

Strengths, weaknesses, and parameters

MultinomialNB and BernoulliNB have a single parameter, `alpha`, which controls model complexity. The way `alpha` works is that the algorithm adds to the data `alpha` many virtual data points that have positive values for all the features. This results in a “smoothing” of the statistics. A large `alpha` means more smoothing, resulting in less complex models. The algorithm’s performance is relatively robust to the setting of `alpha`, meaning that setting `alpha` is not critical for good performance. However, tuning it usually improves accuracy somewhat.

GaussianNB is mostly used on very high-dimensional data, while the other two variants of naive Bayes are widely used for sparse count data such as text. MultinomialNB usually performs better than BinaryNB, particularly on datasets with a relatively large number of nonzero features (i.e., large documents).

The naive Bayes models share many of the strengths and weaknesses of the linear models. They are very fast to train and to predict, and the training procedure is easy to understand. The models work very well with high-dimensional sparse data and are relatively robust to the parameters. Naive Bayes models are great baseline models and are often used on very large datasets, where training even a linear model might take too long.

Decision Trees

Decision trees are widely used models for classification and regression tasks. Essentially, they learn a hierarchy of if/else questions, leading to a decision.

These questions are similar to the questions you might ask in a game of 20 Questions. Imagine you want to distinguish between the following four animals: bears, hawks, penguins, and dolphins. Your goal is to get to the right answer by asking as few if/else questions as possible. You might start off by asking whether the animal has feathers, a question that narrows down your possible animals to just two. If the answer is “yes,” you can ask another question that could help you distinguish between hawks and penguins. For example, you could ask whether the animal can fly. If the animal doesn’t have feathers, your possible animal choices are dolphins and bears, and you will need to ask a question to distinguish between these two animals—for example, asking whether the animal has fins.

This series of questions can be expressed as a decision tree, as shown in [Figure 2-22](#).

In[56]:

```
mglearn.plots.plot_animal_tree()
```

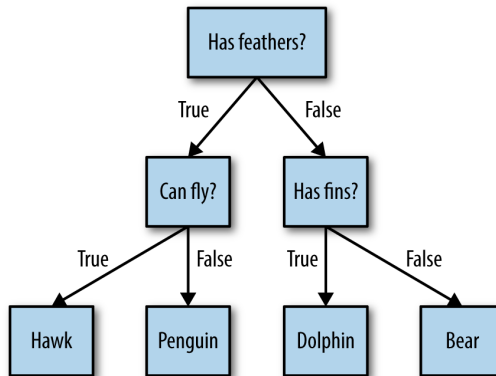


Figure 2-22. A decision tree to distinguish among several animals

In this illustration, each node in the tree either represents a question or a terminal node (also called a *leaf*) that contains the answer. The edges connect the answers to a question with the next question you would ask.

In machine learning parlance, we built a model to distinguish between four classes of animals (hawks, penguins, dolphins, and bears) using the three features “has feathers,” “can fly,” and “has fins.” Instead of building these models by hand, we can learn them from data using supervised learning.

Building decision trees

Let’s go through the process of building a decision tree for the 2D classification dataset shown in [Figure 2-23](#). The dataset consists of two half-moon shapes, with each class consisting of 75 data points. We will refer to this dataset as `two_moons`.

Learning a decision tree means learning the sequence of if/else questions that gets us to the true answer most quickly. In the machine learning setting, these questions are called *tests* (not to be confused with the test set, which is the data we use to test to see how generalizable our model is). Usually data does not come in the form of binary yes/no features as in the animal example, but is instead represented as continuous features such as in the 2D dataset shown in [Figure 2-23](#). The tests that are used on continuous data are of the form “Is feature i larger than value a ?”

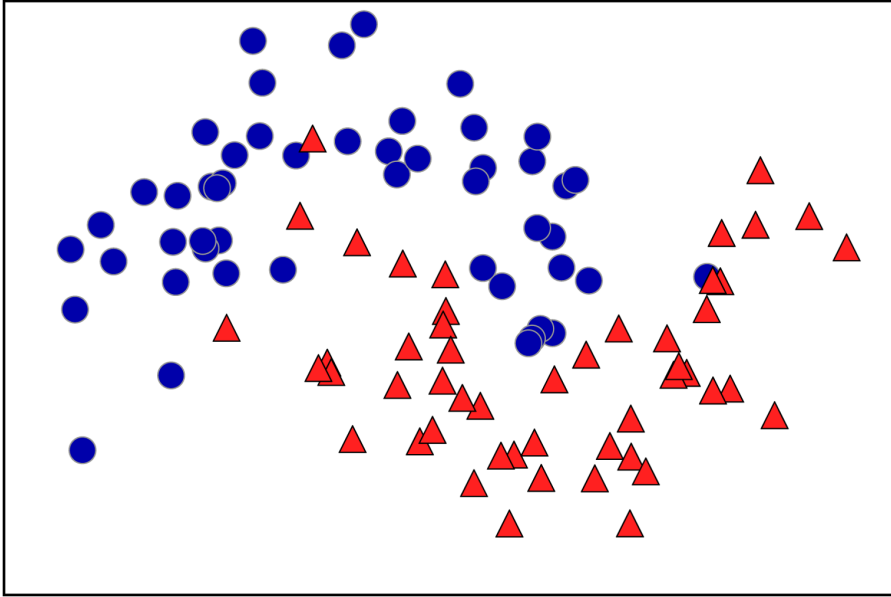


Figure 2-23. Two-moons dataset on which the decision tree will be built

To build a tree, the algorithm searches over all possible tests and finds the one that is most informative about the target variable. Figure 2-24 shows the first test that is picked. Splitting the dataset vertically at $x[1]=0.0596$ yields the most information; it best separates the points in class 1 from the points in class 2. The top node, also called the *root*, represents the whole dataset, consisting of 75 points belonging to class 0 and 75 points belonging to class 1. The split is done by testing whether $x[1] \leq 0.0596$, indicated by a black line. If the test is true, a point is assigned to the left node, which contains 2 points belonging to class 0 and 32 points belonging to class 1. Otherwise the point is assigned to the right node, which contains 48 points belonging to class 0 and 18 points belonging to class 1. These two nodes correspond to the top and bottom regions shown in Figure 2-24. Even though the first split did a good job of separating the two classes, the bottom region still contains points belonging to class 0, and the top region still contains points belonging to class 1. We can build a more accurate model by repeating the process of looking for the best test in both regions. Figure 2-25 shows that the most informative next split for the left and the right region is based on $x[0]$.

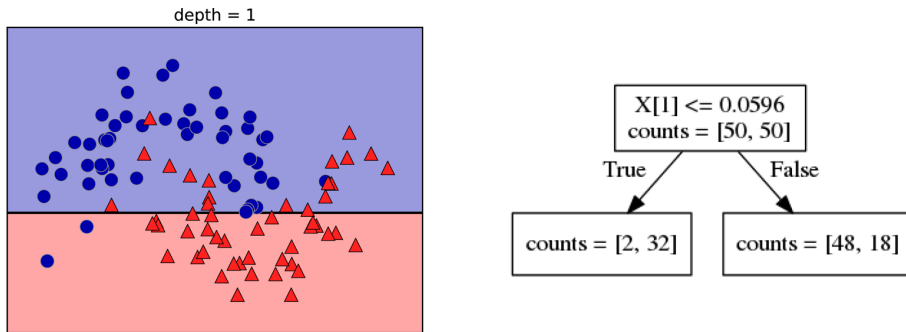


Figure 2-24. Decision boundary of tree with depth 1 (left) and corresponding tree (right)

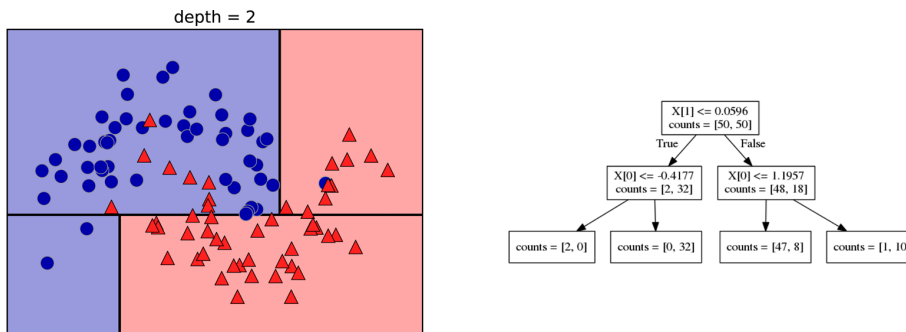


Figure 2-25. Decision boundary of tree with depth 2 (left) and corresponding decision tree (right)

This recursive process yields a binary tree of decisions, with each node containing a test. Alternatively, you can think of each test as splitting the part of the data that is currently being considered along one axis. This yields a view of the algorithm as building a hierarchical partition. As each test concerns only a single feature, the regions in the resulting partition always have axis-parallel boundaries.

The recursive partitioning of the data is repeated until each region in the partition (each leaf in the decision tree) only contains a single target value (a single class or a single regression value). A leaf of the tree that contains data points that all share the same target value is called *pure*. The final partitioning for this dataset is shown in Figure 2-26.

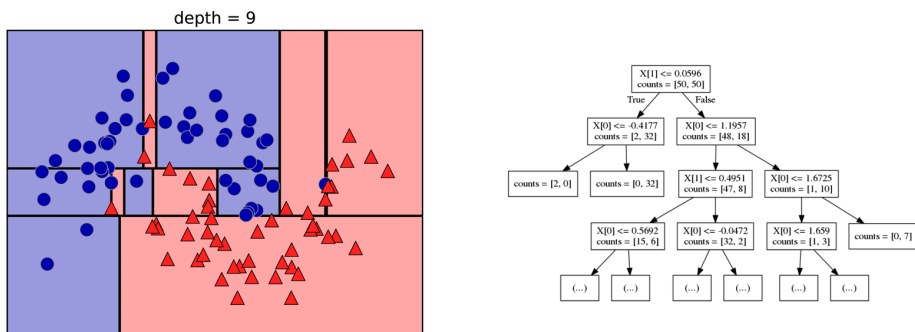


Figure 2-26. Decision boundary of tree with depth 9 (left) and part of the corresponding tree (right); the full tree is quite large and hard to visualize

A prediction on a new data point is made by checking which region of the partition of the feature space the point lies in, and then predicting the majority target (or the single target in the case of pure leaves) in that region. The region can be found by traversing the tree from the root and going left or right, depending on whether the test is fulfilled or not.

It is also possible to use trees for regression tasks, using exactly the same technique. To make a prediction, we traverse the tree based on the tests in each node and find the leaf the new data point falls into. The output for this data point is the mean target of the training points in this leaf.

Controlling complexity of decision trees

Typically, building a tree as described here and continuing until all leaves are pure leads to models that are very complex and highly overfit to the training data. The presence of pure leaves mean that a tree is 100% accurate on the training set; each data point in the training set is in a leaf that has the correct majority class. The overfitting can be seen on the left of Figure 2-26. You can see the regions determined to belong to class 1 in the middle of all the points belonging to class 0. On the other hand, there is a small strip predicted as class 0 around the point belonging to class 0 to the very right. This is not how one would imagine the decision boundary to look, and the decision boundary focuses a lot on single outlier points that are far away from the other points in that class.

There are two common strategies to prevent overfitting: stopping the creation of the tree early (also called *pre-pruning*), or building the tree but then removing or collapsing nodes that contain little information (also called *post-pruning* or just *pruning*). Possible criteria for pre-pruning include limiting the maximum depth of the tree, limiting the maximum number of leaves, or requiring a minimum number of points in a node to keep splitting it.

Decision trees in `scikit-learn` are implemented in the `DecisionTreeRegressor` and `DecisionTreeClassifier` classes. `scikit-learn` only implements pre-pruning, not post-pruning.

Let's look at the effect of pre-pruning in more detail on the Breast Cancer dataset. As always, we import the dataset and split it into a training and a test part. Then we build a model using the default setting of fully developing the tree (growing the tree until all leaves are pure). We fix the `random_state` in the tree, which is used for tie-breaking internally:

In[58]:

```
from sklearn.tree import DecisionTreeClassifier

cancer = load_breast_cancer()
X_train, X_test, y_train, y_test = train_test_split(
    cancer.data, cancer.target, stratify=cancer.target, random_state=42)
tree = DecisionTreeClassifier(random_state=0)
tree.fit(X_train, y_train)
print("Accuracy on training set: {:.3f}".format(tree.score(X_train, y_train)))
print("Accuracy on test set: {:.3f}".format(tree.score(X_test, y_test)))
```

Out[58]:

```
Accuracy on training set: 1.000
Accuracy on test set: 0.937
```

As expected, the accuracy on the training set is 100%—because the leaves are pure, the tree was grown deep enough that it could perfectly memorize all the labels on the training data. The test set accuracy is slightly worse than for the linear models we looked at previously, which had around 95% accuracy.

If we don't restrict the depth of a decision tree, the tree can become arbitrarily deep and complex. Unpruned trees are therefore prone to overfitting and not generalizing well to new data. Now let's apply pre-pruning to the tree, which will stop developing the tree before we perfectly fit to the training data. One option is to stop building the tree after a certain depth has been reached. Here we set `max_depth=4`, meaning only four consecutive questions can be asked (cf. Figures 2-24 and 2-26). Limiting the depth of the tree decreases overfitting. This leads to a lower accuracy on the training set, but an improvement on the test set:

In[59]:

```
tree = DecisionTreeClassifier(max_depth=4, random_state=0)
tree.fit(X_train, y_train)

print("Accuracy on training set: {:.3f}".format(tree.score(X_train, y_train)))
print("Accuracy on test set: {:.3f}".format(tree.score(X_test, y_test)))
```

Out[59]:

Accuracy on training set: 0.988
Accuracy on test set: 0.951

Analyzing decision trees

We can visualize the tree using the `export_graphviz` function from the `tree` module. This writes a file in the `.dot` file format, which is a text file format for storing graphs. We set an option to color the nodes to reflect the majority class in each node and pass the class and features names so the tree can be properly labeled:

In[61]:

```
from sklearn.tree import export_graphviz
export_graphviz(tree, out_file="tree.dot", class_names=["malignant", "benign"],
                feature_names=cancer.feature_names, impurity=False, filled=True)
```

We can read this file and visualize it, as seen in [Figure 2-27](#), using the `graphviz` module (or you can use any program that can read `.dot` files):

In[61]:

```
import graphviz

with open("tree.dot") as f:
    dot_graph = f.read()
graphviz.Source(dot_graph)
```

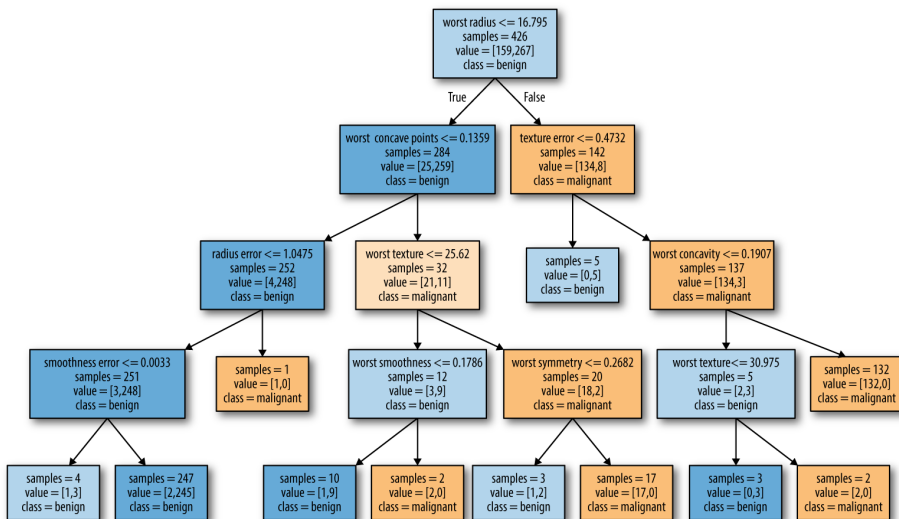


Figure 2-27. Visualization of the decision tree built on the Breast Cancer dataset

The visualization of the tree provides a great in-depth view of how the algorithm makes predictions, and is a good example of a machine learning algorithm that is easily explained to nonexperts. However, even with a tree of depth four, as seen here, the tree can become a bit overwhelming. Deeper trees (a depth of 10 is not uncommon) are even harder to grasp. One method of inspecting the tree that may be helpful is to find out which path most of the data actually takes. The `n_samples` shown in each node in [Figure 2-27](#) gives the number of samples in that node, while `value` provides the number of samples per class. Following the branches to the right, we see that `worst radius <= 16.795` creates a node that contains only 8 benign but 134 malignant samples. The rest of this side of the tree then uses some finer distinctions to split off these 8 remaining benign samples. Of the 142 samples that went to the right in the initial split, nearly all of them (132) end up in the leaf to the very right.

Taking a left at the root, for `worst radius > 16.795` we end up with 25 malignant and 259 benign samples. Nearly all of the benign samples end up in the second leaf from the right, with most of the other leaves containing very few samples.

Feature importance in trees

Instead of looking at the whole tree, which can be taxing, there are some useful properties that we can derive to summarize the workings of the tree. The most commonly used summary is *feature importance*, which rates how important each feature is for the decision a tree makes. It is a number between 0 and 1 for each feature, where 0 means “not used at all” and 1 means “perfectly predicts the target.” The feature importances always sum to 1:

In[62]:

```
print("Feature importances:\n{}".format(tree.feature_importances_))
```

Out[62]:

```
Feature importances:
[ 0.    0.    0.    0.    0.    0.    0.    0.    0.    0.01
 0.048 0.    0.    0.002 0.    0.    0.    0.    0.727 0.046
 0.    0.    0.014 0.    0.018 0.122 0.012 0.    ]
```

We can visualize the feature importances in a way that is similar to the way we visualize the coefficients in the linear model ([Figure 2-28](#)):

In[63]:

```
def plot_feature_importances_cancer(model):
    n_features = cancer.data.shape[1]
    plt.barh(range(n_features), model.feature_importances_, align='center')
    plt.yticks(np.arange(n_features), cancer.feature_names)
    plt.xlabel("Feature importance")
    plt.ylabel("Feature")

plot_feature_importances_cancer(tree)
```

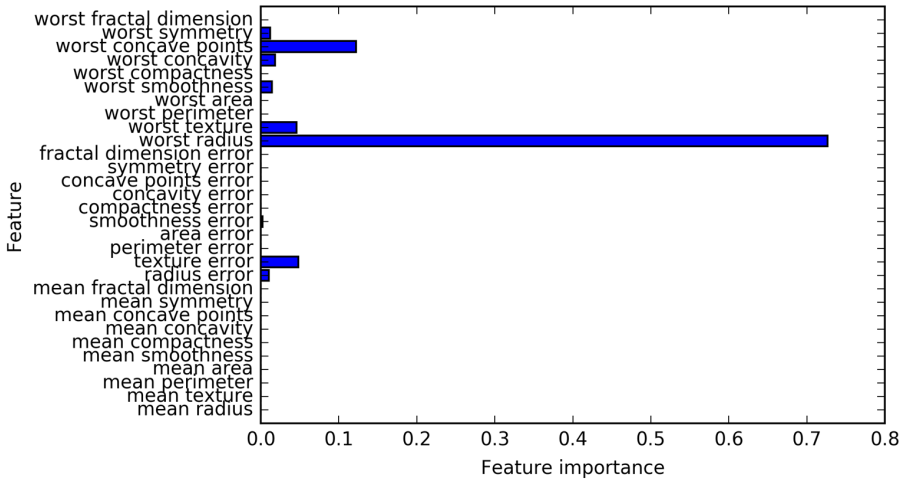



Figure 2-28. Feature importances computed from a decision tree learned on the Breast Cancer dataset

Here we see that the feature used in the top split (“worst radius”) is by far the most important feature. This confirms our observation in analyzing the tree that the first level already separates the two classes fairly well.

However, if a feature has a low `feature_importance`, it doesn’t mean that this feature is uninformative. It only means that the feature was not picked by the tree, likely because another feature encodes the same information.

In contrast to the coefficients in linear models, feature importances are always positive, and don’t encode which class a feature is indicative of. The feature importances tell us that “worst radius” is important, but not whether a high radius is indicative of a sample being benign or malignant. In fact, there might not be such a simple relationship between features and class, as you can see in the following example (Figures 2-29 and 2-30):

In[64]:

```
tree = mglearn.plots.plot_tree_not_monotone()
display(tree)
```

Out[64]:

```
Feature importances: [ 0.  1.]
```

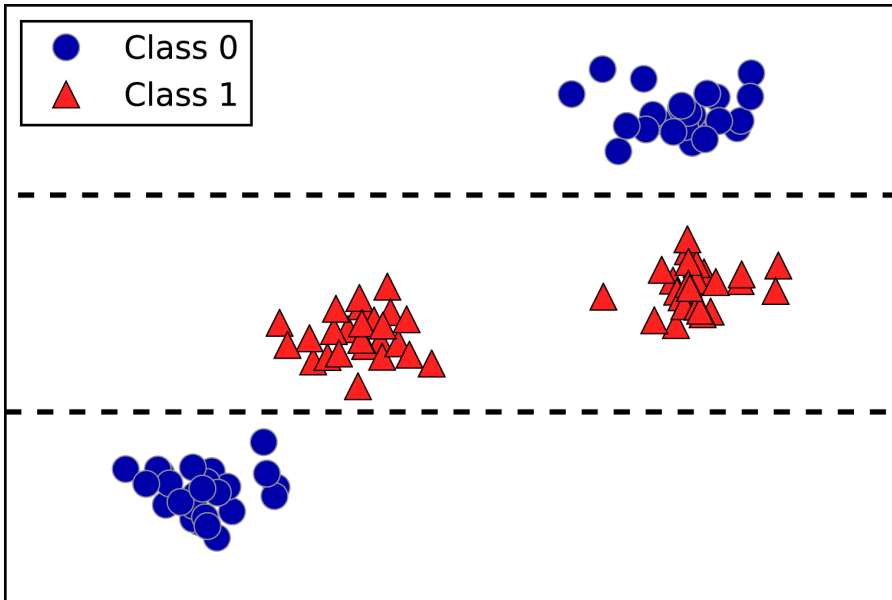


Figure 2-29. A two-dimensional dataset in which the feature on the y-axis has a nonmonotonous relationship with the class label, and the decision boundaries found by a decision tree

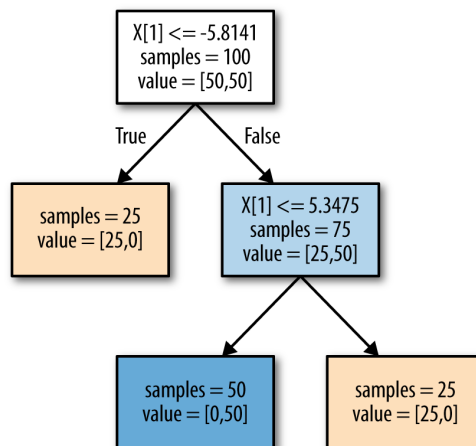


Figure 2-30. Decision tree learned on the data shown in *Figure 2-29*

The plot shows a dataset with two features and two classes. Here, all the information is contained in $X[1]$, and $X[0]$ is not used at all. But the relation between $X[1]$ and

the output class is not monotonous, meaning we cannot say “a high value of $X[0]$ means class 0, and a low value means class 1” (or vice versa).

While we focused our discussion here on decision trees for classification, all that was said is similarly true for decision trees for regression, as implemented in `DecisionTreeRegressor`. The usage and analysis of regression trees is very similar to that of classification trees. There is one particular property of using tree-based models for regression that we want to point out, though. The `DecisionTreeRegressor` (and all other tree-based regression models) is not able to *extrapolate*, or make predictions outside of the range of the training data.

Let’s look into this in more detail, using a dataset of historical computer memory (RAM) prices. **Figure 2-31** shows the dataset, with the date on the x-axis and the price of one megabyte of RAM in that year on the y-axis:

In[65]:

```
import pandas as pd
ram_prices = pd.read_csv("data/ram_price.csv")

plt.semilogy(ram_prices.date, ram_prices.price)
plt.xlabel("Year")
plt.ylabel("Price in $/Mbyte")
```

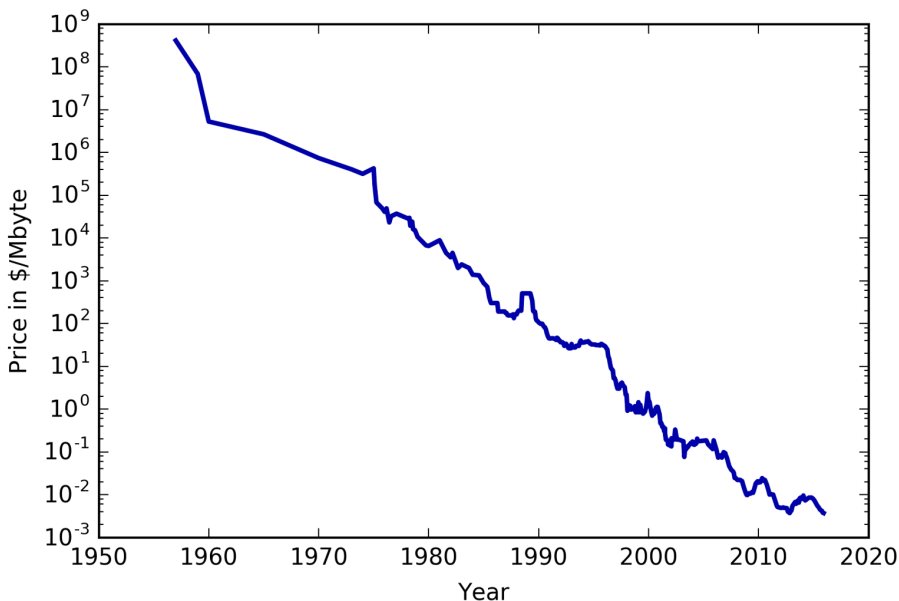


Figure 2-31. Historical development of the price of RAM, plotted on a log scale

Note the logarithmic scale of the y-axis. When plotting logarithmically, the relation seems to be quite linear and so should be relatively easy to predict, apart from some bumps.

We will make a forecast for the years after 2000 using the historical data up to that point, with the date as our only feature. We will compare two simple models: a `DecisionTreeRegressor` and `LinearRegression`. We rescale the prices using a logarithm, so that the relationship is relatively linear. This doesn't make a difference for the `DecisionTreeRegressor`, but it makes a big difference for `LinearRegression` (we will discuss this in more depth in [Chapter 4](#)). After training the models and making predictions, we apply the exponential map to undo the logarithm transform. We make predictions on the whole dataset for visualization purposes here, but for a quantitative evaluation we would only consider the test dataset:

In[66]:

```
from sklearn.tree import DecisionTreeRegressor
# use historical data to forecast prices after the year 2000
data_train = ram_prices[ram_prices.date < 2000]
data_test = ram_prices[ram_prices.date >= 2000]

# predict prices based on date
X_train = data_train.date[:, np.newaxis]
# we use a log-transform to get a simpler relationship of data to target
y_train = np.log(data_train.price)

tree = DecisionTreeRegressor().fit(X_train, y_train)
linear_reg = LinearRegression().fit(X_train, y_train)

# predict on all data
X_all = ram_prices.date[:, np.newaxis]

pred_tree = tree.predict(X_all)
pred_lr = linear_reg.predict(X_all)

# undo log-transform
price_tree = np.exp(pred_tree)
price_lr = np.exp(pred_lr)
```

Figure 2-32, created here, compares the predictions of the decision tree and the linear regression model with the ground truth:

In[67]:

```
plt.semilogy(data_train.date, data_train.price, label="Training data")
plt.semilogy(data_test.date, data_test.price, label="Test data")
plt.semilogy(ram_prices.date, price_tree, label="Tree prediction")
plt.semilogy(ram_prices.date, price_lr, label="Linear prediction")
plt.legend()
```

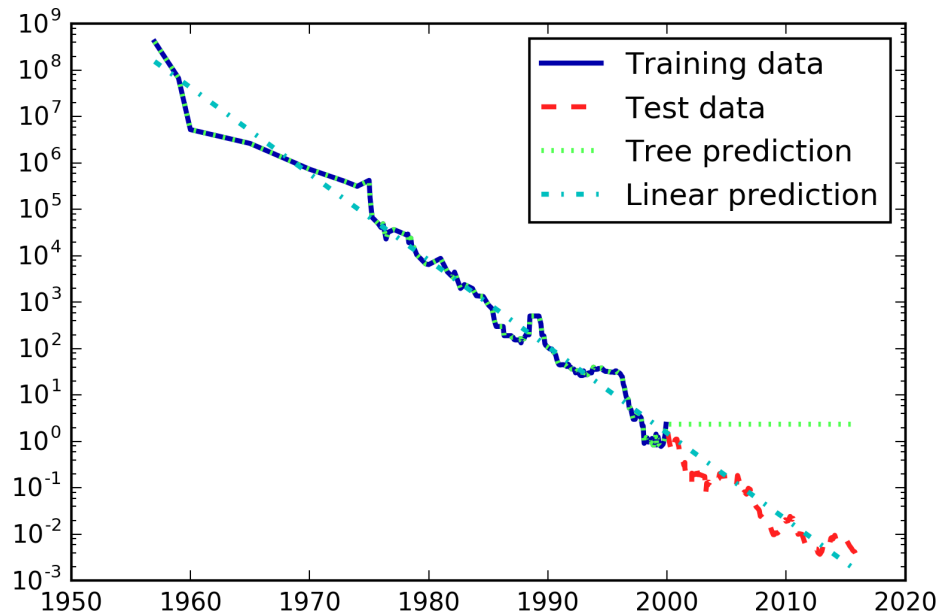


Figure 2-32. Comparison of predictions made by a linear model and predictions made by a regression tree on the RAM price data

The difference between the models is quite striking. The linear model approximates the data with a line, as we knew it would. This line provides quite a good forecast for the test data (the years after 2000), while glossing over some of the finer variations in both the training and the test data. The tree model, on the other hand, makes perfect predictions on the training data; we did not restrict the complexity of the tree, so it learned the whole dataset by heart. However, once we leave the data range for which the model has data, the model simply keeps predicting the last known point. The tree has no ability to generate “new” responses, outside of what was seen in the training data. This shortcoming applies to all models based on trees.⁹

Strengths, weaknesses, and parameters

As discussed earlier, the parameters that control model complexity in decision trees are the pre-pruning parameters that stop the building of the tree before it is fully developed. Usually, picking one of the pre-pruning strategies—setting either

⁹ It is actually possible to make very good forecasts with tree-based models (for example, when trying to predict whether a price will go up or down). The point of this example was not to show that trees are a bad model for time series, but to illustrate a particular property of how trees make predictions.

`max_depth`, `max_leaf_nodes`, or `min_samples_leaf`—is sufficient to prevent overfitting.

Decision trees have two advantages over many of the algorithms we've discussed so far: the resulting model can easily be visualized and understood by nonexperts (at least for smaller trees), and the algorithms are completely invariant to scaling of the data. As each feature is processed separately, and the possible splits of the data don't depend on scaling, no preprocessing like normalization or standardization of features is needed for decision tree algorithms. In particular, decision trees work well when you have features that are on completely different scales, or a mix of binary and continuous features.

The main downside of decision trees is that even with the use of pre-pruning, they tend to overfit and provide poor generalization performance. Therefore, in most applications, the ensemble methods we discuss next are usually used in place of a single decision tree.

Ensembles of Decision Trees

Ensembles are methods that combine multiple machine learning models to create more powerful models. There are many models in the machine learning literature that belong to this category, but there are two ensemble models that have proven to be effective on a wide range of datasets for classification and regression, both of which use decision trees as their building blocks: random forests and gradient boosted decision trees.

Random forests

As we just observed, a main drawback of decision trees is that they tend to overfit the training data. Random forests are one way to address this problem. A random forest is essentially a collection of decision trees, where each tree is slightly different from the others. The idea behind random forests is that each tree might do a relatively good job of predicting, but will likely overfit on part of the data. If we build many trees, all of which work well and overfit in different ways, we can reduce the amount of overfitting by averaging their results. This reduction in overfitting, while retaining the predictive power of the trees, can be shown using rigorous mathematics.

To implement this strategy, we need to build many decision trees. Each tree should do an acceptable job of predicting the target, and should also be different from the other trees. Random forests get their name from injecting randomness into the tree building to ensure each tree is different. There are two ways in which the trees in a random forest are randomized: by selecting the data points used to build a tree and by selecting the features in each split test. Let's go into this process in more detail.