Anudeep's blog

MO’s Algorithm (Query square root decomposition)

28 December 2014 anudeep2011 Algorithms
59 Comments 27,843 views

Once again I found a topic that is useful, interesting but has very less resources online. Before writing this I did a small survey and surprised that almost none of the Indian programmers knew this algorithm. Its important to learn this, all the red programmers on codeforces use this effectively in div 1 C and D problems. There were not any problems on this an year and half back, but from then there is a spike up! We can expect more problems on this in future contests.

1) State a problem
2) Explain a simple solution which takes O(N^2)
3) Slight modification to above algorithm. It still runs in O(N^2)
4) Explain an algorithm to solve above problem and state its correctness
5) Proof for complexity of above algorithm – O(Sqrt(N) * N)

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6) Explain where and when we can use above algorithm
7) Problems for practice and sample code

State a problem

Given an array of size N. All elements of array <= N. You need to answer M queries. Each query is of the form L, R. You need to answer the count of values in range [L, R] which are repeated at least 3 times.
Example: Let the array be {1, 2, 3, 1, 1, 2, 1, 2, 3, 1} (zero indexed)
Query: L = 0, R = 4. Answer = 1. Values in the range [L, R] = {1, 2, 3, 1, 1} only 1 is repeated at least 3 times.
Query: L = 1, R = 8. Answer = 2. Values in the range [L, R] = {2, 3, 1, 1, 2, 1, 2, 3} 1 is repeated 3 times and 2 is repeated 3 times. Number of elements repeated at least 3 times = Answer = 2.

Explain a simple solution which takes O(N^2)

For each query, loop from L to R, count the frequency of elements and report the answer. Considering M = N, following runs in O(N^2) in worst case.

```
for each query:
  answer = 0
  count[] = 0
  for i in {l..r}:
    count[array[i]]++
    if count[array[i]] == 3:
      answer++
```

Slight modification to above algorithm. It still runs in O(N^2)

MO's Algorithm (Query square root decomposition)
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tanmoy1228 on MO's Algorithm (Query square root decomposition)

Xyleth on Persistent

```
add(position):
  count[array[position]]++
  if count[array[position]] == 3:
    answer++

remove(position):
  count[array[position]]--
  if count[array[position]] == 2:
    answer--

currentL = 0
currentR = 0
answer = 0
count[] = 0
for each query:
  // currentL should go to L, currentR
  while currentL < L:
    remove(currentL)
    currentL++
  while currentL > L:
    add(currentL)
    currentL--
  while currentR < R:
    add(currentR)
    currentR++
  while currentR > R:
    remove(currentR)
    currentR--
  output answer
```

Initially we always looped from L to R, but now we are changing the positions from previous query to adjust to current query.

If previous query was L=3, R=10, then we will have currentL=3 and currentR=10 by the end of that query. Now if the next query is L=5, R=7, then we move the currentL to 5 and currentR to 7.

add function means we are adding the element at position to our current set. And updating answer accordingly.

remove function means we are deleting the element at position from our current set. And updating answer accordingly.

Pushkar Kr Raijada on Machine learning everywhere, why not in Competitive programming?

ibahadir.altun on Heavy Light Decomposition

Contact Me

Ask about something or report me a bug or just say Hi. I will be happy to get back.

Explain an algorithm to solve above problem and state its correctness

MO’s algorithm is just an order in which we process the queries. We were given M queries, we will re-order the queries in a particular order and then process them. Clearly, this is an offline algorithm. Each query has L and R, we will call them opening and closing. Let us divide the given input array into Sqrt(N) blocks. Each block will be N / Sqrt(N) = Sqrt(N) size. Each opening has to fall in one of these blocks. Each closing has to fall in one of these blocks.

A query belongs to P’th block if the opening of that query fall in P’th block. In this algorithm we will process the queries of 1st block. Then we process the queries of 2nd block and so on.. finally Sqrt(N)’th block. We already have an ordering, queries are ordered in the ascending order of its block. There can be many queries that belong to the same block.

From now, I will ignore about all the blocks and only focus on how we query and answer block 1. We will similarly do for all blocks. All of these queries have their opening in block 1, but their closing can be in any block including block 1. Now let us reorder these queries in ascending order of their R value. We do this for all the blocks.

How does the final order look like?
All the queries are first ordered in ascending order of their block number (block number is the block in which its opening falls). Ties are ordered in ascending order of their R value
For example consider following queries and assume we have 3 blocks each of size 3.

{0, 3} {1, 7} {2, 8} {7, 8} {4, 8} {4, 4} {1, 2}
Let us re-order them based on their block number.
{0, 3} {1, 7} {2, 8} {1, 2} {4, 8} {4, 4} {7, 8}
Now let us re-order ties based on their R value.
{1, 2} {0, 3} {1, 7} {2, 8} {4, 4} {4, 8} {7, 8}

Now we use the same code stated in previous section and solve the problem. Above algorithm is correct as we did not do any changes but just reordered the queries.

Proof for complexity of above algorithm – O(Sqrt(N) * N)

We are done with MO's algorithm, it is just an ordering. Awesome part is its runtime analysis. It turns out that the O(N^2) code we wrote works in O(Sqrt(N) * N) time if we follow the order i specified above. Thats awesome right, with just reordering the queries we reduced the complexity from O(N^2) to O(Sqrt(N) * N), and that too with out any further modification to code. Hurray! we will get AC with O(Sqrt(N) * N).

Have a look at our code above, the complexity over all queries is determined by the 4 while loops. First 2 while loops can be stated as "Amount moved by left pointer in total", second 2 while loops can be stated as "Amount moved by right pointer". Sum of these two will be the over all complexity.

Most important. Let us talk about the right pointer first. For each block, the queries are sorted in increasing order, so clearly the right pointer (currentR) moves in increasing orc
During the start of next block the pointer possibly at extrer

end will move to least R in next block. That means for a given block, the amount moved by right pointer is O(N). We have O(Sqrt(N)) blocks, so the total is O(N * Sqrt(N)). Great!

Let us see how the left pointer moves. For each block, the left pointer of all the queries fall in the same block, as we move from query to query the left pointer might move but as previous L and current L fall in the same block, the moment is O(Sqrt(N)) (Size of the block). In each block the amount left pointer movies is O(Q * Sqrt(N)) where Q is number of queries falling in that block. Total complexity is O(M * Sqrt(N)) for all blocks.

There you go, total complexity is O((N + M) * Sqrt(N)) = O(N * Sqrt(N))

Explain where and when we can use above algorithm

As mentioned, this algorithm is offline, that means we cannot use it when we are forced to stick to given order of queries. That also means we cannot use this when there are update operations. Not just that, there is one important possible limitation: We should be able to write the functions add and remove. There will be many cases where add is trivial but remove is not. One such example is where we want maximum in a range. As we add elements, we can keep track of maximum. But when we remove elements it is not trivial. Anyways in that case we can use a set to add elements, remove elements and report minimum. In that case the add and delete operations are O(log N) (Resultin in O(N * Sqrt(N) * log N) algorithm).

There are many cases where we can use this algorithm. In few cases we can also use other Data Structures like segment trees, but for few problems using MO's algorithm is a must. Lets discuss few problems in the next section.

Problems for practice and sample code

DQUERY – SPOJ: Number of distinct elements in a range = number of elements with frequency >= 1. So it is the same problem we discussed above.
Click here for sample code
Note: That code will give TLE on submission, it will give AC if fast I/O is added. Removed fast I/O to keep code clean.
Powerful array – CF Div1 D: This is an example where MO's algorithm is a must. I cannot think of any other solution. CF Div1 D means it is a hard problem. See how easy it is using MO's algorithm in this case. You only need to modify add(), remove() functions in above code.
GERALD07 – Codechef
GERALD3 – Codechef
Tree and Queries – CF Div1 D
Powerful Array – CF Div1 D
Jeff and Removing Periods – CF Div1 D
Sherlock and Inversions – Codechef

I am sure there are more problems, if you know any of them, do comment, i will add them.
While this algorithm has a special name "MO", it is just smart square root decomposition.

Signing off! Wish you a happy new year 🙂

Share this post! Learn and let learn 🙂

← When 2 guys talk, its not always about girls/sports ;)

Machine learning everywhere, why not in Competitive programming? →

59 Comments

tanmoy1228

June 26, 2016 at 2:31 AM

can anyone give me any idea about
https://www.codechef.com/problems/GERALD07
this problem….
can not understand the editorial
thanks

Reply

Abhishek Kumar

April 10, 2016 at 11:38 PM

why I am getting WA 🙁
can anyone provide some testcase where my co
gives wrong answer,I have tried many testcases
but couldn't find any mistake.

problem- http://www.spoj.com/problems/DQUERY/
code-https://ideone.com/4RJi9p

Reply

Ashish Tilokani

January 31, 2016 at 8:52 PM

Short and easy to understand explanation.

This problem is from a recent contest which motivated me to understand this algo.

http://codeforces.com/contest/617/problem/E

Reply

Jaswant Singh

January 15, 2016 at 1:15 PM

Another very useful and new algo … thanks anudeep for such a nice effort .

Reply

karthik

December 31, 2015 at 12:19 PM

With fast I/o and inline declarations
Powerful array : http://ideone.com/mc0fNT
Dquery :http://ideone.com/vwX6mY

Reply

Sahil Arora

March 20, 2016 at 12:20 PM

Powerful array does not need fast IO, if you use cin.tie(NULL);

Reply

Suraj

April 1, 2016 at 7:27 PM

Can you please tell mistake in my add() and remove() function? My Solution
Thank You.

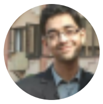
Reply

sunildinday

December 22, 2015 at 2:49 PM

Sir why we take block size always 555

Reply



Karan Kapoor

December 18, 2015 at 11:41 PM

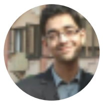
Anudeep i could not understand the update function in Powerful Array

I used the following which gave me TLE :

```
void add(long long int position) {
```

```
cnt[a[position]]++;
answer=answer+cnt[a[position]]*cnt[a[position]]*a[position]-(cnt[a[position]]-1)*(cnt[a[position]]-1)*a[position];
}
```

Reply



Karan Kapoor

December 19, 2015 at 11:53 AM

Understood 😛 It was a small modification 🙂

Reply

Sahil Arora

March 6, 2016 at 10:42 PM

Hey, Look who I found! 😀

Reply

Pratik Singhal

December 13, 2015 at 6:15 AM

We can use a heap with square root decomposition to find the minimum of the range.

Reply

Pandu

October 26, 2015 at 12:16 AM

What should do if sqrt(n) is floating point value?

Sharing is caring!

Reply

abhishekbind2013

October 17, 2015 at 11:42 PM

http://www.spoj.com/problems/ZQUERY/
This problem can be done using Mo’s algorithm.

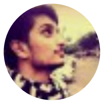
Reply

prakhar

August 20, 2015 at 2:29 PM

very nice approach to MO’s algorithm….i just dont understand why do we get wrong answer if we change the code in DQUERY for the right pointer to
while(current_right right) {
remove(current_right]);
current_right–;
}
and give the initial value of current_right =-1 and current_left=0
its give the correct answer for test cases but wrong on SPOJ

Reply



Akshay Aradhya

July 18, 2015 at 8:36 PM

Hey Anudeep I tried to solve this :

https://www.hackerearth.com/july-clash-15/algorithm/something-genuine Using the above method.

But the only difference in this problem is that I have to calculate the number of unique elements in each sub array and there are N*(N+1)/2 possible sub arrays.How do I solve it ?

Reply

kichu_pari_na

June 12, 2015 at 12:52 PM

another problem on MO’s algorithm:

http://www.spoj.com/problems/ZQUERY/

Reply

Shubham

June 2, 2015 at 3:45 PM

In your solution of DQUERY you fixed the block size according to max no of elements possible. So we do not have to change the block size according to no of elements given in the question every time? Like if n = 10 then also block size remains 555 ?

Reply

free soul

June 2, 2015 at 4:05 AM

no need for fast i/o for dquery.. got acc using your concept just declare remove and add inline functions.

Reply

Shubham Sharma

May 29, 2015 at 4:50 PM

http://ideone.com/mpSL6G
Can you please tell me where am I going wrong ..
I implemented Mo's Algorithm and yet I get a TLE…
Is the implementation correct…
COmpared with your code I found it almost identical.…

Reply

ma5termind

May 10, 2015 at 11:47 AM

hello Anudeep,

problem : powerful array can be solved online with the same complexity O((Q+N)*sqrt(N)). we can discuss it if u want.

Reply

Sharing is caring!

darkshadow

April 24, 2015 at 8:52 PM

Hi, can you explain me how to modify ADD and REMOVE function to solve powerful array problem of codeforces.

Reply

Peeyush Yadav

March 16, 2015 at 7:16 AM

How to find max. element in range. can you please explain the remove part in detail ?
I am not able to understand this “we can use a set to add elements, remove elements and report minimum”

Reply

mukul gupta

March 15, 2015 at 3:00 PM

why we can’t sort on basis of l value instead of block value.please explain????

Reply

Rajat De

March 10, 2015 at 10:52 PM

http://www.spoj.com/problems/FREQUENT/
Sir i solved this problem using segmentree but i think it can be solved with MO’s algorithm too . if

can be solved using MO’s algorithm , can you tell how to do so?

Reply

Abhishek Yadav

March 7, 2015 at 10:42 AM

Thank you for this valuable post

Reply

Rajon Bardhan

February 17, 2015 at 12:29 PM

Link of two problems . I solved the problems by MO’s algorithm .
http://lightoj.com/volume_showproblem.php?problem=1188
http://lightoj.com/volume_showproblem.php?problem=1339

Reply

Himanshu Jaju

February 13, 2015 at 2:56 PM

Hey, the links to the problems are not linked properly I guess.

Reply

Rajon Bardhan

February 4, 2015 at 4:10 PM

Sir ,
Tanks for a nice tutorial !!

Reply

Utkarsh Saxena

January 12, 2015 at 11:50 AM

Another example can be
Number of inversions in the subarray [L,R]
(O(N * Sqrt(N) * log N) algorithm)

Reply

★ anudeep2011

January 22, 2015 at 2:33 AM

Yes, I added a problem from codechef which asks for the same. Thank you

Reply

Amarjeet Singh

January 8, 2015 at 4:02 PM

we get Merge Sort Algorithms code please

Reply

Sanu Kumar Gupta

January 2, 2015 at 2:41 AM

Sharing is caring!

G+

update your solution of DQUERY.there are lot of bugs there.
ie:
range of N is 3*10^4 not 3*105^5
BLOCK should be 174 not 555
here 1<=a[i]<=10^6 not a[i]<=N ..so size of the arrays that you declared are wrong.
BTW thank you for this nice explanation on MOs algorithm.Learnt a lot 🙂

Reply

Deepak

January 2, 2015 at 2:41 AM

This question also uses the same concept, I guess 😉 :
http://codeforces.com/contest/86/problem/D

Reply

★ anudeep2011

January 22, 2015 at 2:26 AM

Thank you.

Reply

Aditya Shah

December 31, 2014 at 9:25 PM

Here are two other problems :

http://www.codechef.com/problems/IITI15

http://codeforces.com/contest/351/problem/D

Reply

★ anudeep2011

January 22, 2015 at 2:26 AM

Thanks for the links.

Reply

cbgiri

December 31, 2014 at 5:11 PM

Sir,
I think there is bug in your slide modification code which run in 0(n*n).

if suppose if have an array of 9 element indexing from 0

1 2 3 1 2 3 1 2 3
and there are
6 queries
query:-
1) L=0 R=8
answer should be 3 but came 2 //wrong answer
2) L=1 R=7
answer should be 1 and also came 1 //right ansv
3) L=2 R=8
answer should be 1 and also came 1 //right ansv

4) L=4 R=7
answer should be 0 but came 1 //wrong answer
5) L=2 R=8
answer should be 1 and also came 1 //right answer
3) L=2 R=7
answer should be 0 but came 1 //wrong answer

Please correct me if i am wrong.

Reply

cbgiri

December 31, 2014 at 5:22 PM

Sir, u wrote it correctly in sample code but in blog u wrote it wrongly . so please update this 🙂 .

Reply

★ anudeep2011

January 1, 2015 at 3:39 PM

I know it, but to keep things simple i did not include the border cases clearly.

Reply

★ cbgiri

January 1, 2015 at 10:55 PM

k sir.
and happy New Year 🙂

Reply

★ anudeep2011

January 22, 2015 at 2:31 AM

Yes, it can be wrong. I really did not look into the boundary conditions and implementation details, scope of this article is to introduce and explain the algorithm and provide enough code so that the reader can implement it on its own.

Reply

Dhruv

December 30, 2014 at 10:08 AM

Hi Anudeep,
Thanks a lot for this blog, after attempting RRJAM in Dec cookoff I had been trying to read about sqrt decomposition, fenwick trees, etc. I also looked at your solution, but couldn't follow it completely. I know its a lot to ask, but if you could explain your RRJAM solution, it would really be helpful 🙂
PS: If you have any relevant resources to learns these related topics, pls do share them too.

Thanks

Reply

Pulkit

December 30, 2014 at 3:03 AM

Hi! Wonderful explanation. 🙂
I have been trying GERALD07 http://www.codechef.com/MARCH14/problems/GERALD07 , but unable to understand . Can you post something for the problem to help more like me facing the same issue.
Thank you 🙂

Reply

red

December 29, 2014 at 5:49 PM

very good and clean explanation thanks Anudeep 🙂

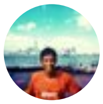
Reply

programmer

December 29, 2014 at 3:58 PM

DQUERY Done 🙂

Reply



Karthik Abinav (@karthikabinav)

December 29, 2014 at 3:56 AM

Out of curiosity, is this name used commonly in the literature? Or is this name used specifically by competitive programming community? I searche online for a reference to MO's algorithm and cou not find it.

Reply

★ **anudeep2011**

December 29, 2014 at 9:58 AM

MO name is used only in china.

Reply

Aditya Kumar Ghosh

December 28, 2014 at 5:38 PM

i just confused with : what are these lines saying ??

How does the final order look like?
All the queries are first ordered in ascending order of their block number (block number is the block in which its opening falls). Ties are ordered in ascending order of their R value.
For example consider following queries and assume we have 3 blocks each of size 3.
{0, 3} {1, 7} {2, 8} {7, 8} {4, 8} {4, 4} {1, 2}
Let us re-order them based on their block number.
{0, 3} {1, 7} {2, 8} {1, 2} {4, 8} {4, 4} {7, 8}
Now let us re-order ties based on their R value.
{1, 2} {0, 3} {1, 7} {2, 8} {4, 4} {4, 8} {7, 8}

Reply

★ **anudeep2011**

December 28, 2014 at 11:35 PM

What exactly you did not understand? Those lines summarize the paras above those and gives an example.

Reply

★ **Sriharsha Sammeta**

December 29, 2014 at 3:32 AM

Hi Anudeep,
Firstly, good initiative with posting algorithms that are not very familiar to most. Please do continue this.
I too didn’t understand how (1,2) came after (2,8) after block wise ordering. Isn’t the division of inputs into blocks is based on L-value And then ties are sorted based on R-value?

Reply

★ **anudeep2011**

December 29, 2014 at 9:57 AM

Initial ordering is block based. As i told the block size is 3, L = 0,1,2 will be in the same block.

Reply

★ **Aditya Kumar Ghosh**

December 29, 2014 at 8:30 PM

how the order has been done means
firstly it has been done on blocks
and then on block numbers and on
the basis of R values
so could you help please

★ **anudeep2011**

December 29, 2014 at 11:47 PM

1) Block number.
2) On ties with block number, then on R value

Aditya Kumar Ghosh

December 28, 2014 at 5:37 PM

Awesome algo !!!
Thnk u for posting sir 🙂

Reply

ma5termind

December 28, 2014 at 3:07 PM

if some one feel need here is link to my solution .. I think i have coded in such a manner that one can understand .. after putting some efforts ..

https://www.hackerrank.com/contests/w12/challenges/white-falcon-and-tree/submissic/code/2445084

Reply

ma5termind

December 28, 2014 at 3:05 PM

Hello Anudeep2011

Add this to the above list . This one is a tough problem which i solved a month ago using sqrt decomposition + heavy light decomposition ..

Reply

ma5termind

December 28, 2014 at 3:08 PM

Problem link : https://www.hackerrank.com/contests/w12/challenges/white-falcon-and-tree

Reply

★ anudeep2011

December 28, 2014 at 3:40 PM

Hi, Thank you.
This post is for MO's algorithm specially, where as the problem you mentioned is sqrt decomposition but different from MO's.

Reply

Leave a Reply

Enter your comment here...