

**// Flow Adj 1-based**

```

int res[201][201];
int find_path(int src, int dest, int n){
    int ret = -1, backlink[n+1];
    queue<int> BFS;BFS.push(src);
    memset(backlink, 0, sizeof(backlink));
    backlink[src] = -1;
    while(!BFS.empty()){
        int x = BFS.front();BFS.pop();
        if(x == dest){
            ret = 1000;
            while(backlink[x] != -1){
                int pre = backlink[x];
                ret = min(ret, res[pre][x]);
                x = pre;
            }
            x = dest;
            while(backlink[x] != -1){
                int pre = backlink[x];
                res[pre][x] -= ret;
                res[x][pre] += ret;
                x = pre;
            }
            return ret;
        }
        for(int i = 1;i<=n;i++){
            if(i != x && !backlink[i] && res[x][i])
                backlink[i] = x, BFS.push(i);
        }
    }
    return ret;
}
int max_flow(int src, int sink, int n){
    int ret = 0, path_capacity;
    while((path_capacity = find_path(src, sink, n)) != -1)
        ret += path_capacity;
    return ret;
}

```

**// Max Matching Bipartite**

```

vector<vector<int>> neigh;
int bw[100000]; // all -1s
int vis[100000];
int find_path(int n){
    if(vis[n])
        return 0;
    vis[n] = 1;
    for(int i = 0;i<neigh[n].size();i++){
        if(bw[neigh[n][i]] == -1 || find_path(bw[neigh[n][i]]))
            bw[neigh[n][i]] = n;
    }
    return 1;
}

```

**// Flow General Graph**

```

struct node{
    map<node*, int> edges;
}nodes[1000000];
int cnt = 0;
node* nxtNode(){
    nodes[cnt].edges.clear();
    return &nodes[cnt++];
}
int find_path(node *src, node *dest){
    int ret = -1;
    map<node*, node*> backlink;
    queue<node*> BFS;BFS.push(src);
    backlink[src] = 0;
    while(!BFS.empty()){
        node *x = BFS.front();BFS.pop();
        if(x == dest){
            ret = 1000;
            while(backlink[x] != 0){
                node *pre = backlink[x];
                ret = min(ret, pre->edges[x]);
                x = pre;
            }
            x = dest;
            while(backlink[x] != 0){
                node *pre = backlink[x];
                pre->edges[x] -= ret;
                x->edges[pre] += ret;
                x = pre;
            }
            return ret;
        }
        for(map<node*, int>::iterator it = x->edges.begin();
            it != x->edges.end(); it++){
            if(backlink.find(it->first) == backlink.end() &&
                it->second)
                backlink[it->first] = x, BFS.push(it->first);
        }
    }
    return ret;
}
int max_flow(node *src, node *sink){
    int ret = 0, path_capacity;
    while((path_capacity = find_path(src, sink)) != -1)
        ret += path_capacity;
    return ret;
}
//GCD
int GCD(int a,int b){
    if(b != 0) while(b^=a^=b^=a%=b);
    return a;}

```

```

    }
    return 0;
}
int max_flow(){
    int res = 0;
    for(int i = 0; i < neigh.size(); i++){
        if(neigh[i].size() == 0)
            continue;
        memset(vis, 0, sizeof(int)*neigh.size());
        res += find_path(i);
    }
    return res;
}

//MATPOW
void matpow(long long M[2][2], int n, long long x)
{
    if(n > 1)
    {
        matpow(M, n/2, x);
        long long a = M[0][0], b = M[0][1], c = M[1][0], d
= M[1][1];
        M[0][0] = ((a*a)%mod + (b*c)%mod)%mod;
        M[0][1] = ((a*b)%mod + (b*d)%mod)%mod;
        M[1][0] = ((a*c)%mod + (c*d)%mod)%mod;
        M[1][1] = ((b*c)%mod + (d*d)%mod)%mod;
    }
    if(n & 1 && n > 1)
    {
        long long a = M[0][0], b = M[0][1], c = M[1][0], d
= M[1][1];
        M[0][0] = (x*(a+b))%mod;
        M[0][1] = (a*x)%mod;
        M[1][0] = (x*(c+d))%mod;
        M[1][1] = (c*x)%mod;
    }
}
long long fib(int n, long long start)
{
    long long M[2][2] = {{1,1},{1,0}};
    matpow(M, n, 1);
    return start*M[0][0];
}

// TARJAN SCC
int n;
struct vertex {
    vector<int> neigh;
    int index, component, lowlink;
} v_tmp, vertices[10000];
bool visited[10000];

```

```

// Dinic Adjacency
int max_flow_dinic(int src, int sink, int n){
    int ret = 0;
    int prev[n+1];
    while(1){
        queue<int> BFS;
        memset(prev, 0, sizeof(prev));
        prev[src] = -1;
        BFS.push(src);
        while(!BFS.empty() && !prev[sink]){
            int curr = BFS.front();
            BFS.pop();
            for(int i = 1; i <= n; i++){
                if(!prev[i] && res[curr][i])
                    prev[i] = curr, BFS.push(i);
            }
        }
        if(!prev[sink])
            break;
        for(int i = 1; i <= n; i++){
            if(res[i][sink] && prev[i]){
                int inc = res[i][sink];
                for(int u = prev[i], v = i; u >= 0; inc = min(inc,
res[u][v]), v = u, u = prev[u]);
                for(int u = i, v = sink; u >= 0; res[u][v] -= inc,
res[v][u] += inc, v = u, u = prev[u]);
                ret += inc;
            }
        }
    }
    return ret;
}

//BIT
int BIT[1000001];
int query(int idx, int n)
{
    int ret = 0;
    while(idx > 0)
    {
        ret += BIT[idx];
        idx -= idx & -idx;
    }
    return ret;
}

void update(int idx, int n, int val)
{
    while(idx <= n)
    {
        BIT[idx] += val;
        idx += idx & -idx;
    }
}

```

```

stack<int> DFS;
int index;
int connected;
void strongconnect(int id){
    vertices[id].index = vertices[id].lowlink = index;
    index++;
    DFS.push(id);
    visited[id] = true;
    int w;
    for(int j=0;j<vertices[id].neigh.size(); j++){
        w = vertices[id].neigh[j];
        if(vertices[w].index == -1){
            strongconnect(w);
            vertices[id].lowlink = min(vertices[id].lowlink,
vertices[w].lowlink);
        } else if(visited[w])
            vertices[id].lowlink = min(vertices[id].lowlink,
vertices[w].index);
        }
        if(vertices[id].lowlink == vertices[id].index){
            do{
                w = DFS.top();
                DFS.pop();
                visited[w] = false;
                vertices[w].component = connected;
            } while(id != w);
            connected++;
        }
    }
}
int scc(){
    index=0;
    connected = 0;
    for(int i=0;i<n;i++){
        if(vertices[i].index == -1)
            strongconnect(i);
    }
}

```

### //Stable Matching

```

function stableMatching {
    Initialize all  $m \in M$  and  $w \in W$  to free
    while  $\exists$  free man  $m$  who still has a woman  $w$  to
propose to {
         $w = m$ 's highest ranked such woman to whom he has
not yet proposed
        if  $w$  is free
             $(m, w)$  become engaged
        else some pair  $(m', w)$  already exists
            if  $w$  prefers  $m$  to  $m'$ 
                 $(m, w)$  become engaged
                 $m'$  becomes free
            else

```

### //POWER

```

int mod = 1000000007;
int POW(long long r, long long n){
    int ans = 1;
    while(n>0){
        if(n&1)
            ans = (ans*r)%mod;
        n >>= 1;
        r = (r*r)%mod;
    }
    return ans;
}

```

### //SEGMENT TREE

```

#define ARR_SIZE 510000
struct node{
    //Node variables
};
node TREE[ARR_SIZE<<2];
int n,arr[ARR_SIZE];
using namespace std;
void build_segment_tree(int NODE,int a,int b)
{
    if(a==b){
        // Single node condition
        return;
    }
    int mid=(a+b)/2;
    int left=2*NODE,right=2*NODE+1;
    build_segment_tree(left,a,mid);
    build_segment_tree(right,mid+1,b);

    // Merge Logic for left and right
}
node query_segment_tree(int NODE,int a,int b,int x,int
y)
{
    if(x<=a && y>=b)return TREE[NODE];
    // lazy_prop(a, b);
    int mid=(a+b)/2;
    int left=2*NODE,right=2*NODE+1;
    if(y<=mid)return query_segment_tree(left,a,mid,x,y);
    if(x>mid)return
query_segment_tree(right,mid+1,b,x,y);

    node left_query=query_segment_tree(left,a,mid,x,y);
    node
right_query=query_segment_tree(right,mid+1,b,x,y);

    node ans;
    // Merge left_query and right_query into ans
    return ans;
}

```

<pre>         (m', w) remain <i>engaged</i>     } }  // KMP COMPUTE_PI(P):     m = P.length     pi[1] = 0     k = 0     for q = 2 to m:         while k&gt;0 and P[k+1] != P[q]:             k = pi[k]         if P[k+1] == P[q]:             k++         pi[q] = k     return pi KMP_MATCH(S, P):     n = S.length     m = P.length     pi = COMPUTE_PI(P)     q = 0     for i = 1 to n         while q&gt;0 and P[q+1] != T[i]:             q = pi[q]         if P[q+1] == T[i]:             q++         if q == m:             print "Pattern occur with shift " i-m             q = pi[q]  //Suffix Array #define MAX_STR_SIZE 50001 #define MAX_AUX 18 int ranks[200]; int LOG2[MAX_STR_SIZE]; class suffix_array{ public:     char str[MAX_STR_SIZE];     struct node{         int pos, tmp1, tmp2;         friend bool operator&lt;(const node &amp;a, const node &amp;b){             if(a.tmp1 == b.tmp1)                 return a.tmp2&lt;b.tmp2;             return a.tmp1&lt;b.tmp1;         }     }SA[MAX_STR_SIZE];     int lcp_arr[MAX_STR_SIZE];     int aux_lcp[MAX_STR_SIZE][MAX_AUX];     int aux[MAX_AUX][MAX_STR_SIZE];     int steps, cnt, len; </pre>	<pre> } void update_segment_tree(int NODE,int a,int b,int x) {     if(a==b){ // l&lt;=a &amp; r&gt;=b for lazy         //Single node logic         // Set lazy bit         return;     }     // lazy_prop(a, b)     int mid=(a+b)/2;     int left=2*NODE,right=2*NODE+1;     if(x&lt;=mid)         update_segment_tree(left,a,mid,x);     else         update_segment_tree(right,mid+1,b,x);      //Merge left and right nodes }  //RMQ int n; int aux[100002][22]; int LOG2[100002]; int RMQ(int x, int y){     int k = LOG2[y-x+1];     return min(aux[x][k], aux[y-(1&lt;&lt;k)+1][k]); } void init(){     int curr = 2;     LOG2[1] = 0;     for(int i = 2; i&lt;=100001;i++){         if(i == curr){             curr &lt;&lt;= 1;             LOG2[i] = LOG2[i-1] + 1;         }else             LOG2[i] = LOG2[i-1];     }      for(int i=1;i&lt;=n;i++)         in_I(aux[i][0]);     for(int j=1; (1&lt;&lt;j)&lt;=n+1; j++)         for(int i=1;i+(1&lt;&lt;j)&lt;=n+1;i++)             aux[i][j] = min(aux[i][j-1], aux[i+(1&lt;&lt;(j-1))][j-1]); }  //Miller Rabin long long n,s,d,a,lt,k,x,j; long long mul(long long a,long long b){     long long x=0,y=a%n;     while(b&gt;0){         if(b%2)x=(x+y)%n; </pre>
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```

void create();
void gen_lcp_arr();
int lcp(int x, int y);
int lcp_RMQ(int x, int y);
}S;
void suffix_array::create(){
    len = 0;
    while(str[len]){
        aux[0][len] = ranks[str[len]];
        len++;
    }
    int pre = 0;
    cnt = 1;
    for(steps = 1;pre<len;steps++, cnt<=&1){
        for(int i=0;i<len;i++){
            SA[i].pos = i;
            SA[i].tmp1 = aux[steps-1][i];
            SA[i].tmp2 = (i+cnt<len)?aux[steps-1][i+cnt]:-1;
        }
        sort(SA, SA+len);
        aux[steps][SA[0].pos] = 0;
        for(int i=1;i<len;i++){
            if(SA[i].tmp1 == SA[i-1].tmp1 && SA[i].tmp2
== SA[i-1].tmp2)
                aux[steps][SA[i].pos] = aux[steps][SA[i-
1].pos];
            else
                aux[steps][SA[i].pos] = i;
        }
        pre = cnt;
    }
}
int suffix_array::lcp(int x, int y){
    int ret = 0;
    if(x == y)return len-x;
    for(int k=steps-1;k>=0 && x<len && y<len; k--){
        if(aux[k][x] == aux[k][y]) x += 1<<k, y += 1<<k,
ret += 1<<k;
    }
    return ret;
}
void suffix_array::gen_lcp_arr(){
    lcp_arr[0] = 0;
    for(int i=1;i<len;i++){
        lcp_arr[i] = lcp(SA[i-1].pos, SA[i].pos);

    }
    for(int i=0;i<len;i++){
        aux_lcp[i][0] = lcp_arr[i];

    }
    for(int j=1; 1<<j <= len; j++){
        for(int i=0; i+(1<<j)<=len; i++){
            aux_lcp[i][j] = min(aux_lcp[i][j-1],

```

```

        y=(y+y)%n;
        b>=>1;
    }
    return x%n;
}
long long pow_1(long long r2,long long n1){
    long long ans=1;
    while(n1>0){
        if((n1&1)>0)
            ans=mul(ans,r2);
        n1>>=1;
        r2=mul(r2,r2);
    }
    return ans%n;
}
bool miller_rabin(){
    if(n==1)return false;
    if(n==2)return true;
    if(n%2==0)return false;
    s=0;d=n-1;
    while(d%2==0){
        s++;d/=2;
    }
    for(int i=0;i<20;i++){
        a=rand()%(n-1)+1;
        x=pow_1(a,d);
        if(x==1 || x==n-1)continue;
        for(j=1;j<=s;j++){
            x=mul(x,x);
            if(x==1)return false;
            if(x==(n-1))break;
        }
        if(j==s+1)return false;
    }
    return true;
}

// Trie
struct node{
    // 0 - new 1 - end 2 - not end
    int status;
    node* kids[10];
}nodes[10000000];
int cnt = 0;
int n;
char str[12];
bool insert(node *curr, int idx){
    if(!str[idx]){
        if(curr->status == 0){
            curr->status = 1;
            return true;

```

<pre> aux_lcp[i+(1&lt;&lt;(j-1))][j-1]); } int suffix_array::lcp_RMQ(int x, int y){     x = aux[steps-1][x];     y = aux[steps-1][y];     if(x&gt;y)         swap(x, y);     int k = LOG2[y-x];     return min(aux_lcp[x+1][k], aux_lcp[y-(1&lt;&lt;k)+1][k]); } int main(){     int cnt = 0;     for(char ch = 'a';ch&lt;='b';ch++)         ranks[ch] = cnt++;     int x = 0;     for(int i=0;i&lt;MAX_STR_SIZE;i++){         if(1&lt;&lt;x &lt;= i)             x++;         LOG2[i] = x-1;     } } </pre>	<pre>     }else         return false;     }     if(curr-&gt;kids[str[idx]-'0']){         return insert(curr-&gt;kids[str[idx]-'0'], idx+1);     }else{         if(curr-&gt;status == 1)             return false;         else{             if(curr-&gt;status == 0)                 curr-&gt;status = 2;             for(int i=0;i&lt;10;i++){                 nodes[cnt].kids[i] = 0, nodes[cnt].status = 0;                 curr-&gt;kids[str[idx]-'0'] = &amp;nodes[cnt];                 cnt++;                 return insert(curr-&gt;kids[str[idx]-'0'], idx+1);             }         }     } } </pre>
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## //BIGNUM

```

class BIGNUM{
    long long *N;
    int BASE, BASE_pow, max_size;
    bool sign;

```

```

    int int_len(long long x);
    void do_carry();
    void compress();

```

```

public:
    int size;
    BIGNUM();
    ~BIGNUM(){delete []N;}
    BIGNUM(const BIGNUM &x);

```

```

    void print();
    string get_str();
    void load_from_LL(long long x);
    void load_from_str(const char *tmp);
    BIGNUM add_mag(const BIGNUM &x);
    BIGNUM sub_mag(const BIGNUM &x);
    int cmp(const BIGNUM &x); // 0-> x is equal, -1 -> x is smaller, 1 -> x is greater
    int cmp_mag(const BIGNUM &x);

```

```

    BIGNUM operator-(){
        if(this->size == 1 && this->N[0] == 0)
            return *this;
        BIGNUM ret = *this;
        ret.sign = !this->sign;
        return ret;
    }

```

## \* Binomial Coefficient

$C(n, r) = n! / (r! * (n-r)!)$

$C(n, r) = C(n-1, r-1) + C(n-1, r)$

## \* Number of derangements

Given an arrangements of n objects, derangements are the number of arrangements such that none object are in their original places

$D(n) = n! * (1 - 1/1! + 1/2! - 1/3! + \dots + (-1)^n 1/n!)$

Recurrence ->  $n * D(n-1) = -(-1)^n + D(n)$

->  $D(n) = n * D(n-1) + (-1)^n$

$D(n) = (n-1) * (D(n-1) + D(n-2))$

## \* Bayes Theorem

$P(A_i/B) = P(A_i)P(B/A_i) / \sum(P(A_i)P(B/A_i))$

-  $rC(n, r) = n * C(n-1, r-1)$

-  $C(n, r)/(r+1) = C(n+1, r+1)/(n+1)$

## // Series

### \* AGP

$a_1, a_2, \dots$  in AP(a, d).  $b_1, b_2, \dots$  in GP(b, r)

$S(n) = ab / (1-r) + dbr(1-r^{n-1}) / (1-r)^2 - (a+(n-1)d)br^n / (1-r)$

\*  $(1-x)^{-1} = 1+x+x^2+\dots \quad (-1 < x < 1)$

\*  $(1-x)^{-2} = 1+2x+3x^2+\dots \quad (-1 < x < 1)$

```

}

BIGNUM operator+(){return *this;}

BIGNUM operator+(const BIGNUM &x);
BIGNUM operator-(const BIGNUM &x);
BIGNUM operator*(const BIGNUM &x);

bool operator<(const BIGNUM &x){return cmp(x) == 1;}
bool operator>(const BIGNUM &x){return cmp(x) == -1;}
bool operator==(const BIGNUM &x){return cmp(x) == 0;}
bool operator<=(const BIGNUM &x){return cmp(x)+1 > 0;}
bool operator>=(const BIGNUM &x){return cmp(x)-1 < 0;}

friend void test();
};

BIGNUM::BIGNUM():BASE(1000000), BASE_pow(6), max_size(3500), sign(false){
    // Default setting is enough for operations on 10000 digits
    N = new long long[max_size];
    N[0] = 0;
    size = 1;
}

BIGNUM::BIGNUM(const BIGNUM &x){
    size = x.size; BASE = x.BASE; BASE_pow = x.BASE_pow; max_size = x.max_size; sign = x.sign;
    N = new long long[max_size];
    for(int i=0;i<size;i++) N[i] = x.N[i];
}

int BIGNUM::int_len(long long x){
    int count=0;
    do { x/=10; count++; } while(x!=0);
    return count;
}

void BIGNUM::print(){
    if(sign) printf("-");
    for(int i=size-1;i>=0;i--){
        int len=int_len(N[i]);
        for(int j=0;j<BASE_pow-len && i!=size-1;j++) printf("0");
        printf("%d",N[i]);
    }
}

string BIGNUM::get_str(){
    string ret;
    int i, len;
    for(i=size-1;N[i]<=0 && i>0;i--);
    int start=i;
    if(sign) ret.push_back('-');
    for(;i>=0;i--){
        int len=int_len(N[i]);
        for(int j=0;j<BASE_pow-len && i!=start;j++) ret.push_back('0');
        string foo;
        int x = N[i];
        do { foo.push_back(x%10 + '0'); x/=10;} while(x);
    }
}

```

#### \* Trigo

```

- sin(A+B) = sinAcosB + cosAsinB
- cos(A+B) = cosAcosB - sinAsinB
- tan(A+B) = (tanA + tanB)/(1-tanAtanB)
- sin(A+B)sin(A-B) = sin^2(A) - sin^2(B)
- cos(A+B)cos(A-B) = cos^2(A) - sin^2(B)
- sin(3A) = 3sinA - 4sin^3(A) = 4sin(60-A)sinAsin(60+A)
- cos(3A) = 4cos^3(A) - 3cosA = 4cos(60-A)cosAcos(60+A)
- tan3A = (3tanA - tan^3(A))/(1-3tan^2(A)) = tan(60-A)tanAtan(60+A)
- sinA + sinB = 2sin((A+B)/2)cos((A-B)/2)
- sinA - sinB = 2sin((A-B)/2)cos((A+B)/2)
- cosA + cosB = 2cos((A-B)/2)cos((A+B)/2)
- tanA + tanB = sin(A+B)/(cosAcosB)

```

```

        reverse(foo.begin(), foo.end());
        ret += foo;
    }
    return ret;
}

void BIGNUM::load_from_LL(long long x){
    if(x<0){sign = true;x *= -1;}
    else   sign = false;
    size = 0;
    do { N[size++] = x%BASE; x /= BASE;} while(x);
}

void BIGNUM::load_from_str(const char *tmp){
    if(tmp[0] == '-'){sign = true;tmp++;}
    else   sign = false;
    int len=strlen(tmp);
    size=(len-1)/BASE_pow + 1;
    for(int i=0;i<size;i++) N[i] = 0;
    for(int i=0;i<len;i++) N[((len-1-i)/BASE_pow)]=N[(len-1-i)/BASE_pow]*10+tmp[i]-'0';
    if(size == 1 && N[0] == 0) sign = false;
}

int BIGNUM::cmp(const BIGNUM &x){
    if(sign && !x.sign) return 1;
    if(!sign && x.sign) return -1;
    int LT = -1, GT = 1;
    if(sign && x.sign) LT = 1, GT = -1;
    if(size<x.size) return GT;
    if(size>x.size) return LT;
    int idx = size-1;
    while(idx>=0){
        if(N[idx] < x.N[idx]) return GT;
        if(N[idx] > x.N[idx]) return LT;
        idx--;
    }
    return 0;
}

int BIGNUM::cmp_mag(const BIGNUM &x){
    int LT = -1, GT = 1;
    if(size<x.size) return GT;
    if(size>x.size) return LT;
    int idx = size-1;
    while(idx>=0){
        if(N[idx] < x.N[idx]) return GT;
        if(N[idx] > x.N[idx]) return LT;
        idx--;
    }
    return 0;
}

BIGNUM BIGNUM::add_mag(const BIGNUM &x){
    BIGNUM ret;
    int size1 = this->size, size2 = x.size, lt = min(size1, size2), i;
    ret.size = max(size1, size2);
    for(i=0;i<lt;i++) ret.N[i] = this->N[i]+x.N[i];

```

#### \* Euler Totient Function

- $\phi(n)$  = number of  $k$  ( $1 \leq k \leq n$ ) such that  $\text{GCD}(k, n) = 1$
- Multiplicative, i.e. if  $\text{GCD}(m, n) = 1$ , then  $\phi(mn) = \phi(m) * \phi(n)$
- $\phi(n) = n * (1 - 1/p_1) * (1 - 1/p_2) \dots$   $p_1, p_2, \dots \rightarrow$  distinct primes dividing  $n$
- $\phi(p^k) = p^k - p^{(k-1)}$
- $\phi(d_1) + \phi(d_2) + \dots = n$   $d_1, d_2, \dots \rightarrow$  positive divisors of  $n$
- if  $a$  and  $n$  are coprime, then  $a^{\phi(n)} \equiv 1 \pmod{n}$
- $b \% a = 0$  implies  $\phi(b) \% \phi(a) = 0$
- $n$  divides  $\phi(a^n - 1)$  ( $a, n > 1$ )
- $\phi(mn) = \phi(m) * \phi(n) * d / \phi(d)$   $d = \text{gcd}(m, n)$ 
  - $\phi(2m) = (m \text{ is even? } 2: 1) * \phi(m)$
  - $\phi(n^m) = n^{(m-1)} * \phi(n)$
- $\phi(\text{LCM}(a, b)) * \phi(\text{GCD}(a, b)) = \phi(a) * \phi(b)$
- for  $n \geq 3$ ,  $\phi(n)$  is even
- if  $n$  has  $r$  distinct odd prime factors, then  $2^r$  divides  $\phi(n)$



```

if(ret.size == this->size)
    for(;i<ret.size;i++)    ret.N[i] = this->N[i];
else
    for(;i<ret.size;i++)    ret.N[i] = x.N[i];
ret.do_carry();
return ret;
}

BIGNUM BIGNUM::sub_mag(const BIGNUM &x){
    BIGNUM ret;ret.size = 0;
    int c = 0, lt = min(x.size, size), i;
    for(i=0;i<lt;i++){
        ret.N[ret.size] = this->N[i]+(BASE-(i!=0)-x.N[i])+c;
        c = ret.N[ret.size]/BASE;
        ret.N[ret.size] %= BASE;
        ret.size++;
    }
    lt = max(x.size, size);
    if(lt == x.size){
        for(;i<lt;i++){
            ret.N[ret.size] = (BASE-1-x.N[i])+c;
            c = ret.N[ret.size]/BASE;
            ret.N[ret.size] %= BASE;    ret.size++;
        }
    }else
        for(;i<lt;i++){
            ret.N[ret.size] = this->N[i]+(BASE-1)+c;
            c = ret.N[ret.size]/BASE;
            ret.N[ret.size] %= BASE;    ret.size++;
        }
    ret.compress();
    if(!c){
        ret.sign = true;
        for(int i=0;i<ret.size;i++) ret.N[i] = BASE-(i!=0)-ret.N[i];
    }
    ret.compress();
    return ret;
}

void BIGNUM::compress(){
    while(N[size-1] == 0 && size>1)size--;
}

void BIGNUM::do_carry(){
    int c = 0;
    for(int i=0;i<size;i++){
        N[i] += c;
        c = N[i]/BASE;
        N[i] %= BASE;
    }
    if(c)
        N[size++] = c;
}

inline BIGNUM BIGNUM::operator+(const BIGNUM &x){
    if(!sign && !x.sign){    // (+) + (+)

```

#### \* Fermat's Little Theorem

if  $m = n \pmod{p-1}$ ,  $p \rightarrow \text{prime}$

$$m = b(p-1) + n$$

$$a^m = a^{b(p-1)} \cdot a^n = 1^b \cdot a^n = a^n \pmod{n}$$

$a^p = a \pmod{p}$ ,  $p \rightarrow \text{prime}$

$a^{p-1} = 1 \pmod{p}$ ,  $p$  and  $a \rightarrow \text{coprime}$

#### \* Euler Theorem

$$a^{\text{ETF}(n)} = 1 \pmod{n}$$

#### \* Primitive root modulo n

$$g^k = a \pmod{n}$$

$a$  is coprime to  $n$  (all coprimes should satisfy this if  $g$  is primitive root modulo  $n$ )

$g$  is the primitive root modulo  $n$

$k$  is the index/discrete logarithm of  $a$  to the base  $g \pmod{n}$

-The product of all primitive roots of prime  $p \neq 3$  is  $1 \pmod{p}$ , for  $p=3$ , it is  $2$

-If multiplicative order of a number  $m \pmod{n}$  is  $\phi(n)$ ,  $m$  is primitive root.

- Multiplicative order of  $a \pmod{n}$  is the smallest positive int  $k$  with  $a^k = 1 \pmod{n}$ ,  $\gcd(a, n)=1$

$$-m^{\phi(n)/p_i} \pmod{n}$$

where  $p_i$  is a prime factor of  $\phi(n)$ . If for a  $m$ , all results for  $p_i$  are different from 1, then it is a primitive root

$$-\text{number of primitive root mod } n = \phi(\phi(n))$$

- $g$  is a primitive root modulo  $p$ , then  $g$  is a primitive root modulo all powers  $p^k$  unless  $g^{p-1} \equiv 1 \pmod{p^2}$ ; in that case,  $g + p$  is

```

    BIGNUM ret = this->add_mag(x);
    return ret;
}
if(sign && x.sign){    // (-) + (-)
    BIGNUM ret = this->add_mag(x);
    ret.sign = true;
    return ret;
}
if(sign && !x.sign){    // (-) + (+)
    BIGNUM ret = sub_mag(x);
    if(!(ret.size == 1 && ret.N[0] == 0))ret.sign = !ret.sign;
    return ret;
}
// (+) + (-)
BIGNUM ret = sub_mag(x);
return ret;
}
inline BIGNUM BIGNUM::operator-(const BIGNUM &x){
    if(!sign && !x.sign){    // (+) - (+)
        BIGNUM ret = this->sub_mag(x);
        return ret;
    }
    if(sign && x.sign){    // (-) - (-)
        BIGNUM ret = this->sub_mag(x);
        if(!(ret.size == 1 && ret.N[0] == 0))ret.sign = !ret.sign;
        return ret;
    }
    if(sign && !x.sign){    // (-) - (+)
        BIGNUM ret = add_mag(x);
        ret.sign = true;
        return ret;
    }
    // (+) - (-)
    BIGNUM ret = add_mag(x);
    return ret;
}
BIGNUM BIGNUM::operator*(const BIGNUM &x){
    BIGNUM ret;
    ret.size = size+x.size-1;
    for(int i=0;i<ret.size;i++) ret.N[i] = 0;
    for(int i=0;i<x.size;i++)
        for(int j=0;j<size;j++) ret.N[i+j] += x.N[i]*N[j];
    ret.do_carry(); ret.compress();
    ret.sign = (sign ^ x.sign);
    if(ret.sign && (ret.size == 1 && ret.N[0] == 0))    ret.sign = false;
    return ret;
}

```

## //POINT

```

template <class T>
class POINT{
public:

```

## TRIANGLE

R -> circumradius

r = inradius

r1, r2, r3 = exradius

\* sine rule

$\sin A/a = \sin B/b = \sin C/c = 1/2R = 2(\text{area})/abc$

\* cosine rule

$\cos A = (b^2 + c^2 - a^2)/2bc$

..

\*  $\sin(A/2) = \sqrt{(s-b)(s-c)/bc}$

$\cos(A/2) = \sqrt{s(s-a)/bc}$

\*  $a = b\cos C + c\cos B$

\*  $r = \text{area}/s$

\*  $2r \leq R$

\*  $r1 + r2 + r3 = 4R + r$

$r1 = \text{area}/(s-a) = \tan(A/2) =$

$4R \sin(A/2) \cos(B/2) \cos(C/2)$

$r1r2 + r2r3 + r3r1 = s^2 = r1r2r3/r$

\*  $r + 2R = s$  in right triangle

\* **PnC**

\* Circular Permutation of n things taken r at a time =  $P(n, r)/r$  (clock+anticlock)

\* selection of n distinct objects taken r at a time with repetition =  $C(n+r-1, r)$

\* no of ways to divide n distinct objects into r unequal groups of size a1, a2,...  
 $= n!/(a1!a2!...)$

\* divide m.n distinct objects equally into n groups =  $(mn)!/(m!^n)n!$

\* Multinomial

$x1 + x2 + ... + xm = n, a1 \leq x1 \leq b1,$

$a2 \leq x2 \leq b2, ...$

number of solutions = coefficient of  $x^n$  in  $(x^{a1} + ... + x^{b1})(x^{a2} + ... + x^{b2})...$

coefficient of  $x^r$  in  $(1-x)^{-n} = C(n+r-1, r)$

```

T P[2];
POINT(){}
POINT(T x, T y){P[0] = x, P[1] = y;}
//dot
T operator*(const POINT &a){
    return a.P[0]*this->P[0] + a.P[1]*this->P[1];
}
POINT operator-(const POINT &b){
    return POINT(this->P[0] - b.P[0], this->P[1] - b.P[1]);
}
//cross
T operator^(const POINT &b){
    return this->P[0]*b.P[1] - this->P[1]*b.P[0];
}
};
template <class T>
vector<int> convex_hull(POINT<T> points[],int n)
{
    points[0].P[0]=20000;
    points[0].P[1]=20000;
    bool used[n+1];
    for(int i=1;i<=n;i++) used[i]=false;
    int bot_left=0;
    vector<int> pos;
    for(int i=1;i<=n;i++){
        if(points[bot_left].P[1]>points[i].P[1])
            bot_left = i;
        else if(points[bot_left].P[1]==points[i].P[1] && points[bot_left].P[0]>points[i].P[0])
            bot_left = i;
    }
    pos.push_back(bot_left);
    int start = bot_left;
    do{
        int n2=-1;
        int dis = 0;
        for(int i=1;i<=n;i++){
            if(i==bot_left)continue;
            if(used[i])continue;
            if(n2==-1)n2=i;
            T cross = (points[i]-points[bot_left])^(points[n2]-points[bot_left]);
            T d = (points[i]-points[bot_left])*(points[i]-points[bot_left]);
            if(cross>0)n2=i,dis=d;
            else if(cross==0)
                if(d>dis)dis=d,n2=i;
        }
        bot_left = n2;
        used[n2]=true;
        pos.push_back(n2);
    }while(start!=bot_left);
    return pos;
}
// Distance of line b-c from point a

```

### Extended Euclid

```

pair<int, pair<int, int> > GCD(int a,int b)
{
    if(a == 0)
        return make_pair(b, make_pair(0, 1));
    else{
        pair<int, pair<int, int> > foo = GCD(b%a,
a);
        return make_pair(foo.first,
make_pair(foo.second.second-
b/a*foo.second.first, foo.second.first));
    }
}
long long modInv(long long a, long long m){
    pair<long long, pair<long long, long long> >
foo = GCD(a, m);
    return ((foo.second.first % m) + m)%m;
}

```

```

int linedist(POINT<int> &a, POINT<int> &b, POINT<int> &c)
{
    int dot1 = (b-a)*(c-b);
    if(dot1>0)return 1;
    int dot2 = (a-b)*(c-a);
    if(dot2>0)return 1;
    return (b-a)^(c-a);
}
// sign of ba x ca
int ccw(POINT<int> a, POINT<int> b, POINT<int> c)
{
    long long foo = ((b-a)^(c-a));
    return foo > 0?1:(foo<0?-1:0);
}
// If a-b intersects with c-d
bool intersect(POINT<int> a, POINT<int> b, POINT<int> c, POINT<int> d)
{
    if(ccw(a,b,c) == 0 && ccw(a,b,d) == 0){
        if((((b-a)*(c-b))>0 && ((b-a)*(d-b))>0) || (((a-b)*(c-a))>0 && ((a-b)*(d-a))>0))
            return false;
        else
            return true;
    }
    return ccw(a,c,d) != ccw(b,c,d) && ccw(a,b,c) != ccw(a,b,d);
}
// Number of points from points1[] at which a-points2[b] intersects.
int getans(POINT<int> &a, int b, POINT<int> points1[], POINT<int> points2[], vector<int> &points)
{
    int cnt = 0;
    for(int i=0;i<-1+points.size();i++)
        if(intersect(a, points2[b], points1[points[i]], points1[points[i+1]]))
            cnt++;
    return cnt;
}
// if pt (in points2) is inside the polygon denoted by pts (in points1)
bool is_inside(vector<int> &pts, int pt, POINT<int> points1[], POINT<int> points2[], int xmin, int xmax, int ymin, int ymax)
{
    if(points2[pt].P[0]<xmin || points2[pt].P[0]>xmax || points2[pt].P[1]<ymin || points2[pt].P[1]>ymax)
        return false;
    POINT<int> coord;
    for(int i=0;i<pts.size()-1;i++)
        if(linedist(points1[pts[i]], points1[pts[i+1]], points2[pt]) == 0)
            return true;
    while(1){
        POINT<int> candi;
        candi.P[0] = rand()%200000 + 200000;
        candi.P[1] = rand()%200000 + 200000;
        bool flag = true;
        for(int i=0;i<pts.size()-1;i++){
            if(linedist(candi, points2[pt], points1[pts[i]])==0){
                flag = false;
            }
        }
    }
}

```

```

        break;
    }
}
if(!flag)continue;
return getans(candi, pt, points1, points2, pts)%2;
}
}

```

## //Splay Tree

```

/*
Insert x
- foo.insert(x)
Remove x
- foo.remove(x)
Kth smallest number
- if(foo.root == NULL || k>foo.root->size+1)
    printf("invalid\n");
else
    find_kth(foo.root, k);
Number of occurrence of x
- find_c(foo.root, x)
*/
class node{
public:
    int data;
    node *left,*right,*parent;
    node(int val, node *l, node *r, node *par):data(val),left(l),right(r),parent(par),size(0){}
    int size;
};
class BST_splay{
public:
    node *root;
    node *insert(int value);
    void remove(int value);
    node *find(int value, bool getClosest);
    node *successor(node *n);
    void inorder(node *x);
    BST_splay():root(NULL){}
    void splay(node *x);
    void rotate_left(node *x);
    void rotate_right(node *x);
};
void BST_splay::rotate_left(node *x){
    x->parent->right = x->left;
    if(x->left != NULL)
        x->left->parent = x->parent;
    x->left = x->parent;
    if(x->parent->parent!=NULL){
        if(x->parent == x->parent->parent->left)
            x->parent->parent->left = x;
        else
            x->parent->parent->right = x;
    }
}

```

```

    }
    node *tmp = x->parent;
    x->parent->size = (x->parent->left == NULL?0:(1+x->parent->left->size)) + (x->parent->right == NULL?0:(1+x->parent->right->size));
    x->parent = x->parent->parent;
    tmp->parent = x;
    x->size = (x->left == NULL?0:(1+x->left->size)) + (x->right == NULL?0:(1+x->right->size));
    if(x->parent == NULL)
        root = x;
}

void BST_splay::rotate_right(node *x){
    x->parent->left = x->right;
    if(x->right != NULL)
        x->right->parent = x->parent;
    x->right = x->parent;
    if(x->parent->parent!=NULL){
        if(x->parent == x->parent->parent->left)
            x->parent->parent->left = x;
        else
            x->parent->parent->right = x;
    }
    node *tmp = x->parent;
    x->parent->size = (x->parent->left == NULL?0:(1+x->parent->left->size)) + (x->parent->right == NULL?0:(1+x->parent->right->size));
    x->parent = x->parent->parent;
    tmp->parent = x;
    x->size = (x->left == NULL?0:(1+x->left->size)) + (x->right == NULL?0:(1+x->right->size));
    if(x->parent == NULL)
        root = x;
}

void BST_splay::splay(node *x){
    if(x == root)
        return;
    if(x == NULL)
        return;
    if(x->parent == root){ //zig
        if(x == x->parent->left)
            rotate_right(x);
        else
            rotate_left(x);
    }else if(x == x->parent->left && x->parent == x->parent->parent->right){ //zig-zag
        rotate_right(x);
        rotate_left(x);
    }else if(x == x->parent->right && x->parent == x->parent->parent->left){ //zig-zag
        rotate_left(x);
        rotate_right(x);
    }else if(x == x->parent->right && x->parent == x->parent->parent->right){ //zig-zig
        rotate_left(x->parent);
        rotate_left(x);
    }else if(x == x->parent->left && x->parent == x->parent->parent->left){ //zig-zig
        rotate_right(x->parent);
        rotate_right(x);
    }
}

```

```

    }
    splay(x);
}
node *BST_splay::insert(int value){
    node *ret;
    if(root == NULL){
        root = new node(value,NULL,NULL,NULL);
        ret = root;
    }else{
        node *parent, *child = root;
        while(child!=NULL){
            parent = child;
            if(child->data > value)
                child = child->left;
            else if(child->data < value)
                child = child->right;
            else{
                splay(child);
                return child;
            }
        }
        if(parent->data > value){
            parent->left = new node(value, NULL, NULL, parent);
            ret = parent->left;
        }else{
            parent->right = new node(value, NULL, NULL, parent);
            ret = parent->right;
        }
        parent = ret->parent;
        while(parent!=NULL){
            parent->size++;
            parent = parent->parent;
        }
    }
    splay(ret);
    return ret;
}
node *BST_splay::find(int value, bool getClosest = false){
    if(root == NULL)
        return NULL;
    else{
        node *parent, *child = root;
        while(child!=NULL){
            parent = child;
            if(child->data > value)
                child = child->left;
            else if(child->data < value)
                child = child->right;
            else
                return child;
        }
        return getClosest?parent:NULL;
    }
}

```

```

    }
}
node *BST_splay::successor(node *n){
    node *parent, *child = n->right;
    while(child != NULL){
        parent = child;
        child = child->left;
    }
    return parent;
}
void BST_splay::remove(int value){
    if(root == NULL)
        return;
    node *x = find(value, true);
    if(x->data != value){
        splay(x);
        return;
    }
    node *p = x->parent;
    if(x->left == NULL && x->right == NULL){
        if(root == x)
            root = NULL;
        else if(x == x->parent->left)
            x->parent->left = NULL;
        else
            x->parent->right = NULL;
        node *tmp = x->parent;
        while(tmp != NULL){
            tmp->size--;
            tmp = tmp->parent;
        }
        delete x;
    } else if(x->left != NULL && x->right == NULL){
        if(root == x){
            root = x->left;
            x->left->parent = NULL;
        }
        else if(x == x->parent->left){
            x->parent->left = x->left;
            x->left->parent = x->parent;
        }
        else{
            x->parent->right = x->left;
            x->left->parent = x->parent;
        }
        node *tmp = x->parent;
        while(tmp != NULL){
            tmp->size--;
            tmp = tmp->parent;
        }
        delete x;
    } else if(x->left == NULL && x->right != NULL){

```

### //3D

```

* Point dividing line joining P(x1, y1,..) and Q(x2,
y2,..)
    - (mx2+nx1/(m+n), ...)
* direction cosine -> unit vector along the line
    l^2+m^2+n^2 = 1
* cos(angle bet 2 lines) = (a1a2+b1b2+c1c2)/
(sqrt(a1^2+b1^2+c1^2)sqrt(a2^2+b2^2+c2^2))
* sin is 0 when l1/l2 = m1/m2 = n1/n2
* Projection of line seg joining P(x1,y1,z1) and
Q(x2,..)
    on a line(l,m,n) is l(x2-x1)+m(y2-y1)+n(z2-z1)
* perpendicular dist of a point P(x,y,z) from a line
passing through A(a,b,c) and
(l,m,n). AN = projection, PN = sqrt(AP^2-AN^2)
* ar(x) = 1/2((y1,z1,1),(y2,z2,1)..). area^2 = ar(x)^2 +
ar(y)^2 + ar(z)^2
* eq of plane -> ax+by+cz+d = 0
    normal form -> lx+my+nz = p, p is length of normal
from origin to plane, l,m,n is DC of normal
    plane passing through P(x1,y1,z1) and perpendicular
to line (l,m,n) is l(x-x1)+m(y-y1)+...=0
    plane passng though three nin-collinear pts |(x-x1,y-
y1,z-z1),(x2-x1,y2-y1,...)| = 0
    intercept form -> x/a+y/b+z/c = 1
* perpendicular dist of point to plane =
mod((ax1+by1+cz1+d)/sqrt(a^2+b^2+c^2))
* bisector planes
    plane1/sqrt(a1^2+..) = +/- plane2/sqrt(a2^2..)

* Line
(x-x1)/l = (y-y1)/m = (z-z1)/n
2 point-> replace l with (x2-x1) and so on
shortest dist bet 2 lines
|(x2-x1,y2-y1,z2-z1),(l1,m1,n1),
(l2,m2,n2)|/sqrt((l1m2-m1l2)^2+(m1n2-n1m2)^2+
(l1n2-l2n1)^2)

```



```

if(root == x){
    root = x->right;
    x->right->parent = NULL;
}
else if(x == x->parent->left){
    x->parent->left = x->right;
    x->right->parent = x->parent;
}
else{
    x->parent->right = x->right;
    x->right->parent = x->parent;
}
node *tmp = x->parent;
while(tmp != NULL){
    tmp->size--;
    tmp = tmp->parent;
}
delete x;
} else if(x->left != NULL && x->right != NULL){
    node *s = successor(x);
    node *tmp = s->parent;
    p = tmp;
    while(tmp != NULL){
        tmp->size--;
        tmp = tmp->parent;
    }
    s->size = x->size;
    if(x == root){
        if(s == x->right){
            p = s;
            s->parent = NULL;
            x->left->parent = s;
            s->left = x->left;
            root = s;
        } else {
            if(s->right == NULL)
                s->parent->left = NULL;
            else {
                s->parent->left = s->right;
                s->right->parent = s->parent;
            }
            s->parent = NULL;
            s->left = x->left;
            s->right = x->right;
            x->right->parent = s;
            x->left->parent = s;
            root = x;
        }
    }
} else {
    if(s == x->right){
        p = s;

```

Divides a given range 1..n into set of disjoint ranges which union to 1..n

eg 1..1234 breaks down to

1230..1234

1200..1229

1000..1199

100..999

10..99

1..9

# len = number of digits of n

calc(a,b,len):

# a>b

# len = length of a and b

# calc performs the query

divide(n, len):

ans = 0

lt = 10\*\*(len-1)

for i=0..len-1 && n!=lt-1 :

foo = n

n = pow\_10[i+1]\*(n/pow\_10[i+1])

ans += calc(foo, n, len)

n--

if n!=lt-1:

foo = n

n = pow\_10[len-1]

ans += calc(foo, n, len)

n--

if n>1:

ans += divide(n, len-1)

return ans

```

s->parent = x->parent;
x->left->parent = s;
s->left = x->left;
if(x == x->parent->left)
    x->parent->left = s;
else
    x->parent->right = s;
} else {
    if(s->right == NULL)
        s->parent->left = NULL;
    else {
        s->parent->left = s->right;
        s->right->parent = s->parent;
    }
    s->parent = x->parent;
    if(x == x->parent->left)
        x->parent->left = s;
    else
        x->parent->right = s;
    s->left = x->left;
    s->right = x->right;
    x->right->parent = s;
    x->left->parent = s;
}
}
delete x;
}
if(p!=NULL)
    splay(p);
}
void BST_splay::inorder(node *x){
    if(x!=NULL){
        inorder(x->left);
        cout<<x->data<<endl;
        inorder(x->right);
    }
}
BST_splay foo;
void find_kth(node *x, int k){
    while(1){
        int q = 1+(x->left==NULL?0:(1+x->left->size));
        if(k==q)
            break;
        if(q<k){
            if(x->right == NULL)
                return;
            k-=q;x = x->right;
        } else {
            if(x->left == NULL)
                return;
            x = x->left;
        }
    }
}

```

```

int pow_10[10];
int len_int(int n){
    int cnt = 0;
    while(n){
        cnt++;
        n/=10;
    }
    return cnt;
}
int postfix_divide(int n, int len){
    // cout<<n<<endl;
    // if(n==10)return 5;
    if(n==0)return 0;
    int ans = 0;int foo;
    int lt = pow_10[len-1];
    for(int i=0;i<len-1 && n != lt-1; i++){
        // cout<<n<<" ";
        foo = n;
        n = pow_10[i+1]*(n/pow_10[i+1]);
        // cout<<n<<endl;
        ans+=calc2(foo,n,len);
        // cout<<calc2(foo,n,len)<<" -- "<<ans<<endl;
        n--;
    }
    if(n!=lt-1){
        // cout<<n<<" ";
        foo = n;
        n = pow_10[len-1];
        ans+=calc2(foo,n,len);
        // cout<<n<<endl;
        // cout<<calc2(foo,n,len)<<" -- "<<ans<<endl;
        n--;
    }
    if(n>1)
        ans += postfix_divide(n, len-1);
    // cout<<ans<<endl;
    return ans;
}

```

```

    }
    foo.splay(x);printf("%d\n",x->data);
}
int find_c(node *x, int val){
    if(x==NULL)return 0;
    int ans = 0;
    while(1){
        if(x->data<val){
            ans += 1+(x->left == NULL?0:(1+x->left->size));
            if(x->right != NULL)
                x = x->right;
            else break;
        }else if(x->data == val){
            ans += x->left == NULL? 0:(1+x->left->size);
            break;
        }else{
            if(x->left != NULL){
                x = x->left;
            }else break;
        }
    }
    foo.splay(x);
    return ans;
}

```

```

//Heavy Light
struct vertex;struct edge;struct chain;
struct vertex{
    vector<edge*> adj;
    edge* parent_edge;
    int heavy_chain_id;
    int size,depth;
    void init(){
        adj.clear();
        parent_edge = NULL;
        heavy_chain_id = -1;
        size = depth =0;
    }
};
struct edge{
    vertex *u,*v;
    chain* host_chain;
    int weight;
    void init(){
        u = v = NULL;
        host_chain = NULL;
    }
};
struct chain{
    vector<int> weights;
    vertex *head;int size;
}

```

```

int rank[10004],sum[10004],p,n,m,a,b,c;
struct foo
{
    int a,b,wt;
    friend bool operator<(const foo &a,const foo &b)
    {
        return a.wt<b.wt;
    }
}bar;
vector<struct foo> path;
void kruskal(){
    // Answer in sum[1]
    for(int i=1;i<=n;i++){
        {rank[i]=i;sum[i]=0;}
    }
    sort(path.begin(),path.end());
    for(int i=0;i<path.size();i++){
        if(rank[path[i].a]!=rank[path[i].b]){
            if(rank[path[i].a]<rank[path[i].b]){
                int tmp=rank[path[i].b];
                for(int j=1;j<=n;j++){
                    if(rank[j]==tmp)
                        rank[j]=rank[path[i].a];
                }
                sum[rank[path[i].a]]+=path[i].wt+sum[tmp];
            }else{
                int tmp=rank[path[i].a];
                for(int j=1;j<=n;j++){
                    if(rank[j]==tmp)
                        rank[j]=rank[path[i].b];
                }
                sum[rank[path[i].b]]+=path[i].wt+sum[tmp];
            }
        }
    }
}

```

```

int *TREE;

// Chain Methods
void clear(){
    weights.clear();
}
void build(){
    size = weights.size();
    TREE = new int[size*4];
    build_segment_tree(1, 0, size-1);
}
int query(vertex *u, vertex *v){
    int x,y;
    x = u->depth-head->depth,
    y = v->depth-head->depth-1;
    return query_segment_tree(1, 0, size-1, x, y);
}

//Segment Tree Methods
void build_segment_tree(int NODE,int a,int b){
    if(a==b){
        TREE[NODE] = weights[a];
        return;
    }
    int mid=(a+b)/2;
    int left=2*NODE,right=2*NODE+1;
    build_segment_tree(left,a,mid);
    build_segment_tree(right,mid+1,b);

    TREE[NODE] = TREE[left]+TREE[right];
}
int query_segment_tree(int NODE,int a,int b,int x,int y){
    if(x<=a && y>=b)return TREE[NODE];
    int mid=(a+b)/2;
    int left=2*NODE,right=2*NODE+1;
    if(y<=mid)return query_segment_tree(left,a,mid,x,y);
    if(x>mid)return query_segment_tree(right,mid+1,b,x,y);

    int left_query=query_segment_tree(left,a,mid,x,y);
    int right_query=query_segment_tree(right,mid+1,b,x,y);

    return left_query + right_query;
}
};
vertex *root = NULL;
int chain_cnt = 0;
vertex V[10002];
edge E[10002];
chain heavy_chains[5002];
void build(vertex *V, vertex* par=NULL){
    if(par == NULL){
        root = V;

```

```

    V->depth = 0;
} else
    V->depth = par->depth+1;

V->size = 1;
for(int i=0;i<V->adj.size();i++){
    vertex *nxt = (V->adj[i]->u == V)?V->adj[i]->v:V->adj[i]->u;
    if(nxt != par){
        nxt->parent_edge = V->adj[i];
        build(nxt, V);
        V->adj[i]->u = V;
        V->adj[i]->v = nxt;
        V->size += nxt->size;
    }
}
}

void heavy_light_decomposition(vertex *V, chain *curr_chain = NULL){
    for(int i=0;i<V->adj.size();i++){
        vertex *nxt = V->adj[i]->v;
        if(nxt!=V){
            if(2*nxt->size >= V->size){
                if(curr_chain == NULL){
                    curr_chain = &heavy_chains[chain_cnt++];
                    curr_chain->head = V;
                }
                V->adj[i]->host_chain = curr_chain;
                curr_chain->weights.push_back(V->adj[i]->weight);
                heavy_light_decomposition(nxt, curr_chain);
                nxt->heavy_chain_id = V->heavy_chain_id = curr_chain->heavy_chains;
            } else
                heavy_light_decomposition(nxt);
        }
    }
}

vertex* LCA(vertex *u, vertex *v){
#ifdef VVVV
    cout<<"FINDING LCA..."<<endl;
#endif // VVVV
    int du,dv;
    vertex *hu, *hv;
    while(u!=v && (u->heavy_chain_id==-1 || u->heavy_chain_id!=v->heavy_chain_id)){
        if(u->heavy_chain_id == -1){
            du = u->depth;
            hu = u;
        } else
            du = heavy_chains[u->heavy_chain_id].head->depth, hu = heavy_chains[u->heavy_chain_id].head;

        if(v->heavy_chain_id == -1){
            dv = v->depth;
            hv = v;
        } else
            dv = heavy_chains[v->heavy_chain_id].head->depth, hv = heavy_chains[v->heavy_chain_id].head;
    }
}

```

```

    if(du<dv){
        v = hv->parent_edge->u;
    }else{
        u = hu->parent_edge->u;
    }
}
if(u->depth < v->depth)
    return u;
return v;
}
int query(vertex *u, vertex *v){
    //u is ancestor of v
    if(u == v)
        return 0;
    if(v->parent_edge->host_chain == NULL){ //Light
        return v->parent_edge->weight + query(u, v->parent_edge->u);
    }else{ //Heavy
        chain* host_chain = v->parent_edge->host_chain;
        if(host_chain->head->depth <= u->depth)
            return host_chain->query(u,v);
        else
            return host_chain->query(host_chain->head, v) + query(u, host_chain->head);
    }
}
int query_k(vertex *u, vertex *v, int k)
{
    if(k == 1)
        return v-V;
    if(v->parent_edge->host_chain == NULL) //Light
        return query_k(u, v->parent_edge->u, k-1);
    else{ //Heavy
        chain* host_chain = v->parent_edge->host_chain;
        if(host_chain->weights.size() >= k)
            return query_k(u, v->parent_edge->u, k-1);
        else
            return query_k(u, host_chain->head, k-(v->depth-host_chain->head->depth));
    }
}
int main()
{
    in_T{
        int n,a,b,c;
        char str[10];
        in_I(n);
        root = NULL;
        chain_cnt = 0;
        for(int i=1;i<=n;i++)
            V[i].init();
        for(int i=0;i<n-1;i++){
            E[i].init();
            scanf("%d%d%d", &a, &b, &c);
            E[i].u = &V[a];

```

```

    E[i].v = &V[b];
    E[i].weight = c;
    V[a].adj.push_back(&E[i]);
    V[b].adj.push_back(&E[i]);
}
build(&V[1]);
heavy_light_decomposition(&V[1]);
for(int i=0;i<chain_cnt;i++)
    heavy_chains[i].build();
#ifdef VVVV
cout<<"Edges:"<<endl;
for(int i=0;i<n-1;i++){
    cout<<i<<": "<<(E[i].u-V)<<" "<<(E[i].v-V)<<" "<<(E[i].host_chain == NULL?"light":"heavy")<<endl;
}
cout<<endl<<"Vertices:"<<endl;
for(int i=1;i<=n;i++){
    cout<<i<<": "<<V[i].size<<" "<<V[i].depth<<" "<<(root == (V+i)?"root":"")<<endl;
}
#endif
while(1){
    in_S(str);
    if(str[1] == 'I'){
        // Distance between two nodes
        scanf("%d%d", &a, &b);
        #ifdef VVVV
        cout<<"QUERY "<<a<<" "<<b<<endl;
        #endif
        vertex *lca = LCA(&V[a], &V[b]);
        long long ans = 0;
        #ifdef VVVV
        cout<<"LCA: "<<lca-V<<endl;
        #endif
        ans = query(root, &V[a]);
        ans += query(root, &V[b]) - 2*query(root, lca);
        printf("%lld\n", ans);
    }
    else if(str[0] == 'K'){
        // cth node on path from a to b
        scanf("%d%d%d", &a, &b, &c);
        #ifdef VVVV
        cout<<"QUERY "<<a<<" "<<b<<" "<<c<<endl;
        #endif
        vertex *lca = LCA(&V[a], &V[b]);
        int total_nodes = V[a].depth-lca->depth + V[b].depth-lca->depth + 1;
        if(V[a].depth-lca->depth + 1 >= c)
            P_I(query_k(lca, &V[a], c));
        else
            P_I(query_k(lca, &V[b], total_nodes-c+1));
    }else
        break;
}
for(int i=0;i<chain_cnt;i++)

```

```
        heavy_chains[i].clear();
    cout<<endl;
}
}
```

### Euler Path

An undirected graph has an Eulerian cycle if and only if every vertex has even degree, and all of its vertices with nonzero degree belong to a single [connected component](#).

An undirected graph has an Eulerian trail if and only if at most two vertices have odd degree, and if all of its vertices with nonzero degree belong to a single connected component.

A directed graph has an Eulerian cycle if and only if every vertex has equal [in degree](#) and [out degree](#), and all of its vertices with nonzero degree belong to a single [strongly connected component](#).

directed graph has an Eulerian trail if and only if at most one vertex has  $(\text{out-degree}) - (\text{in-degree}) = 1$ , at most one vertex has  $(\text{in-degree}) - (\text{out-degree}) = 1$ , every other vertex has equal in-degree and out-degree, and all of its vertices with nonzero degree belong to a single connected component of the underlying undirected graph.