and Patrick Haffner, which introduced the famous *LeNet-5* architecture, widely used to recognize handwritten check numbers. This architecture has some building blocks that you already know, such as fully connected layers and sigmoid activation functions, but it also introduces two new building blocks: *convolutional layers* and *pooling layers*. Let's look at them now.

Why not simply use a regular deep neural network with fully connected layers for image recognition tasks? Unfortunately, although this works fine for small images (e.g., MNIST), it breaks down for larger images because of the huge number of parameters it requires. For example, a 100×100 image has 10,000 pixels, and if the first layer has just 1,000 neurons (which already severely restricts the amount of information transmitted to the next layer), this means a total of 10 million connections. And that's just the first layer. CNNs solve this problem using partially connected layers.

Convolutional Layer

The most important building block of a CNN is the *convolutional layer*:[6] neurons in the first convolutional layer are not connected to every single pixel in the input image (like they were in previous chapters), but only to pixels in their receptive fields (see Figure 13-2). In turn, each neuron in the second convolutional layer is connected only to neurons located within a small rectangle in the first layer. This architecture allows the network to concentrate on low-level features in the first hidden layer, then assemble them into higher-level features in the next hidden layer, and so on. This hierarchical structure is common in real-world images, which is one of the reasons why CNNs work so well for image recognition.

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6 A convolution is a mathematical operation that slides one function over another and measures the integral of their pointwise multiplication. It has deep connections with the Fourier transform and the Laplace transform, and is heavily used in signal processing. Convolutional layers actually use cross-correlations, which are very similar to convolutions (see *http://goo.gl/HAfxXd* for more details).
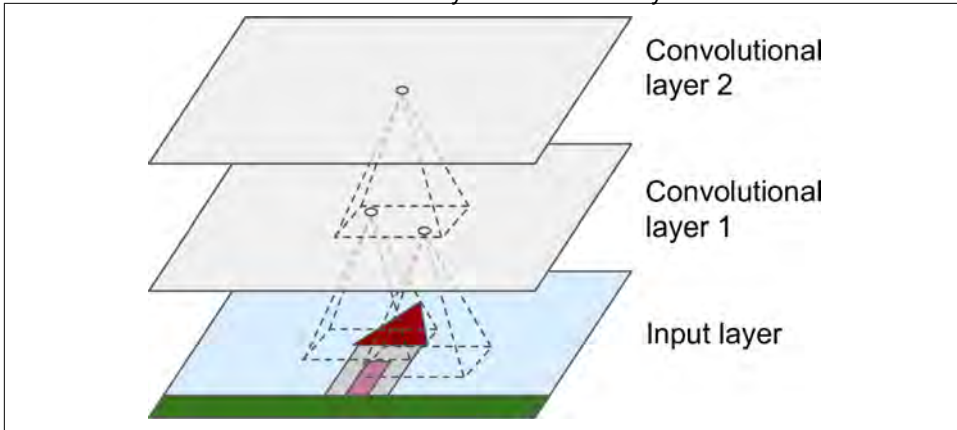
Figure 13-2. CNN layers with rectangular local receptive fields

> Until now, all multilayer neural networks we looked at had layers composed of a long line of neurons, and we had to flatten input images to 1D before feeding them to the neural network. Now each layer is represented in 2D, which makes it easier to match neurons with their corresponding inputs.

A neuron located in row i, column j of a given layer is connected to the outputs of the neurons in the previous layer located in rows i to $i + f_h - 1$, columns j to $j + f_w - 1$, where f_h and f_w are the height and width of the receptive field (see Figure 13-3). In order for a layer to have the same height and width as the previous layer, it is common to add zeros around the inputs, as shown in the diagram. This is called *zero padding*.
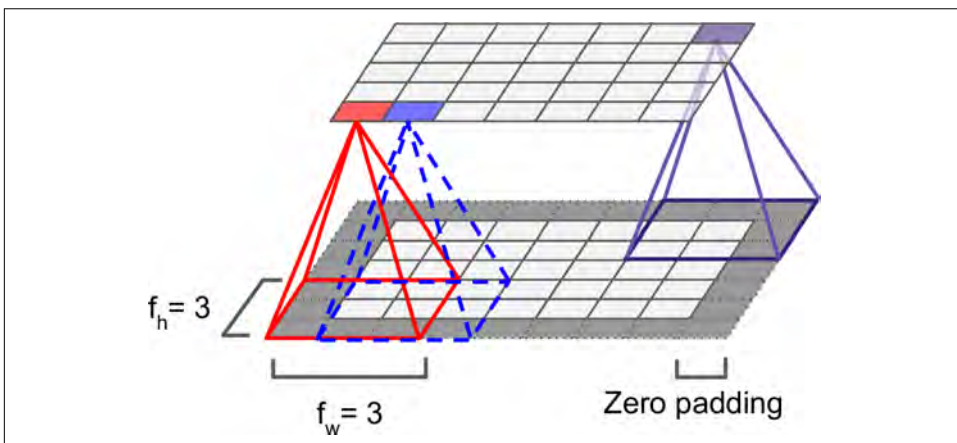

Figure 13-3. Connections between layers and zero padding

It is also possible to connect a large input layer to a much smaller layer by spacing out the receptive fields, as shown in Figure 13-4. The distance between two consecutive receptive fields is called the *stride*. In the diagram, a 5 × 7 input layer (plus zero padding) is connected to a 3 × 4 layer, using 3 × 3 receptive fields and a stride of 2 (in this example the stride is the same in both directions, but it does not have to be so). A neuron located in row i, column j in the upper layer is connected to the outputs of the neurons in the previous layer located in rows $i \times s_h$ to $i \times s_h + f_h - 1$, columns $j \times s_w + f_w - 1$, where s_h and s_w are the vertical and horizontal strides.
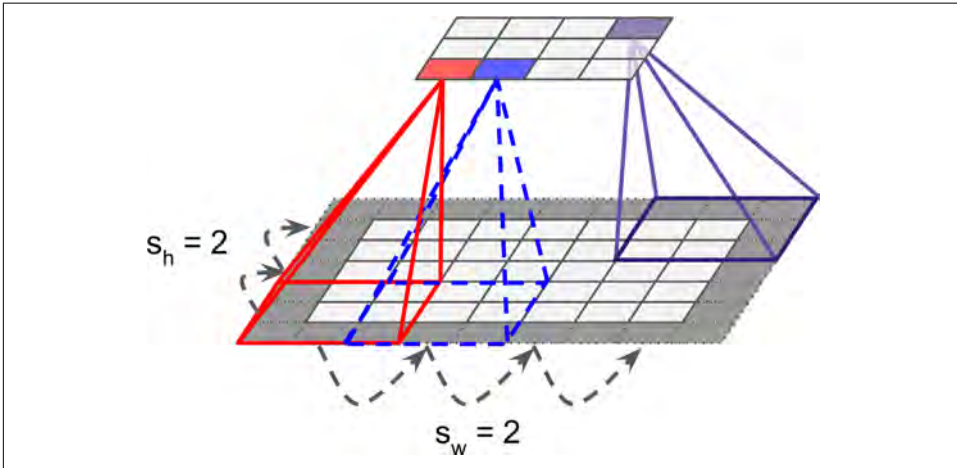

Figure 13-4. Reducing dimensionality using a stride

Filters

A neuron's weights can be represented as a small image the size of the receptive field. For example, Figure 13-5 shows two possible sets of weights, called *filters* (or *convolution kernels*). The first one is represented as a black square with a vertical white line in the middle (it is a 7 × 7 matrix full of 0s except for the central column, which is full of 1s); neurons using these weights will ignore everything in their receptive field except for the central vertical line (since all inputs will get multiplied by 0, except for the ones located in the central vertical line). The second filter is a black square with a horizontal white line in the middle. Once again, neurons using these weights will ignore everything in their receptive field except for the central horizontal line.

Now if all neurons in a layer use the same vertical line filter (and the same bias term), and you feed the network the input image shown in Figure 13-5 (bottom image), the layer will output the top-left image. Notice that the vertical white lines get enhanced while the rest gets blurred. Similarly, the upper-right image is what you get if all neurons use the horizontal line filter; notice that the horizontal white lines get enhanced while the rest is blurred out. Thus, a layer full of neurons using the same filter gives