of shape (224, 224, 3). Continuing the analogy, consider a color video. Suppose that each frame of the video is a 224 × 224 color image. Then a minute of video (at 60 fps) would be a rank-4 tensor of shape (224, 224, 3, 3600). Continuing even further, a collection of 10 such videos would then form a rank-5 tensor of shape (10, 224, 224, 3, 3600). In general, tensors provide for a convenient representation of numeric data. In practice, it's not common to see tensors of higher order than rank-5 tensors, but it's smart to design any tensor software to allow for arbitrary tensors since intelligent users will always come up with use cases designers don't consider.

Tensors in Physics

Tensors are used widely in physics to encode fundamental physical quantities. For example, the stress tensor is commonly used in material science to define the stress at a point within a material. Mathematically, the stress tensor is a rank-2 tensor of shape (3, 3):

$$\sigma = \begin{pmatrix} \sigma_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \sigma_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \sigma_{33} \end{pmatrix}$$

Then, suppose that n is a vector of shape (3) that encodes a direction. The stress T^n in direction n is specified by the vector $T^n = T \cdot n$ (note the matrix-vector multiplication). This relationship is depicted pictorially in Figure 2-5.

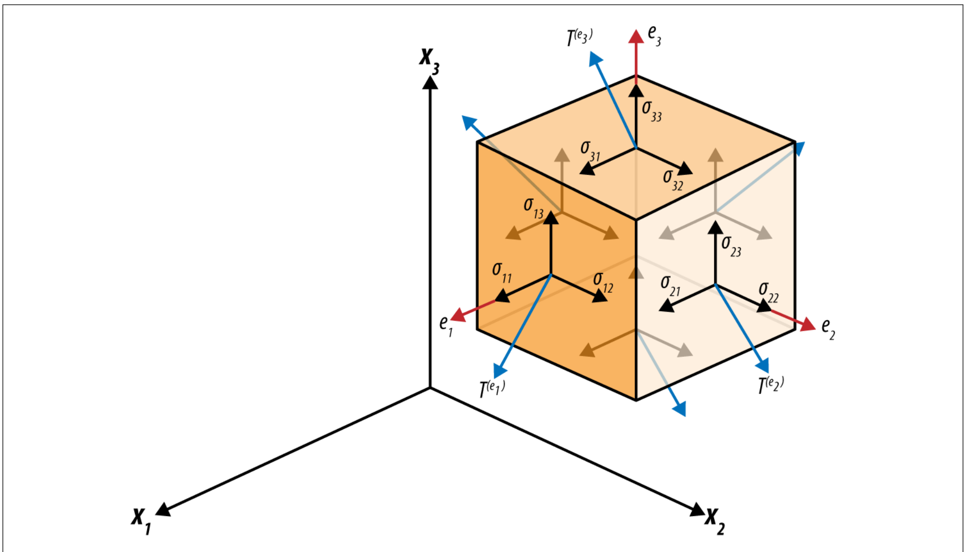

Figure 2-5. A 3D pictorial depiction of the components of stress.

As another physical example, Einstein's field equations of general relativity are commonly expressed in tensorial format:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Here $R_{\mu\nu}$ is the Ricci curvature tensor, $g_{\mu\nu}$ is the metric tensor, $T_{\mu\nu}$ is the stress-energy tensor, and the remaining quantities are scalars. Note, however, that there's an important subtlety distinguishing these tensors and the other tensors we've discussed previously. Quantities like the metric tensor provide a separate tensor (in the sense of an array of numbers) for each point in space-time (mathematically, the metric tensor is a tensor field). The same holds for the stress tensor previously discussed, and for the other tensors in these equations. At a given point in space-time, each of these quantities becomes a symmetric rank-2 tensor of shape (4, 4) using our notation.

Part of the power of modern tensor calculus systems such as TensorFlow is that some of the mathematical machinery long used for classical physics can now be adapted to solve applied problems in image processing and language understanding. At the same time, today's tensor calculus systems are still limited compared with the mathematical machinery of physicists. For example, there's no simple way to talk about a quantity such as the metric tensor using TensorFlow yet. We hope that as tensor calculus becomes more fundamental to computer science, the situation will change and that systems like TensorFlow will serve as a bridge between the physical world and the computational world.

Mathematical Asides

The discussion so far in this chapter has introduced tensors informally via example and illustration. In our definition, a tensor is simply an array of numbers. It's often convenient to view a tensor as a function instead. The most common definition introduces a tensor as a multilinear function from a product of vector spaces to the real numbers:

$$T: V_1 \times V_2 \times \cdots V_n \to \mathbb{R}$$

This definition uses a number of terms you haven't seen. A vector space is simply a collection of vectors. You've seen a few examples of vector spaces such as $\mathbb{R}^3$ or generally $\mathbb{R}^n$. We won't lose any generality by holding that $V_i = \mathbb{R}^{d_i}$. As we defined previously, a function f is linear if $f(x + y) = f(x) + f(y)$ and $f(cx) = cf(x)$. A multilinear function is simply a function that is linear in each argument. This function can be