

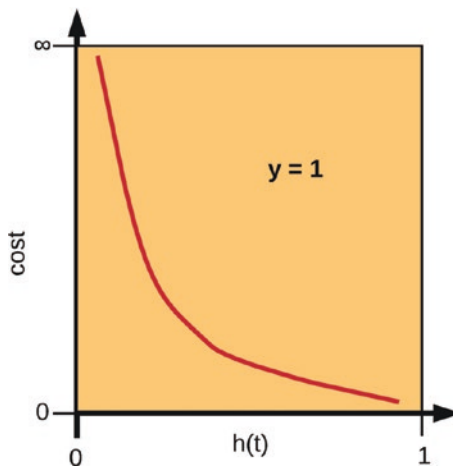
The sigmoid function, which looks like an [S](#) curve, rises from 0 and plateaus at 1. From the sigmoid function shown in Figure 20-4, as  $\hat{y}$  increases to positive infinity, the sigmoid output gets closer to 1, and as  $t$  decreases toward negative infinity, the sigmoid function outputs 0.

## Training the Logistic Regression Model

The logistic regression cost function is formally written as

$$Cost(h(t),y) = \begin{cases} -\log(h(t)) & \text{if } y = 1 \\ -\log(1-h(t)) & \text{if } y = 0 \end{cases}$$

The cost function also known as **log-loss** is set up in this form to output the penalty of the algorithm if the model predicts a wrong class. To give more intuition, take, for example, a plot of  $-\log(h(t))$  when  $y = 1$  in Figure 20-5.



**Figure 20-5.** Plot of  $h(t)$  when  $y = 1$

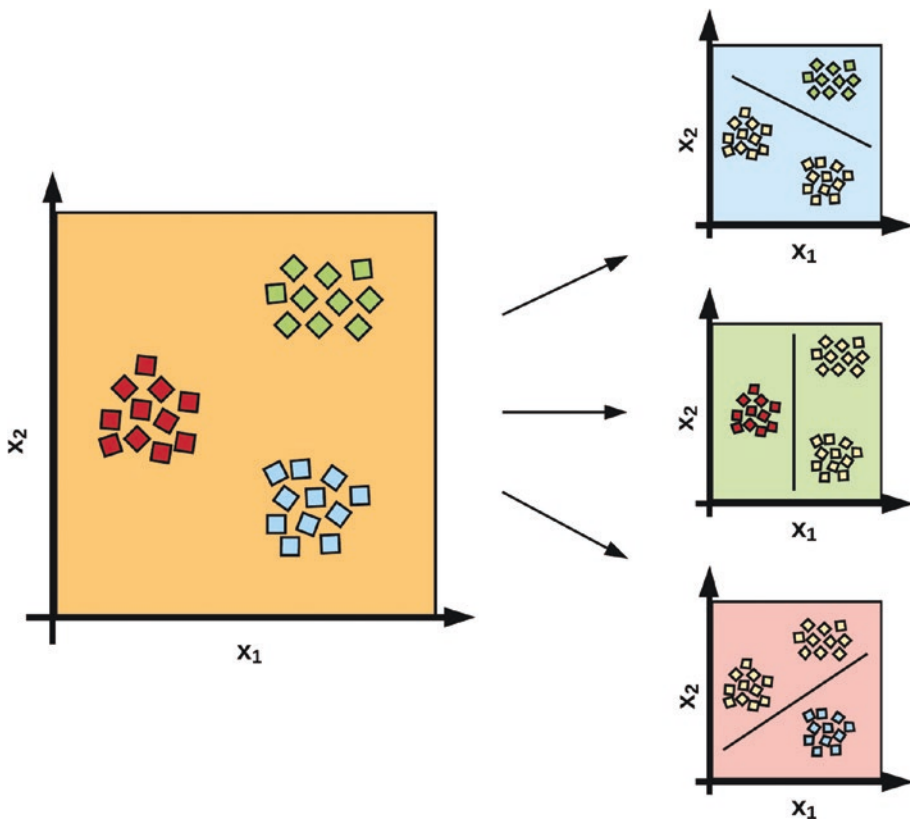
In Figure 20-5, if the algorithm correctly predicts that the target is 1, then the cost tends toward 0. However, if the algorithm  $h(t)$  predicts incorrectly the target as 0, then the cost on the model grows exponentially large. The converse is the case with the plot of  $-\log(1-h(t))$  when  $y = 0$ .

The logistic model is optimized using gradient descent to find the optimal values of the parameter  $\theta$  that minimizes the cost function to predict the class with the highest probability estimate.

# Multi-class Classification/Multinomial Logistic Regression

In multi-class or multinomial logistic regression, the labels of the dataset contain more than 2 classes. The multinomial logistic regression setup (i.e., the cost function and optimization procedure) is structurally similar to logistic regression; the only difference is that the output of logistic regression is 2 classes, while multinomial has greater than 2 classes (see Figure 20-6).

In Figure 20-6, the multi-class logistic regression builds a one-vs.-rest classifier to construct decision boundaries for the different class memberships.



**Figure 20-6.** An illustration of multinomial regression