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Digital Signal Processing

Assignment: 03

Questions:

Question: 1

The signal $x_c(t)$ be given by

$$x_c(t) = 2\cos(650\pi t) + 4\cos(700\pi t) + 6\cos(750\pi t) + 8\cos(800\pi t)$$

Solution:

$$x_c(t) = 2\cos(650\pi t) + 4\cos(700\pi t) + 6\cos(750\pi t) + 8\cos(800\pi t)$$

As we know sampling is done by:

$$f_s \geq 2 f_m$$

$$\Omega_s \geq 2\Omega_N$$

As we know F_s is given but maximum frequency (F_m) which is not given so we have to find F_m : we need to find frequency of each component

$$w_1 = 650\pi$$

$$2\pi f_1 = 650\pi$$

$$f_1 = \frac{650\pi}{2\pi}$$

$$f_1 = 325\ Hz$$

$\omega_2 = 700\pi$

$2\pi f_2 = 700\pi$

$f_2 = \dfrac{700\pi}{2\pi}$

$f_2 = 350\ Hz$

$\omega_3 = 750\pi$

$2\pi f_3 = 750\pi$

$f_3 = \dfrac{750\pi}{2\pi}$

$f_3 = 375\ Hz$

$\omega_4 = 800\pi$

$2\pi f_4 = 800\pi$

$f_4 = \dfrac{800\pi}{2\pi}$

$f_4 = 400\ Hz$

$f_m = \max(f_1, f_2, f_3, f_4)$

so; $f_m = 400\ Hz$; This is maximum frequency. from Sampling Theorem and Nyquist criteria;

$\Omega_s \geq 2\Omega_N$

$f_s \geq 2 f_m$

To avoid Aliasing Effect

$f_s \geq 2(400)$

$f_s \geq 800$

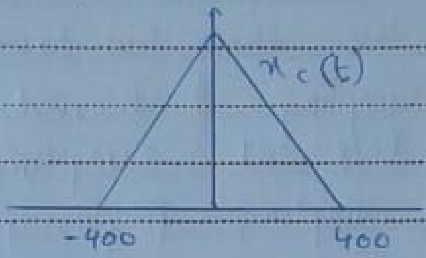
a) If $x_c(t)$ is sampled at $F_s = 801$ samples/sec, determine and plot the spectrum of the sampled signal as a function of frequency.

$$f_1 \geq 2F_m$$

so; $$801 \geq 2(400)$$

$$801 \geq 800$$

condition is fine and there is no Aliasing

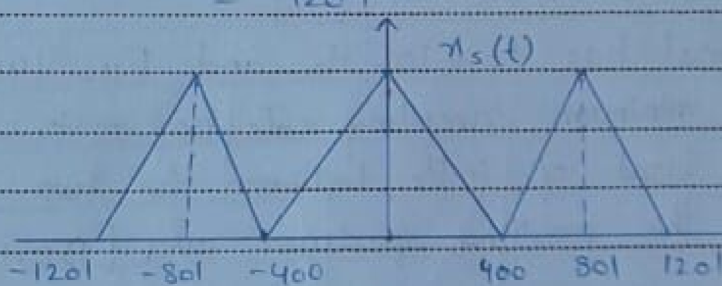


After sampling every spectrum repeated at f_s

$$F_s - F_m = \text{periodic value}$$

$$801 - 400 = 401 + 800$$

$$= 1201$$

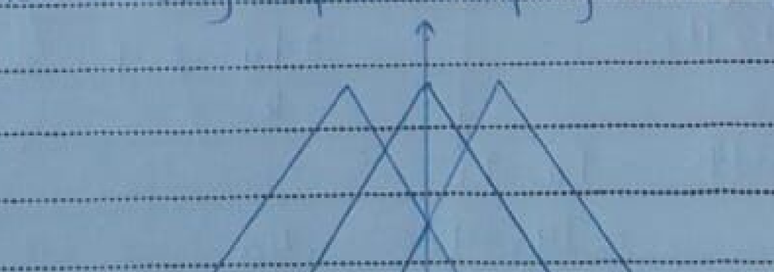


b) If $x_c(t)$ is sampled at $F_s = 201$ samples/sec, determine and plot the spectrum of the sampled signal as a function of frequency.

$$F_s \geq 2F_m$$

$$201 \ngeq 800$$

So there is Aliasing after sampling the spectrum



Sampling frequency is much less than maximum frequency. This is a distorted signal.

c) what is your observation of the baseband signals after sampling in each of the above two cases.

In both cases after sampling I observed that we got different output on different sampling rate and aliasing occurs when sampling frequency is less than two times of maximum frequency, so to avoid Aliasing the criteria must be

$$\Omega_s \geqslant 2\,\Omega_N$$

Question: 2

A bandpass signal has $f_L = 105\ Hz$ and $F_H = 145\ Hz$. Determine the minimum sampling rate so as to have a minimum guard band of $10\ Hz$ between the two spectrum replicas. Draw the resulting spectrum over $[-150, 150]\ Hz$ range.

Solution:

Given:

$$F_L = 105\ Hz$$

$$F_H = 145\ Hz$$

calculating the minimum sampling rate

$$F_S = \frac{2\ F_H}{(F_H/B)} + 10\ Hz \longrightarrow (1)$$

where; $F_H = 145\ Hz$; $\qquad \frac{2F_H}{k} \leq F_S \leq \frac{2F_L}{k-1}$

$$B = \text{Bandwidth} = F_H - F_L$$

$$= 145 - 105 = 40$$

put the values of

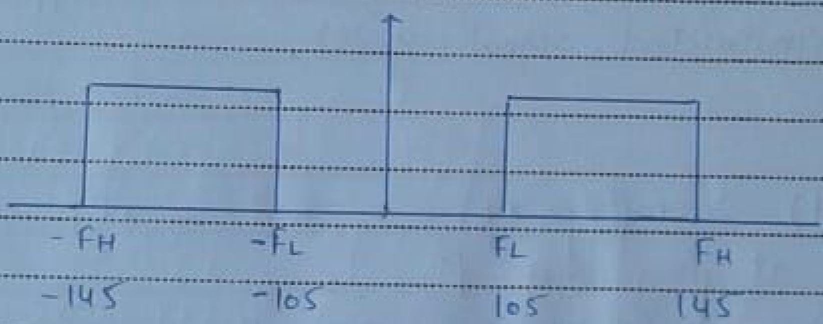
F_L, F_H, B in eq ①

$$F_s = 2\left(\frac{145}{\frac{145}{40}}\right) + 10\ Hz$$

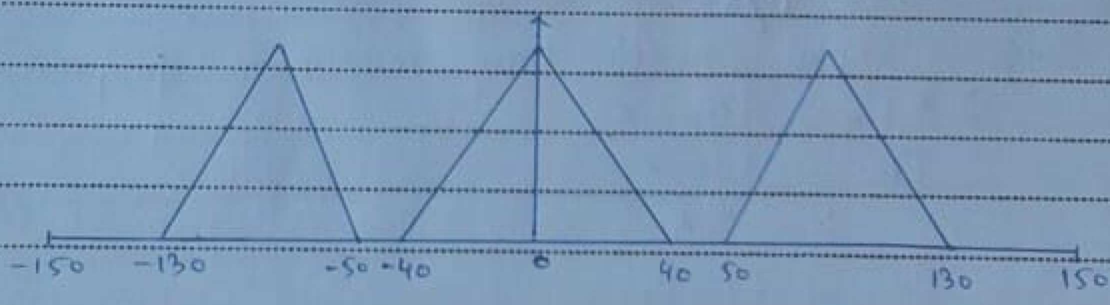
$$= 2\frac{(145)}{3.625} + 10\ Hz$$

$$= 90\ Hz$$

$\therefore k = F_H/B$



The minimum sampling rate $F_s = 2f_m$ required to recover the signal (Nyquist).



So;

within the range [-150, 150], this was the spectrum.

$$\Omega_s - \Omega_N = \text{Periodic value}$$

$$F_s - F_m = 90 - 40$$

$$= 50$$

$$90 + 40 = 130$$

Question: 3

An 8-bit ADC has an input analog range of ± 5 volts. The analog input signal is $x_c(t) = 2\cos(200\pi t) + 3\sin(500\pi t)$. The converter supplies data to a computer at a rate of 2048 bits/s. The computer, without processing supplies these data to an ideal DAC to form the reconstructed signal $y_c(t)$. Determine (a) the quantizer step size, (b) the folding frequency and the Nyquist rate, (c) the reconstructed signal $y_c(t)$.

Solution:

Given:

$$x_c(t) = 2\cos(200\pi t) + 3\sin(500\pi t)$$

sampled at frequency f_s;

as $n = 8$ bit/ samples are used

$$n f_s = \text{Data Rate} = 2048$$

$$f_s = 2048/8 \text{ sample/s} = 256 \text{ samples/sec}$$

Since; Signal $x_c(t)$ have maximum frequency

$$2\pi f_m = 500\pi$$

$$f_m = \frac{500\pi}{2\pi}$$

$$f_m = 250 \text{ Hz}$$

a) The Quantizer Step Size:

$$\text{Quantizer step size} = \Delta = \frac{2m_p}{2^n}$$

Where; $m_p = 5\text{ V}$

$$\Delta = \frac{2\times 5}{2^8}$$

$$= 0.03906\text{ V}$$

b) The folding frequency and the Nyquist rate:

As we know;

Folding frequency (F_N) = Maximum frequency (F_m)

F_N = Maximum frequency of $x_c(t)$

$F_N = F_M$

$F_N = 250\ Hz$ $\qquad \because F_M = 250\ H_z$

Nyquist Rate :

$F_m = 2F_m$

$F_{NS} = 500$ samples / sec

which is as;

$F_{NS} = 2(250)$

$F_{NS} = 500$ sample / sec

c) The reconstructed signal $y_c(t)$:

Since signal $x_c(t)$ samples at instant

$$t = \frac{n}{F_s} = nT_s$$

$$x_c(nT_s) = 2\cos\left(200\pi(nT_s)\right) + 3\sin\left(500\pi(nT_s)\right)$$

$$x_c(nT_s) = 2\cos\left(200\pi\left(\frac{n}{F_s}\right)\right) + 3\sin\left(500\pi\left(\frac{n}{F_s}\right)\right)$$

$$= 2\cos\left(200\pi\left(\frac{n}{256}\right)\right) + 3\sin\left(500\pi\left(\frac{n}{256}\right)\right)$$

$$= 2\cos\left(\frac{25}{32}n\pi\right) + 3\sin\left(\frac{125}{64}n\pi\right)$$

$$= 2\cos\left(\frac{25}{32}n\pi\right) + 3\sin\left(2n\pi + \frac{(-3n\pi)}{64}\right)$$

$$x_c(nT_s) = 2\cos\left(\frac{25}{32}n\pi\right) - 3\sin\left(\frac{3n\pi}{64}\right)$$

After reconstruction ; replace $nT_s = t$

$$y_c(t) = 2\cos\left(200\pi \times \frac{n}{256}\right)\Big|_{t=n/256} - 3\sin\left(12\pi \frac{n}{256}\right)\Big|_{t=n/256}$$

$$y_c(t) = 2\cos(200\pi t) - 3\sin(12\pi t)$$

Question: 4

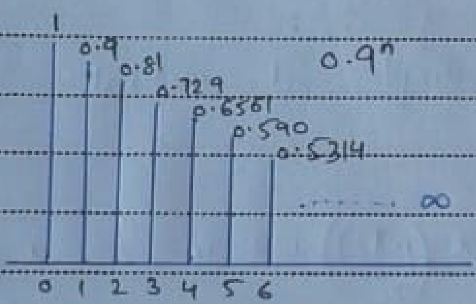
Consider the signal $x[n] = 0.9^n u[n]$. It is to be down-sampled by a factor of $M = 3$ to obtain $x_d[n]$.

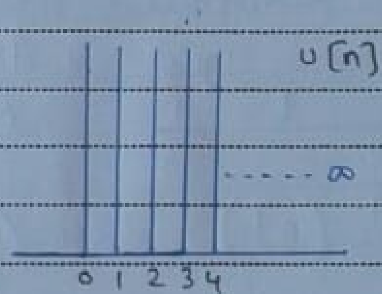
a) Compute the spectrum of $x[n]$ and plot its magnitude.

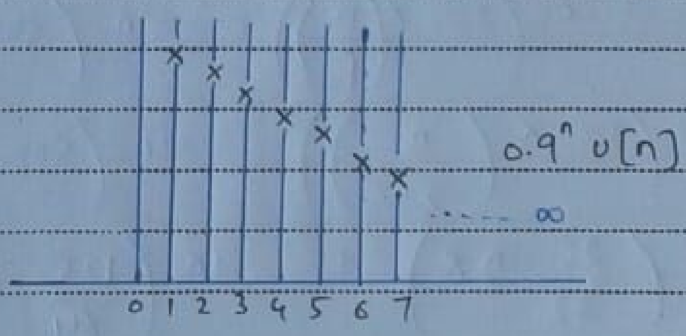
Solution:

Time Domain

$$x[n] = 0.9^n u[n]$$







Frequency Domain:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

This is by Applying DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (0.9)^n u[n] e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} 0.9^n e^{-j\omega n}$$

$\because u[n] = 1$

$$= \sum_{n=0}^{\infty} (0.9 e^{-j\omega})^n$$

$$X(e^{j\omega}) = \frac{1}{1 - 0.9 e^{-j\omega}}$$

Magnitude:

$$|X(e^{j\omega})| = \frac{1}{\sqrt{(1)^2 + (-0.9 e^{-j\omega})^2}}$$

Now, by Applying different values of ω we may obtain the following values

at $\omega = \pi$

$$|X(e^{j\omega})| = \frac{1}{\sqrt{(1)^2 + (-0.9 e^{-j(\pi)})^2}} = 0.72575$$

at $\omega = -\pi$

$$|X(e^{j\omega})| = \frac{1}{\sqrt{(1)^2 + (-0.9 e^{-j(-\pi)})^2}} = 0.72567$$

at $\omega = 0$

$$|X(e^{j\omega})| = \frac{1}{\sqrt{(1)^2 + (-0.9 e^{-j(0)})^2}} = 0.76327$$

b) Compute $x_d[n]$ and plot its magnitude.

$$X_d(e^{j\omega}) = \frac{1}{1 - 0.9 e^{3} e^{-j\omega}} = \frac{1}{1 - 0.729 e^{-j\omega}}$$

Magnitude:

$$|X_d(e^{j\omega})| = \frac{1}{\sqrt{(1)^2 + (-0.729e^{-j\omega})^2}}$$

at $\omega = \pi$

$$|X_d(e^{j\omega})| = \frac{1}{\sqrt{(1)^2 + (-0.729e^{-j(\pi)})^2}} = 0.8081$$

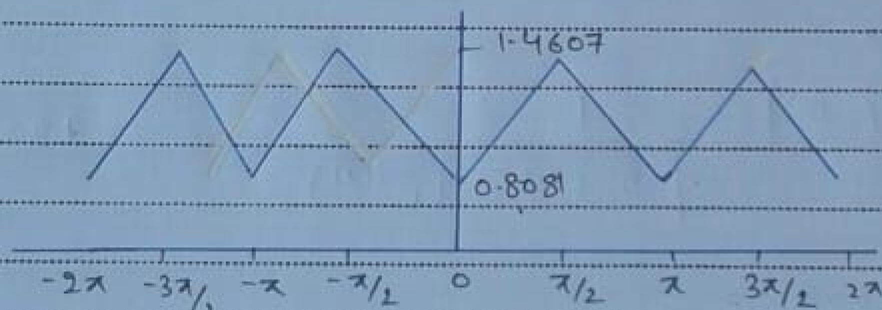
at $\omega = -\pi$

$$|X_d(e^{j\omega})| = \frac{1}{\sqrt{(1)^2 + (-0.729e^{-j(-\pi)})^2}} = 0.8081$$

at $\omega = 0$

$$|X_d(e^{j\omega})| = \frac{1}{\sqrt{(1)^2 + (-0.729e^{-j(0)})^2}} = 0.8081$$

At $\omega = \pi/2, -\pi/2, 3\pi/2, -3\pi/2 = 1.46089$



c) compare the two spectra

By the comparison, It can be seen that $x_d(n)$ is downsampled by the factor of M.