linearly with the number of training instances and the number of features: its training time complexity is roughly $O(m \times n)$.

The algorithm takes longer if you require a very high precision. This is controlled by the tolerance hyperparameter ϵ (called `tol` in Scikit-Learn). In most classification tasks, the default tolerance is fine.

The `SVC` class is based on the *libsvm* library, which implements an algorithm that supports the kernel trick.[2] The training time complexity is usually between $O(m^2 \times n)$ and $O(m^3 \times n)$. Unfortunately, this means that it gets dreadfully slow when the number of training instances gets large (e.g., hundreds of thousands of instances). This algorithm is perfect for complex but small or medium training sets. However, it scales well with the number of features, especially with *sparse features* (i.e., when each instance has few nonzero features). In this case, the algorithm scales roughly with the average number of nonzero features per instance. Table 5-1 compares Scikit-Learn's SVM classification classes.

Table 5-1. Comparison of Scikit-Learn classes for SVM classification

Class	Time complexity	Out-of-core support	Scaling required	Kernel trick
`LinearSVC`	$O(m \times n)$	No	Yes	No
`SGDClassifier`	$O(m \times n)$	Yes	Yes	No
`SVC`	$O(m^2 \times n)$ to $O(m^3 \times n)$	No	Yes	Yes

SVM Regression

As we mentioned earlier, the SVM algorithm is quite versatile: not only does it support linear and nonlinear classification, but it also supports linear and nonlinear regression. The trick is to reverse the objective: instead of trying to fit the largest possible street between two classes while limiting margin violations, SVM Regression tries to fit as many instances as possible *on* the street while limiting margin violations (i.e., instances *off* the street). The width of the street is controlled by a hyperparameter ϵ. Figure 5-10 shows two linear SVM Regression models trained on some random linear data, one with a large margin ($\epsilon = 1.5$) and the other with a small margin ($\epsilon = 0.5$).

2 "Sequential Minimal Optimization (SMO)," J. Platt (1998).

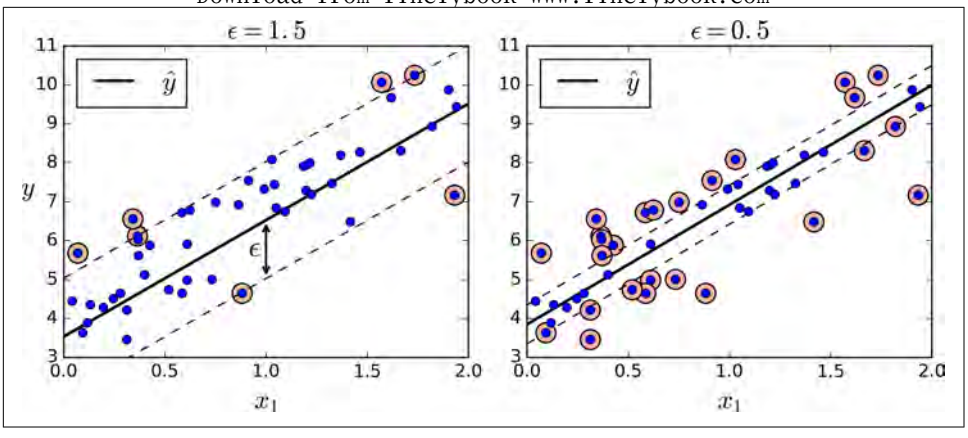

Figure 5-10. SVM Regression

Adding more training instances within the margin does not affect the model's predictions; thus, the model is said to be *ϵ-insensitive*.

You can use Scikit-Learn's `LinearSVR` class to perform linear SVM Regression. The following code produces the model represented on the left of Figure 5-10 (the training data should be scaled and centered first):

```
from sklearn.svm import LinearSVR

svm_reg = LinearSVR(epsilon=1.5)
svm_reg.fit(X, y)
```

To tackle nonlinear regression tasks, you can use a kernelized SVM model. For example, Figure 5-11 shows SVM Regression on a random quadratic training set, using a 2nd-degree polynomial kernel. There is little regularization on the left plot (i.e., a large `C` value), and much more regularization on the right plot (i.e., a small `C` value).

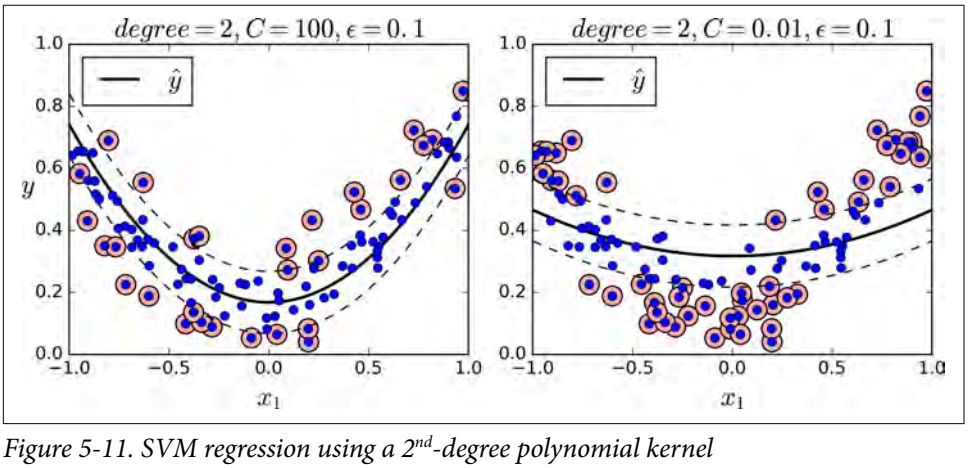

Figure 5-11. SVM regression using a 2nd-degree polynomial kernel

The following code produces the model represented on the left of Figure 5-11 using Scikit-Learn's `SVR` class (which supports the kernel trick). The `SVR` class is the regression equivalent of the `SVC` class, and the `LinearSVR` class is the regression equivalent of the `LinearSVC` class. The `LinearSVR` class scales linearly with the size of the training set (just like the `LinearSVC` class), while the `SVR` class gets much too slow when the training set grows large (just like the `SVC` class).

```
from sklearn.svm import SVR

svm_poly_reg = SVR(kernel="poly", degree=2, C=100, epsilon=0.1)
svm_poly_reg.fit(X, y)
```

SVMs can also be used for outlier detection; see Scikit-Learn's documentation for more details.

Under the Hood

This section explains how SVMs make predictions and how their training algorithms work, starting with linear SVM classifiers. You can safely skip it and go straight to the exercises at the end of this chapter if you are just getting started with Machine Learning, and come back later when you want to get a deeper understanding of SVMs.

First, a word about notations: in Chapter 4 we used the convention of putting all the model parameters in one vector θ, including the bias term θ_0 and the input feature weights θ_1 to θ_n, and adding a bias input $x_0 = 1$ to all instances. In this chapter, we will use a different convention, which is more convenient (and more common) when you are dealing with SVMs: the bias term will be called b and the feature weights vector will be called $\mathbf{w}$. No bias feature will be added to the input feature vectors.

Decision Function and Predictions

The linear SVM classifier model predicts the class of a new instance $\mathbf{x}$ by simply computing the decision function $\mathbf{w}^T \cdot \mathbf{x} + b = w_1 x_1 + \cdots + w_n x_n + b$: if the result is positive, the predicted class $\hat{y}$ is the positive class (1), or else it is the negative class (0); see Equation 5-2.

Equation 5-2. Linear SVM classifier prediction

$$\hat{y} = \begin{cases} 0 \text{ if } \mathbf{w}^T \cdot \mathbf{x} + b < 0, \\ 1 \text{ if } \mathbf{w}^T \cdot \mathbf{x} + b \geq 0 \end{cases}$$