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Digital Signal Processing

Assignment: 02

Questions:

Question: 1

An LTI system has impulse response $h[n] = 5(-1/2)^n u[n]$ use the Fourier transform to find the output of the system when the input is $x[n] = (1/3)^n u[n]$.

Solution:

Given:

$$h[n] = 5\left(-1/2\right)^n u[n]$$
$$x[n] = \left(1/3\right)^n u[n]$$

$$y[n] = x[n] * h[n]$$

$$a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}}$$

$$5\left(-1/2\right)^n u[n] \longrightarrow \frac{5 \cdot 1}{1-\left(-\frac{1}{2}\right)z^{-1}} = 5 \cdot \frac{1}{1+\frac{1}{2}z^{-1}}$$

$$\left(1/3\right)^n u[n] \longrightarrow \frac{1}{1-\left(\frac{1}{3}\right)z^{-1}} = \frac{1}{1-\frac{1}{3}z^{-1}}$$

After Taking z-transform, we have

$Y(z) = X(z) \cdot H(z)$

$$Y(z) = \frac{1}{\left(1-\frac{1}{3}z^{-1}\right)} \cdot \frac{5}{\left(1+\frac{1}{2}z^{-1}\right)}$$

$$\frac{5}{\left(1-\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)} = \frac{A}{1-\frac{1}{3}z^{-1}} + \frac{B}{1+\frac{1}{2}z^{-1}}$$

$$A = \cancel{\left(1-\frac{1}{3}z^{-1}\right)}\left[\frac{5}{\cancel{\left(1-\frac{1}{3}z^{-1}\right)}\left(1+\frac{1}{2}z^{-1}\right)}\right]_{z=1/3}$$

$$= \frac{5}{1+\frac{1}{2(1/3)}} = \frac{5}{1+\frac{3}{2}}$$

$$A = \frac{5}{\frac{5}{2}} = 2$$

$$B = \cancel{\left(1+\frac{1}{2}z^{-1}\right)}\left[\frac{5}{\left(1-\frac{1}{3}z^{-1}\right)\cancel{\left(1+\frac{1}{2}z^{-1}\right)}}\right]_{z=-1/2}$$

$$= \frac{5}{1-\frac{1}{3(-1/2)}} = \frac{5}{1+\frac{2}{3}}$$

$$B = \frac{5}{5/3} = 3$$

$$= \frac{2}{1-1/3\,z^{-1}} + \frac{3}{1+1/2\,z^{-1}}$$

After Taking z-inverse transform, we have

$$y[n] = 2\left(\frac{1}{3}\right)^n u[n] + 3\left(\frac{-1}{2}\right)^n u[n]$$

Question: 2

Determine and plot the DTFT magnitude and phase spectra of the following Signals.

a) $x_1[n] = (1/3)^n u[n-1]$

Solution:

Given that:

$$x[n] = \left(\frac{1}{3}\right)^n u[n-1]$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n u[n-1] e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n (1) e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=1}^{\infty} \left(\frac{1}{3} e^{-j\omega}\right)^n$$

let $n-1 = m$

$n = 1 \qquad m = 0$

$n = \infty \qquad m = \infty$

So;

$$X(e^{j\omega}) = \sum_{m=0}^{\infty} \left(\frac{1}{3}\right)^{m+1} e^{-j\omega(m+1)}$$

$$= \sum_{m=0}^{\infty} \left(\frac{1}{3}\right)^m \left(\frac{1}{3}\right) e^{-j\omega m} e^{-j\omega}$$

$$= \frac{1}{3} e^{-j\omega} \sum_{m=0}^{\infty} \left(\frac{1}{3} e^{-j\omega}\right)^m$$

$$\therefore \sum_{k=0}^{\infty} a r^k = \frac{a}{1-r}$$

$$X(e^{j\omega}) = \frac{1}{3} e^{-j\omega} \cdot \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$X(e^{j\omega}) = \frac{1}{3} e^{-j\omega} \cdot \frac{1}{1 - \frac{1}{3e^{j\omega}}}$$

$$X(e^{j\omega}) = \frac{1}{3} e^{-j\omega} \cdot \frac{1}{\frac{3e^{j\omega}-1}{3e^{j\omega}}}$$

$$= \frac{1}{\cancel{3}} \cancel{e^{-j\omega}} \cdot \frac{\cancel{3} \cancel{e^{j\omega}}}{3e^{j\omega}-1}$$

$$X(e^{j\omega}) = \frac{1}{3e^{j\omega}-1}$$

As we Know;

$$e^{j\omega} = \cos\omega + j\sin\omega$$

$$X(e^{j\omega}) = \frac{1}{3(\cos\omega + j\sin\omega) - 1}$$

Now magnitude and phase;

$$X(e^{j\omega}) = \frac{1}{3\cos\omega + 3j\sin\omega - 1}$$

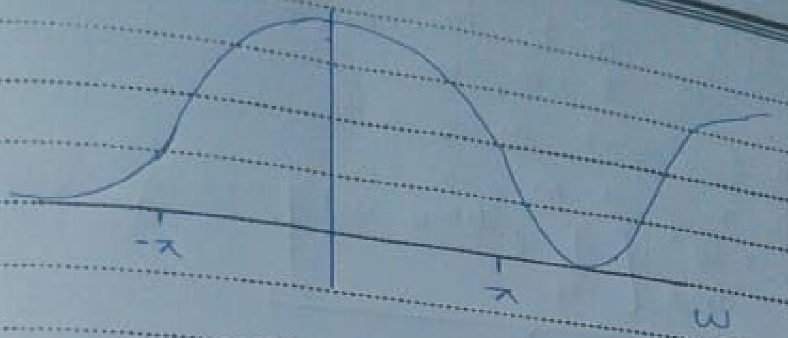
$$|X(e^{j\omega})| = \frac{1}{\sqrt{(3\cos\omega - 1)^2 + (3\sin\omega)^2}}$$

$$= \frac{1}{\sqrt{9\cos^2\omega - 6\cos\omega + 1 + 9\sin^2\omega}}$$

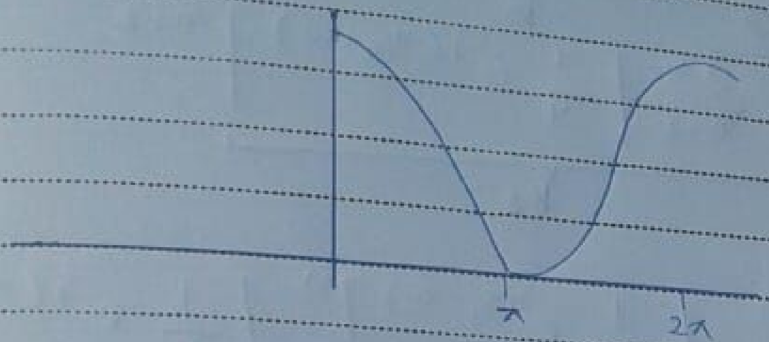
$$= \frac{1}{\sqrt{9 - 6\cos\omega + 1}}$$

$$|X(e^{j\omega})| = \frac{1}{\sqrt{10 - 6\cos\omega}}$$

Magnitude :



Phase :



Question: 2

b) $x_2[n] = \left(\frac{1}{4}\right)^n \cos\left(\pi \frac{n}{4}\right) u[n-2]$

Solution:

As we know

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

So by using this

$$x_2[n] = \left(\frac{1}{4}\right)^n \left[\frac{e^{j\pi n/4} + e^{-j\pi n/4}}{2}\right] u[n-2]$$

$$x_2[n] = \left[\frac{1}{2}\left(\frac{1}{4}\right)^n e^{j\pi n/4} + \frac{1}{2}\left(\frac{1}{4}\right)^n e^{-j\pi n/4}\right] u[n-2]$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$X(e^{j\omega}) = \frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{4}\right)^n e^{j\pi n/4} e^{-j\omega n} + \frac{1}{2}\sum_{n=2}^{\infty}\left(\frac{1}{4}\right)^n e^{-j\pi n/4} e^{-j\omega n}$$

Now;

$$n-2=m$$
$$2+m=n$$
$$n=2 \longrightarrow m=0$$
$$n=\infty \longrightarrow m=\infty$$

$$X(e^{j\omega}) = \underbrace{\frac{1}{2}\sum_{n=2}^{\infty}\left[\frac{1}{4}e^{j(\pi/4-\omega)}\right]^{m+2}}_{K_1} +$$

$$\underbrace{\frac{1}{2}\sum_{n=2}^{\infty}\left[\frac{1}{4}e^{-j(\pi/4+\omega)}\right]^{m+2}}_{K_2}$$

By Solving K_1:

$$K_1 = \frac{1}{2}\sum_{m=0}^{\infty}\left[\frac{1}{4}e^{j(\frac{\pi}{4}-\omega)}\right]^{M}\left[\frac{1}{4}e^{j(\pi/4-\omega)}\right]^{+2}$$

$$K_1 = \frac{1}{2}\left[\sum_{m=0}^{\infty}\frac{1}{4}e^{j(\pi/4-\omega)}\right]^{M}\left[\left(\frac{1}{4}\right)^{2}e^{2j(\pi/4-\omega)}\right]$$

$$K_1 = \frac{1}{2}\left[\frac{1}{1-\frac{1}{4e^{j(\frac{\pi}{4}-\omega)}}}\right]\frac{1}{16}e^{2j(\pi/4-\omega)}$$

$$K_1 = \frac{1}{32}\left[\frac{e^{2j(\pi/4-\omega)}}{\left[1-\frac{1}{4e^{j(\pi/4-\omega)}}\right]}\right]$$

Now Solving K_2:

$$K_2 = \frac{1}{2}\sum_{m=0}^{\infty}\left[\frac{1}{4}e^{-j(\pi/4+\omega)}\right]^{M}\left[\frac{1}{4}e^{-j(\pi/4+\omega)}\right]^{+2}$$

$$K_2 = \frac{1}{2}\sum_{m=0}^{\infty}\left[\frac{1}{4}e^{-j(\pi/4+\omega)}\right]^{m}\left[\left(\frac{1}{4}\right)^{2}e^{-2j(\pi/4+\omega)}\right]$$

$$K_2 = \frac{1}{2}\left[\frac{1}{1-\frac{1}{4e^{-j(\pi/4+\omega)}}}\right]\cdot\frac{1}{16}e^{-2j(\pi/4+\omega)}$$

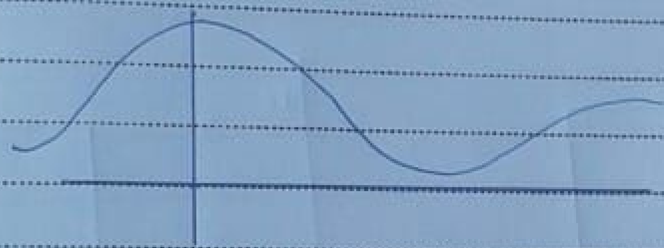
$$K_2 = \frac{1}{32}\left[\frac{e^{-2j(\pi/4+\omega)}}{1-\frac{1}{4e^{-j(\pi/4+\omega)}}}\right]$$

Now;

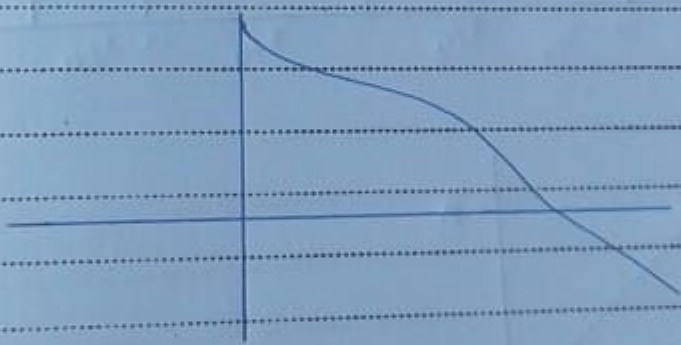
$$X(e^{j\omega}) = K_1 + K_2;$$

$$= \frac{1}{32}\left[\frac{e^{2j(\pi/4-\omega)}}{1-\frac{1}{4e^{j(\pi/4-\omega)}}} + \frac{e^{-2j(\pi/4+\omega)}}{1-\frac{1}{4e^{-j(\pi/4+\omega)}}}\right]$$

Magnitude:



Phase:



c) $x_3[n] = \text{Sinc}\left(2\pi n/8\right) * \text{sinc}\left\{2\pi(n-4)/8\right\}$

Solution:

As we know that convolution in time domain is equal to multiplication in frequency domain

$$x[n-n_d] \longrightarrow e^{-j\omega n_d} X(e^{j\omega})$$

$$X_3(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$X_3(e^{j\omega}) = \int \text{sinc}\left(\frac{\pi n}{4}\right) \times \int \text{sinc}\left(\frac{\pi(n-4)}{8}\right)$$

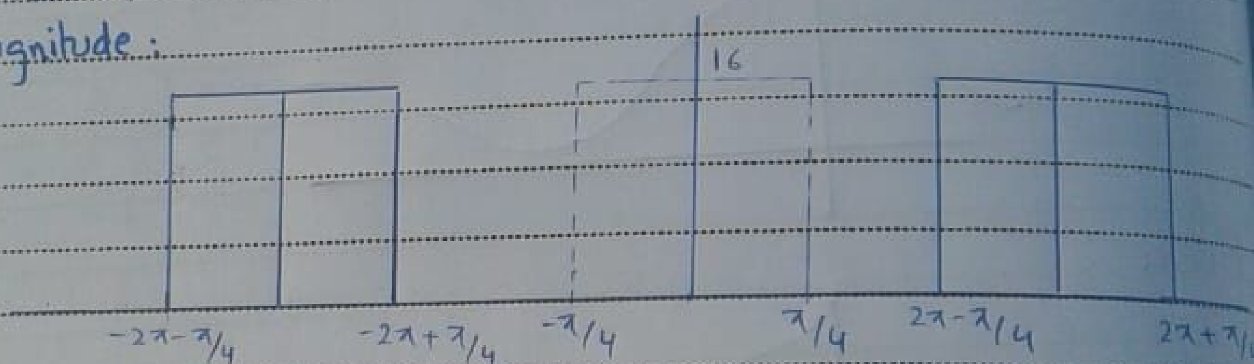
$$X_3(e^{j\omega}) = \int \text{sinc}\left(\frac{\pi n}{4}\right) e^{-j\omega 4}$$

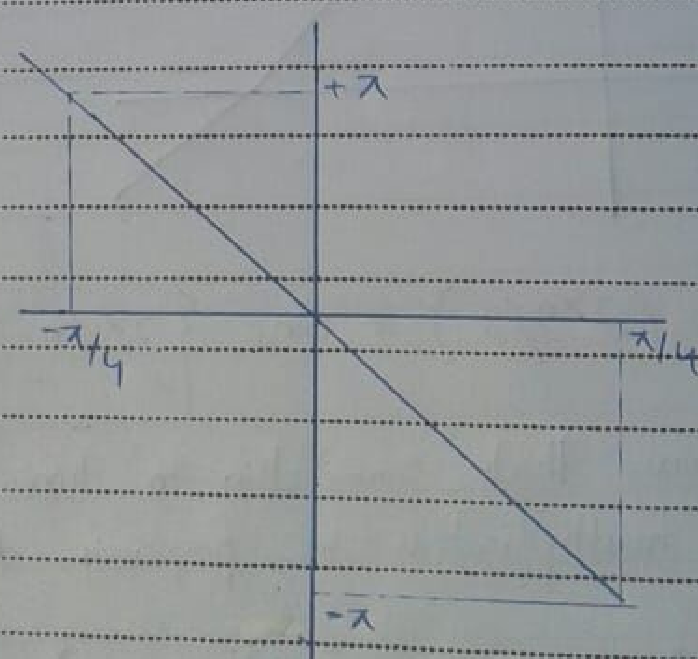
$$X_3(e^{j\omega}) = 16 e^{-j\omega 4} \quad ; \quad 0 \le |\omega| \le \pi/4$$

$$= 0 \quad ; \quad \pi/4 \le |\omega| \le \pi$$

Sinc in one domain corresponds to rectangle in other domain.

Magnitude :





Question: 3

Determine the inverse - z transform of the following Signals.

a) $X(z) = \dfrac{1}{\left(1+\frac{1}{2}z^{-1}\right)^2\left(1-2z^{-1}\right)\left(1-3z^{-1}\right)}$ stable sequence.

Solution:

Given:

$$X(z) = \frac{1}{\left(1+\frac{1}{2}z^{-1}\right)^2\left(1-2z^{-1}\right)\left(1-3z^{-1}\right)}$$

$$X(z) = \frac{A}{\left(1+\frac{1}{2}z^{-1}\right)^2} + \frac{B}{\left(1+\frac{1}{2}z^{-1}\right)} + \frac{C}{1-2z^{-1}} + \frac{D}{1-3z^{-1}} \rightarrow (A)$$

$$A = \left[\cancel{1+\frac{1}{2}z^{-1}}\right]^2\left[\frac{1}{\cancel{\left(1+\frac{1}{2}z^{-1}\right)^2}\left(1-2z^{-1}\right)\left(1-3z^{-1}\right)}\right]_{z^{-1}=-2}$$

$$A = \left[\frac{1}{\left(1-2z^{-1}\right)\left(1-3z^{-1}\right)}\right]_{z^{-1}=-2}$$

$$A = \frac{1}{\left(1-2(-2)\right)\left(1-3(-2)\right)}$$

$$= \frac{1}{(1+4)(1+6)}$$

$$A = \frac{1}{(5)(7)} = \frac{1}{35}$$

$$C = \cancel{\left(1-2z^{-1}\right)}\left[\frac{1}{\left(1+\frac{1}{2}z^{-1}\right)^2\cancel{\left(1-2z^{-1}\right)}\left(1-3z^{-1}\right)}\right]_{z^{-1}=1/2}$$

$$C = \left[\frac{1}{\left(1+\frac{1}{2}z^{-1}\right)^2\left(1-3z^{-1}\right)}\right]_{z^{-1}=1/2}$$

$$C = \frac{1}{\left[1+\frac{1}{2}\left(\frac{1}{2}\right)\right]^2\left[1-3\left(\frac{1}{2}\right)\right]}$$

$$C = \frac{1}{\left[1+\frac{1}{4}\right]^2\left[\frac{2-3}{2}\right]}$$

$$C = \frac{1}{\left(\frac{5}{4}\right)^2\left(\frac{-1}{2}\right)} = \frac{1}{\frac{25}{16}\times\left(\frac{-1}{2}\right)}$$

$$C = \frac{-32}{25}$$

$$D = \cancel{(1-3z^{-1})}\left[\frac{1}{\left(1+\frac{1}{2}z^{-1}\right)^2(1-2z^{-1})\cancel{(1-3z^{-1})}}\right]_{z^{-1}=1/3}$$

$$D = \left[\frac{1}{\left(1+\frac{1}{2}z^{-1}\right)^2(1-2z^{-1})}\right]_{z^{-1}=1/3}$$

$$D = \frac{1}{\left(1+\frac{1}{2}\left(\frac{1}{3}\right)\right)^2\left(1-2\left(\frac{1}{3}\right)\right)}$$

$$= \frac{1}{\left(1+\frac{1}{6}\right)^2\left(\frac{+1}{3}\right)} = \frac{1}{\left(\frac{7}{6}\right)^2\left(\frac{1}{3}\right)}$$

$$= \frac{1}{\frac{49}{108}}$$

$$D = \frac{108}{49}$$

$$1 = A\left(1-2z^{-1}\right)\left(1-3z^{-1}\right) + B\left(1+\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)\left(1-3z^{-1}\right)$$
$$+C\left(1+\frac{1}{2}z^{-1}\right)^2\left(1-3z^{-1}\right) + D\left(1+\frac{1}{2}z^{-1}\right)^2\left(1-2z^{-1}\right)$$

$$1 = A\left(1-5z^{-1}+6z^{-2}\right) + B\left(1+\frac{1}{2}z^{-1}\right)\left(1-5z^{-1}+6z^{-2}\right) +$$
$$C\left(1+z^{-1}+\frac{1}{4}z^{-2}\right)\left(1-3z^{-1}\right) + D\left(1+z^{-1}+\frac{1}{4}z^{-2}\right)\left(1-2z^{-1}\right)$$

$$1 = A\left(1-5z^{-1}+6z^{-2}\right) + B\left(1-\frac{9}{2}z^{-1}+\frac{7}{2}z^{-2}+3z^{-3}\right) +$$
$$C\left(1-2z^{-1}-\frac{11}{4}z^{-2}-\frac{3}{4}z^{-3}\right) + D\left(1-z^{-1}-\frac{7}{4}z^{-2}-\frac{1}{2}z^{-3}\right)$$

Comparing coefficient of z^{-3}, z^{-2} and z^{-1}

z^{-3}:

$$3B - \frac{3}{4}C - \frac{1}{2}D = 0 \quad — 1$$

z^{-2}:

$$6A + \frac{7}{2}B - \frac{11}{4}C - \frac{7}{4}D = 0 \longrightarrow 2$$

z^{-1}:

$$-5A - \frac{9}{2}z^{-1} - 2C - D = 0 \longrightarrow 3$$

Constant:

$$1 = A + B + C + D \longrightarrow 4$$

Now put $A = \frac{1}{35}$; $C = \frac{-32}{25}$; $D = \frac{108}{49}$ in 4

$$1 = \frac{1}{35} + B - \frac{32}{25} + \frac{108}{49}$$

$$B = 1 - \frac{1}{35} + \frac{32}{25} - \frac{108}{49}$$

$$B = \frac{58}{1225}$$

Now put the value of A, B, C and D in eq (A)

$$X(z) = \frac{1/35}{\left(1+\frac{1}{2}z^{-1}\right)^2} + \frac{58/1225}{\left(1+\frac{1}{2}z^{-1}\right)} + \frac{(-32/25)}{(1-2z^{-1})} + \frac{108/49}{1-3z^{-1}}$$

Therefore,

By taking z-inverse transform, we get.

$$x[n] = \frac{1}{35}(n+1)\left(\frac{-1}{2}\right)^{n+1} u[n+1] + \frac{58}{1225}\left(\frac{-1}{2}\right)^{n} u[n]$$

$$+ \frac{32}{25}(2)^n u[-n-1] - \frac{108}{49}(3)^n u[-n-1]$$

b) $X(z) = e^{z^{-1}}$

Solution:

Given:

$$X(z) = e^{z^{-1}}$$

$$X(z) = e^{z^{-1}} = 1 + z^{-1} + \frac{z^{-2}}{2!} + \frac{z^{-3}}{3!} + \cdots$$

Therefore,

After having inverse z-transform

$$x[n] = \frac{1}{n!} u[n]$$

c) $X(z) = \dfrac{z^3 - 2z}{z-2}$; left sided sequence

Solution:

Given:

$$X(z) = \frac{z^3 - 2z}{z-2}$$

$$\begin{array}{r|l} & z^2 + 2z \\ \hline z-2 & \cancel{z^3} - 2z \\ & \pm \cancel{z^3} \qquad \mp 2z^2 \\ \hline & -2z + \cancel{2z^2} \\ & \mp 4z \pm \cancel{2z^2} \\ \hline & +2z \end{array}$$

$$X(z) = z^2 + 2z + \frac{2z}{z-2}$$

$$= z^2 + 2z + \frac{\cancel{z}(2)}{\cancel{z}(1-2z^{-1})}$$

$$= z^2 + 2z + \frac{2}{1-2z^{-1}}$$

$$z^2 \longleftrightarrow \delta[n+2]$$

$$2z \longleftrightarrow 2\delta[n+1]$$

$$\frac{2}{1-2z^{-1}} \longleftrightarrow -2(2)^n u[-n-1]$$

$$\therefore -a^n u[-n-1] \longleftrightarrow \frac{1}{1-az^{-1}} \quad \because z < a$$

So;

$$x[n] = \delta[n+2] + 2\delta[n+1] - 2(2)^n u[-n-1]$$

Question: 4

Consider a causal system with input $x[n]$ and output $y[n]$ if the input is given:

$$x[n] = -\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^n u[n] - \left(\frac{4}{3}\right) 2^n u[-n-1]$$

the output has a z-transform given by

$$Y(z) = \frac{1-z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}$$

a) Determine the z-transform of the input $x[n]$

Solution:

Given:

$$x[n] = -\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^n u[n] - \left(\frac{4}{3}\right) 2^n u[-n-1]$$

$$a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}} \qquad |z| > |a|$$

$$-a^n u[-n-1] \longleftrightarrow \frac{1}{1-az^{-1}} \qquad |z| < |a|$$

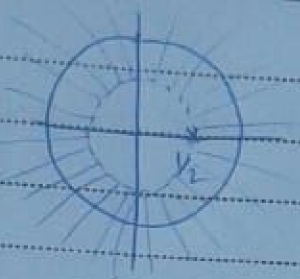
$$\frac{-1}{3}\left(\frac{1}{2}\right)^n u[n] \longrightarrow \frac{-1/3}{1-\frac{1}{2}z^{-1}} \qquad |z|>1/2 \left(\begin{matrix}\text{right - sided}\\ \text{sequence}\end{matrix}\right)$$

$$-\frac{4}{3}(2)^n u[-n-1] \longrightarrow \frac{+4/3}{1-2z^{-1}} \qquad |z|<2 \left(\begin{matrix}\text{left sided}\\ \text{sequence}\end{matrix}\right)$$

$$X(z) = \frac{-1/3}{1-\frac{1}{2}z^{-1}} + \frac{4/3}{1-2z^{-1}}$$

right sided sequence

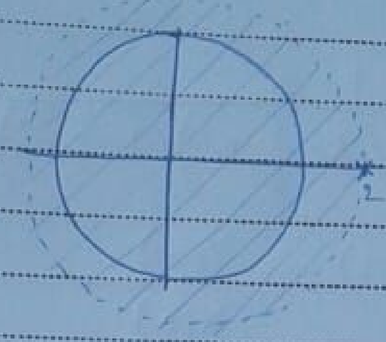
$|z| > \frac{1}{2}$



Roc lies outside the outermost Pole

left sided sequence

$|z| < 2$



Roc lies inside the innermost pole.

let $z^{-1} = x$

$$X(z) = \frac{-1/3}{\left(1 - \frac{1}{2}x\right)} + \frac{4/3}{(1-2x)}$$

$$= \frac{-1/3(1-2x) + 4/3(1 - x/2)}{\left(1 - \frac{x}{2}\right)(1-2x)}$$

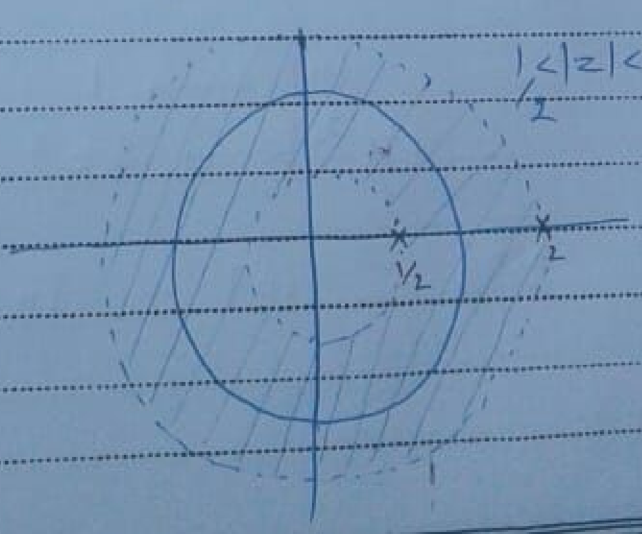
$$= \frac{-1/3 + 2/3x + 4/3 - \cancel{4x/6}}{\left(1 - \frac{x}{2}\right)(1-2x)}$$

$$= \frac{1}{\left(1 - \frac{x}{2}\right)(1-2x)}$$

Now put $x = z^{-1}$

$$= \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - 2z^{-1}\right)}$$

$\frac{1}{2} < |z| < 2$



poles at: $z = 1/2$ and

$z = 2$

b) find all the possible choices for the impulse response of the system.

Solution:

Given:

$$Y(z) = \frac{1-z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}$$

As $H(z) = \frac{Y(z)}{X(z)}$

So;

$$H(z) = \frac{1-z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)} \cdot \frac{1}{X(z)} \quad -① $$

$$\therefore X(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}$$ put in eq ①

$$H(z) = \frac{1-z^{-2}}{\cancel{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}} \cdot \frac{1}{\frac{1}{\cancel{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}}}$$

$$H(z) = 1 - z^{-2}$$

As.

$$\delta[n] \longleftrightarrow 1$$

$$\delta[n-2] \longleftrightarrow z^{-2}$$

$$h[n] = \delta[n] - \delta[n-2]$$

Question: 5

Sketch the pole-zero plot for each of the following z-transform and shade the region of convergence.

a) $X_1(z) = \dfrac{1 - \frac{1}{2} z^{-1}}{1 + 2 z^{-1}}$ ROC: $|z| < 2$

Solution:

Given: $X_1(z) = \dfrac{1 - \frac{1}{2} z^{-1}}{1 + 2 z^{-1}}$ ROC: $|z| < 2$

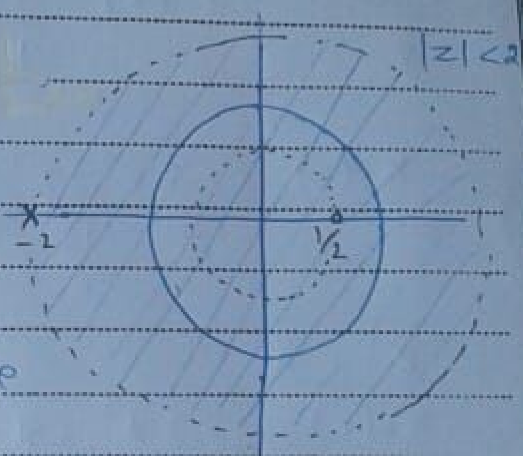
Zero: $1 - \frac{1}{2} z^{-1} = 0$ $1 = \frac{1}{2} z^{-1}$

$z = 1/2$

pole: $1 + 2z^{-1} = 0$ $1 = -2z^{-1}$

$z = -2$

ROC lies inside the inner most pole



b) $X_2(z) = \dfrac{1 - \frac{1}{3} z^{-1}}{\left(1 + \frac{1}{2} z^{-1}\right)\left(1 - \frac{2}{3} z^{-1}\right)}$ $x_2[n]$ causal

Solution:

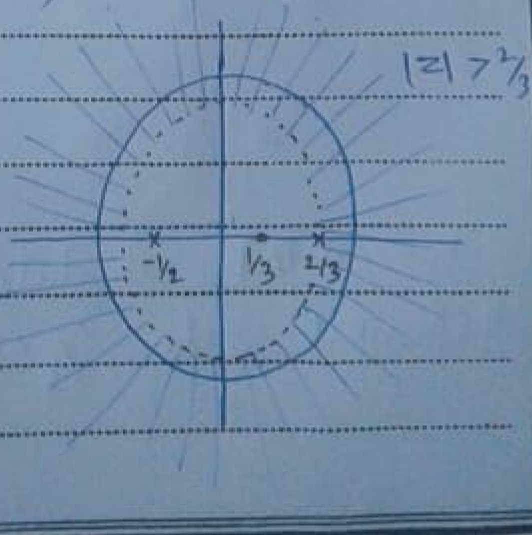
Given: $X_2(z) = \dfrac{1 - \frac{1}{3} z^{-1}}{\left(1 + \frac{1}{2} z^{-1}\right)\left(1 - \frac{2}{3} z^{-1}\right)}$

zero: $1 - \frac{1}{3} z^{-1} = 0$ $1 = \frac{1}{3} z^{-1}$

$z = 1/3$

poles: $1 + \frac{1}{2} z^{-1} = 0$ $z = -1/2$

$1 - \frac{2}{3} z^{-1} = 0$ $z = 2/3$



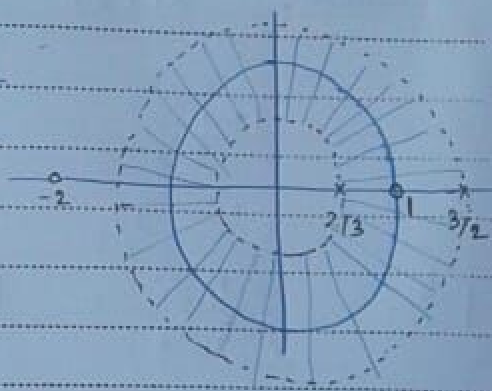
Since $x_2[n]$ is causal, the ROC extends from the outermost pole $|z| > 2/3$.

c) $X_3(z) = \dfrac{1 + z^{-1} - 2z^{-2}}{1 - \frac{13}{6} z^{-1} + z^{-2}}$, $x_3[n]$ absolutely summable

$2/3 < |z| < 3/2$

Solution:

Given: $X_3(z) = \dfrac{1 + z^{-1} - 2z^{-2}}{1 - \frac{13}{6} z^{-1} + z^{-2}}$



$X_3(z) = \dfrac{-2z^{-2} + z^{-1} + 1}{z^{-2} - \frac{13}{6} z^{-1} + 1}$

$z^{-1} = x$

$X_3(z) = \dfrac{-2x^2 + x + 1}{x^2 - \frac{13}{6} x + 1} = \dfrac{(x + 1/2)(x - 1)}{(x - 3/2)(x - 2/3)}$

zero: $z^{-1} + 1/2 = 0 \quad z^{-1} = -1/2 \quad z = -2$

$z^{-1} - 1 = 0 \quad z^{-1} = 1 \quad z = 1$

Pole: $z^{-1} - 3/2 = 0 \quad z^{-1} = 3/2 \quad z = 2/3$

$z^{-1} - 2/3 = 0 \quad z^{-1} = 2/3 \quad z = 3/2$

Since $x_3[n]$ is absolutely summable, ROC must include the unit circle $2/3 < |z| < 3/2$