t_f, so we have got some other conditions like theta equals to θ_f. So, at time t equals to t_f, theta equals to θ_f. Then, comes your theta^dot, $\dot{\theta}$ is nothing but $\dot{\theta}_f$. Then comes your $\ddot{\theta}$ is nothing but $\ddot{\theta}_f$ that is the angular acceleration. So, if I just make the correction, so this particular thing theta^dot will become equal to $\dot{\theta}_f$ and your $\ddot{\theta}$ is nothing but is your $\ddot{\theta}_f$, ok?

So, at time t equals to t_i, we have got three conditions like angular displacement, velocity and angular acceleration. At time t equals to t_f, we have got three conditions like your theta equals to θ_f, angular velocity is $\dot{\theta}_f$ and angular acceleration is $\ddot{\theta}_f$. So, there are six conditions. So, we can fit actually the fifth order polynomial like $\theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + C_4 t^4 + C_5 t^5$.

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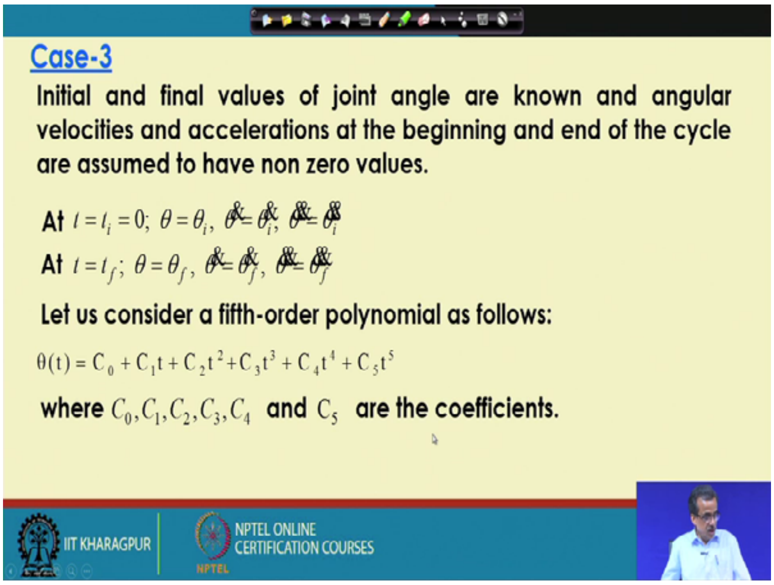

Now, here actually, what I can do is, so we can put these conditions and just to find out all such coefficients.

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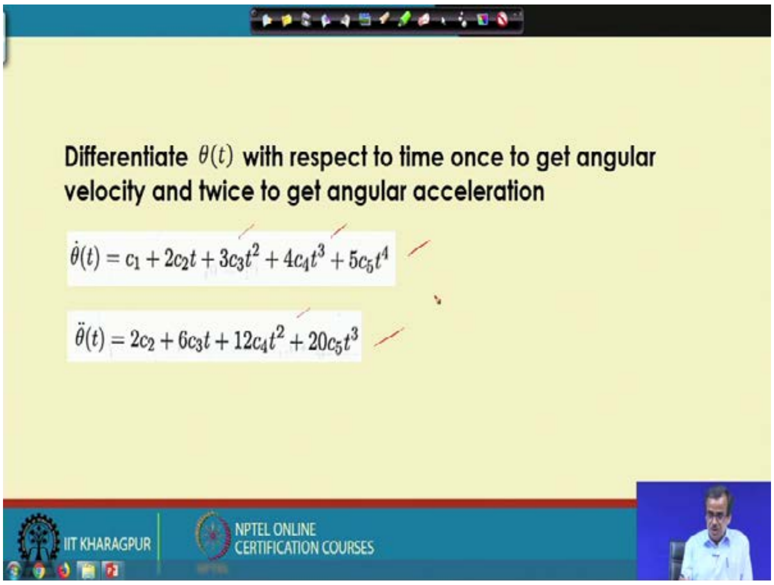

So, there are six coefficients and there are six known conditions. So, I will be getting actually the equations. Now here, so theta (t) we have seen, now if I find out the derivative, the first order derivative, so this is nothing but the first order derivative $\dot{\theta}(t) = C_1 + 2C_2t + 3C_3t^2 + 4C_4t^3 + 5C_5t^4$, then $\ddot{\theta}(t) = 2C_2 + 6C_3t + 12C_4t^2 + 20C_5t^3$.

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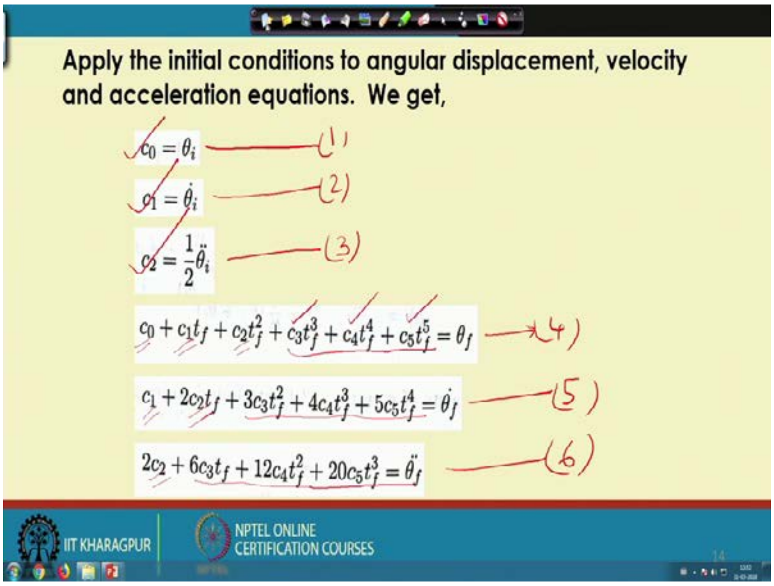

Now here, if I just put all such conditions, I will be getting the equations like this and I will be getting these six equations; for example, say $C_0 = \theta_i$. So, this is equation (1).

$C_1 = \dot{\theta}_i$ is equation (2) and $C_2 = \frac{1}{2}\ddot{\theta}_i$ is equation (3). Similarly, I will be getting equation (4) here, then comes equation (5) here and equation (6) here. Now, out of these 6 unknowns, 3 we have already got like C_0, C_1 and C_2. Now, if I put in this particular equation like C_0, C_1 and C_2, similarly if I put C_1 and C_2 here, so I will be getting another equation in terms of C_3, C_4, C_5 and here also, I will be getting C_3, C_4, C_5.

Now, the same thing you do here, you put this expression of C_2. So, you will be getting another equation in terms of C_3, C_4 and C_5. So, there are three unknowns C_3, C_4 and C_5 and there will be three derived equations. So, there are three unknowns and three equations. So, those can be solved and if you solve it then finally, we will be getting the expression like this.

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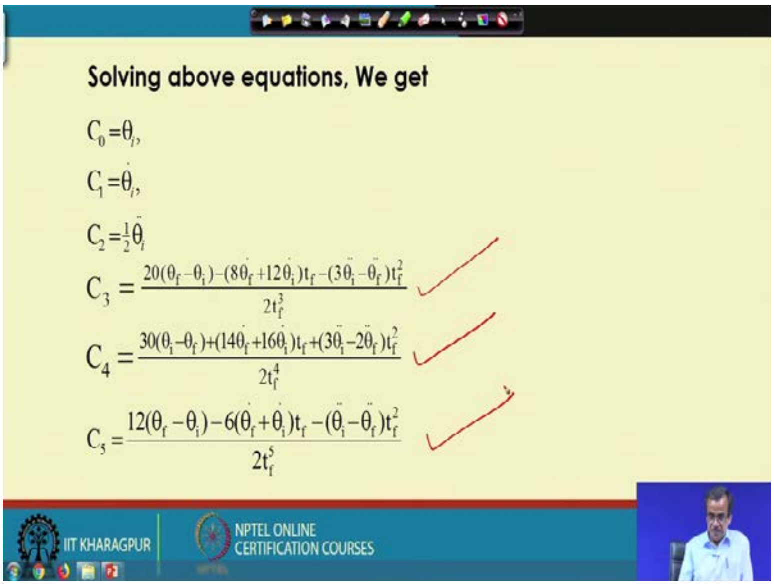

So, your $C_0 = \theta_i$, $C_1 = \dot{\theta}_i$, $C_2 = \frac{1}{2}\ddot{\theta}_i$, then comes your C_3 will be getting this particular big expression involving your θ_f, θ_i, $\dot{\theta}_f$, $\dot{\theta}_i$, $\ddot{\theta}_i$, $\ddot{\theta}_f$, ok? So, I will be getting this big expression for C_3. Similarly, I will be getting another big expression for C_4 and another big expression for C_5.

So, this is the way actually we can utilize the known condition just to find out the smooth variation of this particular theta, that is theta as a function of time and this is actually the purpose of trajectory planning.

Thank you.

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Lecture - 22
Trajectory Planning (Contd.)

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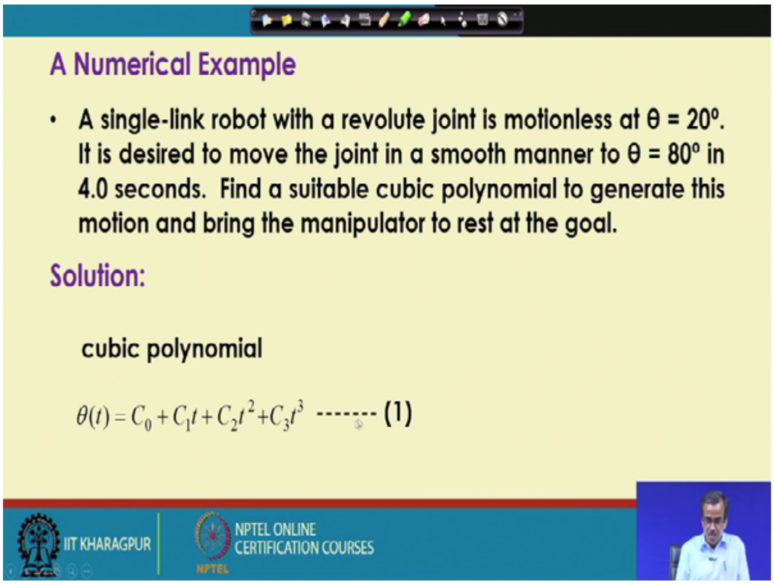

Now, I am going to discuss one numerical example based on this trajectory planning. The statement of the problem is as follows: A single link robot with the revolute joint is motionless at theta equals to 20 degree. It is desired to move the joint in a smooth manner to theta equals to 80 degrees in 4 seconds. Find a suitable cubic polynomial to generate this motion and bring the manipulator to rest at the goal.

So, this is a very simple problem. And, we know the displacement initial displacement initial velocity is equal to 0. We know the final displacement and the final velocity is once again equal to 0. And, we know that the total time, that is nothing but the 4 second. So, very easily, you can fit one cubic polynomial, and this particular problem actually relates to case 1. So, $\theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3$.

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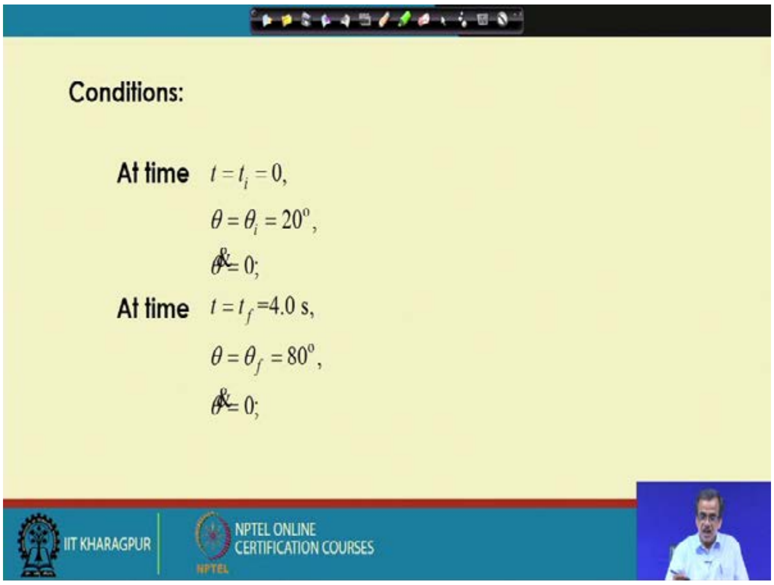

And, the conditions are as follows. For example, say, at time t equals to t_i equals to 0, theta equals to θ_i is equal to 20 degree, and this is nothing but $\dot{\theta}$ is equal to 0. And, similarly, at time t equals to t_f, that is, equal to 4 seconds, theta equals to θ_f equals to 80 degree. And, this particular $\dot{\theta}$ is equal to 0, so these are the conditions. And, based on this particular condition, we can find out what should be the values for these coefficients.

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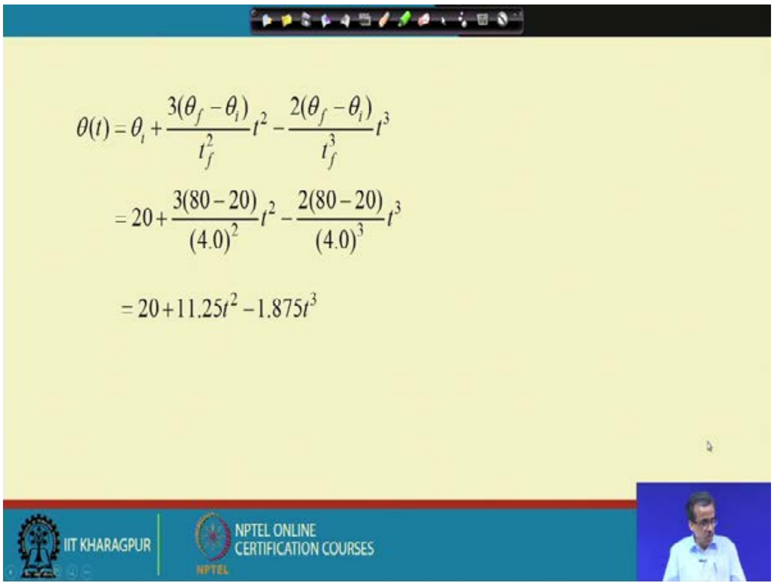

And, we can find out the final expression of this particular cubic polynomial. So, the final expression is coming as $\theta(t)$, is nothing but the initial theta ok, and this is actually the known condition. Now, what you can do is, so we can actually fit, we can just put the numerical values. And, if I put the numerical values, then I will be getting the final expression.

And, the final expression we will be getting as something like this. So, $\theta(t) = 20 + 11.25t^2 - 1.875t^3$, so this type of expression you will be getting. Now, very easily, you can plot this particular theta as a function of time. So, this is how to find out the cubic polynomial.

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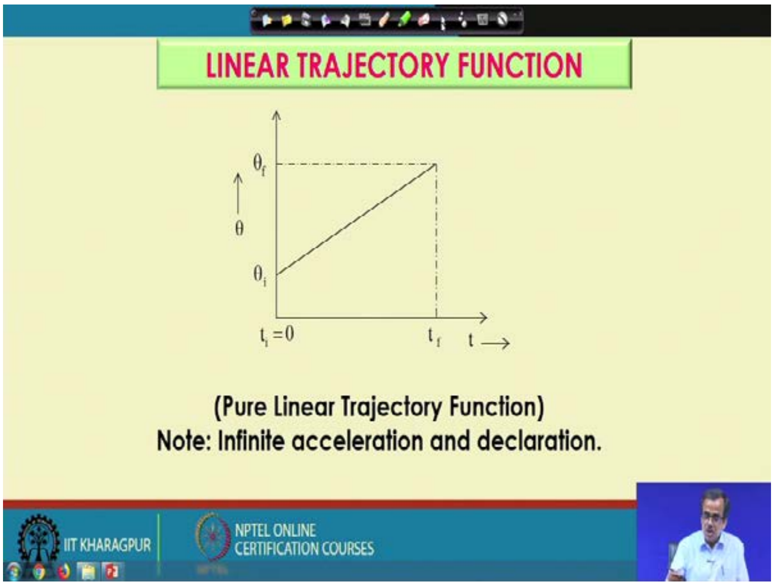

Now, if you see the literature on trajectory planning, there is another form of the function, which is also in use that is called the linear trajectory function. Now, here the variation of theta as a function of time as a linear function of time; so, theta is going to vary linearly with this particular trajectory function.

But, here, we should remember although the theta is varying in the linear fashion like this, so, with time the movement of the end-effector cannot be linear, because if you see the expression for the position, that particular position we have got the $\cos\theta$, $\sin\theta$ terms. And, this particular $\cos\theta$, $\sin\theta$ terms are non-linear. So, the variation of the end-effector with time will become non-linear, of course, the theta is varying linearly.

Now, here, as I told that this particular linear trajectory function is also in use. But, we have got a problem in this type of pure linear trajectory function. Now, the problem is as follows for this pure linear trajectory function, at the beginning, there will be infinite acceleration; and at the end there will be infinite deceleration.

Now, this particular infinite acceleration, and infinite deceleration is not desirable. Now, what will happen, there will be some sort of jerky movement initially, and at the end. And, due to this jerky movement actually, there will be mechanical vibration, and there could be the failure of the robotic joint also. So, just to overcome this particular problem, actually what we do is, we try to fit some sort of parabolic blend at the two ends of this particular linear joint.

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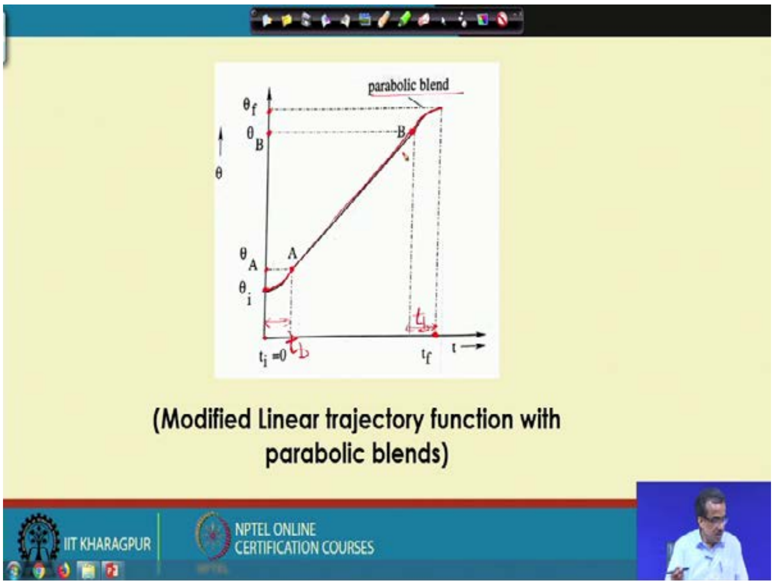

Now, this is actually the modified linear trajectory function. So, here, actually what we can do is, from point A to point B, this is nothing but a pure linear function. But, at the beginning, we put one parabolic blend that is the second order curve, and at the end we put another parabolic blend that is the second order curve.

Now, this parabolic blend is put at the beginning and at the end, just to avoid that particular infinite acceleration here, and infinite deceleration here; just to avoid that jerky movement at the beginning, and at the end. Now, this particular parabolic blend, actually we put during some duration, and supposing that, that particular duration is denoted by t b. So, this particular time is denoted by, say, t_b.

Similarly, here also, we have got the duration, and that is your t_b, ok? And, t_f is nothing but the finishing time. And, t_i is the starting time. So, we are going to start from 0, in terms of time. And, here, at time t equals to t_i, the angular displacement is θ_i. And at time t equals to t_b, the angular displacement is actually theta_A.

Similarly, at time t equals to t_f, the angular displacement is nothing but θ_f. And, at time corresponding to this particular point, actually the angular displacement is theta_B. So, we use this parabolic blend at the two ends as I told, just to avoid that the infinite acceleration and deceleration. And, this is the modified linear trajectory function, which is generally used, and we do not use the pure linear trajectory function, the reason behind that I have already explained.

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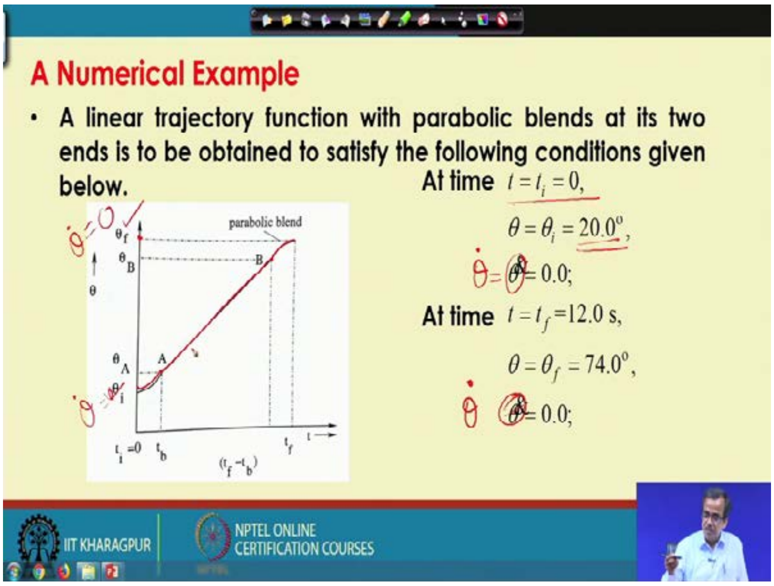

Now, actually, what I am going to do is, I am just going to solve one numerical example, based on this type of the linear trajectory function with parabolic blends. Now, here, the statement of the problem is as follows. Actually, from A to B once again, this is the pure linear trajectory function. And, we have got the parabolic blend here.

Now, here, actually what we are going to do. And, let us see the conditions at time t equals t_i equals to 0, theta equals to θ_i that is equals to 20 degree. Then, comes your $\dot{\theta}$, this is actually $\dot{\theta}$ is equal to 0.0. So, here, the angular velocity is equal to 0. Then, at time t equals to t_f equals to 12 seconds, theta is equal to your θ_f. So, this is given as 74 degree, and $\dot{\theta}$ is equal to 0.0, that means, at the end, the angular velocity becomes equal to 0. So, this is the problem.

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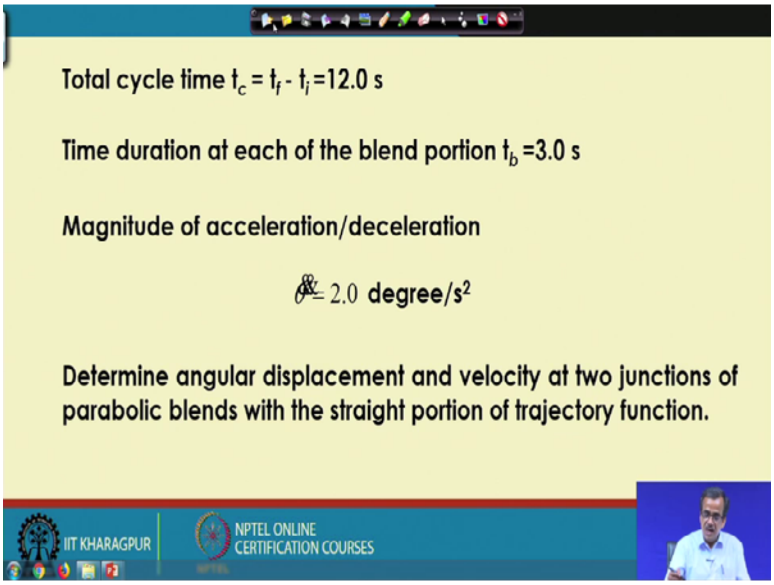

Now, let us see how to determine the different parameters for this type of trajectory planning. Now, here actually, the total cycle time, that is nothing but t_c is t_f finishing time minus t_i is the initial time, and that is nothing but 12.0 seconds. Then, time duration at each of the blend portion, that is, t_b is denoted by 3 second.

And, the magnitude of acceleration and deceleration, that is nothing but is your theta ^double dot, that is the angular acceleration, and that is equal to 2.0 degree per second square. So, these are all known conditions. Now, what I will have to do is, so I will have to find out the angular displacement. So, determine angular displacement and velocity at the two junctions like A and B of the parabolic blend with straight portion of the trajectory function.

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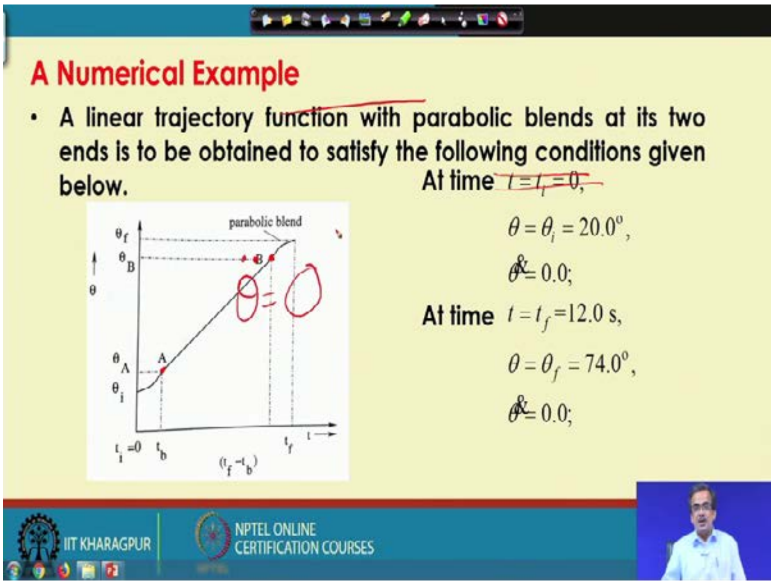

So, if you see that earlier, that earlier picture, our aim is actually to determine the displacement and velocity at the two points, that is actually, at this point A and point B. So, I will have to find out the displacement as well as the velocity.

Now, here one thing is ensured, like if I want to keep this particular continuous trajectory, the velocity at the end of the blend portion should be equal to the velocity at the beginning of the linear portion. Similarly, if I want to maintain continuity at point B, the velocity at the end of the linear portion should be equal to the velocity at the beginning of this particular blend portion. So, these conditions are to be fulfilled.

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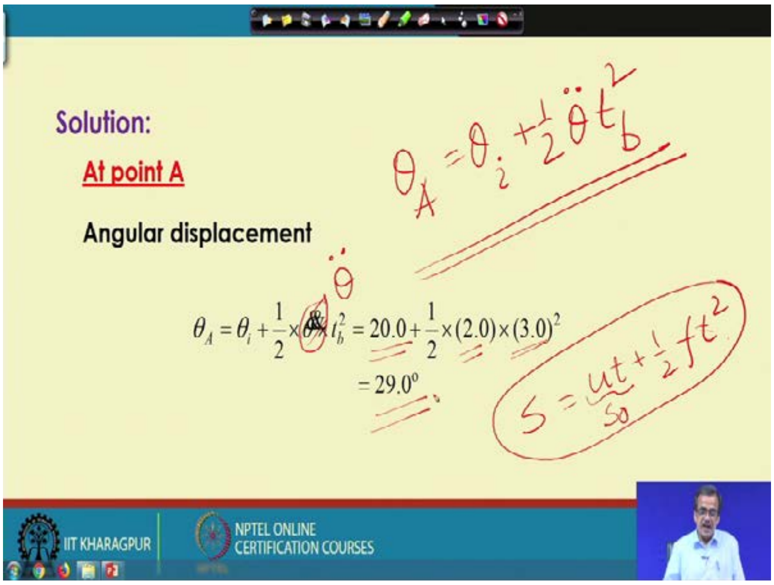

Now, let us see how to find out, and it can be determined very easily. Now, at point A, that is the junction point between the first parabolic blend, and the linear portion. The angular displacement can be determined as follows: like your this angular displacement is nothing but $\theta_A = \theta_i + \frac{1}{2}\ddot{\theta} t_b^2$, that means, here there is one typographical error. So, this will be $\ddot{\theta}$, ok?

This is similar to the equation like, if you remember in your school level, we have studied, that displacement $S = ut + \frac{1}{2} ft^2$, so this is a very well-known formula. The displacement is the initial displacement plus half f t square; f is the acceleration, t is the time and u t is nothing but S_0, this is the initial displacement.

So, exactly in the same way, so we are writing $\theta_A = \theta_i + \frac{1}{2}\ddot{\theta} t_b^2$. So, using this particular formula, so very easily, you can find out what should be the angular displacement at point A, that is, theta_A is nothing but the initial displacement is 20 and half and theta ^double dot is 2 and t_b is nothing but equal to 3.0. So, you will be getting 29.0 degree. So, this is the way actually, we can find out the angular displacement at point A.

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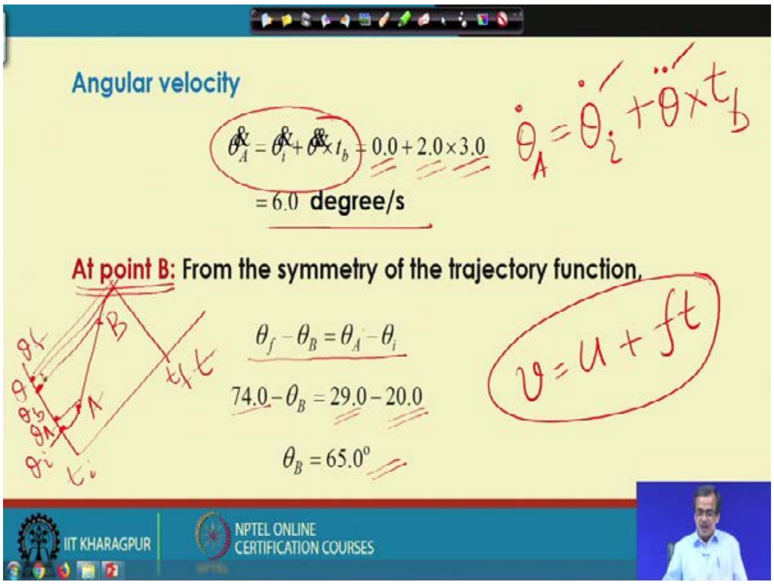

Now, let us see how to find out the velocity that is the angular velocity. And, once again, I am sorry we have got the typographical error here. So, let me just write down. So, this particular part that is your theta_A^dot, that means, I will have to find out the angular velocity at A, that is $\dot{\theta}_A = \dot{\theta}_i + \ddot{\theta} \times t_b$. And, this is similar to the well-known formula, that is, $v = u + ft$, so that particular formula, ok?

Now, here, so this angular displacement sorry angular velocity is $\dot{\theta}_A = \dot{\theta}_i + \ddot{\theta} \times t_b$. And, if we just put the numerical values: theta_i^dot is equal to 0, theta ^double dot is equal to 2, t_b is equal to 3.0, so this angular velocity will be 6.0 degree per second. So, this is the way actually, we can find out the angular displacement, and angular velocity at the point A.

Now, I am just going to concentrate on point B. Now, if you remember the point B is the junction point between the linear and pure linear trajectory, and the second parabolic blend or the last parabolic blend. So, at point B, actually, I am trying to find out what should be the angular displacement and velocity.

Now, if you see this particular plot, if you remember, so this $\theta_f - \theta_B$. So, this particular expression $\theta_f - \theta_B = \theta_A - \theta_i$. Now, roughly actually, if I just plot it here, for example, say if I plot one rough sketch. So, for example, it will be something like this. So, here, we have got the parabolic blend, and here also we have got the parabolic blend. Supposing that, so this particular time, so this is t_f and this is t_i, ok?

Now, here actually this theta_f is the finishing, and the blend portion starts here. So, this is point B, this is your point A. So, this corresponds to theta_B, this is theta_B, and this is your theta_f. So, $\theta_f - \theta_B = \theta_A - \theta_i$. So, here we have got actually theta_A corresponding to these, and initial theta is nothing but theta_i. So, $\theta_f - \theta_B = \theta_A - \theta_i$. And, here we insert all the numerical values known numerical values like theta_f is 74 degrees theta_B will I have to find out theta_A is 29 and theta_i is 20. And, if I calculate you will be getting theta_B equals to 65. So, theta_B can be determined as 65 degree.

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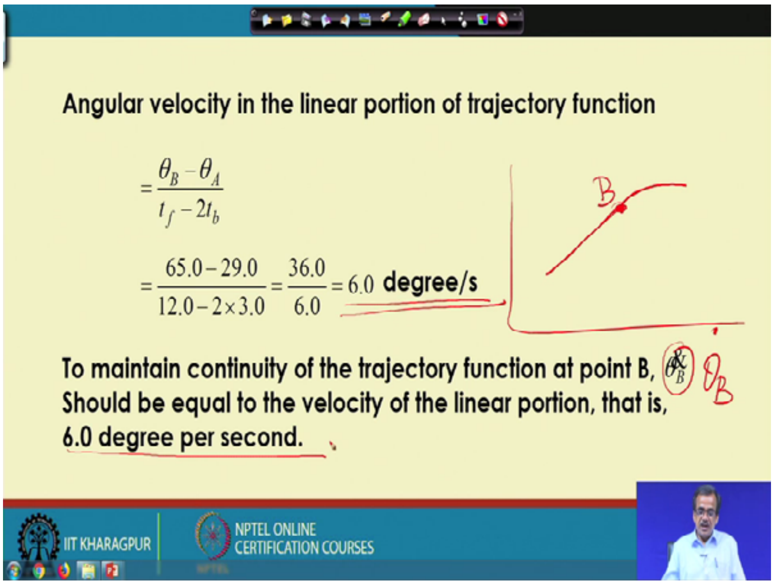

Now, I will try to find out what should be the angular the velocity at point B. Now, to determine the angular velocity of this particular point B; so what I am going to do is, I am trying to find out the angular velocity in the linear portion. Now, how to find out the velocity at the linear portion? To find out the velocity at the linear portion, let us try to concentrate on $\theta_B - \theta_A$.

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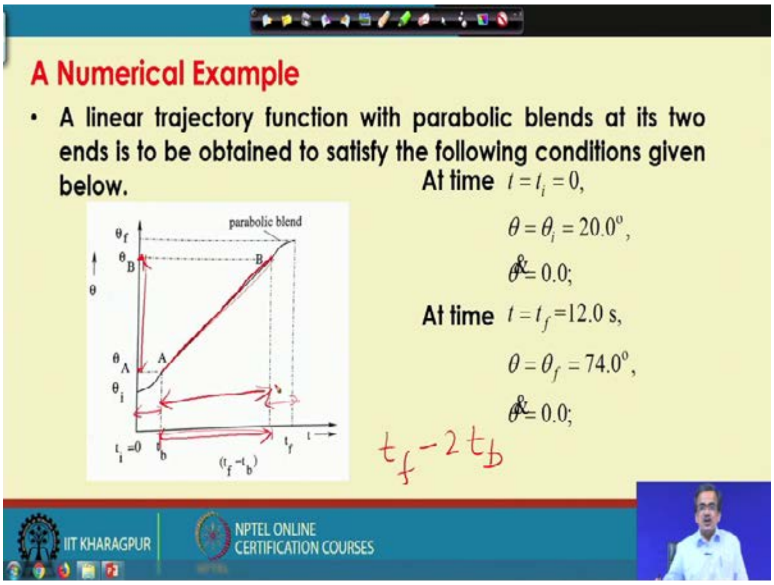

So, if you see in this particular figure. So, if you see this particular figure, $\theta_B - \theta_A$, that is, your this thing: $\theta_B - \theta_A$. So, this is actually the change in angular displacement, and it has happened actually during this particular time. So, this is so this during this particular time, so this particular variation has come. So, if I see this particular thing, so I can find out if this is the change in displacement, and this is the change in time. And, what is this change in time, it is very simple. So, this particular part is t_b. So, this and this is also t_b.

So, the time is actually $t_f - 2t_b$, so this particular the duration. So, I know the change in displacement, I know the change in time. So, very easily I can find out what should be this velocity.

And, we try to find out the linear velocity at this particular portion. So, at the linear portion, so we have got $\frac{\theta_B - \theta_A}{t_f - 2t_b}$, and that is nothing but the angular the velocity. And, if we just insert the numerical values you will be getting your 6.0 degree per second, and that is nothing but the angular velocity at the end of the linear portion.

And, as I mentioned that at point B, so if this is the linear portion, so this is the point B. So, at point B, what you will have to do is, the continuity has to be maintained So, the velocity at the end of this linear portion will become equal to the initial velocity of the blend portion, and that is why, the initial velocity of the blend portion that that particular theta ^dot_B is nothing but is equal to 6.0 degrees per second.

So, here, I have written to maintained the continuity of the trajectory function at the point B, $\dot{\theta}_B$ this is $\dot{\theta}_B$ should be equal to the velocity of the linear portion, that is nothing but 6.0 degrees per second. So, this is the way actually we can fit the trajectory depending on the requirement

So, till now we have considered the polynomial functions like cubic polynomial, and fifth order polynomial and the linear trajectory function with parabolic blend. But, here, we can fit some other types of non-linear function. For example, some sort of log logarithmic function or some exponential function or sometimes a combination of say logarithmic and exponential function, we can use just to find out the trajectory.

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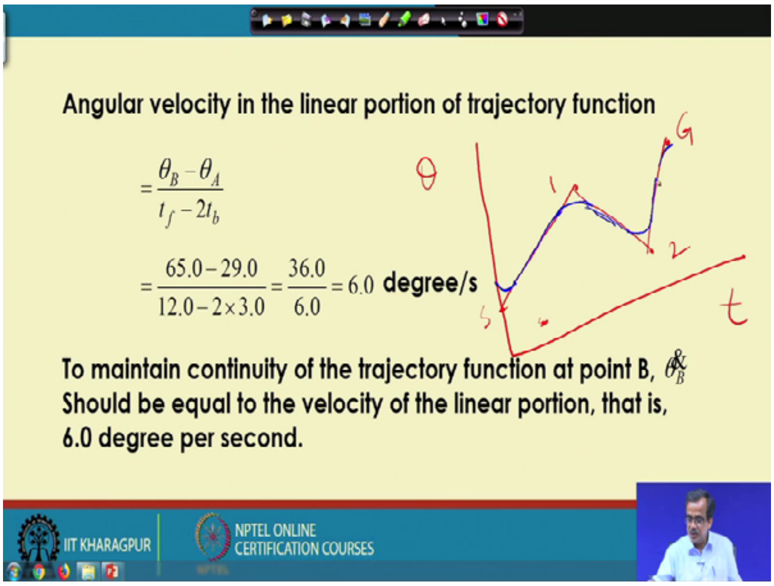

Now, another small thing actually I just want to discuss. Now, supposing that, say I have got this type of distribution of theta. Let me just prepare one a rough sketch here; so theta as a function of time. Supposing that I have got, so this is the initial position of the theta, and say the next point could be say here or let me consider the next point is say here, next point is here, the next point is here, next point is here. So, this could be the goal, and this could be the starting, ok?

Now, if I just fit one the linear curve. So, you forget about this particular point. So, this is starting point goal; the first intermediate, second intermediate. Now, if I just do it like this, so if I put one pure linear, so it will be something like this. If I put one pure linear, it will be something like this. And another pure linear, it is something like this. But, as I told that the pure linear is not possible, and that is why, actually what I will have to do is, I will have to put some parabolic blend.

So, what I can do is, I can put one parabolic blend like this. For example, here I can put one parabolic blend up to this. Here, I can put one parabolic blend up to this, so this type of parabolic blends, ok? Then, here also, I can put one parabolic blend something like this, and at the end also, I can put some sort of parabolic blend. So, the actual distribution of theta as a function of time will look like this. So, this is actually the actual distribution of theta, if I just use that particular parabolic blend with linear trajectory function.

So, this is the way actually, we try to find out a smooth curve, a smooth trajectory for each of the robotic joints and the purpose of which, I have already discussed.

Thank you.

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Lecture - 23
Singularity Checking

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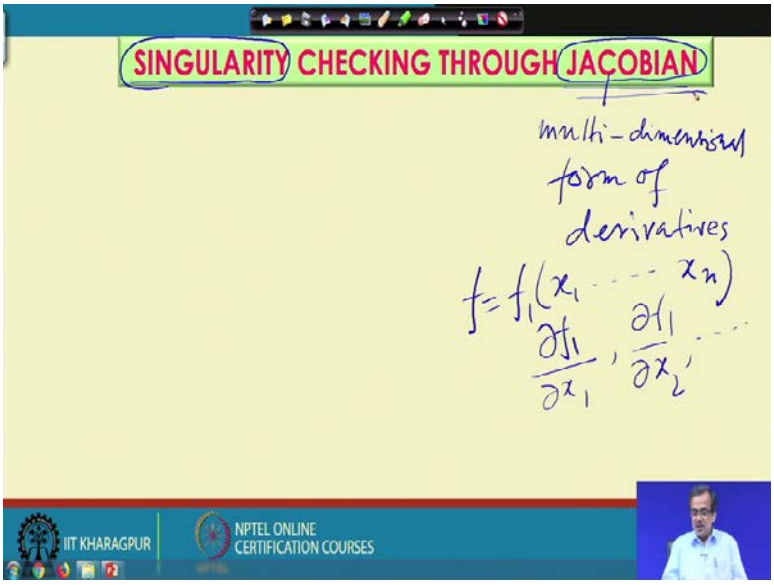

Now, I am going to start with another subtopic, that is, how to determine the singularity condition of a particular manipulator. Now, remember one thing that singularity condition is a condition, during which the manipulator will lose either one or more degrees of freedom. Let me take one example. Supposing that I have got a manipulator having 6 degrees of freedom and out of 6 joints, so might be say 1 or say 2 joints got locked. And due to this the locking condition, the manipulator is going to lose its 1 or 2 degrees of freedom depending on the situation. This is what, we mean by the singularity condition of a particular manipulator.

Now, let us see, how to check the singularity condition using the concept of the Jacobian. Now, what is Jacobian ? You might have studied in the partial differential equation. Now, this particular Jacobian is nothing but the multi-dimensional form of derivatives. Supposing that, I have got a function, which is a function of many variables. For example, say, f is a function of say n variables.

So, if I want to find out the derivative, we will have to take the help of partial derivative. So, we will have to find out partial derivative of f_1 with respect to x_1 partial derivative of f_1 with respect to x_2, and so on. And, by Jacobian, we mean the multi-dimensional form of derivative. Now, this particular mathematical concept of Jacobian can be used in robotics, just to check the singularity. Moreover, we can also use it just to control the manipulator.

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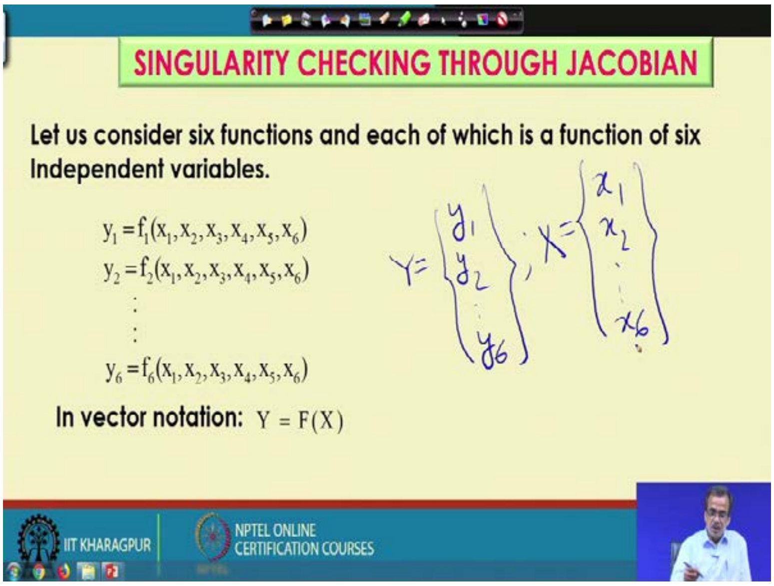

Now, let us try to define the Jacobian first. And, let us see, how to use the concept of Jacobian to check the singularity condition of a manipulator. Now, here, I am just going to take the help of six function, say y_1, y_2 up to y_6. Now, each of these six functions is a function of six independent variables. For example, x_1, x_2, up to x_6, so these are all independent variables and this y_1 is a dependent one. So, $y_1 = f_1(x_1, x_2, ..., x_6)$.

Similarly, y_2 is another function, f_2 of the same independent variable. Similarly, this y_6 is a another function of the same independent variable x_1, x_2, up to x_6. So, I am considering six function and each function is a function of 6 independent variables. Now, here, if you see, this particular thing in the vector form can be written like capital Y is nothing but capital F of capital X. So, this Y is a collection of all small y values. Similarly, X is a collection of all small x values. For example, I can write down this particular Y is nothing but a collection of small y, that is, y_1, y_2 up to say y_6. Similarly, this X is nothing but a collection of all small x values.

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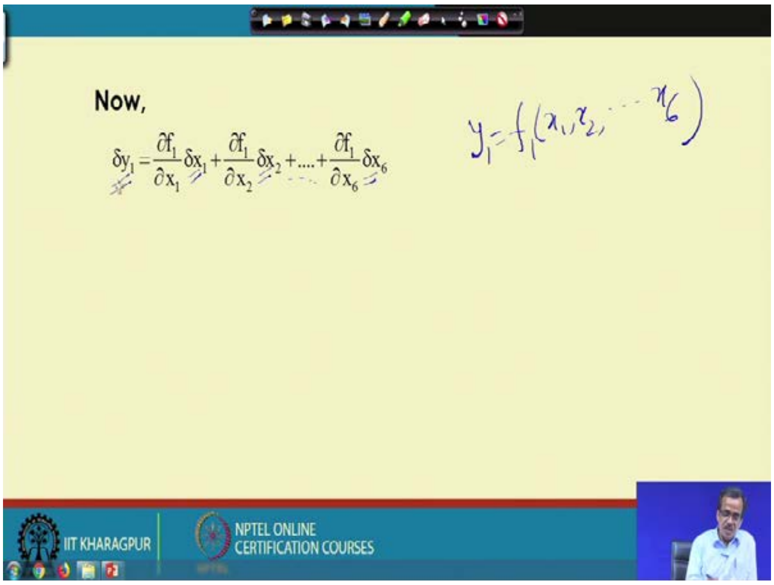

Now, here, now let us see how to proceed further. So, what you can do is, we will try to find out how to find out a change in y_1, a small change delta change in y_1. So, if we just write down. So, $y_1 = f_1(x_1, x_2, ..., x_6)$. So, I am just going to find out a small change in y_1, that is, delta y_1 is nothing but the partial derivative of f_1 with respect to x_1 multiplied by a small change in delta x_1. Similarly, the partial derivative of f_1 with respect to x_2 multiplied by a small change in x_2, that is, delta x_2, and so on. And, the last term that is your partial derivative of f_1 with respect to x_6 multiplied by delta x_6. So, this is the way actually, we can represent the small change in your y_1.

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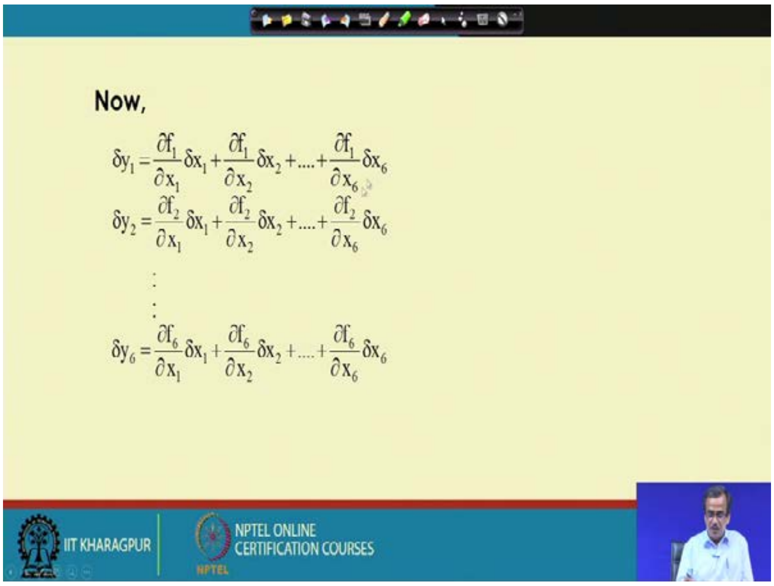

Following the same principle, so I can also write down, what is the small change in your y_2. Similarly, we can also find out, what is the small change in y_6 by following the same method.

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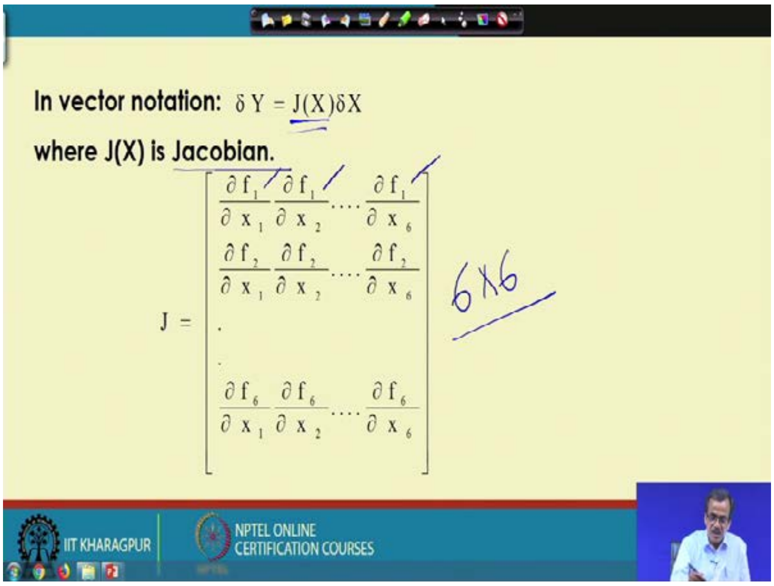

And, actually in vector notation, this can be written something like this, the change in Y, that is, delta capital Y is nothing but J (X) multiplied by the delta X, that is small change in X. Now, here, this particular J (X) is actually nothing but the Jacobian. And, this J matrix, that is, the Jacobian matrix is nothing but the partial derivative of f_1 with respect to x_1, partial derivative of f_1 with respect to x_2, partial derivative of f_1 with respect to x_6. So, this particular J matrix is having the dimension here of 6 cross 6. So, this is the way, actually, we can find out the Jacobian matrix.

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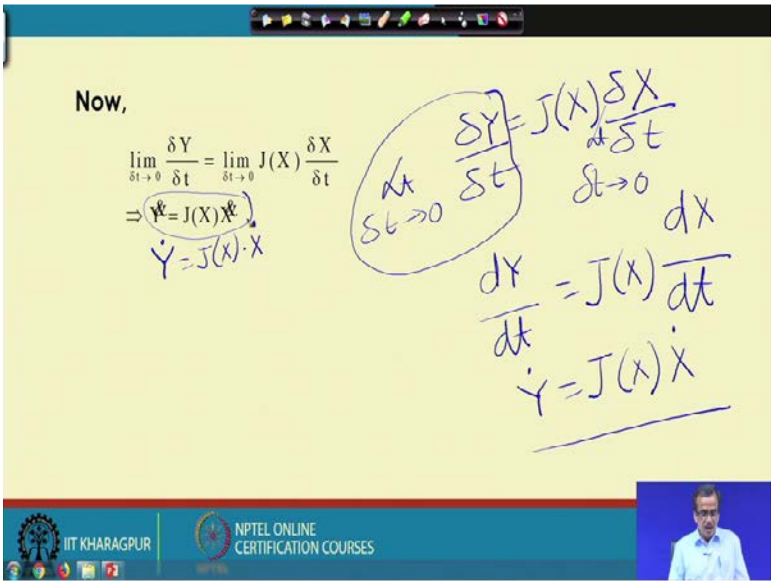

Now, let us see, how to use this particular Jacobian matrix further in robotics. Now, we have got this particular thing like the change in Y that is $\delta Y = J(X)\delta X$. Now, both the sides you divided by a small time that is delta t, and small time delta t, and put limit delta t tends to 0, and here, also you put limit delta t tends to 0.

Now, if I put limit delta t tends to 0, so by definition, so this is nothing but the definition of derivative that is $\frac{dY}{dt} = J(X)\frac{dX}{dt}$, so this is by definition the derivative. And, this can be written as $\dot{Y} = J(X)\dot{X}$, so this is the expression. And here, once again there is some typographical error, this is in fact, $\dot{Y} = J(X)\dot{X}$.

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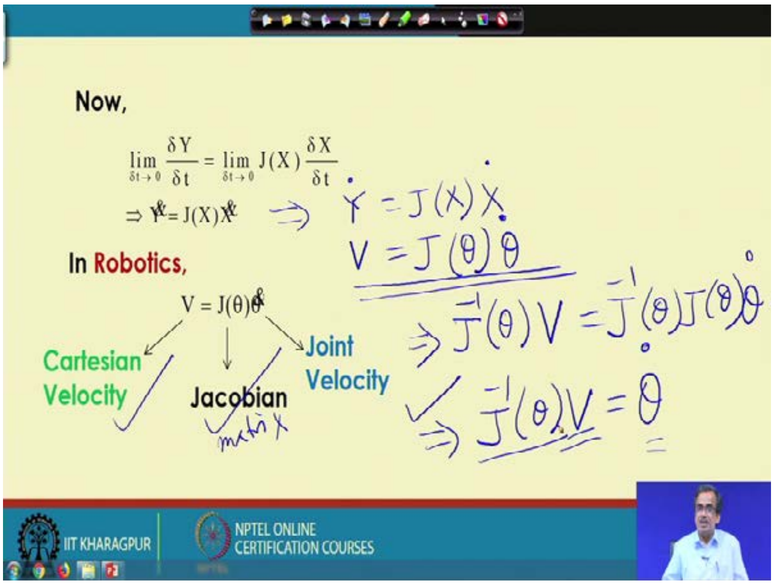

Now, here in robotics, actually, we are going to use this particular expression like this is nothing but $\dot{Y} = J(X)\dot{X}$. So, in robotics actually, this Y^dot we can replace by V, and this J (X), we can replace by J (θ) and this is nothing but $\dot{\theta}$.

Now, why theta, because theta is independent and X was also independent; so this joint angles, supposing that I have got a robot having six joints, all six are rotary joints, so joint angles are the variables. And, these particular theta are determined with the help of say six motors. And, actually, these thetas are nothing but the independent things. So, this can be written as $V = J(\theta)\dot{\theta}$.

Now, V is nothing but the Cartesian velocity, and J (θ) is nothing but the Jacobian matrix. And, $\dot{\theta}$ is nothing but the joint velocity. So, this is the way in robotics, we can connect the Cartesian velocity with the joint velocity with the help of Jacobian matrix. Now, here, we have written $V = J(\theta)\dot{\theta}$. Now, what you can do is, so we can multiply both the sides by $J^{-1}(\theta)V = J^{-1}(\theta)J(\theta)\dot{\theta}$. So, this becomes your $J^{-1}(\theta)V = \dot{\theta}$, ok?

So, this particular relationship we can find out very easily, that means, your we can find out, we can relate the Cartesian velocity with the joint velocity provided we know, the inverse of this particular matrix, that is, your $J^{-1}(\theta)$.

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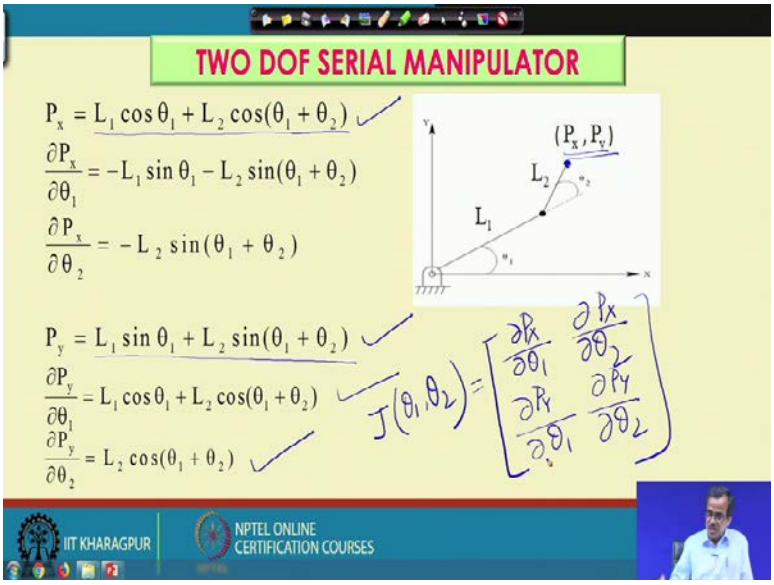

Now, let us try to concentrate on this particular J inverse (theta) thing. Now, let me try to take the help of one manipulator. But, before that let me tell one more thing, this V is nothing but the $V = J(\theta)\dot{\theta}$, and $J^{-1}(\theta)V = \dot{\theta}$.

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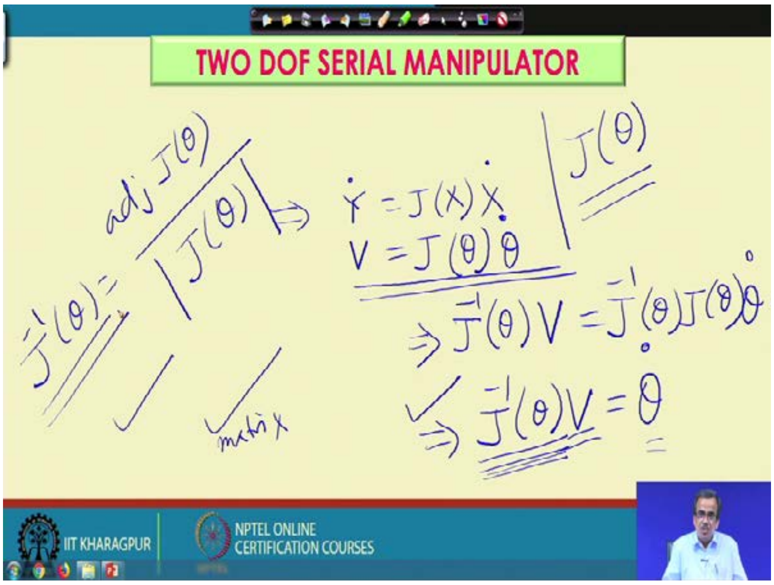

And, as I told that this particular J (θ), that is your Jacobian matrix, we should be able to find out the inverse of that. And, if you want to find out the inverse of this particular Jacobian matrix, which is nothing but adjoint of J (θ) divided by is your determinant of J (θ).

So, this particular J inverse theta to exist, that means, if you want to find out the inverse of J (θ), that is the Jacobian matrix, it has to be invertible, that means, it has to be non-singular. And, that is why, the determinant of J (θ) has to be non-zero. So, to exist this particular inverse of J (θ), the determinant of J (θ) has to be non-zero. Now, I can state in the different way that if I want to check the singularity condition of the manipulator, what I will have to do is, I will have to put the determinant of this particular J (θ) equals to 0.

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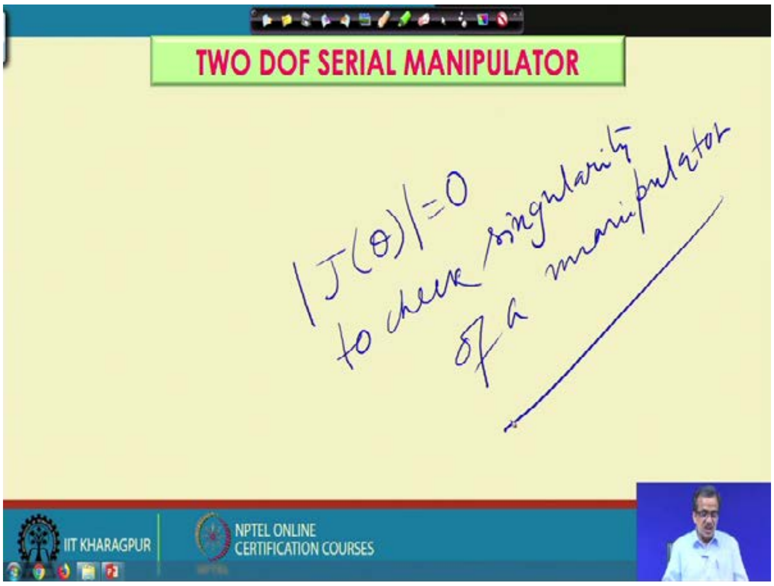

Now, here actually, what we can do is, so the determinant of this particular J (θ), we can put equal to 0, just to find out the singularity condition or to check singularity condition of a manipulator. So, this is the condition, which I am going to use. Now, let us let us take one example. And, let us check, how to determine the singularity of a particular manipulator.

Now, here, I am just going to take the example of a two degrees of freedom serial manipulator, ok? Now, once again, this is the well-known two degrees of freedom serial manipulator having the length of the link L_1 and L_2. And this is the position of the end-effector, whose coordinate is nothing but P_x and P_y. And, all of us we know that very easily you can find out the expression for P_x, $P_x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$.

Similarly, this $P_y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$. So, what I am going to do here is, I am trying to find out the partial derivative of these P_x with respect to theta_1. So, partial derivative with respect to theta_1, I will be getting as $-L_1 \sin\theta_1 - L_2 \sin(\theta_1 + \theta_2)$. Similarly, the partial derivative with respect to theta_2, so it will have no contribution, and here I will be getting $-L_2 \sin(\theta_1 + \theta_2)$. Then, comes your $P_y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$, this is a general expression. I can find out the partial derivative of P_y with respect to theta_1, that is, $L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$.

Then partial derivative of P_y with respect to theta_2 is nothing but $L_2 \cos(\theta_1 + \theta_2)$. Now, how to write down its Jacobian? Now, the Jacobian matrix J is a function of the variables: theta_1, theta_2 is nothing but the partial derivative of P_x with respect to theta_1, then partial derivative of P_x with respect to theta_2, then comes partial derivative of P_y with respect to theta_1 partial derivative of P_y with respect to your theta_2, ok?

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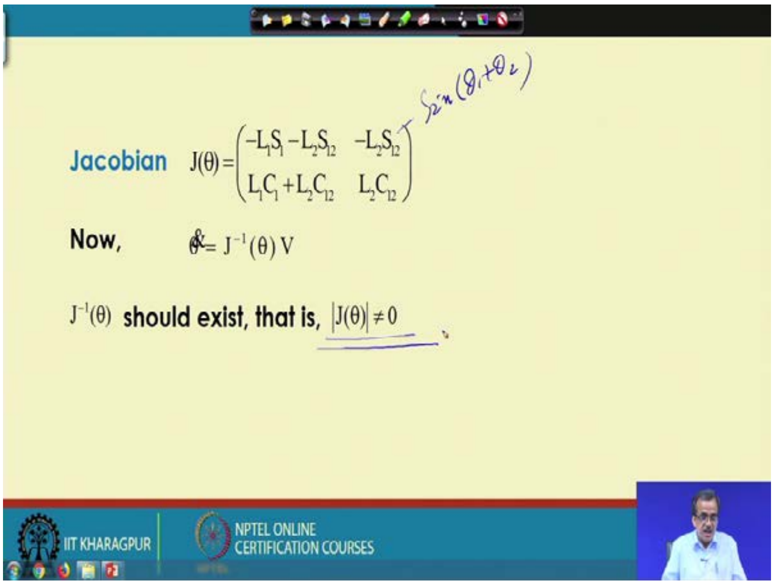

Now, we can insert those values for the partial derivative. And, if we insert the expression for the partial derivative, we will be getting the Jacobian matrix like this. So, this is the Jacobian matrix like your minus L_1 sin theta_1 minus L_2 sin of theta_1 plus theta_2 minus L_2 sin of actually theta_1 plus theta_2; similarly, L_1 cos theta_1 plus L_2 cos of theta_1 plus theta_2 and L_2 cos of theta_1 and theta_2.

Now, this thing I have already discussed that for J inverse (θ) to exist, so this particular condition has to be fulfilled, that means, the determinant of J (θ) has to be non-zero. So, if I want to check the singularity condition, what I am going to do is, I am just going to put the determinant of J (θ) equals to 0.

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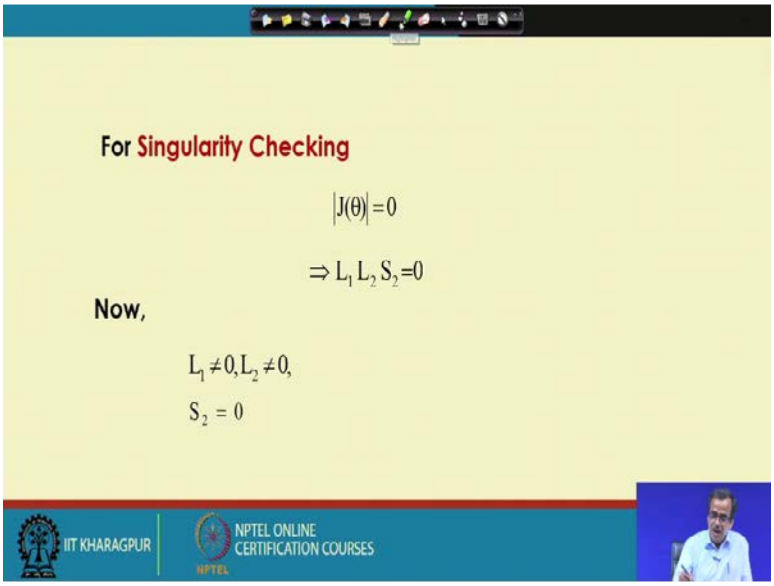

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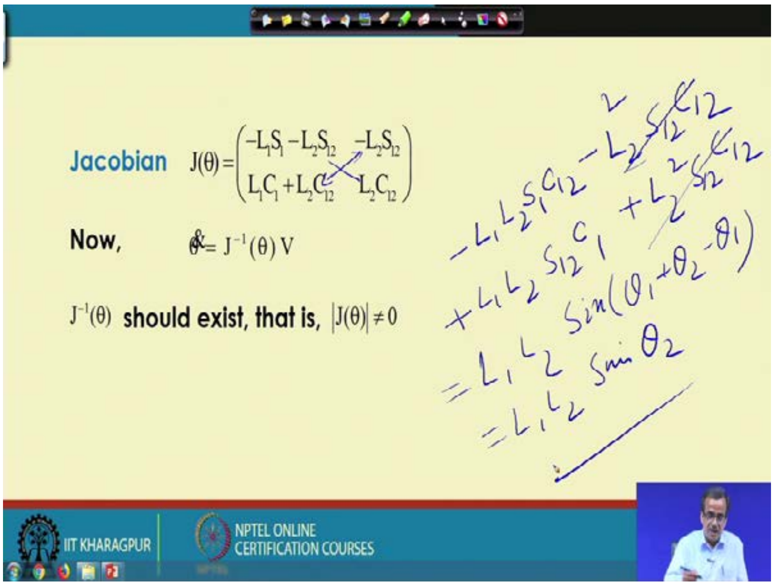

Now, if I put the determinant of J (θ) equals to 0, then what you can get is your, so very easily you can find out the determinant, determinant of this particular is very simple, so what we do is, so we multiply, so this one, that is, $-L_1L_2s_1c_{12} - L_2^2s_{12}c_{12} + L_1L_2s_{12}c_{12} + L_2^2s_{12}c_{12}$.

So, this minus and plus gets cancelled. And, now, we are we are having L_1 L-2. Now sin of theta_1 plus theta_2 cos theta_1 minus your cos of theta_1 plus theta_2 sin theta_1; so sin of theta_1 plus theta_2 minus theta_1. So, I will be getting your $L_1L_2 \sin\theta_2$. So, I will be getting this particular as the determinant. Now, if I put equal to 0, if I put, so this particular determinant equal to 0. So, $L_1L_2 \sin\theta_2$ equals to 0. But, L_1 L_2 are nothing but the length of the links, so that cannot be 0. So, the only possibility is your s_2 equals to 0, that is, $\sin\theta_2$ is equal to 0.

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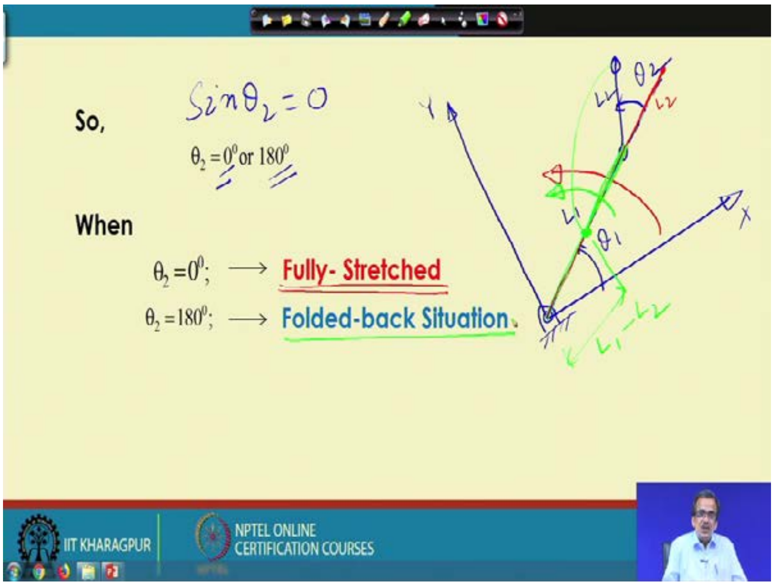

Now, if I put $\sin\theta_2 = 0$, in fact, there are two solutions for theta_2. There are two solutions for theta_2; one is theta_2 equals to 0; another is theta_2 equals to 180 degree. Now, let us see what happens for this particular the two degrees of freedom serial manipulator. So, this is your say I am just drawing here roughly that two degrees of freedom serial manipulator. So, this is the Cartesian X the Y coordinates. And, this is your L_1, and this is say L_2. So, this particular angle is your theta_1, this particular angle is your theta_2.

Now, let us see what happens, if I consider theta_2 equals to 0, now if I consider that theta_2 equals to 0, so what will happen is your, so this theta 2 if I put equals to 0, so this point will come here. So, the link will become this is up to this, it is L_1, and this will become L_2, so the total length of this particular link will be L_1 plus L_2. And, as if at this particular point, it is locked, ok? Now, it will behave just like a manipulator having only one degree of freedom, and the length of the link is like L_1 plus L_2, because theta_2 is locked to 0. Now, this will rotate, and there will be only one variable, one degree of freedom that is your theta_1. So, this is actually is known as the fully-stretched condition.

Now, then comes, if I consider that your theta_2 is equals to 180 degree. So, this particular point is going to come here, so my theta_2 will be here. So, this is my L_1, this is L_2. So, I will have one link, whose length is nothing but L_1 minus L_2 provided L_1 is greater than L_2. Now, this is once again is going to have, so this is my L_1, this is L_2 the tip will be here, and it will have only one movement that is theta_1, and practically it will have only one degree of freedom. So, although it is having two degrees of freedom, so one degree of freedom is lost, and this is known as the folded-back situation of this particular the manipulator.

So both, at fully stretched condition of this particular two degrees of freedom serial manipulator and the folded-back situation of this particular manipulator; so it is going to lose one degree of freedom out of two. And, this will behave as a manipulator having only one degree of freedom, and that is what you mean by the singularity condition of this particular the manipulator.

Thank you.

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Lecture - 24
Robot Dynamics

Now, I am just going to start with another new topic, that is, topic 4 and it is on Robot Dynamics. Now, the purpose of dynamics is to determine the amount of force, if it is a linear joint or the amount of torque, if it is a rotary joint, which is the reason behind the movement of the robotic links and the robotic joint.

Now, let us see, how to carry out the dynamics. Now, to carry out the dynamics, the prerequisite is the robot kinematics and trajectory planning, these things we have already discussed. Now, we are in a position to carry out the dynamic analysis, that means, we are in a position to find out the expression for the joint torque or the joint force, which is going to create that particular movement. Now, let us try to find out the mathematical expression for the joint torque or the joint force.

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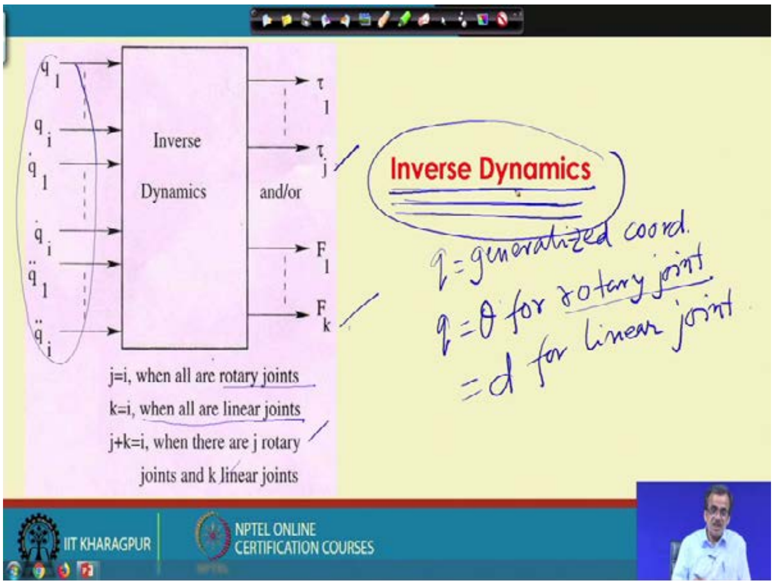

Now, before I proceed further, I am just going to define one term, that particular term is actually the inverse dynamics. Now, while discussing the kinematics, we have already discussed the meaning of two terms: one is called the forward kinematics and another is the inverse kinematics. Now, I am just going to start with another term, that is called the inverse dynamics. Now, let us see, what do we mean by this particular term: the inverse dynamics.

Now, let us concentrate on this particular block diagram: the input side and the output side. Now, in the input side, I have written like q_1 q_2 up to q_i, then q_1^dot q_2^dot up to q_i^dot then q_1^double dot q_2^double dot up to q_i^double dot. Now, this particular q actually represents the generalized coordinate. And, this particular generalized coordinate for a rotary joint, this q is nothing but theta for rotary joint, and this is equal to d for the linear joint, and this q^dot is nothing but the first time derivative. So, if it is theta, then that is the angular velocity, and if it is d, that is a linear velocity, then q^double dot, if it is theta, then it is angular acceleration; and if it is d, that is the linear acceleration.

So, if I consider that all i joints are rotary for a particular manipulator, then this is nothing but $\theta_1, \theta_2, ..., \theta_i$, $\dot{\theta}_1, \dot{\theta}_2, ..., \dot{\theta}_i$, $\ddot{\theta}_1, \ddot{\theta}_2, ..., \ddot{\theta}_i$. So, these are nothing but the inputs and what are the outputs? The outputs are $\tau_1, \tau_2, ..., \tau_j$ and or $F_1, F_2, ..., F_k$. Now, here, all the joints are rotary joints, then if I write that k is equal to i that means all the joints are linear joints. For example, say Cartesian coordinate robot has all linear joints. If j plus k is kept equal to i that means, we have got a combination and we have got j number of rotary joints and k number of linear joints, ok?

So, here, the inputs are the independent variables and outputs are nothing but either the joint torque or the force or a combination, so this particular problem of dynamics is defined as your inverse dynamics, but remember, this is not the forward dynamics. So, this is, in fact, the inverse dynamics and I am just going to discuss in this particular course the inverse dynamics problem of this particular the robots and I am not going to discuss the forward dynamics. So, I am just going to discuss the inverse dynamics. So, this is what we mean by the inverse dynamics problem, which I am going to tackle in this particular course.

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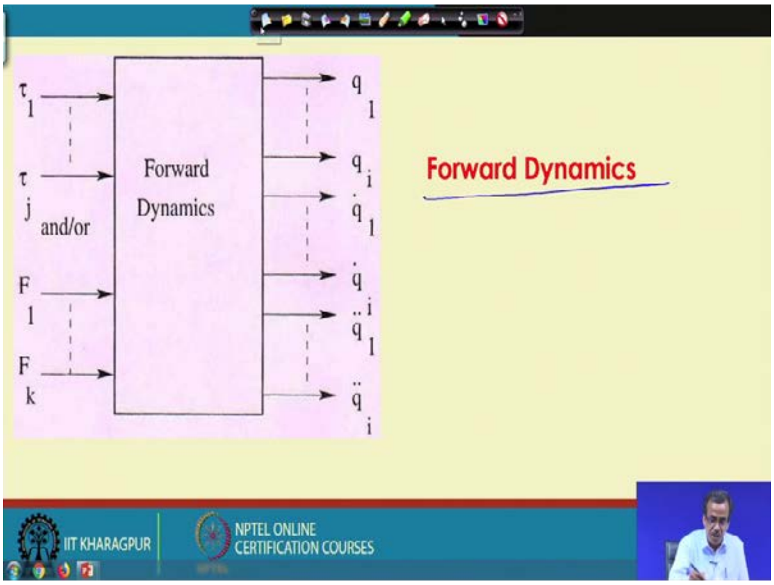

And, as I told, the reverse of that particular previous problem is the problem of the forward dynamics and here, in forward dynamics, as usual, as I told. So, the joint torques and forces will be the inputs and the outputs will be your joint angles, that angular velocity, angular acceleration, ok, so, this is nothing but the forward dynamics. Now, here in this course, as I told, I am just going to consider only the inverse dynamics, but not forward dynamics. If we want to solve the forward dynamics in fact, you will have to take the help of the tools like neural networks, fuzzy logic, which I am not going to discuss in this particular course. So, I am just going to concentrate on the inverse dynamics.

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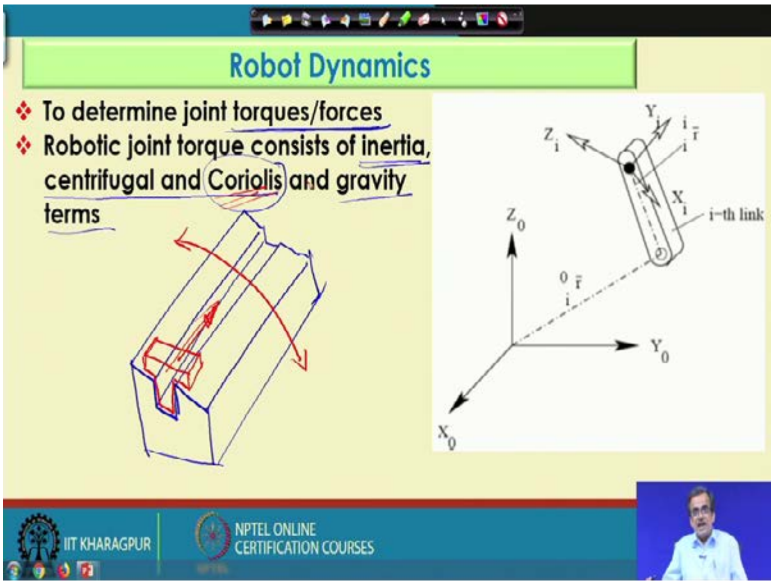

Now, as I told, the purpose is to determine actually the joint torque or the joint force, and if I concentrate on the joint torque, the joint torque consists of a few terms: one is called the inertia terms, another is called the centrifugal and the Coriolis term, another is called the gravity terms. So, gravity terms depends on the acceleration due to gravity, inertia terms depends on the mass distribution of the robotic link and that is expressed in terms of the moment of inertia matrix and centrifugal force, all of you know the concept, and there is another thing that is called the Coriolis component or the force.

Now, this Coriolis force actually requires some explanation, now this Coriolis component will come whenever there is a sliding joint on a rotary link, let me try to prepare one very rough sketch for that just to explain the concept of this Coriolis force, let me try to prepare one very rough sketch. I am, in fact, going to draw one robotic link, so this is nothing but a robotic link, and this end is actually connected to the motor and this is the other end and supposing that on this robotic link. So I have got one sliding component like this, let me try to prepare one sketch for the sliding component. So, this is roughly the sketch for one sliding component and this particular sliding component is going to slide here on this groove. So, this sliding component is sliding in this particular direction and this link is rotating, and this part here is connected to the motor.

So, roughly, I think, it can be visualized, so this particular sliding member is sliding and the link is rotating. So, now, in that case, so it will be subjected to some amount of force which is known as the Coriolis force, this is the concept of the Coriolis force. And, of course, if I want to consider the friction, so that friction also we can consider. Now, here, I am just going to concentrate on this particular figure, so these are x_0, y_0, z_0, nothing but the base coordinate system of this particular robot, and I am just going to concentrate on a particular link, that is nothing but the i-th link.

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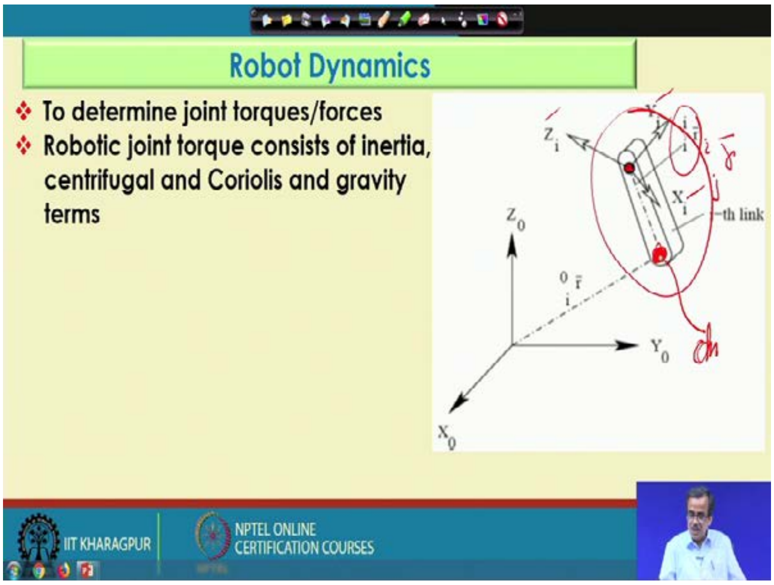

Now, here, this is the i-th link and for this particular i-th link, the motor is connected at this particular end but very purposefully. So, I have put the coordinate system here but not here, the reason is very simple: we, mechanical engineers, we always try to think about the reaction force, reaction torque. So, whenever we calculate any force or torque that is nothing but the reaction force/torque.

Now, here, actually, if I just want to put one motor, the motor is going to generate torque, it is going to generate some angular displacement, velocity and acceleration sorts of thing, now, if I want to measure that, what I will have to do is, I will have to measure the reaction torque. How to measure? To measure the reaction torque, I will have to concentrate here, that other end, and that is why, actually this particular coordinate system is attached here but not here, and here, the coordinate system is denoted by X_i, Y_i, Z_i.

Now, supposing that, on this particular link, I am just going to concentrate on a particular point, say let me consider this particular point and supposing that this point is having the differential mass, say dm, small mass dm. Now, this particular point lying on the i-th link can be represented in its own coordinate system by a position vector, which is nothing but r_i with respect to i^bar; that means, here I am consider considering i-th point lying on the i-th link, I can also consider the j-th point lying on the i-th link in that case the representation will be r_j with respect to i^bar, ok? But, here, I am just going to use r i with respect to i; that means, the i-th point lying on the i-th link, I am just going to consider and it is position is denoted by r_i with respect to i^bar.

The same point can be represented in the base coordinate system of the robot by another position vector, which is nothing but r_i with respect to 0^bar. So, the same point in its own coordinate system, in its base coordinate system. And, we have studied in robot kinematics and we know how to relate this particular r_i with respect to 0 and this r_i with respect to i.

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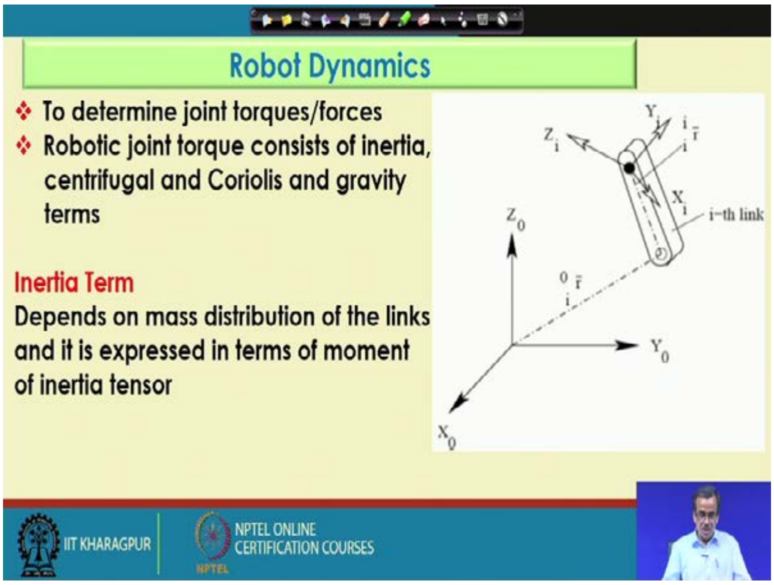

Now, if you see this particular relationship, so we can find out that I am going to discuss in the next slide and in fact, I am just going to concentrate first on the inertia terms. Now, here, on the inertia terms, actually, we try to find out the mass distribution that is the moment of inertia term. So, this inertia terms, I am just going to concentrate first.

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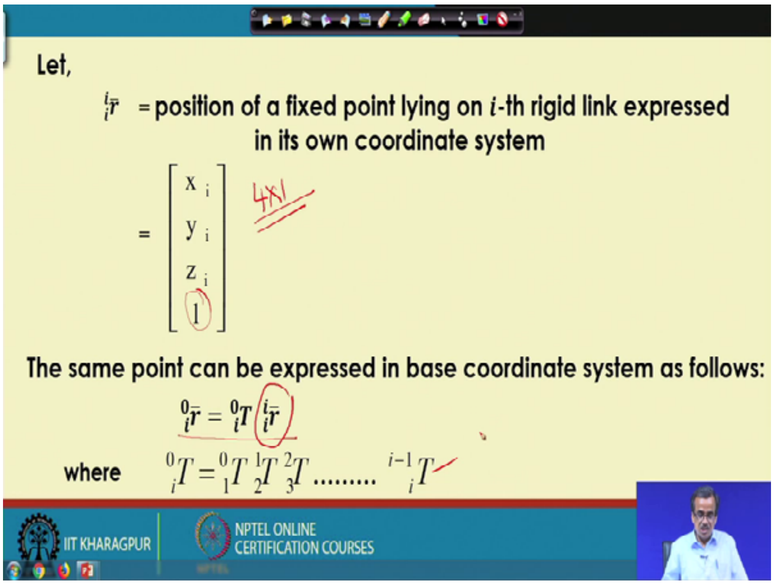

Now, here, this r_i with respect to , that is the position of the i-th particle lying on the i-th link is denoted by x_i y_i z_i, so, this is the coordinate and this particular 1 actually I am just putting to make this position vector as a 4×1 matrix, I hope you remember, this we followed, while deriving the expression for homogeneous transformation matrix. So, at the bottom of this particular position terms, we put 1.

Now, this r_i with respect to i is nothing but x_i y_i z_i 1 and this is nothing but a 4×1 matrix, now this particular expression let us try to concentrate this is how to relate that position of i-th particle lying on the i-th link with respect to the base coordinate system provided, I know this r_i with respect to i.

So, what I will have to do is, I will have to multiply T_i with respect to 0, that is the transformation matrix of i with respect to the base coordinate frame by r_i with respect to i. And, all of us, we know and we have studied in kinematics that this T_i with respect to 0 is nothing but T_1 with respect to 0 multiplied by T_2 with respect to 1 multiplied by T_3 with respect to 2, and so on, and the last term is T_i with respect to i minus 1. So, T stands for the transformation matrix and all of us we know how to derive and how to find out this particular expression.

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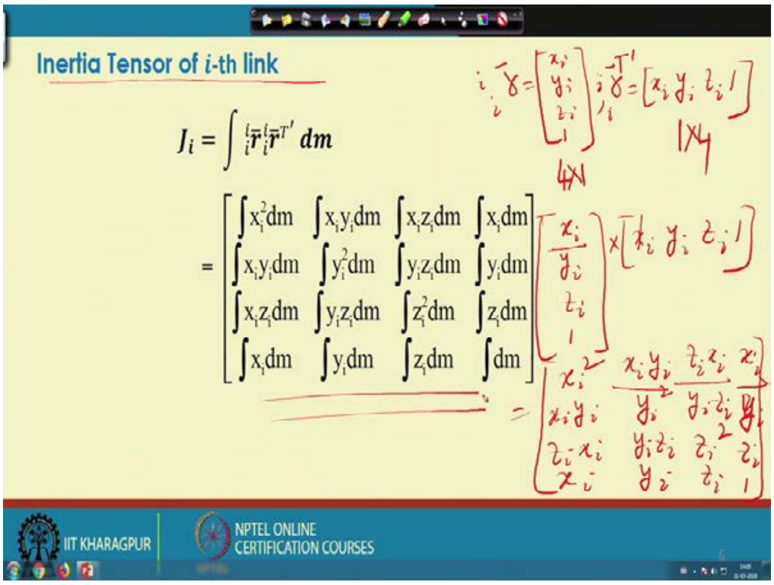

Now, I am just going to concentrate on the concept of this particular inertia, that is the moment of inertia term, how to define this moment of inertia and how to find out the inertia tensor for a particular the robotic link or the i-th link. Now, moment of inertia, all of us we know by definition that is, m r square and for this particular i-th link, the moment of inertia that is denoted by J_i, that is, integration r_i with respect to i multiplied by r_i with respect to i transpose dm.

So, here, actually, we will have to find out this particular the tensor. Now, let us see, how to find out this particular tensor. So, this r_i with respect to I, as we have seen, is nothing but is your x_i y_i z_i 1 and this r_i with respect to i transpose is nothing but is your x_i y_i z_i 1, ok? Now, if I multiply so r_i with respect to i by r_i with respect to i transpose; that means, this is nothing but a 4 cross 1 matrix and this is nothing but a 1 cross 4 matrix and if you multiply then you will be getting a 4 cross 4 matrix.

So, if I just multiply, this is something like this: x_i y_i z_i 1 multiplied by your this particular matrix x_i y_i z_i 1. So, first row and first column, I will be getting your x_i square then, first row second column is x_iy_i, first row third column z_i x_i, first row fourth column is x_i, similarly y_i x_i so x_i y_i then comes here y_i square y_i z_i and this is nothing but y_i, next is your z_i x_i then comes your y_i z_i then comes z_i square then comes z_i then x_i y_i z_i 1. So, this is the 4×4 matrix, which we will be getting and this is exactly same as this.

So, this J_i is nothing but that is the moment of inertia tensor for i-th link is $\int x_i^2 dm, \int x_i y_i dm, \int z_i x_i dm, \int x_i dm$.

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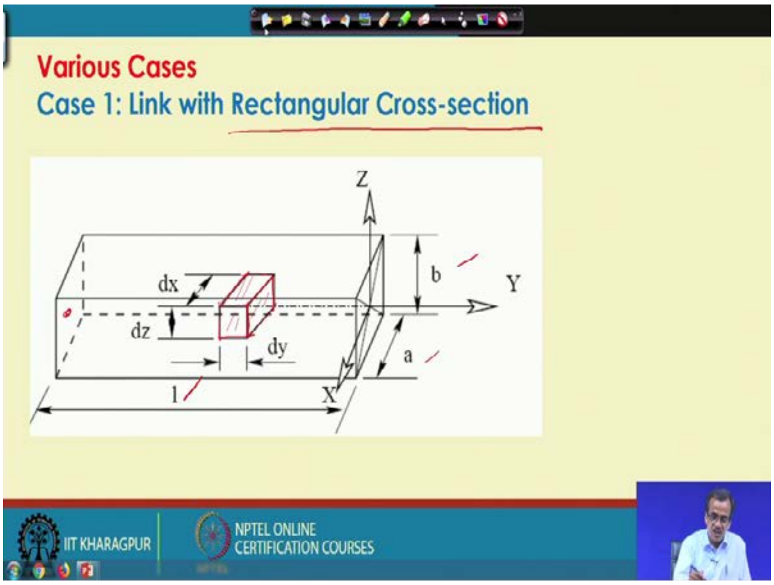

Now once you have got this particular inertia tensor, now what you can do is, we can consider the different cases like the different types of the links we can consider for example, say if I consider the robotic link with rectangular cross section. So, very easily we can find out what should be the expression of this particular the inertia tensor.

Now, let me consider 1 robotic link with rectangular cross section, truly speaking the robotic the motor the motor is connected here and I will have to put the coordinate system or the other end this is the length of the link that is l and it is having the dimension a and b the cross section. And I will have to find out the inertia tensor for this particular the robotic link and for simplicity. So, we are considering a robotic link with constant cross section and that 2 rectangular cross section with sides a and b.

Now, let us see how to find out the inertia tensor we concentrate on this particular differential mass. So, this particular differential mass the small mass having the dimension of the dx then comes here dy and this is dz, so we try to concentrate on this particular the element. Now if we concentrate on this particular element, so very easily can find out the differential mass, we are trying to find out the moment of inertia and that is by definition m r square so always it has to be positive.

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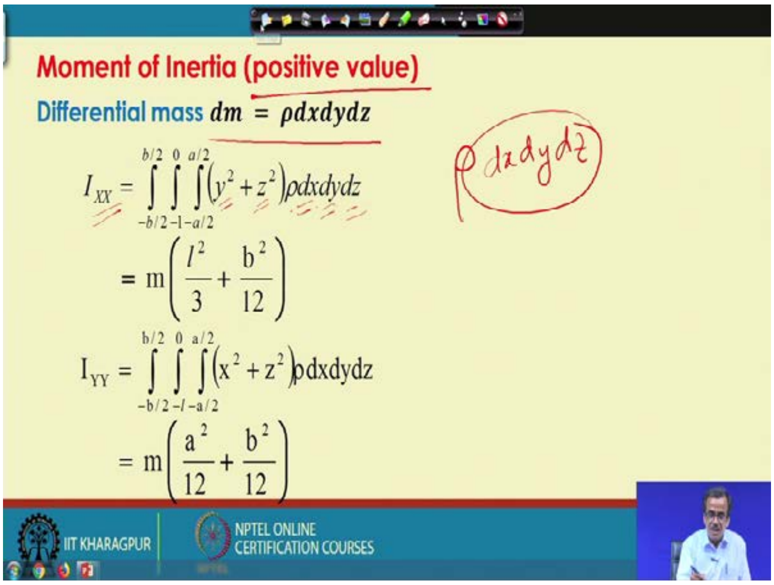

So, it could be positive and differential mass dm is nothing but the volume is dx dy dz. So, this is the volume of the differential mass and multiplied by the density rho this is nothing but the differential mass. Now, moment of inertia about xx, that is, I_xx is nothing but triple integration, so this is about I xx as so r square m r square. So, $r^2 = y^2 + z^2$ and $\rho dxdydz$ and now we will have to decide the limit of integration first you concentrate on dx then dy and after that dz.

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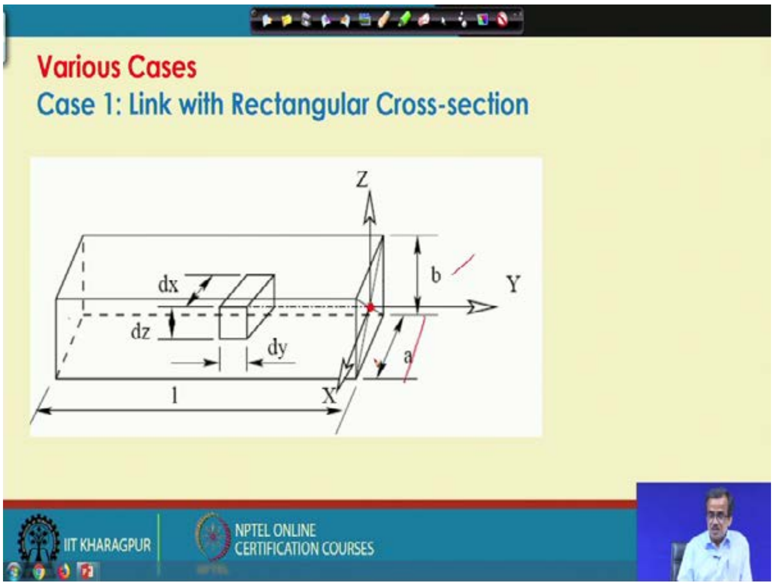

Now, let us go back to the previous slide now here and we are just going to find out the limit for dx, now this is the x direction and along this particular x direction the total dimension is a and this is at the midpoint. So, x will vary from $-a/2$ to $+a/2$ and what should be the range for the limit for y. Now, the coordinate system is here, so if it is here this corresponds to 0 value of y. So, this particular part is minus so from - l to 0. So, the range for this particular y, it is - l to 0 and the range for the z along the z direction the total dimension is b, so from – b/2 to + b/2.

Now, you see. So, what you can find out, we can find out the expression for this particular i the limit for this integration. And, if we just solve this particular integration for example, x is – a/2 to + a/2, then dy is -l to 0, then dz –b/2 to +b/2 and all of us know how to carry out this particular integration, and all of you please practice to find out, to derive this particular expression and if you carry out this particular integration.

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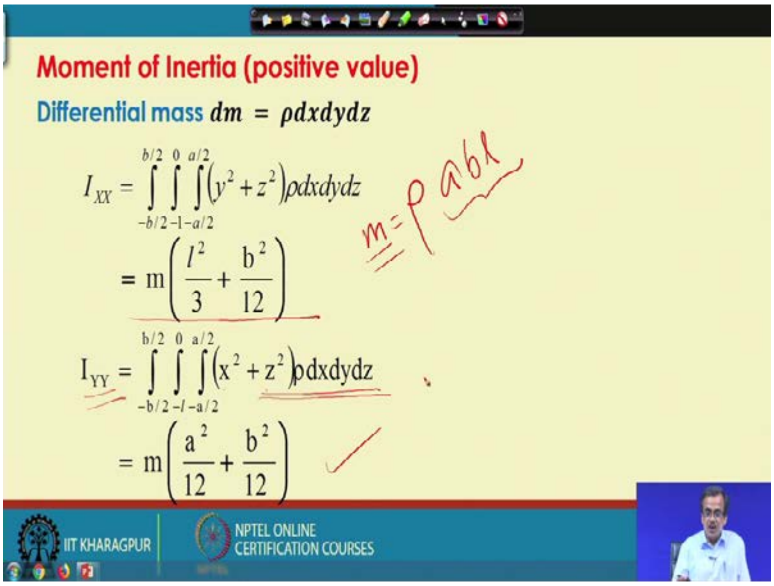

So, you will be getting the expression that is nothing but is your $I_{XX} = m(\frac{l^2}{3}+\frac{b^2}{12})$, where m is nothing but the mass of this particular link that is nothing but $m = \rho abl$. So, abl is nothing but the volume of this rectangular link multiplied by ρ is nothing but the mass.

Now, by following the similar procedure, so we can also find out this I yy that is nothing but triple integration in place of r square we will have to write down $x^2 + z^2 and\, \rho dxdydz$ and the limit for integration will be the same as I discussed earlier and if we carry out this particular integration, I will be getting $I_{YY} = m(\frac{a^2}{12} + \frac{b^2}{12})$. So, all of you please try to derive this particular expression and you will be getting it.

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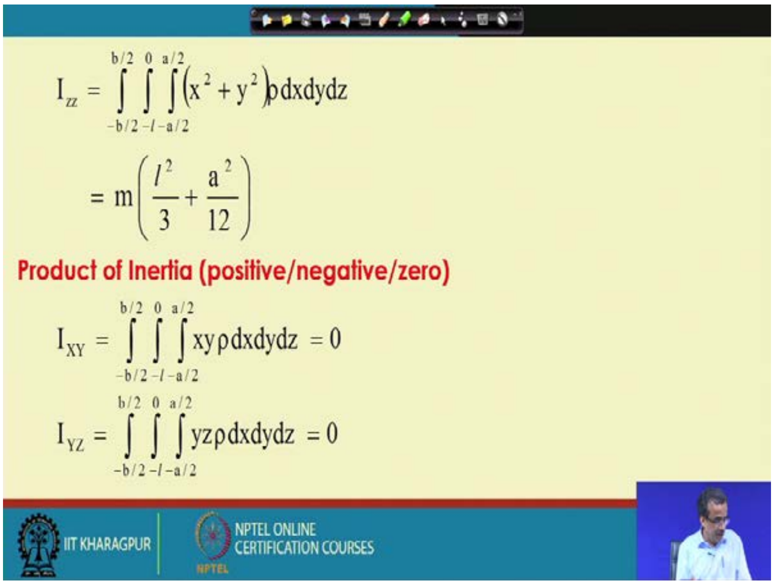

Similarly, by following this we can also find out what is your I zz and that is nothing but in place of r square, so I will have to write down $x^2 + y^2$ and then if you carry out this integration. So, you will be getting $m(\frac{l^2}{3} + \frac{a^2}{12})$, then comes the concept of product of inertia. Now this product of inertia in place of r square so I will have to write down so xy yz zx like this and this product of inertia all of you know it could be either 0 or it could be negative or it could be positive.

So, all 3 possibilities are there for example, if we calculate $I_{XY} = \iiint xy\rho dxdydz$ and following the same limit of this integration and if you carry out this particular integration. So, we will be getting that is equals to 0. Please try to derive this and check then I yz. So, you will have to write down yz here, so I will be getting this is equal to 0.

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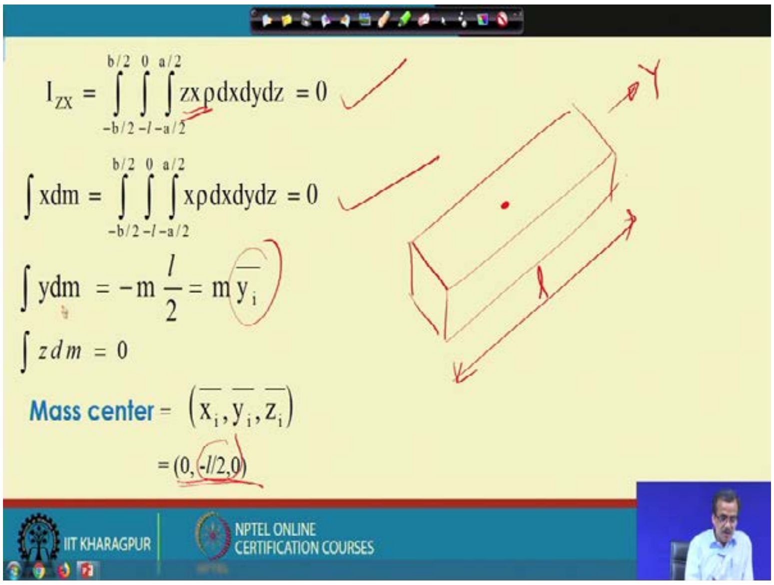

Now, similarly so I can find out, so this I_zx is nothing but this and it will be getting this particular expression. So, I here will have to write down zx and if you carry out this integration we will be getting equals to 0. Similarly, we will have to find out $\int xdm$. So, this is nothing but $x\rho dxdydz$, the same limit for integration and if you calculate we will be getting that is equals to 0, then comes your integration y dm so this will become - m l /2.

Now, if you see this particular you are if you if you just see this rectangular cross section link it is something like this, now here so the mass center is here ok. Now this is the y direction and the total length is your l ok. So, if you see the mass center so x coordinate of the mass center is 0 the y coordinate of the mass center is nothing but - l by 2 and the z coordinate is 0. So, this is actually the coordinate of the mass center and that is why, $-m\frac{l}{2}$ can be written as $m\overline{y}_i$ and then $\int zdm$ is equal to 0.

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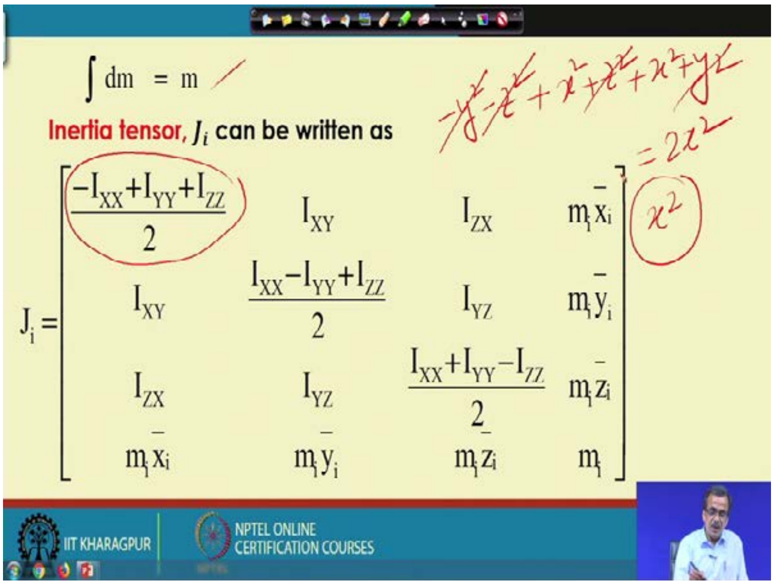

So, if you carry out this integration now we can also find out $\int dm$ equals to m, now I am just going to concentrate on this particular very well known inertia tensor, which is available in all the textbooks of Robotics, ok. But, how to derive this particular inertia tensor the general expression, so let us try to concentrate on this.

Now, while determining this I_xx if you remember we consider y square plus z square and there is a minus. So, I can write down minus y square minus z square. While determining I yy I consider x square plus z square; while determining I zz I consider x square plus y square. So, minus y square plus y square minus z square plus z square gets cancelled so I am getting 2 x square divided by 2, so I will be getting x square.

Now, if you see the previous expression of the inertia tensor, so this particular expression of the inertia tensor the first term is your x_i^2. So, to get this x i square actually I am using so and to get the other terms, so we are using this particular the final expression for this inertia tensor. So, this particular inertia tensor can be derived starting from the first principle and now if I just put all the expression the values of I_xx, I_yy, I_zx and all such things.

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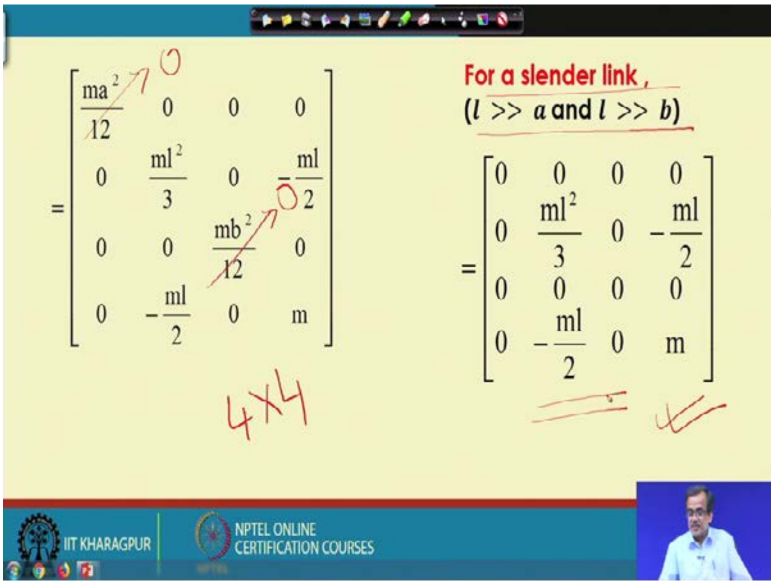

So, I will be getting this particular the matrix, this is nothing but 4×4 inertia tensor for this particular the rectangular link, ok. Now, there is a concept of slender link for which l is very large compared to a the length of the link is very large. That means if I consider the slender link. So, $\frac{ma^2}{12}$ will tend to 0, $\frac{mb^2}{12}$ will tend to 0 and I will be getting this particular matrix as a inertia tensor matrix for a slender link, so this is the inertia tensor.

Thank you.

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Lecture - 25
Robot Dynamics (Contd.)

Now, I am going to discuss how to determine the inertia tensor for the robotic link having circular cross-section.

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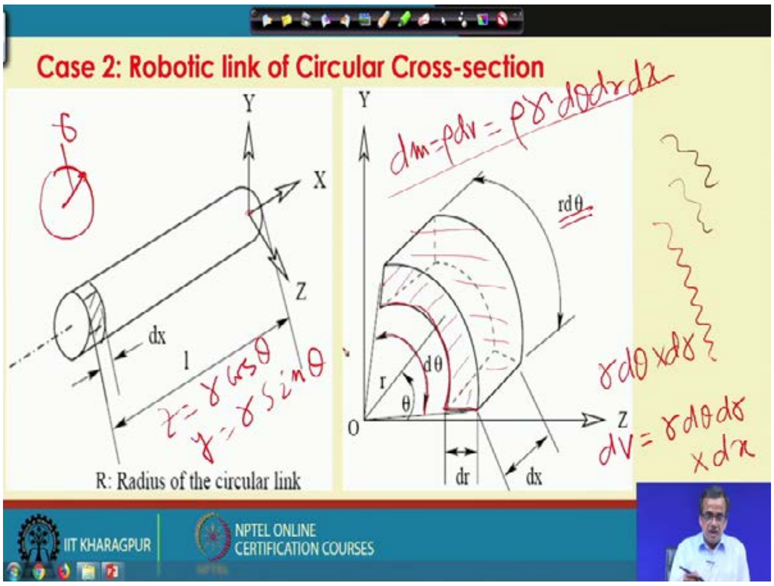

The length of the robotic link is equal to l and it is having the circular cross section with the radius r. Now, here, I am just going to consider a small element, the same element I am just going to redraw here. So, this is a X Y and Z. So, the coordinate system is attached here.

Now, let us try to concentrate on this small element, now this is r, this particular included angle is your $d\theta$. So, this arc is nothing but your $rd\theta$. So, this is nothing but is your $rd\theta$ and this is a dr. So, the cross-sectional area of this part is shaded part and it is nothing but is your $rd\theta$ multiplied by dr and so, to determine the volume that is your dv, what we do is, this area $rd\theta dr$ multiplied by dx. So, this is nothing but the volume.

So, if I want to find out the differential mass, that is, dm is nothing but ρdv and that is nothing but is your rho r then comes your d theta dr dx. So, this is nothing but the differential mass of this small element. So, the differential mass of this small element is nothing but $\rho r d\theta dr dx$. And, now let us try to find out its moment of inertia and before that, let me write down, that this particular z, z is nothing but $r\cos\theta$ and your y is nothing but $r\sin\theta$.

Now, by using this particular expression, we can find out the moment of inertia.

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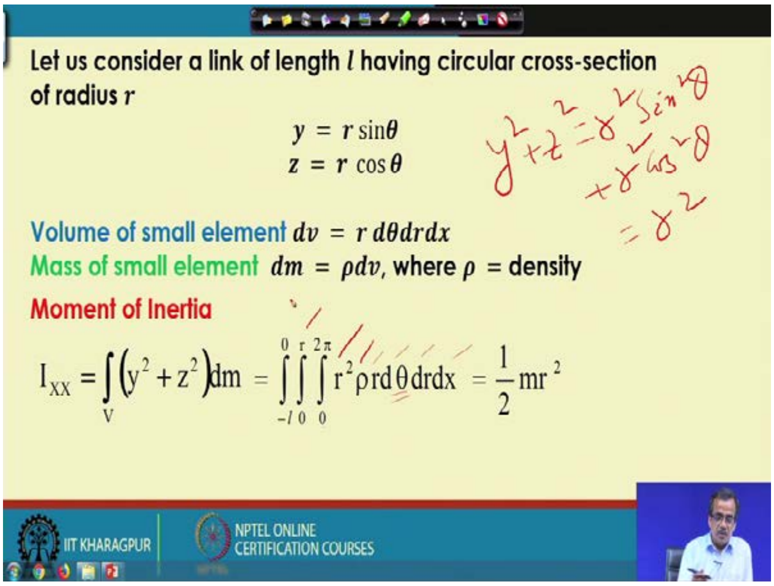

Now, this is your differential mass, as I told ρdv, that is, $\rho r d\theta dr dx$, where ρ is the density. Now, moment of inertia about xx, that is, I_xx is nothing but $\int_v (y^2+z^2)dm$.

Now y square plus z square is nothing but $r^2\sin^2\theta + r^2\cos^2\theta$. So, this is nothing but your r^2.

So, $y^2+z^2=r^2$ and this particular dm is nothing but $\rho r d\theta dr dx$ and let us try to find out the limit of this integration now, $d\theta$. So, θ will vary from 0 to 2π, then comes your r, r will vary from 0 to r and x will vary from -l to 0, now let us try to see, what happens like how to decide the range for this particular x.

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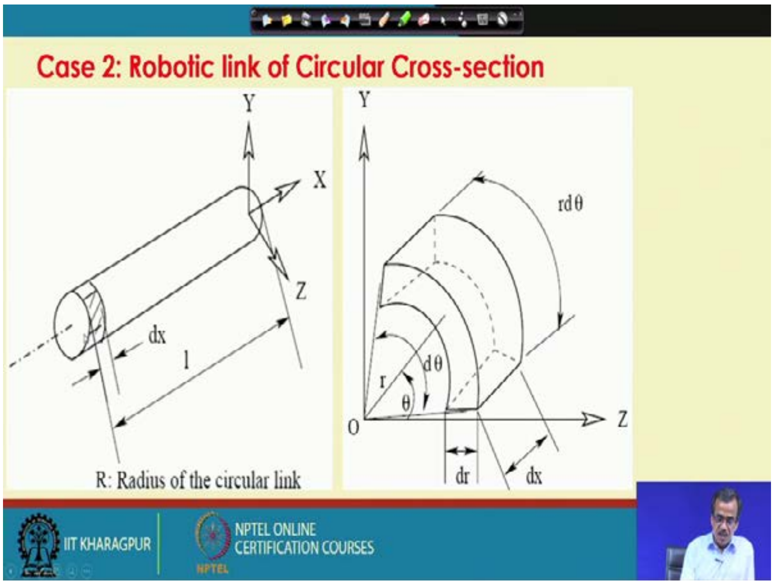

Now, here, we have got the origin of the coordinate system.

So, it is from - l to 0 that is the range for your x.

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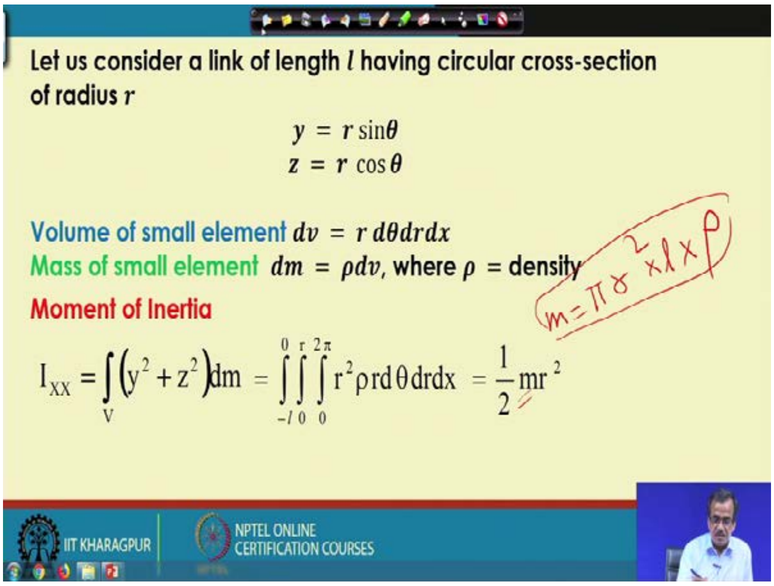

So, we can find out this and we can carry out this integration. And, if we carry out this integration we will be getting $\frac{1}{2}mr^2$, where m is nothing but the mass of this particular the link having a circular cross-section. Now, this mass m can be determined as $\pi r^2 l$ is the volume multiplied by ρ, that is, a density.

So, $\frac{1}{2}mr^2$ is nothing but the moment of inertia about xx.

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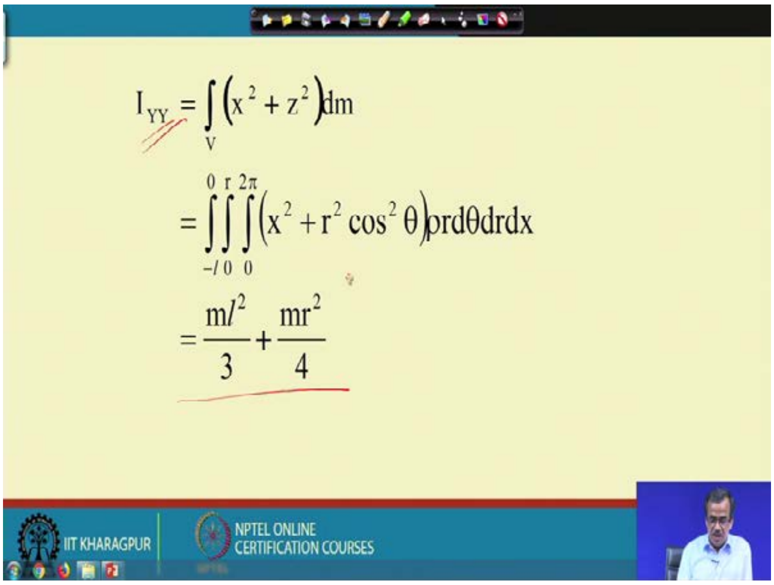

Now, if I see this moment of inertia about YY, that is nothing but the $\int_v (x^2 + z^2)dm$. So, in place of z^2, I am putting $r^2 \cos^2 \theta$, and next is your $dm = \rho r d\theta dr dx$ and if you carry out this particular integration and if these are the limits for integration. So, very easily you can find out. So, this particular expression for I_YY, that is nothing but the moment of inertia about YY, that is, $I_{YY} = \frac{ml^2}{3} + \frac{mr^2}{4}$.

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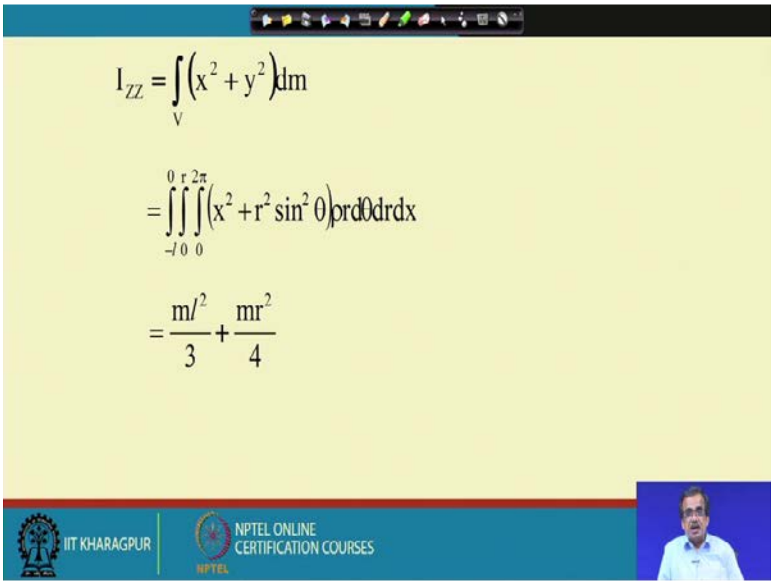

So, following this I can also find out the moment of inertia about ZZ and that is nothing but $\int_v (x^2 + y^2) dm$ and a y square is nothing but $r^2 \sin^2 \theta$ then, $\rho r d\theta dr dx$. And, if we carry out this particular integration we will be getting $\frac{ml^2}{3} + \frac{mr^2}{4}$. So, I can find out the moment of inertia about ZZ, the next is the product of inertia.

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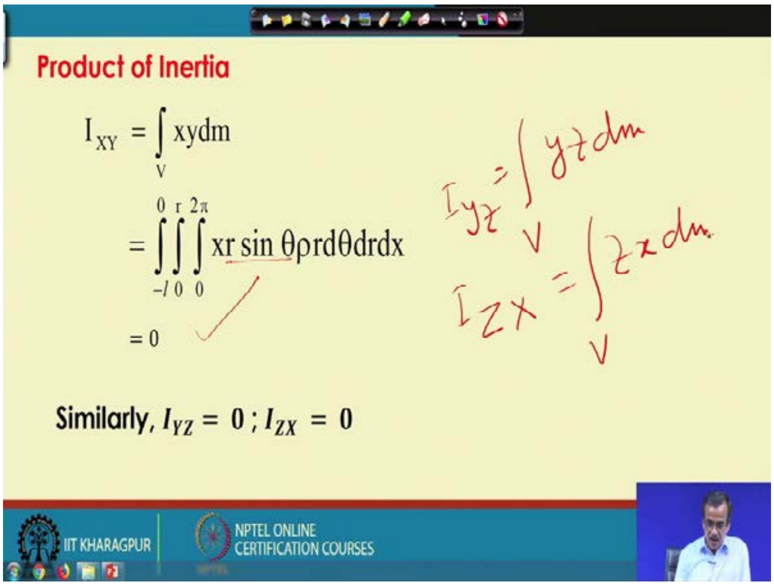

The product of inertia I_XY is nothing but the $\int_v xydm$.

Now here, y is nothing but your $r\sin\theta$ and dm is $\rho rd\theta drdx$ and these are the ranges for the integration, and if you carry out this integration, we will be getting I_XY is equal to 0. Now, by following the similar method, we can also find out what is I_YZ. I_YZ is nothing but is your the $\int_v yzdm$ and if you carry out this integration you will be getting I_YZ is equal to 0. Similarly, I can also find out the product of inertia that is I_ZX and that is nothing but $\int_v zxdm$. And, if I carry out this particular integration and we will be able to find out that I_ZX is equal to 0.

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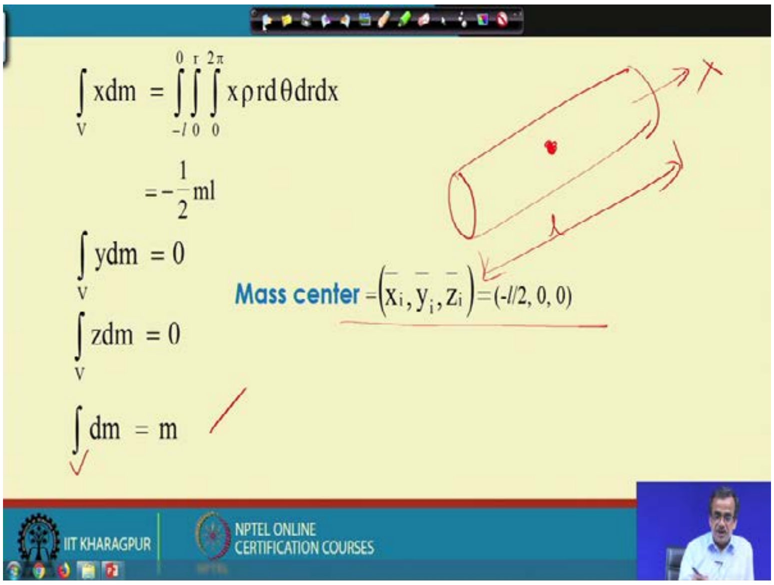

Now, here, we have got this particular product of inertia, now $\int_v xdm$, if you carry out. So, I will be getting $-\frac{1}{2}ml$, then $\int_v ydm$ will become equal to 0, then $\int_v zdm$ will become equal to 0. Now, the mass center, that is, X_i bar, Y_i bar, Z_i bar is nothing but - l / 2, 0, 0 like, if I draw this particular circular cross-section robotic link, my X direction is along this particular direction and the total length is l.

So, its mass center will be here whose coordinate is nothing but - l / 2, 0, 0. So, this is nothing but the coordinate of the mass center. Then, comes your integration, the volume integration of dm, so, this is nothing but m. So, we can find out all such integrations and this is, the volume integrations. So, once you have got all such expressions, now, I can find out the inertia tensor.

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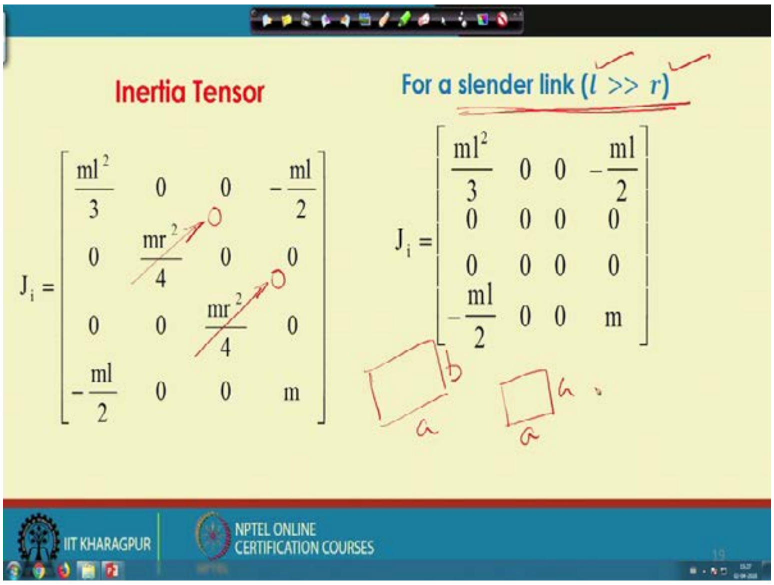

So, this inertia tensor for this particular link with circular cross-section, that is denoted by J_i will become equal to $\frac{ml^2}{3},0,0,-\frac{ml}{2};0,\frac{mr^2}{4},0,0;0,0,\frac{mr^2}{4},0;-\frac{ml}{2},0,0,m$.

Now, if I consider the slender link, where l is very large compared to r, l is very large compared to your r, in that case, this $\frac{mr^2}{4}$. So, this can be neglected. So, this tends to 0, then $\frac{mr^2}{4}$ tends to 0. So, this will become the inertia tensor $\frac{ml^2}{3},0,0,-\frac{ml}{2};0,0,0,0;0,0,0,0;-\frac{ml}{2},0,0,m$. So, this is nothing but the inertia tensor for the robotic link having circular cross section of radius r.

Now here, till now we have considered the rectangular cross-section having the dimensions a and b. So, I can also consider the square cross-section like having the dimensions a and a. So, for this particular cross-section robotic link, we can also find out the inertia tensor. So, this type of robotic link is having the constant cross-section. If you see in the robotic link, the cross-section is varying, the same is true in our hand also.

For example, say if I consider say, this is the robotic links. Now, if I take one cross-section here, if I take another cross-section here, the cross-section is not the same. So, cross-section is going to vary and it is having the varying cross-section. So, determining this inertia tensor is not so easy, the same is true for the actual robotic link. In actual robotic link generally, we do not consider the link with constant cross-section and the cross-section is going to vary along the length.

So, it is a bit difficult to determine the inertia tensor. In fact, we try to take the help of some sort of finite element analysis to find out what should be that particular inertia tensor. So, this is the way, actually we try to find out the inertia tensor for the robotic link.

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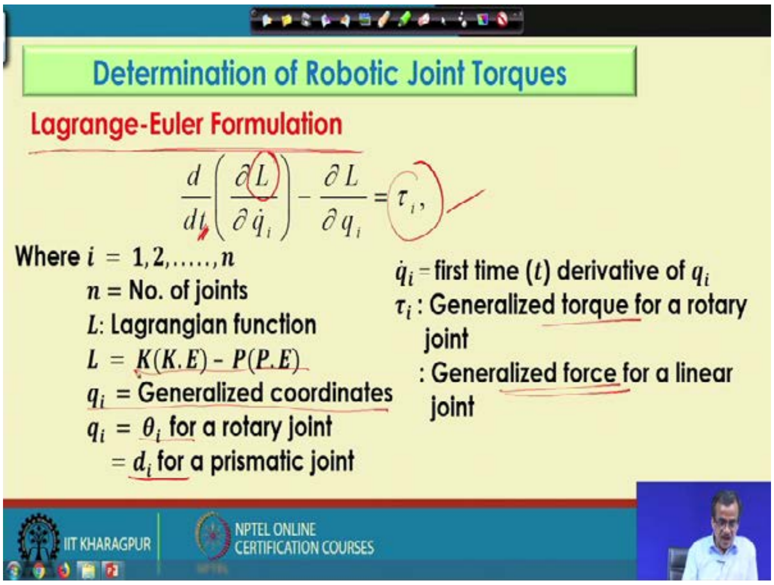

And, once you have got the inertia tensor, now, we are in a position to determine the mathematical expression for the joint torque or the joint force.

Now, here actually, we are going to use Lagrange-Euler formulation, truly speaking this is the Lagrangian method, Lagrange approach. Now, the rule is or the equation is something like this $\frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i}) - \frac{\partial L}{\partial q_i} = \tau_i$. Now, I am just going to define the different terms. So, here, the t is nothing but time. Now L is nothing but the Lagrangian of a system of a robotic system is nothing but the difference between the kinetic energy and the potential energy, that is nothing but the lagrangian of the robotic system.

Now, here this qi is the generalized coordinate for example, if it is a rotary joint. So, q_i is nothing but theta_i, that is the joint angle; if it is a prismatic joint that is nothing but the link offset, that is, di, now q_i dot is nothing but the first time derivative of q_i and your tau_i is nothing but the generalized torque, if it is a rotary joint and if it is a linear joint, that is nothing but the generalized force. So, our aim is to determine the mathematical expression for this particular τ. Now, let us see, how to determine the mathematical expression for this particular τ.

Now, to find out this mathematical expression, the first thing we will have to do is, we will have to find out the expression for this particular the Lagrangian. And, this Lagrangian is nothing but once again the difference between the kinetic energy and potential energy; that means, we will have to determine the kinetic energy for the whole robot. And, we will have to find out the potential energy and this particular difference of kinetic energy and potential energy is nothing but is Lagrangian.

So, my first task is to determine the Lagrangian of this particular robotics system, and now, let us see how to determine the Lagrangian of this particular robotic system.

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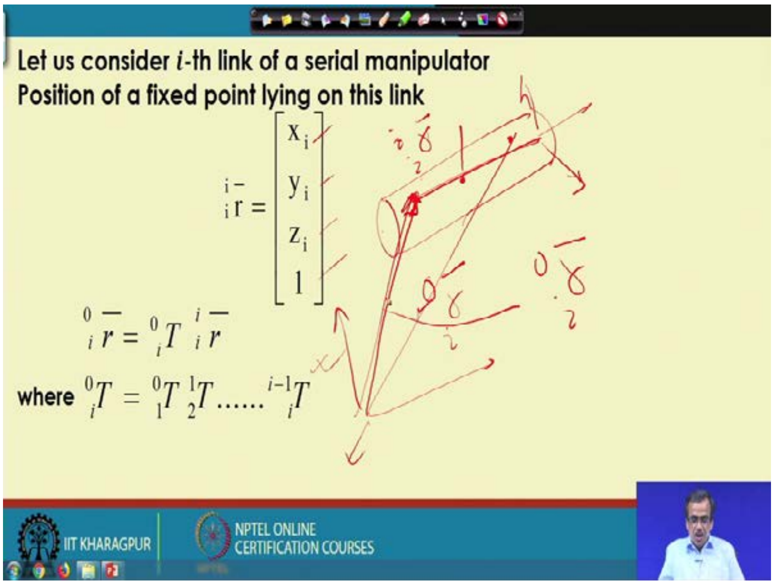

Now, let me once again start with a particular small element, whose coordinates are in its own the coordinate system. For example, say if I consider just like the previous the way, I have got is a robotic link and here, the coordinate system I am having here and the motor is connected here, ok.

So, I am just trying to find out the mass center here, and if I just consider a particular point or say this particular point. So, this point in this coordinate system is nothing but r_i with respect to i. Now, with respect to the base coordinate system, the same point I am just going to find out and that is nothing but is your r_i with respect to 0. So, our aim is to determine this particular r_i with respect to 0, but in this particular point; I am just trying to find out this particular point with respect to the base coordinate system.

So, this is nothing but r_i with respect to 0, and this I am trying to find out provided this is known, that is, r_i with respect to i. So, this r_i with respect to i is nothing but X_i Y_i Z_i and this r_i with respect to 0 is nothing but T_i with respect to 0 multiplied by r_i with respect to i. Now, this T_i with respect to 0 is nothing but T_1 with respect to 0 T_2 with respect to 1 up to T_i with respect to i - 1 and this particular T is nothing but the transformation matrix.

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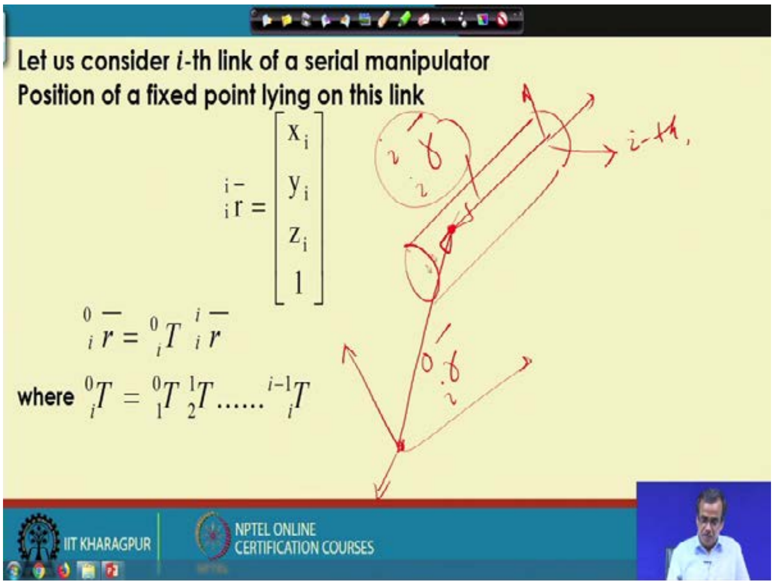

Now, once again, let me let me repeat. So, this is actually the circular link and my coordinate system is here, ok. So, I have got a point here and with respect to this particular coordinate system. So, this dimension is nothing but r_i with respect to i and I have got the base coordinate system here and the same point I am trying to find out and that is nothing but is your r_i with respect to 0. So, r_i with respect to 0 is nothing but T_i with respect to 0 multiplied by this r_i with respect to i and this is nothing but the i-th coordinate system and this is nothing but the base coordinate system.

So, in i-th coordinate system, this particular position vector is known, now, I am trying to find out r_i with respect to the base coordinate point, the same point. So, this is the way actually we can represent your r_i with respect to 0.

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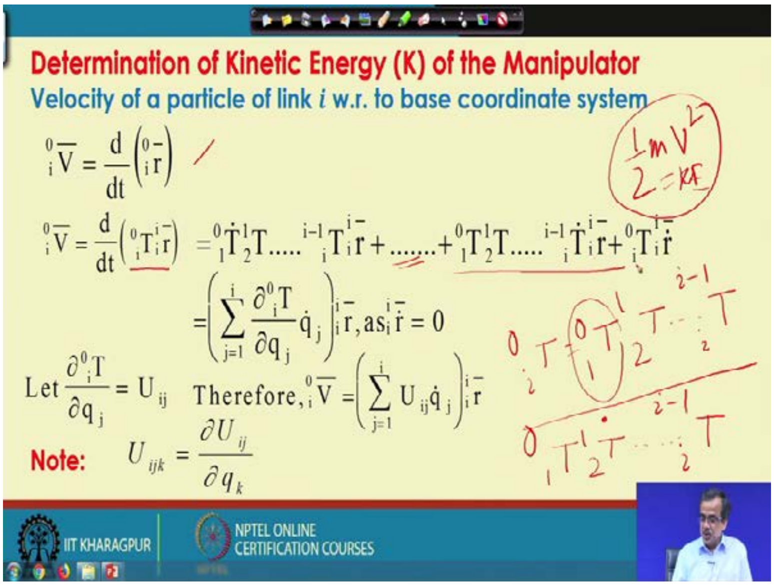

Now, actually, what I am going to do is, I am trying to find out the kinetic energy of the whole robot. Now, if I want to find out the kinetic energy, the kinetic energy is nothing but $\frac{1}{2}mv^2$.

So, the expression for the kinetic energy is nothing but half mass multiplied by V square. So, this is nothing but the kinetic energy. Now, here, actually what we are going to do, first, we are trying to find out the kinetic energy of one small element lying on a particular link, then we are trying to find out the kinetic energy for the whole link, say i-th link and after that, we will try to find out the kinetic energy for the whole robot or the whole robotic system.

Now, to determine the kinetic energy of the particle; so what you do is, you will have to find out the velocity of the particle. Now, the velocity of the particle, that is, V_i with respect to 0, with respect to the base coordinate frame. So, this is nothing but the rate of change of the position; that means, your d/dt of r_i with respect to 0. So, this r_i with respect to 0 is nothing but the position of that particular differential mass with respect to the base coordinate frame, and a rate of change of that particular position with respect to time is nothing but V_i with respect to 0; that means, the velocity of that particular particle with respect to the base coordinate frame.

Now, this r_i with respect to 0 as I told can be written as T_i with respect to 0 multiplied by r_i with respect to i. Now, this T_i with respect to 0; so this can be written as T_i with respect to 0 is nothing but T_1 with respect to 0 multiplied by T_2 with respect to 1 and the last term will be your T_i with respect to your i - 1. T stands for your transformation matrix.

Now, I will have to find out the derivative with respect to time. So, this T_i with respect to 0 is nothing but this. So, if I find out the derivative with respect to time. So, first, I will have to concentrate on this, that is, T 1 with respect to 0 dot multiplied by T 2 with respect to1 and there are a few terms, and the last term is T i with respect to i - 1.

The next term will be like T 1 with respect to 0. So, here T 1 with respect to 0, then T 2 with respect to one dot then comes the last term will be your like T i with respect to i - 1 ok. So, multiplied by r i with respect to i, and the last term will be this that is T 1 with respect to 0 T 2 with respect to 1. And, the last term is your T i with respect to i - 1 dot multiplied by r i with respect to i plus T i with respect to 0, this particular r i with respect to i dot. So, this is another term.

So, this is the way actually, we can find out this particular time derivative, now, let us try to concentrate on this particular term. Now, if we concentrate on this particular term.

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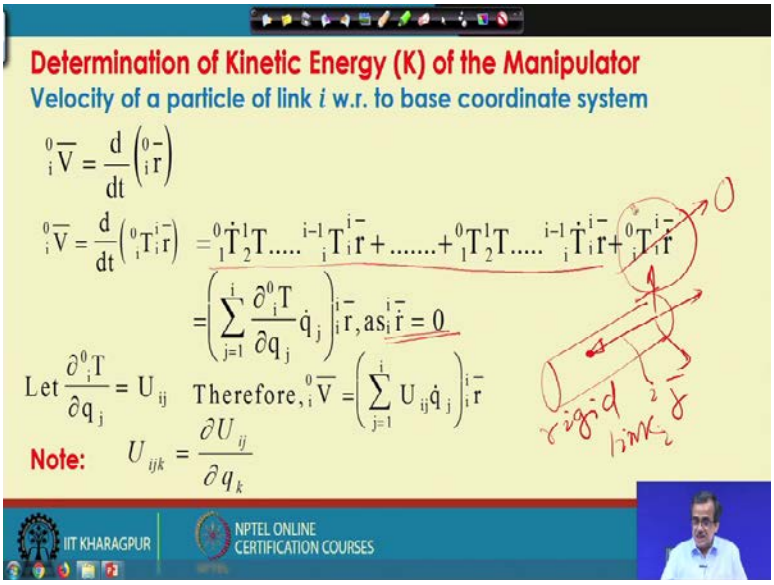

So, this r i with respect to i dot means, supposing that I have got one robotic link, the rigid link something like this. Now, if this is the rigid link and I am just going to concentrate on a particular point, ok. And, supposing that I have got the coordinate system here. So, this is actually the position of this particular differential mass and this is nothing but r i with respect to i.

Now, the rate of change of this particular position with respect to time will be 0, because this is a rigid link. So, this is a rigid link. So, for this particular rigid link, this ${}^{i}_{i}\dot{r} = 0$. So, this particular term will become equal to 0. So, we are left with this up to this. Now, here I just want to mention that if we consider robotic link like flexible robotic link, we cannot assume that this r i with respect to i dot is equal to 0. So, this will become nonzero for a flexible link and we have a few robot having flexible link, also.

And, determining this particular your V i with respect to 0 is not so, easy and we will have to consider this particular your flexible link and this particular term is non-zero and to determine once again, we will have to take the help of finite element analysis. Now, here, actually what we do is so, if you can see. So, this particular term, this particular term can be written like this in a short form.

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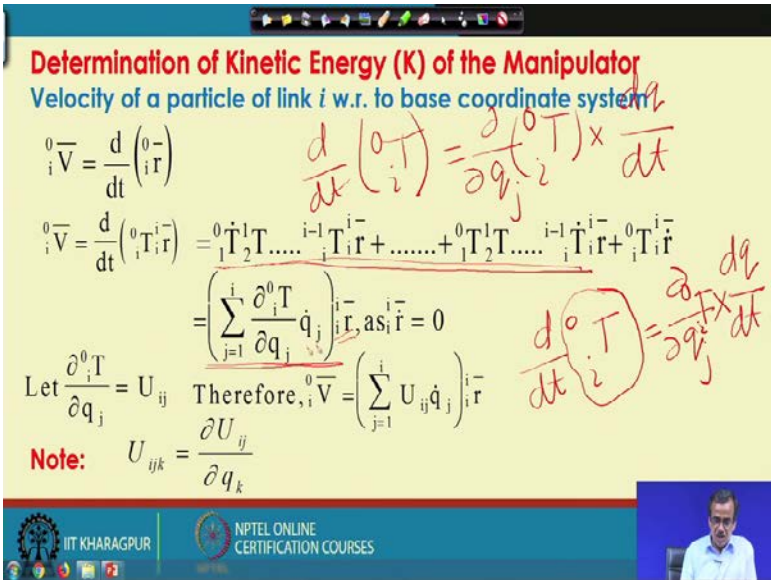

Now if we concentrate on this. So, this is your d/dt, that is, the derivative with respect to time of your T i with respect to 0.

So, this is nothing but the transformation matrix. Now, d/dt of T i with respect to 0, can be written as the partial derivative with respect to your q_j multiplied by dq/dt. So, d/dt of T i with respect to 0 is nothing but the partial derivative of this of T i with respect to q_j multiplied by dq/dt, let me write it once again. So, d/dt of T i with respect to 0 is nothing but the partial derivative with respect to q_j of T i with respect to 0 multiplied by is your dq/dt.

So, it can be written something like this, ok. So, this particular term has been written in this particular form that is your partial derivative with respect to q_j of T i with respect to 0, and dq/dt is nothing but $\dot{q}_j$ and we have got this particular thing, that is, a r i with respect to i. So, this particular expression can be written in short form like this, and this is nothing but your like Vi with respect to 0; that means, the velocity of that particular particle with respect to the base coordinate frame is nothing but this particular expression.

Now, here, I am just going to use another symbol that is partial derivative of T i with respect to 0 with respect to your q_j is nothing but U_ij is a another symbol I am using; that means, your Vi with respect to 0 can be written as summation j equals to 1 to i, then comes U_ij. So, I am using U_ij multiplied by $\dot{q}_j$ multiplied by ${}^{i}_{i}r$ and here, this U_ijk is nothing but the partial derivative of U_ij with respect to q_k. So, these particular symbols, we are using just to write down in a very compact form.

So, let us see, how to write down this particular thing in a very compact form.

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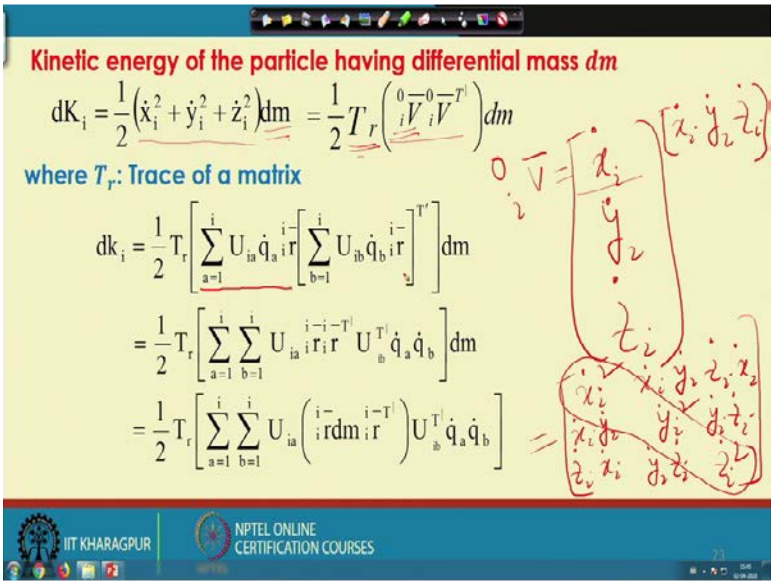

Now, kinetic energy of the particle having the differential mass is nothing but $\frac{1}{2}mv^2$.

Now, here, the mass of this particular differential mass is dm. So, $\frac{1}{2}dmv^2$ now, here this particular velocity, which is a vector having 3 components. So, this particular components velocity has got your 3 components like, say x_i dot then comes your y_i dot and z_i dot. So, these are the 3 components of this particular velocity, ok.

Now, here, this the kinetic energy of the differential mass dm, the small particle, that is $dK_i = \frac{1}{2}(\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2)dm$. Now, this V bar that is nothing but a V i with respect to 0 is nothing but this and now this V i with respect to 0 trace of that. So, this is nothing but a 3 X 1 matrix and I can write down actually here like its trace is nothing but x_i dot, y_i dot then comes your z_i dot now here if I just multiply. I will be getting your first row first column, that is, x_i dot square, then first row second column x_i dot y_i dot then comes your z_i dot x_i dot. So, I will be getting x_i dot y_i dot then comes your y_i dot square then comes your y_i dot z_i dot, then comes your zx, that is, z_i dot x_i dot then comes your y_i dot z_i dot and then comes your z_i dot square.

So, this type of like 3 cross 3 matrix we will be getting. Now, here, we try to concentrate only on the diagonal elements and this diagonal elements is nothing but trace of this particular matrix, ok. So, $\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2$ and that is actually nothing but trace of this particular V i with respect to 0, V i with respect to 0 transpose. So, trace of that is nothing but $\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2$.

Now, this dKi is nothing but your half trace of this V i with respect to 0 now we have already derived that ${}^0_i\bar{V} = \sum_{a=1}^{i} U_{ia}\dot{q}_a\, {}^i_i\bar{r}$, ${}^0_i\bar{V}^{T'} = [\sum_{b=1}^{i} U_{ib}\dot{q}_b\, {}^i_i\bar{r}]^{T'}$. Now, here, I one thing I just want to mention here, this summation I have taken a equals to 1 to i. And, here, I have taken b equals to 1 to i and this I have done very purposefully; for example, say if I consider. So, it is a rotary joint.

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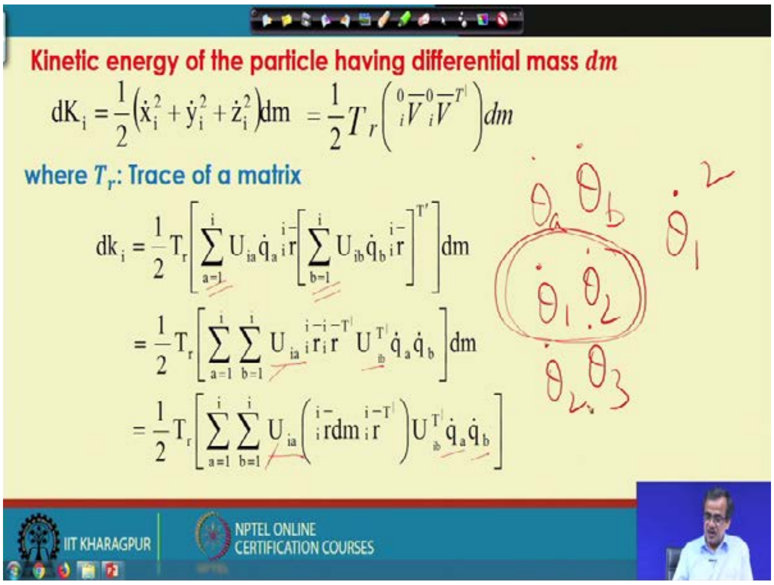

So, in place of this q_a dot I am just going to write down theta_a dot and in place of this q_b dot, I am just going to write down like q_b dot. Now, if you see the final expression, which I am going to derive. So, there is a possibility, there will be a few terms like $\dot{\theta}_1\dot{\theta}_2$ then might be $\dot{\theta}_2\dot{\theta}_3$ something like this, ok. Now if I do not consider the separate range for this particular summation, I am just going to miss this particular the combination of $\dot{\theta}_1\dot{\theta}_2$.

Now, if I consider both are a equals to 1 to i and b equals to 1 to i, there is a possibility that I will be getting $\dot{\theta}_1^{\,2}$. Now, for a very special case, I will be getting $\dot{\theta}_1^{\,2}$ and where a will become equals to b, but just to keep this particular possibility alive. So, I have taken two separate ranges for these particular summations. Now, once you have written a like this we can a rearrange in this particular $dK_i = \frac{1}{2}Tr[\sum_{a=1}^{i}\sum_{b=1}^{i}U_{ia}\,{}_{i}^{i}\bar{r}\,{}_{i}^{i}\bar{r}^{T'}U_{ib}^{T'}\dot{q}_a\dot{q}_b]dm$. This can be further rearranged in this particular format $dK_i = \frac{1}{2}Tr[\sum_{a=1}^{i}\sum_{b=1}^{i}U_{ia}({}_{i}^{i}\bar{r}dm\,{}_{i}^{i}\bar{r}^{T'})U_{ib}^{T'}\dot{q}_a\dot{q}_b]$. So, it can be rearranged in this particular the format.

Thank you.

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Lecture – 26
Robot Dynamics (Contd.)

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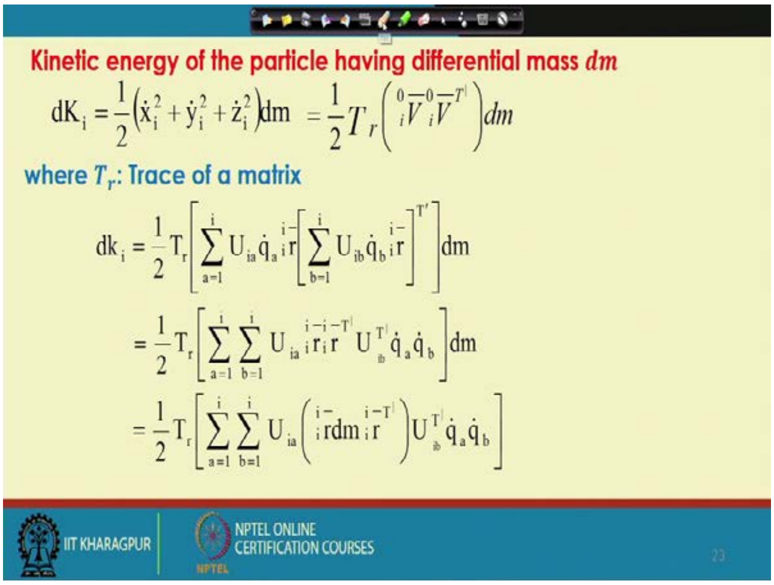

So, this is the expression of this particular kinetic energy of the differential mass, that is, dk_i.

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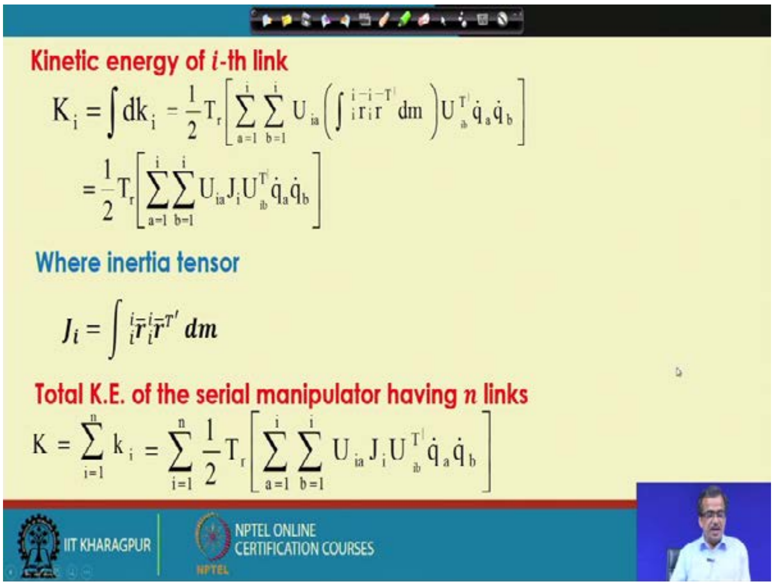

Now, once we have got it, I can find out the kinetic energy of the i-th link, that is, k_i and that is nothing but $\int dk_i$ and that is equal to half trace, then I am just going to write down the expression only thing I have done is I have put one integration sign here.

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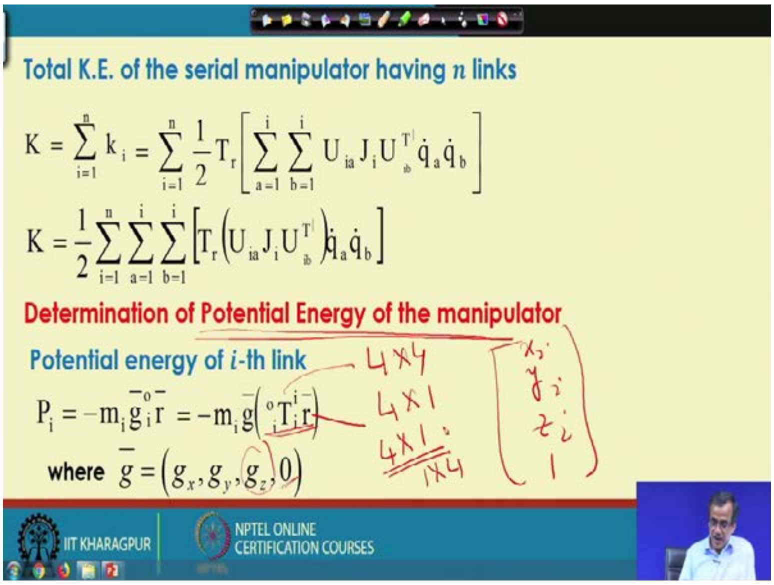

So, this particular integration sign, I have put one integration sign here, and other things are the same. So, this can be written as half trace summation a equals to 1 to i, b equals to 1 to i U_ia. Now, this particular expression is nothing but J_i and this J_i is nothing but the moment of inertia, which I have already derived. Next is U_ib transpose q_a dot q_b dot, ok, where the moment of inertia J_i is nothing but this particular expression.

Now, the total kinetic energy for the whole manipulator is nothing but $K = \sum_{i=1}^{n} k_i = \sum_{i=1}^{n} \frac{1}{2} Tr[\sum_{a=1}^{i} \sum_{b=1}^{i} U_{ia} J_i U_{ib}^{T'} \dot{q}_a \dot{q}_b]$ something like this. So, this is the expression for your the kinetic energy for the whole manipulator.

Now, the kinetic energy for this particular manipulator having n links is nothing but this particular expression. So, this can be rearranged and it can be rewritten in a slightly different way. So, here, we can write down this K is nothing but $K = \frac{1}{2} \sum_{i=1}^{n} \sum_{a=1}^{i} \sum_{b=1}^{i} [Tr(U_{ia} J_i U_{ib}^{T'}) \dot{q}_a \dot{q}_b]$. So, this is the way actually we can find out the expression for the total kinetic energy for the whole robot.

Now, we are going to find out the potential energy of the manipulator. How to determine the potential energy? Now, the potential energy $P_i = -m_i \bar{g}_i \, {}^0_i\bar{r}$. Now, this particular g is nothing but the acceleration due to gravity and truly speaking this is a vector having component g_x, g_y, g_z. And, here, I have put this 0 very purposefully that I am going to tell. Now, this g_x, g_y, g_z are nothing but the three components of the acceleration due to gravity. Now, at a particular place, this particular g_x and g_y are negligible and that is why, we generally consider only your g_z, which is acting vertically downward.

Now, if it is a acting vertically downward, we will have to put here accordingly, we will have to do the sign correction. And, this T i with respect to 0 is the transformation matrix and r i with respect to i. So, this T i with respect to 0 multiplied by this r i with respect to i, so this is nothing but the height of this particular link, that is, r i with respect to 0. So, I can find out the expression of this particular potential energy.

Now, if you see the dimension of this T, T i with respect to 0, so this is nothing but a 4×4. If I see r i with respect to i, so this is nothing but a 4×1 matrix, because x_i y_i z_i and then I put a 1 here, so this is nothing but a 4×1. So, if I multiply 4×4 and 4×1. So, I will be getting 4×1 matrix. And, this 4×1 matrix, I will have to multiply by 1×4 matrix then only I will be able to do this particular multiplication and that is why, in place of g_x, g_y and g_z, I put one 0 here just to make 1×4 matrix, so that I can multiply with this 4×1 matrix.

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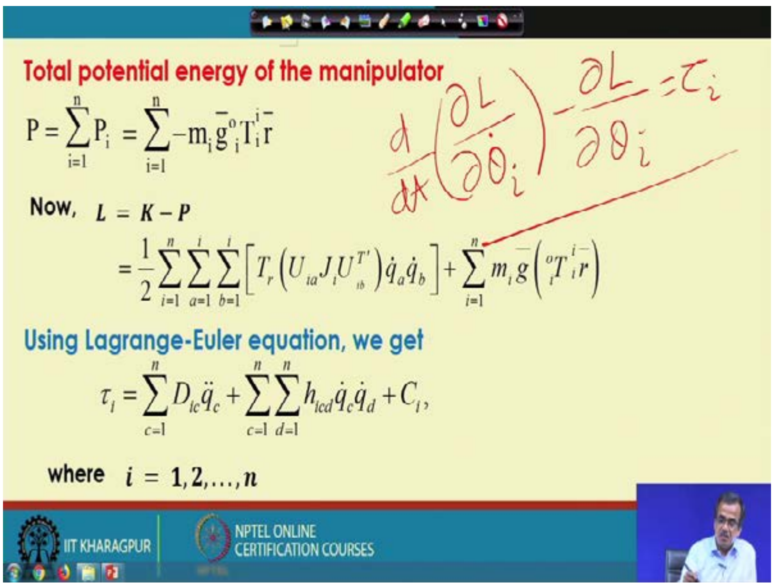

Now, if you multiply, then we will be getting the expression for this particular potential energy. Now, $P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} -m_i \bar{g}\, {}^0_iT\, {}^i_i\bar{r}$. So, this particular Lagrangian is nothing but the kinetic energy minus potential energy. So, we know the expression for kinetic energy and this is the expression for the potential energy.

So, I can write down kinetic energy minus potential energy. So, this is the expression for the whole Lagrangian for the robotic system. And, now actually, what we do is, we try to go back to that particular expression, that is, your that Lagrangian equation: $\frac{d}{dt}(\frac{\partial L}{\partial \dot{\theta}_i}) - \frac{\partial L}{\partial \theta_i} = \tau_i$. So, this particular expression, I am going to use. Now, here in place of theta_i, we can put this particular your q_i. So, this is actually the same expression, which I showed.

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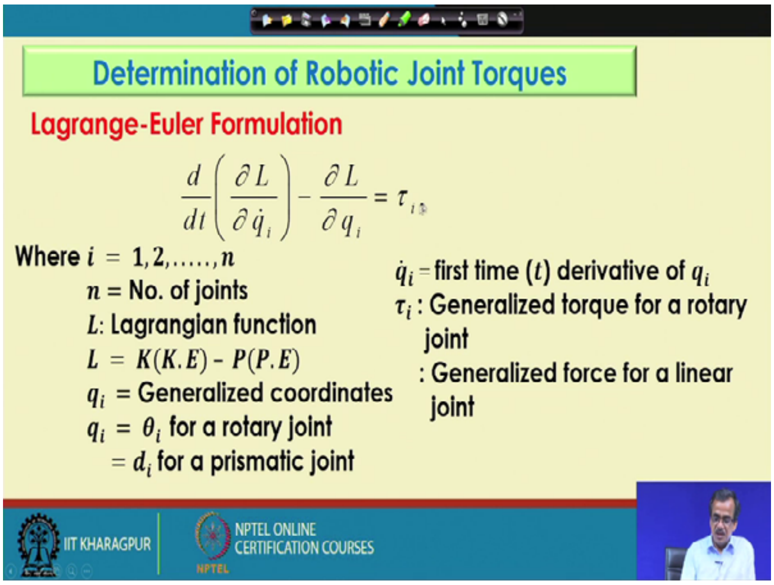

So, let me try to go back to that particular expression once again. So, this is actually the expression. So, I am just going to use this particular expression.

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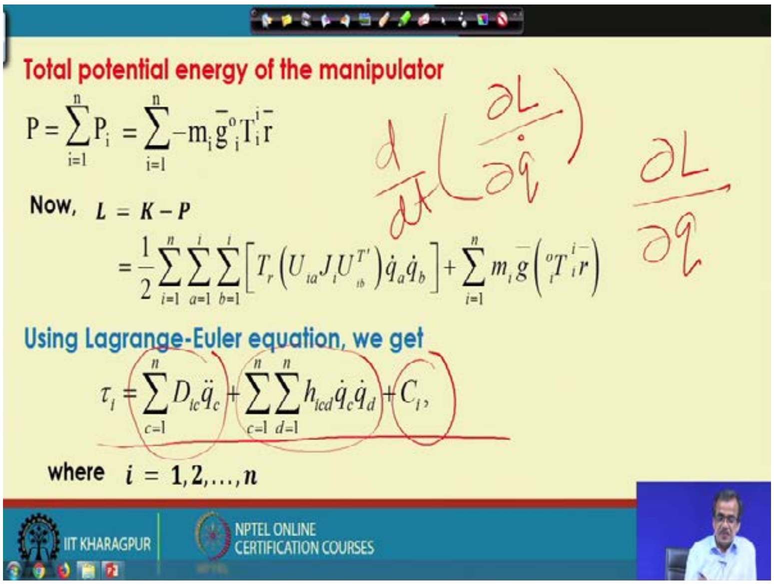

And, now, if I just substitute this particular the Lagrangian and if I just find out the partial derivative of L with respect to this q dot, and if you find out d/dt of that and if you find out separately the partial derivative of Lagrangian with respect to q, then we will be able to find out the final expression of this particular joint torque, which is nothing but this. Now, here in this particular expression for the joint torque, we have got three distinct component. So, this is nothing but the inertia term, which depends on the mass distribution of the link. This is your Coriolis and centrifugal term; and this is nothing but the gravity term.

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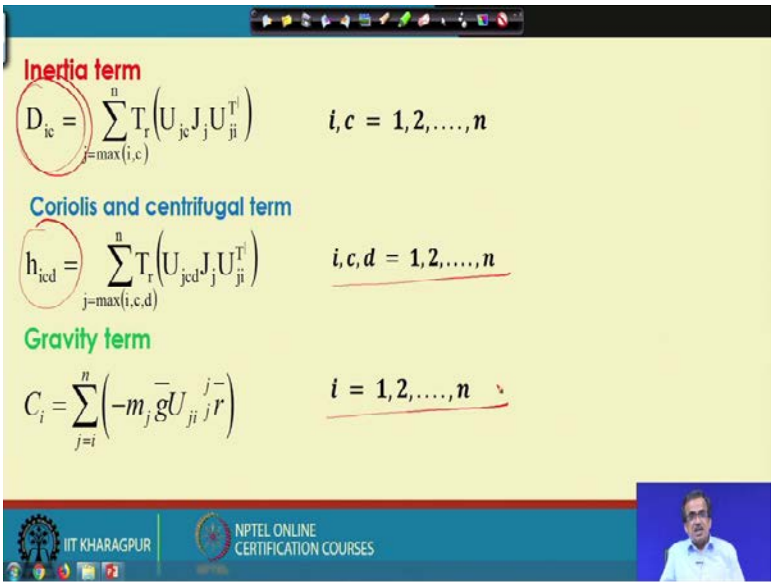

Now, here, so this particular terms like your inertia term $D_{ic} = \sum_{j=\max(i,c)}^{n} Tr(U_{jc} J_j U_{ji}^{T'}), i, c = 1, 2, ..., n$. So, this is nothing but the inertia tensor that we can find out this D_ic. Then, the Coriolis and centrifugal term is $h_{icd} = \sum_{j=\max(i,c,d)}^{n} Tr(U_{jcd} J_j U_{ji}^{T'}), i, c, d = 1, 2, ..., n$. And, the gravity terms, $C_i = \sum_{j=1}^{n} (-m_j \bar{g} U_{ji} \, {}^{j}_{j}\bar{r}), i = 1, 2, ..., n$.

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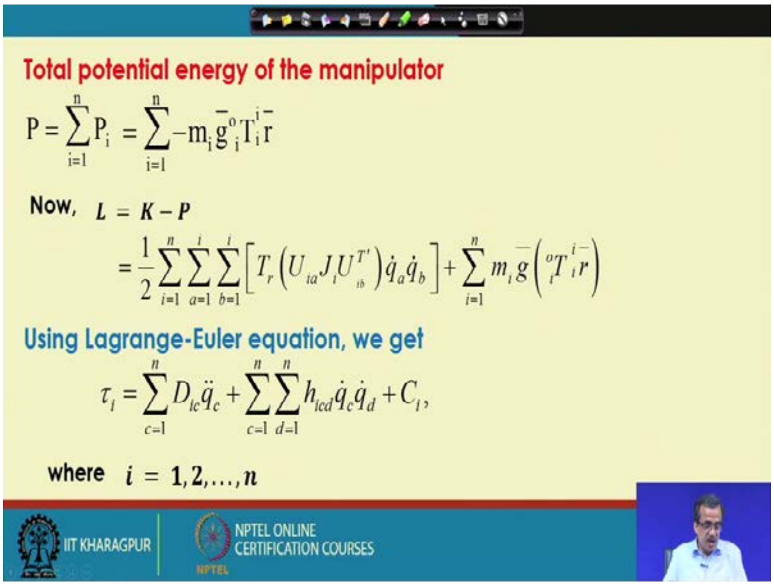

Now, using this particular expression, what you can do is, we can find out the expression for the joint torque or the force, ok. Now, actually what I am going to do by using this particular expression, we will try to derive the expression for the joint torque or the force which is required at the different joints. And, we will take one numerical example and with the help of this particular numerical example, I am just going to find out the big expression for this particular joint torque.

Thank you.

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Lecture - 27
Robot Dynamics (Contd.)

Now, let us see how to determine the joint torque for the different robotic joints, I am using the principle of the Lagrangian method.

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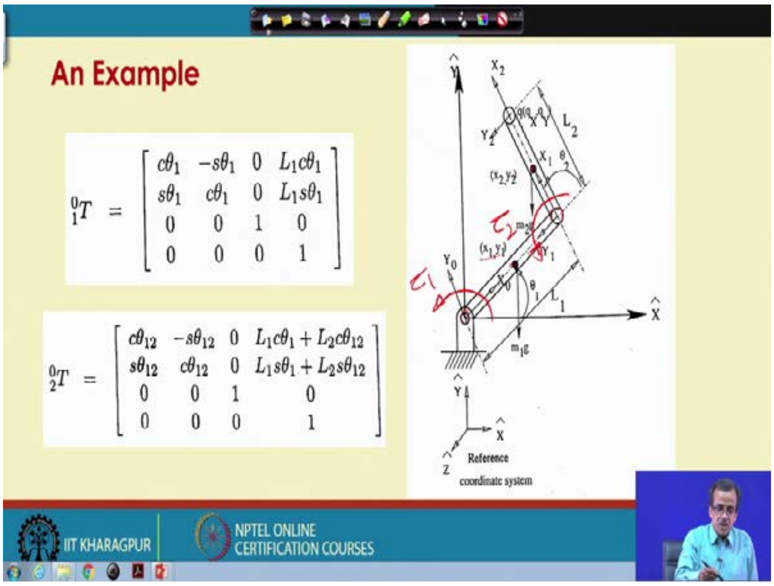

So, we are going to take one example of 2 degrees of freedom serial manipulator. So, this is a 2 degrees of freedom serial manipulator, this is the first joint, and the link 1, the second joint, and link 2, the length of the first link is L_1, and the length of the second link is l_2, the joint angles are theta_1 and theta_2.

Now, here, the link 1 is having the mass m_1. So, m_1g that particular force is acting here at the mass center and x_1, y_1 is the coordinate of the mass center, similarly, for the second link the mass center is x_2, y_2 and this m_2 g is acting here vertically downward and g is nothing but the acceleration due to gravity.

Now, here, our aim is to determine, what should be the joint torque here, that is, τ_1 and what should be the joint torque here, that is your τ_2. So, τ_1 and τ_2, we will have to find out, we will have to derive the mathematical expression for the joint torques, τ_1, τ_2.

Let us see, how to proceed with this, now to determine this particular joint torque actually, what will have to do is, at first we will have to assign the coordinate system at the different joints according to the D-H parameter setting rule, now according to the D-H parameter setting rule, this is nothing but X_naught and this is Y_ naught and Z_naught is perpendicular to the board and away from the board. So, this I have already discussed, so at the joint 2, this is my X_1, this is Y_1 and Z_1 is perpendicular to the board, similarly here at the end, these are X_2, Y_2 and Z_2 is perpendicular to the board.

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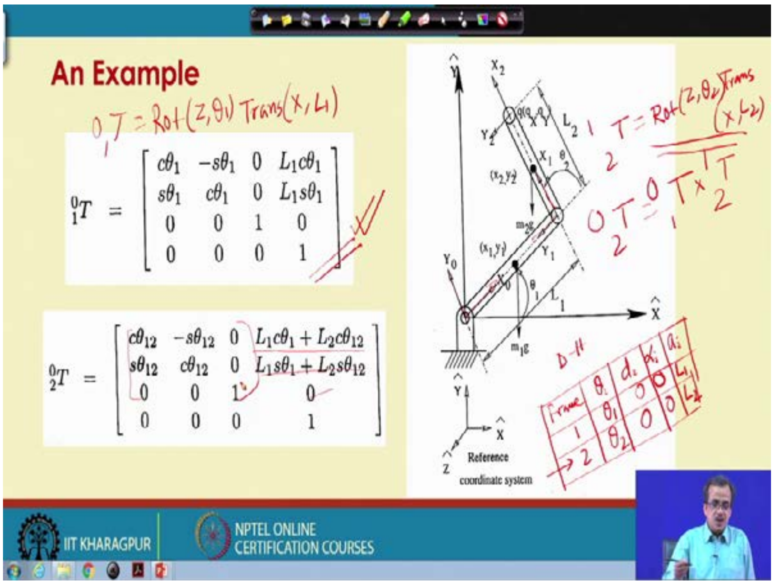

Now, if you just draw this D-H parameters table. So, it looks like this, for example, say if I prepare the D-H parameters table. So, this is nothing but the frame, then we will have to consider the screw z, that is, the rotation about z by an angle θ_1, then translation along z, that is, d_i, then comes your rotation about that x, that is, α_i and translation along x, that is, a_i and here. So, for this particular the first one, so this is the joint angle. So, this particular joint angle is the variable θ_1 then d=0, α is 0 and the length of the link, that is nothing but l_1; for the second one, so the joint variable is θ_2, 0, 0, l_2.

Now, this is the D-H parameters table, now if I know this particular D-H parameters table, very easily, we can determine, what is the transformation metrics, that is T 1 with respect to 0? That is nothing but rotation about z. So, this T_1 with respect to 0 is nothing but rotation about z by angle θ_1, then comes your translation along x by l_1. So, we can express 4 x 4 metrics, which we have already discussed and if you multiply, this will be the final matrix, the 4 cross 4 matrix, which will be getting.

Similarly, this T_2 with respect to 1; that means, I am here. So, I can find out like T 2 with respect to 1 is nothing but rotation about z by an angle θ_2, then comes translation along x by l_2 and these to 4 cross 4 matrices if you multiply, we will be getting another 4 cross 4 metrix and this T_2 with respect to 0 like T_2 with respect to 0 is nothing but T_1 with respect to 0 multiplied by T_2 with respect to 1.

So, this particular matrix and that particular matrix, if you multiply, then I will be getting the 4 cross 4 metrix, that is nothing but T 2 with respect to 0, that is, this particular point with respect to the base coordinate system. And, as we know that these indicate the position terms and this is nothing but the orientation term, that is, 3 cross 3, we have already discussed. So, let us start with here and then, let us see, how to determine the joint torque.

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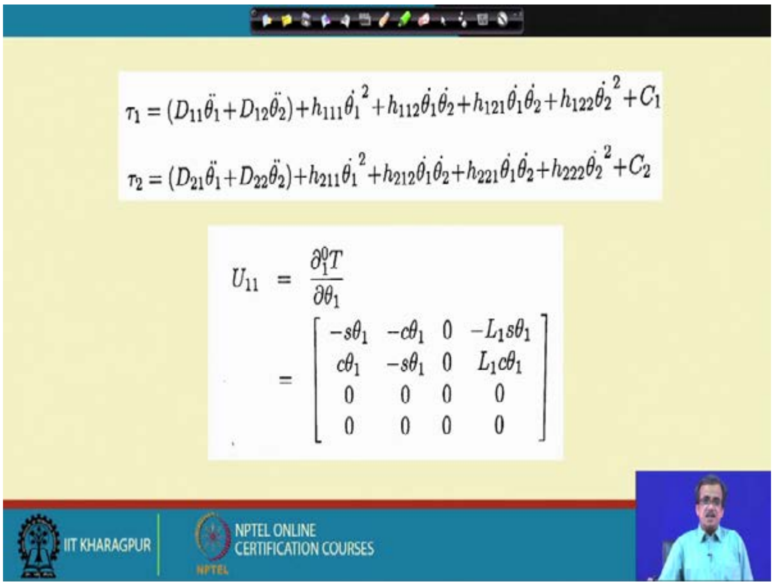

Now, if you see the expression like the expression for the joint torque. So, you will be getting such a big expression for this particular τ_1 and such a big expression for this particular τ_2. Now, these expressions, we can derive from the general expression like if you just go back a few slides.

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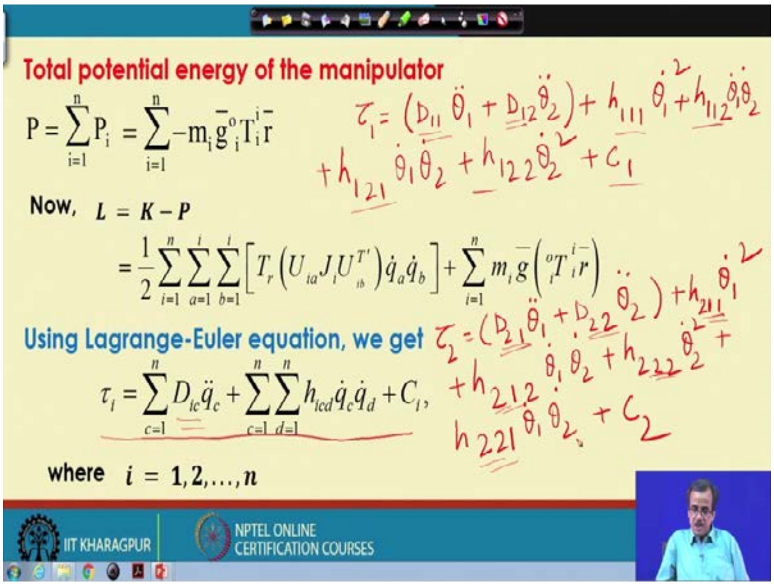

Then, form here, we can, in fact, derive the expression for τ_1 and τ_2. So, let us try to concentrate here, so let us try to concentrate on this particular equation and see how to determine that particular τ_1 and τ_2, now here there are two joints. So, c varies from 1 to 2.

So, let me try to find out the expression for τ_1; that means, I is equal to 1 and c varies from 1 to 2. So, c equals to 1 c equals to 2, so if I take c equals to 1. So, I will be getting D_11 and $\ddot{q}_c$, c equals to 1 to n, in place of q_i, we will be using theta, so $\ddot{\theta}_1$. So, theta_1 double dot plus i equals to 1, so you will be getting D and, but c equals to 2, so D_12, $\ddot{\theta}_2$. So, this I will be getting form this particular expression.

Now, I concentrate here, I put i equals to 1. So, I will be getting h_1, then you consider c equals to 1 to 2, d equals to 1 to 2, you first consider c equals to 1. So, I will be getting 1 here and d varies from 1 to 2, so it is 1 and I will be getting $\dot{\theta}_1^2$, then comes your h, i equals to 1, c equals to 1, d equals to 2. So, I will be getting $\dot{\theta}_1\dot{\theta}_2$ plus then I consider like c equals to 2. So, h_121, then you will be getting $\dot{\theta}_1\dot{\theta}_2$ then comes your h_1 like c equals to 2 and will be getting d is also equal to 2. So, I will getting $\dot{\theta}_2^2$ plus I mean I will be getting c_1.

So, this is the expression for this particular joint torque τ_1 similarly, we can write down the expression for τ_2 also. So, τ_2; that means, i equals to 2 and c is varying from 1 to 2. So, I will be getting D_21 like $\ddot{\theta}_1$, then comes your D_22 then comes your $\ddot{\theta}_2$ plus I will be getting here h_2, c equal to 1, d equals to 1. So, I will be getting $\dot{\theta}_1^2$ then comes your h_2 like c equals to 1, but d equals to 2. So, I will be getting $\dot{\theta}_1\dot{\theta}_2$ plus h_222. So, I will be getting h_222, so I will be getting here $\dot{\theta}_2^2$ and another term h_2 then 2 and I will have to consider this 1 also.

So, I will be getting $\dot{\theta}_1\dot{\theta}_2$ and your c_2. So, this is the way h_211, h_212, h_221, h_222 can be determined. So, this is the way actually, we will be getting the expression for τ_1 and τ_2. So, there will two D terms and there will be four such h terms and one c_1, similarly, here there are two D terms and there will be four such h terms and will be getting your one c terms. So, this is the way actually, we can find out the expression for this particular τ_1 and τ_2, the same expression I have written it here.

So, this is exactly the same expression which I have written it here like τ_1 and τ_2 now I will have to concentrate on, this particular term, that is, D_1, but before I go for this particular D_1 actually what I will have to do is, so I will have to find out another term that is called U_11 and U_11 is nothing but the partial derivative of the transformation matrix t 1 with respect to 0 and this partial derivative with respect to theta_1.

So, if we remember the expression for T 1 with respect to 0 for example, if you see the expression for T 1 with respect to 0. So, this is the T 1 0, so the partial derivative of T 1 0 that is your if you want to find out the partial derivative of T 1 with respect to 0.

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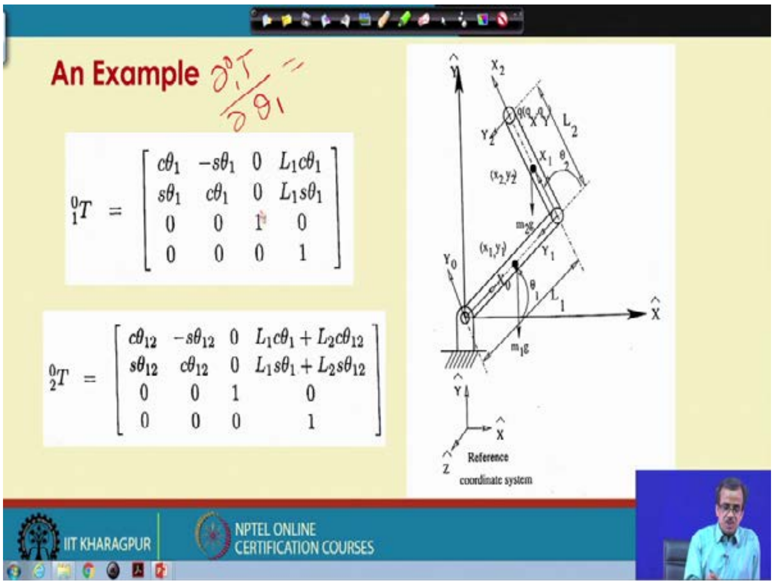

And if this partial derivative is with respect to θ_1, so here in place of $\cos\theta_1$, I will be getting $-\sin\theta_1$. So, here I will be getting $-\cos\theta_1$ then this is 0 and here I will be getting $-L_1\sin\theta_1$.

Similarly, this will be $\cos\theta_1$ this will be $-\sin\theta_1$, 0 and this will be $L_1\cos\theta_1$ and this will be 0 0 this 1 will also become 0 because this is the partial derivative 0 0 0 0 0. So, this type of expression you will be getting for your U_11. So, this U_11 as I told is $-\sin\theta_1$, $-\cos\theta_1$, 0, $-L_1\sin\theta_1$; $\cos\theta_1$, $-\sin\theta_1$ 0 $L_1\cos\theta_1$ and here will be getting all 0 terms.

So, this is the way actually, we can find out this U_11, this is actually the rate of change of these particular transformation metrics with respect to your only θ_1.

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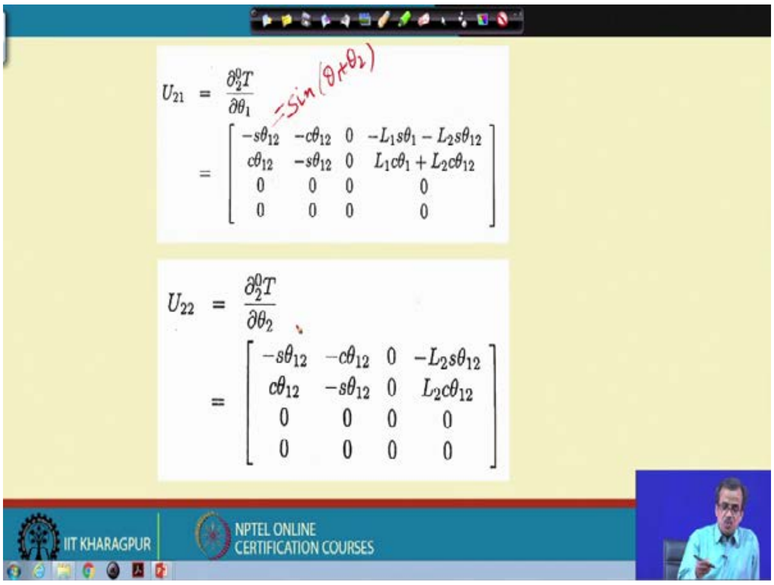

Similarly, we can also find out U_21 and U_21 is nothing but the partial derivative of T 2 with respect to 0 with respect to your theta_1. So, T 2 with respect to 0 like if you see this particular expression like T 2 with respect to 0. So, with respect to theta_1, so we will have to find out its partial derivative.

For example, here, I will be getting in place of $\cos(\theta_1+\theta_2)$, I will be getting $-\sin(\theta_1+\theta_2)$ here I will getting $-\cos(\theta_1+\theta_2)$, and so on. So, this is the way actually, we can find out the partial derivative and this is nothing but $-\sin(\theta_1+\theta_2)$. So, this is nothing but $-\sin(\theta_1+\theta_2)$, then $-\cos(\theta_1+\theta_2)$ and so 1 and these particular terms all terms will become 0 and this will also become is equal to 0.

Now, by following the same method I can also find out the U_22 that is nothing but the rate of change of T 2 with respect is 0 with respect to theta_2. Now, with respect to theta_2 only if you determine then you will be getting something like with respect to theta_2, if you determine. So, then here will be getting like $\frac{\partial}{\partial\theta_2}(C\theta_{12})$, so will be getting $-s\theta_{12}$.

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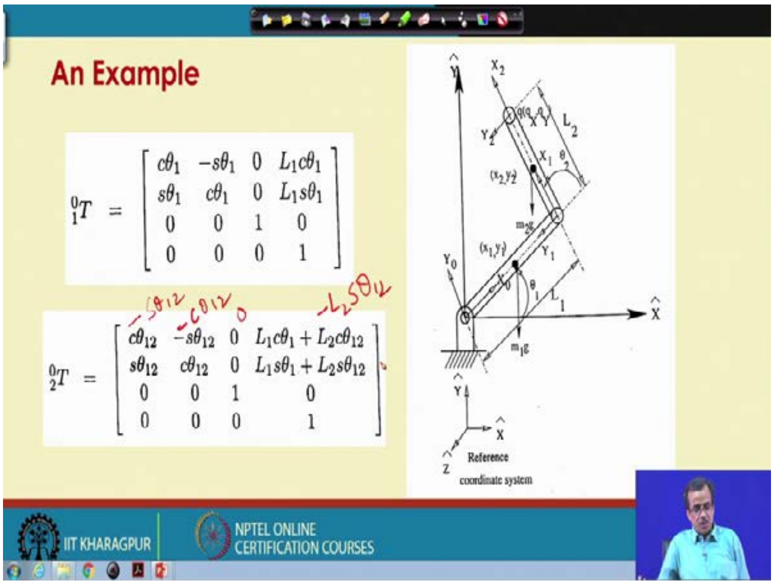

And that means you will getting here $-s\theta_{12}$ here will be getting $-c\theta_{12}$ then this will become 0. And here, there will be no contribution because here there is no theta_2, the only contribution will come here and this will become $-L_2s\theta_{12}$.

Similarly, the other terms also you can find out, and by following the same method actually, we can find out what is your U_22. So, this is nothing but is your U_22 and once I have got this particular U_2. So, let us try to concentrate on the inertia tensor. So, these are nothing but the inertia tensor inertia tensor for the link_1 and link_2.

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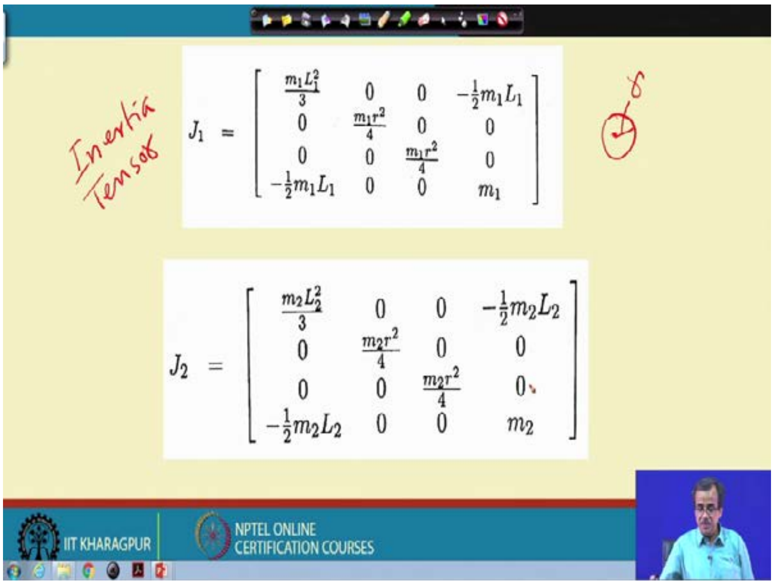

Now, if we remember, you have already derived this particular expression. So, this particular J_1 is the inertia tensor for the first link. And, we have considered that this particular link is having actually your circular cross-section with radius r and for that will be getting the inertia tensor like this like $\frac{m_1 L_1^2}{3}, 0, 0, -\frac{1}{2} m_1 L_1; 0, \frac{m_1 r^2}{4}, 0, 0; 0, 0, \frac{m_1 r^2}{4}, 0; -\frac{1}{2} m_1 L_1, 0, 0, m_1$.

So this, I have already derived this particular inertia tensor. Now, similarly, for the link 2, I can find out what is J_2 and exactly in the same way I can find out this 4 cross 4 matrix. And, this I have already discussed in much more details in one of the previous classes, how to determine that this particular inertia tensor.

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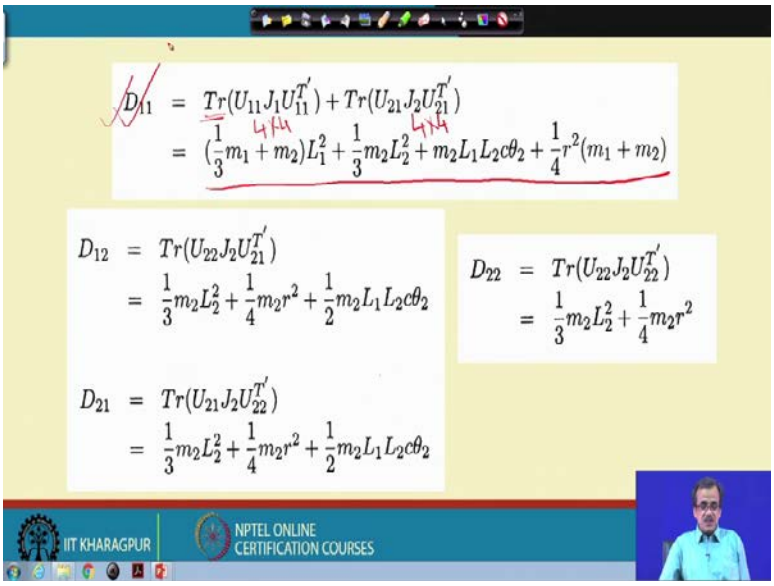

Now, I am just going to derive that particular the D_11 term. Now, if you see, this particular D_11. So, this is nothing but actually this particular D_11 term I am just going to find out. So, I am just going to derive what should be the expression for this particular D_11.

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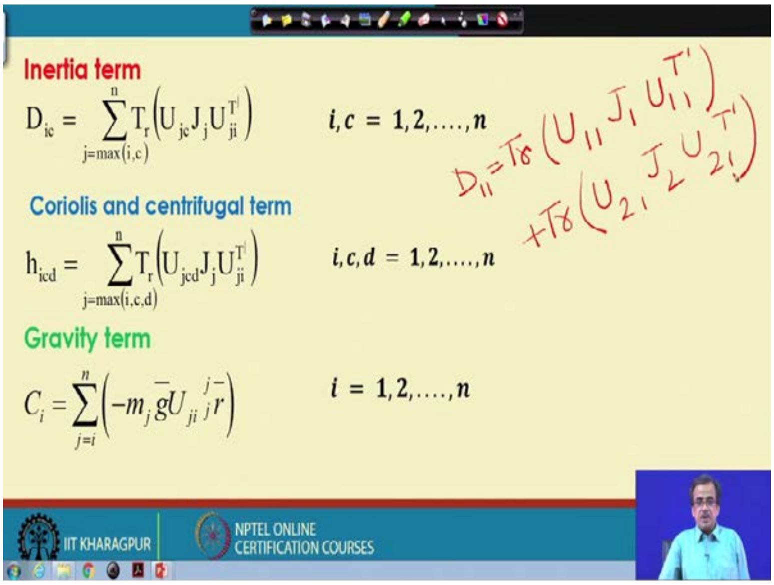

Now, to derive this particular expression, let me try to go back to this particular expression for D_ic. Now, if you concentrate on this particular D_ic there is an inertia term and our aim is to determine what should be your the expression for D_11 ; that means, I equals to 1 c equals to 1 and j is the maximum between 1 coma 1, that is 1. So, j will vary from 1 to 2 and now with the help of this, I can write down. So, when j equals to 1, so it is nothing but trace u the j equals to 1 here and c is 1 here, then comes your J_1 then comes u j equals to 1, i equals to 1. And, this is the symbol for the transpose like Tr that is the trace.

Now, I am just going to use j equals to 2, so this will become U_21 J_2 U_21 transpose of that. So, this is the expression for this particular D_11, now, the same expression I am using here. So, the same expression I am using here for this particular D_11, that is trace of U_11 J_1 U_11 transpose plus trace of U_21 J_2 U_21 transpose.

Now you see all the terms you know to us for example, say this U_11, we have already derived J_1 you have derived. So, you know U_11 transpose of that then comes U_21 we have derived J_2 you have derived U_21 transpose is also known to us. So, each of these matrices are 4 cross 4 matrix, ok. So, if you multiply 2 times, then you will getting finally, 4 cross 4 matrix. So, you will be getting one 4 cross 4 metrics here and here also, you will be getting one 4 cross 4 matrix and here actually this is a trace. So, for this 4 cross 4 matrix, so what will have to do is, you will have to consider only the diagonal elements. So, by trace we mean the sum of the diagonal elements. So, we consider the sum of the diagonal elements here and some of the diagonal elements here and if you just add them of you will be getting the final expression for this particular D_11.

So, the expression for D_11 is known, now the same method we will have to follow for the other example, say you find out D_12 once gain we go back to the expression, so this particular expression.

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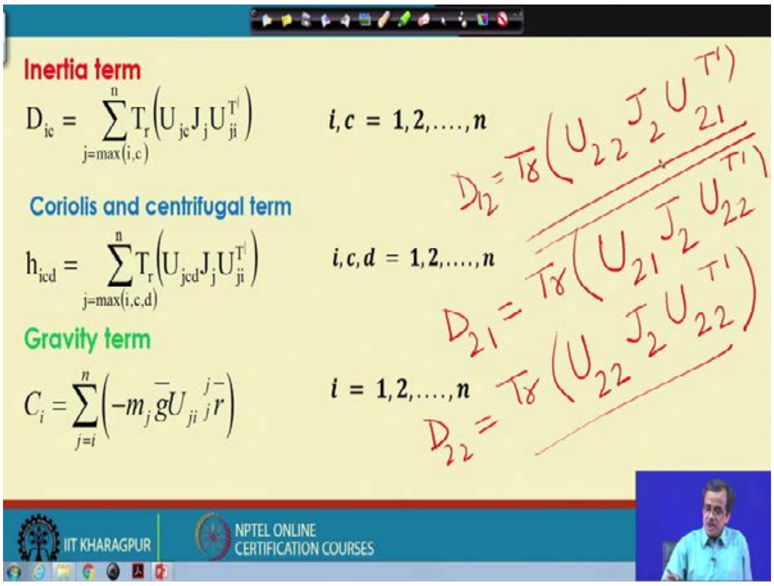

So, here D_12; that means, your D_1 to i equals to 1 and c equals to 2, as the maximum between 1 and 2 is 2. So, there will be only 1 term here, so this is nothing but the trace of U_22 then comes J_2 then comes U_21 transpose of that, so this is the expression for your D_12. Now, similarly, we can also find out the expression for D_21, now this D_21; that means, i equals to 2 and c equals to 1 and the maximum between 2 and 1 is 2. So, there will be only 1 term because j varies from 2 to 2. So, you will be getting the trace of like U_22 and then comes your J_2, then the transpose of U_22.

So, this is nothing but your D_21 and I can also find out the expression for D_22 now for this particular D_22, i equals to 2, c equals to 2, j equals to 2 to 2. So, trace of your u j equals to 2 here and c is also 2 then comes your J_2 then comes j equals to 2 and i is also 2 and transpose of that. So, I can find out the expression for this D_12, D_21 and D_22, and all the terms I know and very easily actually we can find out the expression for these particular the D terms, the way I discuss.

So, this is the expression for your D_12 and once again you write down the 4 cross 4 matrix here, 4 cross 4 matrix for J_2, 4 cross 4 for U_21 transpose, you multiply them consider the sum of the diagonal elements. So, will be getting this is the expression for your D_12.

Similarly, your D_21 the expression we have already seen and if you just follow the same method like matrix multiplication and then, if you consider the sum of the diagonal elements. So, we will be getting the same expression of D_12. So, D_21 becomes equals to D_12, but D_22, so once again you follow the same method write down the expression, multiply the matrices and consider the trace, so you will be getting this expression.

So, till now, all the D values are calculated, now, if I have got all the D values; that means, you're in this particular expression like your all D values are known, now we will have concentrate on the h values that is your h_111, h_112, h_121 and h_122. Similarly, we have got 4 more, so there are 8 such h terms and that I will have to derive ok. Now, how to derive, so what we do is, once again we go for the expression for this h_icd, so here, if you write down like your h_111.

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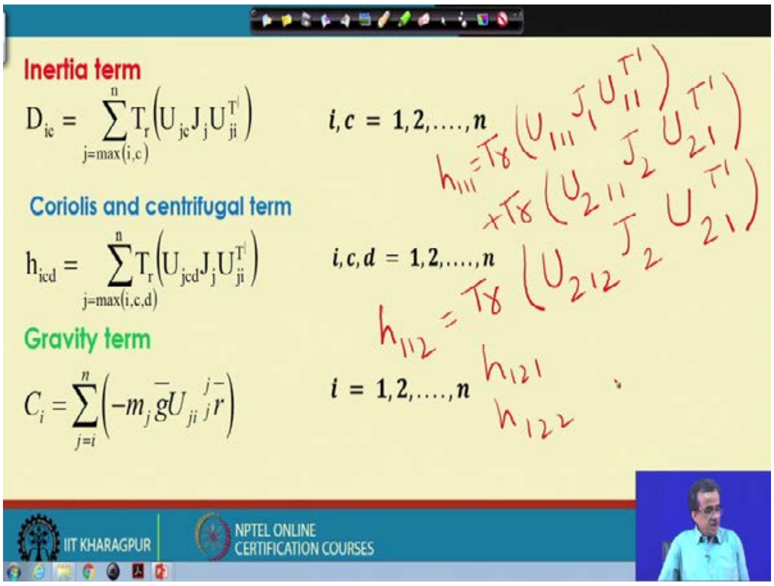

So, h_111, for example, here, i,c,d all are 1, so j varies from 1 to 2, so there will be 2 such terms. So, this is nothing but trace of U_111, multiplied by J_1 and then multiplied by U_11 transpose and then the trace of U_211, then comes J_2, then comes U_21 transpose.

So, this is nothing but is your h_111, similarly, I can also write down the expression for h_112, now h_112 is where i equals to 1, c equals to 1, d equals to 2. So, j is the maximum of icd; that means, 2, so there will be only one term here. So, this is nothing but the trace of U_212 and then J_2 and U_21 transpose. your u.

So, similarly I can find out the other your h terms like your say h_121, h_122 and other terms we can find out. So, we can we can determine other expressions of this particular h terms, now if you see this particular h_111.

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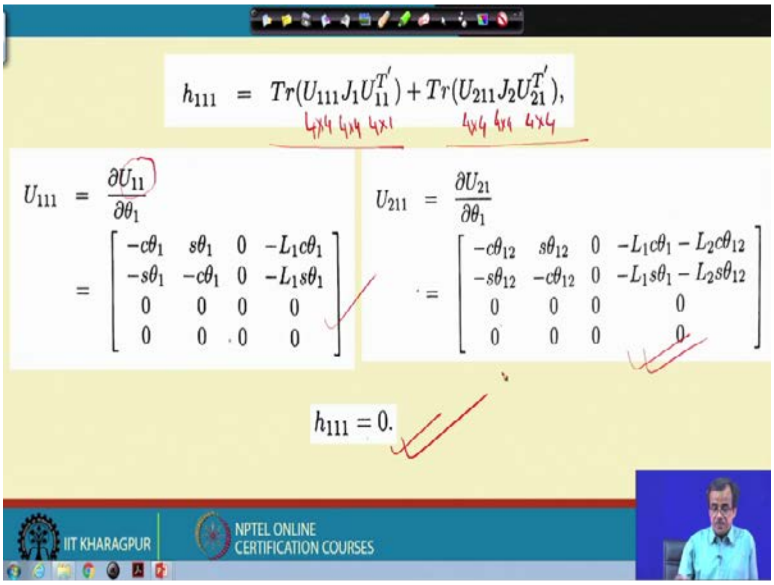

Let us see how to derive this h_111 is nothing but as we have seen trace of this particular thing plus trace of this particular thing, now this U_111 is nothing but the partial derivative of U_11 with respect your theta_1.

We know the expression of U_11 as 4 cross 4 matrix. So, again will have to find out the partial derivative with respect to 1 ok, then you will be getting this U_111, once again the 4 cross 4 matrix. Similarly, you know this U_21 we have already seen that 4 cross 4 matrix and that matrix we will have to find out the partial derivative with respect to theta_1 and that will become U_211. And, we will be getting this particular partial derivative of 4 cross 4 matrix.

Now, U_111 is a 4 cross 4 matrix, J_1 is one 4 cross 4 matrix and U_11 is another 4 cross 4 matrix. Similarly, U_211 is 4 cross 4 matrix, J_2 is 4 cross 4 matrix and this is also 4 cross 4 matrix. So, ultimately, I will be getting one 4 cross 4 matrix, another 4 cross 4 matrix, we consider the trace value and if we just add them up fortunately we will be getting this h_111 is equal to 0.

So, this is the way actually, we can find out the other h values by following the same method like h_112, the expression I have already seen.

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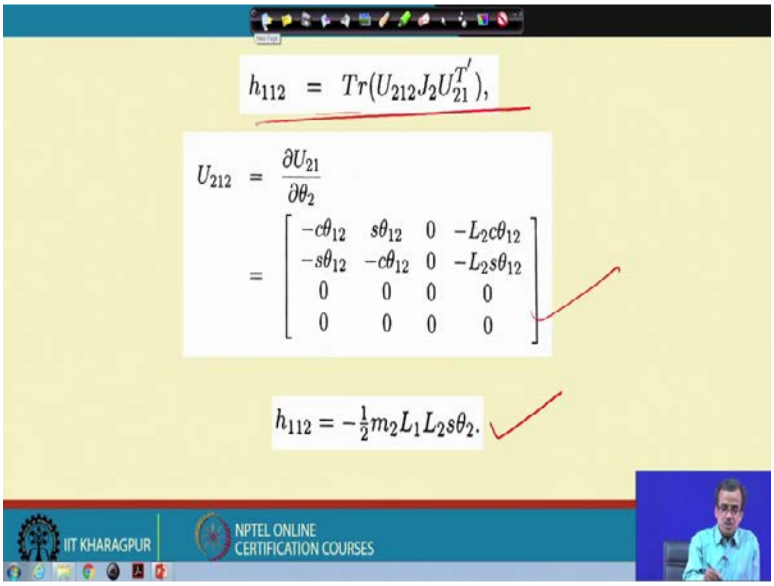

So, this is the expression for h_112 and this U_212 is nothing but the partial derivative of U_21 with respect to theta_2. So, this is the expression and if you just substitute the metrics says multiply find out the sum of the principle diagonal elements. So, you will be getting h_112, is nothing but this, so this is actually the expression for h_112.

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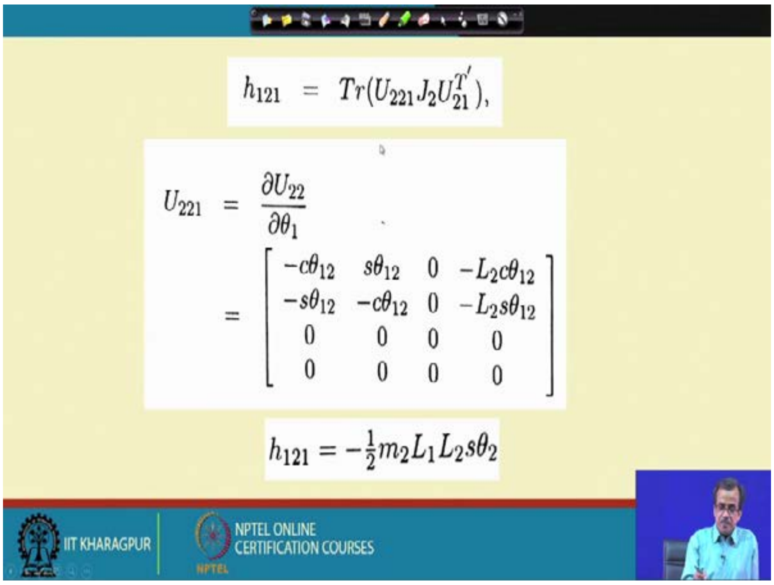

Then, h_121, so this is the expression and if you just follow the same method and this U_221 is nothing but partial derivative of U_22 with respect your theta_1. So, this particular matrix we will be getting and by following the same principle, I can find out h_121 is nothing but this.

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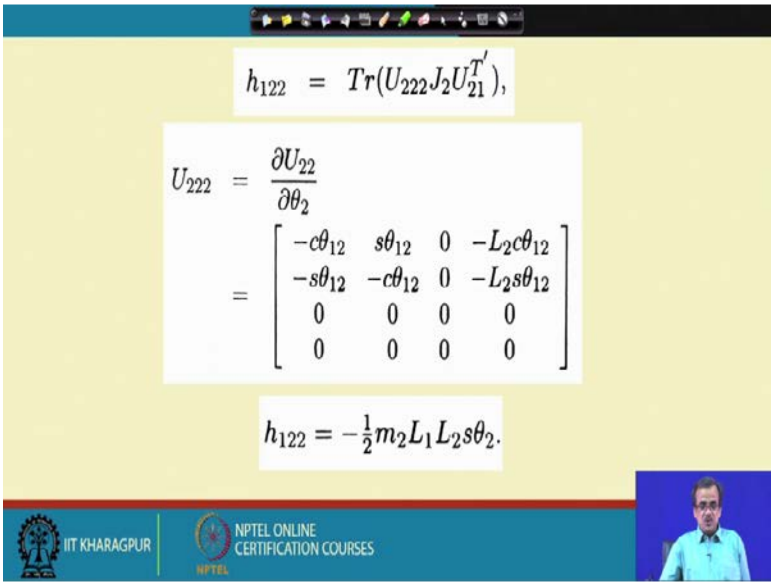

Now, here actually, once again let us try to recapitulate our purpose is to determine the expression for the joint torques like τ_1 and τ_2. So, we are following the same procedure, then comes your h_122. U_222 is nothing but partial derivative of U_22 with respect to theta_2. So, this is the matrix you will be getting and this is h_122. So, this is the expression of this h_112. So, till now 4 h values we have calculated and we will have to determine the remaining 4 h values by following the same procedure.

Thank you.

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Lecture – 28
Robot Dynamics (Contd.)

Now, I will try to derive the expression for the other h terms.

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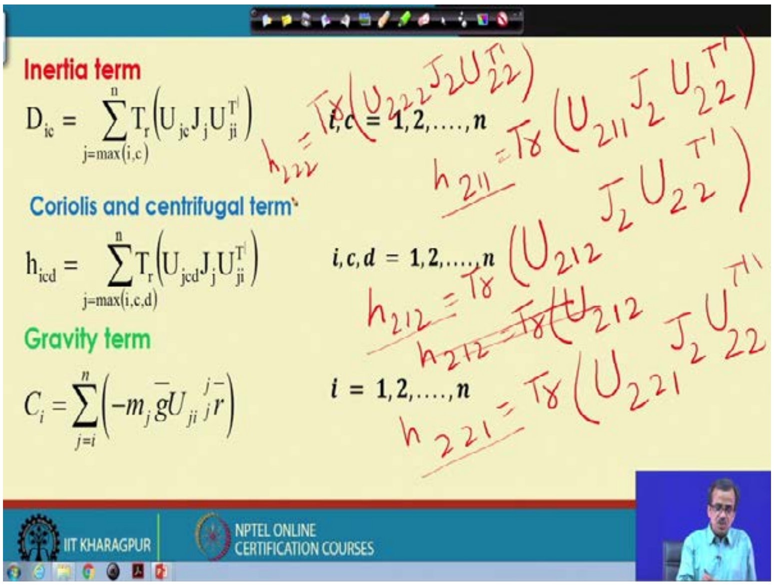

That is your h_211; now, this h_211, that is, i equals to 2, C and D they are equal to 1. So, j equals to maximum among your 2, 1, 1. So, this is 2. So, there will be only one term. So, this is nothing, but trace of like U j equals to 2, c, d are 1, 1, then comes your J_2 then comes your U_j; j equals to 2 and your this particular i is also equal to your 2. So, U_22 transpose. So, this is the way you can find out the expression for this particular h_211.

Similarly, h_212, that is nothing, but, once again 2 1 2 the maximum is 2. So, 2 2 2 there will be only one term then comes your U j is 2 here and c, d are 1, 2 , then comes your J 2 then U, j is nothing, but equal to 2 and i is also equal to your 2. So, i is also equal to your 2 and I will be getting this particular the expression.

Now, I am just trying to find out actually h_221. So, so, this is nothing, but U. So, j equals to how much j equals to 2 and c, d; c, d are nothing, but 2 1, then comes your J_2 then comes U, j equals to 2 and i is equal to your 2, i is 2 here. So, this is the expression, then I can also find out what is h_222.

Now, this is h_222. So, here once again there will be only one term. So, is nothing, but trace of U then j equals to 2 then comes c, d equals to 2 2 then J_2 then comes your U, j equals to 2 and i is also equal to 2 and transpose of that. So, this is the way actually we can find out the expression for h_211, h_222, h_221 and h_222. So, using this actually I can find out the expression for your the final expression for this particular the h terms.

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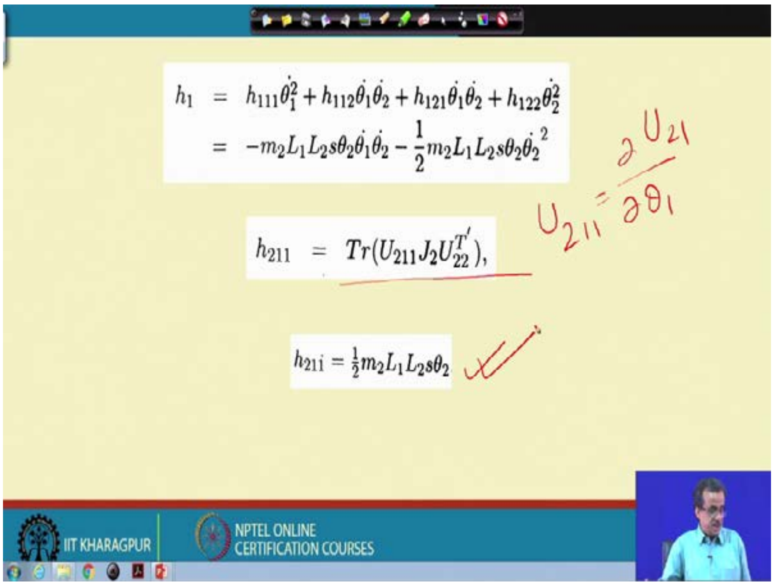

So, this is your h_211. So, this is the expression as we have already seen. Now, once again we know that this U_211 that is nothing, but actually partial derivative with respect to θ_1 of U_21, ok. J_2 you know U_22 transpose we know. So, these matrices we can multiply and we can find out the trace and the trace will become equal to this. So, this is nothing, but the expression for your this h expression for h_211.

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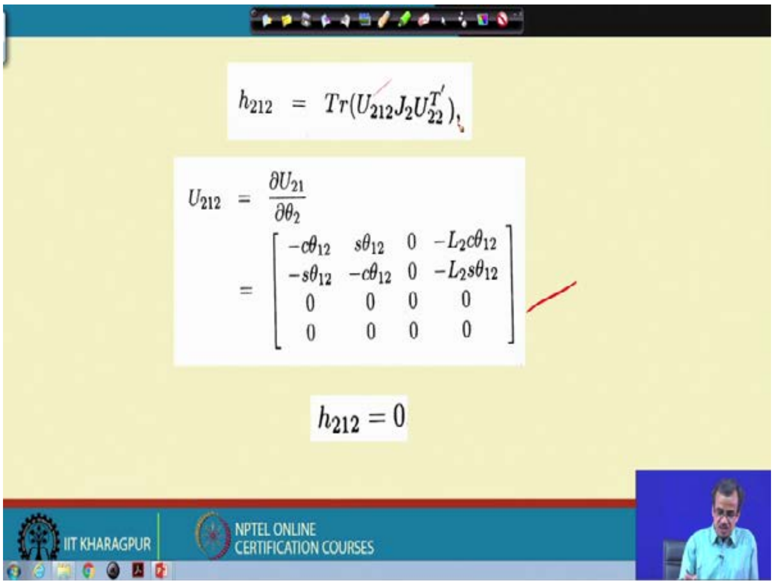

Now, h_212 so, this is the expression and once again you can find out what is U_212, that is nothing, but the partial derivative of U_21 with respect to θ_2. So, this is the expression. So, this is known all the terms are known and if you just multiply these three matrices and if you find out the trace of that, this will become equal to 0.

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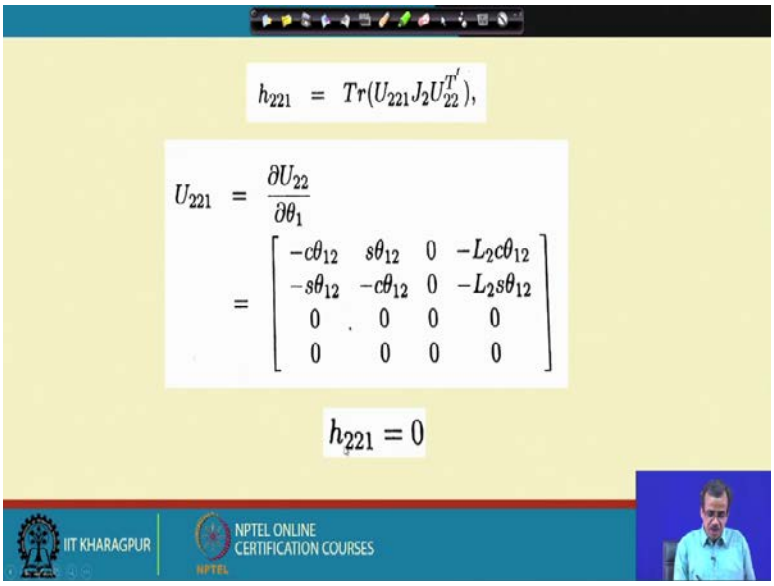

So, what I do is so, we can find out this particular h term, by following the same method, we can find out what is h_221, the same method and we will be getting h_221 is equal to 0.

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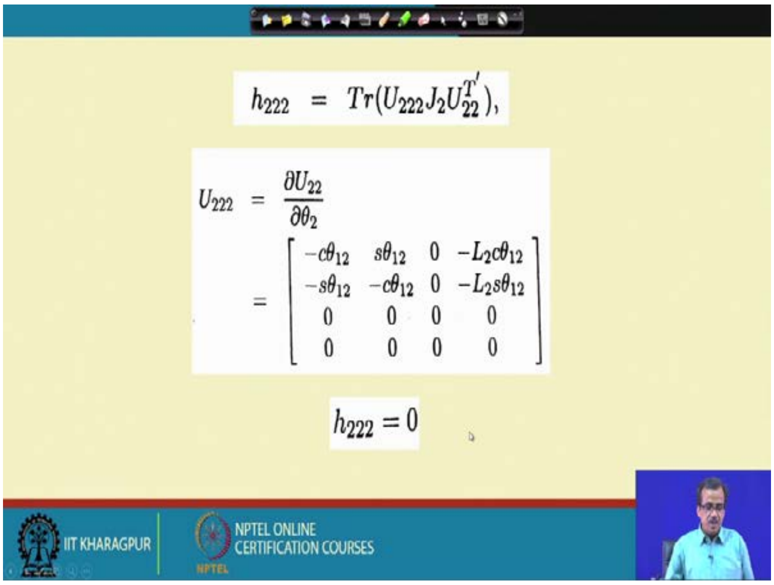

Then comes your h_222. So, this is nothing, but this particular expression, which you have already derived and here so, this U_222 is nothing, but the partial derivative of U_22 with respect to θ_2 and if you just find out so, you will be getting this particular matrix and the other matrices are also known. So, these three 4×4 matrices if you multiply take the trace value you will be getting that is equal to 0.

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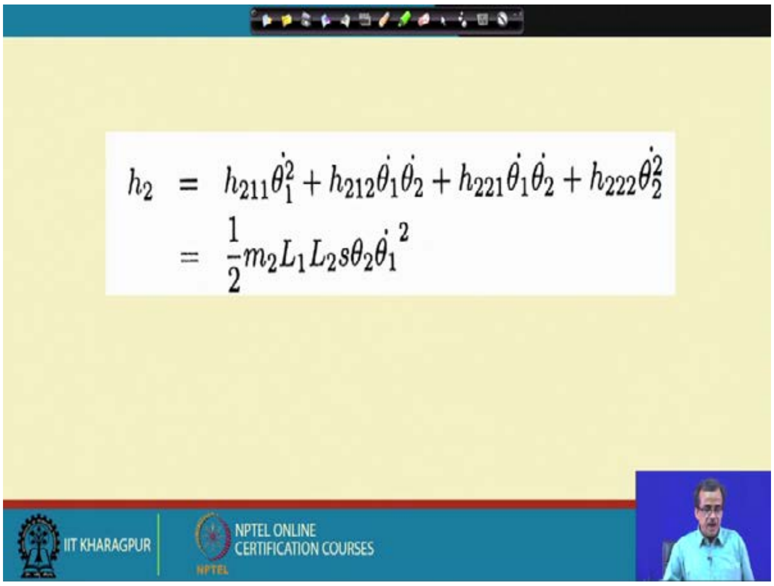

So, by following this particular method, we can find out actually all the h values so, the only thing, which is left if you see this particular expression for your τ_1, τ_2 so, this is the expression for this τ_1, τ_2.

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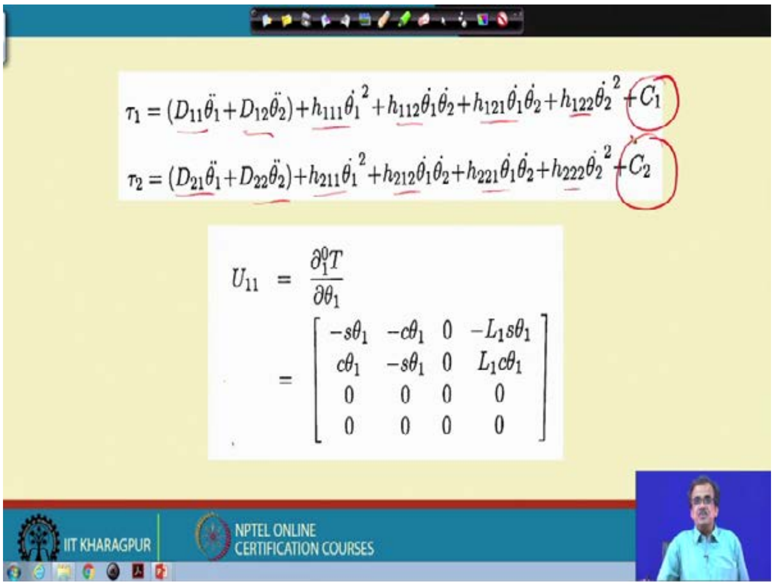

So, all such D terms, all the D terms, all the h terms, all eight h terms we have determined, only thing which is left is your C_1 and C_2; that means, your these two gravity terms are yet to be determined. Now, let us see, how to determine, this particular the gravity term now, to determine the gravity terms actually I am just going back to this particular expression and let us try to find out the expression for C_1 and C_2 from here so, this is C_i, so, let me try to find out the expression for C_1.

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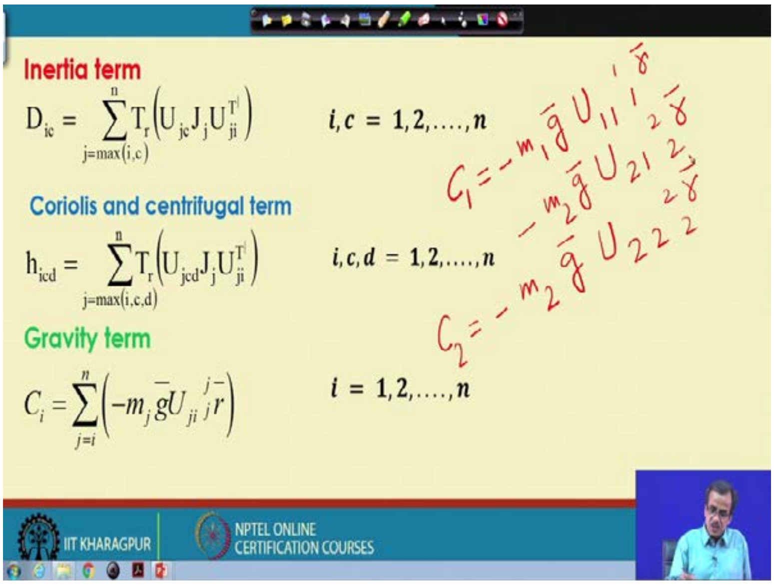

So, i equals to 1 there is j equals to what 1, 2. So, there will be two such terms. So, I can write down like $-m_1\bar{g}U_{11}\,{}^{1}_{1}\bar{r}$ then there is another term, that is, your j equals to 2 now. So, $-m_2\bar{g}U_{21}\,{}^{2}_{2}\bar{r}$.

Now, let us see, how to determine and I can also find out the expression for C_2 also. So, let me write down the expression for C_2 also. So, i equals to 2. So, j equals to 2 to 2. So, there will be only one term that is $-m_2\bar{g}U_{22}\,{}^{2}_{2}\bar{r}$. So, these are the expression for C_1 and C_2.

Now, let us see how to derive further using this particular expression. So, how to derive further?

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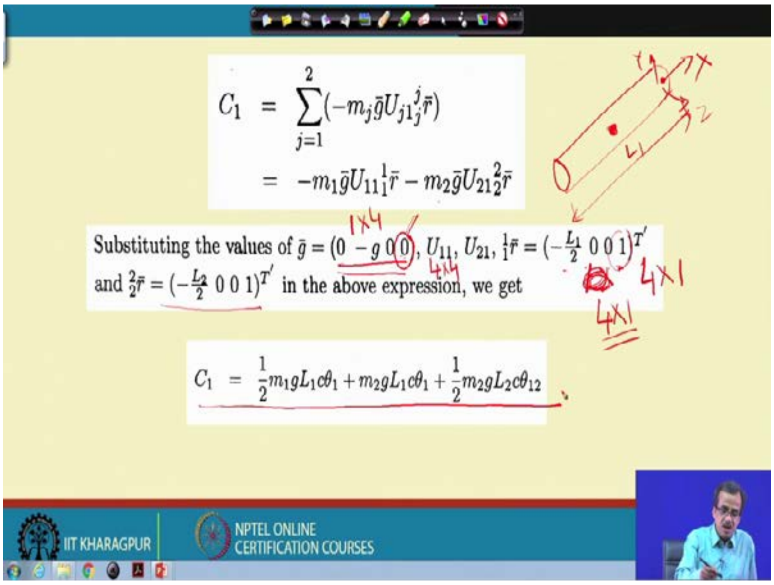

Now, to derive further actually let us see this particular C_1. So, exactly the same expression I got whatever I wrote. So, this is actually C_1.

Now, here m_1 is the mass of the first link, but g is the acceleration due to gravity. Now, here if you see, as we discussed g has three components like X, Y and Z components. Now, here actually, let us see the coordinate system once we will understand, if you see the coordinate system.

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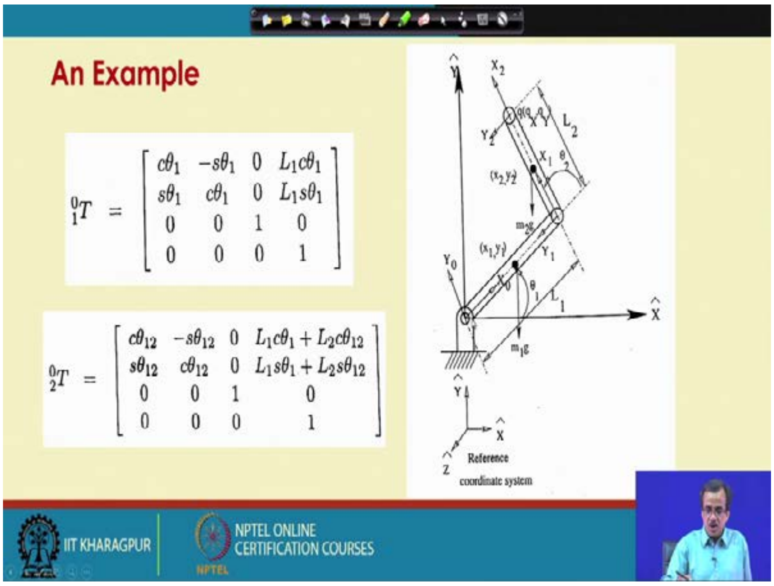

So, this is actually the Y direction. So, g is acting opposite to the Y direction vertically downward and that is why, actually, here will have to put so, this particular expression 0, minus g (in place of y I have written minus g) and Z is 0, but here one extra 0 I have put, for which the reason I will tell you.

Now, here, this U_11 we have already determined and this is nothing, but a 4×4 matrix, ok, then comes this r 1 with respect to 1, how to determine r 1 with respect to 1. So, if I consider that this is my link 1 and for this particular link supposing that the total length is nothing, but L_1, ok. So, its mass center is here, and the coordinate system as I told because we are interested to determine how much will be the reaction torque. So, the coordinate system is attached here, ok. So, with respect to this say this is the X axis, with respect to this what will happen is your, this will be –L/2. So, this is Y and this is your Z. So, this will be –L/2.

So, here, this r 1 with respect to 1 bar is nothing, but the coordinate of the mass center, that is, your minus L_1 by 2, 0, 0 that is, Y is 0, Z is 0 corresponding to this particular point and this one is actually just below the position vector, we put 1 so, that particular 1. Now, you check the dimension. So, U is having 4×4 and this particular r 1 with respect to 1. So, it is having 1×4 this is here there is a transpose so, this will become your then 4×1 matrix. So, this is the 4×1 matrix and this is your 4×4 matrix, so, if you multiply ultimately I will be getting one 4×1 matrix.

Now, this particular g matrix has to be 1×4, otherwise we cannot multiply and that is why, actually we have made it 1×4. So, this is nothing, but the 1×4. So, g has got three components and that is why this particular 0 has been added as an extra, ok, just to make it that particular1×4. So, this 1×4 can be multiplied by this 4×1, ok. That is why so, this particular extra 0 has been considered here exactly in the same way here also, we can determine.

The only thing is the expression of this U_21 is different and this r 2 with respect to 2 is nothing, but this, because in place of link 1, now will have to consider link 2. For this link 2, the mass center is $-L_2/2$, 0, 0, 1 transpose and if we just put all such things here and multiply then we will be getting the expression for C_1 and this is nothing, but this one. So, this is the way, we will be getting the expression for this particular C_1.

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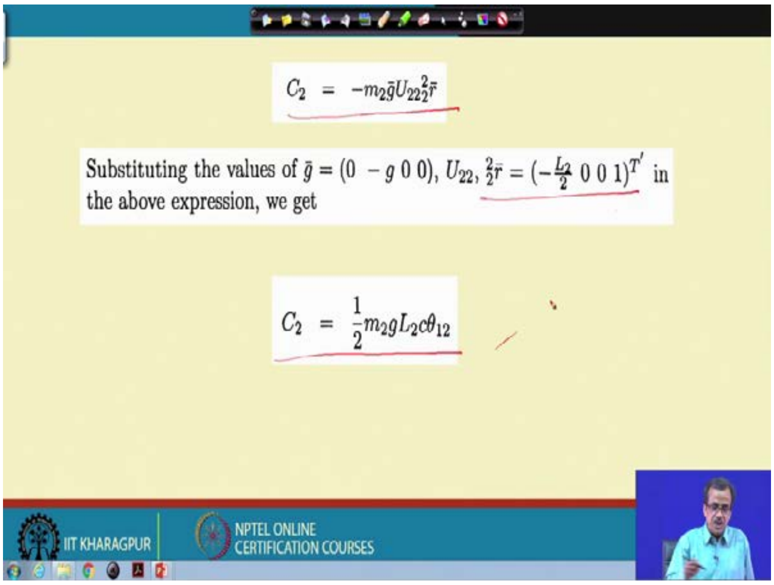

Now, let us see, how to determine the expression of C_2 following the same method. C_2 expression, I have already got and exactly in the same way, g has to be written, U_22 is known r 2 with respect to 2, this is also known and if you multiply then, you will be getting this your $C_2 = \frac{1}{2}m_2gL_2c\theta_{12}$. So, this particular expression we will be getting for this C_2.

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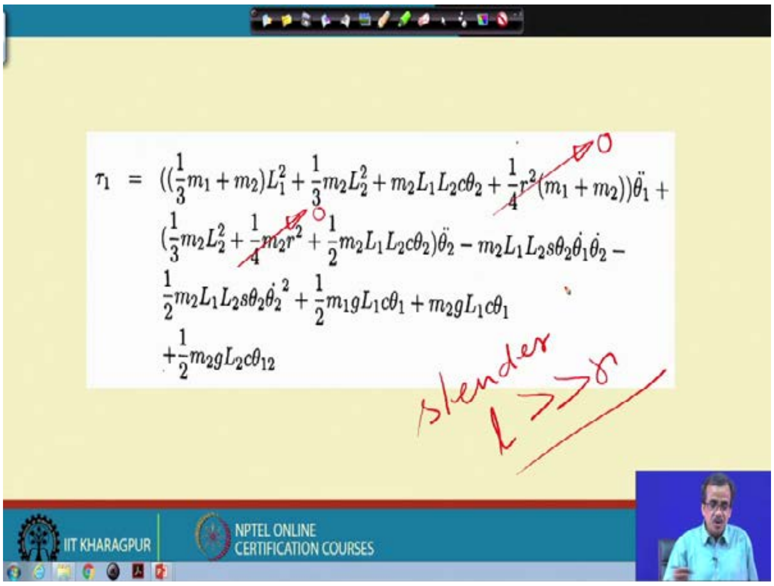

And, once I have got this particular C_2, we are in a position to write down the expression for this particular joint torque, ok. Now, in this particular expression, these particular terms, if you just add them up whatever we are like two values of D, four values of h and one value of C, we have got, if we just write them up and if we just arrange then, you will be getting this type of expression. So, this big expression multiplied by $\ddot{\theta}_1$, that is, you see $((\frac{1}{3}m_1+m_2)L_1^2+\frac{1}{3}m_2 l_2^2+m_2 L_1 L_2 c\theta_2+\frac{1}{4}r^2(m_1+m_2))$ and you can see that all such terms are related to the mass, length of the link. So, these are all actually related to the geometric. Are you getting my point? The geometry and this is nothing, but the inertia term. These are all inertia terms multiplied by this theta_1 double dot.

Similarly, so, these particular terms are multiplied by $\ddot{\theta}_2$; $\ddot{\theta}_2$ is nothing, but angular acceleration of the second joint. Now, here once again you can see m_2, L_2 then comes L_1 and we have got r, that is, the radius of the second the link and/or the first link. So, here, the radius terms are also there, ok. So, this is once again actually the inertia terms. Now, then comes your these particular terms like theta terms involving $\dot{\theta}_1\dot{\theta}_2$ then $\theta_2\dot{\theta}_2^2$. So, these are nothing, but is your the Coriolis and centrifugal term sort of thing and this particular part your $c\theta_1$, $c\theta_{12}$ these are nothing, but is your the gravity term.

So, we have got inertia term. So, these up to these actually we have got the we have got the inertia term. So, these are nothing, but the inertia terms then we have got your the Coriolis and the centrifugal term and here so, these terms are nothing, but your the gravity terms and another observation we should we should have a look to see we are trying to find out the expression for the joint torque at joint 1, ok.

Now, here, if you see carefully, there are a few terms related to your θ_2. For example, I have got $\cos\theta_2$, I have got $\ddot{\theta}_2, \cos\theta_2$ then your $\dot{\theta}_2, \dot{\theta}_2^{\,2}$, then $c\theta_{12}$; that means, although you are trying to find out the joint torque, that is, torque at joint 1 the second joint angle has got some contributions towards this particular τ_1.

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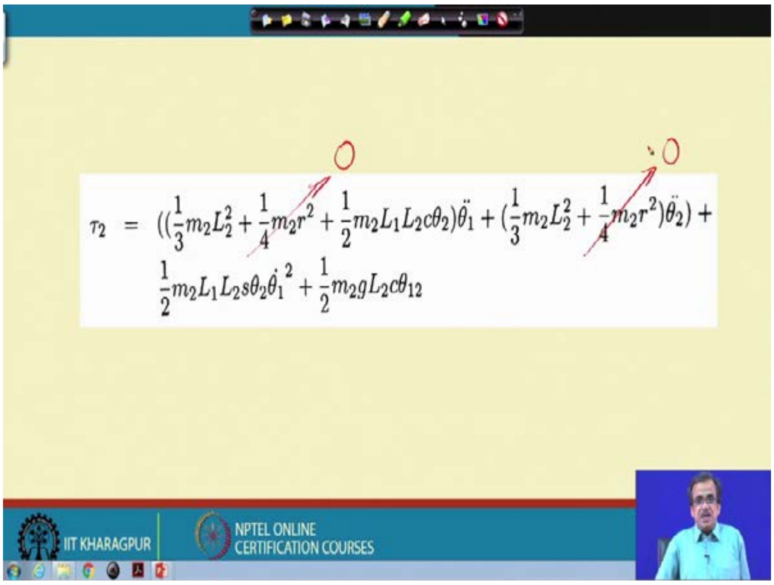

Now, we are just going to see the expression for the second joint term, that is, your these particular terms will be nothing, but the inertia terms like $(\frac{1}{3}m_2L_2^2 + \frac{1}{4}m_2r^2 + \frac{1}{2}m_2L_1L_2c\theta_2)\ddot{\theta}_1 + (\frac{1}{3}m_2L_2^2 + \frac{1}{4}m_2r^2)\ddot{\theta}_2$. So, these are all inertia terms, ok. So, this is multiplied by $\ddot{\theta}_1$, this is multiplied by $\ddot{\theta}_2$ and then, we have got actually your another term here, we can see this term is actually the centrifugal term involving $\dot{\theta}_1^2$ and we have got the gravity terms, ok.

So, this particular expression for τ_2, once again it contains three terms and once again if we look into this, we are trying to find out the expression for the joint torque that is τ_2 and here, this $\ddot{\theta}_1$ has got significant contribution then here we have got $c\theta_{12}$. So, θ_1 has got significant contribution on the joint torque 2 and that is why actually, this particular the contributions are coupled contribution and due to this particular coupled contribution the better ways should be to determine this particular joint torque like to consider the multi-body dynamics and so that the coupling terms we can consider very efficiently and you will be getting very good expression for this particular τ_1 and τ_2.

But, this particular method, the method which I have discussed, it has got one advantage I should say, because if you use this particular method there is a possibility that you will be getting very structured form of this particular expression for the joint torque and which may not be available in other method, but this method gives a very structured form.

Now, another thing I am just going to mentioned like if I consider the slender link; that means the link L is very large compared to your r. So, the terms involving r square are small. So, those terms actually we can neglect. For example, from this particular the expression here there is one r square term, ok. So, this particular term, you can neglect and it will tend to 0, if we consider the slender link.

Similarly, here, there is another r square term. So, this will also tend to 0, if we consider the slender link, and the expression for τ_1 will become simpler, but here you will not find any such terms involving r or that type of thing. So, this is the way, we can make it simple by considering the links to be slender.

Similarly, here, if we consider the slender link, this term will tend to 0. Similarly, here, there is another term, which will become equal to 0. So, the expression for this particular tau_1 and tau_2 will become simpler. Now, in robotics, actually what we do, at each of the robotic joints, we just put the DC motor and the motor is going to provide this particular torque, and this particular torque is going to generate the joint angle and it will have to be very accurately generated. Now, how to generate that very accurate joint angle that I will be discussing after some time, while discussing the control scheme.

Now, here, once you have got the variation of this torque as a function of time now, we can think about, what should be the power requirement for a particular joint and if you know the power requirement, we can prepare the specifications of the motor, which we are going to put at that particular robotic joint. So, that the robotic joint will be able to provide that particular torque and it will be able to generate that particular rotation very smoothly.

Thank you.

Robotics
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Lecture – 29
Robot Dynamics (Contd.)

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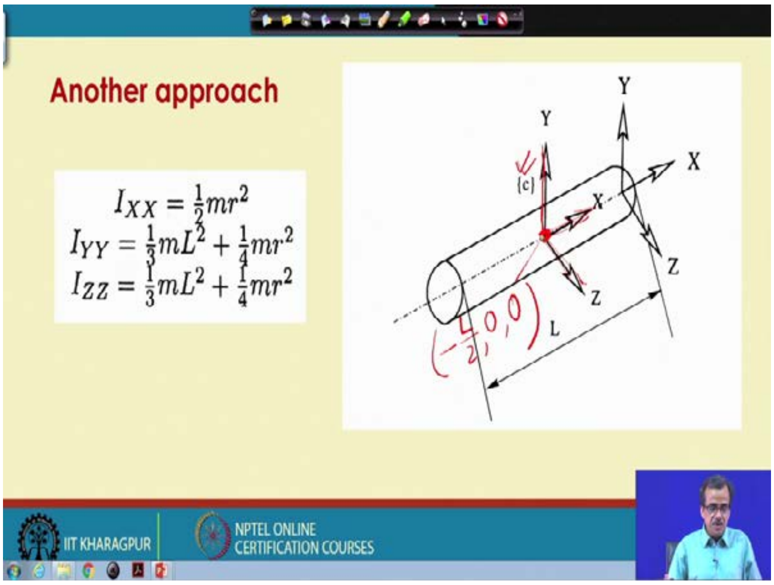

Now, I am going to discuss another approach using the Lagrangian method to derive that particular expression for the joint torque, that is, τ_1, τ_2 for a say 2 degree of freedom serial manipulator. Now, here actually one modification, we will have to do. Now, till now, if this is the robotic arm having the length L and the coordinate system is attached here, so this is your X, Y and Z, this is the coordinate system. But, the motor is connected at this particular joint. And, as I told several times that we are trying to find out the reaction torque.

Now, what you will have to do is, we consider the moment of inertia with respect to your, this coordinate system. For example, we got $I_{XX} = \frac{1}{2}mr^2$. So, m is the mass of this particular link, r is the radius. So, this is having circular cross-section with radius r.

Then, $I_{YY} = \frac{1}{3}mL^2 + \frac{1}{4}mr^2$, so this expression we have already got, we have already derived. Then, $I_{ZZ} = \frac{1}{3}mL^2 + \frac{1}{4}mr^2$, so this expression we have already derived.

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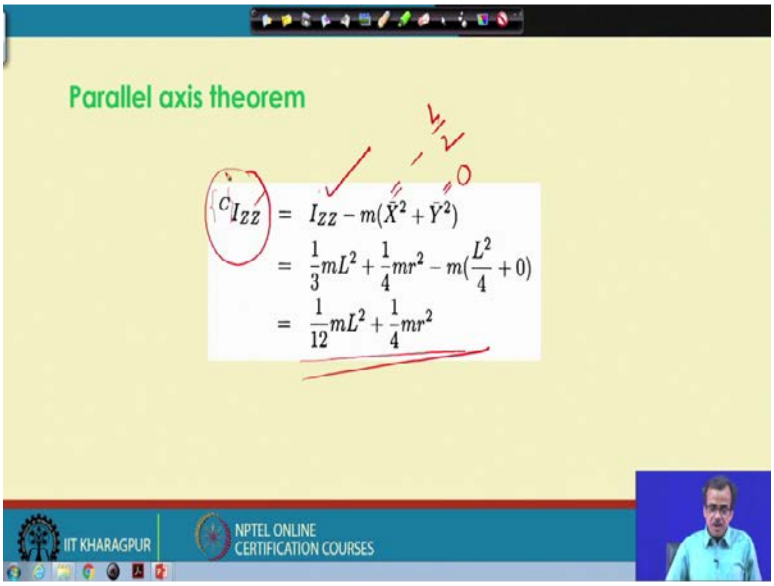

Now, here, if we just go for this particular approach, what will have to do is, we will have to express the moment of inertia with respect to this particular coordinate system, which is attached at the mass center. Now, this is the coordinate system, which is attached at the mass center that is denoted by c. And what is the coordinate of this particular mass center, the coordinate of the mass center is nothing but X equals –L/2, Y equals to 0, Z equals to 0, so this is the coordinate of this particular mass center.

So, what I am going to do is, I am just going to represent the moment of inertia with respect to this particular coordinate system and I am just going to transfer from here to here using the parallel axis theorem. So, by using the parallel axis theorem, I can find out the moment of inertia about ZZ with respect to C; C is the coordinate system, which is attached to the mass center is nothing but $I_{ZZ} - m(\bar{X}^2 + \bar{Y}^2)$.

Now, here, this X bar is nothing but, actually –L/2 and Y bar is is equal to 0. And, so this can be written as $\frac{1}{3}mL^2 + \frac{1}{4}mr^2 - m(\frac{L^2}{4} + 0)$ and if you simplify, so you will be getting this particular the expression. So, this is nothing but the expression for, this I_ZZ about the coordinate system which is attached to the mass center, so knowing this actually, what I can do is, we are going to derive the same τ_1, τ_2.

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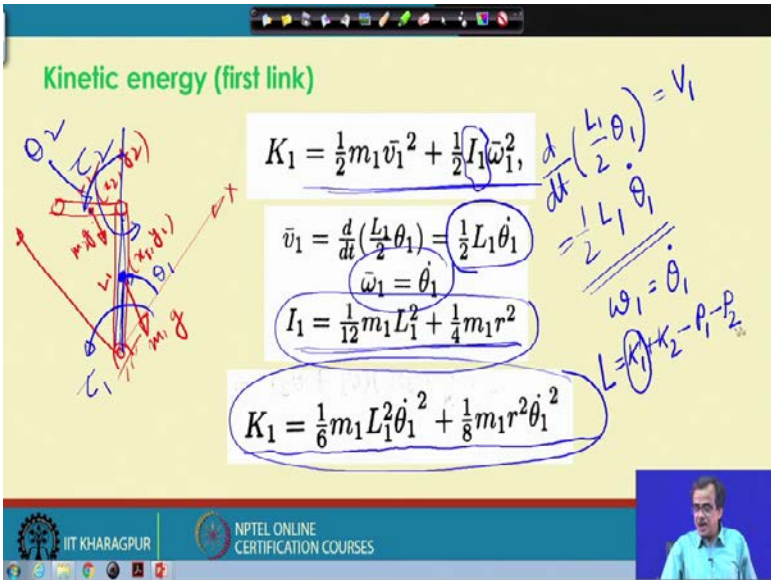

Now, if I just draw this picture once again, the same manipulator having 2 degrees of freedom. So, this is your X, this is Y, so here I have got say L_1, this is L_2. So, length of this particular is L_1, this is L_2. Now, here, this is actually the mass center here m_1 g will be acting, this is the mass center where m_2 g will be acting.

Now, here, this mass center is having the coordinates say x_1, y_1; this is having the coordinate say x_2, y_2. And our aim is to find out, our aim is to derive the expression for so this particular τ_1, τ_2, so that is our aim. The problem is the same; I am using a slightly different approach.

Now, here, let us first try to concentrate on the first link, whose length is L_1. Now, here, we try to find out the kinetic energy of the first link and by definition $K_1 = \frac{1}{2}m_1\bar{v}_1^2 + \frac{1}{2}I_1\bar{\omega}_1^2$. So, I_1 is the moment of inertia, ω is nothing but angular velocity and v_1 is a linear velocity and m_1 is the mass. So, what I will have to do is, the mass is assumed to be concentrated at the mass center. So, I will have to find out, what should be the linear velocity of this particular the mass center.

Now, to find out the linear velocity, what we will have to do is, this is the joint angle say θ_1; and this is nothing but, the joint angle that is your θ_2. So, if this is θ_1 and up to this is actually $\frac{L_1}{2}$, then how much is the arc, arc is nothing but, is your $\frac{L_1}{2}\theta_1$ and the rate of change of this with respect to time is nothing but is your V_1 so that is nothing but the linear velocity at this particular point.

So, this L_1 theta_1 by 2, d/dt of that, so this is actually I have determined here d/dt of $\frac{L_1}{2}\theta_1$ and that is nothing but, is your half L_1. So, d theta_1/dt that is nothing but $\dot{\theta}_1$, so this is nothing but the expression. And here, so this particular ω_1 is the angular velocity and that is nothing, but $\dot{\theta}_1$. Are you getting my point? And, this particular I_1 the expression for this particular $I_1 = \frac{1}{12} m_1 L_1^2 + \frac{1}{4} m_1 r^2$, so this I have already derived.

Now, if I substitute here, m_1 is the mass, m_1 will remain same as m_1, V_1, so this is the expression for V_1 then I_1, so this is the expression for I_1 and omega_1 is nothing but $\dot{\theta}_1$. So, if I substitute all the terms here and if I simplify then, I will be getting, this particular expression for the kinetic energy of the first link, that is, $K_1 = \frac{1}{6} m_1 L_1^2 \dot{\theta}_1^2 + \frac{1}{8} m_1 r^2 \dot{\theta}_1^2$. So, this is nothing but the expression for this particular the kinetic energy, this is the kinetic energy for the first link.

Now, I am just going to derive the potential energy for the first link and then, I am going to derive the kinetic energy for the second link and potential energy for the second link and if I can find out the kinetic energy and the potential energy for both the links I can find out, what should be the Lagrangian for this particular robotic system. For example, say the Lagrangian for the robotic system L will be nothing but kinetic energy of the first link plus kinetic energy of the second link minus potential energy of the first link minus potential energy of the second link. So, till now I have derived only this particular K_1 so, I am just going to derive K_2, P_1, P_2.

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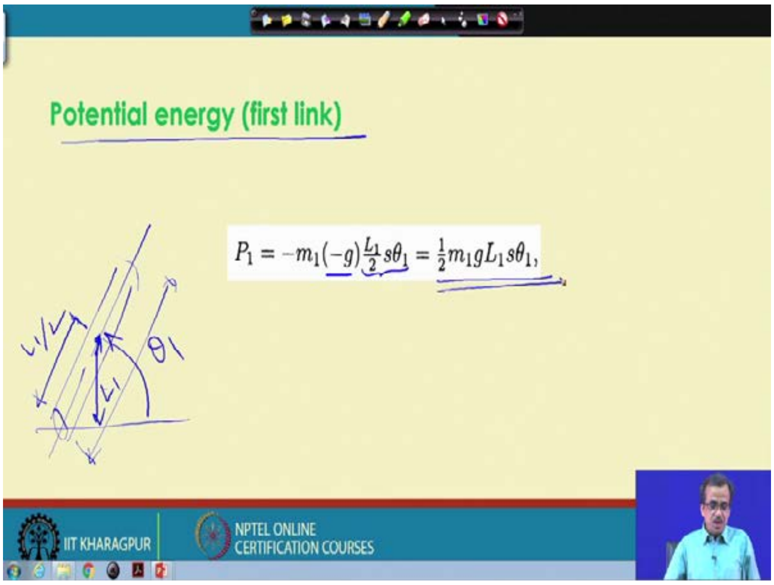

So, let us see, how to derive this particular the P_1, that is the potential energy for the first link. Now, the potential energy for the first link, that is, P_1 is nothing but, minus m_1 then multiplied by minus g once again g is acting vertically downward opposite to the that Y direction and this particular the height. If you see, so that was nothing but if this is the first link, if this is actually the first link the total length was say L_1 and this particular angle, angle was actually with theta 1 and this is nothing but L_1 by 2, so up to this is your L_1 by 2. So, $\frac{L_1}{2}\sin\theta_1$ is actually this particular the height, so this is nothing but $\frac{L_1}{2}\sin\theta_1$.

And, if you just simplify, so you will be getting this as the expression for the potential energy. Now, till now, we have derived the expression for the kinetic energy and the potential energy for the first link. Now, I am just going to derive the kinetic energy and potential energy for the second link.

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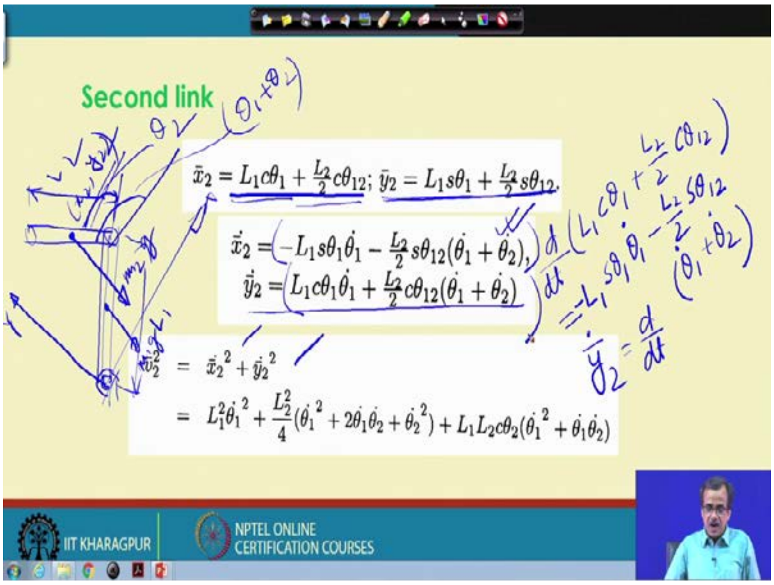

Now, once again, if you go back like if you just once again draw this particular picture once again, so this is actually your. So, I have got first link here and I have got the second link here this is your X, this is your Y, ok. So, the length of this particular link is your say L_1 and this particular link the length is your L_2 and its mass center is here whose coordinate is nothing but x_2, y_2 and here m_2 g is acting and here m_1 g is acting.

Now, here this x_2, y_2 is the coordinate of the mass center for the second link. Now, the general expression for the coordinate of the mass center for the second link that is, $\bar{x}_2 = L_1 c\theta_1 + (L_2/2)c\theta_{12}$, because with respect to this is your theta_2, this particular angle is theta_2, but with respect to x this is theta_1 plus theta_2, ok. Then, $\bar{y}_2 = L_1 s\theta_1 + (L_2/2)s\theta_{12}$.

Now, we can find out the time derivative, that is, x_2 dot, that is, d/dt of this, now, d/dt of your $L_1 c\theta_1 + (L_2/2)c\theta_{12}$. So, I will be getting $-L_1 s\theta_1\dot{\theta}_1$.

Similarly, here, we will be getting $-\frac{L_2}{2}s\theta_{12}(\dot{\theta}_1 + \dot{\theta}_2)$. So, I will be getting this particular expression. By following the same method, so y_2, I know the general expression for $\bar{y}_2 = L_1 s\theta_1 + (L_2/2)s\theta_{12}$. So, $\dot{\bar{y}}_2$, that is nothing but, your d/dt of this particular expression and you will be getting $L_1 c\theta_1\dot{\theta}_1 + (L_2/2)c\theta_{12}(\dot{\theta}_1+\dot{\theta}_2)$, so we will be getting this particular $\dot{y}_2$.

Now, v_2 square is nothing but x_2 dot square plus y_2 dot square. So, square of this plus square of this and if you just add them up, then you will be getting the expression like $\bar{v}_2^2 = \dot{\bar{x}}_2^2 + \dot{\bar{y}}_2^2 = L_1^2\dot{\theta}_1^2 + (L_2^2/4)(\dot{\theta}_1+\dot{\theta}_2)^2 + L_1L_2 s\theta_1 s\theta_{12}\dot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2) + L_1L_2 c\theta_1 c\theta_{12}\dot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2)$.

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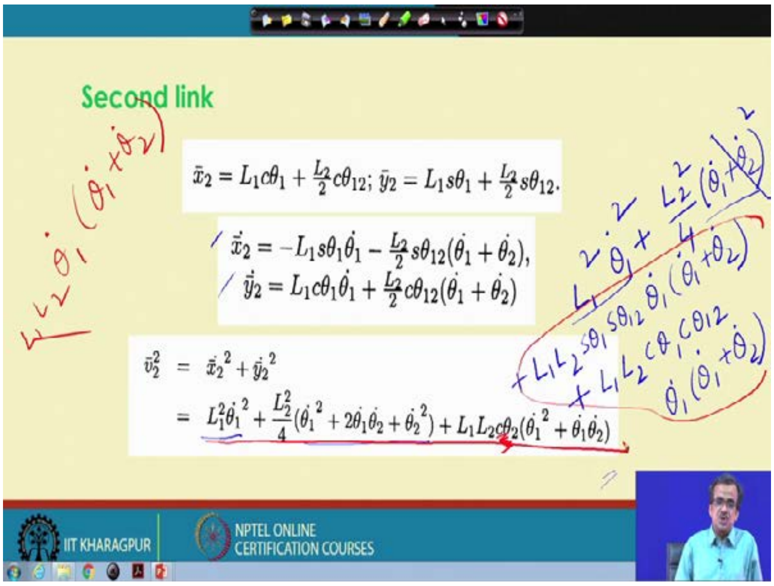

So, using this actually I can if you simplify further. So, I will be getting L_1 square theta_1 dot square. So, these I am getting L_2 square by 4 then this is expanded theta_1 dot square plus 2 theta_1 dot theta_2 dot plus theta_2 dot square. Now here we will have to simplify, now to simplify this part actually, what I can do is, we can take $L_1L_2\dot{\theta}_1(\dot{\theta}_1+\dot{\theta}_2)$ as common. So, I can write down like this particular part I am just going to concentrate on say this particular part.

This particular part if I want to make it simple, so this will become your $\cos(\theta_1+\theta_2)\cos\theta_1 + \sin(\theta_1+\theta_2)\sin\theta_1$, so $\cos(\theta_1+\theta_2-\theta_1)$. So, this is nothing but $\cos\theta_2$. So, I will be getting this particular $\cos\theta_2$ term, ok. So, I am getting this particular expression for v_2 square and once you have got the expression for v_2 square.

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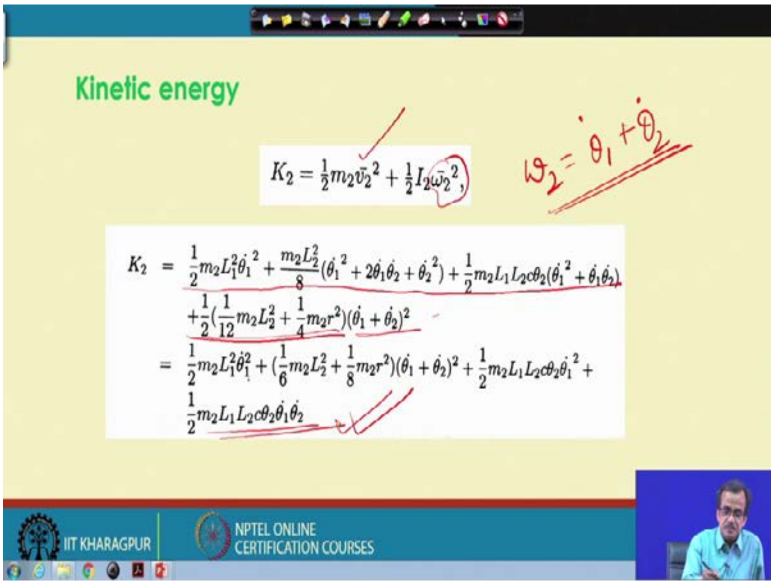

So, very easily, I can find out the kinetic energy for the second link $K_2 = \frac{1}{2} m_2 \bar{v}_2^2 + \frac{1}{2} I_2 \bar{\omega}_2^2$, so this $\omega_2 = \dot{\theta}_1 + \dot{\theta}_2$, not only $\dot{\theta}_2$. So, we will have to be careful.

Now, if we just write down the expression v_2 square we have already derived and we just put it here I_2, I know the expression of I_2, I know, so this is the expression of I_2. In fact, $K_2 = \frac{1}{2} m_2 \bar{v}_2^2 + \frac{1}{2} I_2 \bar{\omega}_2^2$ and if you simplify. So, you will be getting this particular the expression ok, only thing you need some practice to find out whether you are getting the same expression or not. So, you will be getting this particular expression for the kinetic energy of the second link.

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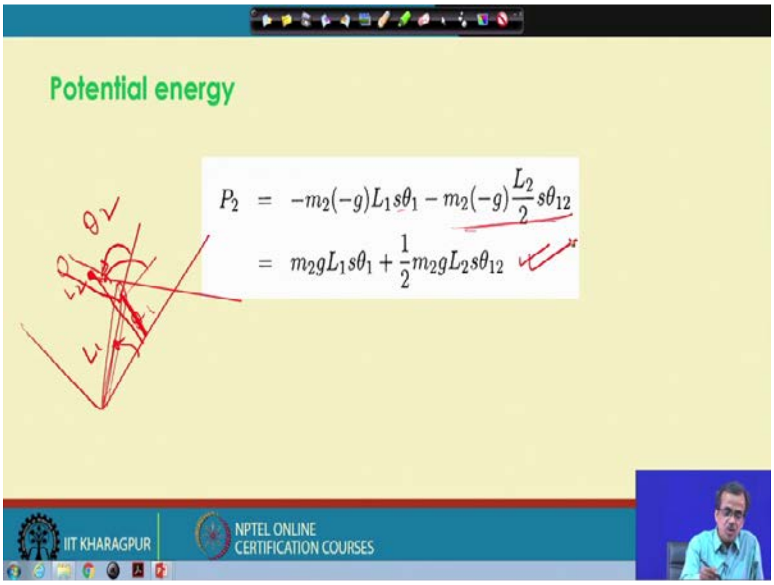

And, once I got the kinetic energy for the second link, now, we are in a position to determine the expression for the potential energy for the second link. Now, for the second link, actually once again if you draw this particular picture, so this is L_1 link and this is another link. So, your $-m_2(-g)L_1 s\theta_1$ because this particular angle is your theta_1 and this is your theta_2 ok, this length is L_1, this is L_2 and here $-m_2(-g)\frac{L_2}{2}s\theta_{12}$. So, this is actually your, the total height will be getting. So, this is the way actually you can find out the potential energy. So, this is the expression for the potential energy, now as I told that we have got the expression for the kinetic energy of the two links and potential energy of the two links.

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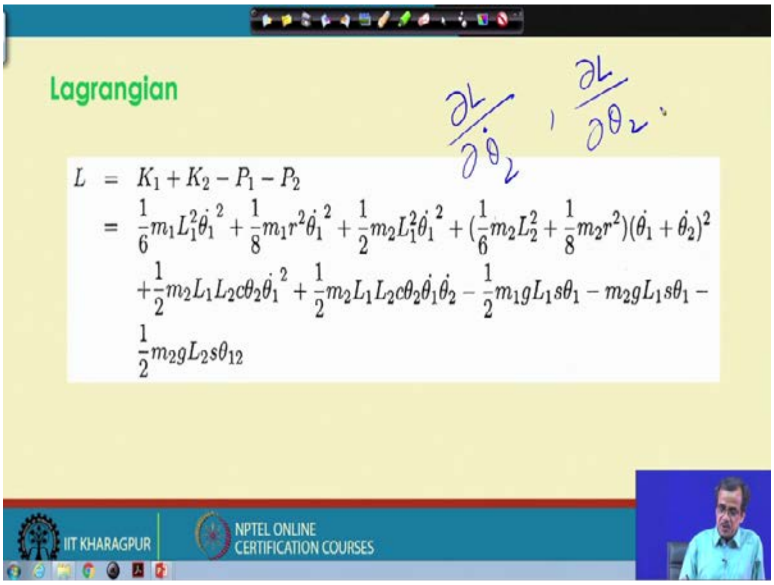

Now, we are in a position to find out the expression for this particular Lagrangian. So, this Lagrangian L is nothing but K_1 plus K_2 that is kinetic energy of the first link plus kinetic energy of the second link minus potential energy of the first link minus potential energy of the second link.

So, whatever expressions you got you just write down and then you do a little bit of rearrangement of $\dot{\theta}_1^2$. So, in this particular way, you try to arrange these particular the terms and this is the expression for this particular Lagrangian for the whole robotic system. And, once I have got this particular Lagrangian, now you know the expression, you know how to determine actually the τ_1, τ_2.

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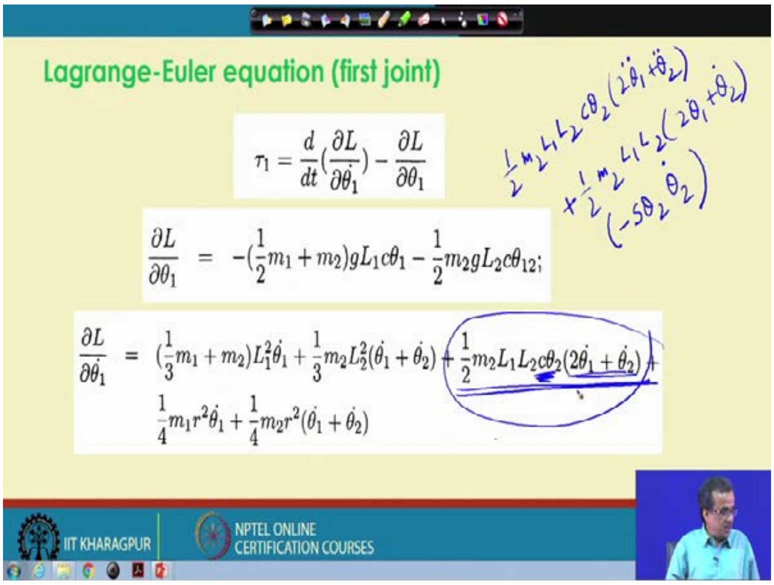

So, using the same formula, the τ_1 is d/dt a partial derivative of Lagrangian with respect to your theta_1 dot minus partial derivative of L with respect to this particular θ_1. So, we know the expression for this particular L, that is, the Lagrangian. So, once again, let us go back to the expression of Lagrangian and here, the derivative which we will have to find out is nothing but the derivative which we will have to find out is nothing but your partial derivative of L with respect to your θ_1 and to determine τ_1, another partial derivative of L with respect to your θ_1 is to be known.

Now, you see, you concentrate on this particular expression. So, here, we have got one $\dot{\theta}_1^2$ term, I have got another $\dot{\theta}_1^2$ term, $\dot{\theta}_1^2$ term and here I have got $\dot{\theta}_1$ term, $\dot{\theta}_1^2$ term, $\dot{\theta}_1$ term. So whenever I am just going to find out this particular partial derivative. So, starting from here up to this, we will have some contribution. Are you getting my point? But, these terms will not have any contribution.

Similarly, whenever we are going to find out your partial derivative of L with respect to θ_1, now $\dot{\theta}_1^2$ will have no contribution, here there will be no contribution, no contribution, no contribution ok, and this is θ_2, but this is with respect to θ_1. So, there will be no contribution here. Here also, no contribution, but the contribution will come from here and it will come from here in this particular partial derivative ok, so that we will have to understand

Now, exactly the same thing whatever I told, the partial derivative of the Lagrangian with respect to θ_1. So, this is the thing which we will be getting. And, this particular partial derivative with respect to $\dot{\theta}_1$, so these are the terms which you will be getting. And, once I have got it, now we are in a position, like now we will have to find out the time derivative of this.

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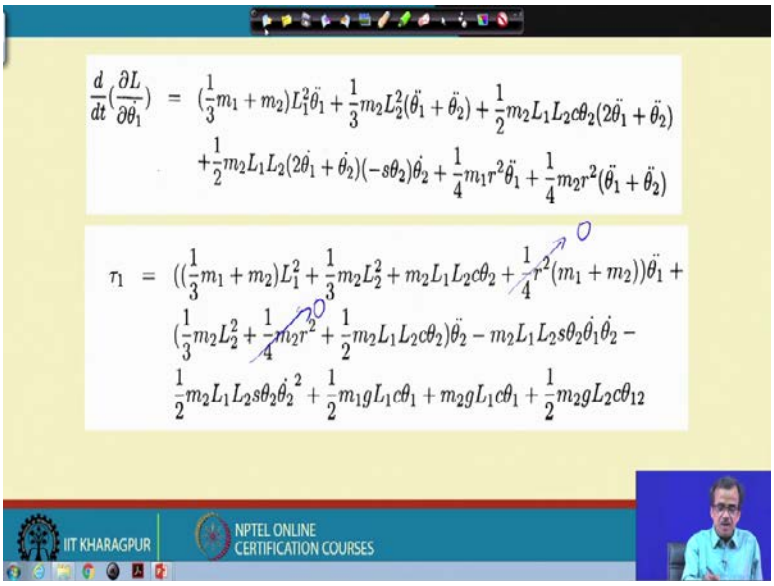

So, the d/dt of the partial derivative of L with respect to $\dot{\theta}_1$, so this particular previous term it was what $\dot{\theta}_1$, now it will become $\ddot{\theta}_1$. This was $\dot{\theta}_1$, $\dot{\theta}_2$, so this will become $\ddot{\theta}_1, \ddot{\theta}_2$. So, here it was $\dot{\theta}_1$, $\dot{\theta}_2$, so this will become $\ddot{\theta}_1, \ddot{\theta}_2$ and next this particular term, So, once again, it will take the form like this, and your so this particular term has got has got two terms here you can see just a minute. So, this term we will be getting from here and this particular term will be getting from here, but this particular term if you want to find out, the time derivative.

So, finding this particular term, the time derivative, so these terms and these terms you will have to consider separately. So, once you will have to consider this as constant, and you will have to find out the time derivative. For example, if I concentrate only here, so its time derivative will be something like this like $\frac{1}{2}m_2L_1L_2c\theta_2(2\ddot{\theta}_1 + \ddot{\theta}_2) + \frac{1}{2}m_2L_1L_2(2\dot{\theta}_1 + \dot{\theta}_2)$. Now, you will have to find out d/dt of

$\cos\theta_2$, that is nothing but d/d theta_2 that is $-\sin\theta_2$, d theta_2/dt, that is your $\dot{\theta}_2$ ok. This is multiplied.

So this particular thing you will have to do. And, if you do that, then you will be getting, this particular expression for this joint torque that is τ_1. And, once again, you can see that, this is nothing but this is nothing but is your the inertia terms multiplied by $\ddot{\theta}_1$ this is inertia terms multiplied by $\ddot{\theta}_2$. This is the centrifugal terms the centrifugal terms and you will be getting some gravity terms something like this.

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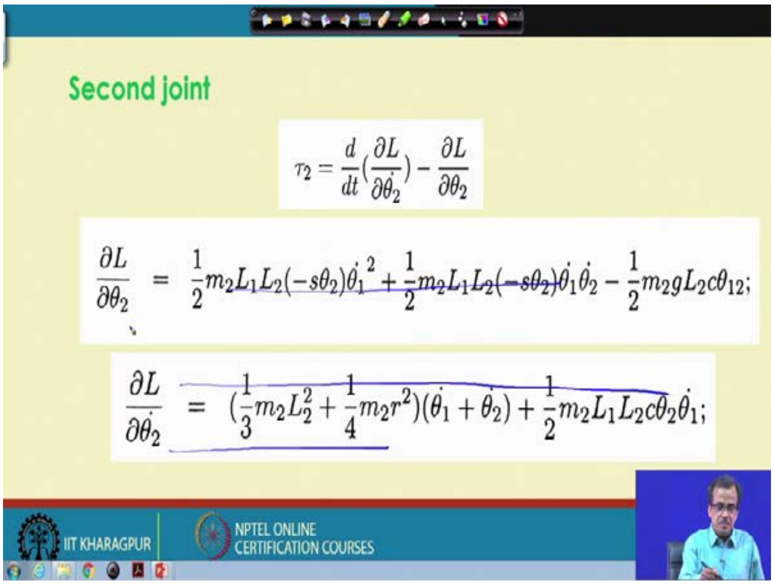

Now, let us try to find out the expression for the torque for the second joint. Now, here this τ_2 is nothing but d/dt of partial derivative of Lagrangian with respect to $\dot{\theta}_2$ minus partial derivative of L with respect to θ_2. Now, exactly in the same way, so we will have to find out the partial derivative of L with respect to θ_2.

Now, if I see the expression of this particular Lagrangian, this is the expression of the Lagrangian. Now, we will have to find out actually the partial derivative of L with respect to $\dot{\theta}_2$; this is one partial derivative another partial derivative with respect to your θ_2.

Now, if you see with respect to $\dot{\theta}_2$ here there is no such $\dot{\theta}_2$ here. So, here the $\dot{\theta}_2$ comes here, so it will have some contribution. Then, $\dot{\theta}_2$ here it has got $\dot{\theta}_2$, so it will have some contribution ok. On the other hand, this θ_2 here, there is no θ_2 here, no here, $\cos\theta_2$, so it will have some contribution, it will have some contribution, this is θ_1 and here, we have got θ_2. So, it will have some contribution.

So, this is the way actually, we will have to find out the partial derivative of this Lagrangian with respect to θ_2 ok and if you find out the partial derivative of this particular Lagrangian with respect to θ_2. So, we can find out like partial derivative of L with respect θ_2, so this is the expression which will be getting this is the expression which we will be getting. And, partial derivative of L with respect to $\dot{\theta}_2$, so this is the expression, which I will be getting. And, now, we will have to find out d/dt of this.

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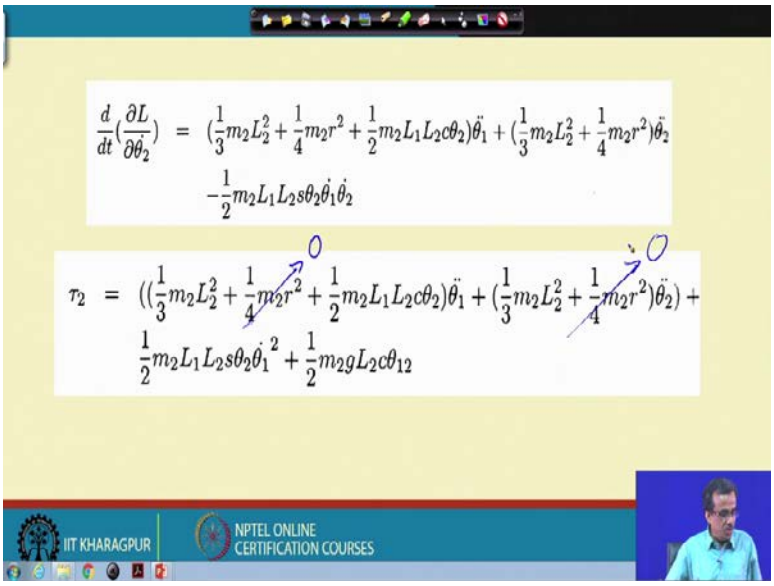

And, if you find out the d/dt of this then what will happen is, so this $\dot{\theta}_1$ term will become your $\ddot{\theta}_1$. So, this will become $\ddot{\theta}_1, \ddot{\theta}_2$. And, here also, dot will become double dot sort of thing. And, then, if we just arrange you will be getting this particular expression. This is the expression, which we will be getting for this. And, then, you substitute you will be getting the expression for the joint torque, that is, τ_2. And, this particular τ_2, if you see once again, so this is nothing but the inertia term, this is also inertia term, this is your the centrifugal term, and this is the gravity terms.

So, we have got this particular expression for this particular τ_1, τ_2 for the same problem using two different methods. And, if we compare, we are getting exactly the same expression, what we got earlier. So, the same expression, I will be getting like if we compare these particular τ_2 for the same problem, whatever τ_2, I got a few minutes ago exactly the same expression I got. The same is true for your τ_1. The expression, which I got using this particular method and the expression which I got a few minutes ago are exactly the same ok. And, it proves that both the methods are correct and we are getting exactly the same expression for joint torques: τ_1, τ_2.

Now, if I consider the slender link exactly in the same way. So, what you can do is, this r squared term we can neglect for the slender link, this particular r square term, we can neglect. Similarly, what you can do is, from this particular τ_2, we can neglect this particular r^2 terms, here also r^2 terms will become 0, and you will be getting this particular final expression for τ_1, τ_2 for the slender link.

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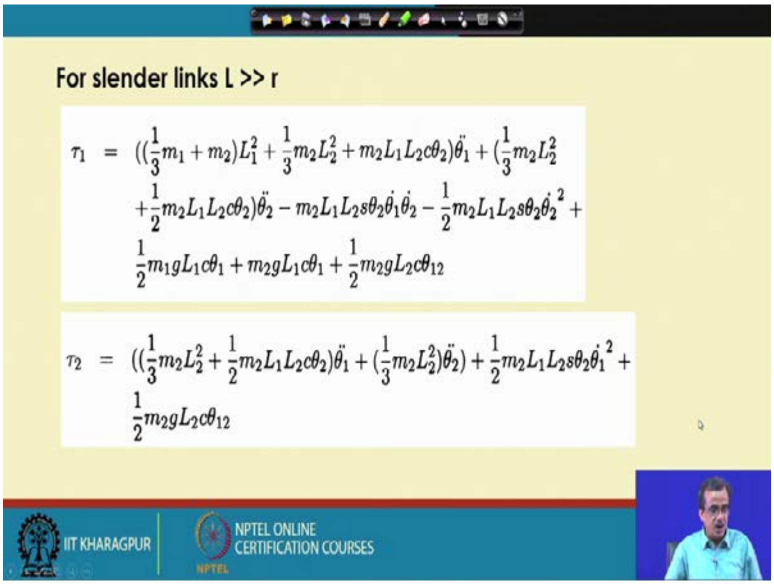

Now, we have got the expression for this particular joint torque. And, as I told, once I have got the expression, the next task will be your how to implement in the robot, in the robotic joint, so that the motor can generate this particular joint torque, which will be discussed in the next class. But, before that, one thing I just want to mention.

Now, today I discussed actually two approaches, two methods to determine the joint torques. The first method is more structured but the second method is very easy to implement; but if we just compare for a manipulator having two degrees of freedom, three degrees of freedom, we can go for the second approach.

But, for a manipulator having say 6-5 or more than 6 degrees of freedom, my recommendations would be the first approach. And, by using this, you can find out the joint torque, then we will see how to control in future like how to control these particular motors to generate that particular torque and to generate the joint angle as accurately as possible.

Thank you.

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Lecture – 30
Control Scheme

We are going to start with a new topic, that is, topic 5, it is on Control Scheme. Now, we have seen starting from the kinematics like how to derive the expression for the joint torque and at each of the robotic joints, we put a motor and this particular motor is going to supply that particular torque.

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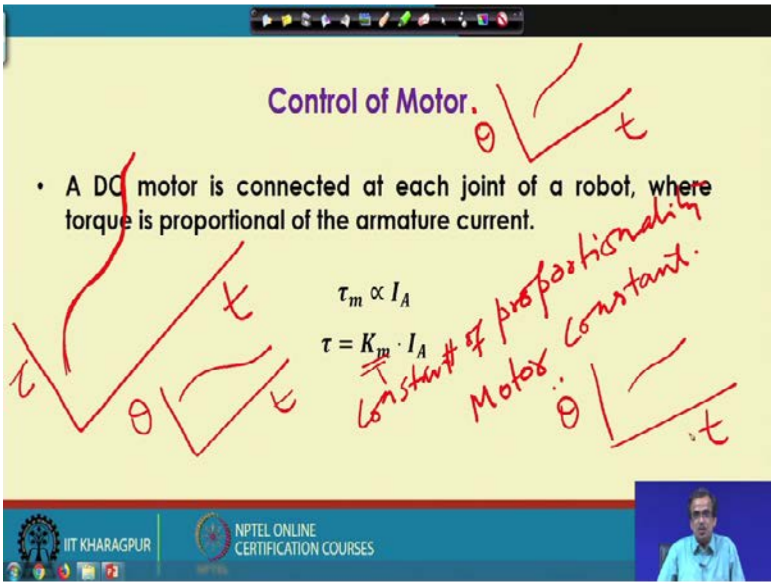

We generally use the DC motor and this particular DC motor is connected at each of the robotic joints. And, here, the generated torque is proportional to the armature current.

Now, if you see the motor torque, that is denoted by say τ_m and the armature current is I_A. So, τ_m is proportional to I_A, that is the armature current and this can be written as your $\tau_m = K_m I_A$. Now, this particulate K_m is nothing, but the constant of proportionality. So, this is nothing, but the constant of proportionality and this is also known as your the motor constant. So, this is also known as the motor constant.

So, this $\tau_m = K_m I_A$. Now, here, actually what will have to do is, at each of the robotic joint will have to generate this particular τ, that is the torque as a function of time, for example, if I plot for E particular robotic joint. So, this is the joint torque, so this particular joint torque as a function of time, will have to generate. Now, supposing that this particular distribution is something like this, a very random distribution, I have considered.

And, at the same time, what will have to ensure is the joint, the angle, that is, θ as a function of time. So, there must be some continuous curve something like this and at the same time, the first time derivative of θ that is your $\dot{\theta}$ as a function of time and your $\ddot{\theta}$, that is, acceleration as a function of time so, some plot we will have to find out, some distribution we will have to find out.

Then, how to ensure that this particular motor, the DC motor is going to generate this amount of torque with time. This is what is required, if I want to or create some angular displacement at the robotic joint in a particular the cycle time, so will have to ultimately generate this particular θ as a function of time, $\dot{\theta}$ as a function of time, $\ddot{\theta}$ as a function of time. Now, I am just going to discuss how to ensure that this particular DC motor is going to generate this torque within this cycle time.

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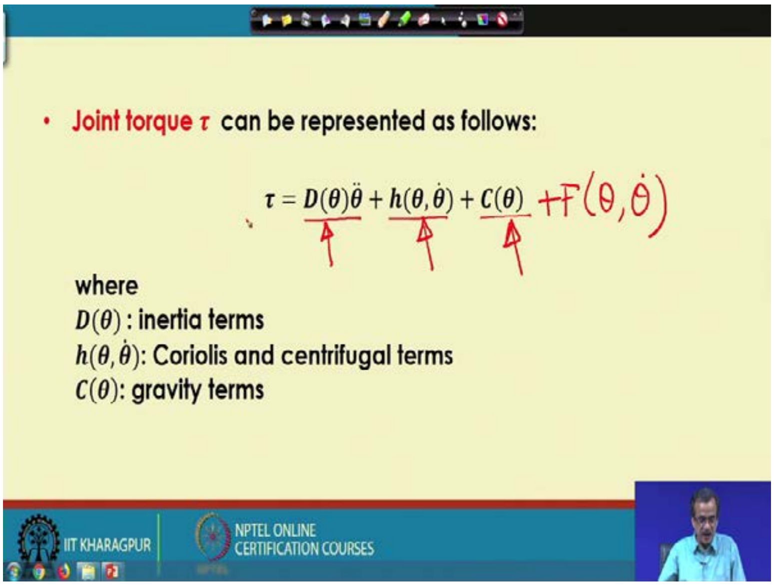

Now, let us see how to generate this particular torque. Now, to generate this particular torque let me once again go back to the expression of the joint toque, which I have already discussed while discussing the dynamics. That this particular joint torque $\tau = D(\theta)\ddot{\theta} + h(\theta,\dot{\theta}) + C(\theta)$. Now, I have already discussed that this is nothing, but the inertia term, this is Coriolis and centrifugal term and this is the gravity term, and if I add the friction term.

So, this frictional term is to be added here, which I am not adding for simplicity, but we can add this particular friction term. Now, this particular torque has to be generated by the motor, how to generate that I am going to discuss.

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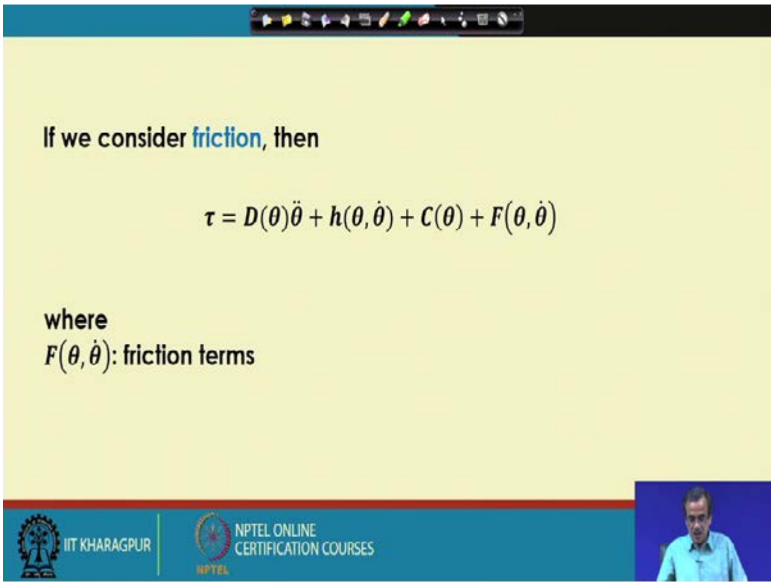

Now, to generate this particular torque, so what will have to do is.

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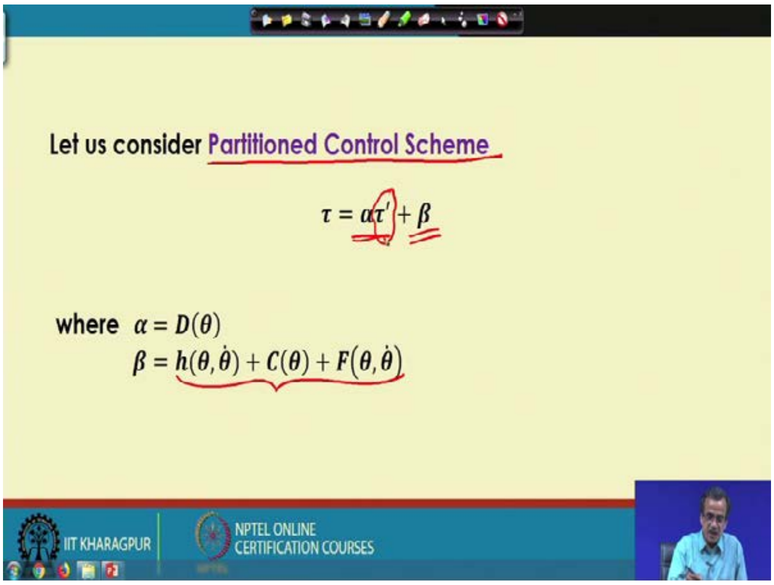

So, we will have to consider a particular control scheme, we will have to take the help of a control scheme. Now, here if you see the literature, we have got a few very popular control schemes and out of all such control schemes, this is the most important one, and this is known as the partitioned control scheme.

Now, here in partition control scheme, what I do is, the torque to be generated, that is, τ that is distributed that is divided into two parts one is nothing, but $\alpha\tau'$, so this is one part and another is the β. Now, if you say that expression of the joint torque, this $h(\theta,\dot{\theta})+C(\theta)+F(\theta,\dot{\theta})$, so this particular terms taken together we call it β and then $\alpha = D(\theta)$, $D(\theta)$ is nothing, but that inertia terms, ok. And, this particular τ' that will have to generate with the help of a controller. Now, each of this particular motor is having its own controller.

Now, let us try to explain how can it generate, so that particular τ' with the help of its controller and this particular controller is inbuilt. So, let us try to explain that how to generate that particular the required τ'.

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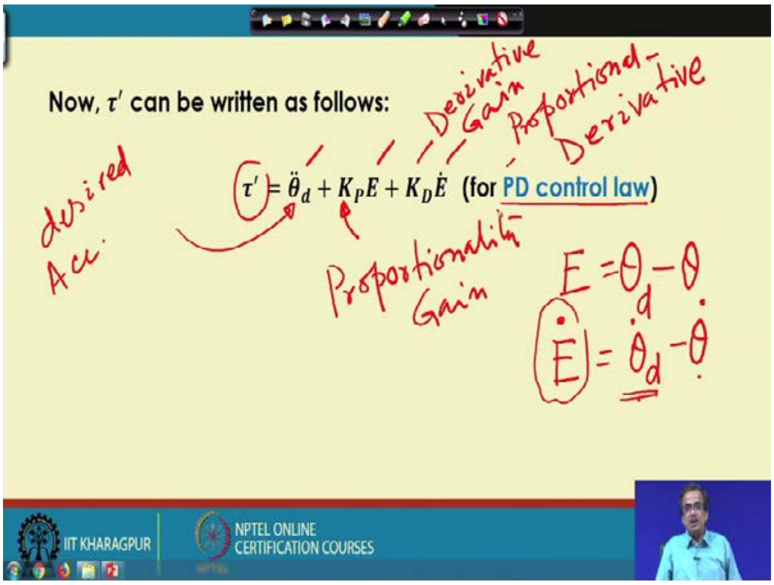

Now, this $\tau' = \ddot{\theta}_d + K_P E + K_D \dot{E}$, if I consider PD control law. Now, PD control law means Proportional Derivative control law. So, proportional derivative control law, that is called the PD control law. Now, here, I am using two symbols: one is called this K_P, now K_P is nothing, but actually the proportionality gain.

So, this is nothing, but proportionality gain value and this K_D is nothing, but the derivative gain value, derivative gain. Now, here, I am also using the terms like E, E is nothing, but the error and that is the difference between the desired theta and the theta which is actually created with the help of the motor. So, this is nothing, but theta and this particular difference is nothing, but the error E and $\dot{E}$ that is your that rate of change of this particular error or we can do something like this, we can find out the deviation or the difference between the angular the velocity, that is, $\dot{\theta}_d - \theta_d$.

So, this is nothing, but actually what I do is, so this is the desired angular velocity and this is the actually obtained angular velocity and this particular difference is nothing, but is your $\dot{E}$. And, what is this $\ddot{\theta}_d$? $\ddot{\theta}_d$ is nothing, but the desired acceleration so this is nothing, but the desired acceleration. Now, let us see, how to generate this particular τ' using this PD control law.

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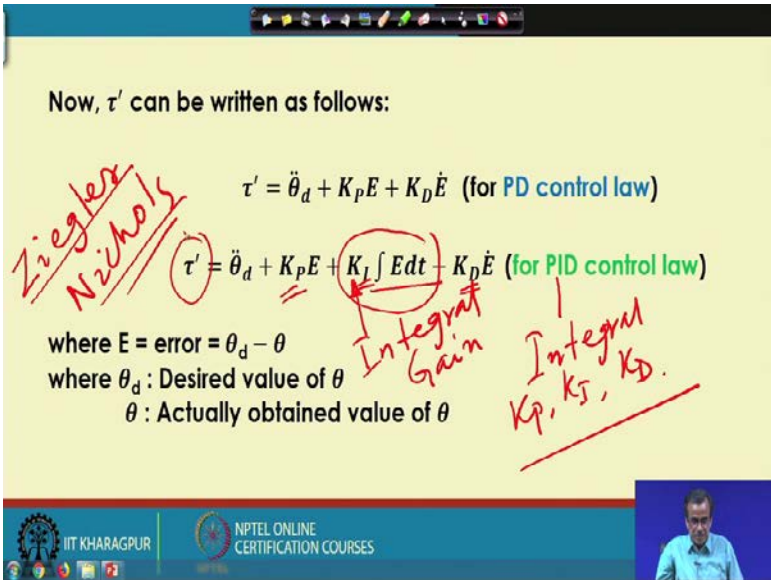

Now, before I go for that let me just tell you that if I use PID controller in place of the PD controller, then $\tau' = \ddot{\theta}_d + K_P E + K_I \int Edt + K_D \dot{E}$ for the PID controller that is Proportional Integral Derivative controller. So, P stands for proportional, I stands for integral so this is integral. So, proportional integral and derivative control law and here, we are adding one extra term that is your K_I multiplied by your that integration Edt.

Now, here this K_I is nothing, but your integral grain. So, this is nothing, but integral gain value. Now the values for this particular K_P, K_I and K_D are, in fact, can be determined mathematically also, and there is one well-known method, that is called Ziegler Nichols rule.

Now, using this Ziegler Nichols rule, we can find out what should be the numerical value for this particular K_P, K_I and K_D. Now, once I have determined the values for these K_P, K_I and K_D those values are kept constant, those are not altered. So, they are using this Ziegler Nichols method we can find out all such K_P, K_I and K_D values and once you have got that now I can implement this particular the τ'.

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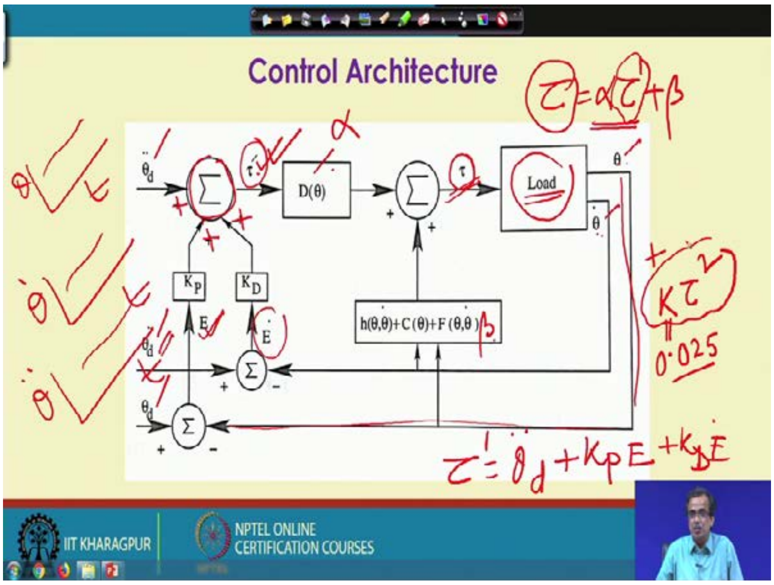

Now, let us see how to implement this particular τ'. Now let us see the controls architecture and this is the block diagram of the control architecture. Now, this particular $D(\theta)$ as I mentioned, so this is nothing, but is your α and this τ as you mentioned according to this partitioned control rule, this is $\alpha\tau' + \beta$. So, $\alpha = D(\theta)$ and this particular thing whatever you have written here h theta, theta^dot plus C theta plus F theta theta^dot this is nothing, but the beta, ok.

So, this is α, this is β, now I will have to generate this particular the τ'. Now, the way it is done is as follows, we take the help of some sort of the closed loop control system. So, initially, there could be some error, but this particular error will be compensated. Now, here, I have mentioned $\ddot{\theta}_d$ that is nothing, but the desired acceleration, then comes your $\dot{\theta}_d$, that is the desired velocity and θ_d that is nothing, but the desired displacement or the angular displacement. Now, here, with the help of these, we are just going to generate this particular τ'. Let us see how to generate this particular τ'.

Supposing that say this is the summing junction say I am passing, so this particular the τ', now, how to determine this particular τ' according to this PD control rule? Now, as I have discussed that this particular, according to the PD control rule, this particular $\tau' = \ddot{\theta}_d + K_P E + K_D \dot{E}$. So, let me write it here, so this particular τ' is nothing, but $\ddot{\theta}_d$.

Now, this is the summing junction and we can see, I am putting three such plus sign here. So, $\ddot{\theta}_d$ is coming from their plus K_P multiplied by the error.

So, K_P multiplied by the error, so this particular thing next is your K_D multiplied by $\dot{E}$, so this K_D multiplied by $\dot{E}$ so those things are summed up here and that is nothing, but is your τ'. And, once you have got this particular τ' we multiply τ' by α so I will be getting $\alpha\tau' + \beta$, so I will be getting the complete τ. Now, here we have got a load, load means this is the mechanical load so, if it is a robot, now actually, the load is to be carried to generate that particular the angular displacement that is nothing, but the mechanical load. And, supposing that I am using a DC motor, DC motor is generating this particular torque. That means, it is generating θ, $\dot{\theta}$ etc.

So, the moment I am just applying, I am just putting that particular motor on. So, what will happen is, it will try to generate this particular torque. This is how to realize the torque that has been implemented for getting the θ, $\dot{\theta}$ and all such things. And, using some sensor, we can measure this particular θ and $\dot{\theta}$, how to measure θ and $\dot{\theta}$ that I will be discussing after sometime.

Now, supposing that we are able to measure so this particular θ and $\dot{\theta}$. So, that will be brought here to this junction for the purpose of comparison. So, theta will be brought here for the purpose of comparison with θ_d, that is, the desired theta and I will be getting this particular error, and this error will be multiplied by K_P and it will be added here.

Similarly, whatever $\dot{\theta}$ we are getting that we can measure. So, this will be brought here for the purpose of comparison at this particular summing junction. And, it will be compared with your $\dot{\theta}_d$ and will be getting this $\dot{E}$ and this $\dot{E}$ is multiplied by K_D and that is also be summed up here. And, I will be getting this particular τ'. So, this process will go on and go on. So, in the first cycle, we may not get the accurate θ and $\dot{\theta}$, but as I told that we are going to use the closed loop control system. So, there will be error compensation and at this particular robotic joint we are going to generate this particular θ, $\dot{\theta}$ accurately as a function of time.

And, that is why, as I mentioned that will be getting the θ as a function of time, $\dot{\theta}$ as a function of time and of course, you will be getting $\ddot{\theta}$ as a function of time, ok. So, will be getting sum distribution here ok. So, this particular joint torque will be realized in the form of θ, $\dot{\theta}$ and $\ddot{\theta}$, ok.

So, this is the way actually we can generate the desired motion at the robotic joint with the help of your the DC motor, but this particular DC motor is having some loss. So, whenever we try to calculate the power rating for this particular DC motor, which you are going to put, so that particular loss will have to be considered. Now, if I know the torque history, and if I know this particular your theta^dot as a function of time, so, very easily, we can find out what should be the power rating.

And if I know this particular power rating so we can prepare the specification of this particular motor, which I am going to put at the robotic joint, but as I told while preparing the specification we will have to consider that one is the torque required and another is the loss of toque, and this particular loss of torque is general,y we try to calculate loss of torque is nothing, but K tau square. So, what about torque is required to generate this theta dot and theta double^dot and all such things after that I will have to add this particular the loss that is K tau square, tau is nothing, but the torque and K is the constant.

Generally we considered a small value for the DC motor. So, 0.025 or so, and we try to find out this particular loss, and then we decide what should be the power rating for this particular motor. So, this is the way actually, we control the different joints of the robots with the help of DC motor.

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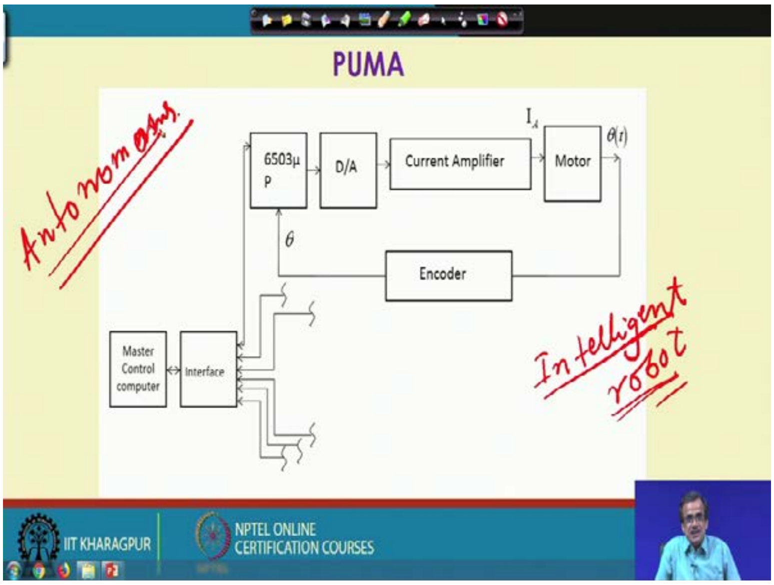

For example, say if you consider the PUMA the way we control PUMA. So, here I am just going to discuss briefly the control architecture for this particular PUMA, that is a programmable universal machine for assembly. Now, here there are 6 joints all 6 are rotary joints and at each of the joint actually we have got this type of the control system. So, if we see the control architecture for each of this particular joint.

So, this is actually the block diagram for the control architecture for a particular the joint. Now, here you can see that we have got that 6503 microprocessor. So, to control a particular joint, so I have got a motor and to control that actually we use this type of control architecture or the control scheme. So, we have got the microprocessor here, then this digital to analogue conversion.

Then, we have got the current amplifier because this armature current is going to enter and will be getting the joint torque. And, this torque will be generated and it will be realized in the form of your theta as the function of time, that is the joint angle. Now, here I am using one encoder so this particular encoder optical encoder is nothing, but the feedback device.

Now, with the help of this optical encoder, we can determine what should be the joint angle and so this particular thing is compared with the desired value. So, if there is any such error so that particular error will be amplified here and will be getting some current here armature current, and once again it will generate and this particular error will be compensated. So, at each of the robotic joint will be getting very accurate movement with the help of this particular the motor.

Now here, for this PUMA there are six motors, now this is the control scheme for a particular the joint. Similarly, for the second joint, I have got another such control scheme, third joint, fourth joint, fifth and sixth joint. And, all such movement of the joints that will be controlled by one centralized computer and that is your the master control computer.

So, here to control this particular PUMA, we have got one controller or the director and we can use one master control computer. So, with the help of this master control computer the all the movement of all the joints actually, can be the controlled. So, this is the way, actually we control this particular the PUMA. Now, here, so till now whatever I have discussed, let me tell you, in short, like your till now starting from the kinematics, we have discussed how to carry out the dynamic analysis and we have seen how to generate that particular the torque with the help of your, say DC motor. And, the motor will be equipped with one controller, and generally, we use either PID controller or PI controller or PD controller and once you know the gain values of this particular controller, we can control the movement at the different robotic joint.

So, till now, actually the robot is ready and we have already discussed how to teach a particular robot. So, if I just want to give a task to the robot, we are in a position to give task to the robot and the robot will try to follow that and try to perform that particular the task. So, till now, whatever we have discussed is this, but one thing we have not a discussed, can a robot take decision, how can you make the robot capable of taking decision?

So, that particular thing, we have not yet discussed that means, how to make a robot intelligent? How to make a robot autonomous? What do you mean by robot intelligence? Those things we have not yet discuss, we have not yet discuss in this course.

So, gradually actually we are moving towards that how to make that particular robot intelligent. Now, before I go for that particular intelligent issues in robotics that is actually, the fourth module or the last module of robotics. Now, let me tell you what do you mean by an intelligent robot. So, by an intelligent robot actually what is meant, so intelligent robot will be able to take the decision as the situation demands.

That means, in a varying situation, in a varying environment an intelligent robot should be able to take the decision, and there is another term that is called the autonomous robot. Now, this autonomous robot is actually, a robot, which has got the permission to perform as the situation demands.

So, if the robot is having the ability to perform in a varying situations that may be called an intelligent robot. However, if it if it is having the permission to perform in the intelligent way or take the decision in the varying situations. So, if it is having that permission then only it is called the autonomous robot. Now, all the autonomous robot should be intelligent robot, but all the intelligent robot may not be autonomous.

Now let me take a very simple example just to find out the difference between the intelligent system and autonomous system. Now, if you see that under one university there are 10 engineering colleges. Now, these engineering colleges will have to follow the rules and regulations of this particular university. Now, at each of the engineering colleges, there could be intelligent people, faculty members, students, but there unable to take any decision they will have to depend on the university rules.

So, they are intelligent, but they are not autonomous on the other hand if you see the institutes like IIT, NIT they are intelligent at the same time autonomous, they are having the capability to take the decision and their having the permission also to take the decision, they are intelligent they are autonomous. In robotics actually we call it is intelligent and autonomous robot, now how to design and develop an intelligent and autonomous robot.

So, those issues actually will be discussed in details one after another and as I told that we copy everything from human being in robotics, we also try to copy the intelligence. That means, the way we collect information, the way we take the decision, the way we implement all such decisions all such things, in fact, we are going to copy in the artificial way in the intelligent and autonomous robot. Now, if we see, we collect information with the help of sensors, but the robot doesn't have any such sensor.

So, will have to put some sensors for example, say will have to put some camera will have to put some sensors and with the help of this sensors and cameras, the robot will be able to collect information of the environment. And, once it is got that particular information, now it will try to do the analysis try to use some sort of motion planning algorithm, which I am going to discuss, what should be the course of action.

And, depending on the course of action, now the movement should be there, the movement of the wheels of the robot should be there or the movement of the different links or the limbs should be there. Now, how to move it actually, we take the help of motor, we take the help of controller, just to get that particular motion implemented. So, here, actually to make it intelligent, all such issues will have to be discussed one after another. So, we are going to discuss, in details, how to make the robot intelligent.

Thank you.

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Lecture – 31
Sensors

We are going to discuss another topic, that is, topic 6, that is on Sensors. Now, let us see like how to design and develop the sensors, how to use the sensors, what are the different types of sensors used and how can we collect information with the help of sensors?

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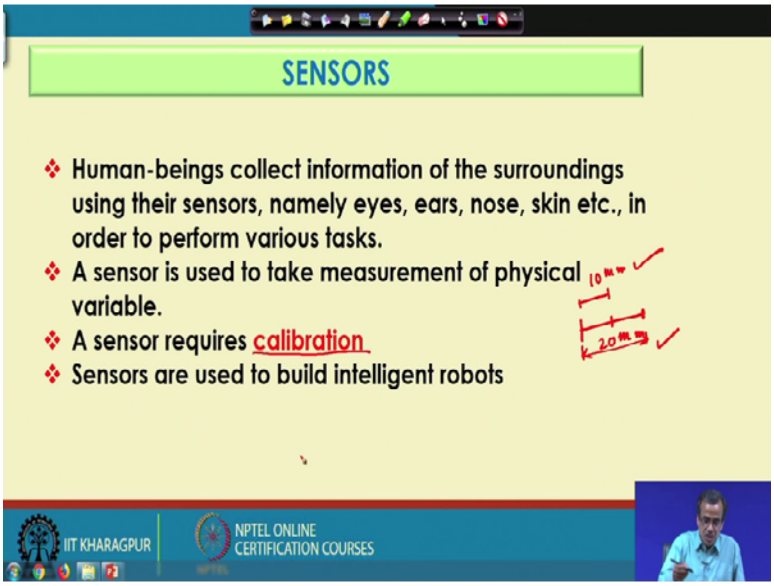

Now, we human-beings, we use different types of sensors like we have got the eyes, ears, nose, skin. In fact, we use multiple sensors to collect information of the environment. And, the data collected with the help of this multiple sensors are actually processed in our brain and with the help of this particular processing, we can collect information of this particular environment.

Similarly, if you want to make robot intelligent, we should put a few sensors and these sensors will help the robot to collect information. Now, here, let me define that the sensor is nothing, but a transducer and we generally use sensor to take some measurements of physical parameter or physical variable and here this sensor; if you want to use as a measuring device; so, definitely there must be some calibration.

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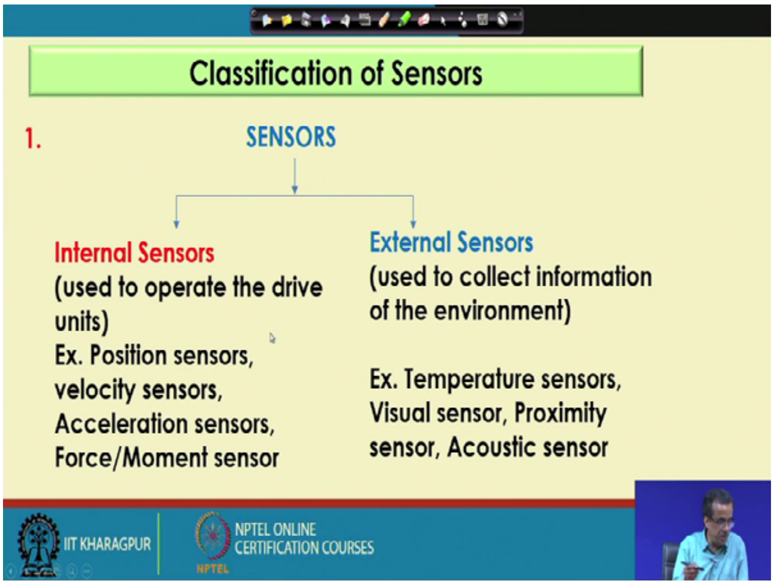

And, by calibration actually we mean, it is actually the comparison with some known data. Now, through comparison with the known data, we will be able to calibrate a particular measuring device or a particular sensor.

Let me take a very simple example, supposing that I will have to draw a straight line of say 10 millimeter. So, starting from here; so I am going to draw supposing that this is my 10 millimeter straight line, ok. Now, if I am told that can you not draw one another straight line which is 20 millimeter long. So, what I will do is if this is 10; so my eyes are going to measure with the previous one and might be this is 20. So, this will be 20 millimeter.

That means, my eyes are following some sort of calibration; if this is 10 millimeter; so, this will become the 20 millimeter just double of that. So, our eyes while taking this particular information; it is following some calibration; it is following some calibration scale and the same is true for any such sensor. Now, you might be knowing that we use different types of sensors, we use different sensors to take some measurement, for example, to measure the joint torques, we use sensor, to measure force, we can use some sensor, but will have to calibrate.

Now, this calibration is a must for any such sensor, now here actually if you want to make it intelligent; as I have already discussed that sensors are to be used or cameras are to be used to collect information with the help of to collect information of the environment and then only we can do some processing to take some decision in a very intelligent way; so, this particular calibration is a must for any such sensor. Now, if you see the literature, we have got different types of sensors for example, say, if you classify the sensors in this way, we can classify like internal sensor and external sensor.

Now, these internal sensors are nothing, but the sensors, which are used to operate the drive units. For example, we have got some position sensors, then we have got the velocity sensor, acceleration sensors, then force or the moment sensors; these are all internal sensors. And, on the other hand, we have got a few other sensors, which are used to collect information of the environment and those are known as the external sensors. For example, we can use some sort of proximity sensor, acoustic sensor, then comes your visual sensor, temperature sensor; these are all external sensors.

Now, here, if you see in our human body, we have got a few internal as well as a few external sensors. For example, say, whenever we try to collect information of the environment; we try to use our eyes. So, with the help of the eyes, we collect information of the environment, but supposing that we are getting some pain in the muscle of leg; now how can we feel that there is some pain? So, to feel that particular pain in the muscle; we use some other types of sensors and those are known as the internal sensors. So, in our human body, we use internal as well as external sensors, the same is true in robots.

In robots also, we use a few internal sensors, we use a few external sensor. Now, the working principle of the different internal and external sensor used in robots, I am just going to discuss one after another, in details.

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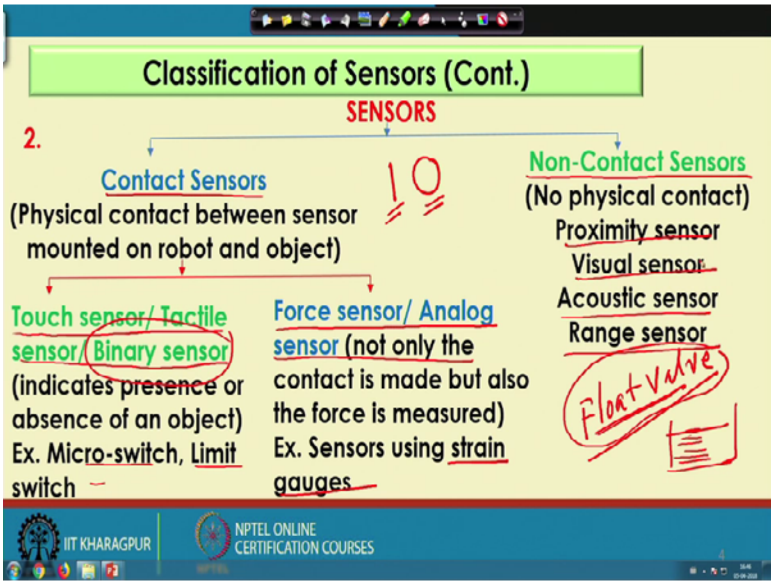

Now if you see the literature, the sensors are also classified in different ways for example, the sensors could be named as contact sensor or the non-contact sensors. So, by contact sensors, we mean that there is a physical contact between the sensor and this particular object, whose distance I am going to measure. And, if there is no such contact, the physical contact between the sensor and the object it is called the non-contact sensor.

Now, this contact sensor can be further classified into two subgroups, now, one is called the touch sensor or tactile sensor or the binary sensor. So, we have got the touch sensor or tactile sensor or this binary sensor, it is almost similar to our skin, skin is nothing, but our touch sensor. So, in robots also we use some touch sensor or the binary sensor for example, say the micro-switch or the limit switch, which are generally used in robots are nothing, but the tactile sensor or touch sensor.

Now, with the help of this touch sensor; so, it is simply going to tell that the robotic finger has touched a particular object, but it is not going to measure the force required to grip that particular object or how much is the force required to or how much is the torque required to manipulate that particular object.

So, it only indicates whether the contact has been made or not now let me take a very simple example. Now in all the water tanks or the oil tanks we use a valve that is called the float valve. Now this particular float valve, what is the function of the float valve? If this is the tank, the moment the water height reaches a particular level the peaks level, this particular float valve will be activated and it is going to indicate that we should stop the pump. And, the pump will be stopped and the water supply to this particular water pump will be stopped. So, this indicates the highest limit, the highest permissible limit for this particular the water. And, that is nothing, but this float valve is an example of the limit switch or either the touch sensor or the tactile sensor.

Now, this is also known as the binary sensor for example, say the micro switch or the limit switch which is generally used in robotic hand. I am just going to take one example, the next slide I will show you that we can use some sort of micro-switch or the limit switch along with your robotic hand.

Now, this is also called the binary sensor because it generates 1s and 0s. The moment it touches; it will generate, it will generate this particular 1 and otherwise it will generate this 1; so, it is going to generate 1 or 0s ok. So, this is known as binary because sometimes it generates 1, sometimes it generates 0; if there is a contact it will generate 1, if there is no contact it will generate 0. So, it is some sort of 1 0 0 1 something like this and that is why, this is also known as the binary sensor.

Now, I am just going to concentrate on the force sensor or this analogue sensor. Now, as I told that with the help of this force sensor or this analogue sensor, we are just going to actually measure the force or the torque. So, this particular portion torque which is required to grip this particular the object and generally, we use some sort of the force sensor or the analogue sensor.

And, here in force sensor or analog sensor, we use some sort of strain gauges. So, I am just going to discuss the working principle of this particular strain gauge in details. And, as I told that we have got a few non-contact sensors, for example, say we have got the proximity sensor, the range sensor, the visual sensor, acoustic sensor.

So, these are all non-contact sensors. So, I am just going to discuss, in details, the working principle of these sensors generally used in robots.

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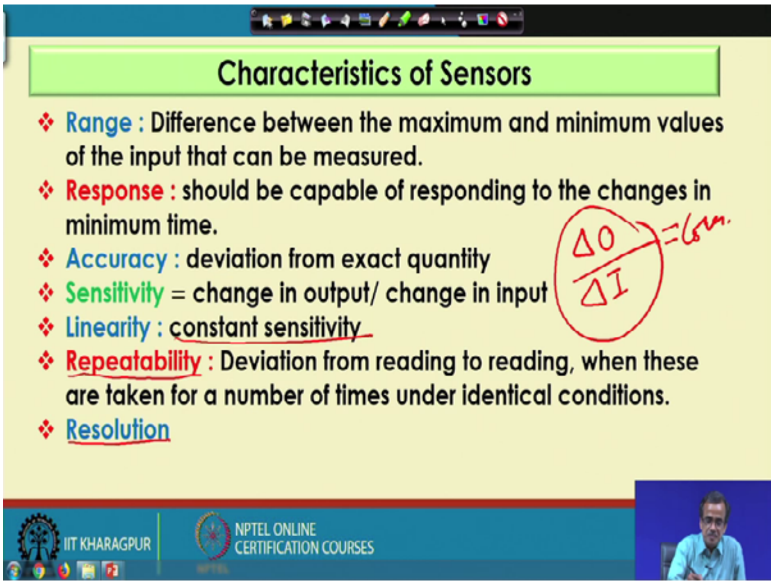

Now, before I go for the discussion of working principle of different sensors; now let me concentrate a little bit on the different characteristics of sensors. Like, if you want to prepare the specification of sensor; what are the information to be provided and what are the numerical values to be provided?.

For example, say the range, response, accuracy, sensitivity, repeatability, resolution all such things, we will have to mention. While preparing the specification of the robot the similar type of information, we also provided. And, here actually or the range for the sensor; that means, what is the maximum and the minimum value that can be measured with the help of this particular sensor that has to be mentioned.

Then, comes your the response, the response should be as quickly as possible, then accuracy is nothing, but the deviation from the exact quantity. So, that we will have to mention, sensitivity we know by definition sensitivity is nothing, but the change in output to the change in input; so, that is nothing, but your sensitivity. So, this particular sensitivity, we will have to mention, how much sensitivity you need and if this particular sensor is having constant sensitivity; now then it is called the linear. So, the sensor is called a linear, if it is having the constant sensitivity.

So, by linearity, we mean constant sensitivity; then comes your repeatability. Now, by repeatability, we know supposing that with the help of the same sensor, I am just going to measure the same thing for say 10 times or 20 times. Now, the same thing, if I measure 10 times or 20 times; there is no guarantee that all 20 times will be getting exactly the same numerical value.

Now, this particular deviation from reading to reading is nothing, but the repeatability. Now, while preparing the specification of this particular sensor; we will have to mention how much is the repeatability we want? And, then comes your the resolution is nothing, but the least count. So, this particular list count for the measuring device or this particular sensor; that we will have to know. Now, let me take a very simple example now, if I take one very simple example for this particular resolution, it will be clear. Supposing that I am using one sensor and in that particular sensor, I am using some electrical signal to generate some angular displacement.

Now, electrical signal cannot be a fraction. So, it could be 1, 2, 3 something like that; corresponding to one electrical impulse, how much is the angular displacement it can generate? That is nothing, but the least count or resolution, the same is true for this sensor, ok. So, these are the information which are to be provided to prepare the specification of this particular sensor.

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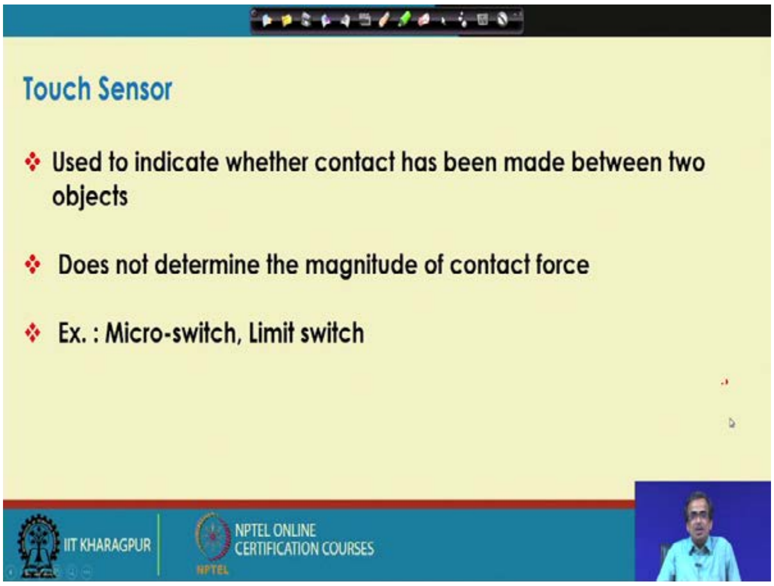

Now, I am just going to discuss the working principle of a few sensors one after another. Now, this touch sensor, I have already discussed; these are use just to indicate whether the contact has been made or not. And, generally, we do not use this sensor to determine how much is the contact force? The examples are micro-switch, limit switch and all such things.

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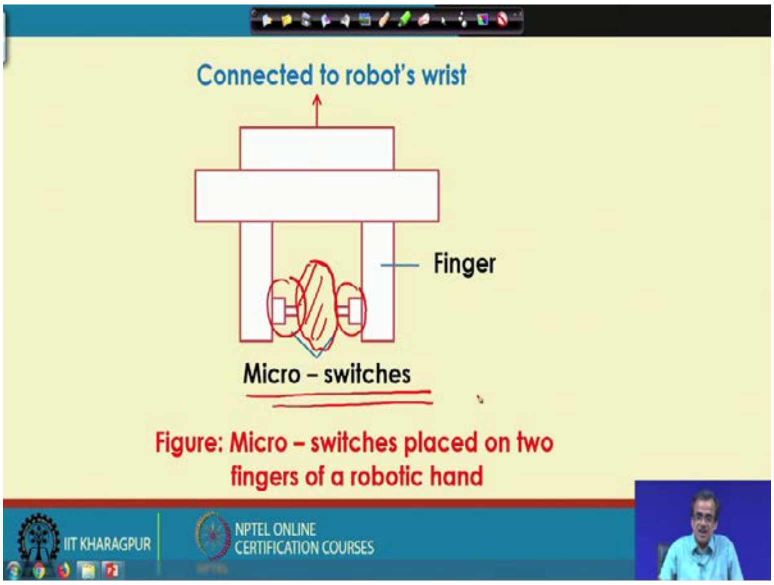

Now, here, I am just going to take one typical example of a micro-switch, which is nothing, but a touch sensor used in robot gripper. Supposing that this is a very simple gripper having two fingers and here we are just going to put some micro switch or the limit switch. Now, with the help of this micro-switch or the limit switch actually, it is going to indicate whether the contact has been made between the object; supposing that I have got the object here, whether the contact has been made between the object and this particular the robotic finger.

So, to serve that type of purpose, we use your the micro-switch or the limit switch and that is nothing, but the touch sensor, the same is true, we have got the skin. So, with the help of this particular skin, we can touch, we can feel the presence of an object, even if we are not using eyes; we can feel the shape and size or the more or less the structure of that particular object, even if we do not see with the help of our eyes.

Because we can use our skin and skin is nothing, but the touch sensor and with the help of this touch sensor; in fact, we can find out, what should be the possible shape and size of this particular object, which I am going to grip.

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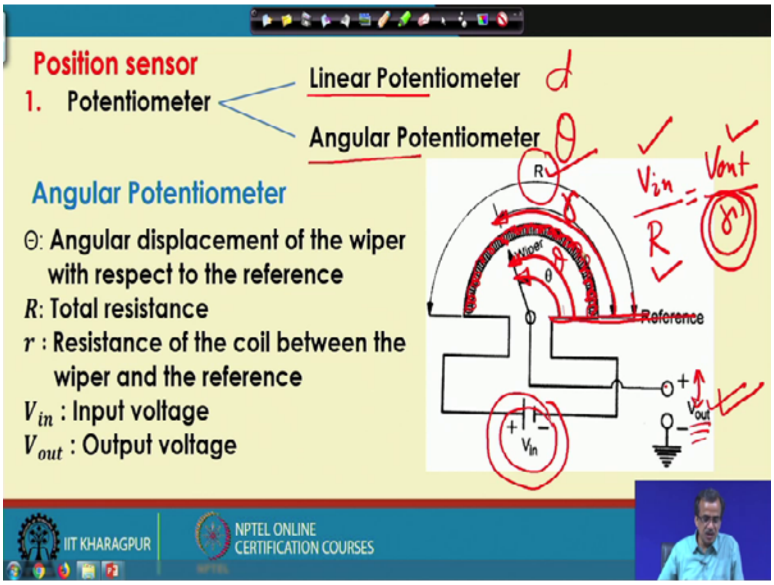

Now, I am just going to discuss one sensor one position sensor which is very frequently used in robotics or we generally use in school level, college level some laboratory classes also. This is your the potentiometer the potentiometer is actually why as I told very well known position sensor. And the potentiometer could be either the linear potentiometer or it could be angular potentiometer.

Now, with the help of linear potentiometer, we can measure the linear displacement, that is, d and with the help of angular potentiometer, we can measure the angular displacement, that is, nothing, but θ. Now, the working principle of this particular potentiometer is very simple; for example, say so here we have got the source for the voltage that is input voltage V_in for example, say we have got the battery or V is connected to some power, ok.

Now, what we can do is supposing that we have got the battery here so, I know the input voltage. So, what I do is, we know the total resistance of this particular wire; so, this is actually nothing, but the wire and it has got a special type of winding. So, capital R is nothing, but the total resistance and the reference here. So, this is nothing, but the reference that means, we are going to measure the angular displacement with respect to this particular reference, ok.

Now, here actually, what I do is, we are going to measure this angular displacement with the help of this pointer or the wiper. So, with respect to the reference; so it is going to generate some angular displacement ok; how to measure this? To measure this, the method which we follow is very simple; so, here with respect to this particular reference. So, I have got the wiper and it has got some displacement and we measure with the help of a voltmeter, how much the output voltage? So, this one point is connected to the wiper and another is your the grounded. And, you can find out, we can measure with the help of voltmeter, how much is the output voltage?

So, we know the total resistance of this particular wire, we know how much is the input voltage, we can measure, how much is the output voltage with the help of the voltmeter or multi-meter. And, if you have measured this particular output voltage; now approximately I can find out what should be this angular displacement. Now, how to find out? It is very simple, because if I know this input voltage and if I know the resistance.

So, the current is nothing, but your $\frac{V_{input}}{R}$ and the same current will also flow here and that is nothing, but is your $\frac{V_{out}}{r}$ and small r is nothing, but the resistance of this winding up to starting from the reference; up to the end of this particular pointer or the wiper. So, starting from here up to this; so this is the resistance small r, so from here see the V_in is known, capital R is known, V_out can be calculated.

So, r can we determined and if I know the value of r and if I know the nature of winding of this particular wire, the electrical wire. I can find out approximately like what should be this particular angular displacement, that is, θ. So, θ can be measured with the help of this particular angular potentiometer. This is the working principle of angular potentiometer, it is very simple and all of us have used.

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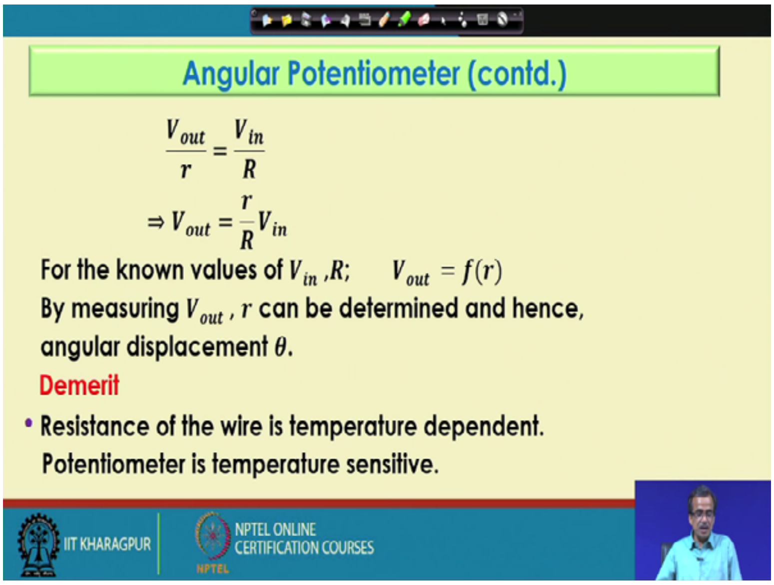

So, this is the working principle of your angular potentiometer, but this angular potentiometer has got one demerit or one drawback; all of us we know that the resistance of wire depends on actually the temperature.

The moment we pass some current through electric wire; so, due to the heating effect of the current, that is, i^2r effect. So, what will happen? There will be some heat generated in that particular electric wire and its temperature is going to increase and as temperature increases; so the resistance of the wire is going to change and if resistance changes, and you will not be getting very accurate measurement with the help of this angular potentiometer.

And, that is why, if we use this particular angular potentiometer at a stretch for a long time; so, initially we may get some accurate results, accurate measurements. But, with time, after might be half an hour or one hour, there is a possibility, we will be getting some erroneous results with the help of this angular potentiometer. So, this is actually the drawback of this angular potentiometer, but its working principle is very simple and this is, in fact, one of the most popular position sensor used nowadays.

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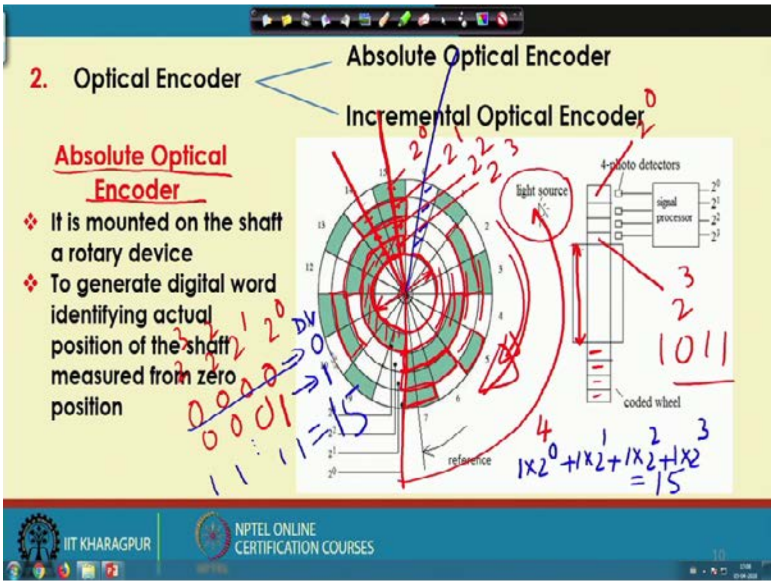

Now, then comes your another position sensor that is called the optical encoder. And, this is also very popular and if you see we have got two types of optical encoder. One is called the absolute optical encoder and we have got the incremental optical encoder. And in robotics actually very frequently this type of optical encoder is used as feedback device. For example like, if it is servo-controlled robot; so, there must be a provision of feedback device and there must be a provision to measure the angular displacement. So, what you can do is; we can take the help of, so this type of optical encoder as a feedback device.

Now, let us see, how does it work? Now, this optical encoder, now, let me first try to explain the principle of this absolute optical encoder first. Now, this absolute optical encoder consists of a number of concentric rings placed one after another. Now, supposing that say this is the output shaft, this is the output shaft of this particular motor I have got the electric motor here.

So, this output shaft is rotated; now I want to measure what should be the angular displacement or what is the rotation of this particular shaft? What I do is here I put this particular absolute optical encoder and absolute optical encoder is nothing, but a collection of a few concentric rings placed one after another and what I do is. So, here we have got the concentric rings and on this particular concentric rings; there will be the marking zone; that means, there will be dark zone and the light zone, and through the dark zone, the light will not pass and for through this particular light zone, the light will pass now on.

So, here, we have got the optical encoder; so, as the shaft rotates the optical encoder mounted on it is also rotating. Now, on one side, I have got the photo source, other side, we have got the photo detector. The moment during the rotation; this particular disc, the circular disc or the rotating disc; if the light zone comes in front of the light source, the light will pass and it is going to activate that particular photo-detector. The same thing I am just going to discuss in more details with the help of this particular the sketch.

Now as I told that it is consisting of a large number of concentric rings. Now here for simplicity I am just going to consider; so, there are only 4 concentric rings. Now supposing that this is nothing, but the diameter; this is the diameter of the shaft whose angular displacement or rotation I am going to measure.

Now, here surrounding this we consider say one concentric ring, another concentric ring, another concentric ring. So, I am considering 4 concentric rings here now you concentrate on the first one that is this particular concentric rings. Now, here what I do is; so this part is made black, this part is made black on the first concentric ring. So, this is made black this part is made black, and this is your the white portion through which the light will pass. Next, we concentrate on the second concentric rings on the second concentric rings starting from here, up to this is made black; so, this part is made black.

So, no light will , then it is white once again, this part is made black and then, there is a white portion, then you concentrate on the third one. So, here, this is the black part and this is the white part, then this is the black part, then comes a white part, then comes this is the black part, this is the white part, this is the black part, then comes the white part and so on, and on the outermost ring, we have got the black portion here.

So, this is the black part, white part, black part, then white, black, white, black and so on. So, this type of marking we have and here we are considering only 4 concentric rings. The outermost ring is going to indicate actually 2^0. And, this particular thing is going to indicate 2^1, this is 2^2 and this indicates 2^3 and so on.

Now, if you see the other view for example, say in this particular view, supposing that this particular shaft it is not drawn here properly. So, this particular shaft supposing that this is actually the diameter of the shaft and on which, I have got this concentric rings mounted one after another ok; that means, this is going to indicate 2^0; that is the outer most and this is going to indicate 2^3.

So, here this is another view; so on this side, we have got the light source and this particular thing is rotating. So, this particular thing is rotating; this is mounted on the shaft, the shaft is rotating, optical encoder is also rotating and the light source is put on; the moment the dark zone is coming. So, here, there will no signal, the moment I will be getting the light zone then only there will be light will pass through this and it is going to activate that particular photo-detector.

So, depending on the relative position of the dark zone and the light zone; so, sometimes light will pass, sometimes it will not pass accordingly it will be generating some 1s and 0 sort of thing, ok. For example, say here, this is the reference line; initially the reference line is here, ok. Now, the reference line is fixed and this particular optical encoder is rotating; now what I do is, here on the screen, I cannot rotate. So, what I can do is; I am just considering as if this optical encoder is fixed and I am rotating this reference in the reverse direction. If I just the rotate the reference in the reverse direction.

The moment my reference is here, the moment my reference is here, truly speaking, the optical encoder is rotating reference is kept fixed. But, here, this is almost equivalent of the situation, my optical encoder is fixed because I cannot rotate it here on the screen; so, I am rotating the reference in the opposite direction. Now, supposing that the reference is here and if it is here, then this is the dark zone, dark zone, dark zone and dark zone..

So, it is going to generate 4 such 0s, say this is 2^0, 2^1, 2^2, 2^3 the moment this particular reference comes here supposing that it is here. So, through the outer-most, the light will pass; this outer most corresponds to a 2^0. So, here it is going to generate 1, but corresponding to 2^1; there will be 0, there will be 0, there will be 0; so 0, 0.

Similarly, the moment we consider that this particular reference is here. So, what will happen? The light will pass through all four; so, through here light will pass, light will pass through this, light will pass through this, light will pass through this. So, it is going to generate actually four such 1s ok; so if it generates four 0s; its decoded value will be equal to 0. Because 0 multiplied by 2^0; so decoded plus 0 plus 0 plus 0 it will be 0.

And, the decoded value for this; so, 1 multiplied by 2^0 that is equals to 1, plus 0 multiplied by this equals to 0, 0 and 0; its decoded value will be 1. And, corresponding to this 1 1 1 1 the decoded value will be your something like this $1\times2^0+1\times2^1+1\times2^2+1\times2^3$.

So, 8 plus 4 that is your 12 plus 2; 14 plus 1; so, 15. So, its decoded value will be 15 ok. So, corresponding to rotation of this particular your optical encoder and depending on the position of angular displacement with respect to the fixed reference; I will be getting some binary. This binary will be generated here and I can just do the decoding and I will be getting, the decoded value corresponding to that particular rotation.

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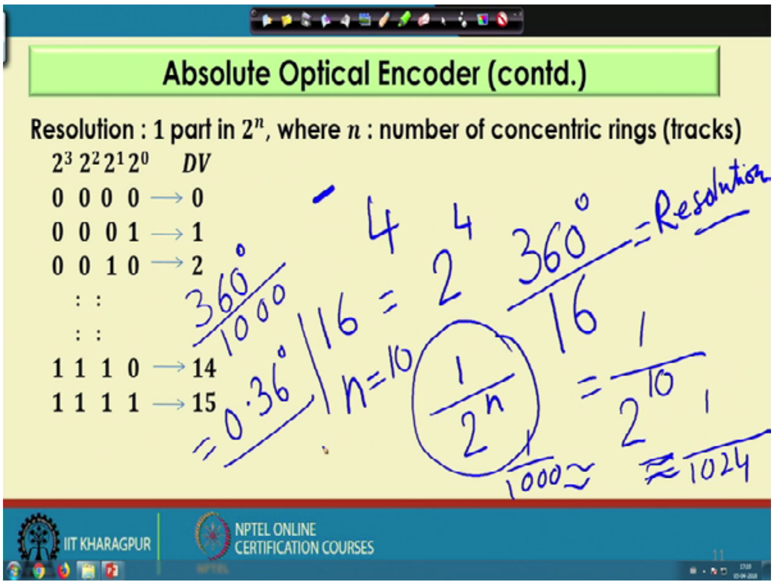

And, whatever I discuss, the same thing I have written it here. So, corresponding to this 0 0 0 0, I will be getting 0 and this is the way actually, I will be getting this particular the decoded value. Now, supposing that I have got some decoded value; now if I use like four concentric rings, if I use four concentric rings then your, how many divisions? We are getting only 16 divisions.

So, 16 divisions is nothing, but your 2^4. So, 2^4 is actually your 16, ok; that means, corresponding to the whole rotation for this particular shaft that is nothing, but 360 degree corresponding to one rotation. So, this particular 360 degree, I am just going to divide equally into 16 parts; that means, your; so, this 360 divided by 16 will be the resolution of this particular optical encoder if I use only 4 concentric rings.

Similarly, if I use n number of concentric rings, then it will have the resolutions like 1 part in 2^n . So, this is nothing, but the resolution of this particular optical encoder for example, say if I take n equals to say 10; then the resolution will be 1 divided by actually 2^{10}, that is nothing, but approximately that is equal to 1024 and approximately that is equal to 1 divided by say 1000, ok.

That means your 360 degrees rotation for one complete revolution will be divided into 1000 equal parts and that is nothing, but 0.36 degree. Now, this particular 0.36 degree will be the resolution of the optical encoder, if we use actually 10 concentric rings. Now, this is the way actually with the help of this absolute optical encoder; we can measure how much is the angular displacement of a particular shaft, which is rotating.

This is the working principle of this absolute optical encoder which is very frequently used in robots as a feedback device.

Thank you.

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Lecture – 32
Sensors (Contd.)

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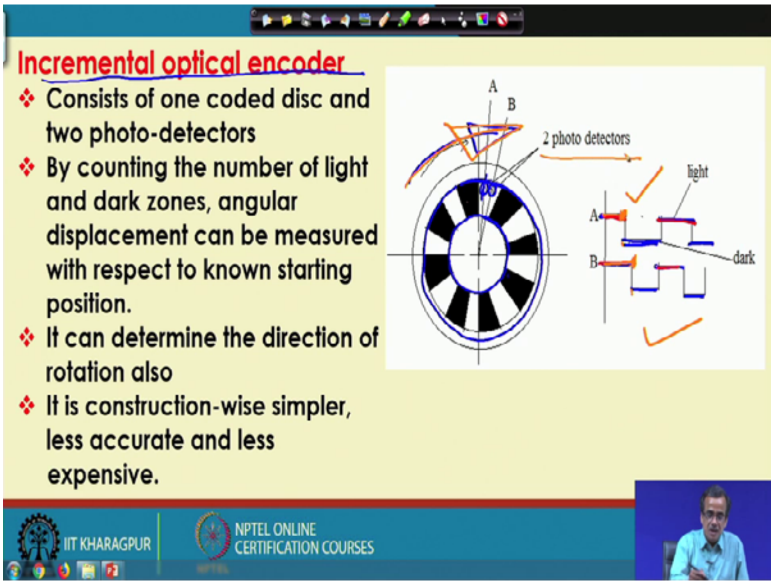

Now, we have already discussed the working principle of one absolute optical encoder. Now, this absolute optical encoder which I have already discussed is actually very precise, but the problem is actually the number of photo-detector should be equal to the number of concentric rings. So, if I use 10 concentric rings, so I will have to use actually 10 photo-detectors, which are very costly, and that is why, actually absolute optical encoder are very costly. And, in place of absolute optical encoder, we use incremental optical encoder.

Now, here in incremental optical encoder, we use only 2 photo-detectors and there is only one coded disc. So, we do not use a large number of coded disc here, and we do not use a large number of photo-detectors here. Now, let us try to understand the working principle of this particular incremental optical encoder.

Now, this incremental optical encoder as I told, we have got only one coded disc. So, this is actually the shaft, whose rotation I am just going to measure. So, here, we mount only one coded disc. So, this is nothing but the coded disc here; so, this is the coded disc.

And, here, on this particular coded disc, we have got the black zone and the white zone, black zone and white zone, black zone and white zone. So, the black zone and white zone are placed here.

Now, if there is black zone, then no light is going to pass; and through this particular white zone, the light is going to pass, ok. Now, here actually what happens, here the principle is slightly different, different in the sense like here, we have got only 2 photo-detectors and these 2 photo-detectors are kept fixed; so, their positions are kept fixed.

Now, what I doing is, here we put one photo-detector, that is A and another is B, and their positions are kept fixed. And, this particular shaft is rotating. The moment it rotates supposing that this particular incremental optical encoder, which is mounted on the shaft, so this is rotating in the clockwise sense, ok. So, if it rotates in the clockwise sense, then photo-detector A will enter the black zone first; and after that, photo-detector B is going to enter the black zone.

Now, if you see the plot, this is the plot, this indicates the black zone, and this is actually the light zone I am sorry; so this is the light zone, and this is actually the dark zone. So, this is the light zone and this is the dark zone. Similarly, this is the light zone; and this is the dark zone; the light zone and this is the dark zone ok.

Now, let me just use another color. So, for this light zone, I am using the red. So, this is the light zone, this is the light zone and this is the light zone, ok. Now, here actually, what happens, the moment it is rotating in the clockwise sense, this black portion, this A is going to face the black portion first, ok; and then B is going to face. So, here actually, what happen this source, now this is the light zone; that means, A will be light zone for small duration compared to B; and B will be in light zone for more amount of time.

Now, here, you can see for this particular A, it is a light zone only up to this; whereas B is in light zone up to this slightly more, that means your A has entered the black zone first. So, this is the starting of the black zone. So, A has entered the black zone first. And, after that, B has entered the black zone, ok. So, once again, let me repeat. So, A will be in light zone only up to this and B will be in light zone up to this that means A will enter the black zone first, and B will enter the black zone after sometime, ok.

So, this type of signal, you will be getting or corresponding to the photo-detector A and photo-detector B. And, now, if I see this particular signal, so if we see this particular signal, and if we count the number of light zone, and the number of black spot, for example the light zone, light zone and the black zone.

Similarly, here also, I am just going to count the number of light zone and number of dark zone. So, by counting the number of light zone and dark zone, in fact, we can find out, how much is the angular displacement, ok. So, the angular displacement can be determined by counting the number of the dark zone, and this particular the light zone.

Now, actually the next thing is, approximately we can find out how much is the angular displacement of this particular the incremental optical encoder, or the shaft whose rotation I am going to measure. Now, here another information we are going to get, that is, it can indicate the direction of rotation. For example, if you see this particular signal once again, here A enters the dark zone first, B enters the dark zone after sometime, that means, this is rotating in the clockwise sense. Now, the reverse will be the situation, if it is rotating in the anticlockwise sense. So, you will be getting the different type of signals here, ok.

So, let me once again repeat. A will enter the dark zone first, and B will enter the dark zone after sometime. It indicates that this particular shaft is rotating in the clockwise sense, ok. So, this is the way actually it can find out, how much is the angular displacement, and what is the direction of movement of that particular shaft. And, as I told that here, we use only one coded wheel; here there is only one coded wheel and only 2 photo-detectors, so it is less costly. And of course, it will be less accurate compared to this absolute optical encoder. But, as it is less costly, it is very frequently used as feedback device in robot; and this is used very frequently as a position sensor. Now, this is actually the working principle of incremental optical encoder.

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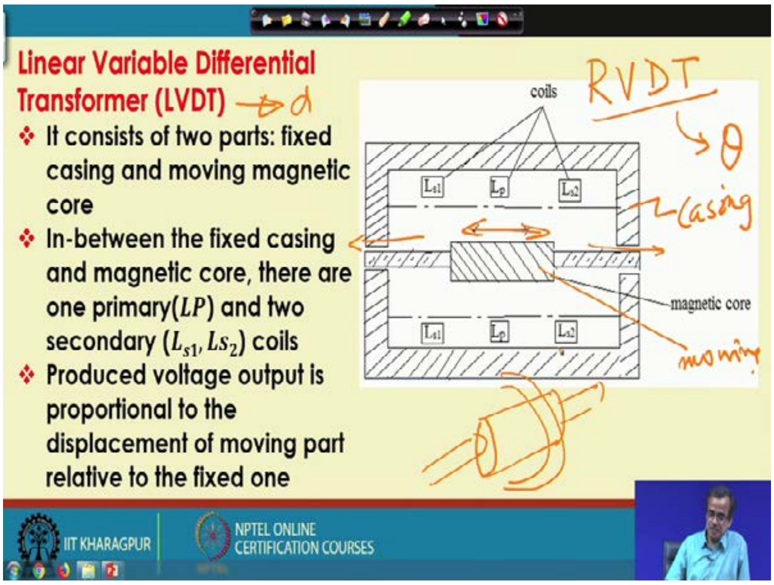

Now, I am just going to discuss the working principle of another very popular position sensor, that is known as LVDT, that is, Linear Variable Differential Transformer. Now, this LVDT stands for linear variable differential transformer, and this is used to measure the linear displacement, that is, d, ok. Now, similarly, we have got RVDT, that is called Rotary Variable Differential Transformer. And, this particular RVDT, that is rotary variable differential transformer is used to measure the angular displacement, that is nothing but θ, ok.

Now, let us try to understand the working principle of this particular LVDT, that is linear variable differential transformer. Now, construction wise, it is very simple, we have got one fixed casing. So, this is nothing but the fixed casing. And, we have got one moving magnetic core, this is actually the moving part, that is the magnetic core. Now, this particular magnetic core, it can slide along these two directions, that means, it can slide towards this or it can slide towards this, ok. And, we have got the fixed casing here.

Now, in between the fixed casing and the moving magnetic core, so here we put one the primary coil, that is, L_P, and two pairs of secondary coil, that is, L_s1 and L_s2, ok. Like if we just draw, here we have got actually the primary coil. Now, if I just draw one very rough sketch sort of thing, for example, say this is the magnetic core, say, if this is the magnetic core, now here surrounding this actually we have got this primary coil; so we have got this particular primary coil, and here, we have got two such secondary coils, ok. Now, let us try to understand the working principle of this, and how can it measure the displacement, that is, the linear displacement with the help of the fixed casing. Let us try to understand the working principle.

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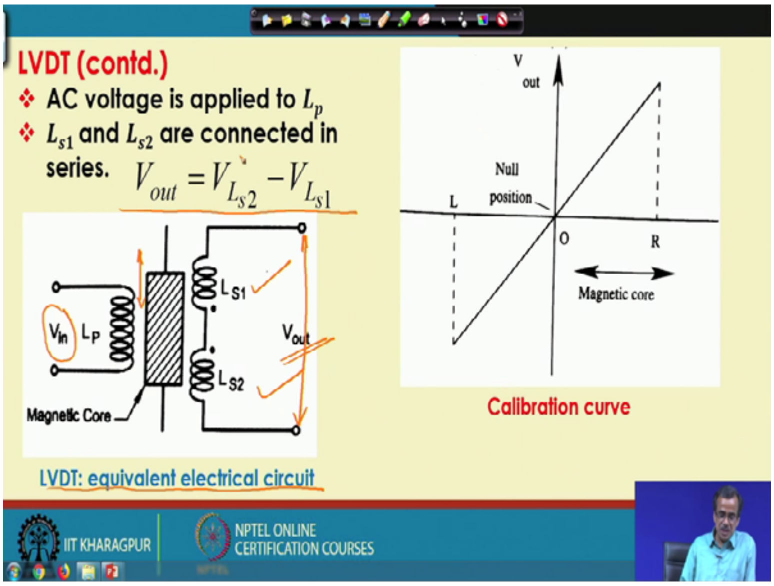

Now, to understand the working principle, actually what we do is, we try to see its equivalent electrical circuit first, ok. Now, this is nothing but the equivalent electrical circuit. So, this equivalent electrical circuit corresponding to that LVDT, this is the magnetic core. Now, here in this particular sketch, the magnetic core can move up and down, ok. And here, we have got the primary coil. And we put the input voltage, that is, V in through the primary coil. And, we have got the secondary coil, the first secondary coil and the second secondary coil. And, here, in between these two points, we try to measure how much is the output voltage, that is, V_out.

Now, let us see how can you measure so this particular displacement or the movement by measuring the output voltage. Now, this V_out is actually nothing but $V_{L_{s2}} - V_{L_{s1}}$. Let me explain, what is this V_L_s1 and V_L_s2.

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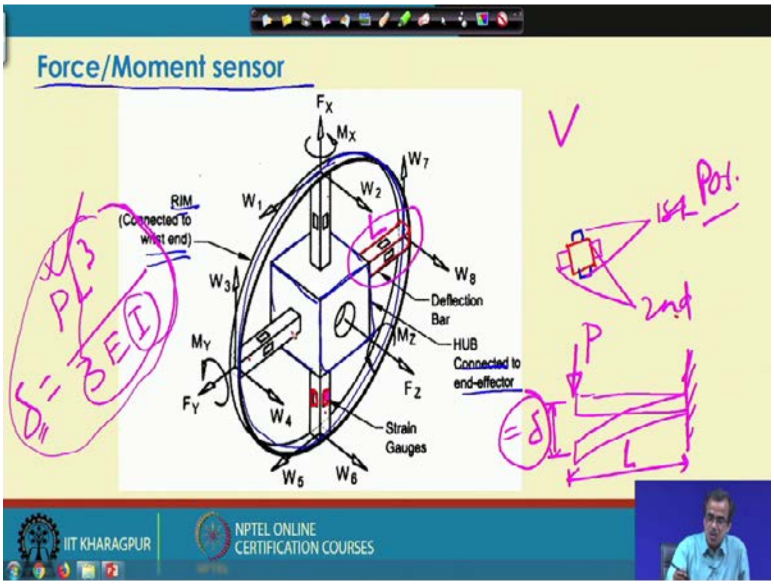

Now, to explain this actually what we do is let us go back to the let us go back to the previous picture first.

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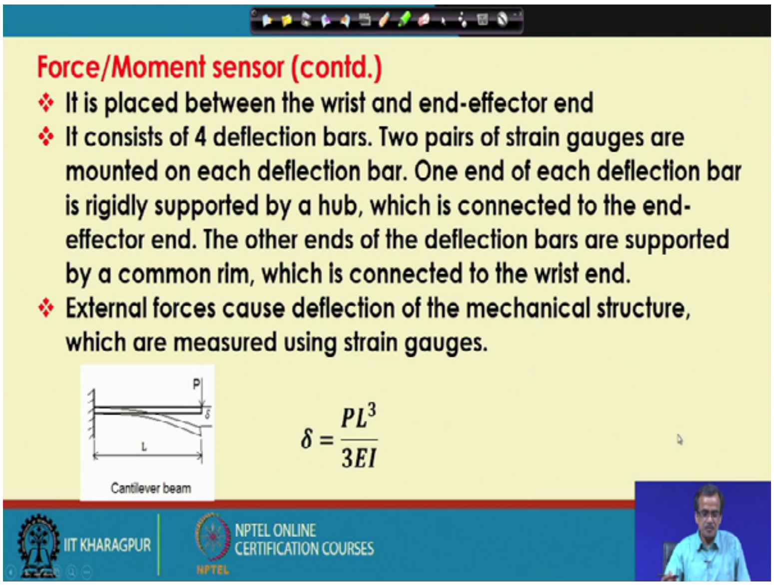

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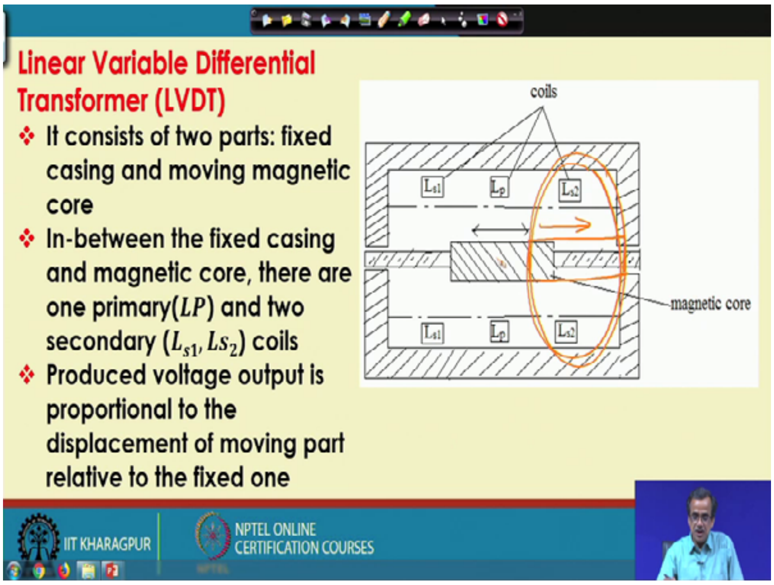

Now, if you see the previous picture, supposing that this is the particular magnetic core, so this is sliding towards my right. So, the magnetic core is here, ok. Now, if the magnetic core is here, it will be closer to the L_s2 compared to your L_s1.

So, coupling between L_s2 and the magnetic core will be stronger compared to the coupling between the permanent magnet, the magnetic core and L_s1. So, here the magnetic strength will be more, the linking will be stronger. And, due to this stronger linking, actually what will happen here is the induced voltage in L_s2 will be more compared to that in L_s1. So, due to this stronger influence of this particular magnetic core, the induced voltage in L_s2 will be more compared that of L_s1.

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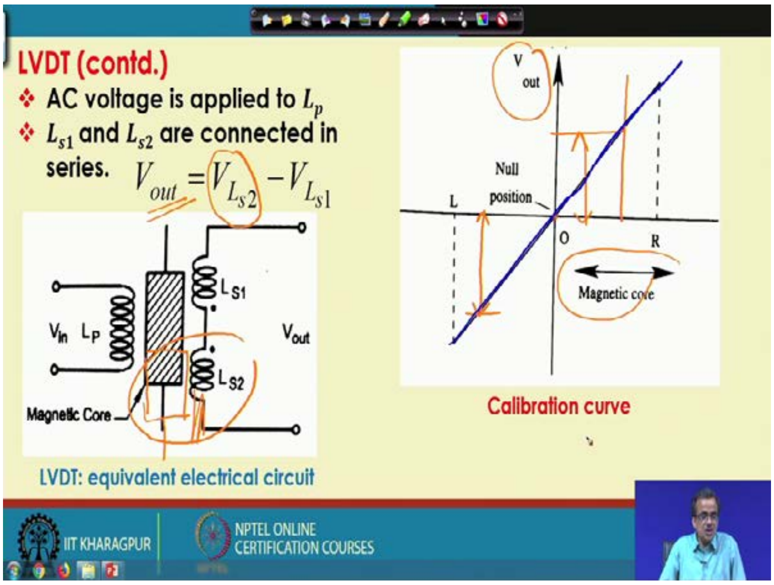

And, now let us see, what happens here. That means your if I just draw it the same situation, it is more towards L_s2, that means my magnetic core is somewhat here ok. That means, this particular coupling is stronger ok, compared to this coupling. That means, the induced voltage in L_s2, that is, V_L_s2 will be more compared to V_L_s1, and I will be getting a positive V_output, ok.

So, I will be getting a positive output voltage, if it is moving downwards, ok. And, reverse is the situation, if it is moving upwards. So, in that case, V_L_s1 will be more compared to V_L_s2, and V_out will become equal to some negative value, ok. Now, here, this shows actually the calibration curve. So, this corresponds to the null position, ok. Now, this is R, indicates as if it is it was moving towards the right, so I will be getting some positive V_out. So, I will be getting some positive V_out.

And, if it is sliding towards L_s1, so I will be getting the negative V_out, that means I am here ok. So, I will be getting some negative V_out here. So, this particular plot like output voltage versus the position of the magnetic core with respect to the fixed casing, ok. So, this is actually the calibration curve. And, once this particular calibration curve is pre-determined, now by measuring this particular V_out, what you can do is, we can find out like what is the position of this particular magnetic core or what is the displacement of the magnetic core with respect to the fixed casing. So, we can measure, how much is the linear displacement of the magnetic core with respect to the fixed casing.

So, this is the way actually this particular LVDT works. And, as I told, this is used just to measure the linear displacement. And, for measuring the angular displacement like we will have to go for RVDT, that is, Rotary Variable Differential Transformer. Now, this particular LVDT is used in robots. It is used very frequently in different machine tools. For example, lathe, milling machine, drilling machine, this type of LVDT are very frequently used. So, this is the working principle of this LVDT.

Now, we are going to discuss the working principle of another sensor that is called the force or moment sensor. Now, the purpose of this force or the moment sensor is to determine, how much will be the force or the moment acting at the robotic joint. Let me take a very simple example.

Supposing that this is my wrist joint; now if I consider say this is the serial manipulator, so this is my wrist joint. And, with the help of this wrist joint, this particular end-effector is connected. Now, I am just drawing something or I am writing with the help of this marker, ok. The moment I am just going to do some manipulation task with the help of this finger, this particular joint is subjected to some amount of moment, some amount of torque, ok.

Now, if I want to measure this particular moment and torque at this wrist joint, how to measure this particular moment or force. To measure this moment or the force, we put this type of the force or the moment sensor. Now, what I do here is, on the wrist end, so this is the wrist end, we put this particular the rim portion. So, this is actually the rim portion, which we put at the wrist end, so this is connected to the your wrist end. And here, we have got one square block sort of thing or cube sort of thing, cuboid sort of thing, ok, so this is called the hub. And, this hub is connected to the end-effector or this particular finger with the help of which I am doing that particular manipulation, while writing, ok.

So, once again let me repeat, the hub is connected to the end-effector, and this particular rim is connected to the wrist end. And, what is our aim, our aim is to determine what should be this particular joint moment or the joint torque or the force. Now, let us see, how to determine that. Now, let me first explain the construction details of this particular force sensor. Now, construction-wise, actually, as I told, this is connected to the wrist end; and hub is connected to the end-effector.

Now, here in between the rim and the hub, we have got some sort of the deflection basr. So, here, we have got one deflection bar. Similarly, I have got another deflection bar here; another deflection bar here; another deflection bar here. Now, if I see this deflection bar, these are actually the bar having some sort of square cross-section, and made of elastic material ok.

So, for example, say elastic material in the sense, we can use some sort of steel, steel will be working within the elastic zone by elastic material, I wanted to mean that it a steel, but it is working within the elastic zone. It has not reached the plastic zone, ok. So, this is actually made of steel in fact and it is having this type of square cross-section.

And, if you see on the deflection bar, you have got some strain gauges. So, here we are putting some strain gauges. In fact, on each deflection bar, we put two pairs of strain gauges. For example, say here, we put say one pair here, so this constitutes one pair of the strain gauge, and there could be another pair of strain gauges; another pair of strain gauge could be something like this, so this is actually the strain gauge, this particular stain gauge.

So, we have got one pair, so this is one pair of strain gauge; this is the second pair of strain gauge. So, this is the first pair and this is the second pair of strain gauges, ok. So, on each of this particular deflection bar, we have got two pairs of strain gauges, and I have got four such deflection bar, so I have got 8 pairs of strain gauges, ok.

And, with the help of this strain gauges, in fact, we are just going to measure, how much is the deflection of this particular deflection bar, and if I know the deflection, so from there, let us try to find out, whether I can find out how much is the load acting on the deflection bar.

Now, let us see, how to determine this, now to determine this with the help of this particular end-effector doing some sort of manipulation job, ok. For example, it is handling some weights, it is doing some sort of pick and place type of operation and something like this, so what will happen is, each of this particular deflection bar will be subjected to some amount of force, ok. How to determine that, now if I concentrate on a particular deflection bar, it is almost similar to the situation as if, so I have got one beam sort of thing. So, this type of beam I have ok. And for this particular beam, so this is the fixed end, and here as if some concentrated load is acting, and this type of cantilever beam, we can assume.

So, here on this particular cantilever beam, there will be some deflection something like this; and if there is some deflection, due to the load, this deflection can be measured; so this delta deflection can be measured, ok. Now, let us see, how to measure this particular delta deflection with the help of strain gauges. The strain gauges are mounted here, ok. And, with this particular strain gauge, actually we count some potentiometer circuit, which I have already discussed with the help of potentiometer, what do you do? We measure the output voltage, and by measuring the output voltage, we can measure how much is the deflection.

So, we use potentiometer, for example, I can use some sort of linear potentiometer to find out, how much is the deflection? And, this particular potentiometer, the output of the potentiometer, that is nothing but the voltage, I can measure with the help of one voltmeter or multimeter.

Let me repeat, for example, say we have got the deflection bar. On each deflection bar, we put two pairs of strain gauges. Now, each pair of strain gauge is connected to the potentiometer circuit. And, on the output side of the potentiometer, we can measure. Actually, the output voltage and that particular output voltage is proportional to the deflection or the displacement.

And, that particular deflection is nothing but this particular delta. And, if I know this particular delta, and this is a cantilever beam, and supposing that I know, the length of this particular beam or the length of this particular deflection bar is L. I know the cross-section, I know the material properties, so very easily I can write down, this particular delta is nothing but $\frac{PL^3}{3EI}$, this is the standard formula.

Now, this is valid, if and only if, this particular bar is working within its elastic limit. Now, here, this delta is known, E is the Young's modulus. So, modulus of elasticity that you know for the material, I is the moment of inertia, I know the cross-section of this particular deflection beam, I know the dimensions, ok. So, I can find out the moment of inertia. L is the length of this deflection bar. So, all the things are known except this particular P, so P can be determined.

So, I can find out, how much is the load coming at this particular point to make that particular deflection possible. So, I can find out, what should be the load acting on each of this particular or deflection bar. And, those particular loads are nothing but the raw readings for these particular strain gauges.

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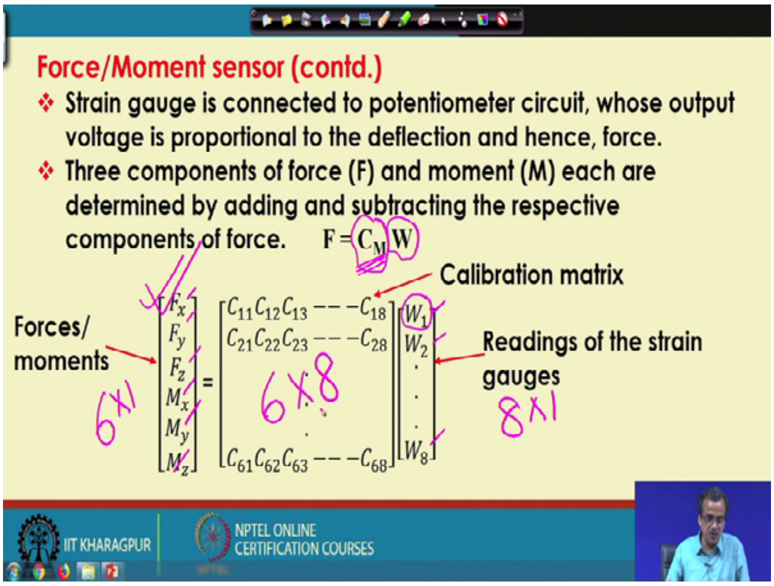

And, those raw readings are nothing but the W values, so these are nothing but the raw readings, which we are getting, ok. Now, these particular raw readings are W_1, W_2 up to W_8, because we have got 8 pairs of strain gauges; so, each pair is going to supply W values like W_1, W_2 up to W_8. And, our aim is to determine F_x, F_y, F_z; moment about x, moment about y, and moment about z.

Now, these particular W values, we can calculate, we can determine with the help of strain gauge and potentiometer circuit. And, our aim is to determine these particular F_x, F_y, F_z; M_x, M_y, M_z but in between there will be some calibration matrix, which is nothing but this particular C_M. So, $F = C_M W$, and this particular C_M is nothing but the calibration matrix.

Now, here in the matrix form, I have shown the calibration matrix, how to determine. I am just going to discuss, now let us see the dimensions of this particular matrix, here, there are 6 such values, so it is 6 cross 1 matrix. And, here we have got 8 such numerical values W values, so it is 8 cross 1 matrix. Now, to make this particular multiplication possible, so this particular matrix has to be 6 cross 8 ok. So, this particular calibration matrix C_M matrix is nothing but 6 cross 8 matrix.

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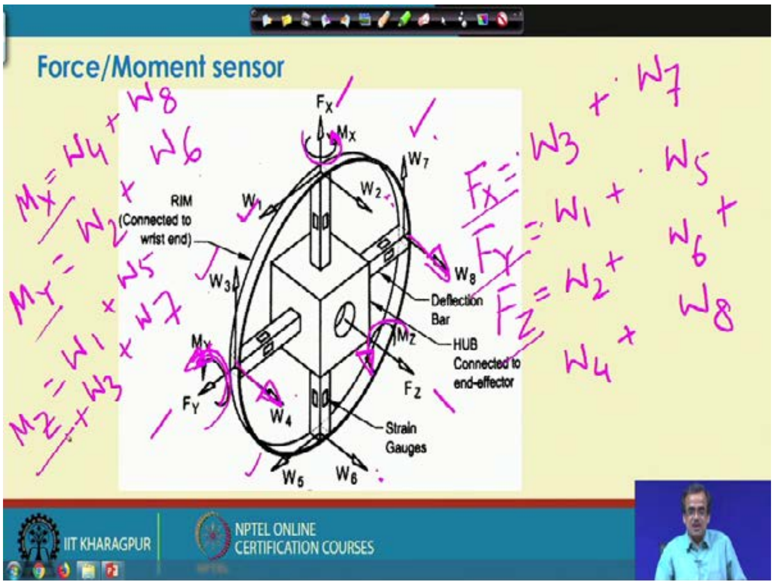

Now, how to determine that, so that I am going to discuss now, if I just concentrate on the previous thing, for example say, this particular thing, if I concentrate, and our aim is to determine your F_x, this is the x direction, so our aim is to determine F_x; this is the Y direction, F_y and F_z, then moment about X; then comes your moment about Y; and moment about Z; I will have to find out. And, these are all raw readings of the strain gauges, that is W_1, W_2 then comes your W_3, W_4 then 5, 6 then comes your 7, 8. So, these are all raw readings of the strain gauges, and I have already discussed how to get these particular the raw readings.

Now, with the help of those raw readings actually, how to determine this particular F_x. F_x is the force along the X direction. Now, here W_1 and W_2 are at 90 degree with F_x, so along F_x, there will have no contribution, no component. Similarly, W_5 and W_6 will have no component along X direction. But, W_3 and W_7 will have some contribution towards X. So, there is a possibility here W_3 will come, and W_7 will come, and of course, there will be some calibration terms, which I am going to discuss after sometime, ok, there will be some calibration terms here.

Next, comes is your F_y that is in this particular direction. So, 3 and 4 will have no contribution; 7 and 8 will have no contribution; but 1 and your 5 will have some contribution, so I am writing W_1 plus W_5, and here I am just going to write something, after sometime ok, so this is actually your F_y. Then, F_z if I want to find out, so this is the F_z direction, so these W_2 and 6 will have some contribution; so W_2 plus something into W_6 plus, so W_2, W_6 then comes your W_4 and W_8, so W_4 plus W_8 will have some contribution then comes your moment about X.

Now, let us try to understand, this is the X direction, so this W_1, W_2, 5 and 6 will have no contribution towards M_X ok. Now, let us see whether 4 and 8 will have some contribution towards M_X or not. Now, W_4 is acting in this particular direction, it is acting in this particular direction ok, so definitely so these two will have some contribution towards M_X.

So, W_4 and 8, so W_4 and W_8 will have some contribution. Then comes your moment about Y, so this is the Y direction this is the moment about Y. So, let us see the moment about Y, the 2 and 6 the W_2 and W_6 will have some contribution towards moment about Y. So, W_2 and W_6 will have some contribution then comes your moment about Z.

Now, this moment about Z, M_Z, so this is the Z direction. So, this 1 and 5 will have some contribution, then comes your, next is 1 and 5 will have some contribution. And, next is your 3 and 7, 3 and 7 will have some contribution, so 3 and 7 will have some contribution, because, this is your Z direction. So, 3, 7 will have some contributions towards M_Z, ok. So, this is the way actually we can find out F_X, F_Y, F_Z; M_X, M_Y, M_Z.

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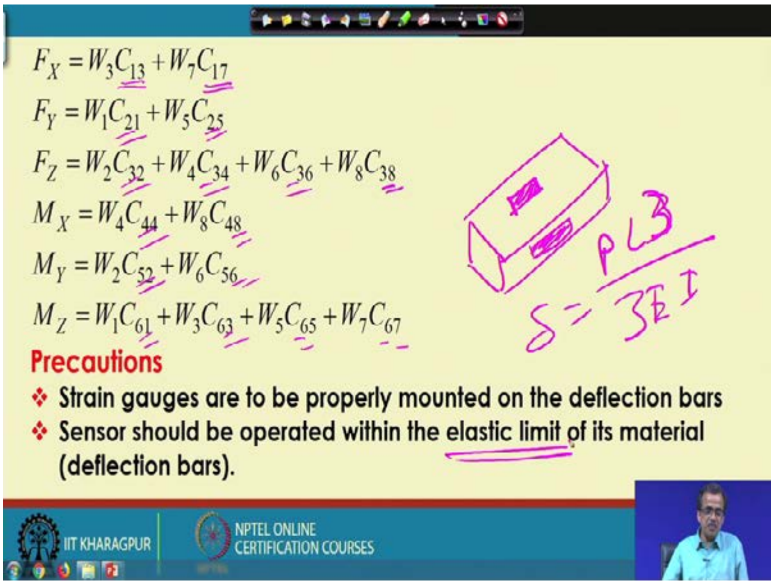

Of course, I have not put the calibration terms. The calibration terms actually, I am just going to show it here. If you see, the calibration terms have been written here. Whatever I discuss that F_X depends on W_3 and W_7 multiplied by this calibration matrix, so W_3 multiplied by calibration matrix C_13 plus W_7 multiplied by C_17.

Similarly, for F_Y, these are the calibration matrix; F_Z these are the calibration matrix; M_X these are the calibration matrix; M_Y depends on your C_52 and C_56, these are the calibration matrix; and M_Z depends on these calibration terms, ok. So, we can find out how much is the force acting its three component what is the moment? There is moment about X; moment about y; and moment about Z.

Now, here if you want to use this type of force or the moment sensor, some precautions are to be taken. For example, strain gauges are to be properly mounted on the deflection bar. So, on the deflection bar, in fact, we are going to mount the strain gauges, and we will have to mount properly, otherwise we may not get the proper reading ok, the strain gauge are to be correctly mounted here, ok. There should not be any such gap sort of thing, properly mounted ok, so this is one precaution.

Another is, your the depletion bar should work within its elastic limit. Otherwise, that particular formula will not be applicable for the deflection that is delta equals to $\frac{PL^3}{3EI}$, so that particular formula will not be able applicable, unless it is working within that particular elastic limit. So, these are the precautions to be taken.

Thank you.

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Lecture – 33
Sensors (Contd.)

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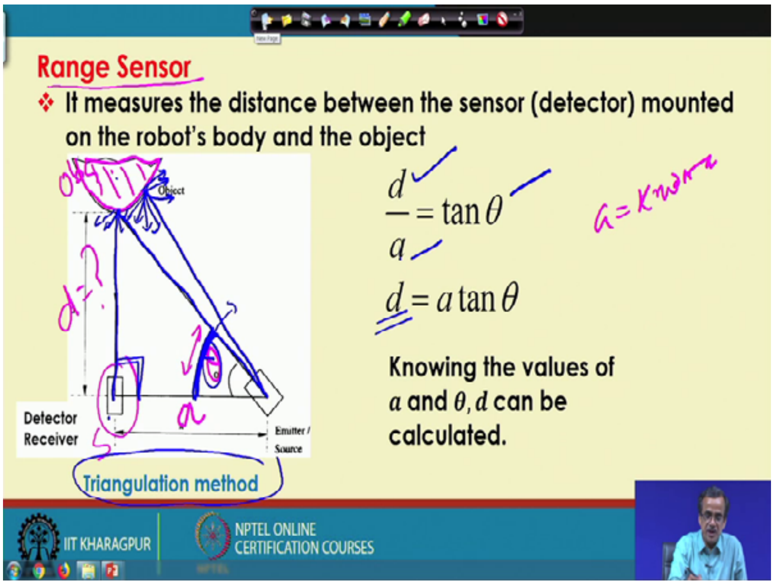

Now, we are going to discuss, the working principle of a Range Sensor. Now, this range sensor is generally used, just to find out the distance between the object and this particular sensor. Now, supposing that this is actually the detector or the sensor. Say, this particular detector or the sensor is mounted on the robotic link or a robotic joint. And, I am just going to ensure the collision-free movement of that particular joint with the obstacle.

Now, supposing that this is the obstacle. So, this obstacle actually I am just trying to find out, what is the distance between this particular obstacle or the object and the sensor or the detector or the receiver. So, I am trying to find out, what is the distance between this particular object or the obstacle and the sensor or the detector. So, this d, I will have to find out, d is equal to what, ok.

Now, here actually on the PPT, I will have to make one correction. So, this particular angle, it is not α, let us write the angle is θ. And, this particular distance is, say a in place of x, ok. Now, here, we have got, there is a light source or a detect or an emitter.

Now, this particular angle θ, so it can be varied. So, I can vary this particular angle θ. Now, supposing that I know the distance between the sensor and this particular light source or the emitter, so this distance is known, that is a is known ok, so a is known and θ is actually a variable and I am just going to vary this particular θ.

Now, by varying θ, so I will be getting different types of responses, here in this particular sensor. For example, say if I use a high value of this particular θ, say for example, I am using a higher value and this light is going to fall here. So, this is the light source; so, light is going to fall here. And, this is not a very smooth surface. So, what will happen is, so for this particular light, there will be some reflection here. So, this type of reflected beams will be getting and it is not a very smooth surface, so I may not get a very bright spot with the help of this detector.

Now, you just go on varying this particular θ. The moment θ becomes equal to this, there is a possibility that I will be getting this type of beam; and here, there will be some reflection this side and that side; but there is a possibility, I will be getting a very bright spot here with the help of this particular sensor. The moment you get a very bright spot with the help of this particular sensor ok, it is more or less correct that this particular angle is your 90 degree and this angle θ we can measure, ok.

And, if you can measure this particular angle θ, that means, I am getting very good reflection here ok, and this angle is 90 degree. I know this particular theta, so very easily I can find out, because d divided by a; so, d, I am going to determine. So, $\frac{d}{a} = \tan\theta$, θ I can measure, a is known. So, very easily, I can find out d, that is the distance between the sensor and this particular obstacle.

Now, let me repeat that particular example once again. Suppose, if this is the robotic joint, here I am just going to put that particular sensor. And, I want to make this particular joint collision-free and supposing that, I have got one object here. So, it will come very near to this particular object, so I will have to find out the distance between the object and this particular the joint. So, this type of sensor, I can use just to find out the distance between this sensor and this object or the obstacle.

Now, here, I can use a light source; I can also use some sort of sound source. And, this method is a very popularly known as the triangulation method. So, in range sensor, we use some sort of the triangulation method. It is very simple. And, using this very simple mathematics, we can find out the distance between the obstacle and this particular sensor, which is mounted on the body of this particular robot, ok. This is the working principle of the range sensor.

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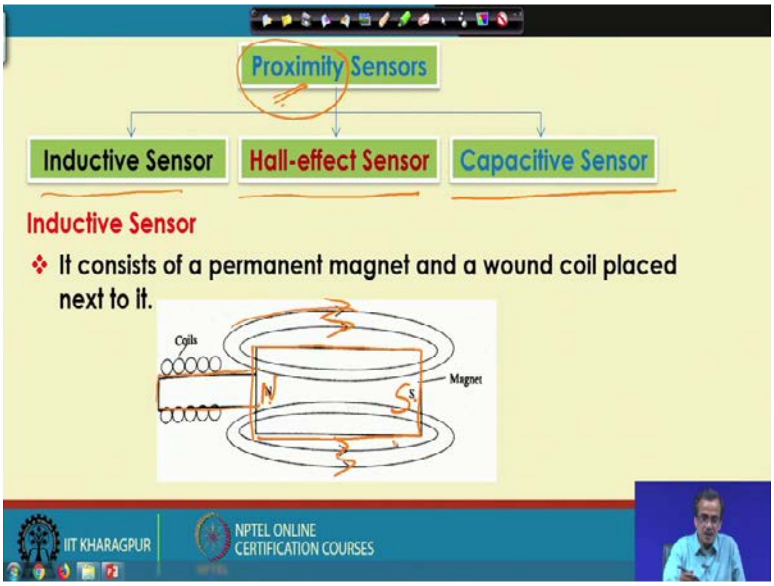

Now, I am going to discuss the proximity sensor; and these proximity sensors are very frequently used. For example, we have got three popular types of proximity sensor; one is called the inductive sensor; we have got the Hall-effect sensor and we have got the capacitive sensor. Now, let me try to explain the working principle of this particular inductive sensor. Now, by proximity we mean, it is the closeness, ok. So, how much closeness we are going to consider between the sensor and this particular object.

So, proximity means, it is actually the closeness. So, the working principle of this inductive sensor is very simple. So, here, we use one permanent magnet, supposing that this is the permanent magnet. And, this is the North Pole, and this is the South Pole of the magnet. And, this is the extended version of this particular magnet, say ok. And, on the extended version, we put some coils for current flow. So, we have got some electric coils wires for current flow.

Now, this is the North Pole and this is the South Pole. So, the magnetic lines of force will come from come out from North Pole and it will move through the South Pole outside that particular magnet; and inside the magnet, it will be from South Pole to North pole.

So, this is actually the direction of the lines of forces. So, these are the direction of lines of forces, ok. These are all fundamentals, all of us we know. And, this is a permanent magnet, ok. Now, so this will have some lines of forces, it will have some magnetic flux. Now, let us see, what happens like if I just bring one ferromagnetic material or the magnetic material closer to this particular inductive sensor. This is nothing but the inductive sensor.

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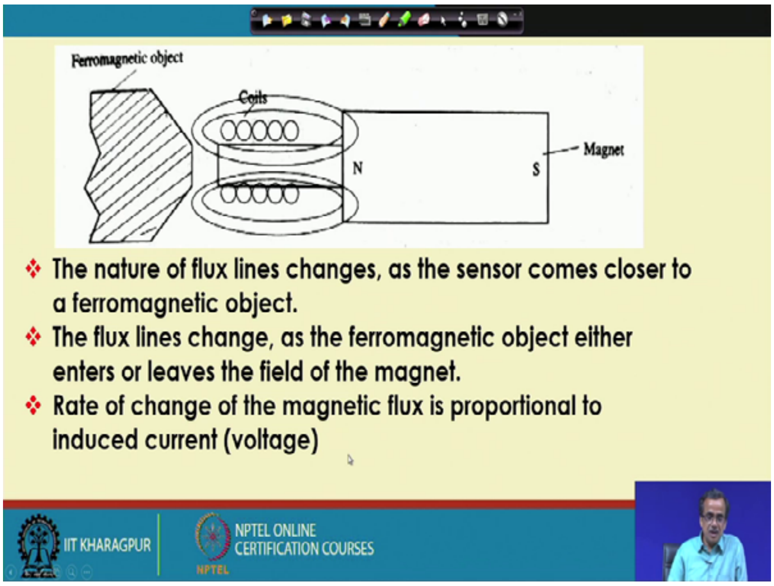

Now, this is the ferromagnetic object, which is brought very near to this particular sensor. So, this is actually the inductive sensor. And, this is the ferromagnetic object. The moment we bring this particular ferromagnetic object very near to this inductive sensor, what will happen to these magnetic lines of forces, the magnetic lines of forces will be deflected.

So, previously the magnetic lines of forces were here ok; it was shown here. Now, there will be a shifting of the magnetic lines of forces; and there will be a change of magnetic flux. Now, this particular rate of change of magnetic flux is proportional to the induced voltage or the induced current. So, due to this change in magnetic flux, what will happen, there will be some induced voltage and there will be some current flow through these particular coils. So, we have got the coils through which the current will flow, and due to this induced voltage, there will be some current flow, ok. So, this particular voltage or this current, we can measure.

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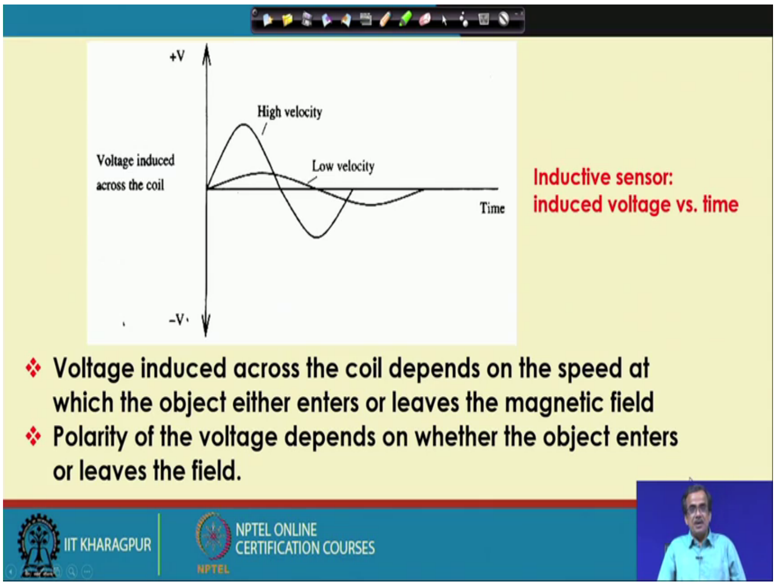

So, let us see the way it works. Now, supposing that say I have got this particular paramagnetic object. So, this is stationary. So, inductive sensor is, say stationary. And, this ferromagnetic object is brought near to the inductive sensor or it is actually thrown away from this particular inductive sensor. The moment it is brought near to this particular ferromagnetic object, near to this inductive sensor, there will be some amount of the voltage induced. And, you will be getting the voltage induced like this.

For example, if I plot the induced voltage with time, supposing that the ferromagnetic object is brought near to the inductive sensor with high speed. So, if it is brought with high speed, so there is a possibility that I will be getting this type of plot for this induced voltage. Now, this positive sign, as if the ferromagnetic object is brought near to the inductive sensor. And, the moment it is thrown away from the inductive sensor, it will be getting this particular the negative induced voltage, ok.

Now, once again let me repeat if this particular ferromagnetic object is brought at high speed, I will be getting this type of plot for the induced voltage. But, if it is brought at low speed, there is a possibility that I will be getting this type of plot for this induced voltage with time, ok. So, the nature of the induced voltage will be changing, and it depends on the speed with which I am just going to bring that particular ferromagnetic object near to the inductive sensor; or I am throwing away this ferromagnetic object from the inductive sensor. So, depending on this particular speed, we will be getting the different types of voltage distribution.

Now, here if I just concentrate on this type of the induced voltage distribution, I will be getting its amplitude. And, if I consider this type of distribution, I will be getting the different amplitudes, that means, I will be getting high amplitude value for this induced voltage, if it is moving with a high speed, and if it is moving with low speed, then I will be getting low amplitude for this particular distribution of voltage. Now, once again, let me repeat that corresponding to the high speed, I will be getting high amplitude that means, corresponding to the high speed.

Now, if I consider a fixed time, I am plotting here with time. So, if I consider a fixed time, and if this is the inductive sensor and this is the ferromagnetic object. Now, if it is moving with high speed towards the ferromagnetic object, at the fixed instant of time a, for at the fixed duration, it will come more closer to this particular inductive sensor compared to the situation, whenever it was moving with slow speed.

Now, once again, let me repeat, if the time duration is same, say Δt or t, if it is the same, so time is the same. And now, once again, I am repeating for the same time. So, this is the fixed position of the inductive sensor, and it is moving with high speed. So, in the same duration, it will be coming closer to this particular inductive sensor compared to the situation, whenever it is moving with the slow speed. So, whenever it is moving with the slow speed, the distance between the sensor and the object will be more; and whenever it is moving with high speed, the distance between the object and the sensor will be small. The same thing is actually is coming here in the calibration curve.

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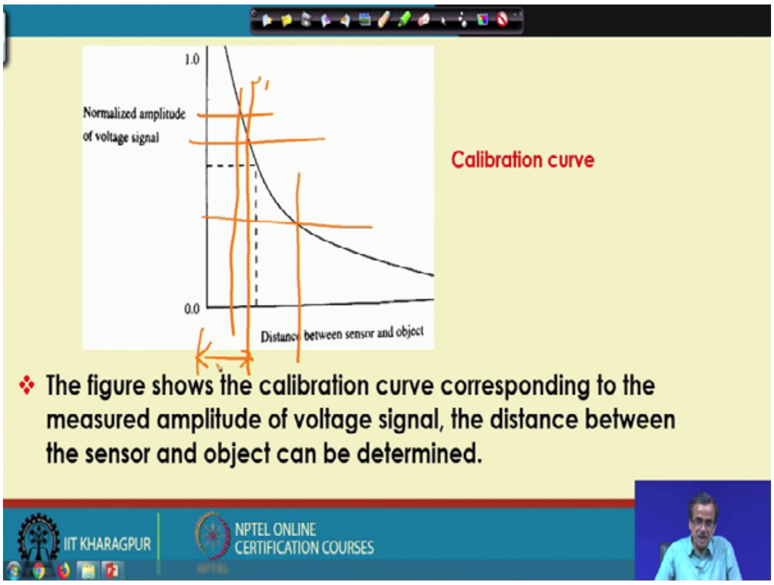

Now, this is the calibration curve. Now, whenever we are getting the higher amplitude that means, the ferromagnetic object is moving with high speed, it is coming very near to the inductive sensor, ok. Now, in that case, corresponding to the high amplitude, I will be getting the smaller or the distance between the sensor and the object and corresponding to the lower amplitude, I will be getting the larger distance between the sensor and this particular the object; now if this particular calibration curve is known, and if I can measure the normalized amplitude or the voltage signal.

Now, why this is the normalized, because in the scale of 0 to 1, I want to represent this normalized amplitude of voltage signal, if we can measure, then very easily I can find out what should be the distance between your object and this particular sensor, ok. So, this is the way, actually, we can use the inductive sensor, just to find out like what should be the distance between the sensor and this particular the object. Now, this is the way actually one inductive sensor is working. Now, this sensor is suitable only for the magnetic material; this is not going to work for the non-magnetic material.

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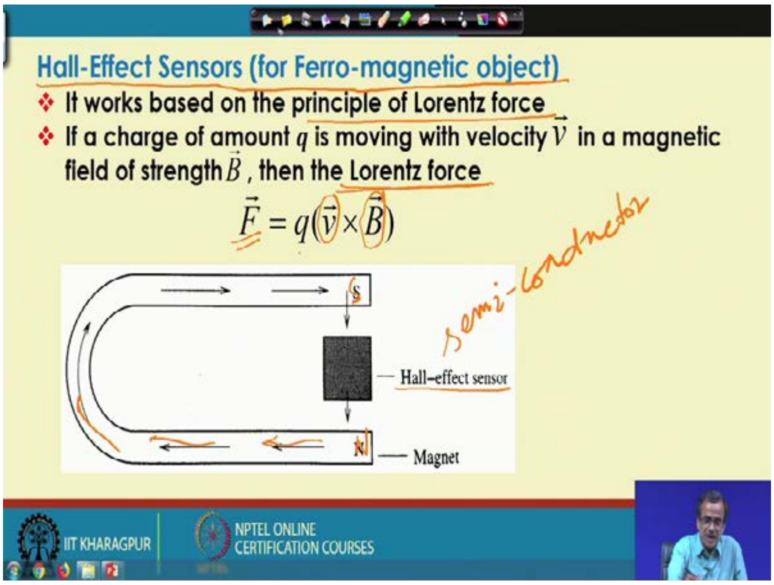

Now, I am just going to discuss another very popular sensor, that is called the Hall-effect sensor. And, this is also suitable only for the ferromagnetic material; this is not suitable for the non-magnetic material. And, this particular Hall-effect sensor is very frequently used in the vertex.

Now, its working principle is based on the principle of the Lorentz force. Now, supposing that an amount of charge q is moving with velocity v in a magnetic field of strength B, then it will be subjected to one force, that force is known as the Lorentz force, ok. Now, this Lorentz force is nothing but $F = q(V \times B)$; V is nothing but the velocity, so this is the vector; and B is nothing but the strength of the magnetic field, that is also a vector. So, we find out the cross product, that is, V cross B multiplied by the amount of charge that is q that is nothing but the Lorentz force.

Now, this particular principle, in fact, we are going to use here to develop the Hall-effect sensor. Now, supposing that this is once again a permanent magnet, say permanent magnet, this is the North Pole, and this is actually the South Pole, North Pole and South Pole, so will be getting the magnetic lines of forces like this. Now, supposing that in between the North Pole and South Pole, supposing that I am just going to put one Hall-effect sensor, now what is a Hall-effect sensor, the Hall-effect sensor is nothing but one semi-conductor material say silicon. So, this is nothing but a semi-conductor material.

Now, this semi-conductor material or say silicon or having some free electrons, the moment we put the semi-conductor material in between the North and South Pole, and within the influence of this magnetic lines of forces, there will be some amount of voltage induced in the your semi-conductor material or this Hall-effect sensor, ok.

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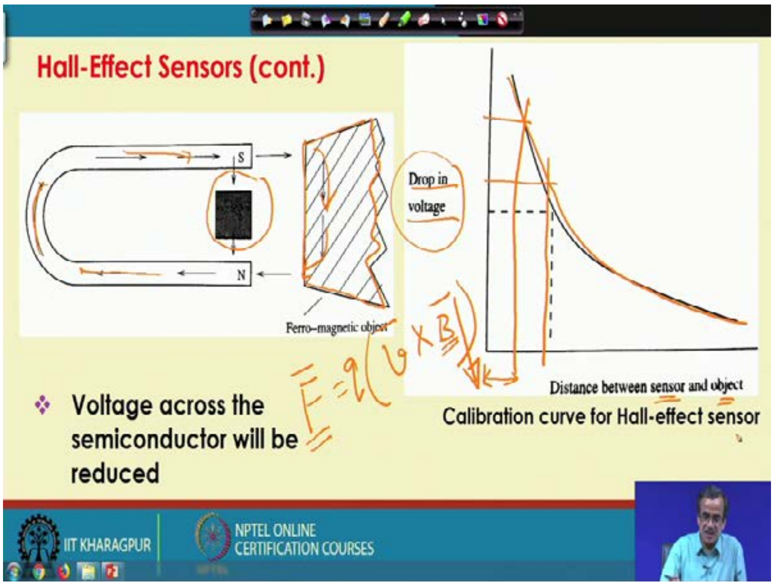

Now, actually, what happens, the moment we just bring one magnetic object near to that. Supposing that this is the magnetic object, which has been brought near to that particular Hall-effect sensor, ok. So, this is the Hall-effect sensor. Now, what will happen is, so this is a ferromagnetic material. And, we have got the permanent magnet here, this is the North Pole, this is the South Pole. So, the magnetic lines of forces will pass through this. And, here, as this is the magnetic material, some of the lines of forces will pass through this particular ferromagnetic object.

And, due to that actually, what will happen is, the strength of the magnetic field here will be reduced. And, the Lorentz force $F = q(V \times B)$. Now, here actually, due to the presence of this particular ferromagnetic object, the strength of the magnetic field will be reduced, the strength of B will be reduced. Then, what will happen to the Lorentz force, the Lorentz force is also going to be reduced.

And, consequently, the amount of induced voltage here in this semi-conductor material is going to be reduced, and there will be some drop in voltage. So, when this particular material was not there in front of the sensor, there was some induced voltage. And whenever it is coming, there is some change in induced voltage, truly speaking there is some drop in induced voltage. And, this particular drop in induced voltage can be measure with the help of voltmeter or multimeter.

And, if I know this particular calibration curve, so this particular calibration curve is known. And, if I know the drop in voltage, I can find out the distance between the sensor and this particular object. Now, if this particular object comes very near to the sensor, what will happen to this drop in voltage, the drop in voltage is going to increase, and the distance between the sensor and the object will be reduced.

And, this is the way by measuring the drop in voltage, I can find out the distance between the sensor and this particular object. This is the working principle of this particular Hall-effect sensor. And, this is also very popular in robotics. But, the main drawback of these sensors like inductive sensors, then comes your Hall-effect sensors is that these are not suitable for the non-magnetic material.

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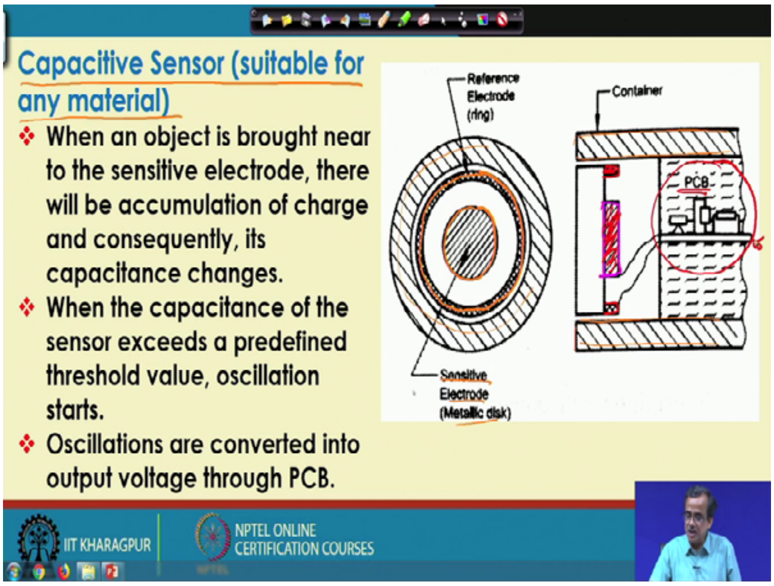

Now, I am just going to discuss the working principle of another sensor that is called the capacitive sensor, it is suitable for any material, whether it is magnetic or nonmagnetic, so this particular sensor is going to work. Now, let us try to understand the construction details at first, it is very simple. Supposing that we have got a container here, so this shows this view, this shows the container, it is just like cylindrical container sort of thing. So, this is another view, so this is the container, ok. So, this shows actually another view.

So, we have got the cylindrical container, and here we have got one sensitive electrode, and that is nothing but a metallic disc. This is a very thin metallic disc. And, we have got one reference electrode, which is nothing but a ring. So, this reference electrode is nothing but a ring. So, this is nothing but a ring, and this is the reference electrode. And we have got your the sensitive electrode. And, this is nothing but is your sensitive electrode.

So, this is the sensitive electrode; and it is very thin and lightweight. If you see in this particular view, this sensitive electrode is shown here, so this is nothing but the sensitive electrode. And, the reference electrode is nothing but this. Now, this reference electrode is kept fixed, but in this sensitive electrode, there could be some oscillations. Now, let us try to find out the reason behind this particular oscillation. Now, as I told that this particular sensitive electrode is a thin metallic disc, and very lightweight, the moment we bring any object in front of this particular the thin metallic disc. So, what will happen is, there will be some amount of charge accumulated.

And, due to this particular accumulation of charge, its capacitance is going to change. And, the moment its capacitance exceeds the threshold value, then oscillation starts. So, there will be some sort of oscillations here. And, this particular charge is due to some sort of static electricity sort of charge. And, there will be some oscillations, as we told. And, for the sensitive electrode, there will be some oscillations, but the reference electrode is kept fixed. So, with respect to the fixed reference electrode, there will be some oscillations in the the sensitive electrode or the metallic disc.

Now, let me repeat a little bit. Now, here if you see so this particular sensitive electrode is a very thin and lightweight, and an object is brought very near to that sensitive electrode. What will happen is, there will be some amount of charge accumulation, static electricity sort of charge accumulation. And, due to this accumulation, the capacitance of this particular metallic disc is going to increase. And, the moment it exceeds the threshold value of capacitance, there will be some oscillations.

And, as I told that this reference electrode is kept fixed, but in this sensitive electrode, there will be some oscillations. So, with respect to the fixed one, there will be some oscillations. Now, this oscillation is converted into some output voltage here with the help of some electronic circuit, some printed circuit board. And, with the help of this printed circuit board, the output side will be getting some output voltage, but the inputs will be some sort of oscillations.

And, here, there are some electronic circuits, and this is the printed circuit board, and this output voltage can be measured with the help of your voltmeter or multi-meter. So, we can find out so by measuring this particular change in capacitance or change in output. In fact, we can determine the distance between the sensor and this particular object.

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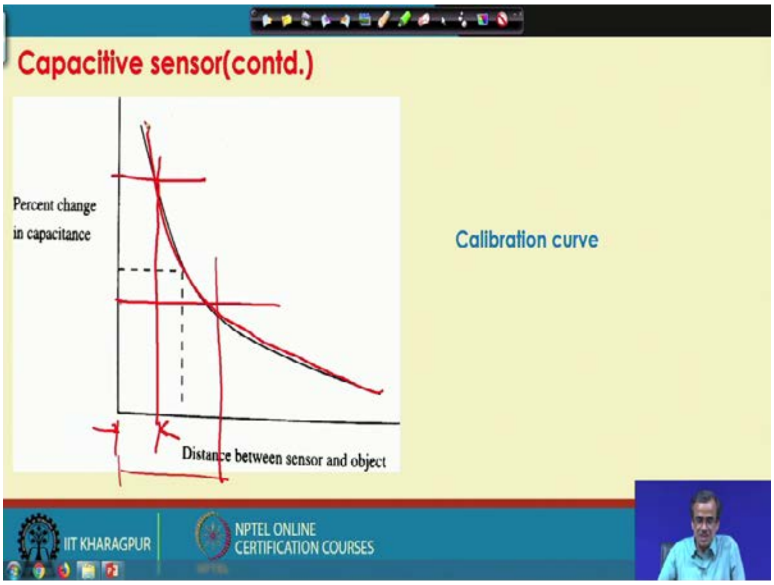

Now, as I told, the moment that object is brought near to that particular sensor, its capacitance is going to be changed due to the accumulation of charge. Now, supposing that the amount of charge accumulation is more, and this is the percentage change in capacitance. If it is more then what will happen? The object has come very near to the sensor. So, this is the distance between the sensor, and this particular object.

And, supposing that the percentage change in capacitance is less that means, the object is far from this particular sensor. So, this is the calibration curve. And, by knowing this particular calibration curve, and knowing the percentage change in capacitance, we can find out the distance between the sensor and this particular object. So, this is the way, we can determine the distance between the sensor and object using the capacitive sensor.

Thank you.

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Lecture – 34
Robot Vision

We are going to start with a new topic, that is, topic 7, it is on Robot Vision. Now, this robot vision is also known as the computer vision.

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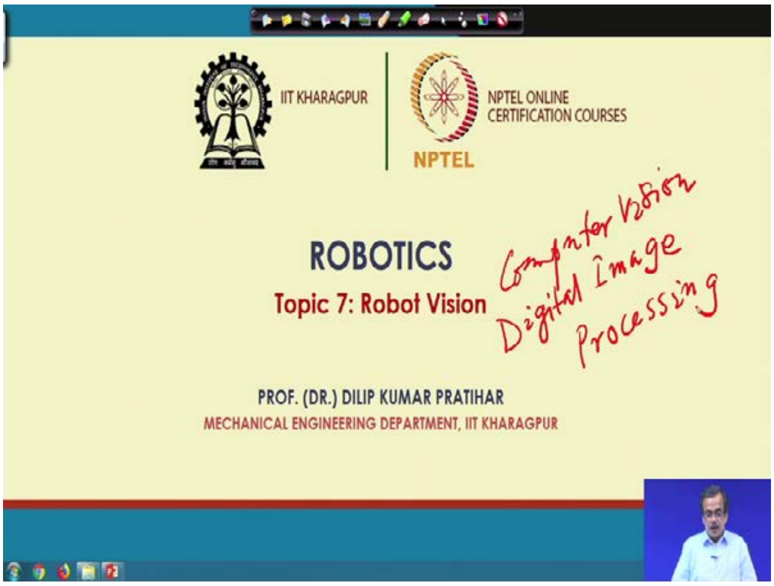

And, in robot vision, we use the principle of the digital image processing. So, we are going to use the principle of digital image processing. Now, the aim of this robot vision, or the computer vision is to help the robot to collect information of this particular environment.

Now, let us see, how a robot can collect information of the environment with the help of camera. Now, before I discuss further, now let us try to see the way we, the human-beings, do collect information of the environment, with the help of our eyes. So, with the help of our eyes, we take the photograph or snap of the environment and there is a lot of processing in our brain and consequently, we could identify that this is object a, this is object b, present in a particular scenario, or project in a particular the image, or photograph.

Now, exactly this particular principle, we are going to copy in the artificial way in robot vision, or the computer vision. Now, once again, let me repeat the purpose of robot vision is to identify and, interpret the different objects present in a particular image or the photograph. Now, let us see how to carry out this particular the digital image processing, or the robot vision or the computer vision.

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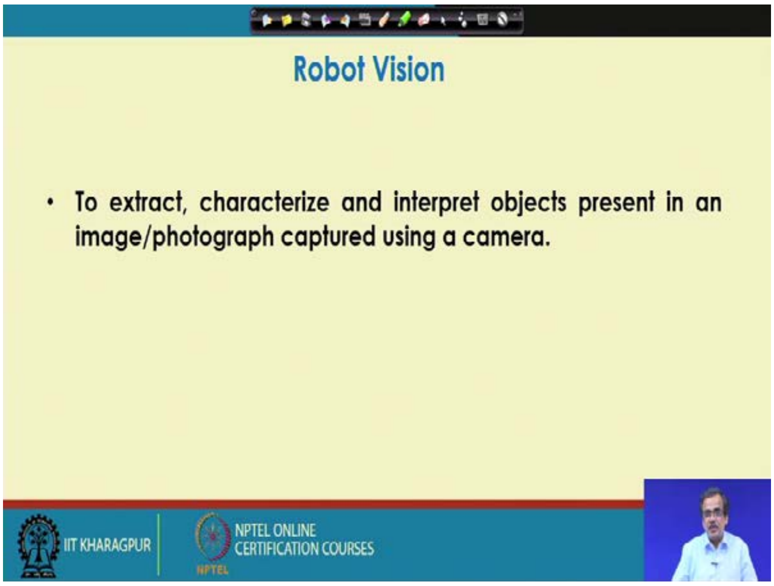

Now, the purpose as I told to extract characterize and interpret objects present in a scenario, or a photograph with the help of camera. Generally, we use some sort of CCD camera, the charged coupled device camera.

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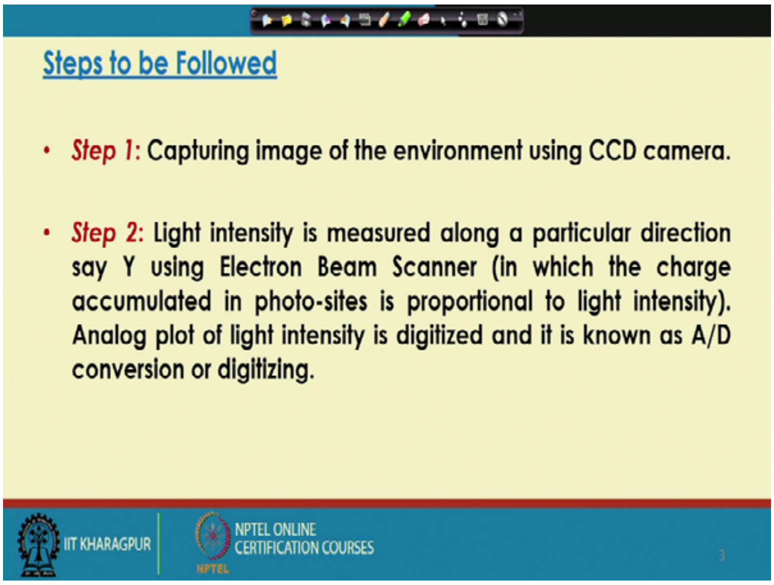

So, we capture image with the help of the CCD camera now, as I told, by CCD we means charge coupled device camera. So, this is actually the step one of this particular computer vision or the robot vision and, once we have got this particular image of the environment collected with the help of this camera, it looks like this.

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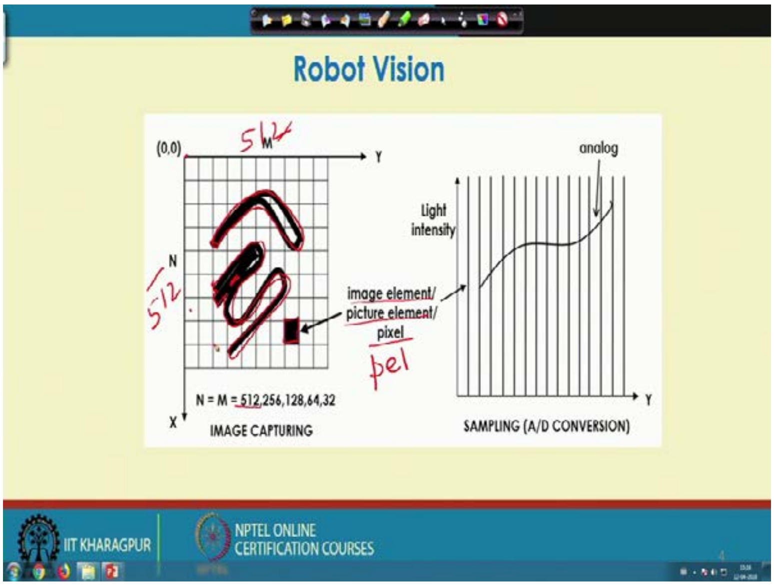

Supposing that, this is the computer screen. So, this is the positive Y direction and this is the positive X direction. Now, here actually, what I do is, with the help of camera, the image or the environment, whose photograph we have taken, is actually transferred to the computer and on the computer screen, we will be able to see this type of image. Now, what I do is, this particular computer screen that is divided into a large number of small segments for example, say along Y direction. So, we take m number of divisions along X direction, we take N number of divisions and this is nothing, but the origin, that is, (0, 0).

Now, if this is the computer screen, that is divided into M cross N. So, so many such small small subdivisions for example, say, I will be getting M equals to, N equals to 512 or 256 or 128 or 64 or 32. Now if I take M equals to, N equals to 512 that means, here there are 512 divisions and here, also along this particular X direction, there will be 512 divisions.

That means, this particular area is divided into like 512 multiplied by 512. So, so many such small small image elements and this image element for example, say this is one small image element and this is known as the image element, or the picture element or the pixel or in some of the literature this is also known as pel.

So, this particular pixel, so many such pixels we have, and now we will have to concentrate on these particular the pixels. Now, supposing that with the help of camera, we have got this type of image so, for example, say this is one image, which I have got on this particular computer screen and this is collected with the help of say camera and that is transferred to the display of this computer, ok.

Now, if this is black and white picture, the difference between the black and white is actually the amount of the light intensity, for example, if I consider that this is the black object means, the light intensity is less and on the white portion, the light intensity will be more.

Now, depending on this particular light intensity and the difference in light intensity, we can identify the black and white for example, say if we take the photograph, the black and white photograph of a human being, the head portion or the hair portion will be black and the face will be slightly white-ish, if I compare the light intensity values of the hair and the face, the light intensity value of the face will be more compared to that of the hair part, that is the black hair part. So, this is the way actually, we can find out the difference between the black object and white object due to the difference in light intensity.

Now, supposing that we have got this particular picture and now, actually we are going to find out, what should be the light intensity at each of these particular pixels. Now to do that actually, what I do is, we try to take the help of step 2.

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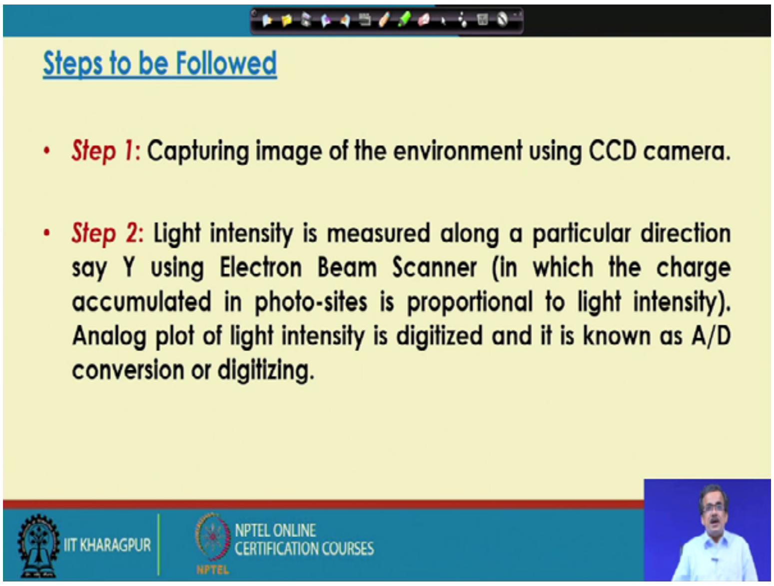

Now, this particular step two is nothing, but actually as follows: we take the help of one electron beam scanner and, we do the scanning along the Y direction and this particular the X direction just to collect the light intensity values at each of these pixels.

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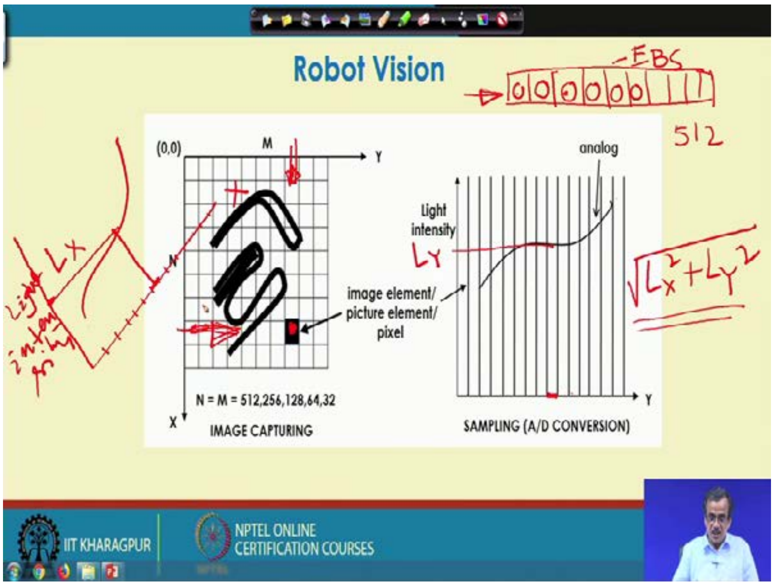

Now, let us say how to do it, now what I do is, say we are just doing scanning in the positive Y direction, supposing that I want to find out what should be the light intensity. So, at this particular pixel, what I do is, we try to actually do the scanning along this particular the Y direction and, how to do this particular scanning, we take the help of one electron beam scanner. Now, this electron beam scanner is something like this, now here on this electron beam scanner there are some photo-sites.

So, we have got a large number of photo-sites here for example, if there are 512 divisions so, I can consider the 512 photo-sites. And, these particular electron beam scanners, this is electron beam scanner and that is put just say below that and we do the scanning, the moment we do this particular scanning along this particular Y direction, what will happen is so, due to the variation of this light intensity, different amounts of electrical charges will be accumulated on these photo-sites, for example, say here I have got a photo-site, I have got another photo-site here, another photo-site. So, these are all photo-sites and now, I am doing the scanning in this particular direction.

Now, if the light intensity is more, the more amount of charge will be accumulated in the photo-site, on the other hand, if the light intensity is less; that means, I am passing through the black region, less amount of charge will be accumulated in this particular photo-site, now what I can do is, we can measure, how much is the amount of accumulated charge at each of the photo-sites. And, here we just prepare one plot and this particular plot is nothing, but light intensity versus the Y direction, now here each of these points indicates actually the pixels.

So, starting from here, pixel-wise I can plot; that means, along this particular Y, ok. So, I can plot, what is the variation of these particular the light intensity values and, this particular information is nothing, but the analogue information. Now, this is what is happening along the Y direction, ok. And, supposing that I am concentrating here so, corresponding to this particular pixel, I am getting that this is the amount of the light intensity and supposing that that is denoted by say L_y. The same thing, we do along this particular X direction. So, what I do is, we try to do the scanning in this particular direction, in the positive X direction and once again, we will pass through the same pixel, ok.

And, exactly in the same way, if I just plot, supposing that this is the light intensity, say for example, this is the light intensity and this is the X direction. So, once again, I have got all such pixels. So, for each of these particular pixels, I will be able to find out what should be the analogue plot for this light intensity.

For example, say I am just moving along this particular the positive X direction. So, there is every possibility that I will be getting some sort of the profile of light intensity like this. And, once again, if I concentrate on the same pixel supposing that I am here. So, I will try to find out what is the light intensity value corresponding to that particular the pixel and supposing that, that particular numerical value is nothing, but L_X.

Now, corresponding to this particular pixel, I have got this L_Y and this particular L_X. So, after that actually what I do is, we try to find out what is this $\sqrt{(L_X^2 + L_Y^2)}$. So, I will be getting some numerical value. So, I will be getting some real value and we try to find out, what is the nearest integer. Now, corresponding to this value, corresponding to this the nearest integer will be the light intensity value, corresponding to this particular the pixel. The same process we follow for each of these pixels.

So, we can find out, what should be the light intensity value ok, corresponding to each of these pixels. Now, here, corresponding to this particular image, so, I have got some sort of the light intensity values, ok.

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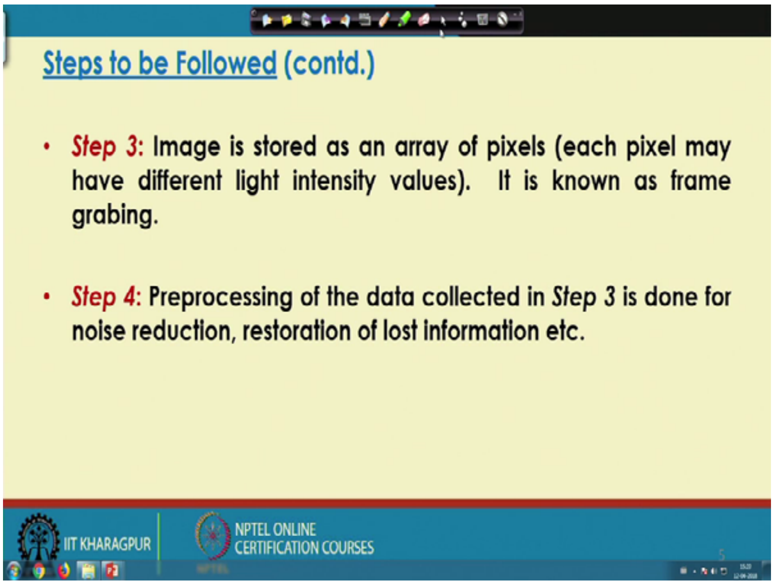

Now, the step three, whatever I mentioned so, that is nothing, but the step 3, that is image is stored as an array of pixels. And, at each pixel actually, we try to mention what is the light intensity value and this particular process of storing an image, or a photograph with the help of some numerical values of light intensity. So, this is what is known as the frame grabbing.

In fact, unless we do the frame grabbing, we will not be able to carry out any such calculation with the help of this computer. Now, because computer does not know anything except the numbers so, what I will have to do is, corresponding to that particular image. So, I will have to find out the corresponding matrix of light intensity values. And, this particular process is known as the frame grabbing. And, once that particular the frame grabbing is done.

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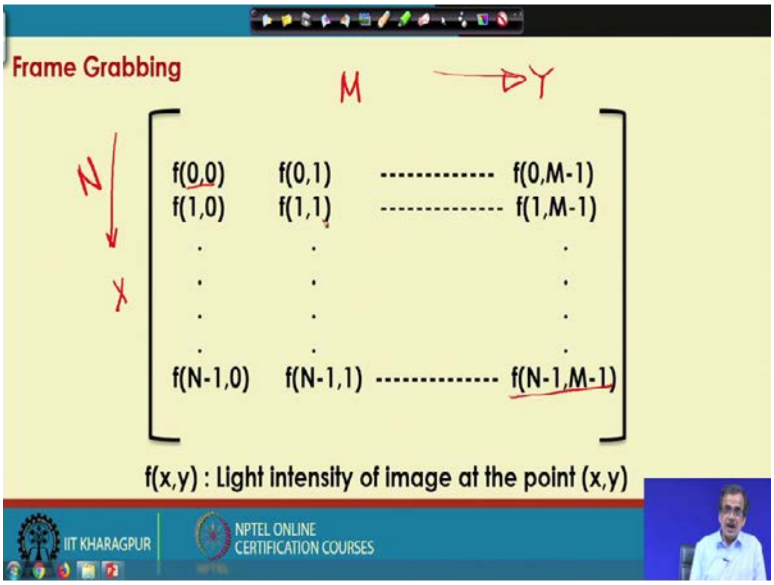

Now, actually, we are in a position to represent this particular image in the form of this type of matrix. Now, here, if you see, for example, say we consider, there are M number of divisions along this particular positive Y direction. So, this is the positive Y direction and, we consider capital M number of divisions. Now, this is actually the positive X direction and we consider capital N number of divisions, ok. Now, here, this f (0, 0) is nothing, but the light intensity corresponding to the pixel, whose coordinate is 0 comma 0. Similarly, this f (N-1, M-1) is nothing, but the light intensity value corresponding to the pixel whose coordinate is (N-1, M-1).

So, for each of these pixels, we can find out the light intensity values and this values are nothing, but the integer values. Now, here I have written so, f (x, y) indicates the light intensity of the image at the point (x, y). So, similarly, this f (1, 1) is a light intensity value at the point, whose coordinate is your (1 comma 1). So, this is the way actually, we can represent one image with the help of a matrix of some numerical values and, this numerical values are nothing, but the light intensity values in the integer form. Now, let us see like how to proceed further.

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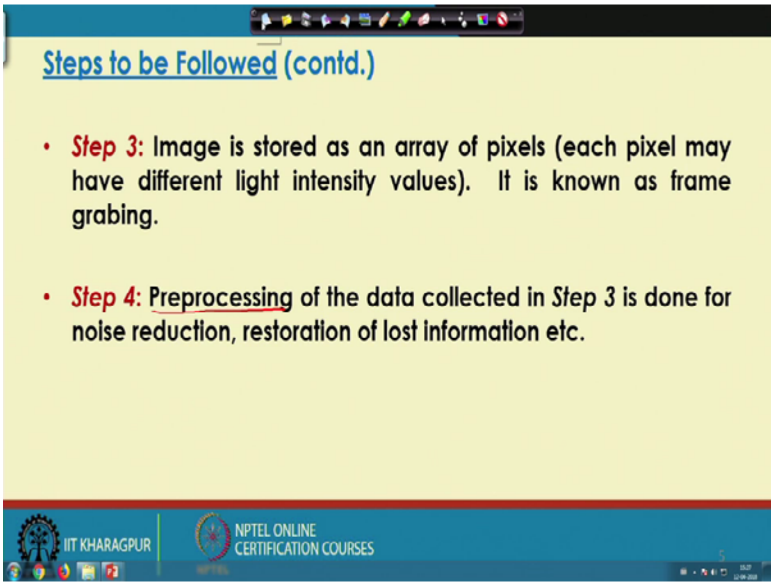

Now, here actually, what we will have to do is, if you see this particular matrix of light intensity values. So, this particular matrix cannot be very accurate, or the correct the reason is very simple. The quality of this particular image, or the light intensity values depends on a number of parameters, for example, it depends on the level of illumination at which we are taking that particular, we are collecting that particular picture.

It depends on the angle at which I am collecting that particular picture; it depends on actually the expertise of the operator. So, this data which you have got corresponding to this particular image may not be very accurate. So, there could be some noise, there could be some sort of in imprecision, there could be some sort of uncertainty and that is why, we take the help of one step, that is called the preprocessing.

So, we try to do some sort of the pre-processing and, if you do this preprocessing actually we can remove this particular noise from this the matrix. So, the purpose of preprocessing is to the remove noise, from this particular the light intensity values, or sometimes there is a possibility that some part of information will be lost from this particular picture, and we try to restore that particular the information.

So, if you want to reduce the noise from this particular data, or if you want to restore some sort of lost information, will have to take that help of some sort of preprocessing. Now, if you see the literature, in fact, we have got different methods for this particular processing.

Now, here I am just going to discuss the principle of each of these particular the pre-processing methods. So, this is actually the thing which we will be getting, this particular matrix corresponds to the image.

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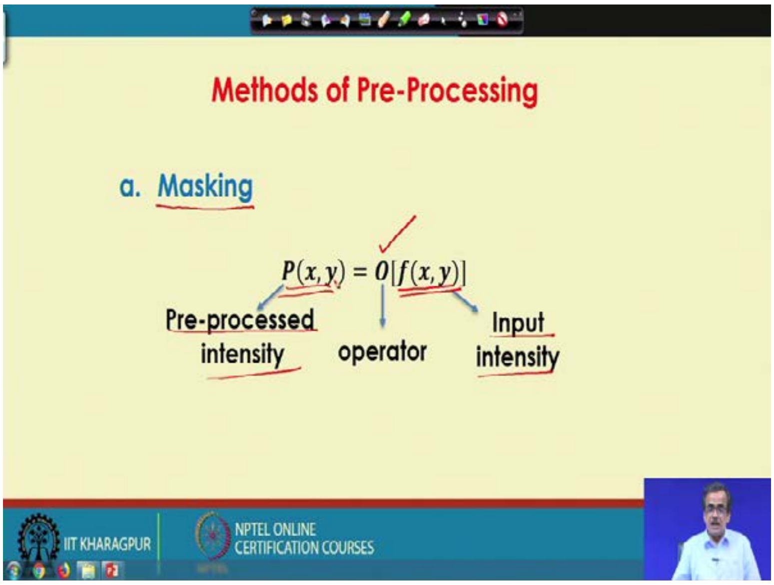

Now, let us see how to do this particular the preprocessing just to reduce that noise from the data. So, methods of preprocessing as I told there are several methods and out of all such methods, I am just going to discuss a few very popular methods, for example, say the masking is a very popular method for preprocessing. Now, the method of masking is a very simple actually, what I do is, supposing that, this f (x, y). So, this is nothing, but the light intensity value at the pixel, whose coordinate is x comma y.

And, on this particular light intensity value, we use one operator that is nothing, but O. So, this operator O is going to work on this f (x, y) that is the light intensity value, which is nothing, but the input intensity. And, we are going to find out what is this preprocessed intensity, that is, P (x, y). So, our aim is to determine this particular P (x, y). Now, let us see like how to determine this particular the P (x, y).

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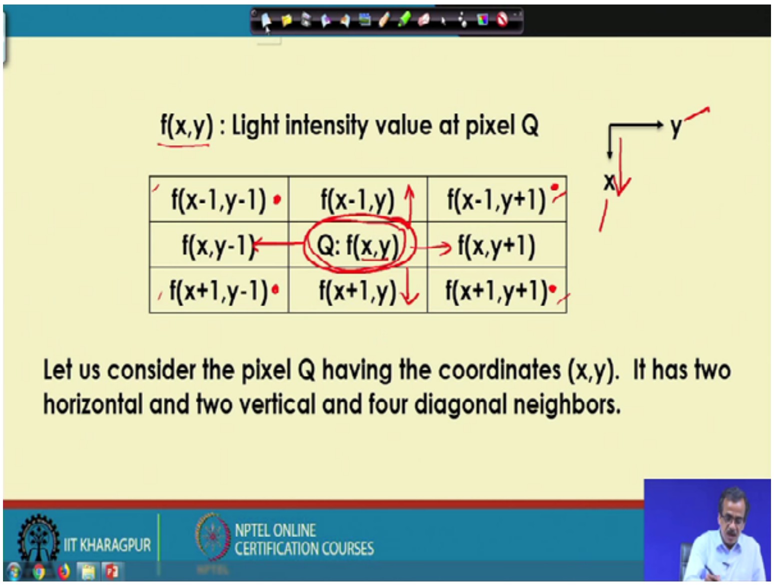

That is the preprocessed data. Now, here, I am just going to concentrate on a particular pixel and its neighborhood. Now, here, as I told that f (X, Y) is going to indicate the light intensity value at the pixel, whose coordinate is nothing, but is your (x, y).

Now, this is the positive direction of Y, this is the positive direction of X. So, starting from here, if I move along this particular direction. So, Y is going to increase, this will be your (X, Y+1). Similarly, here, this will be (X,Y-1), because this is in the negative direction of Y. Now, similarly starting from here, if I just go down, then what will happen is, this particular X is going to increase, because this is the positive direction of X. So, this will become f (X+1, Y) and the coordinate of this particular pixel will be your (X-1, Y).

Similarly, I can also find out the coordinate of this. Now, if I concentrate on this particular pixel, that is denoted by Q, whose coordinate is (X, Y) and whose light intensity is f (X, Y). Now, this particular pixel has got two horizontal neighbors, it has got two vertical neighbors and, it has got four such diagonal neighbors. So, once again, let me repeat, let me repeat that for a particular pixel, there are two horizontal neighbors two vertical neighbors and there are four such your diagonal neighbors. So, we will have to concentrate on this particular horizontal vertical and the diagonal neighbors, ok.

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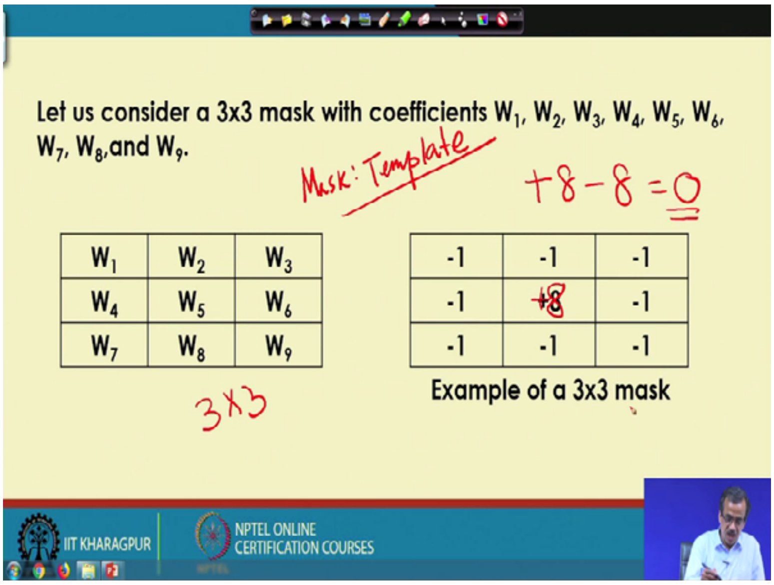

Now, let us see how to carry out this particular the pre-processing. Now, here actually, in masking what we do is, we try to take the help of a mask and this particular mask is nothing, but a template. So, by masking we mean, this is nothing but a template, now with the help of this particular template actually, we can do this masking operation, or we can do this particular the preprocessing..

Now, here, this shows a typical 3 cross 3 mask and these W values are nothing, but the coefficient of the mask, for example, W_1, W_2, W_3 up to say W_9. So, this is a 3 cross 3 mask so, there are 9 such W values and these are nothing, but the coefficients of this particular mask, ok.

Now, here, it shows a typical 3 cross 3 mask, now here we can see that here, I have put plus 8 so, this is plus 8 and here you can see I have put minus 1, minus 1, minus 1, minus 1, here minus 1, minus 1, minus 1, minus 1. So, if we just add all such minus 1 values. So, I will be getting 3 plus 2, that is, 5 plus 3 that is 8. So, I have got plus 8 minus 8, that is equal to 0.

So, the sum of all these particular coefficient values will be equal to 0. So, this mask coefficient values are selected in such a way, that the sum of these particular mask coefficient values becomes equal to 0. So, as I told that this is one typical 3 cross 3 mask, which is very frequently used for the preprocessing. Now, let us see, how to implement.

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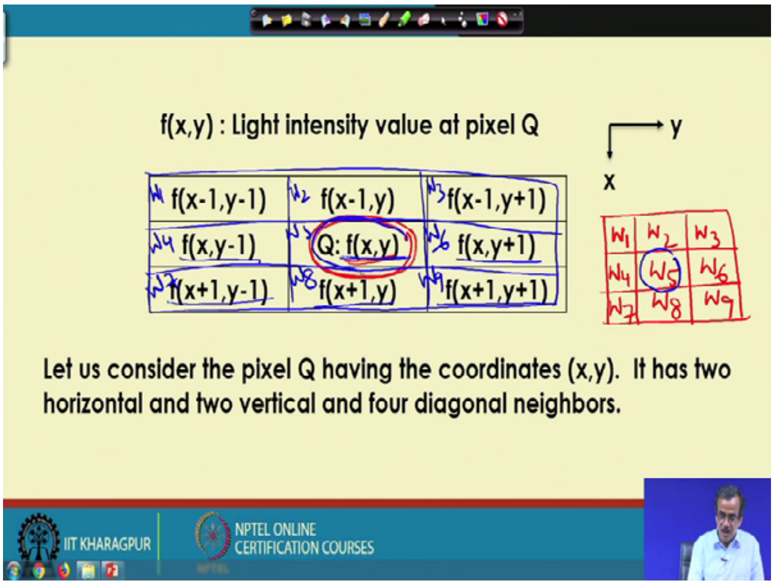

So, this is actually, say one image and these are actually the light intensity values at the different pixels, and our aim is to find out what should be the preprocessed value, corresponding to this particular f (X, Y). And, if I want to find out what should be the corresponding preprocessed value, for this particular f (X, Y), what we do is, we try to take the help of one template, or the mask and let be considered one 3 cross 3 mask, or 3 cross 3 templates. And, as I discuss, the coefficients are W_1; W_2, W_3 then comes W_4, W_5, W_6, then comes W_7, W_8, W_9. So, these are nothing, but the mask coefficients and how to find out the pre-processed value corresponding to this.

The method is very simple actually, what we do is, actually, what I do is so, this is actually its corresponding preprocessed value I will have to find out and, you concentrate on the mask centre, and supposing that this is the mask center. So, this particular template or the mask you bring it here and this particular mask centre is going to coincide with this particular pixel.

So, what I am going to do is, I am just going to put this particular mask here. So, as if I am just going to put the mask something like this and here, I am just going to write down all such mask coefficients: W_1, W_2, W_3, then comes your W_4, W_5, W_6 then W_7, W_8 and W_9 and after that actually what we do is.

So, we multiply this particular W_1 with f of (X-1, Y-1) plus W_2 multiplied by f of (X-1, Y) plus W_3 multiplied by so, this particular light intensity value plus W_4 multiplied by this light intensity value, plus W_5 multiplied by this plus W_6 multiplied by this, W_7 multiplied by this f, W_8 multiplied by this f, plus W_9 multiplied by this f, we sum them up.

And then we will be getting some numerical value and that particular numerical value is nothing, but the preprocessed value corresponding to this particular the light intensity value. So, this is the way actually, we can find out like, what should be the preprocessed value corresponding to that particular the pixel.

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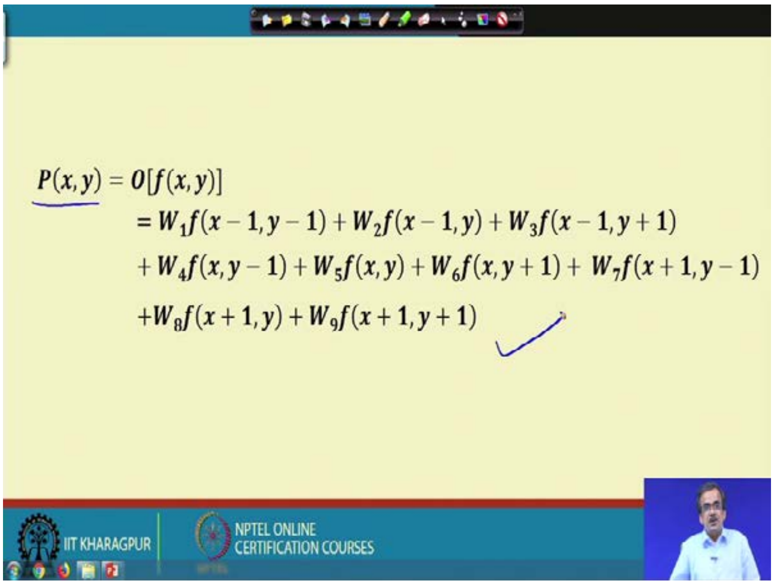

Now, the same thing, whatever I discussed the same thing I have just written it here. So, P (x, y) is nothing, but the operator O that is acting on f (x, y) and if we remember. So, W_1 multiplied by this f, W_2 multiplied by this f, so, whatever I discuss the same thing I have written it here. So, this is the way actually, we can find out the preprocessed value, corresponding to that particular the masking.

Now, here I am just going to take another very small example, like how to determine the preprocessed value corresponding to this particular the light intensity value. So, we have already seen, how to determine the preprocessed value here, but let us discuss, how to determine the preprocessed value corresponding to this particular the pixel whose light intensity value is nothing, but f of (X-1, Y-1).

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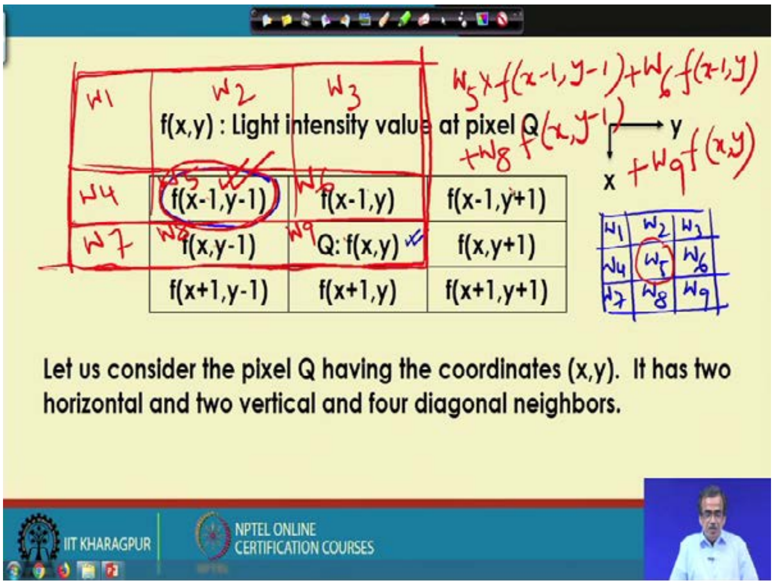

Now, as I told that we will have to take the help of the mask, that is nothing, but the 3 cross 3 matrix. So, these are W_1, W_2, W_3, W_4, W_5, W_6, W_7, W_8 and W_9. So, our aim is to determine actually what should be the preprocessed value corresponding to this particular pixel. So, what we do is, we concentrate on this particular the mask center.

So, once again this is the mask center so, this particular mask center is made coincident with this particular the pixel; that means, W_5 will come here and so, this will be the W_8 sort of thing. So, I will have to put the mask something like this. So, this is the way I can put this particular mask, so here, this particular W_5 will be here so, I will have to do something like this, ok.

And, now we can see that we have got so, this is the way actually we can do, so we put this particular mask here. So, this is the mask, which I am going to put. So, this corresponds to your W_1, this is your W_2, this is W_3, this is your W_4 and here you have got W_5 and we have got W_6 here, then comes your W_7 here, then comes W_8 here and this is your W_9 and our aim is to find out the pre-processed value corresponding to this.

Now, if this is the scenario, the contribution of these particular W_1, W_2, W_3, W_4, and W_7 will be equal to 0 here, ok. So, now, we will have to concentrate only on these particular 1, 2, 3, 4. So, only on these four, we will have to concentrate. Now, if we concentrate only on these particular the four, I will be able to find out the preprocessed value is nothing, but W_5 multiplied by f of (x-1, y-1).

This particular thing plus W_6 multiplied by f of (x-1, y) plus W_8 multiplied by f of (x, y-1) plus W_9 multiplied by f of (x, y). So, we can find out the pre-processed value corresponding to this particular pixel, the same procedure actually, I can follow at each of the pixels, just to find out the preprocessed value corresponding to that particular the pixel.

Thank you.

Robotics
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Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 35
Robot Vision (Contd.)

Now, I am going to solve one numerical example using the method of the masking.

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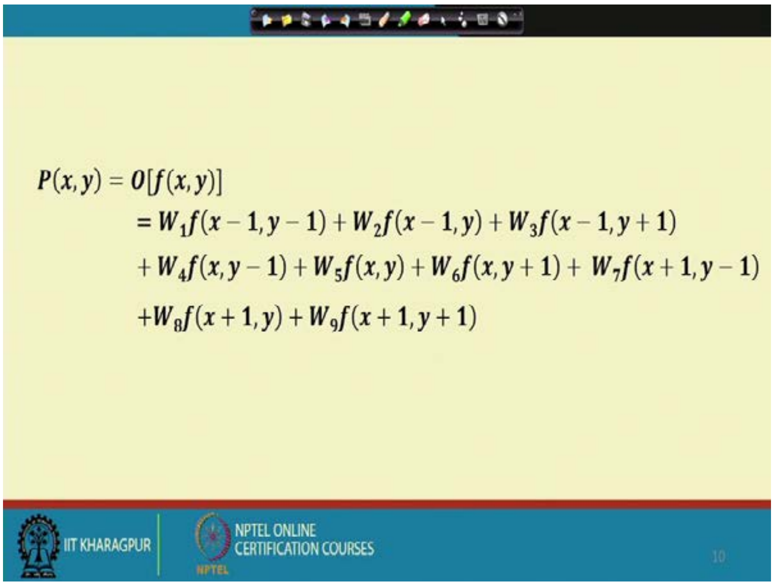

Now, let me take one example, one numerical example.

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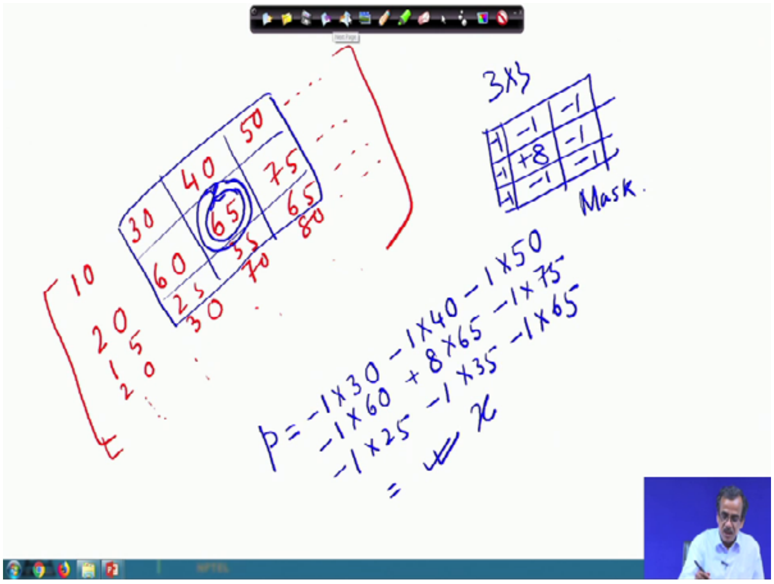

Supposing that we have got one the image and corresponding to this particular image, we have got some the light intensity values pixel-wise; let me take, at random, some example, say 10, 30, 40, 50 and there are some numerical values, which I am not considering 20, 60, 65, 75; there are some numerical values then comes 15, 25, 35, 65 and there are some numerical values. Then comes 20, then comes 30, then comes 70, then comes 80; there are some numerical values and here also, we have got a few other numerical values, ok.

Now, here my aim is to find out, what should be the preprocessed value so, corresponding to this particular any of this pixel? Now, let me try to concentrate that I am just going to find out the preprocessed value corresponding to this particular say 65, ok.

So, how to do it and supposing that I am just going to take the help of one mask like 3 cross 3 mask, the way we discussed the 3 cross 3 mask is something like this. So, here, we have got plus 8 minus 1 minus 1 minus 1. So, minus 1 minus 1 minus 1 minus 1 minus 1.

So, this is nothing, but a 3 cross 3 template or the mask. So, this is nothing, but a mask and our aim is to find out the preprocessed value corresponding to this. So, what I do is, the center of the mask is made coincident with this particular 65. So, I can draw this particular template or the mask here, ok.

Now, I just try to find out, what should be the preprocessed value. So, p the preprocessed value; so here, we can see that this is minus 1; so, if we just draw it here. So this particular template if we just draw it here. So, here we have got minus 1; so, minus 1 multiplied by 30, then comes minus 1 multiplied by 40, then comes minus 1 multiplied by 50, then comes your minus 1 multiplied by 60 plus 8 multiplied by 65 minus 1 multiplied by 75, then comes your minus 1 multiplied by 25, minus 1 multiplied by 35, minus 1 multiplied by 65.

Now, if I calculate; so, that will be the preprocessed value corresponding to this particular the pixel. So, in place of 65 supposing that I am getting x that integer value. So, I am just going to put this particular x in place of your 65. And, this particular the further processing will be done with the help of this particular the preprocessed value.

This is the way actually, we do the masking; now for each of this particular pixel, we will have to follow this particular the method of masking and purpose I have already told, the purpose is nothing, but to remove that particular noise from the image.

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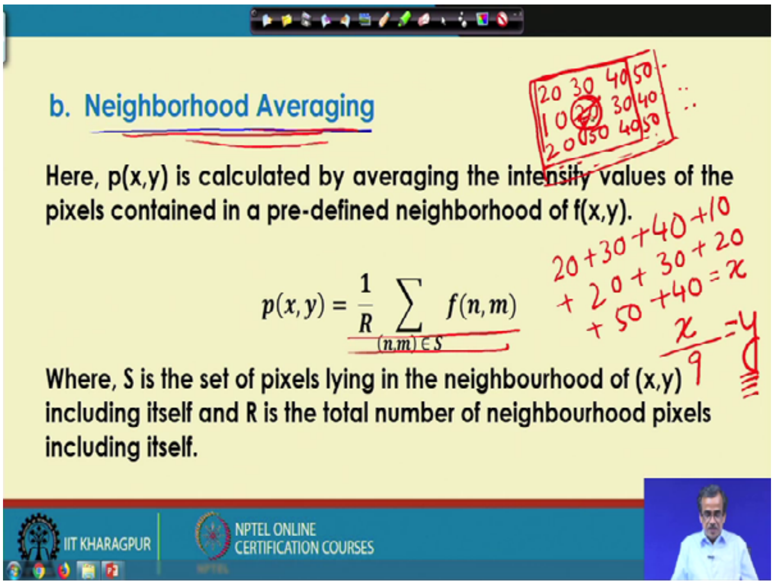

Now, there is another method; this method is also very popular for preprocessing, that is called the neighborhood averaging. Now, here actually what you do is, in neighborhood averaging; so, what you do is. So, we try to find out this neighborhood averaging actually, what do you do?.

So, we try to concentrate on a particular neighbor, we define a neighborhood and we try to find out the average. Now, let me take a very simple example supposing that, I have got one image sort of thing. Now, say this is something like this; this is actually the light intensity values at the different pixels; say let me consider 20 here, 30 here, 40 here, 50 here something like this and there are some numerical values 10, 20, 30, 40; there are some numerical values 20, 30, 40, 50 there are some numerical values and here also there are some numerical values.

Now, let us see how to find out, how to carry out this particular the neighborhood averaging. Now, before we carry out this neighborhood averaging what you do is; we define the neighborhood first. For example, if I concentrate on this, say I will have to find out the neighborhood averaging value corresponding to this particular 20.

So, I will have to define the neighborhood first; supposing that the neighborhood is nothing, but is 3 cross 3 neighborhood. So, if this is the 3 cross 3 neighborhood; so this is nothing, but the 3 cross 3 neighborhood ok. So, surrounding this, actually we have got the 3 cross 3 neighborhood and once I have got defined this particular neighborhood; I know the light intensity values at the different pixels.

So, what you do is, you sum them up all the light intensity values. So, that is 20 plus 30 plus 40 then comes your plus 10 plus 20 plus 30 plus 20 plus 50 plus 40. So, we try to find that and supposing that this is equal to x and how many entries are there? 1 2 3 4 5 6 7 8 9; so, x divided by 9. So, whatever value we get and its nearest integer will be nothing, but a number.

So, that is equal to say y and y is considered as the nearest integer. So, this particular y is going to replace that particular the 20; so, this is the method of neighborhood averaging. Now, mathematically, actually it can be expressed something like this, the sum of all the light intensity values contained in that particular neighborhood divided by the number of neighbors including that particular preprocessed value I am going to calculate, ok. So, that is why, I have divided by 9, but not 8.

So, this 1 by R multiplied by summation f (n, m) ok. So, this is nothing, but like how to determine the neighborhood, how to determine the average of this particular neighborhood. And, this particular average is going to replace that light intensity value at that particular the pixel; now, this is also a very popular method for preprocessing.

Now, this method is very simple ok. So, very easily, we can implement this particular the neighborhood averaging.

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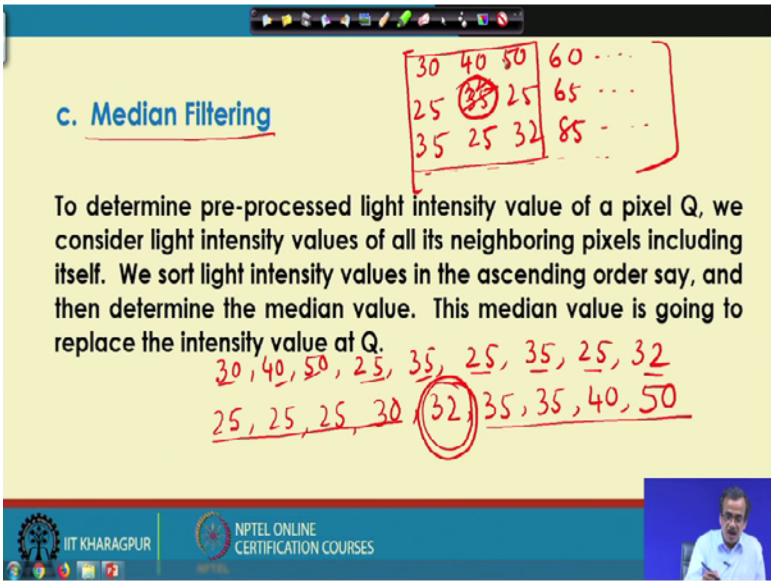

Now, I am just going to discuss another very popular the method of preprocessing and that is called the median filtering. Now, if you see this particular the method of median filtering this is very simple. Now, by median, we mean like supposing there are 9 values; so, what I am going to do is 9 means there are odd number of values.

So, what I will do is, I will leave the first 4 and the last 4 and I will try to find out, what is there at the middle; if there are odd numbers. And, if there are even numbers, supposing that 8; so, what I will do is. So, first 3 and the last 3 we are going to remove and at the middle, we have got 2 remaining because 3 plus 3; 6 plus 2 is 8. So, we try to find out the mean or the average of the middle two; so, this is the way actually we calculate the median.

Let me take a very simple example, the similar type of example; supposing that I have got an image, whose light intensity values are nothing, but say 30, 40, 50, 60, 25, 35, 25, 65 and there are some other values, here. Then, comes 35, 25, then comes your 32, 85; there are some other values and here also, there are some other values, ok.

Now, once again, we will have to define one the neighbor and supposing that I am just going to find out, what should be the preprocessed value corresponding to this particular

35. Now, if I want to find out, what should be the value corresponding to this particular 35? So, what you do is, we define the neighborhood first; so, this is actually the 3 cross 3 neighborhood. And, in the neighborhood, we have got all the light intensity values like 30, 40, 50, 25, 35, 25, 35, 25, 32, ok.

So, the light intensity values, let me write it here. So, we have got 30, then comes 40, then comes your 50, then comes 25, 35, then comes 25 then comes your 35, 25 and 32. So, there are 9 numbers, 9 values: 1, 2, 3, 4, 5, 6, 7, 8, 9; so, what you do is, we sort them in the ascending order, ok. Now, if I want to sort them in the ascending order. So, what I will have to do? I will have to find out the lowest value here, the lowest value is 25 and there are 3 such entries of 25.

So, let me write here 25, 25, 25 then comes your; so, 25; above 25 we have got 30 here. So, let me put 30 here then comes your 32 here, 32 here, then comes 35; 35 and here also we have got 35, then we have got 40 and after that we have got 40. So, all such 9 values these are sorted in the ascending order ok. So, from the lowest to the highest, there are 9 values.

So, there is odd number of values; so, what I do is. So the first 4 you neglect; so, the first 4 you neglect, the last 4 you neglect and whatever is there at the middle; so, that is 32 is the median value corresponding to these 9 values. So, this is the median, we are going to replace. So, this particular 35 by this particular 32; that is what you do in your median filtering ok.

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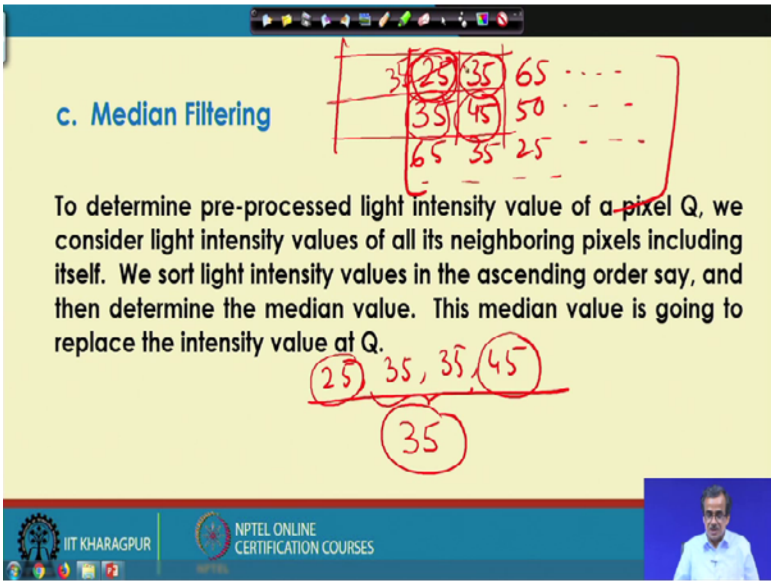

Now, let me take another; another example similar type of example, but slightly different, ok. Now, supposing that I have got, this type of numbers, say 25, 35, 65 and there are something, say I have got 35, 45 and 50 and something then, there are 65 35, 25 and something and something. So, this is nothing, but the image and supposing that I will have to find out, what should be the preprocessed value according to the median filtering corresponding to this particular 25?

So, what I will have to do is. So, 3 cross 3 neighborhood if you define. So, this is the way we can define for example, say. So, this is the way we can define; so, here we have get 3 and here also we can write 3 here. So, those things are missing. So, only thing we have got only these 4 numbers: 25, 35, 35 and 45, are you getting my point? And, other things are 0s, ok.

So, what will have to do it here? So, we will have to concentrate only on your. So, this particular the 4 values; that means, the values which you have here are 25, then comes 35, then comes 35, then comes 45 and they are in actually the ascending order. Now, here, there is an even number of numbers ok; so, what I will have to do is. So, this I will have to leave, this I will have to leave and as there are even numbers.

So, I will have to find out the mean or average of 35 and 35 and that is also equal to 35. So, this 35 is going to replace that particular the 25 as the preprocessed value and following the same method; so I can find out the preprocessed value corresponding to each of these pixels. So, this is the way actually, we can carry out the preprocessing. And, as I told the purpose is to remove the noise and to restore, if there is any such the lost information.

So, these are the methods of preprocessing, which are generally used very frequently and once I have done, this particular preprocessing. So, we will be getting the preprocessed data and on the computer screen actually we have got that particular matrix, the matrix of the light intensity values, ok; now we will have to find out the difference between your the object and the background.

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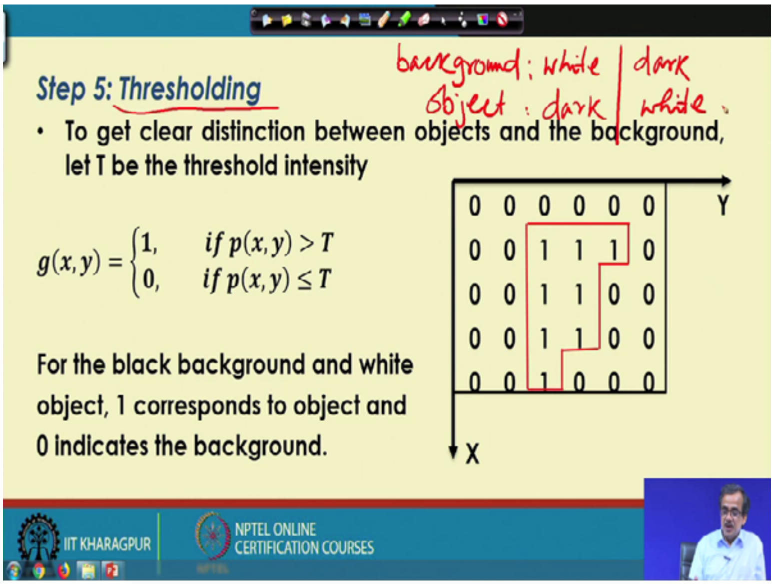

Now, as I told that here I am just going to concentrate only on the black and white picture sort of thing for simplicity ok. Now here we will have to take the help of one operator that is called the thresholding operator. So, using this particular thresholding operator; we can find out the difference between the object and this particular the background. Now, there could be two possibilities in black and white, say, the background and this object, the possibilities are the background could be white and the object could be the dark, ok.

And, there is another option, the background could be dark and this particular object could be the white, ok. So, there are two possibilities and both the possibilities can be solved very easily using the principle of thresholding. So, let us see, let us see the principle of this particular thresholding. So, the purpose is to find out the difference between the object and this particular the background.

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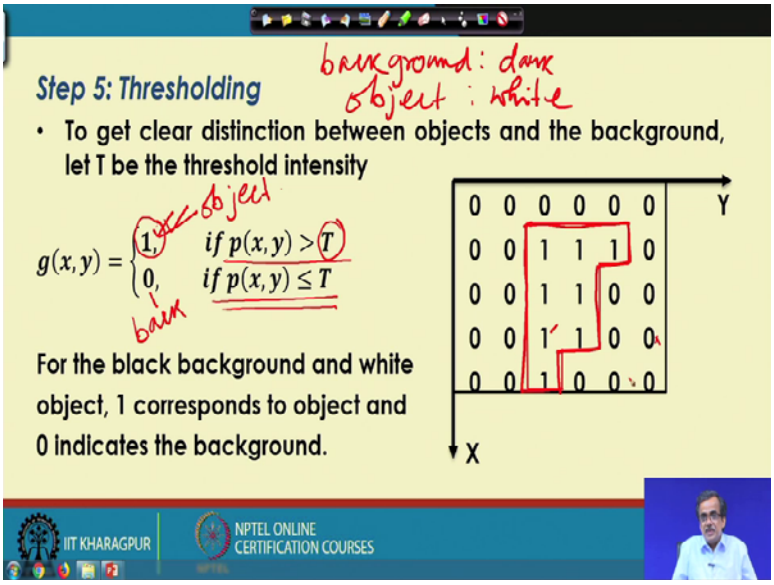

Now, let me consider a particular special case like the background, let me consider the dark background say the dark background. And, the object is nothing, but the white, let us see what happens. White object means what? The light intensity value will be more and dark background means, there the light intensity values will be less.

Now, here for this thresholding, we will have to define one threshold value of light intensity, that is denoted by T. Now, here I am writing after thresholding; supposing that I am getting say g(x, y). So, initially we had f (x, y) is the light intensity value, then we converted into p (x, y) that is the preprocessed value and now, I am just going for g (x, y) after the application of this particular thresholding. And, I am just going to consider white object and dark background; that means, on the object the light intensity value will be more.

Now, here I am just putting the condition that if this particular p (x, y) is found to be greater than the threshold value which is predefined by the user; then it will generate 1, and as I told that object is white. So, its light intensity value is more; that means, 1 is going to indicate the presence of that particular the object. And, on the other hand, if p (x, y) is found to be less than or equal to T, that is the threshold value. So, it is going to generate 0; so 0 means what? So, 0 means it is nothing, but the background and 1 means this is nothing, but the object.

Now, previously on the computer screen; so, this is the computer screen. So, we had the matrix of light intensity values, its preprocessed values. So, on the computer screen, I could have seen that particular matrix of the light intensity values. Now, if you use this thresholding; suddenly, we will find that on this particular computer screen, there will be a collection of 1s and 0s, ok. So, there will be a connection of 1s and 0s and as I told that 1 indicates that this is the object and 0 indicates that this is nothing, but the background.

Now, if I get here all such 1s I get; then what will happen is, so, I can just find out one boundary; so, this is nothing, but the boundary and this boundary will be the approximate boundary of this particular object on 2 D view. So, this is nothing, but an approximate the boundary for the object and 0 indicates that this is nothing, but the background.

So, on the computer screen, corresponding to that particular object, ok, you will be getting actually some sort of approximate picture of the objects something like this and it is in 2D, ok. So, this type of picture will be getting by using the operator like the thresholding, this is the purpose of the thresholding.

Now, the reverse is also possible for example, say if I consider your; the black object and the white background that is also possible. And, accordingly, I will have to actually change this part just to take care of that and that is also possible, ok, I can just consider the reverse also.

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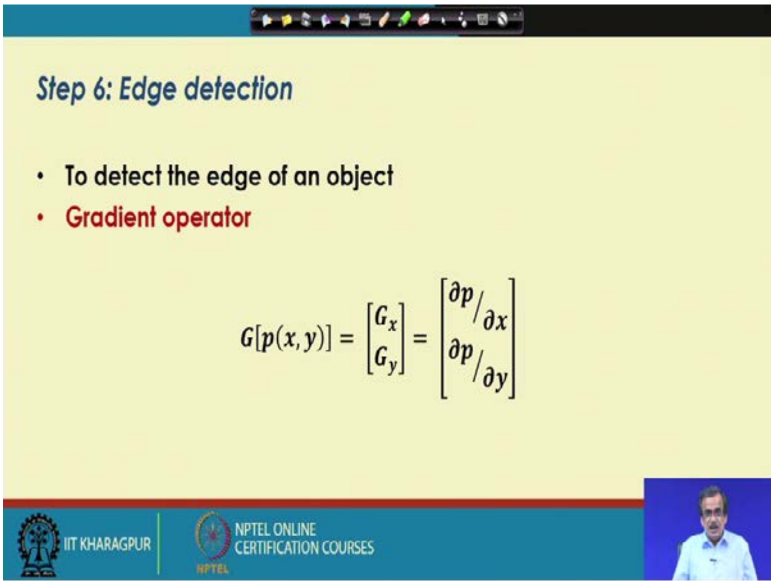

Now, once I have got this particular object, what is the next task? The next task is how to identify or how to actually detect that particular edge that is the edge between the object and this particular background. So, my aim is to detect this particular the edge of this object.

Now, let us see how to detect that particular edge? That is nothing, but the edge detection, that is, step 6. So, in step 6, we try to take the help of some sort of edge detection technique. Now, edge detection techniques are nothing, but the gradient operator; by gradient, we mean the rate of change. So, on this particular boundary, the rate of change will be very prominent and that is why, to detect the edge between the object and the background, we take the help of the gradient operator.

For example, this particular gradient operator is very popular just to find out the difference between the object and this particular the background; that means, to identify the edge. So, this gradient operator is working on, say, this particular the light intensity values and that is nothing, but G_x, G_y and G_x is nothing, but the partial derivative of p with respect to x and partial derivative of p with respect to y is nothing, but actually the G_y.

So, we will have to find out the partial derivative just to detect that particular the edge. Now, derivative is computationally expensive and all such things ultimately, we will have to write in the computer program and in the computer program, if you want to determine this type of derivative, it will be computationally very expensive and that is why, to carry out this particular, the derivative we take the help of some sort of templates and those templates are nothing, but so, this type of template.

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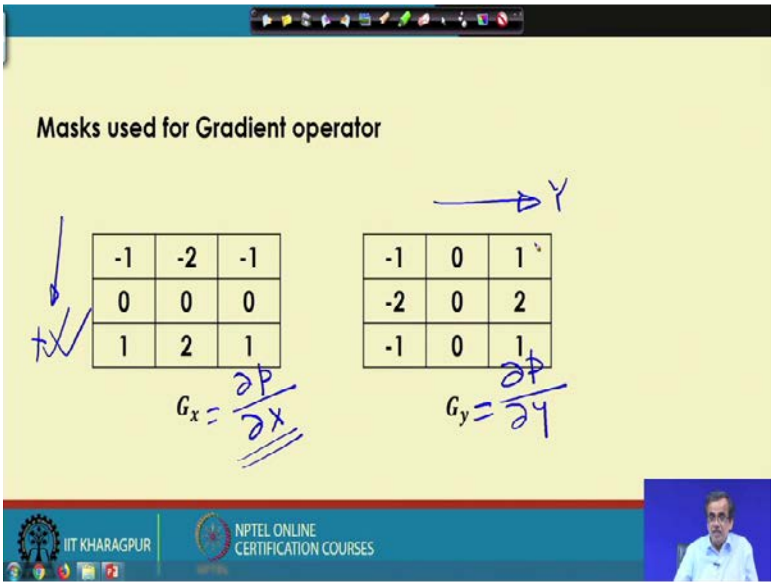

For example, say if we want to carry out the G_x that is nothing, but the partial derivative of p; the preprocessed light intensity with respect to x; so, this type of 3 cross 3 mass or the template, we use. And, here you can see that this is the positive x direction say.

Now, along this positive x direction, we can see that there is change in sign, for example, we have got minus 1 then 0 then plus 1. So, from minus 0 plus there is change in sign whereas, along this particular y direction; there is no change in sign. So, this is minus minus minus 0 0 0 plus plus plus ok. So, this is the way actually, we try to design this particular the mask to carry out this particular G_x.

Now, here once again, if we just add the mass coefficient. So, this will become equal to 0 for example, minus 1 minus 2 minus 1. So, we have got minus 4 plus 0 plus 4; so, this will become equal to 0. Now, similarly, we can also find out this particular G_y and that is nothing, but the partial derivative of p with respect to this y and once again, this is the positive y direction and along this particular y direction there is change in sign.

So, minus 1 0 1, minus 2 0 2, minus 1 0 1 and along this particular x direction; there is no change in sign. So, keeping that in mind actually, we try to design this type of mask or the template just to carry out this type of derivative; so, this is actually the derivative, which is very frequently used just to implement the gradient operator in the computer program.

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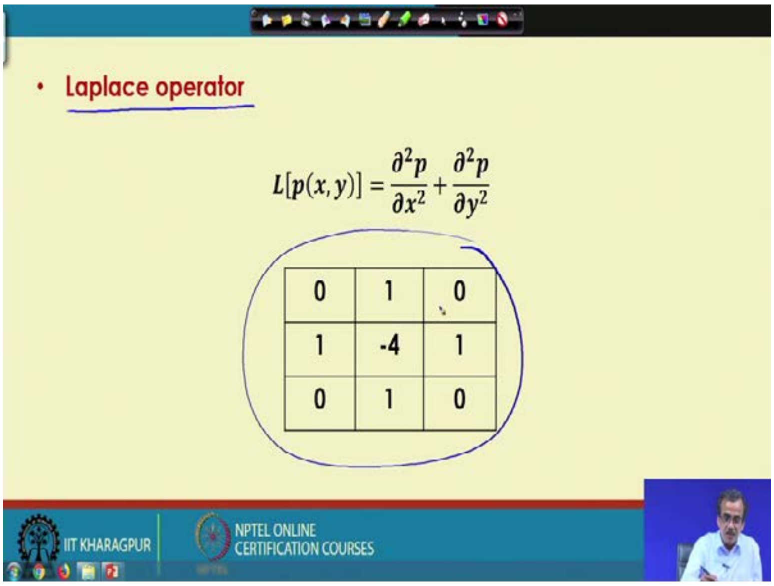

Now, there is another operator, which is also very frequently used that is called the Laplace operator. And, here actually, we generally go for the second order derivative. So, this L on p (x, y) is nothing, but $\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}$. And, once again as I mention that if I want to implement on the computer program that will become computationally very expensive and that is why to implement this Laplace operator in the computer program, what you can do is, like we will have to use some mask or the template and this is a very typical template used for Laplacian operator.

So, at the middle we have got minus 4 and on the horizontal side we have got plus 1 plus 1, vertical side plus 1 plus 1 and in there are 4 such neighboring 0s here. And, the sum of these particular coefficients values will be equal to 0. So, this is a very widely used operator, that is called the Laplacian operator just to find out or just to indicate or identify or detect the edge of an object from the background.

Now, this is the way actually, we can carry out some sort of edge detection and using this gradient operator particularly the Laplacian operator; we can find out the difference between the object and this particular background. And, we can, in fact, identify the edge between the background and this particular the object.

Thank you.

Robotics
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Lecture – 36
Robot Vision (Contd.)

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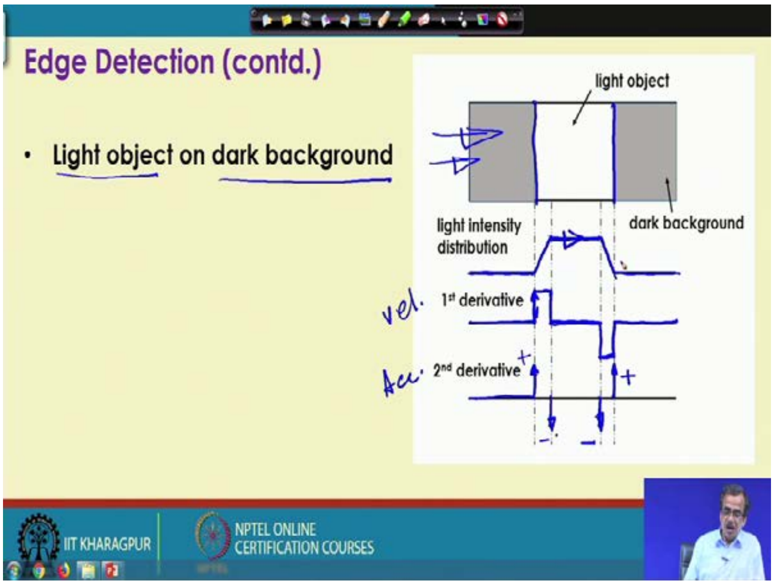

We have discussed that for edge detection, we can use the gradient operator like 1st order gradient or the 2nd order derivative. Now, here, let us see what is happening physically; if I consider the 1st order derivative or the 2nd order derivative during the edge detection.

Now, once again let me consider the light object; let me consider the light object and the dark background. So, this is actually the dark background, the black one is the dark background and here, I have got the light object. Now, if I just do the scanning along this particular direction and try to find out the light intensity value; so if I just do the scanning in this particular direction, I will be getting this type of light intensity distribution.

So, up to this black portion, the light intensity value will be small and in the light zone actually, the light intensity value will be more. And, once again, in the dark zone, the light intensity value will be less and in between actually here, this particular light intensity value is going to increase and it will reach the maximum. And, starting from here, the light intensity value is going to decrease and it is going to reach the minimum value. So, this is actually the distribution of light intensity, if I do the scanning in this particular direction, ok.

Now, the moment we are taking the help of gradient operator, that is 1st order derivative. So, what happens is, you are here, so, there is no change of light intensity value; that means, the rate of change is 0. So, this indicates your 0, then comes from here to here; so, from here to here there is an increment, there is increase in light intensity value and this particular rate is constant, this is the straight line. So, it has got the constant slope; so, this is actually the amount of the 1st derivative.

Then, comes your here; so, from here to here, there is no change in light intensity. So, once again, this will be the distribution for the 1st derivative; then from here to here, there is decrease in light intensity; that means, there is one the rate for decrement and that particular rate is constant. So, this is actually the constant rate of decrement and then, it is your 0; the rate of change is 0. So, this is actually what we mean by the 1st derivative of this particular the change in light intensity.

Now, if I consider the 2nd derivative; let us see what happens. So, up to this, there is no problem, so this will become 0 sort of thing, ok. So, here, there is suddenly a change; so, the rate of change. So, it is actually, if you see, this is nothing, but the positive sort of thing, ok. So, after that, this will remain same, then it is going to be reduced. So, it is going to be reduced; so, I will be getting some sort of negative sign here, ok. Then, from here to here; the rate is 0; so, there is no change, now once again, from here to here, there is further decrease. So, here, there will be an arrow in the negative and here, suddenly there is an increase. So, this will be the positive, that is plus, ok.

So, this is what is happening in the 2nd derivative. Now, if I just compare this particular distribution of light intensity with the displacement. So, this is nothing, but the velocity and this is nothing, but the acceleration sort of thing, ok. This is actually, what is happening, the moment we are using the gradient operator like; 1st order derivative or 2nd order derivative as a tool for the edge detection.

So, this particular derivative is going to detect this particular edge. So, this edge will be able to detect between this light object and the dark background with the help of this gradient operator. So, this is the way actually these particular gradient operators are working just to detect that particular the edge.

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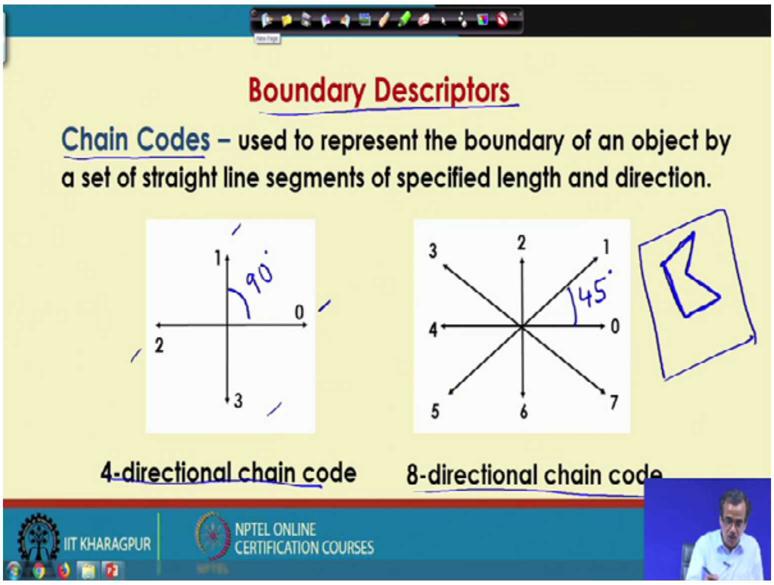

Now, I am just going to discuss the concept of the boundary descriptor. Now, before we start discussing on this particular descriptor, let us try to understand the reason behind going for this boundary descriptor.

Now, these boundary descriptors are used just to represent the boundary of this particular object. Supposing that I have got say on the white background, say this is the background and on this particular background, I have got one object something like this.

So, if I have got this particular object, now the boundary of this particular object, I will have to represent for further processing. Now, how to represent this particular boundary? Now, to represent the boundary, actually we take the help of the boundary descriptor and if you see the literature, we have got a few boundary descriptors. So, here I am just going to discuss two boundary descriptors, in detail, and these are very frequently used.

The first one that is called the chain code; now, here the boundary of the object is represented with the help of some straight line segments of pre-specified length and direction. So, what we do is, we generally take the help of either the 4 directional chain code or 8 directional chain code. Now, let us try to see, what is there in 4 directional chain code; it is very simple, it shows only 4 directions denoted by 0, 1, 2 and 3 and they are 90 degree apart.

So, this is the 4 directional chain code and if I take the 8 directional chain code starting from 0, 1, 2; up to say 7. So, this is called the 8 directional chain code and here, the included angle is 45 degree and here, the included angle is your 90 degree.

Now, to represent the boundary of the object; if I take the help of say 8 directional chain code; so, there is a possibility that we will be able to represent the boundary more accurately compared to the 4 directional chain code. Now, let us see, how to use that the 4 directional chain code or 8 directional chain code to represent the boundary.

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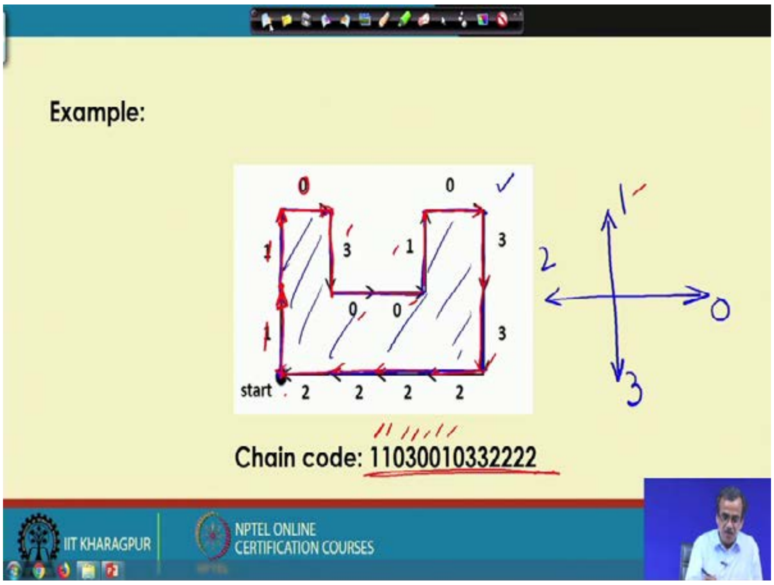

Now, here, I am just going to use one 4 directional chain code just to represent this type of the object. Now, supposing that this particular thing, this is nothing, but say white, this is the white background. So, this is the white background and here, we have got one object, whose boundary is something like this.

Now, this particular boundary, I will have to represent with the help of some numbers or mathematically, so that we can do some sort of processing; the further processing, ok. Now to represent this particular boundary in the computer program; we will have to use the set of numbers, because computer program does not know anything except this particular the numbers, ok.

Now, here supposing that the object, the boundary of the object is something like this; so, this is a very simple example. Suppose that this is the object, the boundary of this particular object and this particular object is, say dark object. So, this is the dark object on say light background, ok.

Now, how to represent this particular the boundary or the object? As I told, we are going to take the help of 4 directional chain code. So, this is 0; this is 1, this is 2 and this is your 3 and let us start from any point; let me start from here. Now, if I just start from here; if I just start from here. So, this is the starting point now from here; so I will have to move along the boundary; now this is the direction of 1. So from here to here; so I move along the direction of 1; by how much amount? By some pre specified the fixed length ok; so, I will be here.

Now, from here; so, I will have to reach this particular point, once again, I will move along this particular 1. So, I will be writing 1 here; then from here; so I will be moving towards this side, ok. This is the direction of 0, so here I will write 0 and from here, I will move along this particular direction this is the direction of 3; so, I will have to write 3, here.

Then, from here, so this is once again the direction of 0; once again second from here to here the direction of 0, then from here to here; so this is nothing, but the direction of 1 ok, this is the direction of 0 this is the direction of your 3. So, this particular direction this is the direction of 3 then from here, I am just going to move towards that towards 2. So, this is the direction of 2, 2, 2 and I am just going to reach this starting point.

Now, to represent this particular boundary; what we do is, we just go on writing all such numerical values in this particular sequence. For example, we start from 1. So, I have got a 1 here next is 1, next is 0, then comes here 3 then 0 0 0 0 then 1, 1 and we follow that and then we will be coming back to this and we have got 2 all such 2s here.

So, this particular numerical value is going to represent this particular boundary of the object, ok. And, in a computer program, this particular object will be represented like this and then, we can do the further processing with the help of the computer programming. So, this is the way actually, we can represent the boundary with the help of some boundary descriptor.

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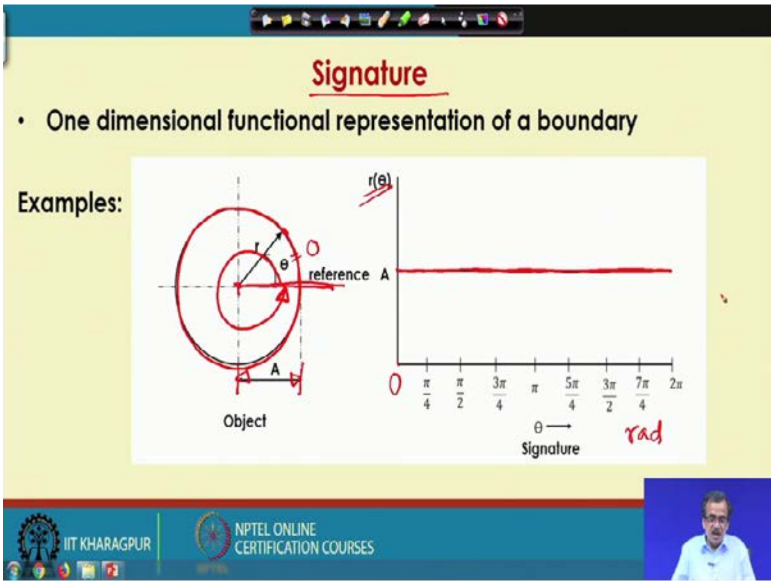

Now, if you see the literature, we have got some other types of boundary descriptor also and that is known as the signature. Now, signature is actually nothing, but the functional representation or the mathematical representation of boundary of the object.

Now, let me take a very simple example of say circular object; now if I take a circular object, for example, say this type of circular object I have got and I want to represent its boundary. Now, how to represent? So, what we do is; we try to find out its center. So, this is the center and this is the reference line with respect to which, I am just going to represent the fixed reference, ok.

And, r is actually the distance between the center and the point lying on this particular boundary of the object. So, this r is the distance between the center and the boundary point and I am just going to start from this particular reference, where θ is made equal to 0. So, corresponding to this particular reference θ is made equal to 0 ok. So, θ is made equal to 0; so, this is in radian.

So, all such things are in say radian this corresponds to your say 0; this is $\frac{\pi}{4}$ means your 45 degree, $\frac{\pi}{2}$ with respect to these; so this is 90 degree; then $\frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}, 2\pi$. So, I will be coming back, ok.

So, 360 degree rotation and here, so as I am moving along this particular θ ok; so, the distance between the center point and the point on the boundary is kept equal to r; A is nothing, but the radius of this particular the circle. So, A is the radius and the distance between center and the boundary that is nothing but r.

So, if I plot r as a function of θ, that is nothing, but is your $r(\theta)$. So, there is a possibility that I will be getting one straight line because starting from here up to here. So, the value of r that will remain same as your equal to A that is nothing, but the radius. Now, this particular straight line can represent the circular object, mathematically, ok.

So, this is nothing, but like say r is equal to A, that type of equation, ok. So, this particular equation is going to represent this circular object lying on the computer screen. Now, this particular circular object with the help of this equation we can represent and then, we can do some sort of the further processing; this is actually the method of signature.

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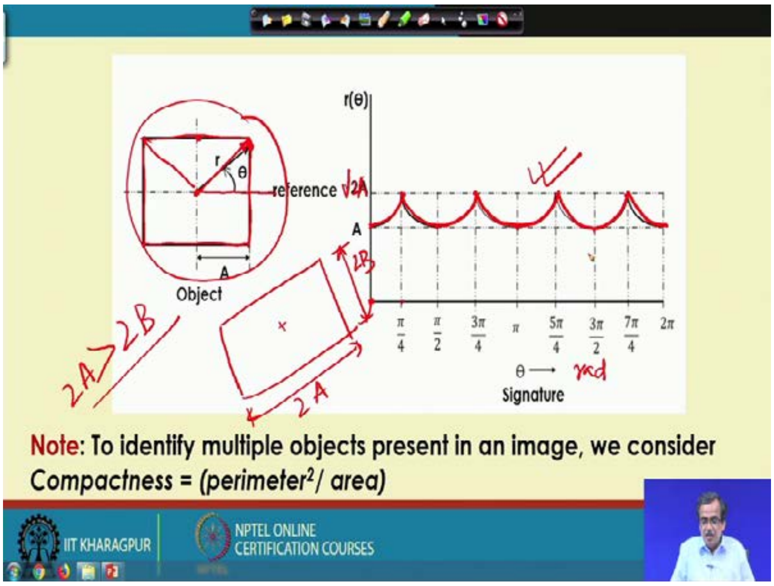

Now, let me take another example just to make it more clear; so, I am just going to take the help of another example ok. Now here supposing that I have got one object and this is nothing, but a square object. So, if I take a square object like this; so this is nothing, but the square object and if I take this type of square object and A; 2 A is actually the dimension of this particular side and that particular side.

So, I can find out the distance between this particular center and the point which is there on this particular boundary and that is denoted by r and this r as a function of θ; so, I can plot. So, this is θ in radian and θ corresponds to 0. So, this is the reference; so I am here, so if I am here then this $r(\theta)$ is nothing, but this is equal to A.

So, corresponding to this particular θ equals to 0 ok. So, r is nothing, but A; then corresponding to θ equals to 45 degree. So, I will be getting; so this is actually the distance between the center and the boundary and that is nothing, but is your $\sqrt{2}A$; so, this is your $\sqrt{2}A$, ok.

Similarly, at $\frac{\pi}{2}$; so, at $\frac{\pi}{2}$; that means, I am here. So, once again r is equal to A then comes $\frac{3\pi}{4}$; that means, I am here. So, once again it is $\sqrt{2}A$, then corresponding to π. So, this will be you're a, then once again here it is $\sqrt{2}A, A, \sqrt{2}A, A$ and so on, ok. And, this particular distribution will be non-linear distribution, so there is a possibility, you will be getting this type of non-linear distribution of $r(\theta)$ with respect to your θ, ok. This type of distribution will be getting for this particular corresponding to this type of square object.

Now, in place of square like if I just take the rectangular shape for example, this type of rectangular shape. So, once again I will be able to find out, for example, this side is say 2A and this is your 2B and supposing that 2A is greater than say 2B or A is greater than B and for corresponding to this, we can also find out another signature.

So, corresponding to this particular square object, this is the signature, which we are getting and once you are getting this type of plot, now it can be expressed mathematically. And, if I can express mathematically, then further processing becomes easier. Now, I am just going to discuss like how to identify the multiple objects which are present in one photograph or in one image.

Now, if I just take; let me take a very simple example, then after doing all such calculations, supposing that I am just going to get one scenario, for example, say this is the background.

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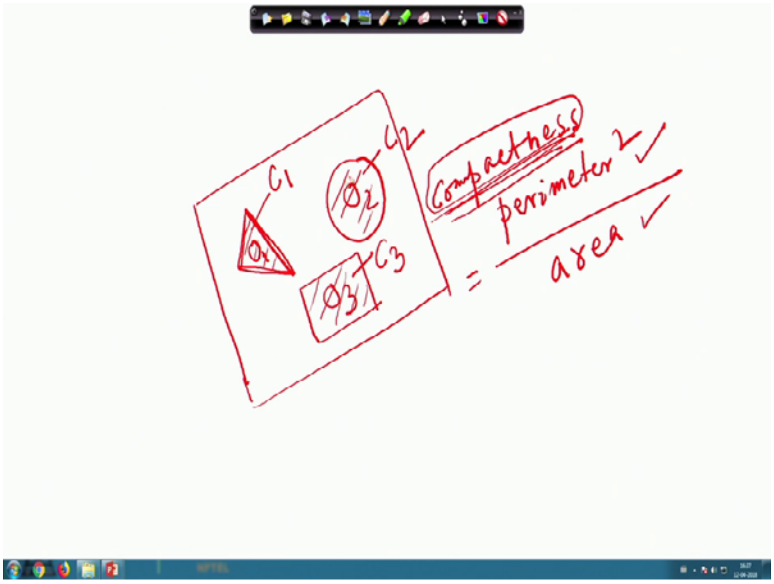

And, on this particular background; supposing that I have got one object like this, I have got another object like this, I have got another object like this, ok.

So, this is object 1, this is object 2 and this is object 3 and I have taken the photograph and the way I explained; I carried out that image analysis and ultimately on the computer screen supposing that I am getting so this type of image. For example, this is nothing, but the object it is a black object and white background sort of thing. So, I am getting say this is the black object and a white background; so, I am getting, so 3 objects on this particular the computer screen.

Now, how can I identify that this is object 1, this is object 2, this is object 3? Now to identify that, what we do is, for example, with the help of our eyes, whenever we see the picture of the environment or the surrounding, we very easily can identify that this is a chair, this is a table, this is a human being and so on. So, immediately within the fraction of a second, there is a lot of processing in the brain and due to this particular processing; we are able to identify these particular objects.

Now, how can computer or how can one robot identify that this is object 1, this is object 2 and this is object 3? Now, the method I have already explained and this type of objects we are getting. Now, actually what we do is, we try to calculate one parameter that is called compactness, now this particular compactness is nothing, but perimeter square divided by the area.

So, for this particular object; we try to find out perimeter square by area, that is nothing, but the compactness. For example, if I have got a chair, if I have got a table, if I have got a human being. So, in our brain, actually this compactness, this particular information is already stored. And, that is why, very quickly within a fraction of second, we can identify that this is a chair, this is the table and so on, ok. So, all such information has been stored in our brain.

Now, here actually for this artificial image processing or the artificial computer vision or the robot vision; what we do is, for each of these particular objects; we predetermine what are their compactness values? Now, on the screen, we are getting the background and the object. So, approximately I can find out what is the perimeter of this, what is the approximate area, I can find out perimeter square by area.

So, I can find out the compactness; supposing that for this particular object 1, the compactness is C_1, object 2 the compactness is C_2 and object 3 it is your C_3. So, this we can calculate from this particular computer screen and we try to match with the known values of compactness of object 1, object 2, and object 3. And, then, we try to recognize and we interpret, we identify that this is object 1, this is object 2, this is object 3.

So, this is the way actually one computer or a robot can identify, interpret the different objects. Now, if the robot wants to do some sort of manipulation task; it will have to identify, it will have to interpret the objects, ok. And, the way we human-beings carry out in our vision system; exactly in the same way, we try to copy in the artificial way, in computer vision or the robot vision just to collect information of this particular environment.

Now, once again, if I just summarize a little bit, this computer vision or the robot vision what they do? The first thing is do is, we try to capture the image. So, we capture image; so, image capturing in the first stage.

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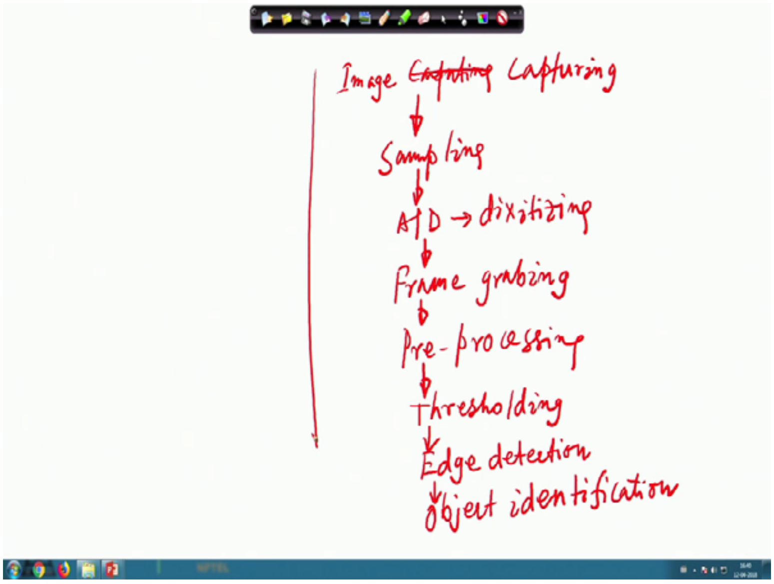

So, image capturing with the help of camera. So, we try to capture; we try to capture this particular image with the help of camera and once I have got this particular image.

So, what we do is; we do some sort of sampling and for this particular sampling, we take the electron beam scanner, ok. And, we generally go for analog to digital conversion and that is nothing, but the digitizing. So, we generally go for the digitizing and once you have done it; we go for the frame grabbing. So, we go for the frame grabbing and once I have got this; we go for the preprocessing; preprocessing.

And, once the data have been preprocessed, then we go for some sort of thresholding. Now, once you have done this thresholding, then we go for some sort of edge detection, and after this edges have been detected; we generally go for object identification.

So, object identification; so these are the actually the steps for the computer vision or the robot vision; exactly the same thing we do; we, human-beings, do. And, in computer vision or the robot vision, we try to copy everything in the artificial way, so that we can collect the information of the environment. The robot can collect the information of the environment with the help of camera and we will have to make this particular process very fast, so that within a fraction of second, we get the information of this particular environment.

Thank you.

Robotics
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Lecture – 37
Robot Motion Planning

So, our aim is to design and develop intelligent and autonomous robot. Now, we have seen how to collect the information of the environment with the help of sensors, with the help of cameras.

Now, this particular camera could be either the on board camera or the overhead camera. There could be multiple sensors, there could be a combination of sensors as well as camera and we collect information of the environment. Once we have collected the information of the environment; now an intelligent robot should be able to take the decision as the situation demands. So, how to take the decision; how can a robot take decision, that I am going to discuss.

So, we are going to start with a new topic that is called the topic 8, Robot Motion Planning. So, in motion planning actually what we do is, we try to plan the motion or try to find out the course of action while moving from an initial position to the final position.

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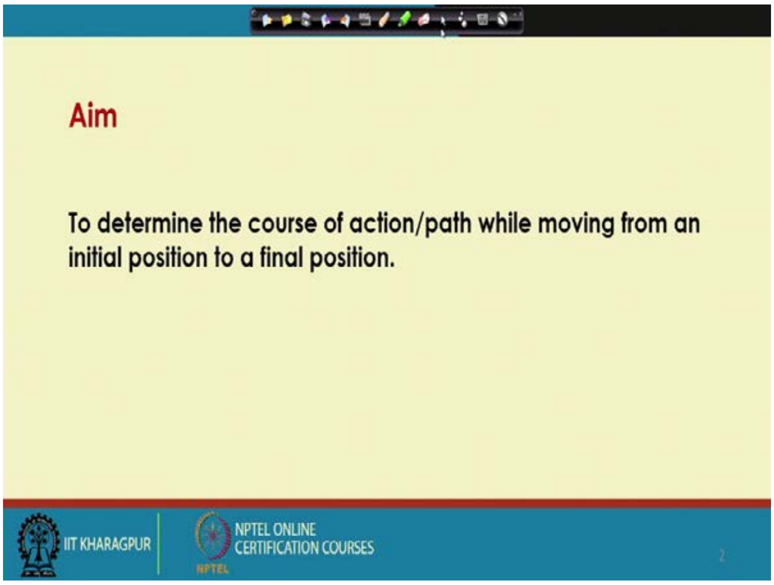

Now, let us concentrate, let me consider that this is nothing, but the tip of the manipulator. So, this is the initial position of the robot and the final position is here. So, starting from here, it is going to reach this particular the final point through a number of intermediate points and might be there could be a few obstacles sort of thing.

So, it will have to avoid collision with the obstacle; so, how to determine the course of action or the path that is the collision-free path that is the task of a robot; that means, to perform that particular task, the robot should have a proper motion planner, the path planner, ok. Now, here I am just going to discuss like how to design and develop a suitable path planner or the motion planner for this particular the intelligent and autonomous robot.

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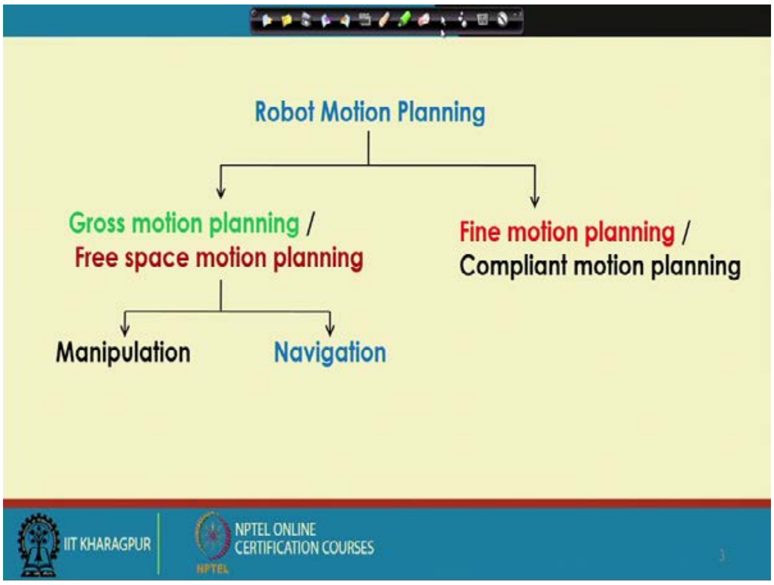

Now, if you see the robot motion planning. So, this robot motion planning is broadly classified into two subgroups.

So, these are broadly classified into two subgroups; so, these are broadly classified into two subgroups. Now, one is known as the gross motion planning or the free space motion planning. And, another is the fine motion planning or the compliant motion planning. Now, let me take one example, let me take one example just to find out the difference between this gross motion planning or the free space motion planning and the fine motion planning.

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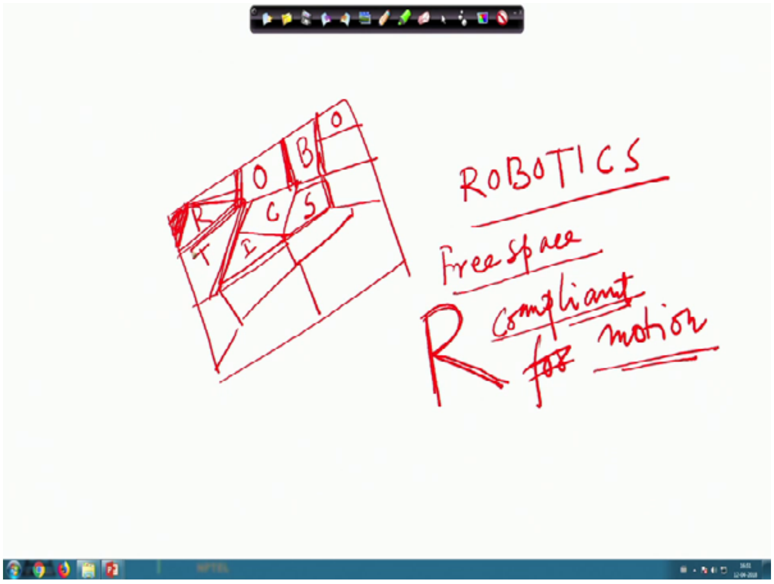

So, let us take one example supposing that say I have got one board something like this. And on this particular board say I will have to write say one word say I am just going to write a word say robot I am just going to write.

Now, this is the black board on which I am just going to write with the help of a marker or a chalk. Now, if you just give the same task to a robot to an intelligent robot; that intelligent robot will first try to find out the free space on this particular board, where it can write that particular the word. For example, say the board could be something like this; this is here, this is not the free zone, so this is not the free zone. So, this is not the free zone ok; so, the board could be something like this.

So, I am just going to show the free zone and the dark zone sort of thing. So, this is actually the structure; so something is written here and this part is not cleaned of this particular board. So, I cannot write anything on this part, where there is something written. So, how to write the word: robot or the robotics? Now, if you give this particular task to the robot; the robot will first try to find out, where is the free space, where I can write down?

So, this particular the word that is the robotics. So, all such letters, I will have to write. So, first thing it will do is, it will try to find out the free space. And, once it has got that particular free space, now it is going to write down the robotics, r will be written here. So, R O B O T I C S; so, the robot is going to write down ROBOTICS on the board, ok.

Now, let me repeat the first thing, you will have to find out the free space and once you have got the free space; now you will have to write this particular word. And, these letters you will have to write down and writing this particular letter on the board is not so easy; particularly for the robot.

Because, whenever I am writing this particular the letter, this particular marker is in touch with the board and there will be compliant motion. So, this particular marker is in touch with the compliant motion; marker is in touch with the board there and I will have to put some force; while writing, some amount of force is to be put that is called the compliant motion. So, here there are two types planning: one is called the free space planning, another is called the compliant motion planning.

Now, compliant motion planning is, if you want to write down, then how to put force, how to manipulate? And, while writing, I am just gripping that particular marker with the help of my finger, then I am doing some sort of manipulation, so that I can write down R O and all such things.

So, I have got a planning to write R; I have got a planning to write O; I have got another planning, to write A; I have got another planning another sequence. And, those things starting from our childhood we learn, we learn through a number of iterations through a lot of practice; that is called the compliant motion planning.

And, free-space motion planning; the purpose of free-space motion planning is to find out the feasible and the infeasible zone. For example, say this is an infeasible zone, but this part is a feasible zone, where I can write down one letter. So, the purpose of free space planning is to determine the feasible space an infeasible space, but the purpose of compliant motion planning is to write that particular the letter.

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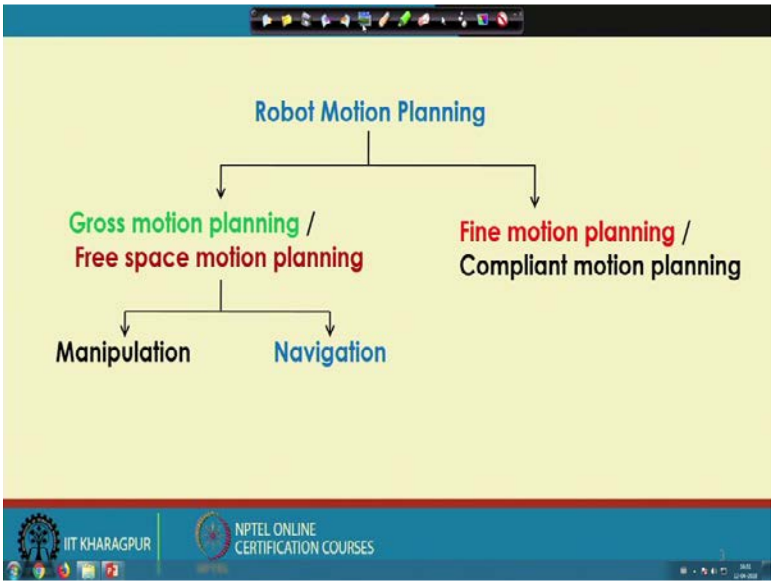

So, this is actually a bit difficult this compliant motion planning or this particular fine motion planning is bit difficult. And, this gross motion planning or the free space motion planning is easy I should say and here, in this particular course, I will be concentrating only on this particular; the gross motion planning or the free space motion planning. But, I will not be discussing the fine motion planning or the compliant motion planning because this has been kept actually beyond the scope of this particular course. So, I will be concentrating on this particular the gross motion planning or the free space motion planning.

Now, this gross motion planning or the free space motion planning can be once again subdivided into two parts: one is called the manipulation problem, another is called the navigation problem. The moment actually I am just going to take the help of one serial manipulator, just to write something on the board that is called the manipulation task or the moment I am just writing something on this particular board, ok.

So, that is nothing, but the manipulation task and the moment that moving robot or the mobile robot is actually working that is nothing, but the navigation task. In fact, we are planning to give some practical examples of this manipulation and navigation in this particular course, might be at the latter part.

So, we are going to concentrate on this particular manipulation and navigation; that means, if I just want to put it in another way; that a serial manipulator or a parallel manipulator solves the manipulation problem. On the other hand, a mobile robot could be a wheel robot or a multi-legged robot or a tracked vehicle tackles the navigation problem. And, let us see, how to proceed further with the different types of the motion.

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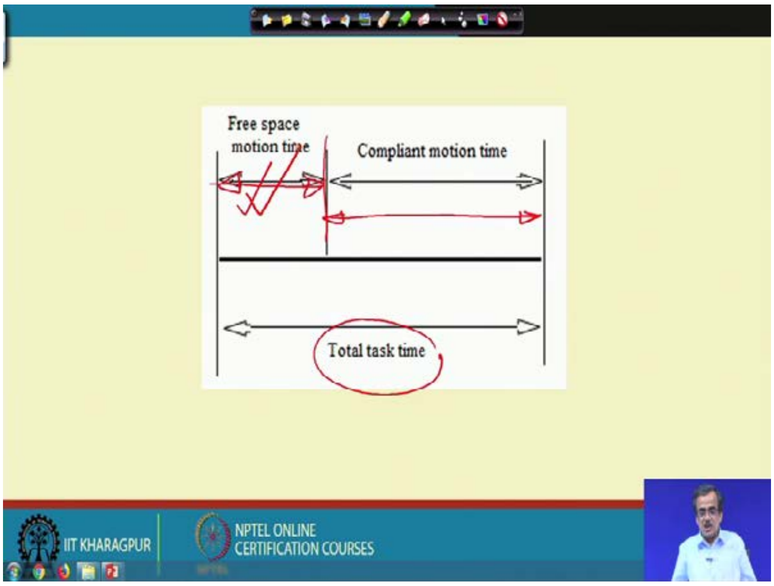

Now, this particular, it shows this sketch shows actually, if this is the total time for planning; now the free space motion planning takes only a small part, small duration. On the other hand, the compliant motion planning takes the larger duration and the total time is nothing, but is the total time for the motion planning or total task time. So, the total task time if I divide the free space motion time is much smaller compared to the compliant motion time, but as I told, I am just going to concentrate on this course only, on the free-space motion planning.

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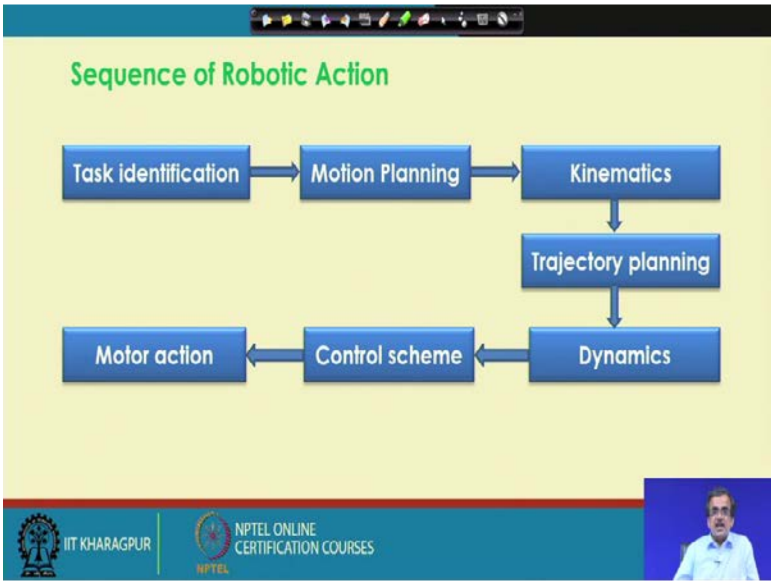

Now, this shows actually the sequence of robotic action and you will see that all such things, all such modules, I have already discussed and now, I am discussing motion planning. So, if I complete this discussion on motion planning, you will see that all such modules of robotics have been touched. For example, say if I just want to solve one robotic task with the help of a robot; the first thing is the task identification.

So, you will have to identify the task; the task which is going to be tackled or solved with the help of a robot. Then, we go for the motion planning, which I am discussing now and once that particular the course of action has been planned, we go for the kinematic analysis that I have already done, already discussed.

Then, we go for the trajectory planning before the dynamics. So, dynamics also I have discussed; the control scheme also, I have discussed because you will have to realize that particular the torque with the help of a motor with a suitable controller. And, once those things are ready; now, we are in a position to generate that particular the motion.

And, this is actually, in short, the necessary modules of robotics; and as I told that in this course, I am just going to touch the fundamentals of all the modules, ok. So, let us try to concentrate more on this particular motion planning, now.

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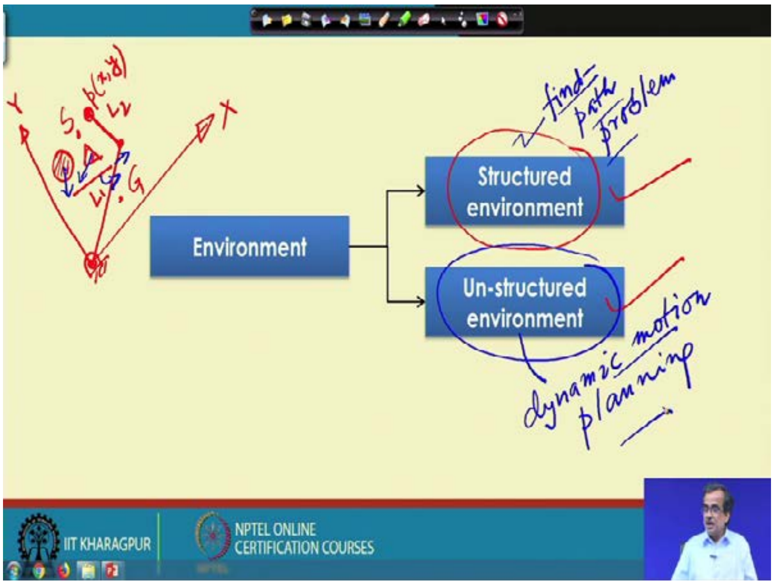

Now, if you see the environment; the environment could be either a structured environment, that is the known environment or it could be unstructured. Now, if the complete information of the environment is known beforehand, that is called the structured environment.

For example, say I am just going to solve a motion planning problem like this, where the environment is known. Let me take a very simple example, supposing that say I have got say X and Y in Cartesian coordinate system and I have got a robot. Say, if the robot having say 2 degrees of freedom, very simple.

So, this is L_1, this is L_2, ok; so, this is the length of the first link , length of the second link and this is the tip of the manipulator, whose coordinates are X and Y. Supposing that I am just going to give a task that you start from here, that is point S and reach the goal, that is point G, ok. Now, the tip of the manipulator is going to start from here and it is going to reach the goal.

And, supposing that I am just going to put one condition or the constraint that the tip of the manipulator should not collide with any of the obstacle; supposing that I have got one triangular obstacle, I have got one circular obstacle, I have got one line obstacle and these are all 2 D stationary or the fixed obstacles.

Now, here the environment is known; this is a structured environment. So, we know this particular the static obstacles, we know their location and I know my problem that the tip of the manipulator should start from here and it will reach this particular the goal, ok. And, this type of environment is known as the structured environment.

Now, if I just modify a little bit, for example, say if I add, if I just consider that these obstacles are moving. For example, say this obstacle is moving in this particular direction with some speed. So, this is moving in this particular direction with some speed so, this is moving in this particular direction or say this particular direction with some speed, ok.

Now, the problem becomes difficult and the position of the obstacles are going to vary with time and that particular problem will become a problem of motion planning in the presence of moving obstacles. So, the path planning or the motion planning in the presence of the structured environment is called the find-path problem.

And, the motion planning in the presence of unstructured environment, this is known as your dynamic motion planning problem; dynamic motion planning problem or the motion planning among dynamic obstacle or dynamic environment. So, I am just going to discuss like how to tackle the problem, that is the find-path problem and this type of dynamic motion planning problem. And, I am just going to discuss the working principle of a few tools.

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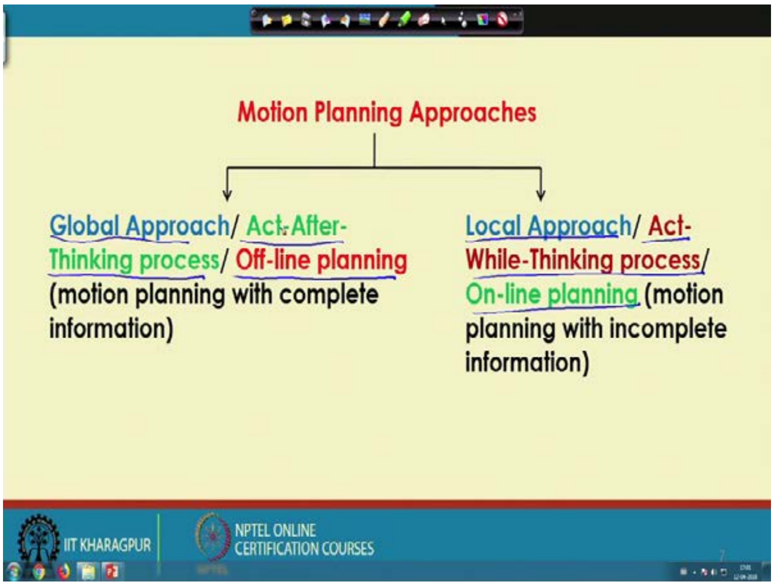

The tools for motion planning and with the help of some example, I am just going to explain. Now, here the motion planning approaches if you see, those are broadly classified into two subgroups. Now, one is called actually the global approach or this is also known as act-after-thinking process or this is known as the offline planning.

And, we have got another that is called the local approach, act-while-thinking process or the online planning. Now, if the environment is known; if the structured environment we have, then we can go for some sort of global planning, global approach or offline planning, ok.

But, supposing that I have got some sort of the moving obstacles in the environment; the environment is dynamic. So, here, this environment is un-structured environment; so we will have to go for the local approach or act-while-thinking process or online planning.

Now, let us try to explain the principle of this particular both global approach and the local approach. Now, let us start with this particular the motion planning scheme.

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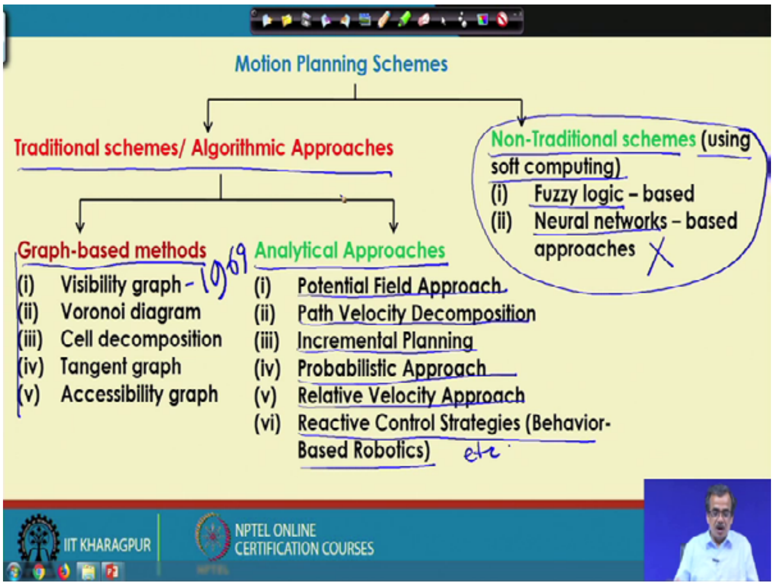

Now, if you see the motion planning schemes; these are broadly classified are into two groups: one is called the traditional schemes or the algorithmic approaches. And, we have got the non-traditional schemes using the principle of soft computing.

Now, here actually the traditional schemes are known also known as algorithmic approaches, these are once again classified into two sub-groups. One is called the graph-based techniques and we have got the analytical approaches. Now, if you see, if we compare this graph-based methods and the analytical approaches; the graph-based method actually it was proposed first. For example, the visibility graph that was proposed first in the year 1969 by Nilsson, then we have got the Voronoi diagram, cell decomposition, tangent graph, accessibility graph; so, these are all graph-based methods.

On the other hand, actually we have got some analytical approaches. For example, say we have got the potential field method, we have got the path velocity decomposition, then we have got the incremental planning, probabilistic approach, then comes relative velocity approach, then reactive control scheme or behaviour-based robotics etc., ok.

So, we have got a large number of approaches, large number of methods to solve this particular the motion planning problem. Now, on the non-traditional side, in fact, we have got a few approaches like the motion planning approaches using the principle of fuzzy reasoning tool, using the principle of the neural networks, using the principle of the combined neuro-fuzzy system, and so on.

Now, those things actually are beyond the scope of this particular course. So, this will not be taught in this particular course; so, here actually I am just going to concentrate only on the traditional schemes or the algorithmic approaches. And, both the graph-based techniques as well as the analytical approaches, we are going to discuss and we will see that how to solve that particular the find-path problem and dynamic motion planning problem.

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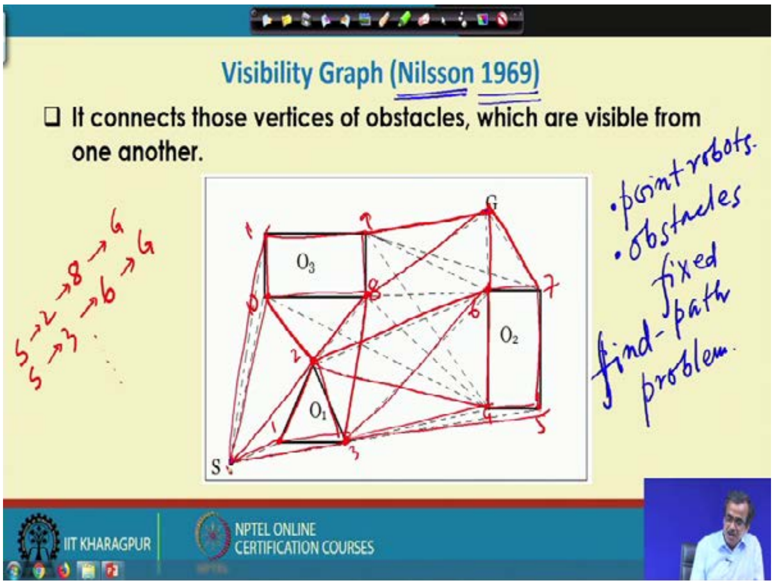

Now, here, we are going to start with say one the graph-based technique, which is known as the visibility graph. And, this particular visibility graph as I told, this is the first approach, which was proposed in the year 1969 by Nilsson and here, the principle is very simple; now, for simplicity, we are considering that the robots are point robots.

So, we are just going to consider the point robots and we are going to consider that the obstacles are stationary, that is the fixed obstacles. Now, the problem scenario is very simple and this type of problem is known as the find-path problem. So, this is known as the find-path problem; that means, starting from an initial position, it will have to reach the goal by avoiding collision with this particular static obstacle.

Now, let us try to see, this is the starting point for this particular point robot and for simplicity, we are going to consider the point robot and this is the goal. And, here, we have got some obstacles. So, this is obstacle 1, here we have got say obstacle 2 and these are all 2D obstacles, stationary obstacles, ok; so this is your another obstacle. Now, according to this particular method, we will start from here, now if there is no such obstacle, very easily you can connect this S and G by straight line.

And, that will be the best path or the optimal path that will be the collision-free and time optimal path could be, because there is no such obstacle. But, due to the presence of this particular obstacle; the robot, we will have to find out a feasible path.

So, that it is not going to collide with this particular the obstacle; the rule is very simple. The rule is as follows: it connects those vertices of obstacle, which are visible from one another. Now, let me start from here and let me try to look towards the goal. So, I starting from here, if I look towards goal, this particular vertex is visible, this is also visible, this is also visible. So, you draw one line here, you draw another line here, you draw another line here.

Now, you come back here, now from here, you look into this. So, this particular vertex is visible, this vertex is visible, this is also visible. So, the visible vertex you connect by the straight line, visible vertex you connect by the straight line and similarly, from here this particular vertex is also visible.

But, this is not visible, this may not be visible, this is also not visible. Now, from here, this particular vertex is visible, so this vertex is also visible. Similarly, from here, this is also visible, this is also visible, but this is not visible this is also not visible, this is also not visible.

Now from here, the goal is visible, then from here, this is visible. So, from here, this is visible from here, this is visible, ok. Now, another thing is from here; this is also visible and this is also visible. Now, if I just do the numbering, say it is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and this is G. So, I can write down the different feasible paths.

For example, one path could be starting from S; you go to 2, then 2 to 8; 2 to 8, then 8 to G. Another path could be that start from S then you go to 3. So, you come here then you go to 6, and then you go to G.

Similarly, there could be many other possible combinations, possible sequences, ok. Now, out of all the possible sequences, you can find out the time-optimal path, these are all collision-free paths. And, if you want to find out collision-free and at the same time optimal path, we will have to take the help of some optimization tool.

But, Nilsson did not use any such optimization tool, he could give all such feasible paths. And, then he concluded that starting from S to reach this particular goal, there are many such feasible collision-free paths. And, out of all the feasible paths; the robot we will have to choose one. This is actually the principle of the visibility graph.

Thank you.

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Lecture – 38
Robot Motion Planning (Contd.)

We are discussing, how to solve the find-path problem using graph-based techniques. Now, I am just going to start with the working principle of this Voronoi diagram, which is a very popular graph-based method to solve the find-path problem.

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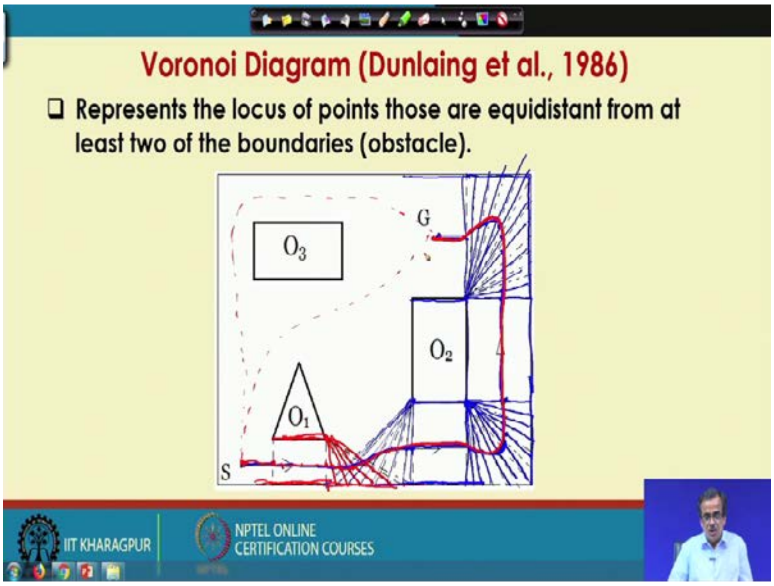

The concept of Voronoi diagram was proposed by Dunlaing et al. in the year 1986; Now, here, the problem is something like this; supposing that this is the field, and I have got one robot, say point robot and this is the starting position of the robot. And, its goal is denoted by the point G; now starting from here the point S, it will have to reach the point G by avoiding collision with the obstacles.

Now, here, we are just going to consider 3 obstacles, 3 static obstacles like O_1, then comes O_2 and O_3. Now, supposing that there is no such obstacle; now if there is no obstacle, then it will start from here and it is going to reach this point G following a path something like this, but unfortunately, there are some obstacles, there are some fixed obstacles. Now, let us see, how to find out the collision-free path using this particular the Voronoi diagram.

Now, in a Voronoi diagram actually, what we do is; we try to find out the locus of the points, which are equidistant from two boundaries. Now, if I start from the point S and our aim is to reach this particular goal. So, here, the most critical obstacle is nothing, but O_1 and we have got the boundary of this particular field and this is nothing, but the boundary of this particular field and at the same time, this is nothing, but the boundary of the obstacle.

So, what we do is; we consider this particular boundary of the obstacle and the boundary of the field and we try to find out the midpoint. So, the midpoint of this is nothing, but this particular point. So, this point is equidistant distance from this obstacle boundary and the boundary of the field. Similarly, I can find out the midpoint of these two boundaries are something like this and I will be able to reach this particular point.

And, once I have reached this particular point; now I will have to consider this particular point that is the vertex of this particular obstacle and the boundary of this field. So, if I consider this is one point and this is another point lying on the boundary, this is the midpoint. Similarly, we are going to consider a few more distances from this particular point and we try to find out what should be the midpoint and what should be the locus of the midpoint.

For example, on this particular straight line, the midpoint could be here; the midpoint could be here, the midpoint could be here, the midpoint could be here and the midpoint could be here. So, what you do is, we start from here and then up to this, we can find out the collision-free path; the collision free path could be something like this. For example, starting from here the collision free path up to this it is something like this, then I can find out a path up to this.

Next, we try to consider this particular vertex of the obstacle, that is O_2 and this is the boundary of this particular field. So, what you do is, we try to draw some straight lines; so, for example, say from this particular boundary to this point, these are the straight lines, we consider. So, these are the straight lines, we consider and try to find out the midpoint; so, the midpoint of this particular straight line is this, here this is the midpoint, this is the midpoint, midpoint and we try to join by a smooth curve and this could be the path.

So, we have reached up to this; that means, the point robot has reached up to this; now we consider this particular boundary and the boundary of this particular field and the locus of the midpoint starting from here; so, this will be the locus up to this. Now, after that we consider this particular vertex of O_2 and this is the boundary; so, we draw the straight lines, we draw all such straight lines here. And, for each of this particular straight line, we try to find out the midpoint. So, these are the midpoints; so, from here, we can just join by a smooth curve. So, the point robot has reached up to this.

Next, we consider this vertex and this is nothing, but the boundary of the field. So, this is the straight line; so, these are the straight lines, we consider. And, we consider the midpoint of this particular straight line; so, from here, there is a possibility that I will be getting this type of path. So, the point robot is here; so from here to here; this is the boundary of the obstacle and this is the boundary of the field. So, it is a midpoint, the locus of the midpoint could be something like this; so, from here, I will be able to reach this particular point.

Next, we consider this vertex and this is the boundary of this particular field; so, we join by straight line. So, these are the straight lines and we try to find out the midpoint; so these are the midpoint and we join by a smooth curve. So, up to this, this is the path now we consider, so this vertex and this is the boundary of the field. So, these are the straight lines; so these are the straight lines, which you are going to consider and once again, we try to find out the locus of the midpoint. So, it will be getting say this type of locus and then from here to this particular goal; so, this is the obstacle boundary and this is the boundary of the field. So, from here, the locus will be something like this.

So, starting from here; starting from the initial point, I can find out a collision-free path, which is something like this. So, this is a collision-free path for this particular point robot; so this is a collision-free path. Similarly, we can start from here and I can move along this particular direction and there is a possibility that I will be able to find out another feasible path something like this or I will be able to find out another path something like this.

So, there is a possibility that we can find out several such feasible paths and out of all such feasible paths actually, we will have to find out the time-optimal path. But, in a Voronoi diagram actually, they did not try to find out the time optimal path; they only tried to find out the obstacle-free, the collision-free path. So, this is one of the possible collision-free path for the point robot. So, this is the way actually, by using the principle of Voronoi diagram, we can find out the collision-free path for the point robot in the presence of some static obstacles. That means, we can solve the find-path problem using the principle of Voronoi diagram.

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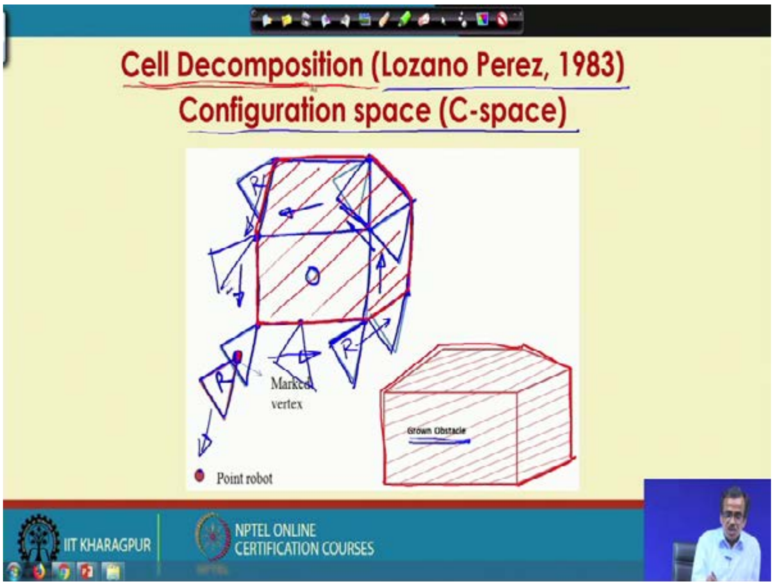

Now, the next is the concept of cell decomposition. Now, this particular cell decomposition is another very popular graph-based technique to solve the find-path problem. The concept was proposed by Lozano Perez in the year 1983, so it was proposed in the year 1983 and Lozano Perez proposed one technique that is called the cell decomposition and he also gave the concept of this particular configuration space or the C-space.

Now, let us try to see the problem; now, here, actually what we do is, we consider one physical robot in place of point robot and we consider one static obstacle. Supposing that the physical robot is something like this, so this is the physical robot which I am going to consider and this is the marked vertex of this particular physical robot. And, supposing that I have got one static obstacle and the boundary of the obstacle is something like this. So, this is nothing, but the obstacle; and I have got a physical robot something like this. So, this is the robot and we have got the obstacle here; how to ensure the collision-free path for this type of the robot, ok?

Now, what you do is; this particular physical robot is converted into a point robot something like this and this obstacle, this type of rectangular obstacle is converted into this type of grown obstacles something like this. Now, how to reach this grown obstacle starting from this particular obstacle? That, I am going to discuss. So, how to replace this physical robot by a point robot and how to replace this particular obstacle by a modified grown obstacle, so that the problem remains the same.

Now, what is the problem? The problem is, we will have to find out a collision-free path for this particular robot, so that it does not collide with this particular obstacle. And, this problem is equivalent to that of determining a collision-free path for the point robot in the presence of this type of the grown obstacle.

Now, let us see, how to achieve this grown obstacle; now here, I am just going to put one condition, the condition is as follows. So, this particular marked vertex; so its orientation will remain the same. So, what you do is. So, I have got this particular obstacle and this particular the robot I just place it here. The robot is placed here and this particular marked vertex is going to coincide with this particular the corner of the obstacle.

Now, actually what you do is, we try to slide in this particular direction; so, the robot is going to slide in this particular direction. So, there is a possibility, this will be the position and once again, it is sliding and then, after sometime, it will reach this particular point. So, this is the marked vertex and once it is reached this, what we do is, this particular robot R can slide along this particular edge. So, what we will be getting is, it can slide in this particular direction.

So, this will be the marked vertex and this will be the position of the robot and once it is reached, now, it can slide in this particular direction keeping this particular marked vertex, the orientation of the marked vertex the same. So, I can slide in this particular direction, then gradually, I am just going to reach this particular position. So, this will be actually the locus of the marked vertex.

Then, here, actually what I can do is, I can slide in this particular direction so, there could be sliding here and there is a possibility, if it slides it will take the position something like this. And, this is the orientation of the marked vertex and this will be the locus of this particular the marked vertex. So, after that, it can slide in this particular direction, then gradually I am just going to reach this particular point. So, the marked vertex will be here and this will be the position of this particular robot R.

So, this will be the locus of this particular the marked vertex and after that, here it is going to slide in this particular direction. So, there is a possibility that this will be the situation. So, this could be the situation and this is the marked vertex. So, the marked vertex is sliding along this and then, the marked vertex is going to reach this particular the starting point and this is the direction along which the robot is going to slide.

Now, if this is the situation, then very easily we can find out the locus of this particular the marked vertex. For example, we started from here then the marked vertex moves like this, then it moves like this, then it moves like this, then it moves like this, then it comes here, then it will come here and then, it will come here. And, this is nothing, but actually the grown obstacle.

So, this particular robot will be replaced by the marked vertex; so, this particular marked vertex is nothing, but a point. So, this is the point and the original obstacle; so, that will be converted to the grown obstacle. So, this particular grown obstacle I have just now redrawn it here; so, this is nothing, but the grown obstacle.

So, now the problem is equivalent to a point robot and this particular the grown obstacle. So, I will have to find out the collision-free path for this particular point robot by considering this type of grown static obstacle. Now, let us see how to tackle this particular problem using the principle of cell decomposition.

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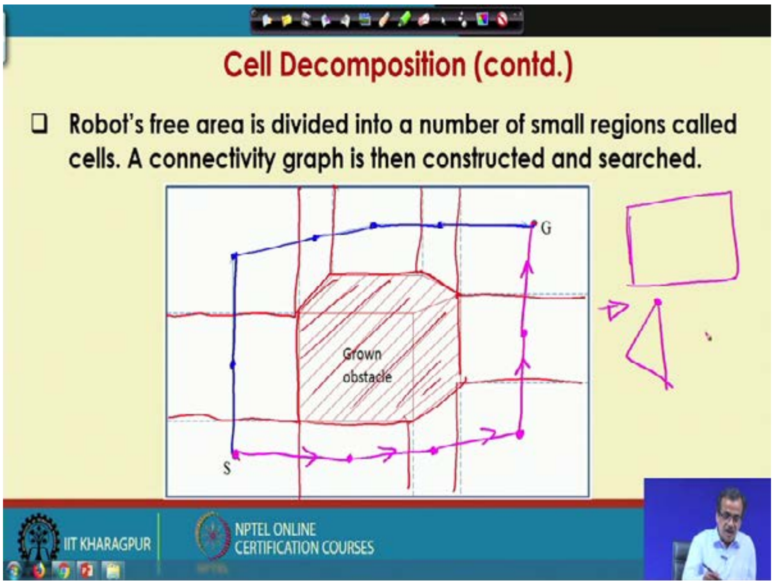

Now, here actually what we do is; the same grown obstacle I just redraw here and this is nothing, but the position of this particular the point obstacle. So, this is the position of the point obstacle, ok; so this is the starting position and the goal could be here that is denoted by G and this is nothing, but the grown obstacle. So, the grown obstacle is something like this; so, this particular grown obstacle represents the infeasible zone. The point robot should not reach here just to avoid that particular the collision with the static obstacle.

Now, if this is infeasible zone, the rest of the zone will be the feasible zone. So, this is actually the boundary of the field. So, this particular white portion will be nothing, but the feasible zone; now this particular feasible zone this is divided into a large number of small sub-regions. For example, starting from here; so what I can do is, this feasible zone, I can just divide something like this.

So, this is one feasible sub-region, similarly, this is another feasible sub-region, this is another feasible sub-region, another feasible sub-region, another feasible sub-region. So, we will be getting some feasible sub-regions, something like this and once we have got this particular the feasible sub-regions, what I do is, we start from here, go to the nearest feasible sub-region; so, this could be the nearest feasible sub-region. So, we try to find out the midpoint of this particular the sub-region; the center of this particular the sub-region.

Similarly, this is the center of this particular sub-region, this could be the center of this particular sub-region, this is the center of the sub-region, center of the sub-region. Then, the path could be something like this, start from here, then you reach this particular point, then you come here, then you come here, you come here and the robot is going to reach that particular the goal.

Now, this is one of the feasible paths; there could be some other feasible paths, for example, another feasible path could be something like this. You just start from here, then you find out this as the sub-region; the feasible sub-region. So, the midpoint could be here, now next, this is the feasible sub-region, the midpoint could be here, the center of this particular sub-region could be here, this could be the sub-region; the center for the sub-region.

So, another feasible path could be something like this; so, this could be another feasible path for this particular point robot. So, this is the way, by using the principle of the cell decomposition method; we can find out the feasible collision-free path for the robot. Now, here, we are trying to find out the collision-free path for the point robot considering this grown obstacle.

Now, by solving this, we are also able to solve the collision-free path for the actual physical robot, which are something like this and this was the marked vertex and original obstacle was something like this. So, if you solve this particular find path problem; so, indirectly we are solving this particular the find-path problem. So, this is the way by using the principle of the cell decomposition method, we can find out the collision-free path.

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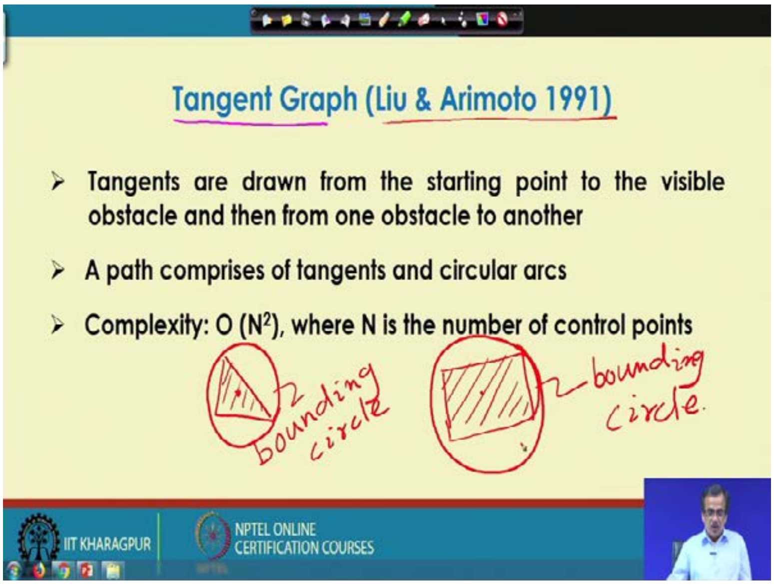

Now, we are going to start with another very popular method for solving the find path problem. And, this is known as the tangent graph technique, now this tangent graph technique was proposed by Liu and Arimoto in the year 1991. Now, here actually, what you do is, we try to move along the tangent of these circles.

Now, supposing that I have got this type of the fixed obstacles, a triangular fixed obstacle or there could be some sort of say, rectangular fixed obstacles sort of thing. So, here actually, what we do is, we try to draw one bounding circle for this particular the obstacle. Now, supposing that this is actually the triangular obstacle; so we try to find out the center and considering this as the center, we try to draw one circle and this particular circle will be the bounding circle for this static obstacle, that is, the triangular obstacle.

So, we try to find out the collision-free path considering this particular the circular boundary and this is nothing, but the bounding circle. So, this is the bounding circle for this particular static obstacle. Similarly, if this is the static obstacle, we try to find out the center of the area and once again we try to draw one circle. And, this will be nothing, but the bounding circle for this particular the static obstacle.

And, once you have got this particular bounding circle, instead of considering the physical dimension of this particular obstacle. So, we are going to consider this type of boundary, the bounding circle. Now, here, let us try to find out like the feasible path for a point robot considering this particular the bounding circle. So, once again, let me repeat like for each of these particular static obstacles, we try to find out the bounding circle. And, considering the bounding circle, we try to determine, what should be the collision-free path for the point robot?

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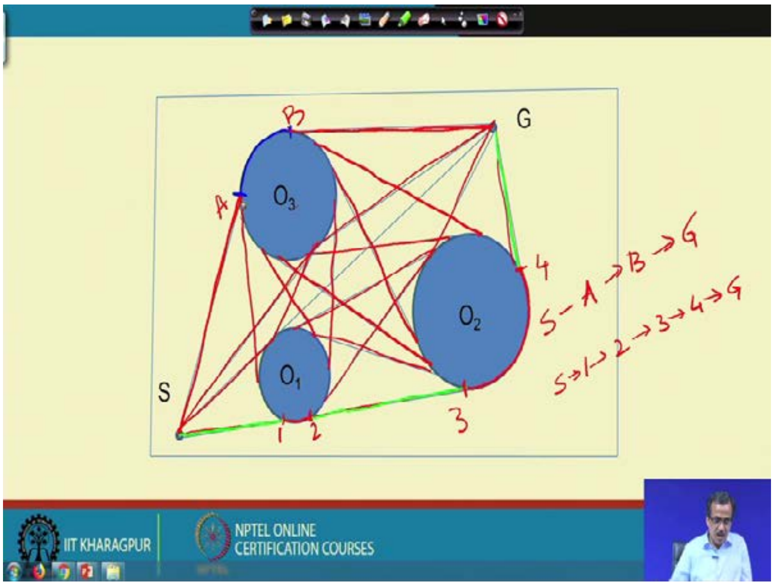

Now, here, if you see, this is the find path problem, say we consider this is the initial position; that is, starting position of the point robot and this is the goal. Now, if there is no such obstacle; so very easily starting from here, it is going to reach that particular the goal. And, absolutely, there is no problem because there is no such obstacle, but if I consider the obstacle here. So, the path will be slightly different; how to find out that particular path? To find out the path, we take the help of the tangent graph technique; the technique is as follows: we start from this particular point S and we try to find out which one is the most critical obstacle.

Now, if I compare O_1, O_2 and O_3, these are all static obstacles; so out of these three obstacles, so O_1 is the physically the closest. So, we first consider this particular O_1 obstacle. So, what I do is; from here, we try to draw the tangent, so from here, we can draw one tangent, which is something like this. From here, we can draw another tangent to this particular circle; then we consider this particular obstacle, that is, O_3.

And, we try to draw all the external and the internal tangents, for example, one tangent could be something like this, another tangent could be something like this and we can also consider this type of tangents. So, these are also tangents, ok; next we try to find out the tangent between this particular obstacle and that particular obstacle. But, before that starting from S, I can also draw this particular tangent, I can draw another tangent here, ok. And, between these two obstacles; so I can find out the tangent something like this. So, this is one tangent, this is another tangent, then we get another tangent here; we get another tangent here.

Now, from here, this obstacle O_2; so, I can find out this type of tangent also between O_1 and O_2, then this is another tangent, ok. Then, I can find out another tangent, another tangent, ok, I can draw the tangent here, I can draw the tangent here. Similarly, this is another and I can draw this type of tangent, also.

Now, once you have got these particular tangents; now, we are trying to find out, what should be a feasible path. Let us see, how to find out a feasible path; now to find out the feasible path actually, what you do is, supposing that I am just going to start and this is the first critical obstacle. So, let me follow this particular path ok; so from here supposing that I am just going to follow this particular tangent. The next tangent is here; so from here, so the next tangent could be here up to this. And, this point obstacle will follow this circular path and it will reach this, then it is going to follow this particular the tangent; so, this is one feasible path.

Now, here in this particular feasible path, we can see that we can draw one circular arc here, for example, say from here to here, there will be a circular arc. So, from here to here, there could be another circular arc something like this. Now, if I draw one circular arc here, another circular arc here; supposing that this is denoted by point 1, this is denoted by point 2, ok, this is denoted by point 3, this is denoted by point 4. So, the feasible path will be from S to 1, 1 to 2, 2 to 3, 3 to 4, 4 to G; so, this is one feasible path.

Similarly, there could be some other feasible paths, for example, say I can also consider some other feasible paths. For example, another feasible path could be something like this; so you start from, here then you reach this particular point, then, from here, you follow these particular circular paths up to this.

Now, from here, actually what you can do is; you can follow this type of the tangent path, ok. So, if I just write down, say this is A, this is B and this is G; the path could be your S to A, A to B, B to G. Similarly, we can find out the suitable such feasible paths and out of all the feasible paths; in fact, we will have to find out what should be the time-optimal and collision-free path.

Now, this particular algorithm actually could reach some popularity and as it is using the principle of tangent. So, there is a possibility that you will be getting the optimal path, but here, the main problem is actually the computational complexity. Now, if you see the computational complexity it has been checked that the computational complexity of this particular algorithm is nothing, but order of N square.

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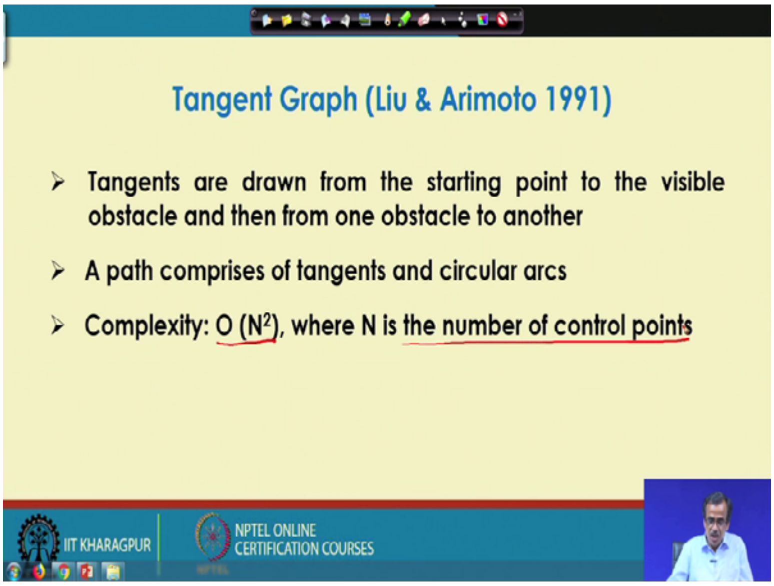

where N indicates the number of control points or the number of circular arcs, ok. So, the more the number of control points or the circular arcs, the more will be the complexity. And, the complexity is actually quadratic, it is in the order of N square; N is nothing, but the number of circular arcs. Otherwise, this method is very good and it could solve the find-path problem very efficiently. So, this particular method, as I told, could reach good popularity.

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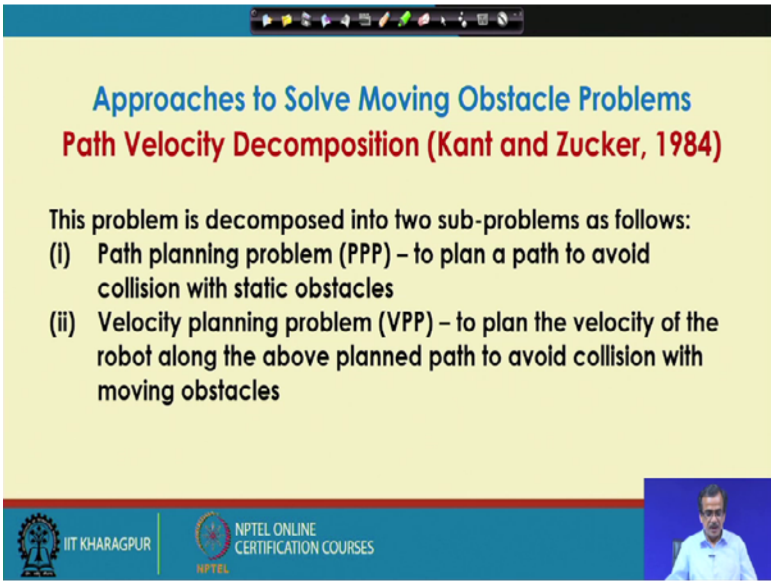

Thank you.

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Lecture – 39
Robot Motion Planning (Contd.)

Now, we are going to discuss how to determine the collision-free path for the robot in the presence of some moving obstacles.

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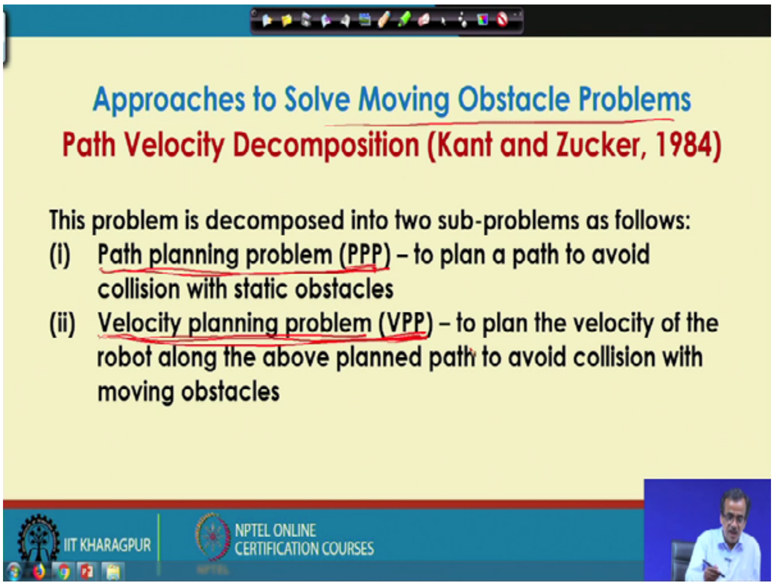

Now here the obstacles are moving, the robot is moving, the obstacles are also moving; how to find out, how to ensure the collision-free path? Now, if you see the literature the first approach which was proposed in the year 1984 by Kant and Zucker is the most popular approach, which is known as the path velocity decomposition.

Now, let us see, how to use the concept of this path velocity decomposition to solve the navigation problem of a mobile robot in the presence of some moving obstacles. Now, this particular problem is popularly known as the dynamic motion planning problem.

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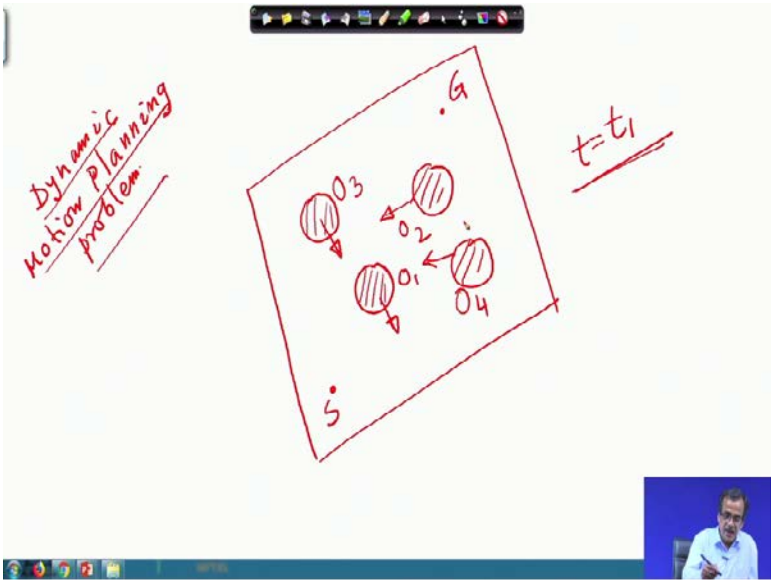

So, this is known as the dynamic motion planning problem; dynamic motion planning problem. Now, the problem is as follows: supposing that I have got one field, say this is the field, and I have got a point robot at position S. So, this is the starting position for the robot and the goal could be here that is denoted by G now, here. So, the robot we will have to start from S and it will have to reach G and it will have to find out some sort of optimal path.

Now, if there is no obstacle, then starting from S, directly it will reach that particular point G. But, supposing that there are some moving obstacles; so, this is obstacle 1, O_1 and this is moving in this particular direction with some speed. I have got another obstacle here, say this is O_2 and this is moving in this particular direction with some speed. I have got say another obstacle; so, this is the direction of movement with some speed say O_3.

Then, how to ensure or how to find out the collision-free path and the time-optimal path for this particular robot? So, this is actually the problem and let me consider one more obstacle here and suppose this is moving in this particular direction with some speed. So, this type of problem is known as the dynamic motion planning problem.

Now, this is a dynamic motion planning problem; that means, it is varying with time, ok? So, it is varying with time; now this particular dynamic motion planning problem can be converted to a find path problem at time t equals to t_1. So, at a particular instant, at time t equals to t_1; so, this will become a find path problem. So, I know at time t equals to t 1; I know the predicted position of this particular obstacle and there is a possibility that I will be getting this type of problem which is nothing but so, this is the field.

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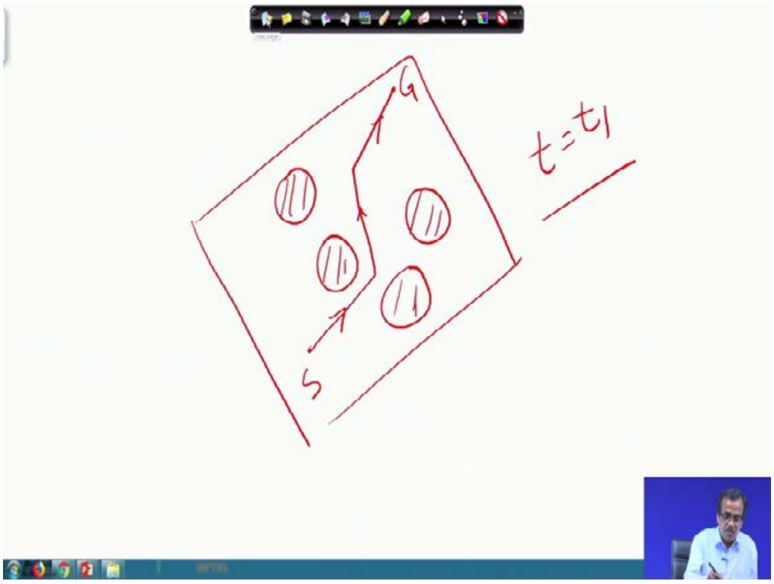

And, I have got the starting point here, the goal is here and this is one obstacle, this is another obstacle, this is another obstacle, this is another obstacle. So, these are all obstacles; so, at time t equals to t_1; this becomes a find-path problem. So, for simplicity; so this particular dynamic motion planning problem is converted into a find path problem at time t equals to t_1 and actually they solved this particular the find-path problem. Now, supposing that for this particular problem, say supposing that it has got a collision-free path something like this. So, this could be one of the possible collision-free paths for this find-path problem, ok.

Now, once it has got this particular find path, this particular feasible path; now it will have to do something to ensure the collision-free movement because truly speaking, these particular obstacles are moving, ok; so, here actually, in this particular method, what we do is, we try to solve this dynamic motion planning problem using two sub-problems. So, this dynamic motion planning problem is actually considered as a combination of two sub-problems: one is called the path planning problem and another is called the velocity planning problem.

So, in path planning problem, we consider that at time t equals to t_1. So, this is nothing, but a find path problem; that means, the robot will have to find out a collision-free path in the presence of some static obstacle. So, considering the obstacles to be stationary; so, it will try to find out a collision-free path and next, we just go for the velocity planning; that means, the robot is going to follow the path obtained through this path planning; that means, in the first stage.

Now, the velocity of this particular robot has to be adjusted, so that it does not collide with the moving obstacles. So, once again, let me repeat the dynamic motion planning problem is converted into two sub-problems; one is called the path planning problem that is PPP and another is the velocity planning problem, that is, VPP, ok. So, we first plan a path, the collision-free path considering the obstacles are stationary and the robot will try to follow that predetermined path by adjusting its velocity, so that it does not collide with the moving obstacles. So, this is the way actually, they implemented the path velocity decomposition method and it became very popular.

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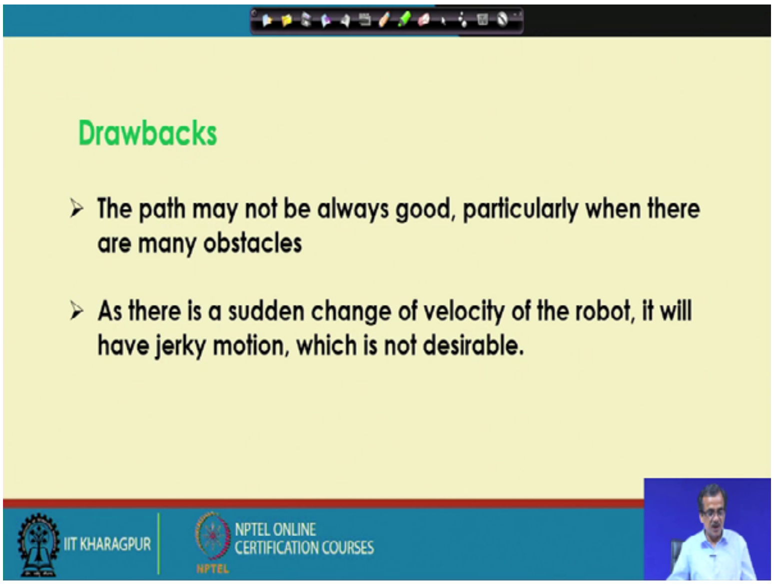

This particular approach became very popular, but this method has got a few drawbacks for example, say if there are so many obstacles, there are so many moving obstacles, this particular method may not find a feasible path; the robot may not be able to find out a feasible path by following this path velocity decomposition method.

Now, here, as I told that in the second stage; we will have to plan the velocity of the robot. So, the velocity of the robot is going to vary with time and there could be a sudden change of the velocity of the robot. And, consequently, there could be some sort of jerky movement of the robot, which is not desirable. So, these are actually the drawbacks of this particular the path velocity decomposition.

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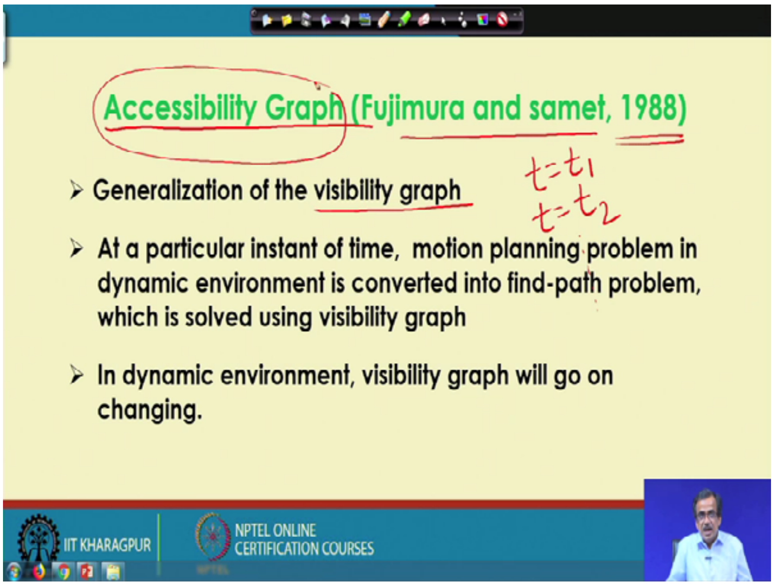

Now, if you see the literature, there are some other algorithms, which have been proposed to solve the dynamic motion planning problem and out of those methods, the accessibility graph also could reach some popularity. Now, let us try to explain the principle of this particular accessibility graph. The accessibility graph, this concept was proposed by Fujimura and Samet in the year 1988.

Now, here actually, what they do is, this is the modified version of the visibility graph, which you have already discussed and which was proposed by Nilsson in 1969. Now, if I just consider a dynamic motion planning problem, as I discussed that at time t equals to t_1. So, this particular dynamic motion planning problem will become a find-path problem; that means, that is the path planning problem for a robot in the presence of static obstacle.

Now, if it is a find-path problem; so, I can use the concept of the visibility graph to find out a path, to find out a collision-free path, which I have already discussed. Now, at time t equals to t_1, we consider that this is a find path problem. So, we will be getting some visibility graph collision-free path; next at time t equals to t_2. So, once again I will be getting another scenario; so, another find path problem I will be getting. So, I will be getting another visibility graph; so, with time, I will be getting a number of visibility graphs, ok.

Now, these particular visibility graphs will go on changing with time. So, this accessibility graph is nothing, but the modified version of this particular the visibility graph. As if we have added one more dimension, that is time to the visibility graph and this particular visibility graph is going to vary with time. And, that is nothing, but the concept of the accessibility graph, but the main drawback of this particular accessibility graph is the computational complexity.

So, it is computationally very complex and it cannot be implemented online. So, what is our aim? Our aim is to determine one collision-free path, time-optimal path, but at less computational time, so that we can implement this particular motion almost online. So, this particular accessibility graph, as I told is computationally very expensive and which may not be suitable for online planning.

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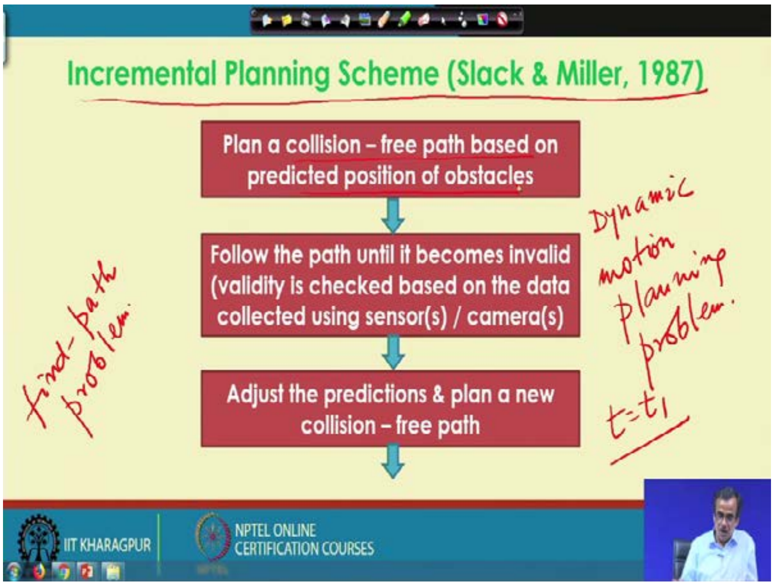

Now, another concept that is the concept of the incremental planning that was proposed by Slack and Miller in the year 1987; Now, this concept is very simple, very simple concept, they used, the problem is the dynamic motion planning problem; that means, the obstacles are moving and the robot is also moving. Now, the robot will have to find out the collision-free time optimal path; so, this is dynamic motion planning problem.

Now, how to solve this particular problem? Now, what I do is, once again at time t equals to t_1, we consider that this particular dynamic motion planning problem is nothing, but a find path problem. So, if it is a find path problem; so very easily actually, we can find out, what should be the collision-free path? So, we plan a collision-free path based on predicted position of the obstacle, ok. Next, we follow that particular path until it becomes invalid, the moment it is found to be invalid, ok. So, what we do is, we replan and we try to find out actually another collision-free path and we follow that particular collision-free path.

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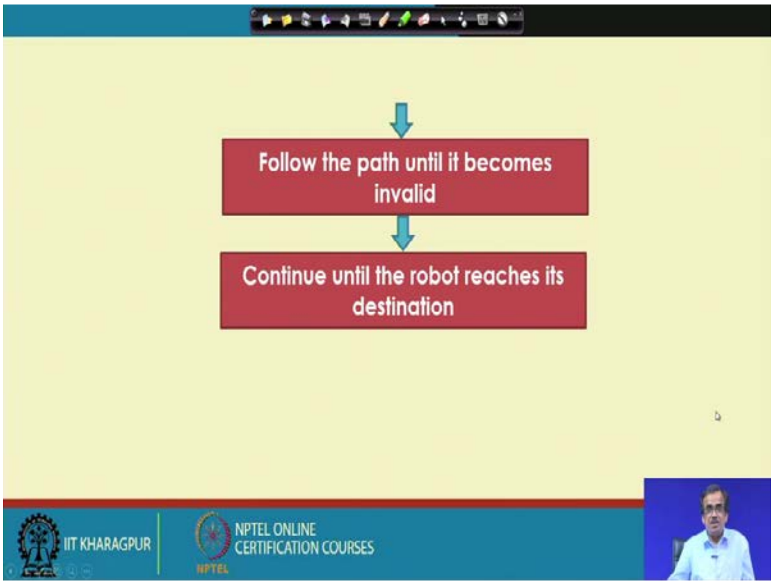

So, till it becomes invalid and the moment it becomes invalid, once again we re-plan and this particular process will continue till the robot reaches the destination. So, this is the way actually, we can implement the incremental planning. Now, incremental planning actually could not reach much attention of the researcher; it is due to the fact that several times, we will have to find out the collision-free path, we will have to re-plan and that is actually not very interesting. So, it could not reach much attention of the researchers working in the field of robot motion planning.

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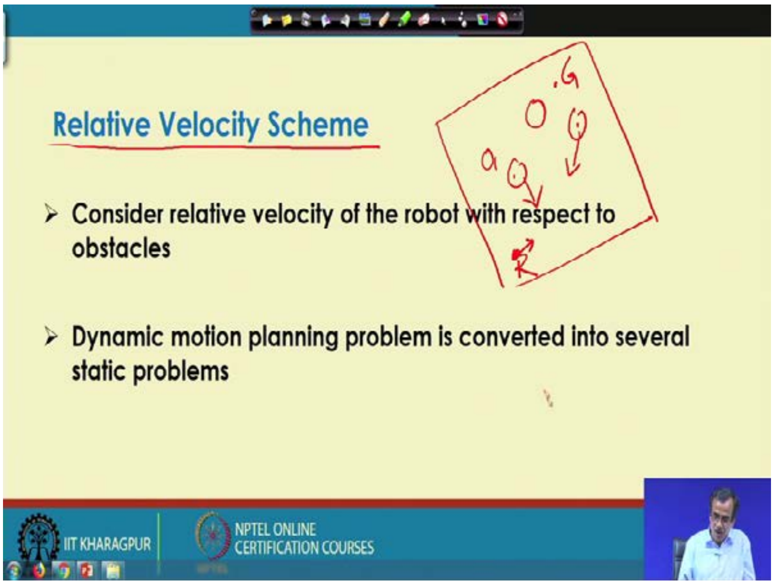

Now, then came actually the concept, another concept that is the concept of the relative velocity scheme. Now, supposing that the robots are moving, the obstacles are also moving; so, we can find out the relative velocity of this particular robot with respect to the moving obstacle. Now, let me just draw it here a little bit, say if this is the field; so, the robot is here, the robot is moving and obstacles are also moving and say, this is the goal. So, what you do is, as both the robots and the obstacles are moving, we can consider the relative velocity of this particular robot with respect to the different obstacles. The moment we consider the relative velocity of this particular robot with respect to obstacle O_1, we consider, as if obstacle O_1 is stationary and we try to find out the relative velocity.

Now, this is the concept of relative velocity. So, like your two bodies are moving with different velocities; so we try to find out the relative velocity of body 1 with respect to body 2; as if we consider the body 2 is kept stationary and we try to find out the relative velocity. Now, the same principle is copied here, both the robot and the obstacles are moving. So, we try to find out the relative velocity of the robot with respect to O_1, then relative velocity of the robot with respect to O_2, relative velocity of the robot with respect to another obstacle O_3, and so on.

So, for the same robot, there will be different relative velocities with respect to the different obstacles. And, its implementation actually becomes a little bit difficult and by following that, so we could convert the dynamic motion planning problem into several static problems. And, once you have got that particular matrix of the relative velocity of a robot with respect to the different obstacles; so, we try to implement just to find out the collision-free path.

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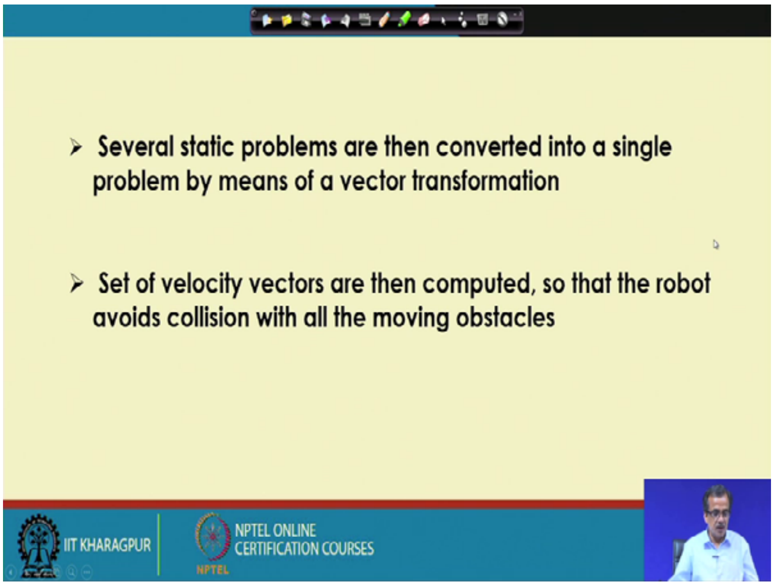

So, the set of velocity vectors, we try to find out, so that the robot avoids collision with all the moving obstacles. So, this is the way actually, we can implement this particular the relative velocity scheme, just to find out the collision-free path, the collision-free the time optimal path.

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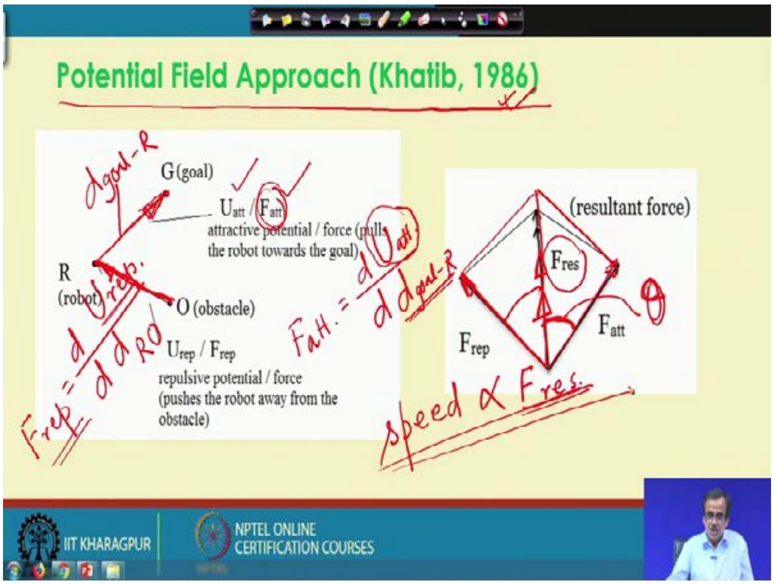

Then, came actually the concept of the potential field approach and that was proposed by Khatib in the year 1986. And, out of all the traditional motion planning algorithms, this particular algorithm could reach the maximum popularity. Now, here, actually in this potential field method, the robot will move under the combined action of attractive and repulsive potentials or attractive and repulsive forces.

Now, let me consider that this is the goal for the robot and this is the present position of the robot and at time say t equals to t_1. So, this is nothing, but the distance between the robot and the goal. The goal is going to attract that particular robot towards it and there will be some attractive potential, that is, U_attractive or there could be some attractive force.

Now, this is nothing but the attractive potential or the attractive force with which this goal is going to attract the robot towards it. Similarly, there could be repulsive potential or the repulsive force between the robot and this particular obstacle. And, due to this repulsive force actually, the obstacle is going to repel that particular robot. So, it is going to repel; so here, there will be some attraction, but here, there will be some repulsion.

Now, this robot will be under the combined action of this particular attraction and this repulsion, ok. Now, here, so before I go for this, how to find out the resultant of these attractive force and the repulsive force. Let me tell you, how to find out this attractive potential; attractive force from this particular the attractive potential. It is very simple, now, this attractive force, that is, F_attractive is nothing, but the derivative of this particular potential, attractive potential with respect to the distance; so this is d_goal-R, that is the distance between the goal and the robot. So, this is nothing, but d_goal-R; so, this is your d_goal-R, that is the distance between the robot and the goal.

So, what we do is, we try to find out the derivative of this particular the attractive potential with respect to d_goal-R; that particular distance; so we will be getting the attractive force. Similarly, from this repulsive force, if you want to find out this repulsive force, that is, F_repulsive is nothing, but the derivative of repulsive potential U_rep with respect to the distance between the robot and this particular obstacle. So, this is the distance between the obstacle and the robot. So, we try to find out the derivative with respect to your d_R-O, and that is nothing but the repulsive force.

Now, as I told that the robot is subjected to both attractive as well as repulsive force. Now, this I am just going to draw it here; so, here I have got the attractive force in this particular direction. So, I am just drawing it here; so this is nothing, but the attractive force and here, there will be a repulsive force. So, here, I am just going to draw the repulsive force, ok; so, I have got the attractive force, I have got the repulsive force.

So, very easily, I can find out what should be the resultant force and this resultant force is denoted by F_res; so, I can find out so this particular the resultant force. And, once you have got this particular the resultant force; so, what do you do is; the robot actually as I told that it is moving under the combined action of attractive and repulsive forces, now it will try to follow this particular the resultant force. That means, the speed of the robot or the acceleration of the robot will be directly proportional to the magnitude of this particular the resultant force.

And, moreover, this is the angle of the resultant force with respect to the attractive; supposing that this is angle θ. And, this particular angle θ with respect to the attractive; so, this will be the angle of deviation for this particular the robot.

So, the robot is subjected to the attractive force and repulsive force and due to this attractive and repulsive force. If you try to move with a speed, which is proportional to the magnitude of this particular resultant force and its angle of deviation will be this particular angle, that is, θ; that is the angle between the resultant force or attractive or we can find out this particular angle, that is the angle between the resultant force and this particular the repulsive force. So, we need these two information for the movement of the robot; one is the speed, another is the angle of deviation. So, this is the way actually, we are determining the speed and the angle of deviation for this particular robot

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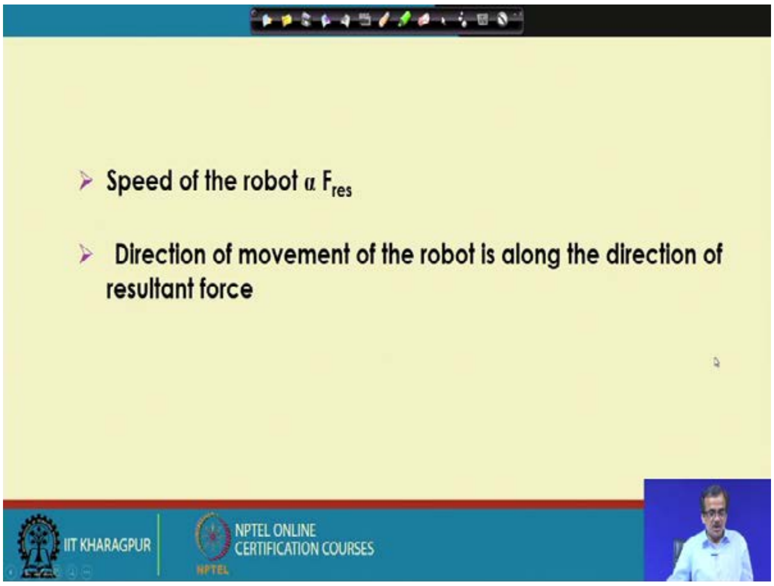

Now, here whatever I mentioned, the same thing I have written it here.

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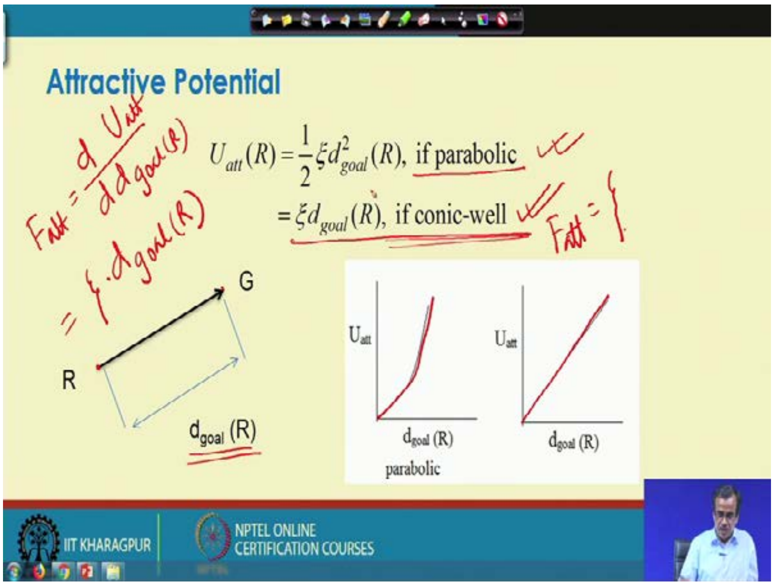

So, what I am going to do is; so, I am just going to assign some attractive potential and repulsive potential, some mathematical expression. And, try to see, how to derive that particular thing like how to derive that attractive potential and the repulsive potential.

Now, as I told that this is nothing, but the position of the robot and this is the goal and this is nothing, but the distance, that is, d_goal ®. Now, this $U_{attractive} = \frac{1}{2}\xi d_{goal}^2(R)$; if it is considered to be parabolic, that is second order curve. And, if I consider that $U_{attractive} = \xi d_{goal}(R)$; so, this is nothing, but a straight line.

Now, if I just draw it here; so this U_attractive as a function of d_goal-R; so this particular thing. So, I will be getting; so these type of plot for the attractive potential; if I consider this type of expression, so this is nothing, but the variation of U_attractive with d_goal-R. And, as I told, how to find out this particular F_attractive; so this F_attractive will be nothing, but d_goal-R of this particular thing. So, that will be d_goal-R of U_attractive potential and this will be nothing, but here, there is a square. So, 2 will come; so this will become $\xi d_{goal}(R)$; so, this will be the attractive force.

Similarly, if I consider this type of distribution for the attractive potential, then your F_attractive will be nothing, but is your constant and that is nothing, but this ξ, ok. So, by differentiating actually, we can find out the attractive force. So, we can use different types of function for this attractive potential and accordingly, we can find out like what should be the expression for attractive force.

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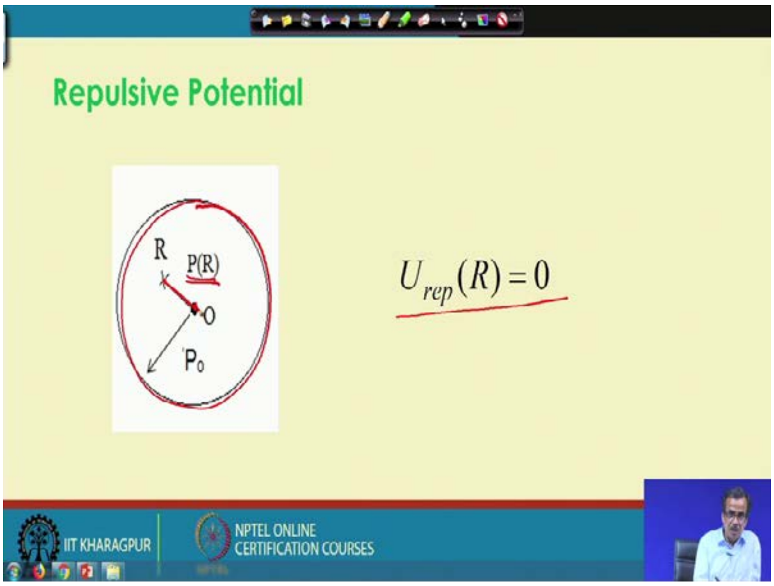

Next, we try to concentrate on the repulsive force.

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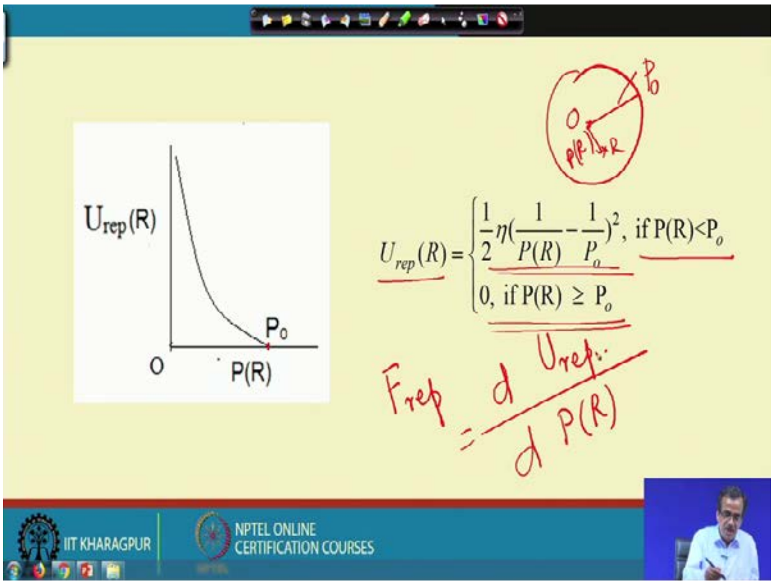

Now, here, this repulsive force is actually is defined in such a way that this is actually the obstacle and the robot is here. And, this is the distance between the robot and the obstacle, that is, denoted by P ®. Now, surrounding this particular the obstacle, we define one circle and on the boundary of the circle and beyond the boundary of the circle the repulsive potential will become equal to 0, but inside this particular circle, there will be some repulsive potential. And, this particular repulsive potential will be maximum, when the robot comes very close to this particular the obstacle.

So, when the robot is very close to the obstacle; the repulsive potential will be more and whenever the robot reaches this particular the boundary of this particular circle, then the repulsive potential will tend to 0. And, beyond which, outside this particular circle, the repulsive potential will become equal to 0. Now, the same thing, I have just plotted in here; so this is actually the plot of the repulsive potential. So, when P ® is small, the repulsive potential is very large and this repulsive potential will become equal to 0, when P ® becomes equals to P_0, ok.

So, when P ® becomes equal to P_0, that is actually the radius of that particular circle, this repulsive potential becomes equal to 0; now this is the mathematical expression like repulsive potential is equal to $\frac{1}{2}\eta(\frac{1}{P(R)}-\frac{1}{P_0})^2$. If P® is found to be less than P_naught;

so, if I just draw it here. So, this is the obstacle say center of the obstacle; so this is your P_naught and supposing that the robot is here. So, this is the robot and this is your P ®.

So, if P ® is less than P_naught; that means, it is inside this particular circle. So, this is the expression for this particular the repulsive potential, otherwise, this particular repulsive potential becomes equal to 0. And, once you have got this particular repulsive potential, by differentiating with respect to this particular P ®, P ® is nothing, but the distance between the obstacle and the robot. So, we can find out what should be F_repulsive. So, F_repulsive is nothing, but d/d P® of your U_repulsive; so, this is nothing, but your F_repulsive, ok. So, this is the way actually, we define this particular attractive and the repulsive potential.

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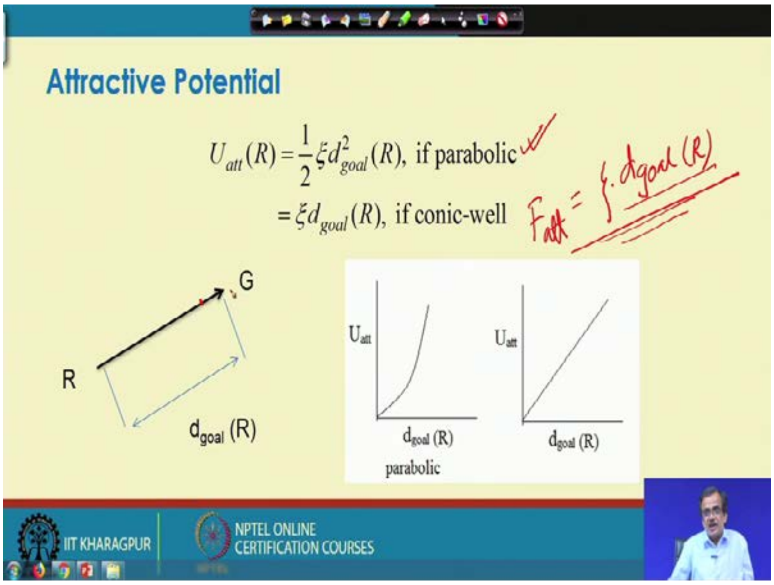

Now, here, actually for this particular attractive potential, there is another point to be considered, that is your, as I told, that is, F_attractive becomes nothing, but zeta d_goal ® ok. So, here, there is another important point to be discussed like say ξ is having some fixed and numerical value. Now, if d_goal ® is small; that means, the robot has reached very near to this particular goal, might be the robot is here. So, if d_goal ® decreases; this particular attractive potential is going to be reduced.

Now, this is done very purposefully, otherwise the robot will not be able to stop at the goal with 0 velocity. So, when d_goal ® is more, then the attractive potential is more. So, when the robot is at far distance from the goal, the active force will be more, but whenever it comes very close to the goal, the attractive potential is going to be reduced, so that the robot can stop at the goal with 0 velocity; and this has been actually done very purposefully; this is the way actually we can determine attractive and repulsive potentials, attractive and repulsive forces.

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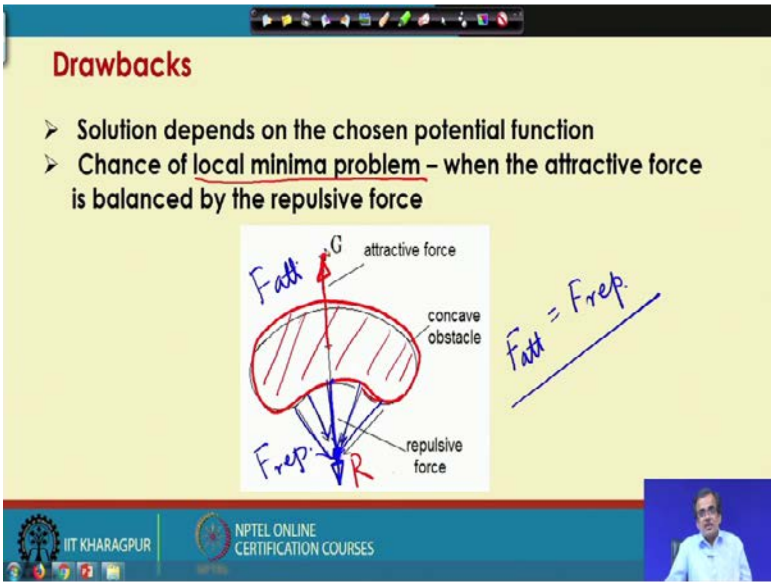

Now, as I told that out of all the traditional tools for the motion planning; this potential field method is the most popular one, but it has got a few drawbacks. For example, say here, the solution depends on the chosen potential function, this is called the artificial potential function. And, depending on the nature of this artificial potential function, we will be getting this solution for the robot. The robot will try to find out a path, the collision-free path depending on the chosen potential function.

Now, here, there is another very big problem, which we are going to face that is actually called the local minima problem. Now, this is a very typical scenario and this happens for the concave obstacle. Now, supposing that this is the concave obstacle; so this is the concave obstacle sort of thing, it is a very hypothetical situation. And, supposing that this is the goal for the robot and fortunately or unfortunately, this is the present position for the robot, ok.

Now, if this is the situation; here, we have got the concave obstacle, the goal will try to attract this particular robot. So, there will be some attractive force in this particular direction, but this particular the obstacle, the obstacle boundary is going to put some sort of repulsive force on this robot. For example, say it is going to put some sort of repulsive force here, some sort of repulsive force here, repulsive force here; so, it is subjected to the repulsive forces.

Now, all such forces are passing through a particular point. So, all of us, we know how to find out the resultant of these particular forces, the set of forces. So, graphically, we can find out, what should be the resultant of these forces. And, supposing that the resultant repulsive force is something like this. So, this is the resultant and repulsive force; so, this is F_repulsive and this is your F_attractive, ok.

Now, fortunately or unfortunately, if F_attractive becomes equal to your F_repulsive then what will happen? So, attractive force becomes equal to the repulsive force; the robot will become stationary here. So, there will be no movement of this particular robot and the robot will not be able to reach this particular goal. So, this type of the problem, we may face in the potential field method.

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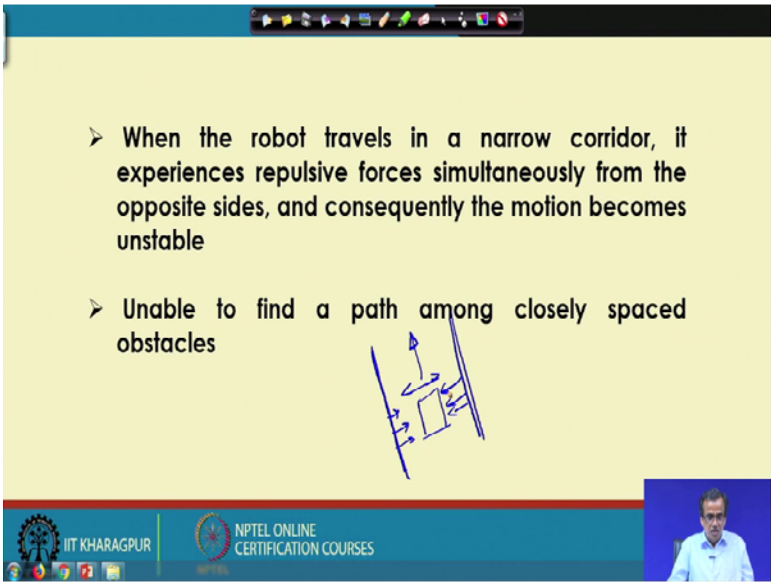

There are some other disadvantages, supposing that I have got one narrow corridor. And, I am just going to find out a collision-free path for this particular robot, say I have got a robot here and I am just going to make a plan for this particular robot.

And, this is one wall, this is another wall. So, there will be some repulsive force here, there will be some repulsive force here and consequently, this particular robot will have some sort of oscillatory movement something like this; so, there will be oscillation and which is not desirable. Moreover, supposing that there are so many such obstacles; large number of obstacles, there is a possibility that it may not be able to find out the time-optimal and collision-free path.

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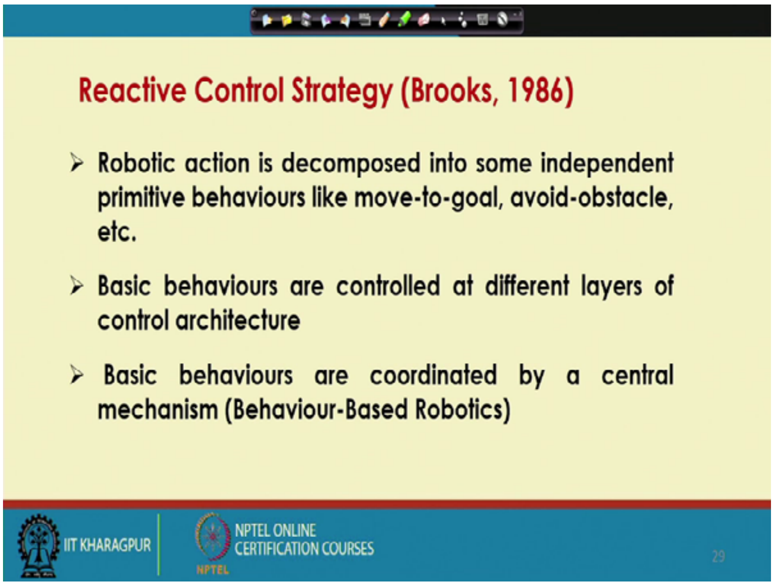

Now, here, there is another very popular scheme that is called the reactive control strategy that I will be discussing later on.

Thank you.

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Lecture - 40
Robot Motion Planning (Contd.)

Now, I am going to discuss the principle of another very popular motion planning algorithm, which is known as the reactive control strategy.

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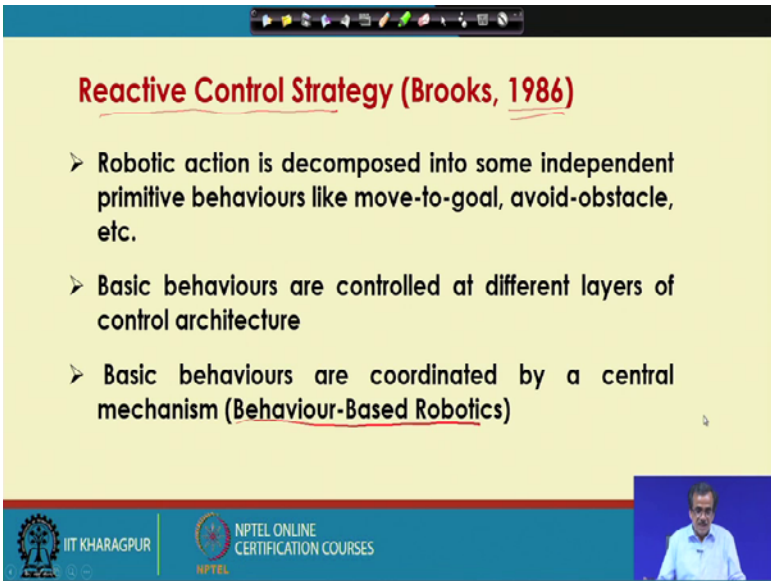

Now, this reactive control strategy was proposed in the year 1986 by Brooks. Now, here, a robotic action is divided into a large number of independent primitive or basic behaviours. For example, the robotic action in a soccer plying robot is to score goal.

Now, to score goal; this particular robotic action is divided into a large number of basic behaviours, and each of these basic behaviours is controlled at a particular layer of the control architecture. Now, if I just consider a complicated robotic action; now this complicated robotic action if I want to control, so, I will have to take the help of a large number of layers in the controlled architecture. And, here, there must be one supervisory controller; that means, there must be one main computer, which is going to control the activities of the different layers.

And, therefore, to handle a complicated task, we will have to take the help of a large number of layers and the computational of complexity is going to increase and moreover, it requires a large amount of computer memory.

Now, this particular control strategy becomes very famous and a particular scheme in robotics was proposed, that is known as the Behaviour-Based Robotics. Now, this behaviour-based robotics could reach the popularity and many people used in different forms, they modified also and they could solve a number of problems related to the motion planning algorithm.

Now, here, this reactive control scheme has got a few drawbacks. For example, say if I consider a complicated robotic task, as I told that there could be a large number of layers and consequently,

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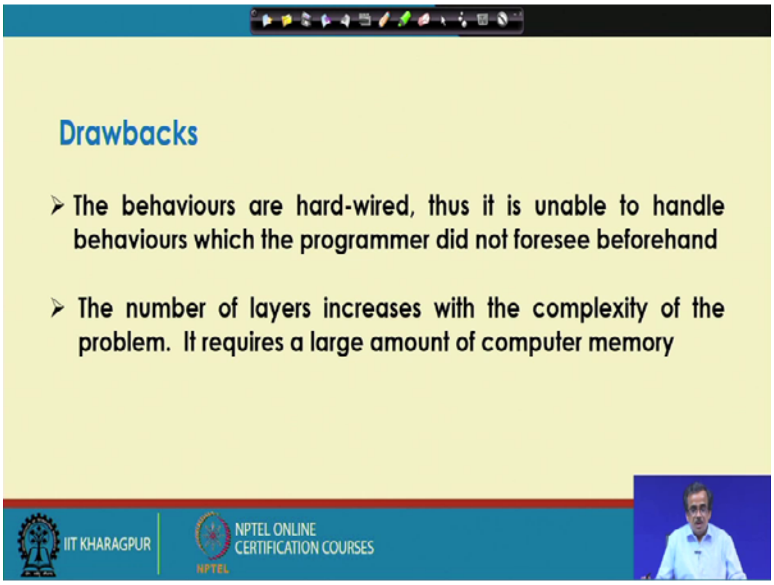

this particular control architecture will become complicated, it requires a large amount of computer memory and this particular process will become slow. There is another drawback now, supposing that the designer say could not foresee a few behaviours, while designing that particular the controller.

Now, if this particular robot is going to face that type of complicated scenarios, which was not consider during the design of that particular controller, the robot will not be able to handle that particular complicated situation very efficiently. And, there is a possibility that the robot is going to fail to tackle that type of scenario.

Now, these are all drawbacks of this particular the reactive strategy.

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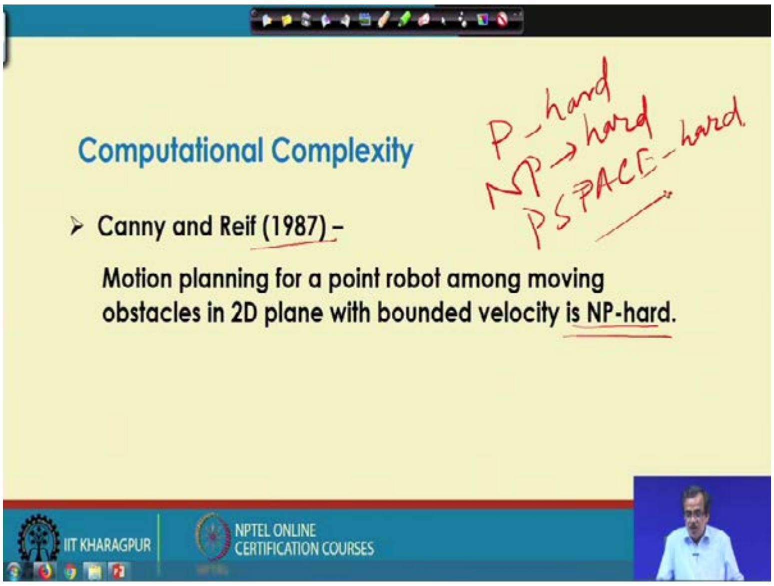

Now, if I see the computational complexity of the various motion planning algorithms used in robotics. So, this particular motion planning algorithm is computationally very expensive and thus, this algorithm could not be actually implemented, online.

For example, say the computational complexity of these motion planning algorithms was studied, in details, by Canny and Reif. Now, in the year 1987, they studied that computational complexity of the motion planning algorithms and according to them, the motion planning for a point robot moving among moving obstacles in 2 D plane with bounded velocity is found to be NP-hard; so, this is computationally very expansive.

Now, the computational complexity of this particular algorithm is expressed in terms of the hardness values like NP-hard, P-hard then P-space hard and all such things. For example, this hardness is represented as P-hard, then comes your NP-hard, that is not polynomial hardness. And, then comes your P-Space hardness; P-space hardness and that is very hard and this particular hardness is the exponential hardness.

Now, here, in 2 D plane and for a very simple scenario like a point robot; moving in the presence of some moving obstacle with some bounded velocity is found to be NP-hard.

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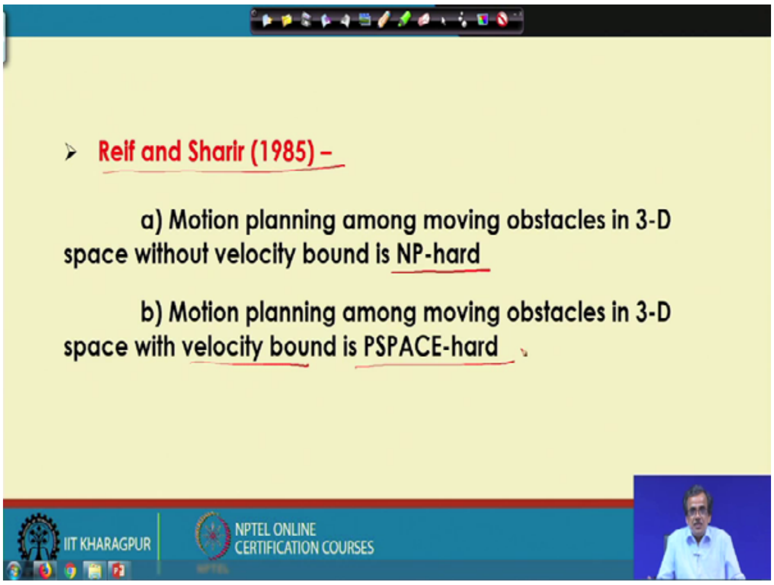

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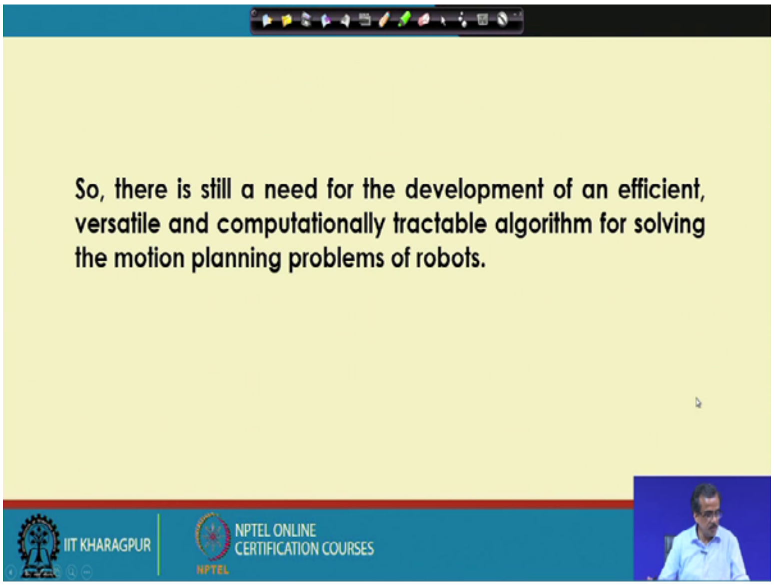

Now, similarly the similar study was carried out by Reif and Sharir in the 1985. And, according to them; they studied this computational complexity in 3-D space for a point robot in 3-D space.

Now, this particular motion planning algorithm with the velocity bound is found to be NP-hard. On the other hand, the motion planning problem of a point robot in 3-D space with the velocity bound is found to be the PSPACE-hard. And, that is why, this type of traditional method for robot motion planning could not be implemented online, because these are all computationally very expensive.

Now, if you see these particular drawbacks of this particular traditional tools for the motion planning.

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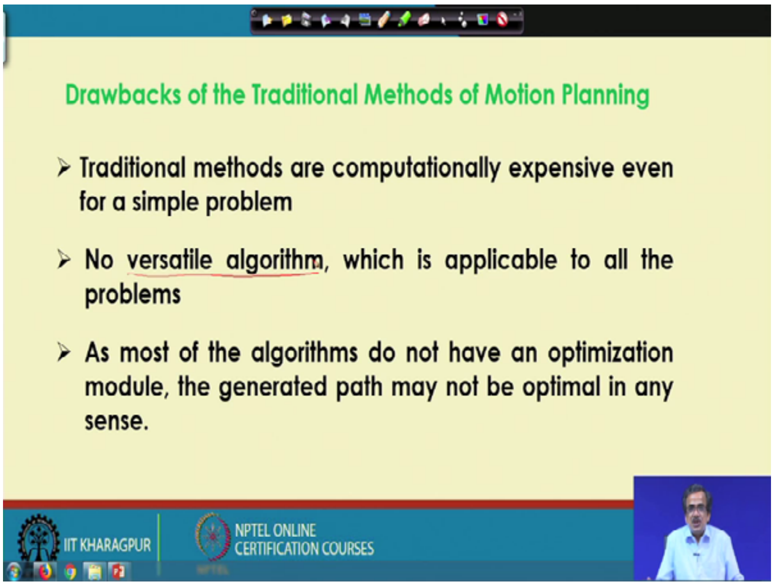

Now, we have seen that these particular traditional tools for robot motion planning are computationally very expansive. Thus, we could not implement online to solve this dynamic motion planning problem. And, moreover, now each of these particular traditional tools for motion planning is suitable to solve a particular problem and that is why, these algorithms are not versatile.

So, for different problems, we will have to use the different algorithms and that is why, in fact, we will have to find out some sort of versatile algorithm or the robust algorithm, which can be implemented to solve a verity of problems.

Now, then comes these particular traditional tools are not equipped with any such optimization module. And, that is why, the planned path or the generated path may not be optimal in any sense. Now, these are all actually the drawbacks of the traditional methods of the robot motion planning.

Now, here, if you see these particular the drawbacks, due to these drawbacks, we could not implement all such traditional tools for robot motion planning in a very efficient way.

And, that is why, there is a need for the development of an efficient, robust and computationally tractable algorithm, which can be implemented online to solve this robot motion planning in a very efficient way.

Now, to understand the computational complexity of this particular robot motion planning; let me try to take one very simple example. Now, if I take this particular example, we will understand that this particular robot motion planning problem is very complicated.

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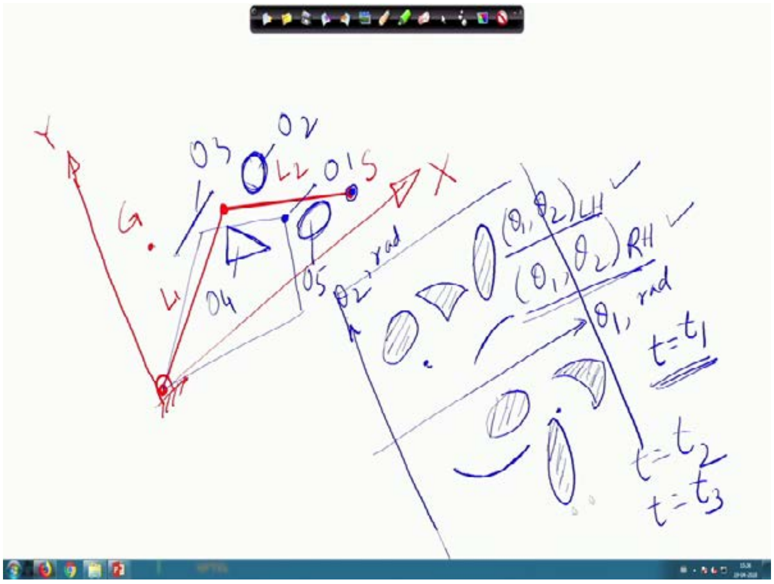

Now, for simplicity, let me just try to consider one path planning problem for a very simple manipulator. Now, supposing that this is the Cartesian coordinate system like X and Y. And, I have got a serial manipulator having say 2 degrees of freedom; say this is the serial manipulator having 2 degrees of freedom.

So, this is the length of the first link and this is the length of the second link. So, this is the length L_1 and the length L_2 and supposing that the starting point for this particular manipulator is nothing, but S and the goal point of this manipulator supposing that is denoted by G. Now, starting from the point S; it will have to reach the goal and while moving so, this particular tip of the manipulator should not collide with some static obstacle.

For example, this is a point obstacle; now, similarly I can also consider this is the point obstacle. I can consider there could be a line obstacle here, there could be some sort of triangular obstacle, there could be some sort of circular obstacle here or say elliptical obstacle here. Now, this particular tip of the manipulator will start from the point S and it will reach the point G; that is the goal.

Now, while moving the tip of the manipulator should not collide with all such static obstacles. Now, how can we ensure this type the movement; the collision-free movement? Now, to solve this actually, we can do it analytically in a very easy way for example, say the moment this particular the tip of the manipulator is going to touch the point obstacle.

So, this could be actually the configuration, this is one configuration. Similarly, there could be another configuration with the help of which the tip of the manipulator can touch this particular point obstacle. And, we can solve for the joint angles, the moment it is going to touch. So, this particular point obstacle, I can find out the joint angles like your θ_1, θ_2, the left hand solution and θ_1, θ_2 and that is nothing, but the right hand solution.

So, corresponding to this particular point, I will have that two sets of values for these particular the joint angles. Now, if I concentrate on this particular the line obstacle; so, this line obstacle will be nothing, but a combination of so many such points. So, we consider a large number of points lying on this particular the line obstacle.

And, corresponding to each of these particular points so, I can find out the two sets of θ values. So, I will be getting; so a large number of; the large sets of θ values corresponding to this particular the line obstacle. Now, if I want to ensure the collision-free path for this particular tip, for this particular triangular obstacle.

So, this tip is going to trace the boundary of the triangle and I can also find out the sets of theta values or the joint angle values. Similarly, the moment is going to trace, the boundary of this particular circular obstacle. So, I can also find out what should be the θ_1, θ_2; the sets of θ_1, θ_2 values. And, following the same principle, the moment this particular tip of the manipulator is going to trace the boundary of this particular the elliptical obstacle. So, I can also find out the combination of this particular the θ values.

Now, if I want to ensure the collision free movement of this particular tip. So, what I will have to do is; so, I will have to plot these particular θ_1, θ_2 like the joint angles. And, these are in say radian, θ_1, θ_2 are in radian. And, there is a possibility for one point I will be getting two points here.

And, for this particular the straight line obstacle; so, I will be getting one curve line here and there is a possibility I will be getting another curve line here. Then, corresponding to this particular triangular obstacle so, there is a possibility I will be getting one curved line, another curved line, another curved line.

Similarly, here, I will be getting one curve line, another curve line, another curve line. Then, corresponding to this particular circular obstacle; so, there is a possibility, I will be getting one elliptical and here, I will be getting another elliptical step. And, corresponding to this particular elliptical so, I will be getting one distorted ellipse sort of thing; so, might be another distorted ellipse sort of thing.

Now, on this particular θ space, these are nothing, but the forbidden zone; that means, if I want to ensure the collision free path for the tip of this particular manipulator. So, what I will have to do is; I will have to select θ_1, θ_2 in such a way that your θ_1, θ_2 should not lie on these particular the forbidden zone.

So, these are all forbidden zones, this is nothing, but a forbidden point, another forbidden point this is nothing, but a forbidden curve, another forbidden curve, ok. So, this is the way actually, we can ensure the collision free movement of this particular the tip of the manipulator.

Now, this is a very simple the motion planning problem or the path planning problem. Because here, we have considered that these particular obstacle, that is, your obstacle 1, then comes obstacle 2, obstacle 3, then comes obstacle 4 and obstacle 5; these are all stationary obstacle and this is in 2 D.

Now, if I consider that these particular obstacles are moving; then, how to ensure the collision free path for this particular the tip of the manipulator? Now, this problem will become more complicated; now this will become more complicated, because the positions of these particular obstacles are going to change with time.

So, might be at time t equals to t_1; so, I will be getting. So, this is the scenario, this is actually the infeasible the region at times a t equals to t_1. Now, if it is a problem of dynamic motion planning; so, there is a possibility.

So, at time t equals to t_2; so, this particular feasible and infeasible the zones that is going to vary. And, similarly at time t equals to t_3; so, I will be getting another combination of these particular the feasible and infeasible zones. And, through this particular feasible zone supposing that this is nothing, but the total area.

Now, this white portion is nothing, but the white portion is nothing, but the feasible zone. And, through this particular feasible zone actually, what I will have to do is; so, I will have to go for, I will have to find out the feasible and infeasible zones.

Now, once gain let me consider the same example here.

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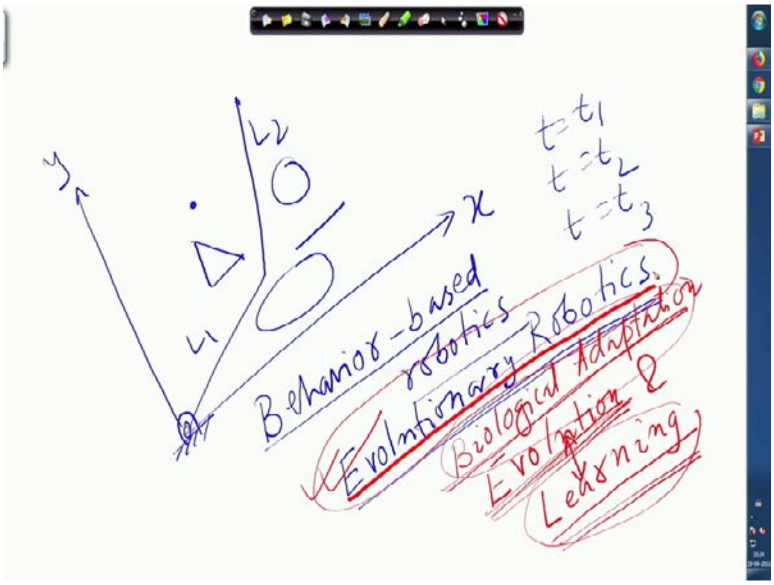

So, if I consider x and y and so this is nothing, but the robot, this is L_1 and L_2 and we consider some static obstacles like point obstacle, line obstacle, triangular obstacle, circular obstacle, elliptical obstacle, something like this and these obstacles are all stationary.

Now, as I consider in the next time, these obstacles are moving in 2 D; the obstacles are moving. So, as I mentioned at time t equals to t_1, at time t equals to t_2, at time t equals to t_3; the scenario that is the feasible and the infeasible scenario; so, that is going to vary.

The problems becomes much more difficult; now if I just make it more complex. Now, supposing that I am just going to add another dimension to these particular obstacles, that means, your obstacles will become 3 D and if I consider that these particular obstacles are moving.

And, if I consider a manipulator having say 6 degrees of freedom. So, if I consider that my hand is a manipulator; the serial manipulator and staring from a particular point. So, tip of this particular finger; so, I just wanted to move this particular the goal point and while moving from here to this particular point; so, supposing that the robot has started moving.

And, while moving, now, there are some moving obstacles, which are going to come in between. And, this particular tip of the manipulator, we will have to find out the collision free path online. And, this particular problem becomes very difficult to handle because here, the motion planning algorithm will have to take the decision within a fraction of second.

And, to solve this type of problem actually, the traditional motion planning algorithm is going to face a lot of problem. Because these algorithms are computationally expansive so, we cannot take the decision online within a fraction of second and there is a chance that the tip of the manipulator is going to collide with the moving obstacle.

Particularly, whenever it is working in the 3 D space and whenever we are working with the robot with 6 degrees of freedom and if it is working in the 3 D space with the moving obstacle; it becomes difficult to find out the collision free movement for the tip of this particular the manipulator.

So, this particular motion planning algorithms are very complex and finding the online solution is very difficult. And that is why nowadays there is a trend to replace this particular the behaviour based robotics, the principle of behaviour based robotics, which I have already discussed using the reactive control scheme.

Now, this particular behaviour-based robotics actually will not be able to solve this type of difficult situation in a very efficient way. And, that is why, nowadays, there is a trend that in place of these particular behaviour-based robotics; we go to another, the motion planning algorithm and that is called actually your evolutionary robotics.

Now, this particular evolutionary robotics, we take the principle of evolutionary principle. So, I am just going to tell you, in short, the principle of this particular evolutionary robotics, but I am not going to discuss, in details, the principle of evolutionary robotics because this is not is not there within the scope of this particular the course.

But, I am just going to tell you the philosophy behind this particular evolutionary robotics and this particular evolutionary robotics could be a possible solution to solve this type of very complicated problem particularly the motion planning in 3 D space for a manipulator having 6 degrees of freedom.

And, if we consider the moving obstacle, there is a possibility that this principle of evolutionary robotics is going to help us to find out a feasible solution. Now, let us see , how does it work? Now, here in this evolutionary robotics actually, what you do is; we use the principle of the biological adaptation. So, this biological adaptation, if you see, there are nothing but two principles of the biological adaptation.

Now, those are nothing, but the evolution and the learning. So, this evolution and learning are going to help in biological adaptation. Now, if you see this particular evolutional and the leaning; so, these operators are working on two different time scales.

For example, say evaluation is working through a large number of iterations. On the other hand; learning takes space in one’s life time and these two operators are going to help each other. For example, say if you see the principle of learning; now while learning, we use the principle of optimization. And, most of the optimization tools actually work using the principle of evolution.

So, this particular principle of learning or the principle of optimization works through a large number of evaluation. And, if I can learn some good things throughout my life; so, I am just going to pass this particular good information to my next generation.

And, there is a possibility, say due to this particular good information, there is a possibility, the rate of evolution is going to increase. So, the evolution is going to help this particular learning and learning is going to help this particular the evolution.

So, they are going to help each other and there is a possibility; it is going to increase the rate of this biological adaptation. Now, this particular principle of biological adaptation has been copied in evolutionary robotics. Now, here in evolutionary robotics actually what we do is; we try to design and develop some motion planning algorithms using the principle of the evolution and that of learning.

So, what you do is, we try to use some evolutionary tool like some sort of biologically inspired optimization tool. For example, it could be genetic algorithm, particle swarm optimization, and so on. And, we use some learning tool like some sort of neural networks, some sort of fuzzy reasoning tool and we try to evolve the more efficient motion planning algorithm for this particular the robots.

And, that is actually the principle of these particular the evolutionary robotics. So, evolutionary robotics is going to solve this particular motion planning, the problem in a very efficient way. And, nowadays, actually there are many such applications to solve the motion planning problem; using the principle of this evolutionary robotics.

Now, as I told that this particular principle of evolutionary robotics is beyond the scope of this particular course. So, I am not going to discuss in more details the principle of this particular the evolutionary robotics, but as I told that this is one of the possible ways to solve this particular motion planning problem of the robots; particularly, for the mobile robots in a very efficient way.

Thank you.

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Lecture – 41
Intelligent Robot

Now, we are going to start with another topic and that is on Intelligent Robots, we will try to see, how to design and develop an intelligent robot. Now, we call a robot an intelligent one, if it can take the decision as the situation demands. Now, let us see how to design and develop this particular the intelligent robots?

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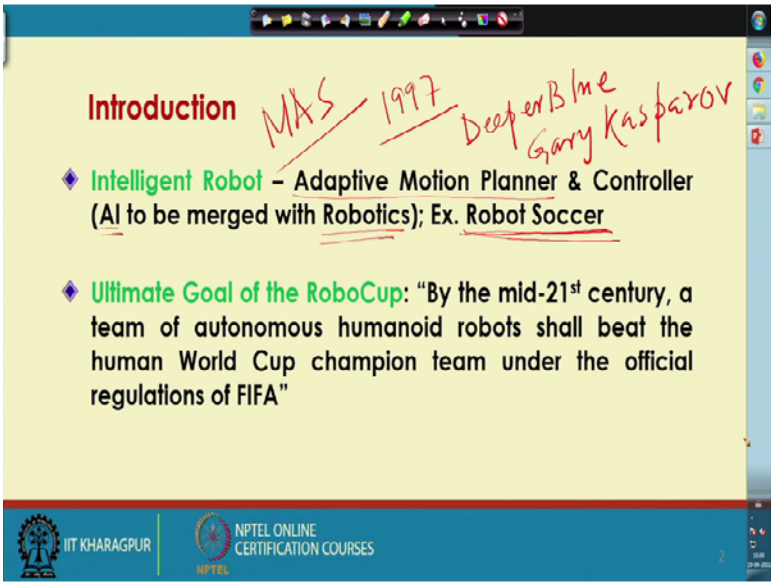

Now, this intelligent robots should have the adoptive motion planner, the reason behind this particular adoptive motion planner I have already discussed that it should be able to take the decision in a varying situation. And, moreover there should be adoptive controller; now while discussing the control scheme of a robot, so we have discussed that each of this particular motor is equipped with one controller and if I use the PID controller, their gain values are to be determined.

Now, if I want to make it intelligent we will have to find out one controller, which is also adaptive; that means, it can take the gain values in an adaptive way, as the situation demands. Now, if you want to design and develop this intelligent robot, we will have to merge the principle of the artificial intelligence; that is AI with this particular robotics and if we can merge the principle of AI or computational Intelligence, that is CI to this particular robotics, we will be able to make the robots intelligent.

Now, here, I am just going to take one example; the example of the soccer playing robots; that means, the football playing robots. Now, before we go for this football playing robots, I am just going to discuss a little bit like the way one expert system, whose name is Deeper Blue could defeat the World Chess Champion; Garry Kasparov. So, we know that in the year 1997, one expert system, the name of the expert system was Deeper Blue.

Now, this Deeper Blue could defeat Garry Kasparov; Garry Kasparov, the world the chess champion, and this particular expert system could defeat Kasparov using the principle of the artificial intelligence. Now, here, this particular chess playing is a very simple task compared to this particular the soccer playing. Because the in chess playing, the environment is static; so, we know what is happening here, we know the position of the different players, ok.

But, here in this particular the soccer playing, that is the football playing task; the field is dynamic. So, the field or the scenarios are going to vary with time, so if the scenarios are going to vary with time; that means the dynamic environment. So, how to tackle? So, let us see and this particular problem is much more difficult actually, what we do in soccer playing robots.

So, there are 2 teams and each team will consist of 11 players and among these players, among the team mates, there will be some sort of cooperation and between the two teams, there will be some sort of competition. And, using the principle of these cooperation, competition and updating, these particular robots are going to play.

Now, each of the robots is having its own goal and that is nothing, but the function of the main goal of the team, that is how to win that particular the game or how to score the goal? Now, each of the robots, as I told is having its own goal and that particular goal is a function of the main goal. And, each of these particular robots are intelligent and they are called agent and that is why, this soccer playing team is nothing, but the Multi-Agent System, that is, MAS; Multi Agent System. Now, here, in this particular multi-agent system, each of these particular robots is intelligent or the agent and they are going to perform in the optimal sense, so that the team can ultimately win that particular the game.

Now, here, as I told that we need adaptive motion planner and adoptive controller; that means, if I want to design and develop, the intelligent and autonomous robot. So, we will have to use the adaptive motion planner and the adaptive controller. And, that is why, the soccer playing robots has becomes much popular and the main purpose of this soccer playing robots is how to design and develop that particular intelligence of these robots.

So, these particular robots can work in a multi-agent system and ultimately, the team of these particular robots can win that particular the game by scoring goal; now, here, the ultimate goal of the robocup was set as follows. So, by mid 21st century, a team of autonomous humanoid robots should beat the human world-cup champion team under the official regulations of FIFA.

So, that particular goal was said by the investigators or the researchers working in this multi-agent system of robotics. Now, to reach that particular goal, actually many people are working throughout the world; some problems have been solved, but still there are many such open research issues, which are to be solved in a very efficient way, so that a team of autonomous humanoid robot will be able to beat the World Cup champion team, the football team, according to the regulations of FIFA.

So, this is a very complicated task and to reach this particular that the target, we will have to make improvement in different areas, so that we can reach that particular the target. Now, let us see a simplified version of that, like how to implement or how to design and develop one intelligent and autonomous robot?

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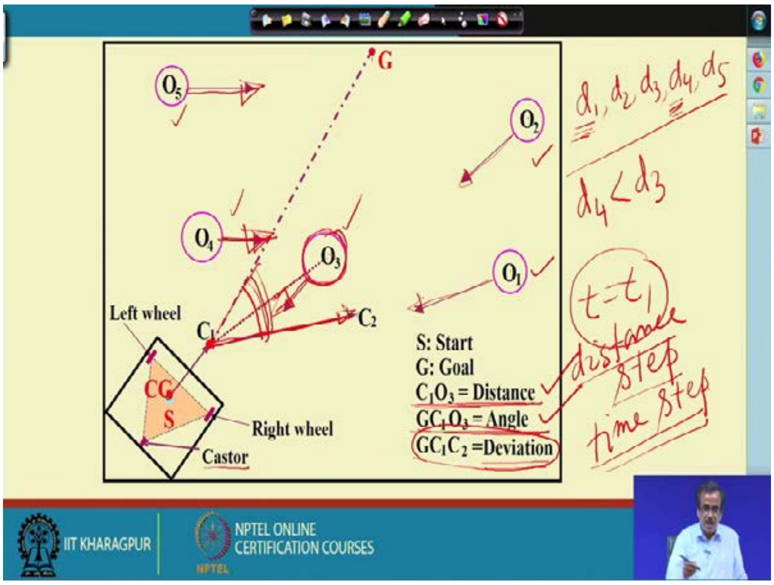

Now, here, I am just going to take one scenario, this is a very simple scenario, but this scenario can be made much more complicated also. So, I am just going to concentrate on this simple scenario; now supposing that this is actually the starting point for a particular robot and this is the goal and here, we are going to consider a 2-wheeled one castor robot.

So, this is the left wheel, the right wheel and we have got a free wheel here or a support sort of thing and that is nothing, but the castor, ok. Now, this point indicates the CG of this particular robot. Now, the physical robot, I am just going to solve and I am just going to explain after sometime in much more difficult. But, before that, let me explain the problem, which is going to be solved with the help of this particular robot.

Now, at time t equals to say t_1, supposing that this is nothing, but the CG of this particular robot and supposing that this is the goal, G is the goal and we have got a few moving obstacles like we have got O_1, O_2, O_3, O_4 and O_5.

Now, for simplicity, I am just going to consider only say 5 obstacles. Now, each of these obstacles is moving with a velocity or speed along a particular direction. For example, O_1 with some speed it is moving in this particular direction; similarly O_2 is moving with this particular direction with some speed, O_3 is moving with this particular direction, O_4 is moving along this direction and O_5 is moving along this direction.

Now, here, to solve this particular motion planning algorithm; our aim is to find out the collision free and time optimal path for this particular mobile robot. Now, to solve this particular problem, we take the help of some sort of the concept of the distance step and the time step. So, we consider the distance step and time step; now during a particular time step, the robot is going to move through a particular distance and that is nothing, but the distance step.

Now, at time t equals to t_1; supposing that this is the CG of the robot and these are all predicted position of this particular the obstacle. Now, here, if it wants to find out what should be the collision free path for this for a particular time step, what it will have to do is, first thing it will have to find out which one is the most critical obstacle?.

Now, here, there are 5 obstacles and we consider the distance between the present position of the robot and the position of this particular obstacle, the different obstacle. And, we try to find out, what should be the distance between the robot and this particular the obstacle. So, here, there are 5 obstacles; so, I will be getting 5 distance values and out of those 5 distance values. So, we will try to find out that which one is the minimum.

Supposing that the distance values are d_1, d_2, d_3, d_4 and d_5; d_1 is the distance between the obstacle 1 and the CG of this particular robot and. So, this is nothing, but the d_1; similarly I will be getting d_1, d_2 up to d_5; we compare all the d values and we try to find out the minimum. Now, supposing that out of all such d values and if you concentrate on this particular scenario. So, this d_4 is found to be the minimum; that means, this particular obstacle that is O_4 obstacle is physically found to be the nearest to this particular robot. But, this particular the obstacle O_4 is moving in this particular direction; on the other hand, the O_3, another obstacle, is moving towards this particular robot.

Now, if I compare the distance here, so, d_4 will be less than the d_3, but as this particular obstacle is moving towards the robot. So, this will be considered the most critical obstacle, but not O_4; that means, your O_3 is considered as the most critical obstacle, because it is moving towards that particular robot. Now, if I just select these as the most critical obstacle, the distance between this particular the present position of the robot and the obstacle. So, this is nothing, but the distance input for the motion planner, so $C_1 O_3$; $C_1 O_3$ is nothing, but the distance input for the motion planner.

Similarly, the angle between the goal; the present position of the robot and the obstacle O_3, that is your G C_1 O_3; so, this particular angle is another input for the motion planner. Now, here, for this motion planner, I have got two inputs; one is the distance, another is the angle. And, to avoid collision, now, we will have to find out like what should be the angle of deviation. Now, here, this angle G C_1, C_2 is nothing, but the deviation angle; so this is nothing, but the deviation angle.

That means, to avoid collision with this particular moving obstacle, the robot is going to follow this particular path or the robot is going to deviate from this particular path just to avoid the collision with the moving obstacle. So, the output of the motion planner will be this particular the deviation angle, that is nothing, but G C_1 C_2. So, this is nothing, but the deviation of the motion planner and we can also consider another output of the motion planner, that is nothing, but the speed of the robot or the speed or the acceleration of this robot.

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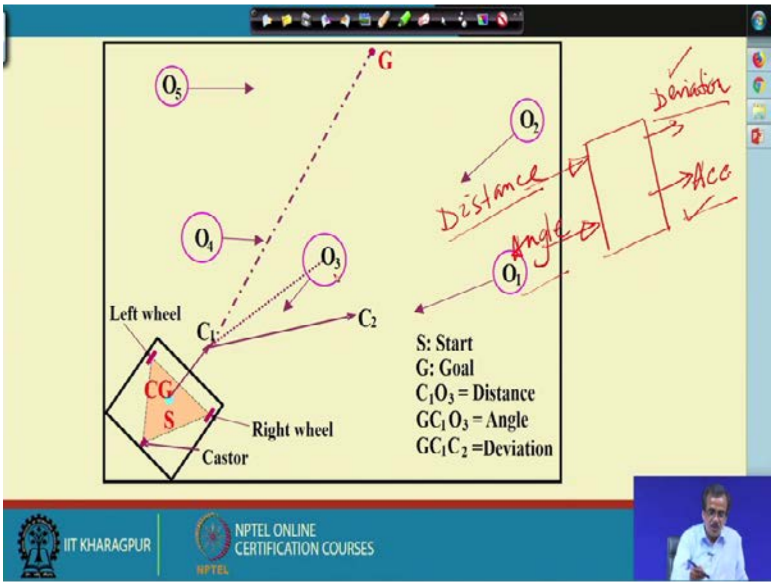

So, if I just try to find out what should be what should be the inputs and outputs of the motion planner; now if I just draw the block diagram of these particular motion planner, there are 2 inputs one is the distance input the distance input and another is your the angle input. And there are 2 outputs one is nothing, but the deviation angle and another could be the speed or the acceleration of these particular the robot.

So, if I know the acceleration, we can also find out the speed. So, there are two inputs: distance and angle and there are two outputs, that is, deviation and acceleration of this particular the robot. And, using this motion planning algorithm; we can find out; what should be the angle of deviation and what should be the acceleration, so that the robot can avoid collision with this particular the moving obstacle. Now, let us see, how to implement it in the real experiment.

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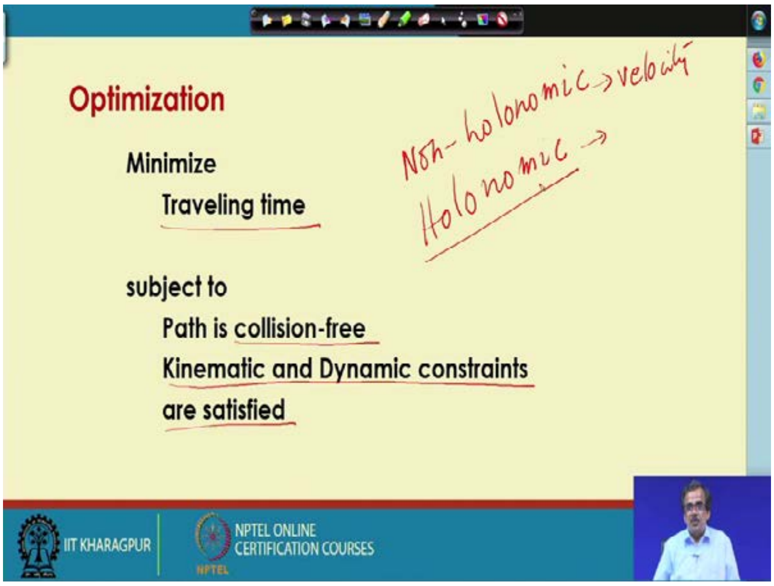

Now, here, we can consider that this is nothing, but an optimization problem and our aim is to minimize travelling time; that means, the robot will be able to reach the goal, starting from the initial position in minimum time. And, at the same time, the path should be collision free; so there should not be any collision between the robot and this particular the moving obstacle. And, moreover, the kinematic and dynamic constraints of this particular robot are to be the fulfilled.

Now, regarding these particular kinematic and dynamitic constraints actually, before I discuss a little bit, I will have to show the physical model of this particular the robot and now, I am just going to show you the physical robot the physical model of this particular robot.

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Now, this is nothing, but a two-wheeled, one castor robot; now, we can see that we have got one wheel here, we have got another wheel here and we have got one support and that is nothing, but the castor. So, this is a two-wheeled, one castor robot and this is nothing, but a two-wheeled one castor differential drive robot.

Now, for each of these particular wheels, we have got a separate motor. So, this motor is connected to the wheel and of course, for each of this particular motor; there must be a controller. And, here, we use some sort of PID controller just to control this particular the motor. So, that we can generate some speed at the two wheels and accordingly, will be getting that movement of this particular the robot.

Now, we will be discussing, in details, how to make it intelligent? But, before that, let me tell you now regarding this kinematic and dynamic constraints; so, the kinematic constraint for these particular robot could be of two types. For example, say, there could be non-holonomic constraints, and there could be holonomic constraints.

So, by non-holonomic constraints, we mean those constraints, which are dependent on velocity. So, these non-holonomic constraints are dependent on velocity of the robot and these holonomic constraints are independent of the velocity. So, these constraints are to be fulfilled, otherwise, we cannot generate the movement of this particular robot particularly, whenever it is taking a turn.

Now, if I just concentrate once again on the physical model; we can see that supposing that it is going to take a turn, the robot is going to take a turn. Now, if it is going to take a turn on the left side; on the right hand side actually the RPM or this speed of this particular wheel should be more compared to that that of the other side, then only I can take a turn.

And, while taking this particular turn; these particular constraints like non-holonomic and holonomic constraints are coming in to the picture. And, moreover, each of these particular the motors is controlled by the controller and this motor is going to generate the torque required just to give some sort of rotation to the wheels. Now, to determine the power rating of this particular motor; we always try to find out, how much is the torque requirement of these particular wheels?.

So, these dynamic constrains are going to tell us like how to decide the power rating of the motor so that your these particular motor will be able to provide the necessary torque; so that it can generate that particular RPM to this particular the wheel. So, these kinematic and dynamic constraints are to be fulfilled and at the same time, the path has to be the collision-free. Now, let us see, how to carry out this particular the real experiment.

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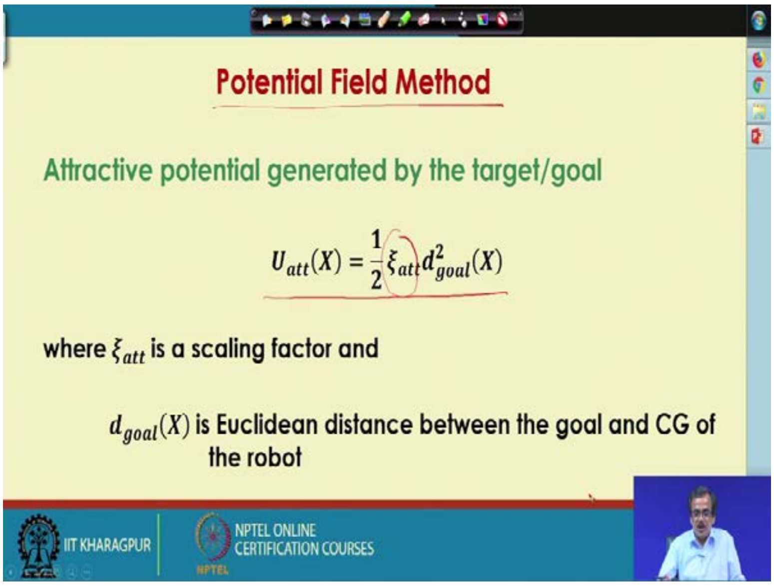

Now, before I go for this particular real experiment, the motion planning algorithm which I am going to use for this experiment is nothing, but the potential field method.

The principle of which I have already discussed in much more details and as I told, this particular potential field method is going to work based on the concept of the attractive potential and the repulsive potential. Now, let me repeat that this attractive potential, $U_{att}(X) = \frac{1}{2}\xi_{att} d_{goal}^{2}(X)$ and this d_goal (X) is nothing, but the distance between the goal and this particular the CG of the robot.

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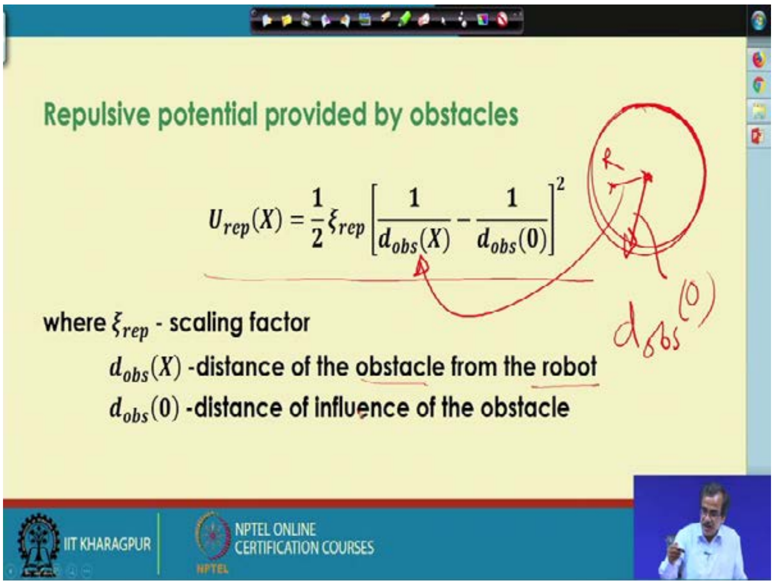

And, this goal is going to attract that particular robot and on the other hand, we have got one repulsive potential, which is nothing, but your U_repulsive (X); this I have discussed in much more details and here I am just going to consider. So, this particular expression for the repulsive potential $U_{rep}(X) = \frac{1}{2}\xi_{rep}[\frac{1}{d_{obs}(X)} - \frac{1}{d_{obs}(0)}]^2$.

Now, this particular d_obstacle (X) is nothing, but the distance between the obstacle and this particular robot. And, your d_obstacle (0) is nothing, but supposing that I have got an obstacle here. And, surrounding that obstacle, we consider one circle and that is called the circle of influence. Now, if this is the center of the circle; so, this is nothing, but is your d_obstacle (O) sort of thing. And, this d_obstacle (X); supposing that the robot is here; so, this particular distance is nothing, but is your d_obstacle (X), ok.

Now, using this actually, the obstacle is going to put some sort of repulsive force on the robot. And, using these attractive and repulsive forces, attractive and repulsive potentials, the robot is going to move towards the target or the goal. And, ultimately, the robot is going to reach the goal and this robot will be under the combined action of attractive and repulsive potentials or attractive and repulsive forces.

And, I have already mentioned that we consider that there are two inputs for these motion planer, that is your distance and angle. And, there are two outputs, that is nothing, but the angle of deviation and acceleration or speed of this particular the robot. Now, this I have already discussed in much more detail. So, I am not going to spend much time on this.

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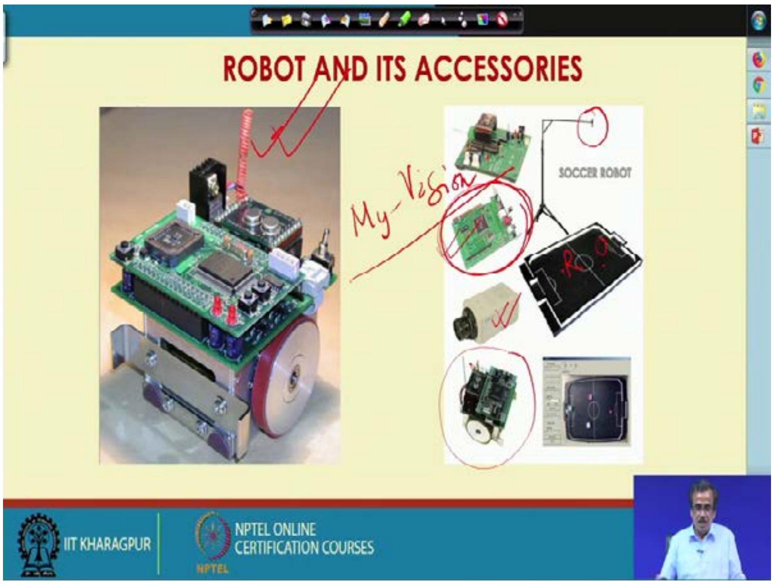

And, now, I am just going to show you the robot, the physical model; I just showed a few minutes ago. Now, this is actually the photograph of the same robot, now here, you can see that these are the wheels and the castor we cannot see.

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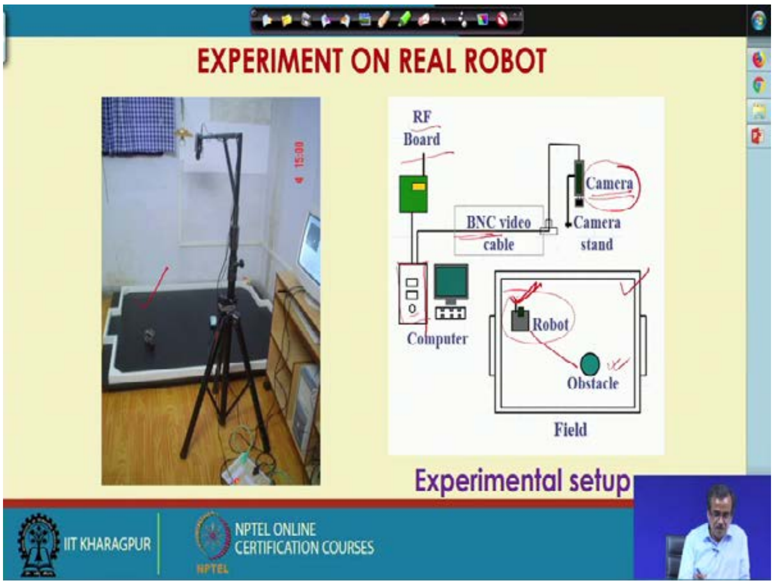

Now, this is nothing, but the antenna used for the radio frequency module. Now, this robot is wireless; so, we will have to use some sort of RF module or the radio frequency module. So, which I am going to discuss in details and with the help of this particular antenna; so, we are going to send signal to the controller for this particular the motor.

Now, as I told that to control the movement of this particular, the wheel, we have got a motor. The motor is not visible, they are inside and each of these motors is connected to the controller and this robot is having actually the micro-controller. So, that a little bit of calculation, a little bit of the decision it can take with the help of that particular motion planning algorithm.

Now, let us see, how does it work? Now, here, actually we are going to take the help of one camera. So, this is nothing, but a CCD camera and this is the stand of the camera. So, here, we are actually inside that particular camera and this is the field, on which we are going to carry out this particular the experiment. So, on this particular field, we have got the said robot, say robot is denoted by R and we have got some obstacle, say it is denoted by O.

Now, let us see, how can this particular robot take the decision just to avoid collision with these particular the moving obstacles. Now, here, we can see that this is nothing, but is actually the my vision board; so, this is my vision board. And, this particular the hardware; now it is for how to carry out that image processing online with in a fraction of second? Now, this particular my vision board, so we will have to put it in the CPU or the computer and I am just going to tell you the method through which, we just carry out this particular the experiment.

So, this my vision board has to be put in this particular the CPU. Now, let us see, how can you send this particular information of the environment, so that we can carry out this particular the experiment and this is actually the whole view of this particular the robot.

Now, let us see, how can you implement this particular the principle of the motion planning to make it intelligent, ok? Now, this shows actually the experimental set-up, we developed; now this is nothing, but the field. So, this is nothing, but the field in a real experiment, the same field I am just drawing it here and as I told that we have got this particular robot. So, this is nothing, but the robot and we have got the obstacle here.

Now, let us see, how can we take the decision to avoid collision with this particular the moving obstacle. Now, here, we use one camera that is called the overhead camera, I can also use onward camera; that means, the camera can be mounted on the body of this particular the robot.

Now, here, we had just going to a put one overhead camera and this camera has to be calibrated. So, the camera calibration is the first task actually the quality of the image collected with the help of camera depends on a number of parameters. For example, say if depends on, what should be the focal length of the lens; it depends on some sort of scaling factors and some other factors, those factors are to be determined with the help of the calibration. So, this particular camera has to be calibrated first and supposing that this camera has been calibrated. Now, with the help of camera; so, we can collect information of this particular the environment; that means, we can take the snap of this particular environment at a regular interval.

Now, depending on this speed of this particular camera; so, we can take snap of this particular environment at regular interval. And, this particular information, that is the information of the environment or this particular image that will pass through the BNC cable and through this BNC cable; the information of the image or the environment it will go to the CPU or this particular the computer.

Now, here inside this particular CPU; we have put that my vision board or the image processing software. So, the information of the image will enter this my-vision board and there will be some sort of image processing. And, through this image processing, the principle of which, I have already discussed in much more details; we can find out the information of this particular the environment.

That means, I can find out the distance between the robot and this particular obstacle and the angle through which this obstacle is moving towards the robot and these two are nothing, but the inputs of the motion planner. And, once we have got the inputs of the motion planner; now the motion planning algorithm, that is, the potential field method is going to determine, what should be the output; that means, what should be the deviation and what should be the acceleration or the speed of the robot.

And, using this particular deviation angle and the speed of the robot; so, through some small programming; we can find out what should be the RPM of the two wheels ? Now, once we have calculated; these particular RPM values, what we do is. So, this particular information will pass it through this particular the radio-frequency module, that is nothing, but the RF board or the RF module.

And, through this RF module; so, we are going to pass the information through this particular robot. And, in this particular robot, we have got your antenna; so, through this wireless communication, the information regarding the RPM or the speed requirement of the two wheels will be passed through the controller of this motor, which is connected to the two wheels of the robot.

And, now with the help of this controller like the PID controller or PI controller; so, this particular motor is going to rotate the wheel and we will be getting this particular movement at the two wheels of the robot. And, consequently, the robot will be able to move in the forward direction or in the backward direction; it will be able to take some turn either clockwise or anti-clockwise. So, this shows actually the photograph of this experimental set up, this is nothing, but the overhead camera.

And, this is the field and we can see that we have got this particular the robot here and we have got the obstacle here, ok. And, this is the CPU, which you use for the carrying out this experiment and this is the display of that particular the computer. And, let us see, how can you carry out this particular the experiment.

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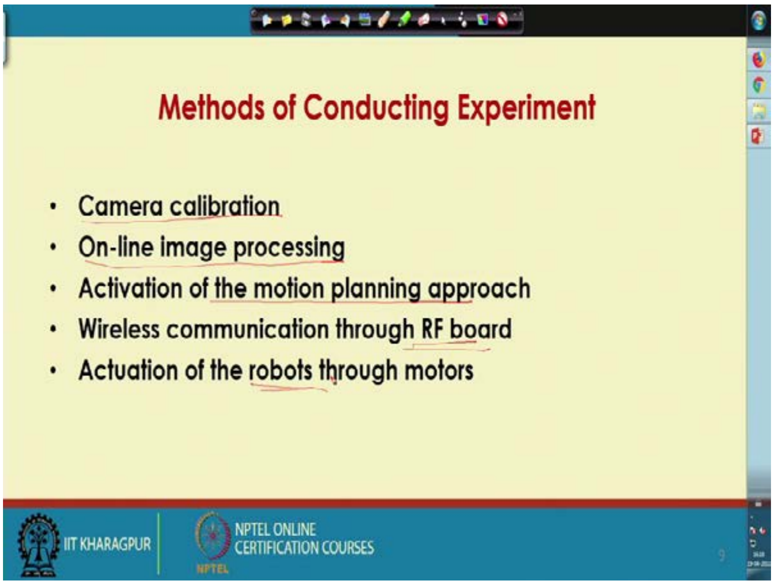

Now, to carry out the experiment, as I told that these are the different steps, which are to be followed; for example, the first step is, we will have to calibrate this particular camera. Then, we can go for some sort of online image processing, which I have already discussed, then we will have to activate the motion planning approach; that means, here we are going to use the potential field approach.

Then, there must be some wireless communication through the radio frequency module. So, that we can pass the information of the required RPM at the two wheels to the respective controller of the motors and with the help of this particular controller, the motor is going to generate that particular the required motion or the required torque. And, then we will be able to get some sort of movement of the robot, ok.

Now, I am just going to show you one video just to show you this particular experiment; the way we carried out this particular the experiment. Now, here, I just want to acknowledge, we got one DST project, that is, Department of Science and Technology; Government of India project.

With the help of this particular project, actually we conducted this particular the experiment and the video of this particular experiment I am going to show you. And, here, one SRF worked on this particular the experiment Dr. Nirmal Baran Hui, who worked on this particular project to develop this particular the experimental set-up.

So, we are just going to show the video of our experiment like how could we develop the intelligence of the robot; that means, how could you design and develop an intelligent wheeled robot, the mobile robot. So, I am just going to show you that particular the video.

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Now, this is the robot, which I actually showed you and here, the robot is now moving in the forward direction. And, it will be able to move in the forward and the backward direction. And, now, I am just going to show you the way, it can avoid the collision with the two static obstacles. Now, this particular robot, the wheel robot can move in the forward direction and backward direction. Now, I am just going to show you that how we can avoid collision with one static obstacle; so this is nothing, but the static obstacle.

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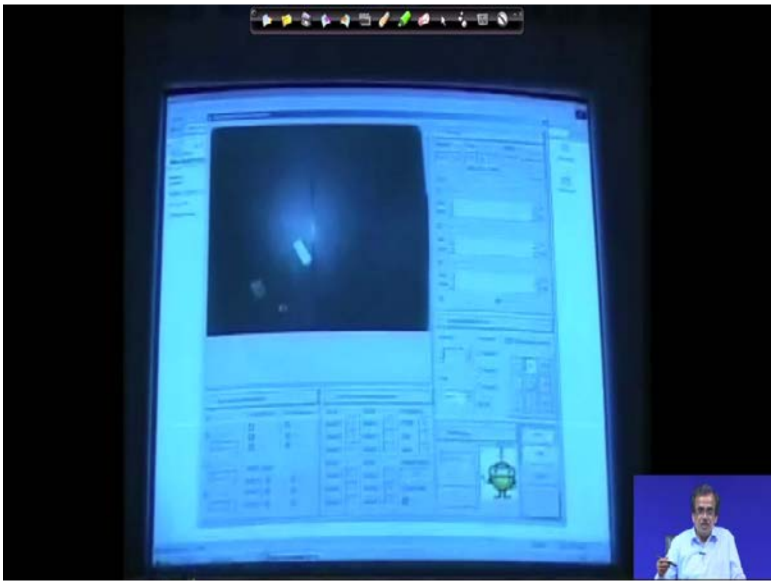

So, with help of this particular the robot; with the help of the motion planning algorithm, we will be able to find out the collision free path.

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Now, this is another example like how to avoid the collision with the two static obstacles? So, the robot is going to a avoid collision with both the static obstacles and it is going to reach the goal.

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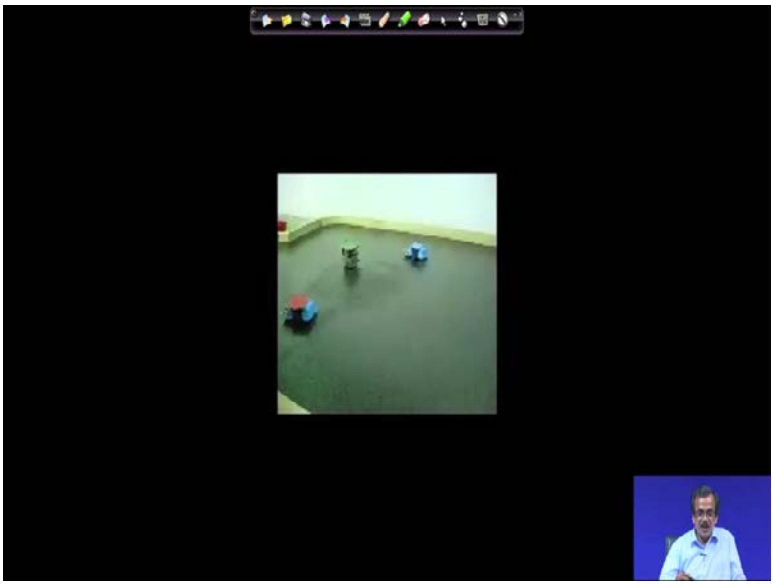

Now, I am just going to show you, how to use the potential field method? Now, you can see that the robot has become almost stationary and after it has crossed the obstacle, the moving obstacle, now it is moving with the high speed to reach that particular the goal.

Now, this is the way with the help of the motion planning algorithm and with the help of this controller; we can incorporate intelligence to this particular the robot. Now, here although we used some sort of motion planning algorithm to make it intelligence, sometimes we take the help of some sort of reactive control along with some sort of the motion planning algorithm; just to incorporate intelligence to this particular the robot.

Now, the principle of which you have used here to make this particular wheeled robot intelligent one, more or less the similar type of principle we can use to make the different types of mobile robot intelligent. For example, we can make, say multi-legged robot like say 4-legged robot, 6-legged robot or 2-legged robot an intelligent one, using more or less the similar type of principle.

Now, here, if I just consider a legged robot, beside this motion planning, we will have to consider the gait planning also. Now, we are not going to discuss the gait planning and other things, in details, ok. Now, if you are really interested, you can have look a look into the textbook, that is, the fundamentals of robotics written by me. Now, that particular book is the textbook for this particular course, as I mentioned earlier. So, we will have to concentrate on that particular textbook just to collect more information.

Thank you.

Robotics
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Lecture – 42
Biped Walking

Now, I am going to discuss on a new topic and that is on Biped Walking. Now, before I start, let me define what do we mean by this biped robot? Now, this biped robot is actually a simpler version of this particular the humanoid robot. So, humanoid robot is very much complicated and this biped robot is actually the simpler version of that particular the humanoid robot.

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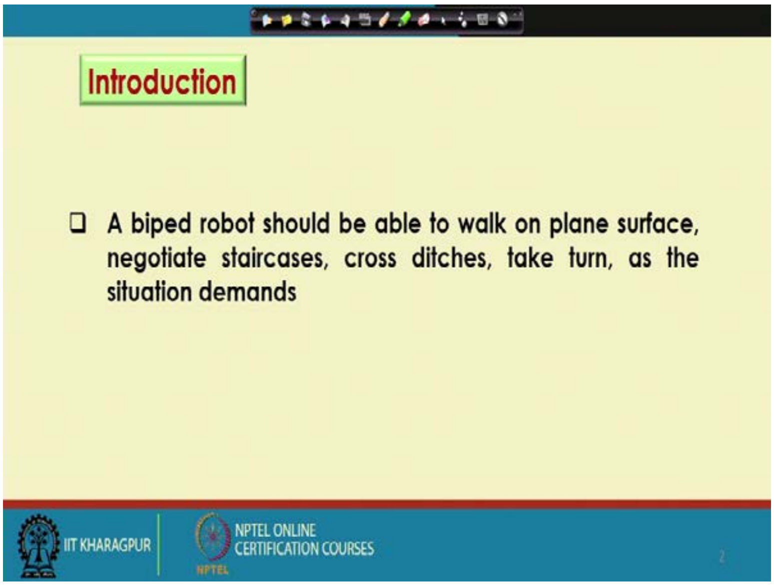

Now, a biped robot should be able to walk on the plain surface, it should be able to negotiate the staircases, take turn, cross ditches as the situation demands. Now, while walking, this particular biped robot should be able to maintain its balance and that balance is nothing, but the dynamic balance. Now, I am just going to discuss in details, like how can it maintain the dynamic balance.

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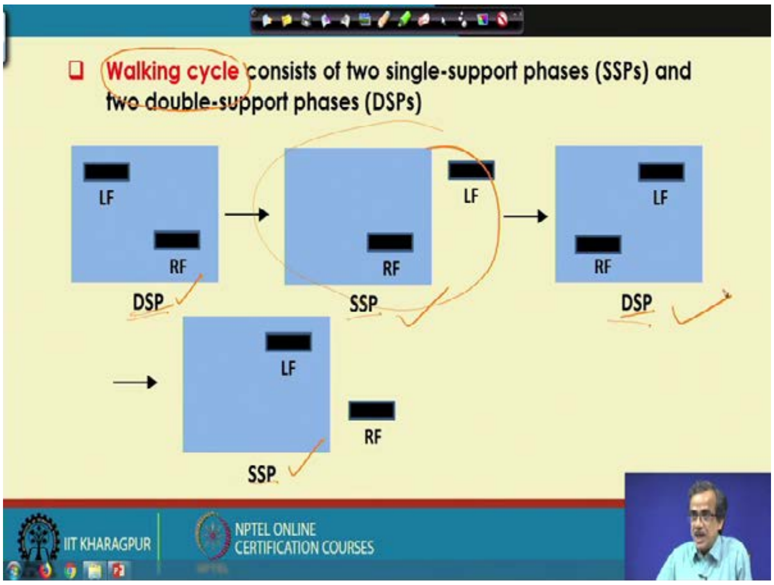

So, here, I am just going to discuss the walking cycle of a biped robot; now if you see, if you concentrate on this particular figure, we can see that these LF and RF are nothing, but the left foot and the right foot and these two feet are the ground feet. Now, let me assume this particular the rectangular box indicates actually it is the ground. So, both the feet are placed on the ground and this is nothing, but a double support phase.

Now, after this double support phase what happens? The right foot will remains at the same position and the left foot that will be taken away from the ground and now, it is an air. So, here, in this particular configuration, for example, this particular configuration that is the single support phase configuration; the right foot is on the ground and the left foot is in air and this is a single support phase.

Now, after that; there will be another double support phase, now here, this particular right foot that is already there on the ground and the left foot that will be placed on the ground. So, here both the feet are on the ground and this is nothing, but the configuration of the double support phase. And, after that, this particular the left foot will be on the ground and the right foot will be put in air and this is once again a single support phase.

Now, starting from this particular double support phase, then there will be one single support phase, then double support phase and single support phase that completes actually one walking cycle. So, one walking cycle consists of 2 single support phases and there will be 2 such double support phases. Now, while walking this particular biped robot should be able to maintain the dynamic balance during its single support phase as well as the double support phases.

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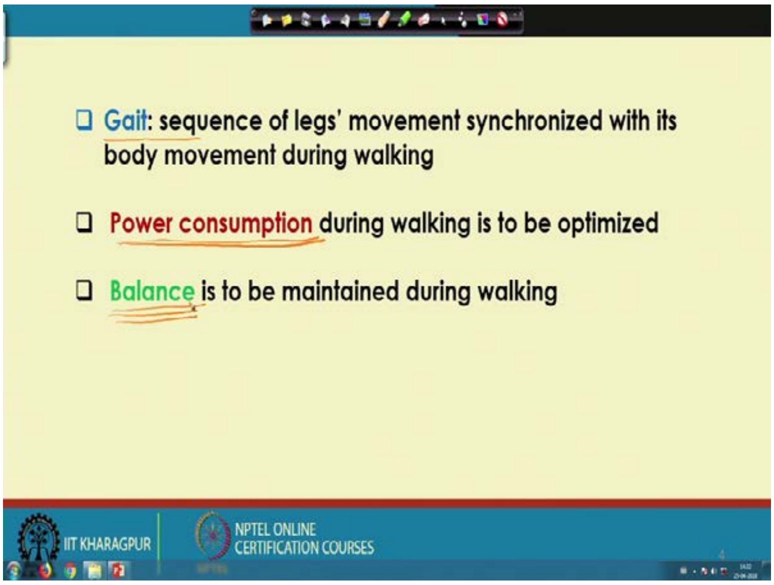

Now, here, I am just going to discuss, how to maintain that particular the dynamic balance. Now, before that, let me define like what do you mean by gait; the term gait is very frequently used in biped walking. Now, by gait, we mean it is the sequence of legs movement in coordination of the body movement, which is required for walking of that particular the biped robot.

Now, while walking, this particular biped robot should be able to consume the minimum amount of power, but at the same time, it should have the maximum dynamic balance margin. So, I am just going to discuss, in brief, how to determine this particular the power consumption during walking and how to maintain this particular the dynamic balance.

Now, here actually, what I am going to do? I am just going to discuss, in brief, just to make it simple, but the exact derivation or the detailed derivation, if you want to have a look, you will have to concentrate on the textbook, that is, the fundamentals of robotics written by me. So, there are all such things are dealt in much more detail, but here, as I told for simplicity I am just going to discuss, in brief, how to determine the power rating for this particular biped robot and how to determine the balance margin.

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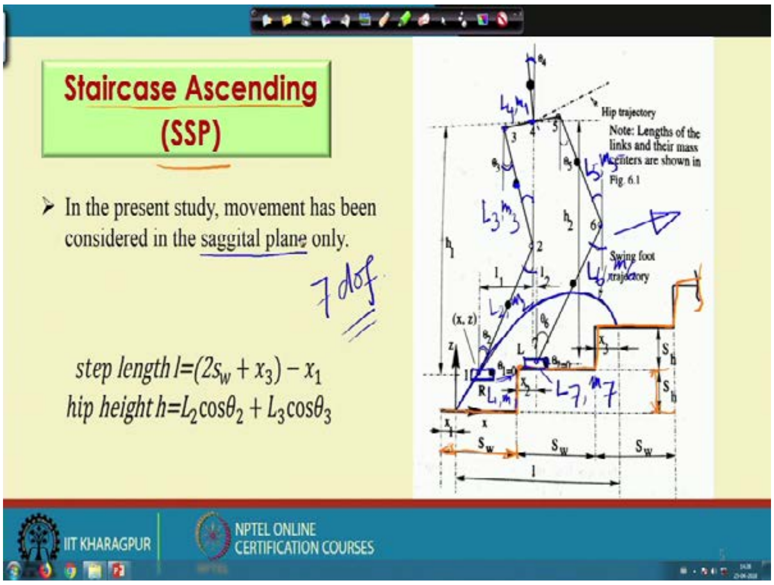

Now, here, I am just going to concentrate first on this single support phase, and here, for simplicity, I am just going to concentrate on a particular task that is nothing, but the ascending of staircase. So, I am just going to discuss staircase ascending and that too for the single support phase. So, let us see, what happens during the single support phase and whenever this particular biped robot is going to negotiate or going to ascend through the staircase. Now, here, on this particular figure, the staircase; that is denoted by these. So, these are nothing, but the steps of the staircases.

So, these are nothing, but the staircases now, here. So, this S_w that particular symbol indicates the width of the staircase and S_h is nothing, but the height of this particular the staircase. And, here, this is the single support phase; so, only one foot will be on the ground and the other foot will be on the air.

Now, here, out of these two feet; this particular foot is on the ground and this particular foot is in air because this is a single support phase. And, this indicates actually the trajectory of this particular the swing foot that is the foot which is in the air. So during this particular walking through the staircase, this is the locus of the swing foot; so this is nothing, but the swing foot trajectory.

Now, this particular swing foot trajectory I am just going to represent with the help of some mathematical expression and we will derive that particular the mathematical expression. Now, before that let me tell you that this indicates say one foot, this is another foot, this is one link, this is another link, another link, another link, another link and this is the foot.

So, let me write here; the length of this particular the foot is denoted by say L_1 and its mass is denoted by m_1. Similarly, for this particular link supposing that the length is your L_2 and the mass is m_2 and this particular mass is concentrated at this particular the point. Similarly, for this particular link, the link length is L_3 and the mass is m_3 and say, m_3 is concentrated here at this particular the point.

Similarly, for this particular link, the length is L_4 and the mass is m_4. Now, here the length is L_5 and mass is m_5, this here the length is L_6 and the mass is your m_6. And, for this particular foot the length is L_7 and mass is nothing, but m_7.

So, here, there are 7 links say L_1, L_2, L_3, L_4, L_5, L_6 and L_7 and here I am just going to consider for simplicity only 7 degrees of freedom. So, here, I am going to consider a biped robot having 7 degrees of freedom and the joint angles. So, all the joints are actually the rotary joints and the joint angles are denoted by say θ_1. Here, θ_1 is equal to 0, then comes here, we have got θ_2; the second joint angle. So, this is also θ_2 then comes your θ_3; so, this is also θ_3 and the joint angle θ_4.

Similarly, the joint angle θ_5 then comes θ_6 and θ_7 and for simplicity, we have assumed that θ_1 is equal to 0 and θ_7 is equal to 0. And, this particular joint is actually the hip joint and here, we consider that this joint, that particular joint and this particular joint all 3 joints are coinciding. So, this is actually the ankle joint, this is the knee joint and this is the hip joint, similarly on the other leg, this is nothing, but the knee joint and this is the ankle joint.

Now, here, let us see, how to determine the power consumption, if this particular robot is planning to negotiate the staircase; in this particular direction. And, here, for simplicity, we are going to consider the movement only along the sagittal plane; that means, the sideways movement, we are not going to consider, for simplicity.

Now, let us see, how to carry out this particular the analysis, but as I told that I am not going to discuss in details, the mathematical derivation which is available in the textbook of this particular the course. Now, here, the step length that is denoted by

$l = 2s_w + x_3 - x_1$. So, here, actually the step length, this is nothing, but the distance between this particular point and this particular point. So, this is actually the step length, that is l and this $l = 2s_w + x_3 - x_1$; so, this is your x_3.

So x_3 plus s_w plus s_w minus this particular x_1, that is from here to here; so, from here to here. So, this is nothing, but the step length, that is nothing, but l. Similarly, the height of this particular hip, that is denoted by h is nothing, but $L_2 \cos\theta_2 + L_3 \cos\theta_3$. Now, this is your L_2; the length of this particular the link; this angle is θ_2. So, your $L_2 \cos\theta_2$ is nothing, but this; so from here; so this will be your $L_2 \cos\theta_2$.

Similarly, this is your L_3 and this particular angle is nothing, but θ_3. So, from here to here is nothing, but is your $L_3 \cos\theta_3$; so $L_2 \cos\theta_2 + L_3 \cos\theta_3$ is nothing, but the height of this particular the hip. So, these two terms actually will have to be defined for the purpose of analysis and here, another thing. So, this is the hip joint, now during this particular walking through the staircase; so the hip should also follow a particular trajectory.

Now, here, for simplicity, we have assumed that the hip is going to follow a straight path. And, the slope of this particular straight line is nothing, but the slope of this particular the staircase. So, if I just try to find out the slope of the staircase and the slope of this particular the hip; so, they are having the same slope. Now, this is the way actually mathematically, we are going to describe this particular the configuration for the purpose of analysis.

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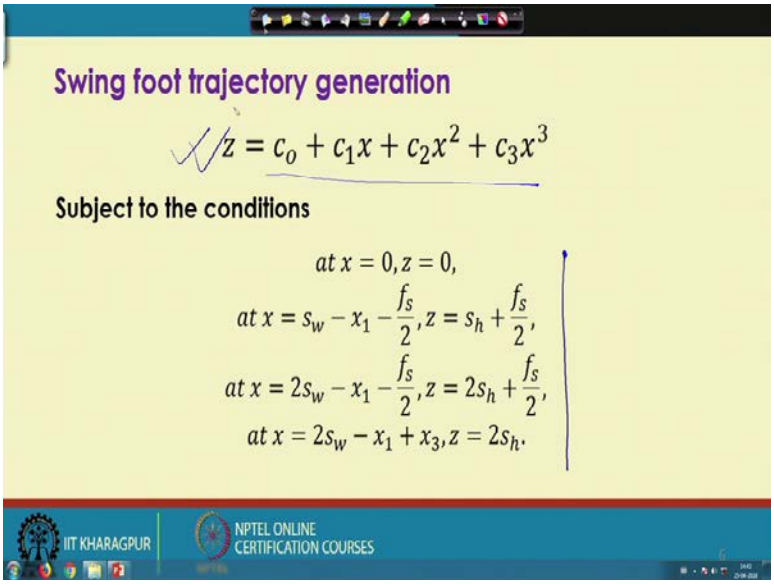

Now, here, as I told that this particular swing foot should have some trajectory, now for simplicity, we have considered that the swing foot is going to follow one cubic polynomial of this particular form; that is $z = c_0 + c_1x + c_2x^2 + c_3x^3$. Now, here actually, for this cubic polynomial, there are 4 unknowns like c_naught, c_1, c_2 and c_3. Now, if I want to solve it; I will have to take the help for such known conditions and these are nothing, but the boundary conditions. Now, if I discuss that let me once again go back now this is nothing, but is actually the hip trajectory, which I am going to represent mathematically.

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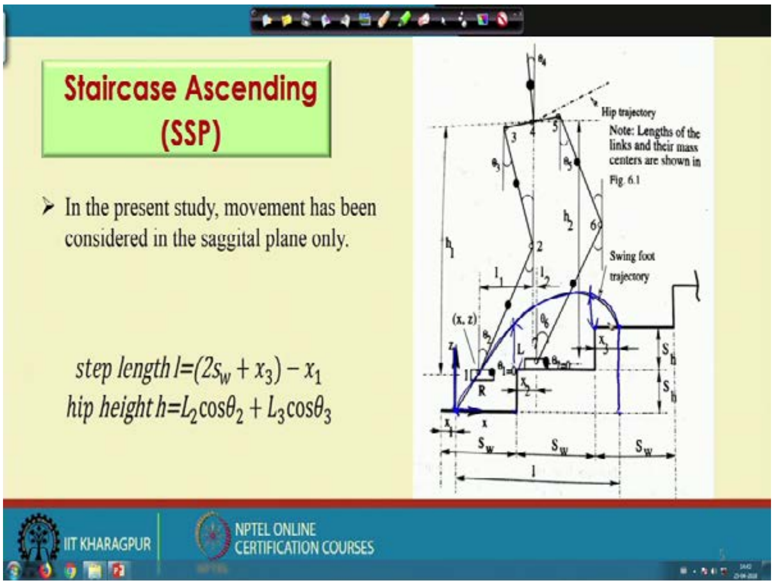

Now, here, this is the coordinate system, this is your x and this is z. So, at this particular point, the z height is actually equal to 0; similarly, when x is this at that particular situation, I can find out that this is nothing, but the height along this particular z and then, we try to find out for a particular value of x.

So, when x is here, I can find out this much is actually your z, when x is this much here. So, this is nothing, but the value of this particular the z. So, using these 4 conditions; so, I can derive this particular the cubic polynomial. So, I am just going to write down all such conditions here; so, the conditions are written here.

So, these boundary conditions, the four boundary conditions are written here, for example, at x equals to 0; z is equals to 0, and so on. And, if you use this boundary condition, we can find out what should be the values for these c_naught, c_1, c_2 and c_3. And, once you have got the values of the coefficient, I can represent this swing foot trajectory.

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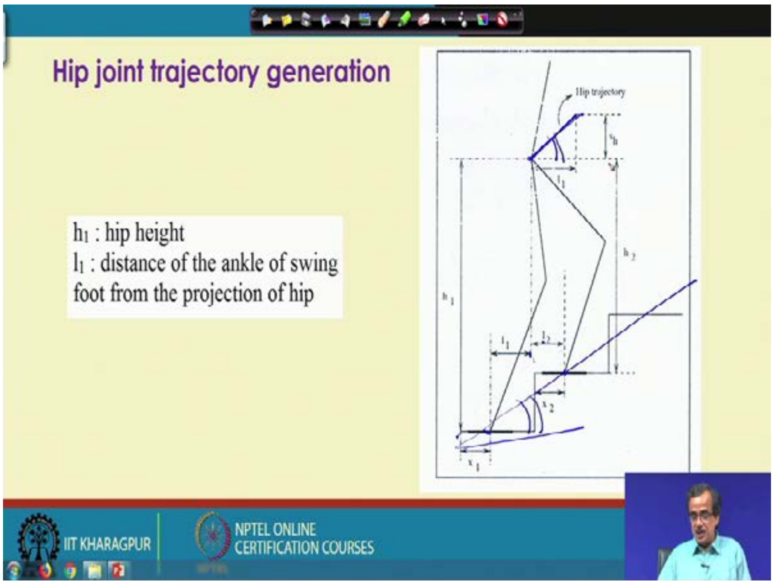

Now, once you have got this particular the swing foot trajectory; next we try to find out the hip joint trajectory. And, I have already mentioned that we have assumed that this particular hip joint is going to follow a straight path and whose slope is nothing, but the slope of this particular the staircase.

So, this particular angle and that particular angle, they are the same. Now, here, actually there is a chance of optimization, we can find out a suitable optimal slope or optimal trajectory for this particular the hip joint. But, here, for simplicity, we consider that the slope of this particular trajectory is same as the slope of the staircase.

Now, if I concentrate on this particular hip joint, I can find out. So, if I take the projection of this particular hip joint; I can find out the distance between this ankle joint and the projection of the hip joint. Similarly, I can find out the distance between the projected point from the hip and the distance of this particular the ankle joint and that is denoted by l_2. And, I can also find out, what is h_1, that is the height of this particular the hip joint and I can also find out what is h_2, that is nothing, but the height of this particular the hip joint.

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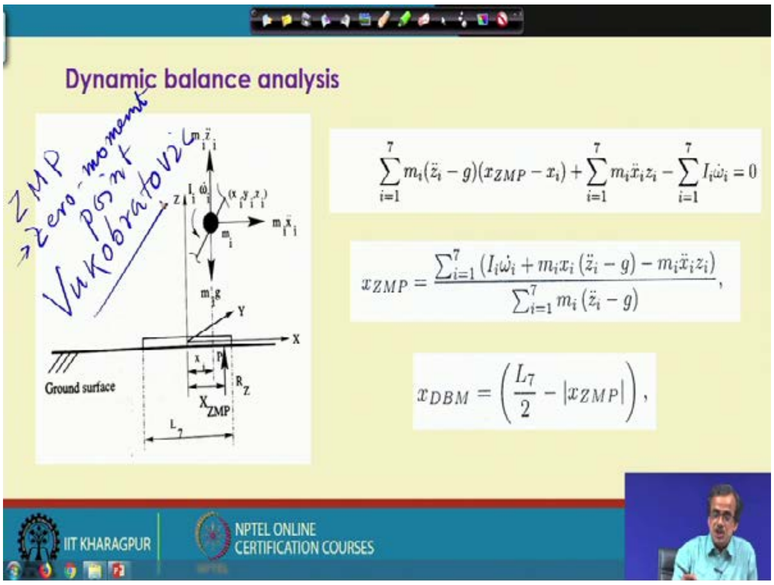

Now, knowing this particular, what you can do is, we can carry out the analysis for this dynamic balance. And, we can also find out the expression for the power consumption. Now, here, I am just going to discuss a little bit, how to maintain the dynamic balance for this particular the biped robot.

Now, before I proceed further, I just want to mention that we human-beings, we are not statically stable; we are dynamically stable. Even if we are standing at a particular location, we are not statically stable, but we are dynamically stable. Now, let us see, how to maintain that this particular the dynamic balance. Now, here, I am just going to use the concept of the ZMP and that is known as the zero moment point. So, ZMP is the zero moment point and the concept of ZMP was introduced by Vukobratovic and this particular concept has become very popular.

Now, let us try to understand, how can you find out this particular ZMP or the zero moment point. Now, to find out this particular zero moment point, actually what I am going to do is, I am just going to consider, say a particular link for the robot. And, for this particular biped robot, I am just going to consider say a particular leg.

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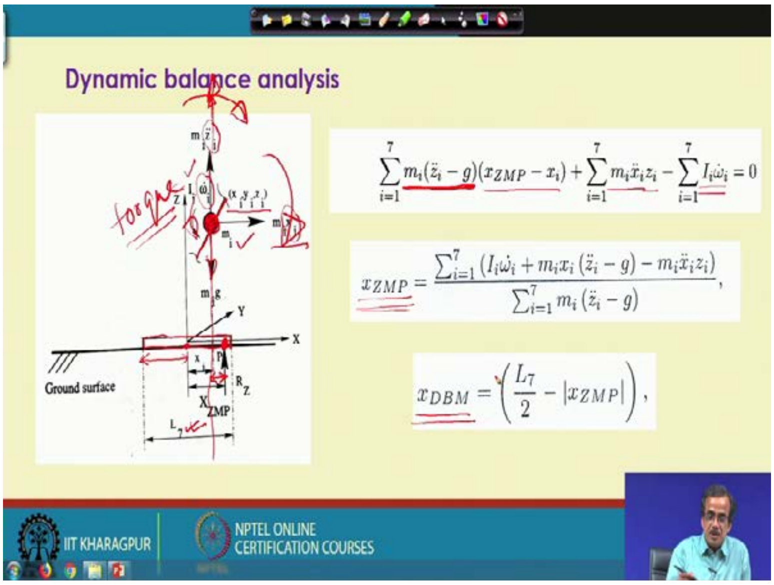

Supposing that that particular leg is denoted by this, this is nothing, but a link or a leg and this particular link or the leg is having one concentrated mass and this particular mass is denoted by this. And, supposing that for this i-th leg or the i-th link; the mass is denoted by m_i and this mass center is having the coordinate that is nothing, but x_i, y_i, z_i. Now, let us see, how to determine the ZMP, that is, the zero moment point. Now, here, this is nothing, but the foot which is in touch with the ground; so, this is nothing, but the foot.

And, this foot is having the length and this length is denoted by L_7. And, this is nothing, but the center of this particular the foot; that means, that is at the midpoint. So, this particular length is nothing, but L_7 by 2; so, this is the midpoint.

Now, let us see, how to derive and how to determine this ZMP. So, before I define actually, let me tell, what do you mean by this particular the ZMP? Now, ZMP is actually a zero moment point, which is a hypothetical point and this is a point about which the sum of all the moments becomes equal to 0. Now, let me repeat; ZMP is a hypothetical point, now this is a point about which the sum of all the moments becomes equal to 0.

Now, here so this particular mass m_i; so this is subjected to a few forces. For example, say; g is the acceleration due to gravity. So, this particular m_i g is acting vertically downward; so this is the direction along which this m_i g is acting vertically downward.

Now, here, if I consider the movement of this particular mass along the x direction and the movement of this particular mass along the z direction. And, if I say that along the x direction, there is one acceleration, that is nothing, but x_i double dot and along this particular z direction, there is one acceleration that is nothing, but z_i double dot. Then, we can say that there is a force acting along the x direction, that is, your m_i x_i double dot, mass multiplied by acceleration is the force.

Similarly, here, along this particular z direction; m_i, z_i double dot, that particular force is acting and moreover, here, this is a rotary movement the link is rotating. So, here, we will have to consider the moment of inertia and this I is nothing, but the moment of inertia of the i-th link or the i-th leg and this $\dot{\omega}_i$, that is nothing, but the angular acceleration.

So, the moment of inertia multiplied by angular acceleration is nothing, but is actually a torque. For example, say force multiplied by linear acceleration, sorry, mass multiplied by linear acceleration in force. Similarly, the moment of inertia multiplied by the angular acceleration is nothing, but is actually the torque.

So, here, this is subjected to the torque, that is, $I_i\dot{\omega}_i$ and then, it is subjected to the force like $m_i g$; then comes a $m_i, \ddot{x}_i$, then $m_i, \ddot{z}_i$. Now, here, I am on this particular the ground foot; so, I am just going to consider a hypothetical point. Supposing that, the point is here; now corresponding to this particular point, let us try to find out what should be the moment and we just put the sum of those particular moments equal to 0.

Now, here, the vertically downward force is nothing, but $m_i g$ and vertically upward is $m_i\ddot{z}_i$. And, truly speaking, this $m_i\ddot{z}_i$ is larger compared to this particular $m_i g$; because this is moving vertically upward direction. So, what is the difference between these two forces; the resultant force is nothing, but $m_i\ddot{z}_i - m_i g$. So, this is nothing, but the resultant force in this particular the direction.

Now, I will have to find out the moment; so, the force is acting, the resultant force is acting in this particular direction and how much is the moment? So, with respect to this particular moment; so, I will have to find out, I will have to multiply it by this particular the distance. And, what is this particular distance? That is nothing, but x_ZMP minus x_i.

So, $ZMP - x_i$; so I am getting the moment due to this particular the vertical force; Now, I am trying to find out the moment due to this horizontal force; now horizontal direction, the force is $m_i \ddot{x}_i$. So, $m_i \ddot{x}_i$ and this particular height is nothing, but is your z_i. So, m_i x_i double dot multiplied by z_i is the moment; now here, so this is going to create some sort of clockwise moment.

This also creates some sort of clockwise moment, but this is going to create some sort of anticlockwise and that particular torque, the summation of torque is nothing, but summation i equals to 1 to 7 because I have got 7 links. So, $I_i \dot{\omega}_i$; so this is nothing, but the torque and summation of that and this is anticlockwise and other things are clockwise.

So, clockwise I have taken as positive and anticlockwise is negative and that is equal to 0 and if I solve, if I simplify; so, I will be getting the expression for the x_ZMP. So, I will be getting the coordinate of this particular point, that is nothing, but the zero moment point. And, once I have got this particular zero moment point, now very easily, I can find out what is this; your dynamic balance margin, that is, x_ZMP. Now, x_ZMP is nothing, but L_7 by 2; now L_7 by 2 if I conside,r now L_7 by 2 if I consider. So, I am here.

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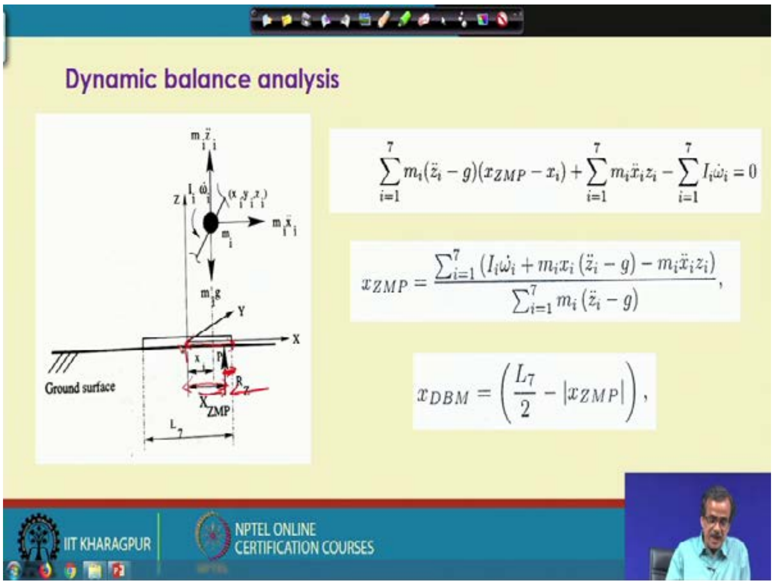

So, this is nothing, but $\frac{L_7}{2} - x_{ZMP}$ is the dynamic balance margin? So, dynamic balance margin is this much. So, this is nothing, but is the dynamic balance margin. Now, if I consider x_ZMP is here, in that case, I will have the maximum dynamic balance margin. Now, this is the way actually, we calculate the dynamic balance margin for this particular during the biped walking.

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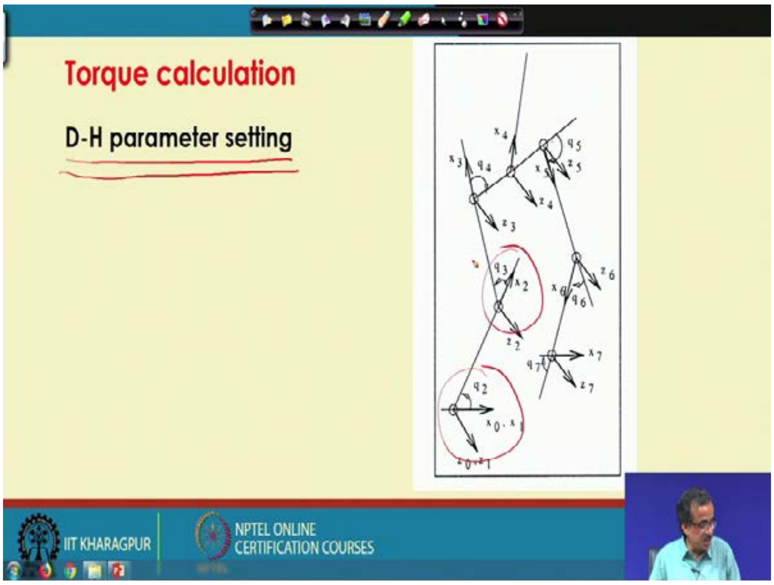

Now, in short, let me discuss; let me tell you the procedure, how to find out the joint torque and how to determine the joint torque, I have discussed in much more details, while discussing the dynamics. Now, let me proceed a little bit faster. So, this is actually, how to how to assign the coordinate system at the different joints; according to this D-H parameter setting rule. Now, this D-H parameter setting rule, I have discussed, in details, in the chapter of robot dynamics and those things, I am not going to repeat.

Now, using that particular principle of D-H parameter setting, at each of these particular robotic joints 1, 2, 3, 4, 5, 6, 7; So, I will have to assign this particular the coordinate system like x axis, y axis and z axis.

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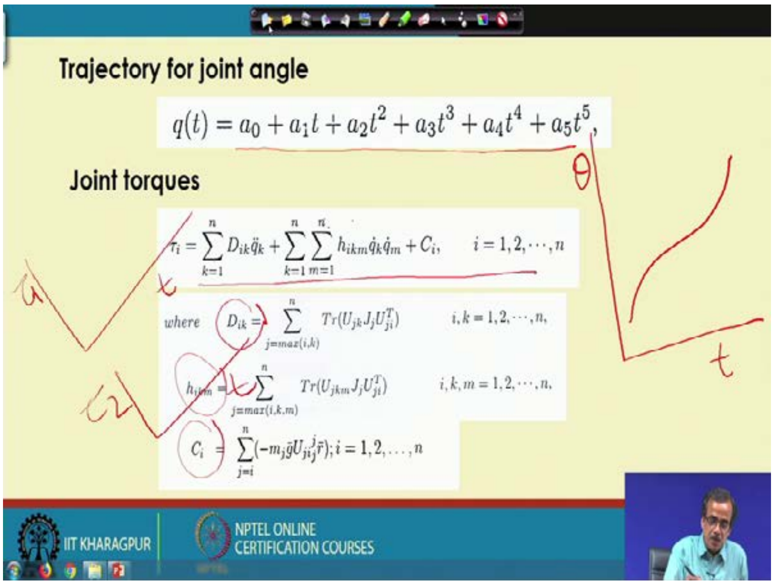

Now, once you have assigned this particular coordinate system; now, if I want to find out the joint torque. So, what I will have to do is, I will have to find out, what should be the variation of θ as a function of time. And, we will have to assume actually the smooth variation of this particular joint angle.

Now, while discussing the trajectory planning, I have discussed in much more details; like how to fit. So this type of fifth under polynomial just to find out a smooth variation of θ; so, here q (t) is nothing, but $\theta(t)$ because this is nothing, but the rotary joint. So, what I will have to do is; I will have to find out θ as a function of time, some sort of a smooth curve I will have to fit.

And, once you have got that particular thing; now I am in a position to find out what should be the variation of this particular joint torque, that is, τ_1 as a function of time, then τ_2 as a function of time and so on, up to τ_7 and how to derive? So, those things I have discussed in much more details; so I am not going to repeat. Now, this is actually the final expression for the torque and this D_ik is nothing, but the inertia terms, which I have already discussed and derived; h_ikm; correlation and centrifugal term and C_i is nothing, but the gravity terms, as we have discussed in much more details in robot dynamics. So, I am not going to spend much time on this.

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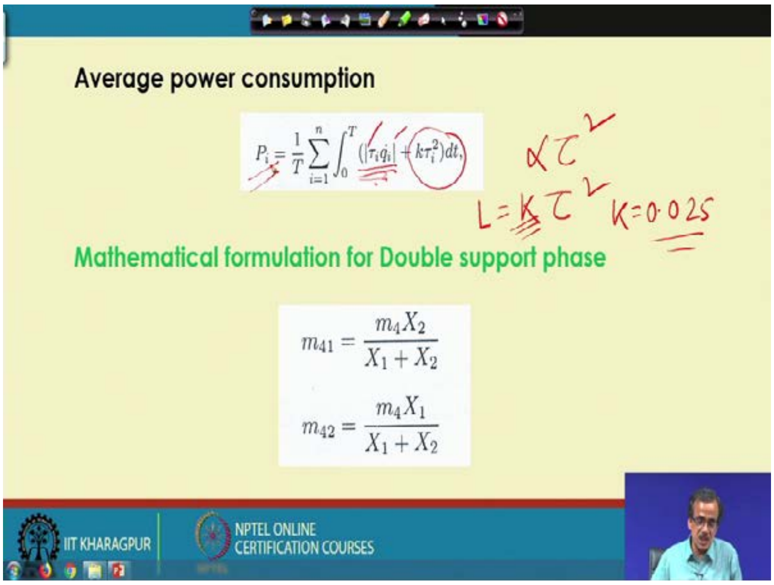

So, now, I am in a position to think that for this particular biped robot; I am able to find out what should be the expression for the joint torque and how to determine actually the dynamic balance margin. Now, here, actually what we can do is, I can discuss like how to determine this power consumption. Now, the expression for the power consumption; power we know that is nothing, but the rate of change of work done or a work done per unit time. So, here; this particular τ_i, τ denotes actually the torque multiplied by q_i dot that is nothing, but the angular velocity. So, torque multiplied with angular displacement is the work done per unit time; so, this is nothing, but the power plus here I have written $k\tau_i^2$.

Now, I have already discussed that at each of the robotic joint, we use some DC motor and whenever we are going to use DC motor, there will be some loss. And, that particular loss is nothing, but the loss in the in this DC motor, that is proportional to τ^2; that means, here the loss $L = k\tau^2$ and k is nothing, but is your constant of proportionality. And, generally for the DC motor; so this particular k is taken to be equal to 0.025 or very close to that.

Now, if I know this particular; so I can find out how much is the loss due to this particular loss in this particular the DC motor. And, this is the requirement of the torque and I can find out what should be the a power rating for the motor, which I am going to put at the different joints.

And, using this particular principle actually, we can find out what should be the power rating for this particular the motor connected at the joint. This is how to carry out the analysis for the single support phase.

Thank you.

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Lecture – 43
Biped Walking (Contd.)

Now, we are going to discuss, how to tackle the scenario for the double support phase.

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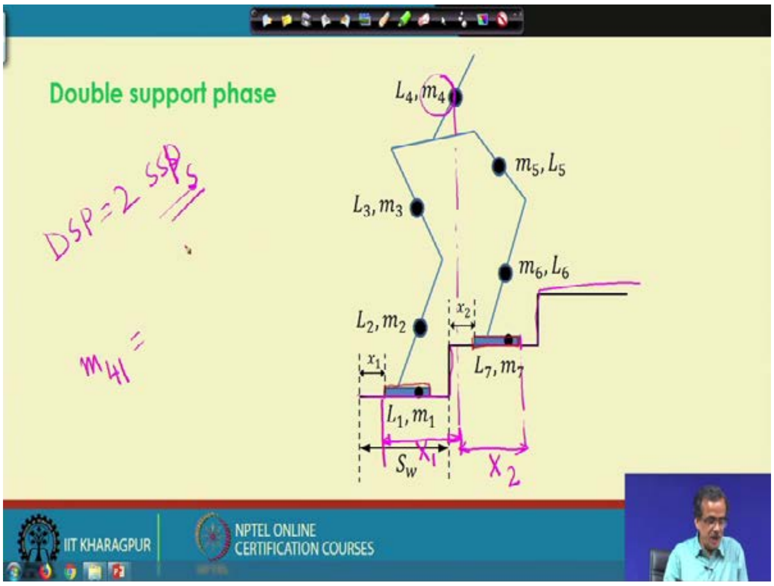

Now, during the double support phase, actually both the feet are on the ground. Now, here, let us see, how to take care and how to carry out this particular the analysis during the double support phase. Now, during the double support phase; this particular foot is on the ground. Similarly, this particular foot is also on the ground and this is nothing, but the staircase.

So, this staircase is denoted by this, this is nothing, but the staircase; so this is the staircase. Now, here, the way I discuss, L_1, m_1 are the length and mass of the first link, that is the foot. Similarly, we have got L_2, m_2, length and mass for the second, L_3 and m_3 for the third, L_4 and m_4, that is length and mass for the fourth link, m_5 and L_5 for fifth, m_6 and L_6 for the sixth one and L_7 and m_7 are the length and mass for the foot.

Now, here to tackle this, your dynamic support phase, it is bit difficult and actually, what we do is, now here, the trunk mass of this particular m_4 has got significant influence on the dynamic balance margin. Now, what you will have to do is, the moment this particular biped robot is walking on the plain surface. So, what you can do is; so this particular m_4, we can distribute or divide into two equal parts. And, the moment, it is negotiating so, this type of staircase, this particular m_4, I can divided into two parts, but the two parts will not become equal.

Now, why do you need it? For the purpose of analysis of this double support phase, we will have to assume that that this is consisting of two single support phases. And, we have already seen, how to carry out the analysis for this particular the single support phase. So, what I do is; so this particular DSP is actually assumed to be consisting of two single support phases (SSPs) and for each of these particular SSPs, we try to carry out the dynamic analysis, we try to find out what should be that particular the ZMP point.

Now, let us see, how to carry out this particular the analysis, now as I told that this particular trunk mass, that is, m_4 has got significant influence on the balance. So, what I do is; we take the projection of this on the ground for this trunk mass and what I do is, we try to find out what is the distance between this particular point, that is, the edge of the leg to this particular point.

And, similarly, we try to find out the edge of this particular leg or the foot from this particular projected point of the trunk mass. And, supposing that this is denoted by capital X_1 and this is denoted by capital X_2. Now, if I know this capital X_1 and X_2, now I can distribute this particular m_4 into two parts. Now, supposing that X_1 is equal to X_2; that means, the biped robot is walking on the plain surface. Now, in that case, we can find out, that is, m_41.

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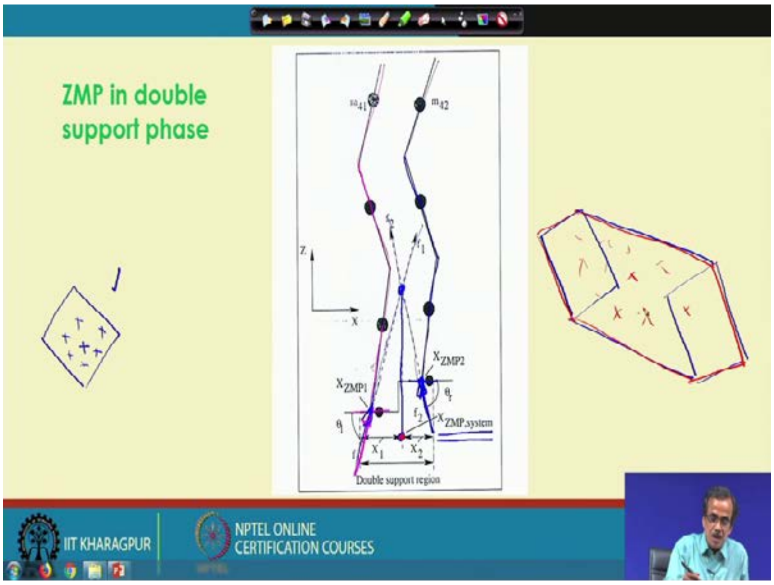

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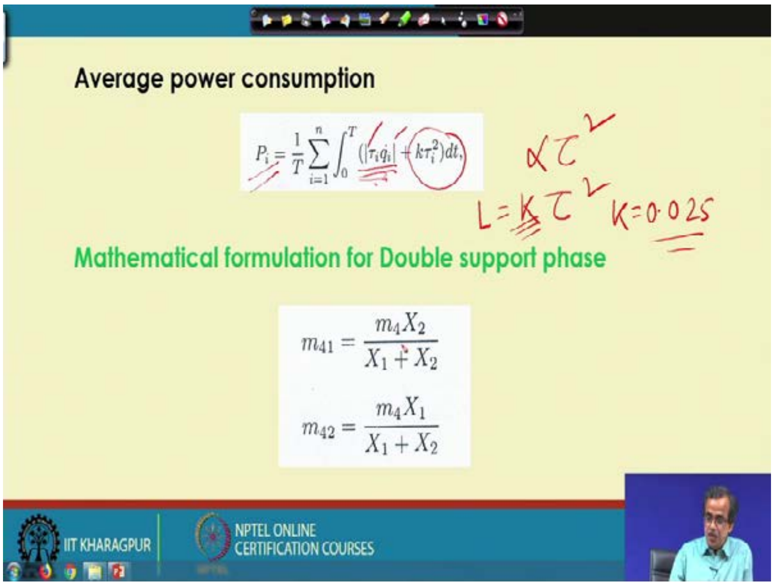

So, m_41 will be equal to this particular expression, that is, m_4 X_2 divided by X_1 plus X_2 and similarly, m_42 is nothing, but m_4 X_1 divided by X_1 plus X_2.

Now, if I get X_1 equals to X_2; so definitely m_41 will become equal to m_42; that means, whenever it is walking on the plain surface, m_41 will become equal to m_42. But, the moment it is negotiating the staircase or it is crossing the ditch or some uneven terrain,

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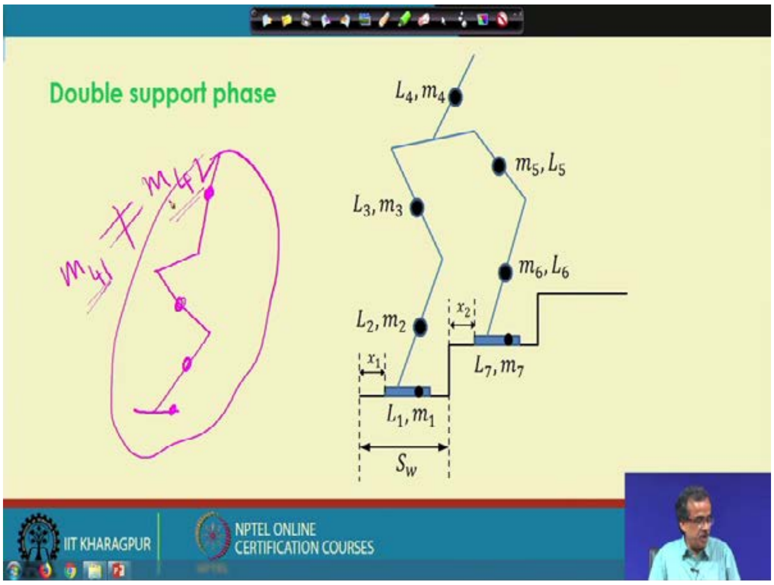

so in that case; this particular m_41 and m_42 are not equal and this particular m_41 does not become equal to m_42. And, supposing that it is negotiating the staircase, now in that case, m_41 and m_42 could be said as 40 percent and 60 percent of m_4 or they could be 30 percent, 70 percent of this particular the m_4.

Now, once I have got this particular the numerical values for these m_41 and m_42, very easily, what we can do is, we can consider this particular double support phase is nothing, but a combination of two single support phases. For example, say one phase will be something like this, so this is one single support phase. So, this will be something like this and this is one mass, similarly I have got one mass here, I have got one mass here, I have got one mass here. So, this is nothing but a single support phase; similarly on the right hand side, I can consider another single support phase. And, once I have got this particular single support phase; by following the same principle, so what I can do is, I can carry out this particular, I can find out what should be the ZMP point.

Now, for example, say if I concentrate on this particular the single support phase. So, this is one single support phase; so for this particular single support phase. So, I can find out the ZMP and this particular ZMP is denoted by this, so this is the ZMP. Similarly, for this particular double support phase, another single support phase, which is nothing, but this, this is one link, this is another link, another link, another link so this is nothing, but the ZMP point and this is your X_ZMP.

Now, remember, this particular X_ZMP is nothing, but a vector and I can also find out, what should be the magnitude, what should be the direction. And, we assume that this particular the reaction force, the ground reaction force, whenever the biped robot is walking on a ground or it is negotiating some staircase; there should be some reaction force. And, this particular ground reaction force is going to act through this particular ZMP.

Now, and due to this ground reaction force only, we are able to walk. So, this is the point through which the ground reaction force will walk and as I told that this is nothing, but a vector. So, this indicates the ground reaction force and it is passing through the ZMP. Now, if I extend, this particular straight line, I will be getting something like this and this vector, if I extend, I will be getting something like this and these two straight lines are going to intersect at this particular the point.

And, we take the projection of this particular intersection point on the ground and that indicates actually the system ZMP, that is, X_ZMP comma system. Now, once again I am just going to concentrate on the single support phase and the double support phase and how to maintain the balance. Now, during the single support phase; supposing that this is the ground foot, ok; now if the X ZMP point or the ZMP point if it is lying within this particular the ground foot ground region then only the dynamic balance is maintained, but if it goes outside, the balance is going to be lost.

Now, if I consider one double support phase something like this. So, this is one ground foot and this is, say another ground foot. Now here; the safe region is denoted by this, so this is nothing, but the safe region. So, the safe region if I just draw, so the safe region is nothing, but is nothing but this; so this is nothing, but the safe region. Now, this particular system ZMP; so this system ZMP should fall within this particular safe region; then only the dynamic balance will be maintained, otherwise, it is going to lose the balance and it is going to fall.

Now, in a particular walking cycle, whether it is in single support phase or in double support phase; the balance has to be maintained, at the same time during the transition of single support phase and the double support phase, that particular balance has to be maintained, then only it will be able to maintain its balance in a particular walking cycle. So, this is the way actually the biped robot maintains balance during the walking.

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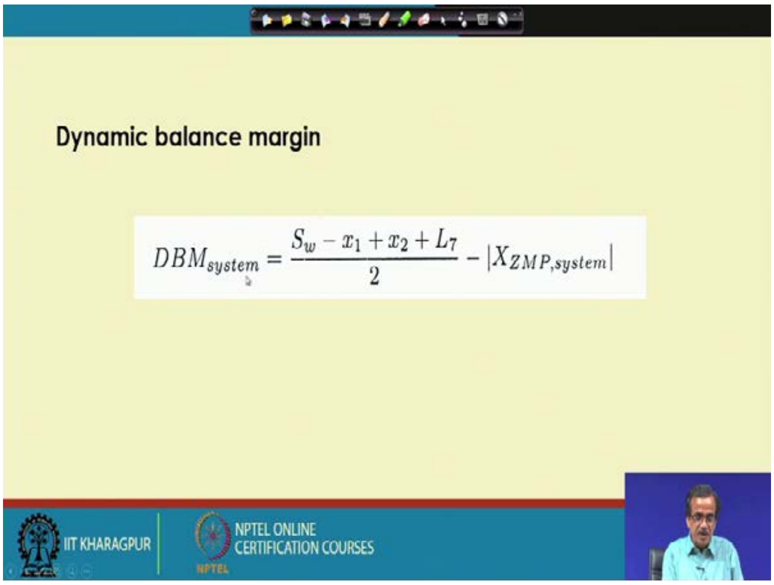

And, this is how to determine the ZMP during the double support phase, this I have already discussed a little bit.

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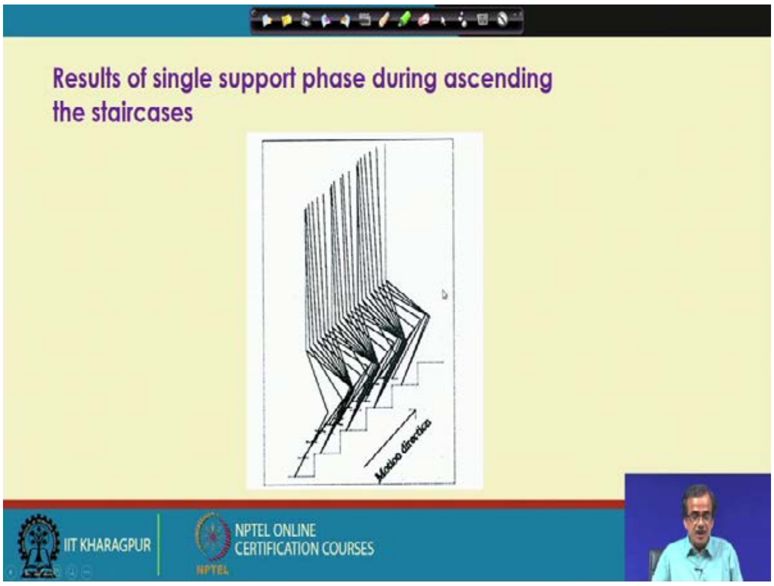

Now, here, I am just going to show you some stick diagram that a biped robot having 7 degrees of freedom is negotiating the staircase during the single support phase. So, one foot is on the ground and the other foot is in air.

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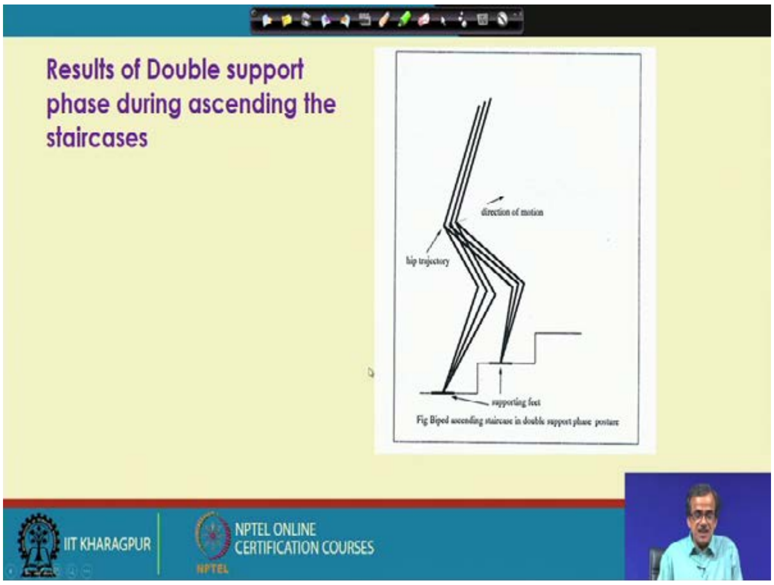

This is another stick diagram, where the same 7 degrees of freedom biped robot is negotiating the staircase and here, both the feet are on the ground.

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Now, I am just going to show you one real experiment on a very simple biped model. Now, I am just going to discuss the different components of the biped robots, which we have in our lab.

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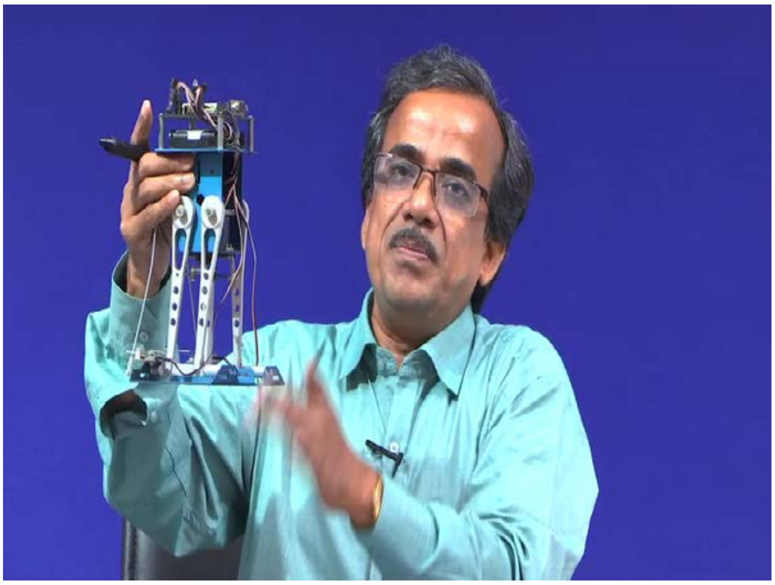

And, actually this is nothing, but the very simple biped robot and we can see that so, here we have got 2 servo motors. So, the servo motors we can see, so this is one servo motor, this is another servo motor, it is a very simple model.

Now, with the help of this particular servo motor; so we can control the movement in the forward and the backward directions. Similarly, with the help of this particular servo motor and with the help of these two tilt rods; so, we can actually go for, we can lift and we can place the foot of this particular the biped robot. And, here, we have got actually one micro controller, so with the help of that, we can control the preprogrammed motion, actually we can control, we can run.

Now, here, if you see, the area of this particular foot is much larger compared to the overall dimension of this very simple setup. Now, the purpose is actually, which I have already discussed, so that we can get more safer region to maintain its dynamic balance. So, that we can get during the double support phase or the single support phase, we can get the larger area for the safe region and that is why, the dimensions of this particular foot has been kept somewhat larger even compared to the overall dimensions of this particular the setup.

Now, with the help of this particular very simple biped model, we are going to show you like how to generate the forward and the backward movement. Or, how can it walk on the plain surface in the forward direction and in the backward directions. So, now, we are going to show you that particular experiment.

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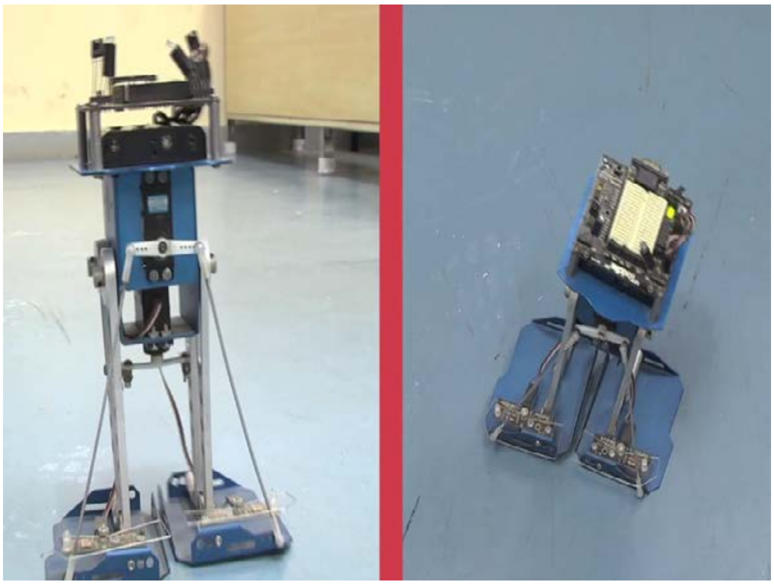

Now, it is showing the forward movement of the biped robot and now it is moving in the backward direction.

Thank you.

Robotics
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Lecture – 44
Summary

Now, I am going to summarize of this course on Robotics.

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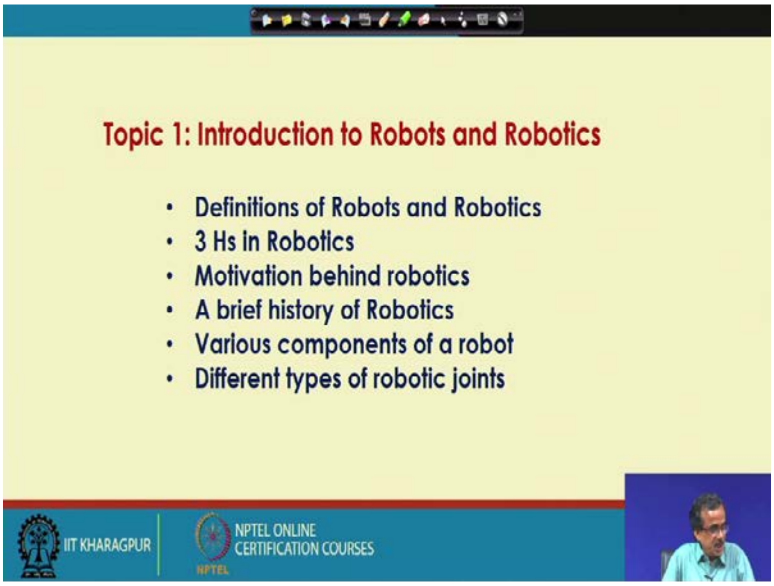

In this course, in fact, there are 10 topics and all 10 topics have been taught. I am just going to summarize topic-wise; the topic 1, the first topic that was on introduction to Robots and Robotics, we started with the very definition of the terms: robots and robotics.

And, to recapitulate, we mean a robot an automatic machine, which can perform a variety of tasks and the term: robot was introduced in the year 1921 by Karel Capek. And, robotics is actually a science, which deals with the issues related to the design, manufacturing and applications of robots. And, the term: robotics was coined in the year 1942 by Isaac Asimov. Now, in robotics we copy everything from human-being. For example, we copy head, heart and hand of a human being in the artificial way and that is actually popularly known as 3 Hs in robotics. Now, why should we study robotics?

Now, we have seen that the today’s market is dynamic and competitive and if we want to survive, so we will have to produce goods at low cost; at the same time, the quality has to be good and the productivity has to be high. And, to avail all these or to reach all the requirements, so we will have to go for automation. And, robotics is actually a flexible automation and that is why, modern industries should go for the robotics.

We discussed a little bit, a brief history of the robotics; now, as we discussed the first robot, the first patent on the robot that was filed in the year 1954 by George Devol and he is known as the father of robot and after that, the different universities particularly different US universities, then NASA, USA then USSR; so, they started manufacturing different types of the robots.

For example, say Stanford Research Institute, they developed robots, Carnegie Mellon University; CMU could develop some robots, Ohio State University could develop some robots, NASA developed a few robots. And, all of us we know NASA sent some intelligent robots to the Mars like spirit and opportunity, curiosity and all such robots are nothing, but the intelligent robots.

Now, here, as I told that the most sophisticated robot as on today might be the Sofia; which was developed in the year 2015 by Hanson Robotics; Hong Kong and Honda has already designed and developed sophisticated humanoid robots; so, this shows the brief history of the robotics.

Now, if you see what are the different components of a robots now, in a robot, we have got a few links, 2 links are joined by a joint; joints could be of 2 types, the linear joint or the rotary joints. Linear joints could be either the prismatic joint or sliding joint; rotary joints could be either the revolute joint or the twisting joint. And, of course, we have got a few special type of joints like the Hook joint, then comes the ball and socket joint, those are also used in robots.

Now, actually in a robot, there will be one controller or the director, there will be drive units, there will be links, joints, there must be some sorts of gripper or the end effector. And, if you want to make it intelligent the robot should be equipped with some sorts of sensors; so, these are the different components of the robots. Now, if you see, we have got different types of robotic joints, which I have already discussed.

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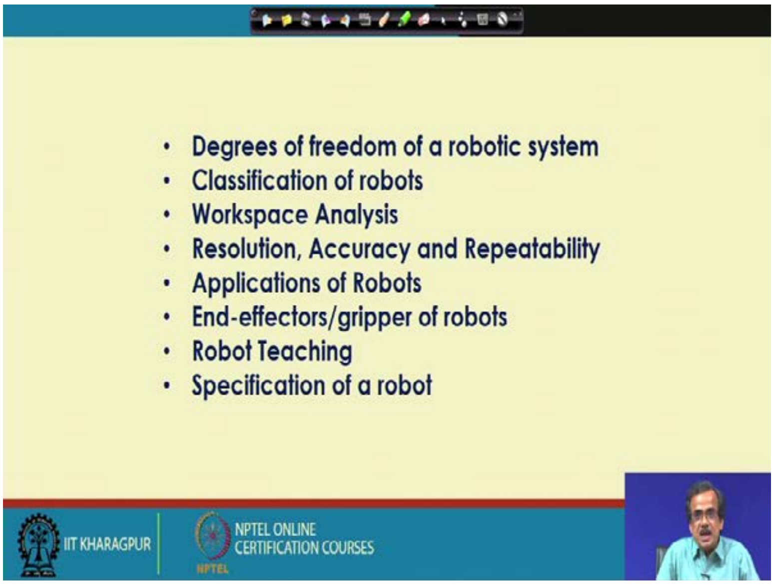

Now, the degrees of freedom of a robotic system; now, before I go for the degrees of freedom of a robotic system, the first thing we will have to find out, what is the degrees of freedom or connectivity of the different robotic joints. For example, if I consider the prismatic joint, it is having 1 degree of freedom or one connectivity, then comes your sliding joint has got 1 degree of freedom linear, revolute joint has got 1 degree of freedom, twisting joint 1 degree of freedom.

Then, if I consider cylindrical joint, which is having 2 degrees of freedom; then comes the Hooke joint or the universal joint has got 2 degrees of freedom. Spherical joint or ball and socket joint has got 3 degrees of freedom. Now, if I know the connectivity or the degrees of freedom of a robotic joint, we can also find out the degrees of freedom of a robotic system and we use actually Gruebler’s criteria just to find out, which I have already discussed, in details, to determine the degrees of freedom of a robotic system.

Now, if it is an ideal planar robot, it should have 3 degrees of freedom; if it is an ideal spatial robot, it should have 6 degrees of freedom. Now, there are a few special robots, which are having more than 6 degrees of freedom; those are called redundant robots. There could be a few special robots, which are having less than 6 degrees of freedom, that is called the under actuated robots. Now, then comes the classification, the robots have been classified in a number of ways; for example, the robots could be either the point-to-point robot or the continuous path robot.

The robots can be classified like the servo-controlled robot, non servo-controlled robot. Another classification could be based on the coordinate system; like Cartesian coordinate robot, then comes the cylindrical coordinate robot, spherical coordinate robot, revolute or articulated coordinate robot. Another classification could be based on the mobility levels; for example, say we have got robots with fixed base those are called the manipulators, manipulators could be either serial manipulator or parallel manipulator.

In serial manipulator, the links are in series; parallel manipulator the links are in parallel. Now, regarding the mobile robots, it could be either the wheel robots or there could be multi-legged robots or there could be tracked vehicles. So, these are, in short, the classifications of the robots. Then, we concentrate on the workspace analysis, we define the terms like what do you mean by the reachable workspace and the dexterous work space.

So, in short, the reachable workspace is that volume of space which can be reached with at least one configuration of the robot. And, dexterous workspace is lying in that particular volume of space which can be reached with the different combinations of the joint angles. So, we discuss, how to determine the workspace of different types of the joints; then we discussed the terms like resolution, accuracy and repeatability.

Resolution is nothing, but the least count of a robot and this particular resolution could be either the programming resolution or the control resolution. By accuracy, we mean the precision with which the end-effector of the robot can reach the computed point. And, by repeatability, we mean the same robot, if I run large number of times, so, there is no guarantee that every time it is going to reach the same point and there could be some deviation that particular deviation is nothing, but the repeatability.

We discussed, in brief, the various applications of robots; for example, the robots are used in manufacturing unit. Robots are used in medical science like telesurgery, orthotic device, prosthetic device or we use some sort of multi-legged very small robots like in the form of capsules. Then, robots are also used as the helping hand for the doctors, robots are used in sea-bed mining; just to find out the valuable stones, then do some underwater repairing, maintenance job we use underwater robots.

Robots are used in space; just to collect information of the Mars or the space, we can use the robots. Nowadays, robots are being used even in agriculture, for example, say just to spray some pesticides, just to spray some sort of fertilizer in liquid form, for cleaning, just pikking the fruits, we can use the different types of robots. So, there are a large number of applications of robots, nowadays. Now, we concentrate on actually the different types of end-effectors or the grippers used in robots, we use different types of mechanical grippers.

For example, the gripper is designed and developed using some sort of mechanisms like piston and cylinder mechanism. We use some sort of gear mechanism to design and develop the end-effector, we use cam and follower mechanism to design and develop the end-effector. Then, we discuss the principle of the vacuum gripper, magnetic gripper used in robots, we also discuss some passive grippers like remote center compliance. In fact, we have got different types of grippers; different types of end-effectors, we have discussed all such things in details.

Then, the teaching methods we discussed, in detail. Basically, we have got 2 types of teaching methods to provide instruction to the robots; one is called the online method, another is called the offline method. So, by online method, we mean those methods, where while giving instruction; we will have to use the robots. And, for offline teaching, we are not using the robots, we are taking the help of some sort of the programming language.

Now, this online teaching could be either the manual teaching or it could be the lead through teaching. The manual teaching is suitable for point to point task and lead through teaching is suitable for the continuous path task. Then, we prepared the specification of a robot; like if I want to purchase a robot; how to prepare the specifications, what are the information to be given, that we have discussed in details. Then, we carried out some sort of economic analysis; through this economic analysis, we tried to take the decision, whether we should purchase a robot by taking loan from the bank.

And, here, we define 2 terms; one is called the payback period of a robot, another is called the rate of return of a robot. So, if the payback period is found to be less than the techno-economic life of a robot and the rate of return on investment, if it is found to be less than the rate of bank interest; I am sorry if the rate of return on investment if it is found to be more than the rate of bank interest, then only we just go for purchasing the robots. All such things, actually I discussed in your the first topic, that is, introduction to robots and robotics.

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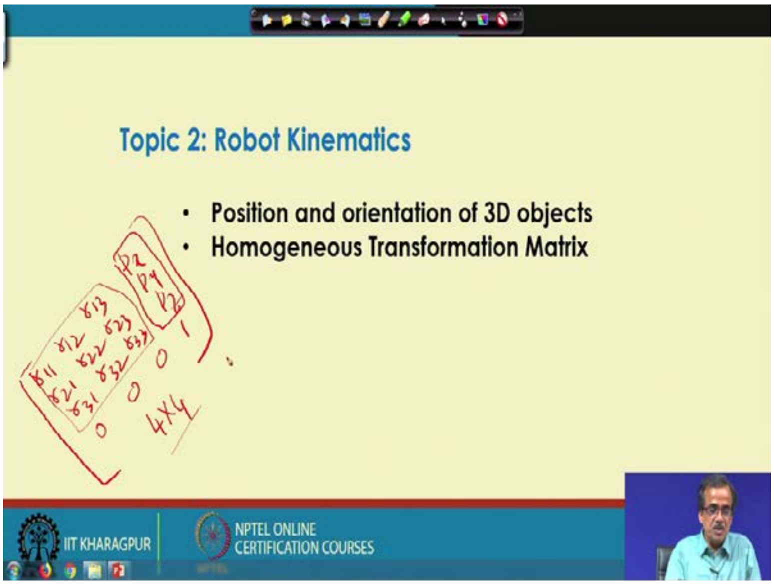

Then, I concentrated on topic 2; that is the robot kinematics. Now, the purpose of kinematics is to study the motion or the movement of the different links, different joints without considering the reason behind that particular the movement, that is the force or the torque; now here, the position and orientation of 3 D object in 3 D space; how to express that we discussed, in details. For example, say the position can be expressed either in Cartesian coordinate system or in cylindrical coordinate system or in spherical coordinate system.

Similarly, the orientation can be represented in 3 coordinate systems, one is the Cartesian or we can use the roll, pitch and yaw system or we can use some sort of Euler angle representation for the orientation. Next, we concentrate on how to derive the matrix that is the homogeneous transformation matrix, which is a 4 cross 4 matrix. And, this particular homogeneous transformation matrix carries information of this particular the position and orientation.

For example, say if I just draw one homogeneous matrix; so, r_11, 12, 13, 21, 22, 23, 31, 32, 33, 0 0 0 1; px, py, pz; so this is nothing, but a typical 4 cross 4 homogeneous transformation matrix. Now, here, these 3 cross 3 matrix carries information of the orientation and this particular the vector; so, this is nothing, but the position vector and this is nothing, but a 4 cross 4 homogeneous transformation matrix.

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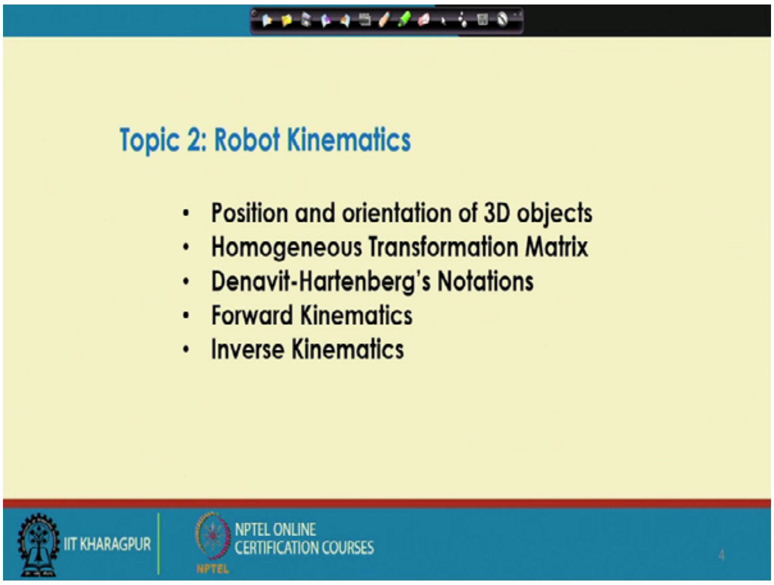

Then, we discussed the Denavit-Hartenberg notation; like how to assign the coordinate system at the different robotic joints so, that we can carry out the kinematic analysis. We concentrate on the problem of forward kinematics; that means, if I know the length of the links and the joint angles, then how to determine the position and orientation of the end-effector with respect to the base coordinate system. Then, we concentrate on the inverse kinematics.

Now, here the positional orientations of the end-effector are known and we will have to find out the joint angles provided the length of the links are known; so, this is the problem of inverse kinematics.

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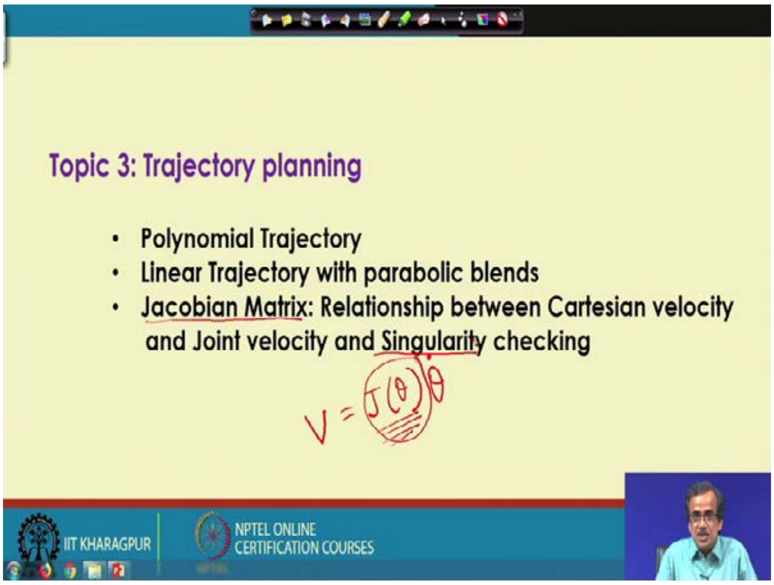

And, once we have completed this inverse kinematics; we started with another topic that is called the trajectory planning. The purpose of trajectory planning is to fit a trajectory so, that we can ensure the smooth variation at the different robotic joints.

Now, this trajectory planning problem can be solved either in Cartesian system or in joint space, but if I solve trajectory planning in Cartesian coordinate system, then I will have to carry out the inverse kinematics online. And, that is why, we try to follow the joint space scheme of trajectory planning; that means, in the space of theta or the joint angle. Now to fit a smooth curve so that it can ensure the smooth variation of this particular; the joint angle, we take the help of some trajectory function.

For example, we take the help of polynomial trajectory; we generally consider cubic polynomial or higher order polynomial like fifth order polynomial and the coefficients of the polynomials are determined with the help of some boundary conditions or the known conditions. Then, we concentrate on the linear trajectory, but we cannot use the pure linear trajectory function because there will be infinite acceleration and infinite deceleration at the ends, if I use the linear trajectory function and that is why, we use 2 parabolic blends at two ends of the linear trajectory function.

Then, we concentrate on the Jacobian matrix; now this particular Jacobian matrix is used to relate the Cartesian velocity with the joint velocity. For example, say if V is the Cartesian velocity, that is nothing, but the Jacobian matrix multiplied by the joint velocity. So, this $J(\theta)$ is nothing, but the Jacobian matrix and moreover, with the help of this Jacobian matrix; we studied the singularity of a manipulator. And, by singularity configuration, we mean a configuration, where the manipulator is going to lose one or more degrees of freedom.

Now, with the help of this Jacobian matrix, we can also carry out this particular the singularity checking. So, all such things, in fact, we have discussed in much more details.

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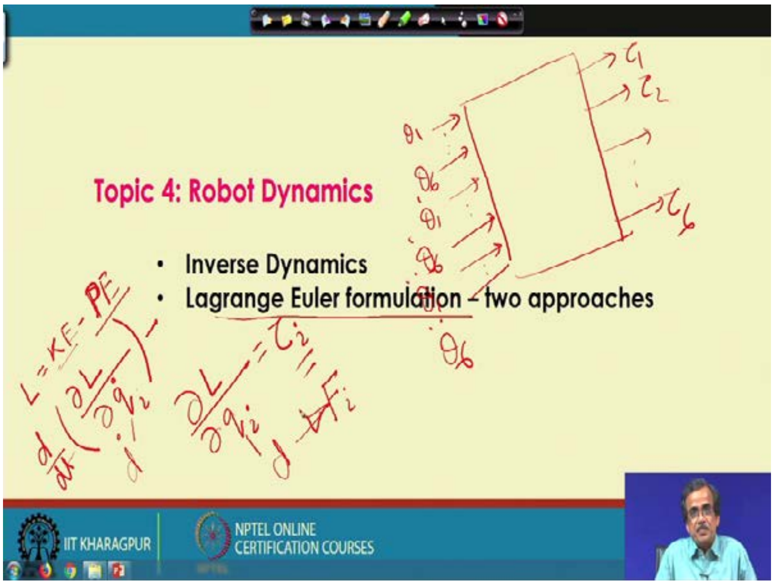

Then, we concentrate on the robot dynamics and truly speaking here; we concentrate it only on the inverse dynamics. And, by inverse dynamics, we mean that we have got all such joint angle values and their velocity and accelerations are known. For example, say $\theta_1.............\theta_6$, then comes your $\dot{\theta}_1.............\dot{\theta}_6$, then comes $\ddot{\theta}_1.............\ddot{\theta}_6$.

So, these are all given and I will have to find out all the torque values your like $\tau_1, \tau_2, ..., \tau_6$. So, this particular problem is nothing, but the inverse dynamics problem. Now, here to solve this inverse dynamics problem; actually what we do is; we took the help of the Lagrange Euler formulation. And, according to the Lagrange Euler formulation, we tried to find out, what is Lagrangian of a robotic system, that is nothing, but the difference between the kinetic energy and potential energy.

So, to derive that particular expression, what we do is, before we determine the Lagrangian for the whole robot, we try to concentrate on a small point or a differential mass lying on a robotic joint; we try to find out the kinetic energy and potential energy. Then, we tried to find out for the whole link and we considered all the links just to find out what should be the kinetic energy for the whole robot and what should be the potential energy for the whole robot and we tried to find out the Lagrangian.

And, once you have got this particular Lagrangian; then we use this Lagrange Euler formulation, which is nothing, but $\tau = \frac{d}{dt}(\frac{\partial L}{\partial \dot{q}_i}) - \frac{\partial L}{\partial q_i}$. So, this is the way we can find out the joint torque and if there is a linear joint so in place of this particular τ; so, I will be getting the force that is nothing, but F_i and of course, this particular q will be replaced by the d; that is the offset. So, this will be replaced by d dot, this will be replaced by d; if it is a linear joint and if I want to find out the force. So, using this actually we tried to find out what should be the joint force or the joint torque in robot dynamics.

Thank you.

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Lecture – 45
Summary (Contd.)

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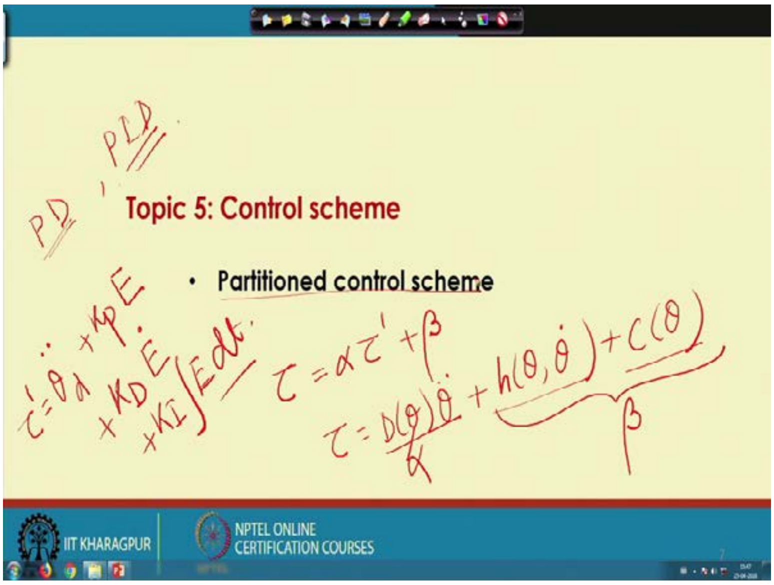

Now, then we concentrated on the control scheme; that means, once you have determine the expression for the joint torque, then how to achieve that? How can the motor supply that type of that amount of torque in a particular cycle time? And, each of this particular motor, generally we use DC motor, which is actually equipped with one controller; say PD controller or PID controller or PI controller. And, with the help of that; so, this particular motor will be able to generate the required torque.

Now, in partitioned control scheme, actually what we do is, the total torque τ, that is divided into 2 parts $\alpha\tau' + \beta$. Now, this α is nothing, but your $D(\theta)$, that is the inertia terms and this B is nothing, but is actually, if I just write down the expression for the torque, that is, $\tau = D(\theta)\ddot{\theta} + h(\theta, \dot{\theta}) + c(\theta)$. So, this is nothing, but the expression for the joint torque.

So, this is the inertia term, correlation centrifugal term and this is the gravity term. So, β is actually take care of this correlation centrifugal and the gravity terms. So, this is nothing, but β and $D(\theta)$ is nothing, but alpha then how to determine this particular τ'? To determine the τ' actually, what we do is; either we take the help of PD controller, that is, Proportional Derivative controller or we take the help of PID controller, that is, Proportional Integral and Derivative controller.

Now, supposing that I am using PD controller; so τ' will be $\ddot{\theta}_d$, that is the desired acceleration plus K_P, that is the gain value for the proportional gain multiplied by error plus your K_D, that is the derivative gain multiplied by is your $\dot{E}$. And, if I use PID controller; so, I will have to add K_I that is the integral gain multiplied by $\int Edt$; E is nothing, but the error and $\dot{E}$ is the rate of error, ok. So, using this particular principle and using the partitioned control scheme and using the closed loop control system, the motor will be able to generate that particular the desired torque; so, these things I have discussed in much more details.

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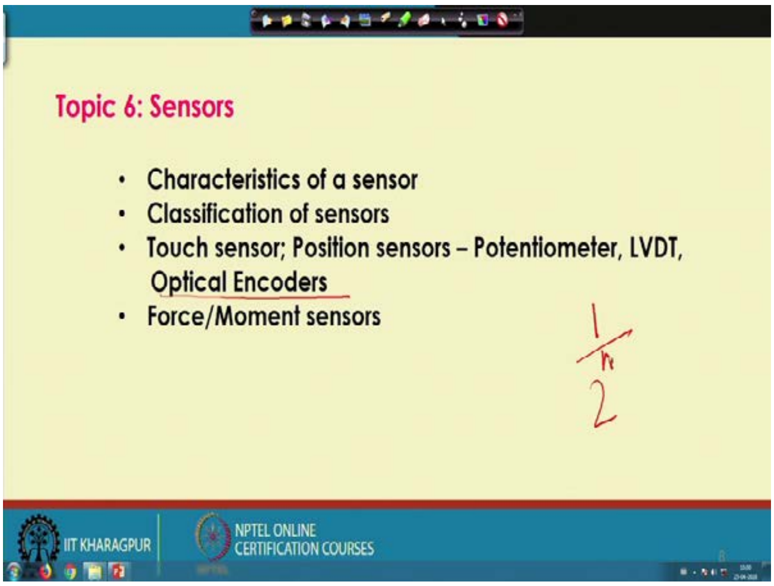

Now, comes your, if you want to make the robot intelligent; we will have to take the help of some sensors because we human being, we use a large number of sensors like eyes, ears, nose, skin and all such things. Sometimes, we collect information with the help of multiple sensors and there will be multi-sensor data fusion also.

Here, also in robotics, we will have to use the different types of sensors and if you want to purchase a sensor, so we will have to prepare the specification; so, we will have to mention, what is the resolution of a sensor, what is the repeatability of a sensor, what is the range of a sensor, and so on. Then, comes your; we discussed the different types of sensors, we generally used in robots, for example, we have got internal sensors, we have got external sensors.

Internal sensors are generally used to operate the drive units and external sensors are generally used to collect information of the environment; we have got some sort of touch sensor, we have got non-contact sensors, ok; in fact, we have got different types of sensors. Now, we discussed, in detail, the principles of touch sensor like limit switch, different types of position sensor like potentiometer; how does it work.

Then, LVDT, that is, Linear Variable Differential Transformer and it can measure the linear displacement and for measuring the rotary displacement; we will have to use RVDT; that is RVDT is a Rotary Variable Differential Transformer. We can use some sort of optical encoders; now optical encoders could be either the absolute optical encoder or there could be incremental optical encoder. Absolute optical encoder is more accurate, more costly because there we use large number of the photo-detectors and here, the resolution, which we get is nothing, but 1 in 2^n; n is nothing, but the number of concentric rings.

And, then comes the force or the moment sensor; now these force or moment sensors are generally used to find out, what should be the component of the force, the component of the moment. Supposing that one robot is doing some sort of manipulation task, that is, say it is doing some sort of pick and place type of operation and while doing this pick and place type of operation; so, the gripper is going to grip, it is going to carry it and place it somewhere.

So, the robotic joints are subjected to some amount of force, moments, torques and to determine that, we can take the help of this type of force or the moment sensor. The working principle of this force and moment sensors, I have discussed in much more details.

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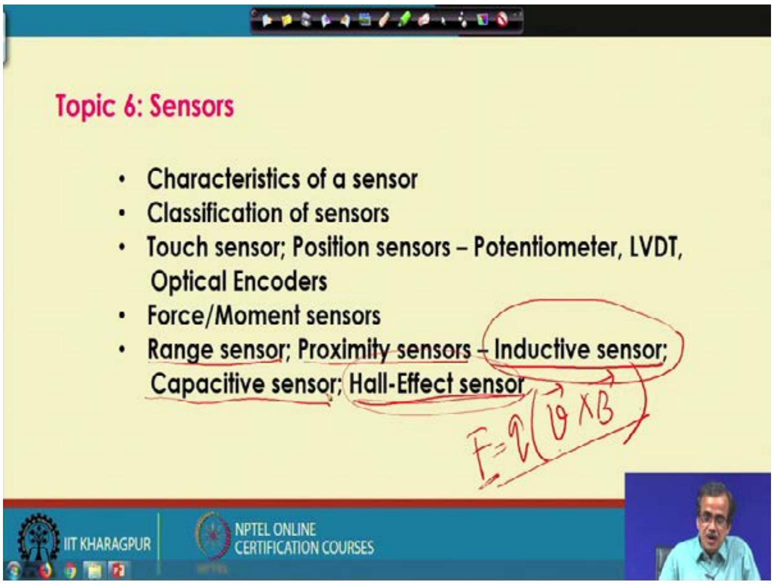

Now, then, comes the range sensor; now in this range sensor, we use the principle of the triangulation. And, using the principle of triangulation, we can determine the distance between the object and this particular the sensor.

We can use light as a source or sound as a source in this type of range sensor to determine the distance between the object and the sensor. Then, comes your proximity sensor; we have got a few proximity sensor, which are very popular. For example, say we have got inductive sensor, then comes we have got the Hall-Effect sensor; these are suitable only for the magnetic material. And, Hall effect sensor works based on the concept of that Lorenz force, that is $F = q(V \times B)$, V is nothing, but the velocity with which a charge q is moving in a magnetic field of strength B; then it will be subjected to the force, that is called the Lorenz force.

So, using the principle of Lorenz force, this Hall effect sensor is working, then using the principle of the law of magnetic induction, that is, the rate of change of magnetic flux is proportional to the induced voltage or the induced current. So, based on that; so this inductive sensor is working, these are suitable for the magnetic material, then comes your capacitive sensor, it is suitable both for your the magnetic as well as the nonmagnetic material. So, these are some of the sensors very frequently used in robots.

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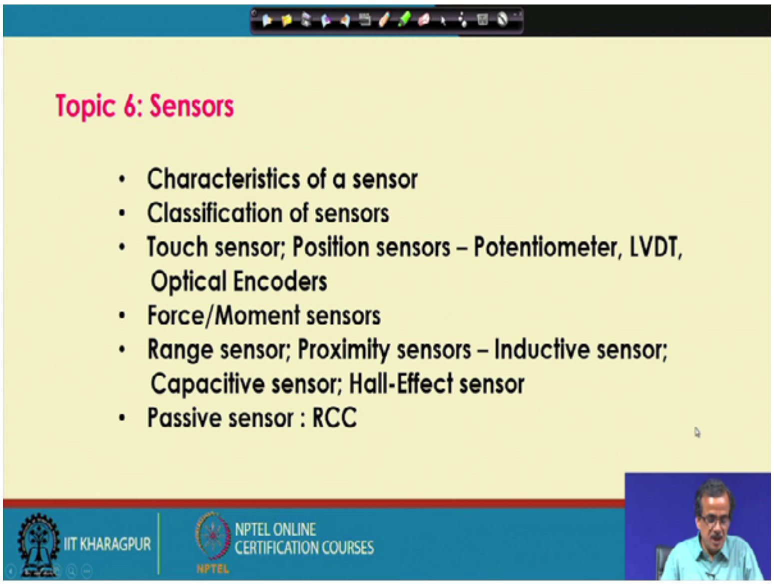

And, there is a another sensor, that is also very frequently used, that is called a passive sensor and in this passive sensor actually, we do not use any feedback for this passive sensor and Remote Center Compliance that is RCC is a typical example of this particular the passive sensor.

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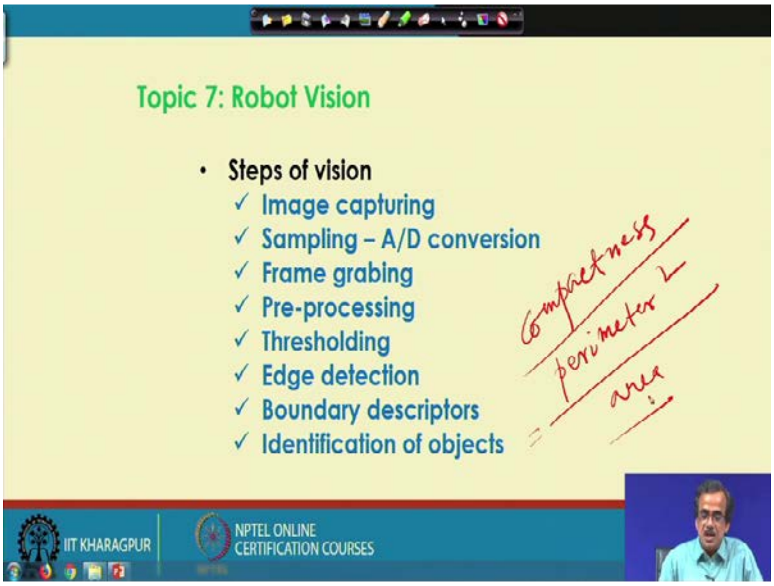

Next, we started with the topic 7, that is, Robot Vision. So, in place of sensor; if the robot is using some camera, then how can it collect information of the environment? So, that particular principle actually, I have discussed in much more detail.

The steps of the robot visions are as follows, for example, we capture image or the photograph with the help of a camera. Generally, we use some sort of CCD camera, then we go for some sort of sampling, that is analog to digital conversion and for this particular sampling, we take the help of some sort of electron beam scanner. And, I have already discussed that we do this scanning along the x direction and y direction and on the electron beam scanner, there are some photo-sites. And, whenever we have doing the scanning, some amount of charge will be accumulated on the photo-sites.

And, amount of charge accumulated is proportional to the light intensity. So, by using that particular information, we can do some sort of sampling that is called analog to digital conversion. We use some sort of digitizer here and once, we have expressed that particular image; say black and white image, in the form of a matrix of numerical values; that means, your image I am just going to represent with the help of one matrix of number and this is what, is known as the frame grabbing.

Now, this particular frame grabbing, if you do so, I will be getting that particular the image in the form of matrix form, but that may not be the correct and there could be some noise, there could be some lost information. So, we will have to go for preprocessing, there are different methods of preprocessing, which I discussed, for example, the masking operation, then neighborhood averaging or median filtering.

So, these are all preprocessing methods, once we got the preprocessed data; next we take the help of thresholding, just to find out the difference between the object and that particular background. And, to find out the boundary, we take the help of edge detection technique; these are nothing, but the gradient operator, we use the second degree gradient, first degree gradient, that is, Laplacian is the second degree gradient, also.

Once you have got that particular boundary; now, we try to express that particular boundary of the object in a mathematical way, that is, we use some sort of boundary descriptor, so that we can do some sort of further processing. And, once you have got that, now we will have to identify, actually as I discussed, we try to find out the compactness of the different objects.

Now, by compactness, we mean that is nothing, but the perimeter square divided by the area. And, by knowing this particular compactness, we try to find out actually, we try to find out the different objects. So, this is actually, how to determine that particular, how to collect information of the environment; the next is your the robot motion planning.

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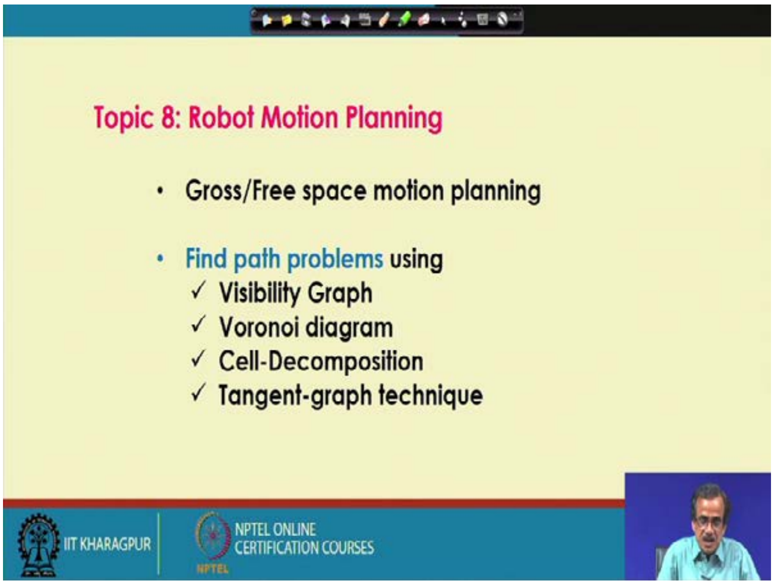

The aim of robot motion planning is to plan; is to determine the course of action. Now, this robot motion planning could be either gross motion planning or free space motion planning. Now, here, we concentrate only on the gross or the free space motion planning.

We solved the find path problem using the different methods, graph-based methods like the visibility graph, which was proposed long back in the year 1969 and in fact, this is the first approach of robot motion planning, that is the visibility graph. Then, came the concept of Voronoi diagram, we tried to find out the locus of the points, which are equidistant from 2 of the boundaries and that could be the feasible path for the robot.

Then, we discussed the cell decomposition; so before we go for the cell decomposition, what we do is; so we try to find out the feasible and the infeasible zones. Now, if I have got a physical robot and an obstacle; we try to convert it into a point robot and a grown obstacle and we will be getting some sort of your the feasible and infeasible zones.

So, the feasible zone that is divided into a large number of small segments and we try to find out, what should be the midpoint for each of the feasible regions.

And, then, we try to connect all the feasible points by the straight line and that will be nothing, but the collision-free path for the robot. Then, we discussed the principle of tangent graph technique; so, we consider the bounding circle for the obstacle and we try to move, the robot will try to move along the tangent and the circular arc to reach that goal and that is the principle of tangent graph technique.

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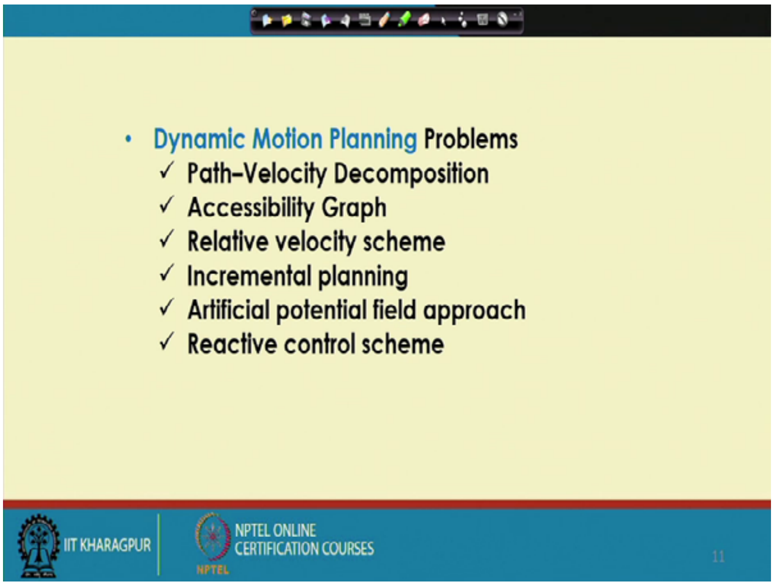

Then, we discussed, in details, the dynamic motion planning problem. Now, here, in dynamic motion planning problem, the robot is moving, at the same time the obstacles are also moving and to solve that particular problem, we took the help of a few approach; one is called the path velocity decomposition.

And, as I mentioned that this is the first approach proposed to solve the dynamic motion planning problem. This path velocity decomposition that consists of 2 sub problems; one is called the path planning problem and another is called the velocity planning problem. Then, comes your accessibility graph, now this accessibility graph is nothing, but the modified version of the visibility graph.

Now, here, the obstacles are moving; so at time t equals to t_1, I will be getting one visibility graph, one collision-free path at time; t equals to t_2, another visibility graph. So, the visibility graph is going to vary with time and that is nothing, but the concept of accessibility graph. Then, comes the relative velocity scheme; so, in dynamic motion planning problem the robot is moving, obstacle is moving.

So, here, actually what we do is, we try to find out the relative velocity of the robot with respect to the obstacle, as if we consider the obstacles are stationary and we try to find out the velocity and considering that, we try to find out the collision-free path. Then, comes your incremental planning, so at time t equals to t_1, say the dynamic motion planning problem will become the find-path problem.

So, we make a plan considering the problem as a find-path problem, then the robot is going to start moving; the moment it faces problem and if there is a chance of collision, the robot is going to stop and it is going to re-plan and once again, it will try to find out the collision-free direction, this is the principle of your the incremental planning. Then, comes the artificial potential field method, here, the robot is going to move under the combined action of attractive force or attractive potential of the goal and a repulsive force or the repulsive potential of the obstacle.

And, due to this combined effect of this attractive and repulsive forces, the robot is going to move towards the goal. So, this is actually the principle of your artificial potential field method, then comes your the reactive control scheme. So, here, each of the robotic action is divided into a large number of the primitive robotic tasks and each of these primitive robotic task is controlled at a particular layer of the control scheme.

Now, supposing that a particular task has been divided into certain primitive behaviors to design the control scheme, there should be 10 layers and over and above, there will be one centralized computer and which is going to control all the 10 layers. So, this is actually the scheme for the reactive control scheme and based on this reactive control scheme, one field of robotic research started that is called the behavior-based robotics.

But, as I told that behavior-based robotics has got a few drawbacks and that is why, currently we are working on evolutionary robotics. And, evolutionary robotics is going to actually overcome the problem faced by the behavior-based robotics, but as it is out of scope of this particular course, I did not discuss the principle of the evolutionary robotics.

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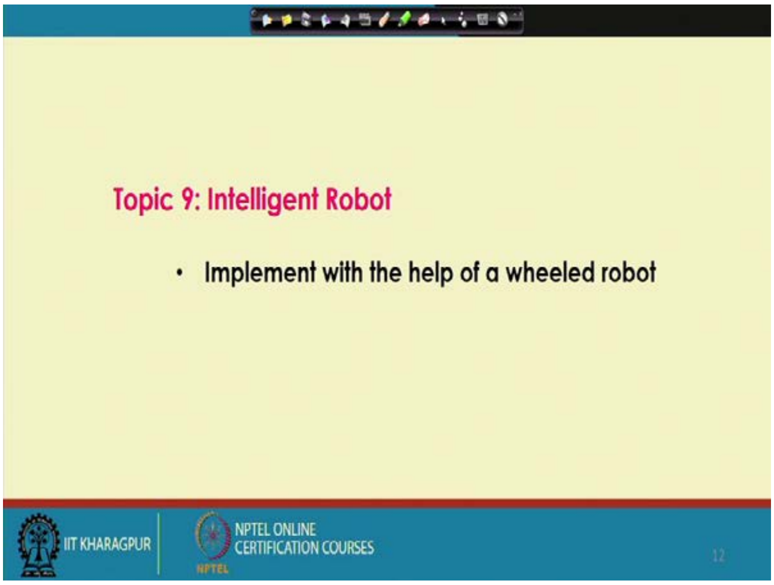

Now, then comes your, we started discussing on intelligent robot like how can a robot take the decision as the situation demands and that we implemented with the help of the wheeled robot.

Now, I am just going to discuss, in short, the way this particular thing was implemented, it is very simple. So, we have got a field and in the field, we have got a robot and some moving obstacles, at the top, we put the camera that is the overhead camera; so, with the help of this overhead camera, we take the snap at a regular interval and that particular picture collected with the help of camera goes to the a computer through BNC cable.

And, in the computer, we have got the my vision board, that is nothing, but the image processing hardware and there, we carry out some sort of image processing within a fraction of second. And, based on that image processing, we try to find out, what is the position of the robot, what is the position of the obstacle, which one is the most critical obstacle, and so on. And, based on the critical obstacle, we use the motion planning algorithm just to find out the angle of deviation and acceleration or the speed so that it can avoid collision.

Now, with the help of motion planner whatever decision we have got; now we will have to implement. To implement that, we need that we will have to give some instructions to the controller of the motor because the motor is connected to the wheel of the robot; now what we do is, we took the help of some sort of wireless communication through radio frequency module. And, with the help of this radio frequency module, the information related to how much RPM is required on the left side wheel, how much RPM is required on the right hand wheel that information we are going to pass to the controller of the motor.

And, the controller of the motor is going to generate that particular RPM so, that this particular wheeled robot can take the left turn or the right turn or it will be able to move in the forward and backward direction, as the situation demands. So, this is the way, actually we could make the robot intelligent and we did real experiment and in this course, we showed some video also for that real experiment.

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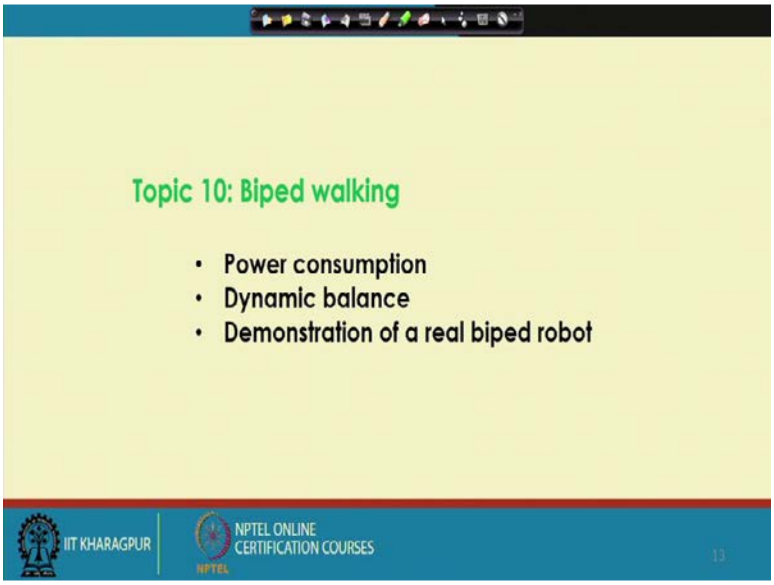

Then, comes the biped walking. So, we concentrated on the biped walking, we consider how to find out the power consumption for the biped walking; how to maintain the balance of this particular, the dynamic balance of the biped walking and I did not discuss the mathematical derivation, in details, like how to derive the power, expression for the power and as I told that is available in the textbook of the course. So, those who are interested they can see from this particular textbook and after that with the help of a small model on a biped robot, we showed some movement, some forward and backward movement of that particular the biped robot.

And, the video of the real experiment has or also been shown here and it has been demonstrated. Now, this particular course, we have come to the end of this particular course on robotics and I am sure all of you have enjoyed this course and have learnt a lot through this particular course, but this is simply the beginning. So, if you want to learn robotics and if you want to be the true roboticist, these are fundamentals, you will have to understand, the all 4 modules of robotics, which I have discussed. Those things we will have to understand, we will have to start learning that will give you the initial momentum, so that you can learn this particular subject in future, in depth.

And, robotics once again I should mention that this is the future, so we will have to go for this type of multidisciplinary fields in future and using the robotics, we can solve different types of real-world problems. So, I think, there is a very good future for this particular the robotics. I thank you all and I wish you all the best.

Thank you.

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