

- $n_c = m$, where m is the number of training instances.
- \mathbf{H} is the $n_p \times n_p$ identity matrix, except with a zero in the top-left cell (to ignore the bias term).
- $\mathbf{f} = \mathbf{0}$, an n_p -dimensional vector full of 0s.
- $\mathbf{b} = \mathbf{1}$, an n_c -dimensional vector full of 1s.
- $\mathbf{a}^{(i)} = -t^{(i)} \dot{\mathbf{x}}^{(i)}$, where $\dot{\mathbf{x}}^{(i)}$ is equal to $\mathbf{x}^{(i)}$ with an extra bias feature $\dot{\mathbf{x}}_0 = 1$.

So one way to train a hard margin linear SVM classifier is just to use an off-the-shelf QP solver by passing it the preceding parameters. The resulting vector \mathbf{p} will contain the bias term $b = p_0$ and the feature weights $w_i = p_i$ for $i = 1, 2, \dots, m$. Similarly, you can use a QP solver to solve the soft margin problem (see the exercises at the end of the chapter).

However, to use the kernel trick we are going to look at a different constrained optimization problem.

The Dual Problem

Given a constrained optimization problem, known as the *primal problem*, it is possible to express a different but closely related problem, called its *dual problem*. The solution to the dual problem typically gives a lower bound to the solution of the primal problem, but under some conditions it can even have the same solutions as the primal problem. Luckily, the SVM problem happens to meet these conditions,⁶ so you can choose to solve the primal problem or the dual problem; both will have the same solution. Equation 5-6 shows the dual form of the linear SVM objective (if you are interested in knowing how to derive the dual problem from the primal problem, see Appendix C).

Equation 5-6. Dual form of the linear SVM objective

$$\begin{aligned} \underset{\alpha}{\text{minimize}} \quad & \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^m \alpha^{(i)} \alpha^{(j)} t^{(i)} t^{(j)} \mathbf{x}^{(i)T} \cdot \mathbf{x}^{(j)} \quad - \quad \sum_{i=1}^m \alpha^{(i)} \\ \text{subject to} \quad & \alpha^{(i)} \geq 0 \quad \text{for } i = 1, 2, \dots, m \end{aligned}$$

⁶ The objective function is convex, and the inequality constraints are continuously differentiable and convex functions.

Once you find the vector $\hat{\alpha}$ that minimizes this equation (using a QP solver), you can compute $\hat{\mathbf{w}}$ and \hat{b} that minimize the primal problem by using [Equation 5-7](#).

Equation 5-7. From the dual solution to the primal solution

$$\hat{\mathbf{w}} = \sum_{i=1}^m \hat{\alpha}^{(i)} t^{(i)} \mathbf{x}^{(i)}$$

$$\hat{b} = \frac{1}{n_s} \sum_{\substack{i=1 \\ \hat{\alpha}^{(i)} > 0}}^m \left(1 - t^{(i)} (\hat{\mathbf{w}}^T \cdot \mathbf{x}^{(i)}) \right)$$

The dual problem is faster to solve than the primal when the number of training instances is smaller than the number of features. More importantly, it makes the kernel trick possible, while the primal does not. So what is this kernel trick anyway?

Kernelized SVM

Suppose you want to apply a 2nd-degree polynomial transformation to a two-dimensional training set (such as the moons training set), then train a linear SVM classifier on the transformed training set. [Equation 5-8](#) shows the 2nd-degree polynomial mapping function ϕ that you want to apply.

Equation 5-8. Second-degree polynomial mapping

$$\phi(\mathbf{x}) = \phi\left(\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}\right) = \begin{pmatrix} x_1^2 \\ \sqrt{2} x_1 x_2 \\ x_2^2 \end{pmatrix}$$

Notice that the transformed vector is three-dimensional instead of two-dimensional. Now let's look at what happens to a couple of two-dimensional vectors, \mathbf{a} and \mathbf{b} , if we apply this 2nd-degree polynomial mapping and then compute the dot product of the transformed vectors (See [Equation 5-9](#)).