```
import numpy.random as rnd

learning_rate0 = 0.05
learning_rate_decay = 0.1
n_iterations = 20000

s = 0 # start in state 0

Q = np.full((3, 3), -np.inf)  # -inf for impossible actions
for state, actions in enumerate(possible_actions):
    Q[state, actions] = 0.0  # Initial value = 0.0, for all possible actions

for iteration in range(n_iterations):
    a = rnd.choice(possible_actions[s])  # choose an action (randomly)
    sp = rnd.choice(range(3), p=T[s, a]) # pick next state using T[s, a]
    reward = R[s, a, sp]
    learning_rate = learning_rate0 / (1 + iteration * learning_rate_decay)
    Q[s, a] = learning_rate * Q[s, a] + (1 - learning_rate) * (
            reward + discount_rate * np.max(Q[sp])
        )
    s = sp # move to next state
```

Given enough iterations, this algorithm will converge to the optimal Q-Values. This is called an *off-policy* algorithm because the policy being trained is not the one being executed. It is somewhat surprising that this algorithm is capable of learning the optimal policy by just watching an agent act randomly (imagine learning to play golf when your teacher is a drunken monkey). Can we do better?

Exploration Policies

Of course Q-Learning can work only if the exploration policy explores the MDP thoroughly enough. Although a purely random policy is guaranteed to eventually visit every state and every transition many times, it may take an extremely long time to do so. Therefore, a better option is to use the *ε-greedy policy*: at each step it acts randomly with probability ε, or greedily (choosing the action with the highest Q-Value) with probability 1-ε. The advantage of the ε-greedy policy (compared to a completely random policy) is that it will spend more and more time exploring the interesting parts of the environment, as the Q-Value estimates get better and better, while still spending some time visiting unknown regions of the MDP. It is quite common to start with a high value for ε (e.g., 1.0) and then gradually reduce it (e.g., down to 0.05).

Alternatively, rather than relying on chance for exploration, another approach is to encourage the exploration policy to try actions that it has not tried much before. This can be implemented as a bonus added to the Q-Value estimates, as shown in Equation 16-6.

Equation 16-6. Q-Learning using an exploration function

$$Q(s,a) \leftarrow (1-\alpha)Q(s,a) + \alpha\left(r + \gamma \cdot \max_{a'} f(Q(s',a'), N(s',a'))\right)$$

- $N(s', a')$ counts the number of times the action a' was chosen in state s'.
- $f(q, n)$ is an *exploration function*, such as $f(q, n) = q + K/(1 + n)$, where K is a curiosity hyperparameter that measures how much the agent is attracted to to the unknown.

Approximate Q-Learning

The main problem with Q-Learning is that it does not scale well to large (or even medium) MDPs with many states and actions. Consider trying to use Q-Learning to train an agent to play Ms. Pac-Man. There are over 250 pellets that Ms. Pac-Man can eat, each of which can be present or absent (i.e., already eaten). So the number of possible states is greater than $2^{250} \approx 10^{75}$ (and that's considering the possible states only of the pellets). This is way more than atoms in the observable universe, so there's absolutely no way you can keep track of an estimate for every single Q-Value.

The solution is to find a function that approximates the Q-Values using a manageable number of parameters. This is called *Approximate Q-Learning*. For years it was recommended to use linear combinations of hand-crafted features extracted from the state (e.g., distance of the closest ghosts, their directions, and so on) to estimate Q-Values, but DeepMind showed that using deep neural networks can work much better, especially for complex problems, and it does not require any feature engineering. A DNN used to estimate Q-Values is called a *deep Q-network* (DQN), and using a DQN for Approximate Q-Learning is called *Deep Q-Learning*.

In the rest of this chapter, we will use Deep Q-Learning to train an agent to play Ms. Pac-Man, much like DeepMind did in 2013. The code can easily be tweaked to learn to play the majority of Atari games quite well. It can achieve superhuman skill at most action games, but it is not so good at games with long-running storylines.

Learning to Play Ms. Pac-Man Using Deep Q-Learning

Since we will be using an Atari environment, we must first install OpenAI gym's Atari dependencies. While we're at it, we will also install dependencies for other OpenAI gym environments that you may want to play with. On macOS, assuming you have installed Homebrew, you need to run:

```
$ brew install cmake boost boost-python sdl2 swig wget
```