NIMRA IDRIS SIDDIQUI

17 EEB 409

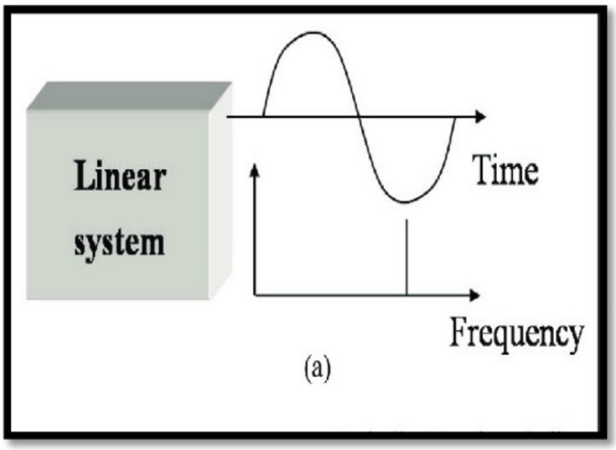
GI-2134

22-11-2020

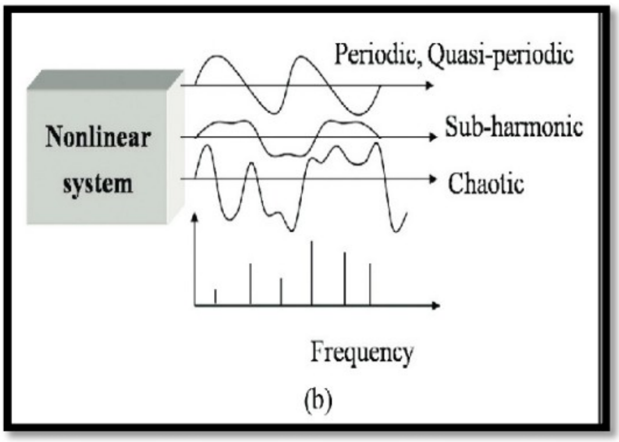
SUBMITTED TO:

DR. MOHD RIHAN

ASSIGNMENT-1



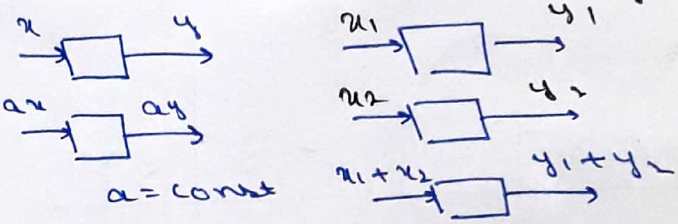
(a)



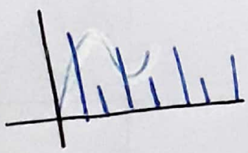
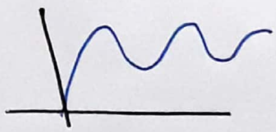
(b)

Q1 Explain the characteristic phenomena of non-linear system which makes it different from a linear system.

→ (1) The non-linear system don't follow the principle of superposition i.e homogenity & additivity while linear system follow.

Non-linear	Linear
Don't obey.	Homogenity Additivity 

(2) Non-linear have discrete time while linear have continuous time

Non-linear	Linear
	

(3) Non-linear have finite escape time i.e at finite interval of time system goes out of hand

while linear control sys. have infinite time.

(4) NLCS have multiple equilibrium point

i.e $\dot{x} = f(x)$, $x(t_0) = x^*$, $x(t) = x^*$

& which equilibrium pt. operate depend on initial condition

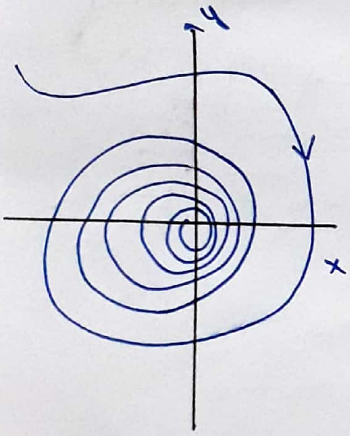
LCS have only one equilibrium pt

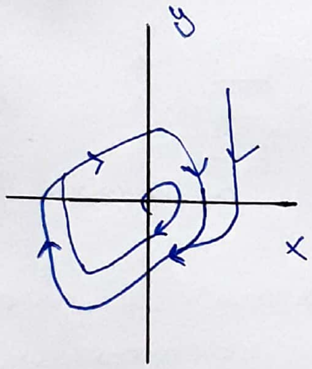
(5) NLCS have limit cycle i.e

→ sustained oscillation in NLCS independent of initial condition.

→ much more common in practical

Non-Linear



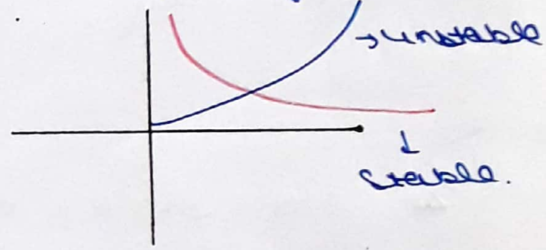


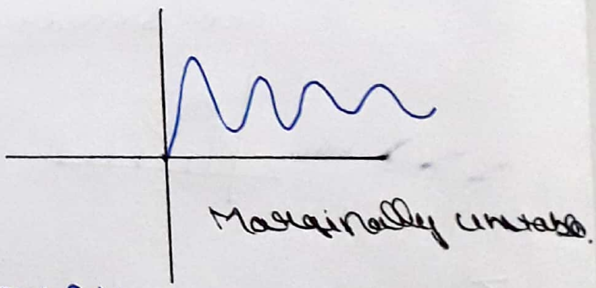
Linear

→ Stable

→ unstable

→ Marginally stable.





Amplitude of these oscillation depends on the initial conditions in LCS

(6) Subharmonic, harmonic, or almost periodic oscillation.

A non linear system can oscillate with frequencies which are submultiple or multiples of i/p freq. It may generate an almost -

Periodic oscillation, i.e sum of periodic oscillations with frequencies which are not multiples of each other.

→ A stable linear system under a periodic i/p produces an o/p of same freq.

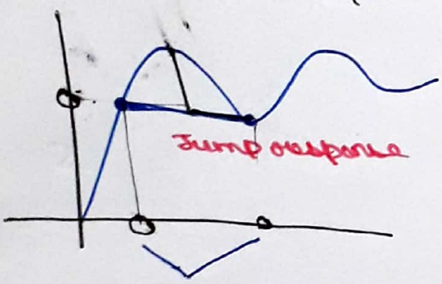
⑦ Jump response & multimodes of behaviour.

→ In NLCS have multimode of behaviour.

→ NLCS with increasing input, output may increase, decrease or remain same.

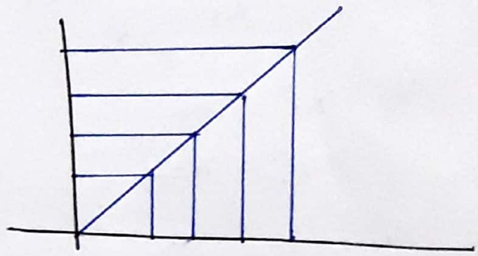
→ In LCS when input inc. output also increase, also when i/p dec, o/p also dec.

Non-linear



2 i/p may have 1 o/p

Linear



unique i/p
unique o/p

Q2 The analysis of non-linear system is different as compare to linear systems why?

Linear system →

The analysis of the L.S is done by two method

(i) State space

(ii) Transfer function.

The analysis of non linear system is different from linear system because

① In NLS functionally output of system is not proportional to input of system

② In NLS there is a presence of non-linearity comes from physical limitation of the component involved.

for a non-linear system output increase up to certain limit after it become constant

① it doesn't response to input when it reaching to saturation.

This is because types of Non-linearity Present

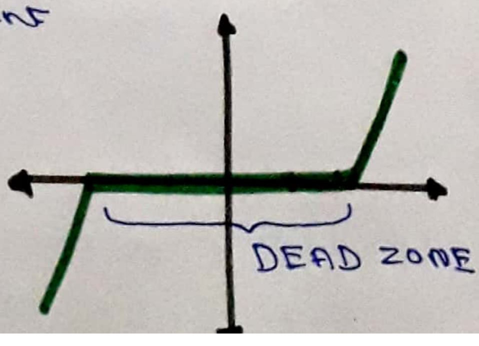
① Incidental → present in system naturally

② Intential → Introduce artificially by an Engineer or designer to achieve certain objective

⇒ Incidential non linearity comes from

① Saturation - Incidental.

② Dead Zone - It is the range of input where no output is present



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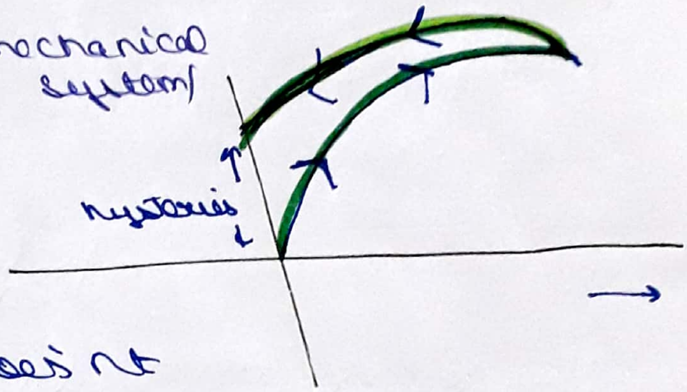
After dead zone system may respond to FIT

Due to dead zone system is NLS

3. friction
- Colombus fric.
- Static fric.

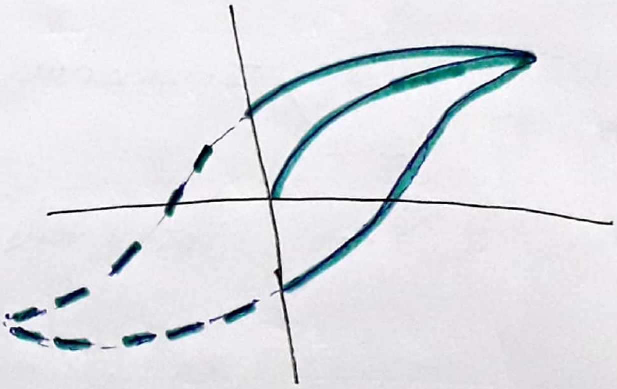
4. Blacklash (mechanical system)

Applying mag. force.
flux is produce.



⇒ Backward curve does not follow forward curve

Energy supply in frwrd drn not completely recoved in bckwrd drn



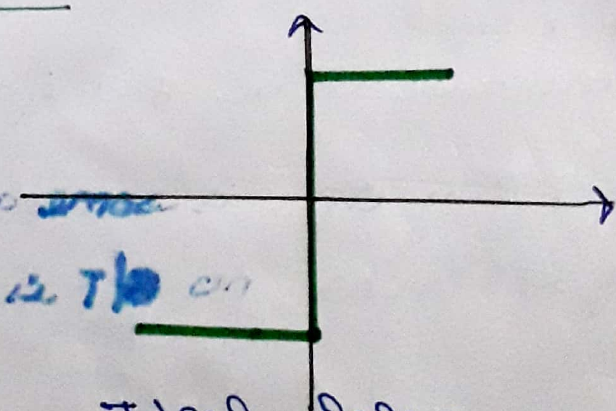
forward charc ≠ backward charac.

Hysterisious loss (Dec. sys.)

Intential NL

Relay

like as switch



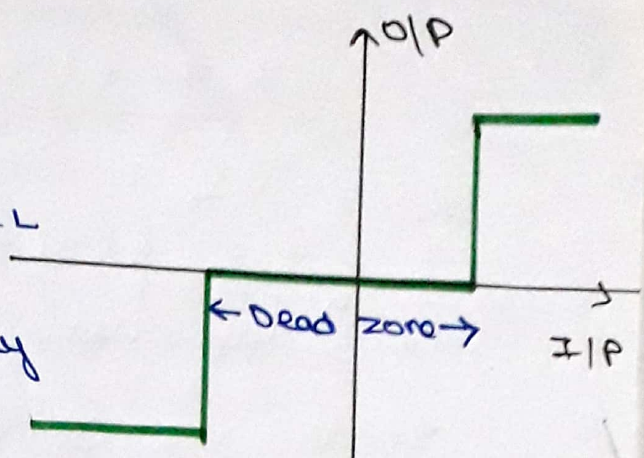
Ideal Relay

combination →

of 2 N.L i.e dead zone + Relay

Intentional L (Relay)

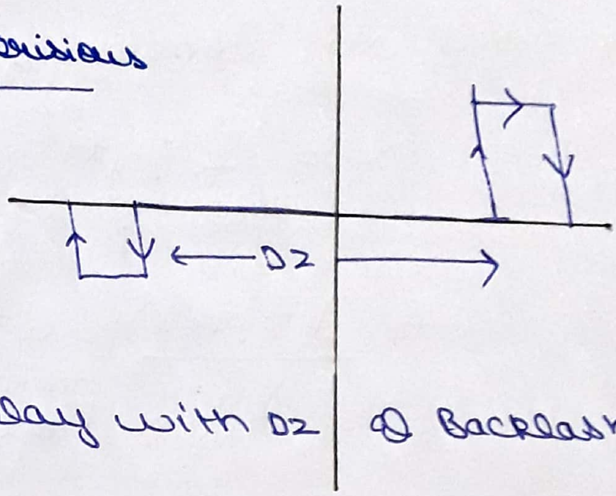
Incidental NL (dead zone)



Relay with dead zone.

Relay + dead zone + hysterisious

Incidential NL



Relay & dead zone can consider 1.

Relay with DZ & Backlash

NLCS → NL → Naturally
NLCS → NL → introduce

combination of NL

Analyzing NLS

1) Linearization

→ Developing an approximate linear model for the NL system

$\dot{x} = f(x, u, t)$ → General mathematic of cntrl sys

$\dot{x}(t) = Ax(t) + Bu(t)$ → Linear time sys

NON-LINEAR SYSTEM

The analysis of NLS is by linearization

Linear approximation of Non-linear Mathmatical model →

To obtain a linear mathmatical model for a NLS,

consider a system where i/p is $x(t)$

$O/P = y(t)$

The relationship b/n $y(t)$ & $x(t)$ is given by

$$y = f(x) \quad -① $$

eq^n ① is expanded into taylor series

$$y = f(x)$$

$$y = f(\bar{x}) + \frac{df}{dx}(x-\bar{x}) + \frac{1}{x!}\frac{d^2y}{dx^2}(x-\bar{x})^2 + \cdots ②$$

where the derivatives

$\frac{dy}{dx}, \frac{d^2y}{dx^2}$ are evaluated $x = \bar{x}$.

eq^n ② can be written as (neglect the infinite order terms in $x-\bar{x}$)

$$y = \bar{y} + k(x-\bar{x}) \quad -③$$

where, $\bar{y} = y(\bar{x})$

$$k = \left.\frac{dy}{dx}\right|_{x=\bar{x}}$$

Above eq^n can also be written as

$$y-\bar{y} = k(x-\bar{x}) \quad -④$$

$$\boxed{\dot{x} = f(x, u, t)}$$ → Non-linear eq^n

TI

$\dot{x} = f(x, u)$

↳ Eq. pt. when $\dot{x} = 0$

Imp. b/c linearization is around eq^n pt.

$$f(x_e, u_e) = 0$$

→ How to lineariz non linear system?

$$\dot{x} = f(x^*, u^*) + \frac{\partial f(x,u)}{\partial x}\Bigg|_{\substack{x=x^* \\ u=u^*}} x^*$$

$$+ \frac{\partial f(x,u)}{\partial u}\Bigg|_{\substack{x=x^* \\ u=u^*}} u^* + \text{Higher order term}$$

$$- f(x^*, u^*)$$

Let the perturbation or variation is small

[valid only around the eq^n pt.]

⇒ HOT may be neglected

$$\dot{x}^* = \frac{\partial f(x,u)}{\partial x}\Bigg|_{\substack{x=x^* \\ u=u^*}} x^* + \frac{\partial f(x,u)}{\partial u}\Bigg|_{\substack{x=x^* \\ u=u^*}} u^*$$

Let assume sys. is of 2nd order

x_1, x_2

$$\dot{x}_1^* = \frac{\partial f_1}{\partial x_1}\Bigg|_{\substack{x_1=x_1^*,\ x_2=x^* \\ u=u_1^*,\ u_2=u_2^*}} + \frac{\partial f_1}{\partial x_2}\Bigg| x_2^*$$

$$+ \frac{\partial f_1}{\partial u_1}\bigg| u_1^* + \frac{\partial f_1}{\partial u_2}\bigg| u_2^*$$

$$\dot{x}_2 = \frac{\partial f_2}{\partial x_1}\bigg|_{\substack{x_1 = x_1^*,\ x_2 = x_2^* \\ u_1 = u_1^*,\ u_2 = u_2^*}} + \frac{\partial f_2}{\partial x_2}\, \delta x_2^*$$

$$+ \frac{\partial f_2}{\partial u_1}\bigg| u_1^* + \frac{\partial f_2}{\partial u_2}\bigg| u_2^*$$

$$\begin{bmatrix} \dot{x}_1^* \\ \dot{x}_2^* \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} x_1^* \\ x_2^* \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \end{bmatrix} \begin{bmatrix} u_1^* \\ u_2^* \end{bmatrix}$$

$$\dot{x}(t) = A^* x^*(t) + B^* u^*(t)$$

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$ defined as eqm pt.

Called Jacobian responsible of converting N.L sys into linear sys.

DESCRIBING FUNCTION -

→ filtering effect will be significant

→ neglect the harmonics

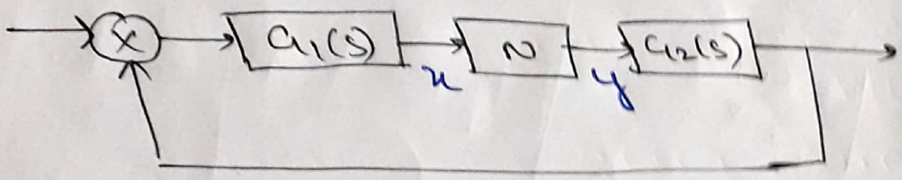
→ Linearized →

2 Requirement

1) No. of LC >> No. of NLS
2) I/T should be Sinusoidal

] Describing func. is used as Linearization technique

Having 2 requirement



$$x = X \sin \omega t$$

$$y = A_0 + A_1 \sin \omega t + B_1 \cos \omega t + A_2 \sin 2\omega t + B_2 \cos 2\omega t + \cdots$$

If y is symmetrical → $A_0 = 0$

B/C of LPF of G_1 & G_2

$$y = A_1 \sin \omega t + B_1 \cos \omega t$$

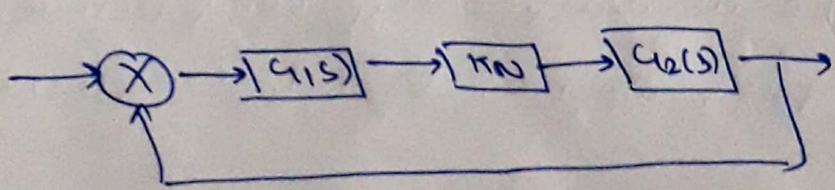
$$y = Y_1 \sin(\omega t + \phi)$$

Y_1 → peak of y & ϕ → phase diff

X = peak of x

Describing func.

$$\boxed{K_N = \frac{Y_1}{X} \angle \phi}$$



Describing fun in NLS = T/f func. L.C.

A_1, B_1

$$Y_1 = \sqrt{A_1^2 + B_1^2}$$

$$\phi = \tan^{-1}\frac{A_1}{B_1}$$

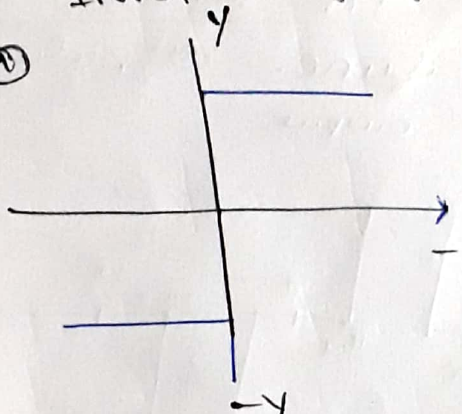
$$A_1 = \frac{2}{\pi}\int_0^{2\pi} y(t)\sin\omega t\, d\omega t$$

$$B_1 = \frac{2}{\pi}\int_0^{\pi} y(t)\cos\omega t\, d\omega t$$

Describing func. of an Ideal Relay

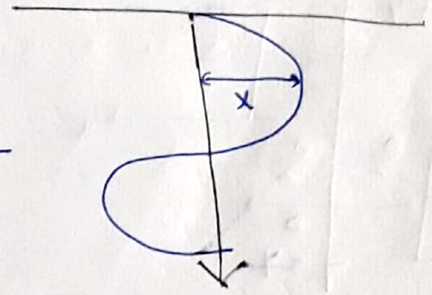
INPUT

I/T



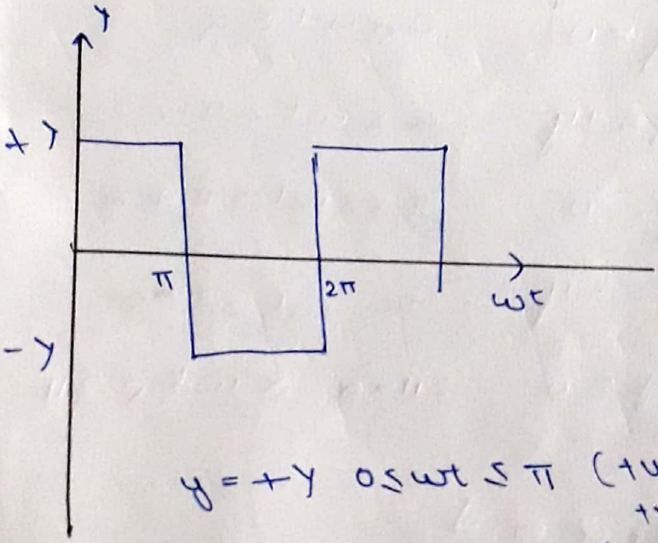
Non-linear.

Apply purely sinusoidal wave to the i/T



OUTPUT

O/T ωt



$$A_1 = \frac{2}{\pi}\int_0^{\pi} Y\cos d\omega t = 0$$

$$B_1 = \frac{2}{\pi}\int_0^{2\pi} Y\sin\omega t\, d\omega t$$

$$= \frac{2}{\pi}\left[-Y\cos\omega t\right]_0^{\pi}$$

$$= \frac{4Y}{\pi}$$

$y = +Y \quad 0 \le \omega t \le \pi$ (+ve for +ve cycle)

$y = -Y \quad \pi \le \omega t \le 2\pi$ (-ve for -ve cycle)

$$Y_1 = \sqrt{A_1^2 + B_1^2}$$

$$= \frac{4Y}{\pi}$$

$$\phi = \tan^{-1}\frac{A_1}{B_1} = 0$$

$$K_N = \frac{Y_1}{X} \angle \phi$$

$$\boxed{K_N = \frac{4Y}{\pi X} \angle 0} \rightarrow \text{Ideal Relay}$$

$$K_N(X, \omega) = \frac{4Y}{\pi X} \angle 0$$

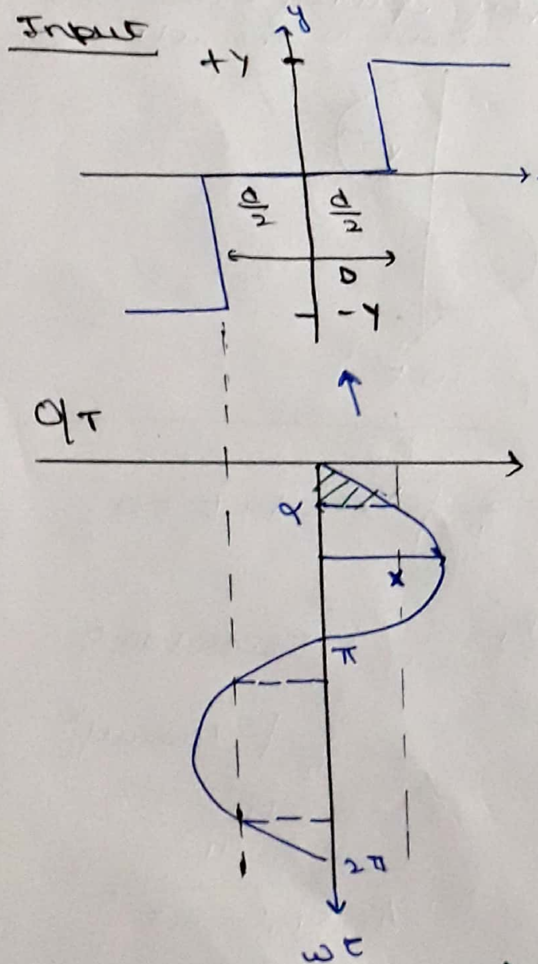
Describing func depend on

X = ampl. of i/p
ω = freq. of i/p

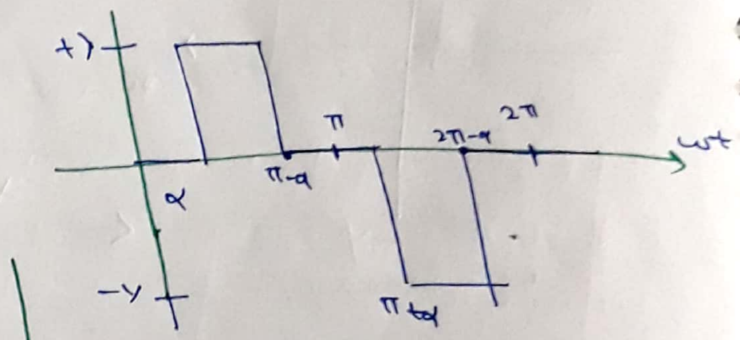
∴ describing func is different from T/f func.

Q3 Determine describing func. of relay with a dead zone

Input



Output



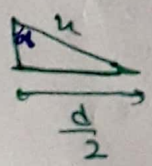
$y = 0 \qquad 0 \le \omega t \le \alpha$

$y = +Y \qquad \alpha \le \omega t \le \pi - \alpha$

$y = 0 \qquad \pi - \alpha \le \omega t \le \pi + \alpha$

$y = -Y \qquad \pi + \alpha \le \omega t \le 2\pi - \alpha$

$y = 0 \qquad 2\pi - \alpha \le \omega t \le 2\pi$



$$\sin\alpha = \frac{d}{2X}$$

$$\alpha \sin^{-1}\frac{d}{2\alpha}$$

$$A_1 = \frac{2}{\pi}\int_{\alpha}^{\pi-\alpha} Y\cos\omega t \, d\omega t = 0$$

$$B_1 = \frac{2}{\pi}\int_{\alpha}^{\pi-\alpha} Y\sin\omega t \, d\omega t$$

$$= \frac{2Y}{\pi}\left(\cos\omega t\right)_{\pi-\alpha}^{\alpha} \Rightarrow \frac{2Y}{\pi}\cos\alpha - (-\cos\alpha)$$

$$B_1 = \frac{4Y\cos\alpha}{\pi}$$

$$\sin\alpha = \frac{d}{2X}, \quad \cos\alpha = \sqrt{1-\left(\frac{d}{2X}\right)^2}$$

$$= \sqrt{\frac{X^2-\left(\frac{d}{2}\right)^2}{X^2}}$$

let $\boxed{\frac{d}{2X} = M}$

$$\boxed{B_1 = \frac{4Y}{\pi}\sqrt{1-M^2}}$$

$$\boxed{\phi = \tan^{-1}\frac{A_1}{B_1} = 0}$$

$$K_N(\alpha,\omega) = \frac{Y_1\angle\phi}{X} \Rightarrow \frac{4Y\cos\alpha}{\pi X}$$

$$Y_1 = \sqrt{A_1^2+B_1^2} \Rightarrow \sqrt{0+\left(\frac{4Y}{\pi}\cos\alpha\right)^2}$$

$X = B_0$

$$Y_1 = \frac{4Y\cos\alpha}{\pi} \quad \text{or} \quad Y_1 = \sqrt{0^2+\left(\frac{4Y}{\pi}\sqrt{1-M^2}\right)^2}$$

$$Y_1 = \frac{4Y}{\pi}\sqrt{1-M^2}$$

$$\boxed{K_n(\alpha,\omega) = \frac{4Y\sqrt{1-M^2}}{\pi X}\angle 0}$$

$$\boxed{K_n(\alpha,\omega) = \frac{4Y}{\pi X}\sqrt{1-\frac{\delta^2}{(2X)^2}}\angle 0} \quad \text{— (3)}$$

Equation (3) is the describing function of a relay with deadzone & saturation.

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