UNIVERSITI TEKNIKAL MALAYSIA MELAKA

FACULTY OF MECHANICAL ENGINEERING (FKM)

TUTORIAL (CHAPTER 1)

1. Complete the table below:

No.	Differential Equation	ODE/PDE	Dependent Variable	Independent Variable
1	$\frac{d^2s}{dt^2} - \sin t = 0$			
2	$\frac{\partial u}{\partial x} = 2 + x\frac{\partial u}{\partial y}$			
3	$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = x^2 + t^2$			
4	$x^2(1+y)\frac{dy}{dx} - (1+x)y^2 = 0$			

2. Determine the order and the linearity of the equations below:

No.	Differential Equation	Order	Linearity	
			Linear	Non-Linear
1	$\left(\frac{dx}{dt}\right)^2 + 2\frac{dx}{dt} = 0$			
2	$\frac{\partial y}{\partial x} + y\tan x = \sin x$			
3	$\frac{\partial^2 f}{\partial x \partial y} + \frac{\partial f}{\partial x} = 0$			
4	$\frac{\partial^3 u}{\partial x^2 \partial y} + x^2 yu = y^2 + x - 1$			
5	$\frac{d^2x}{dt^2} - x\frac{dx}{dt} = 3\cos t$			
6	$3\frac{dx}{dt} + \sin x = 0$			

7	$\frac{ds}{dt}+3t^2s=2t$			
8	$\left(\frac{\partial^3 x}{\partial t^3}\right)^2+\frac{\partial^2 x}{\partial t^2}-6x\cos t=e^t$			
9	$\frac{d^3u}{dx^3}+e^x\frac{du}{dx}-(1+x)u=0$			
10	$\frac{d^2y}{dx^2}+\frac{dy}{dx}+kxy^2=\ln x$			
11	$(z^3+1)\frac{\partial z}{\partial x}-xz^2=x\sinh x$			

3. Determine whether the equations below are separable or not

a) $\frac{dy}{dx}=xe^{y-x}$

ANSWER: separable

b) $x\frac{dy}{dx}=x-2y$

ANSWER: non-separable

c) $\frac{dy}{dx}-xy=x$

ANSWER: separable

4. Solve the given differential equation

a) $(x+2)\frac{dy}{dx}=y$

ANSWER: $y=e^c(x+2)$

b) $\cot y\frac{dy}{dx}=\cot x$

ANSWER: $\sin y=e^c\sin x$

c) $e^x\frac{dy}{dx}=xy^2$

ANSWER: $y=\frac{1}{xe^{-x}+e^{-x}-c}$

d) $\frac{dy}{dx}=\sec^2 y$

ANSWER: $\sin 2y+2y=4x+4c$

5. Solve the initial value problems

a) $x\dfrac{dy}{dx}-3=2\left(y+\dfrac{dy}{dx}\right)\quad;\quad y=0,\ x=3$

ANSWER: $y=\dfrac{3(x-2)^2-3}{2}$

b) $x^2(1-y)\dfrac{dy}{dx}+y^2(1+x)=0\quad;\quad y(1)=1$

ANSWER: $y=xe^{-\frac{1}{x}-\frac{1}{y}+2}$

c) $\dfrac{dy}{dx}+(1+y^2)=0\quad;\quad y(0)=0$

ANSWER: $y=\tan(-x)=-\tan x$

6. By using the given substitution, solve the differential equations

a) $\dfrac{dy}{dx}=\dfrac{x+y+1}{x+y+5}\quad;\quad z=x+y$

ANSWER: $(x+y+3)^2=e^{x-y+c}$

b) $x\dfrac{dy}{dx}+y=2x\sqrt{1-x^2y^2}\quad;\quad z=xy$

ANSWER: $y=\dfrac{\sin(x^2+c)}{x}$

7. Determine whether the equations below are linear or not

a) $x\dfrac{dy}{dx}-2y=x+1$

ANSWER: linear

b) $(1-x^2)\dfrac{dy}{dx}=x(y+\sin^{-1}x)$

ANSWER: linear

c) $\dfrac{dy}{dx}+y^2=e^x$

ANSWER: non-linear

d) $2x^2\dfrac{dy}{dx}+xe^y=\sin x$

ANSWER: non-linear

8. Solve the given differential equation

a) $\frac{dy}{dx} + y = x$

ANSWER: $y = x - 1 + ce^{-x}$

b) $\frac{dy}{dx} + y\tan x = \cos x$

ANSWER: $y = \tan x \ln\sin x + \sin x + c\tan x$

c) $\left(1 - x^2\right)\frac{dy}{dx} - xy = \frac{1}{1 - x^2}$

ANSWER: $y = \frac{x}{1 - x^2} + \frac{c}{\sqrt{1 - x^2}}$

9. Solve the initial value problems

a) $\frac{dy}{dx} - \frac{y}{x} = xe^x \quad ; \quad y(1) = e^1 - 1$

ANSWER: $y = xe^x - x$

b) $\frac{dy}{dx} + \frac{3y}{x} + 2 = 3x \quad ; \quad y(1) = 1$

ANSWER: $y = \frac{3}{5}x^2 - \frac{1}{2}x + \frac{9}{10x^3}$

10. Substitution z into the equation and solve it

a) $x^2 y\frac{dy}{dx} - xy^2 = 1 \quad ; \quad z = y^2$

ANSWER: $y^2 = -\frac{2}{3x} + cx^2$

b) $\frac{dy}{dx} + \frac{\tan y}{x} = \frac{\sec y}{x^2} \quad ; \quad z = \sin y$

ANSWER: $\sin y = \frac{\ln x}{x} + \frac{c}{x}$