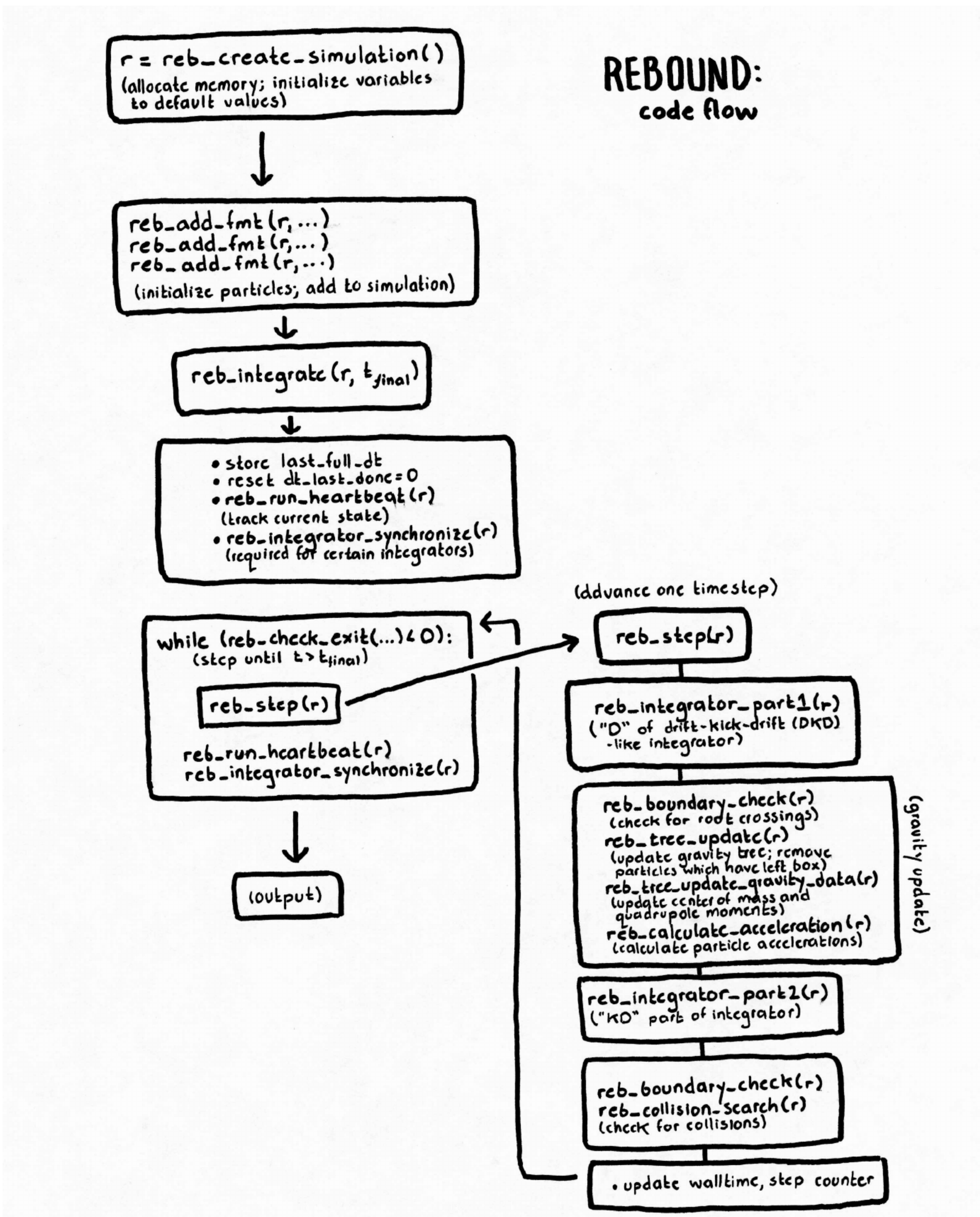
AST 381 Homework 3: N-Body

Nina Filippova

February 25, 2022

1. Let's not be strangers!

(a) *Draw a flowchart for how the code initializes and evolves the interaction of a three-body system.*



(b) *Make a table of the different integration options: list their order of accuracy (if documented), what problems they are best suited for, and note any tunable parameters.*

Integrator	Order	Best suited for:	Tunable parameters

IAS15 (default)	15	non-conservative/velocity-dependent forces
		variational equations
		high-accuracy integration
			`epsilon`
			(integrator accuracy)
			`min_dt`
			(min allowed timestep)
			`epsilon_global`
			(acceleration error)

WHFast	(varies; ≥ 2)	dominant central body
		small orbit perturbations
		variational equations
		(need to set `dt`)
			`corrector`
			(first symplectic corrector)
			`corrector2`
			(second symplectic corrector)
			`kernel`
			(integrator kernel)
			`coordinates`
			(coordinate system)
			`recalculate_coordinates`
			(from particle structure)
			`safe_mode`
			(synchronize every timestep)
			`keep_unsynchronized`

BS	(not given)	arbitrary ODEs
		short integration (medium accuracy)
		close encounters (adaptive timestep)
			`eps_rel`
			(relative tolerance)
			`eps_abs`
			(absolute tolerance)
			`min_dt`
			(min allowed timestep)
			`max_dt`
			(max allowed timestep)

Mercurius	(varies)	hybrid (complex planetary dynamics)
		(WHFast for long-term integrations;
		IAS15 for close encounters)
			switching function
			(between IAS15 and WHFast)
			`hillfac`
			(particle switchover radii)
			`recalculate_coordinates`
			(from particle structure)
			`recalculate_dcrit`
			(switchover distance)
			`safe_mode`
			(synchronize after every timestep)

SABA	(varies)	no close encounters
		long-time evolution
			`type`
			(symplectic integrator type)
			`safe_mode`
			(synchronize after every timestep)
			`keep_unsynchronized`

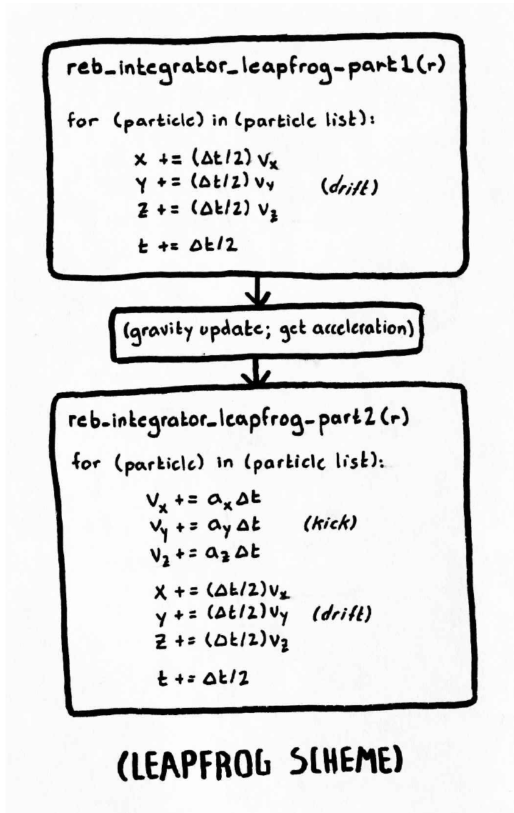
JANUS	even
	(between 2 and 10)
		exact time-reversal symmetry	`scale_pos`
			(scale positions)
			`scale_vel`
			(scale velocities)
			`order`
			(integration scheme order)
			`recalculate_coordinates`
			(integer coordinates from float)

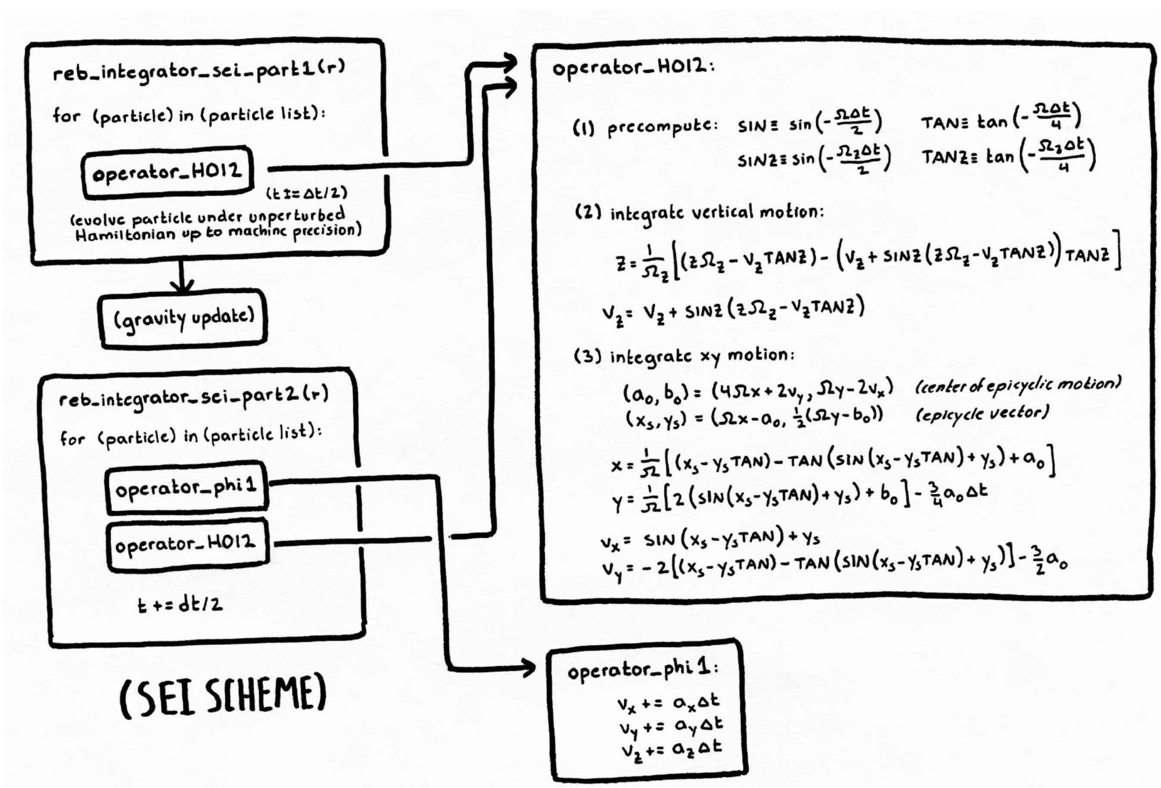
EOS	(varies)	simple algorithm (no Kepler solver)	`phi0`
			(outer operator splitting scheme)
			`phi1`
			(inner operator splitting scheme)
			`n`
			(number of sub-timesteps)
			`safe_mode`

Leapfrog	2	fast, low-accuracy	(none)

SEI	2	dominant central body (rotating frame)
		shear-periodic boundary conditions
			`OMEGA`
			(epicyclic/orbital frequency)
			`OMEGAZ`
			(vertical epicyclic frequency)

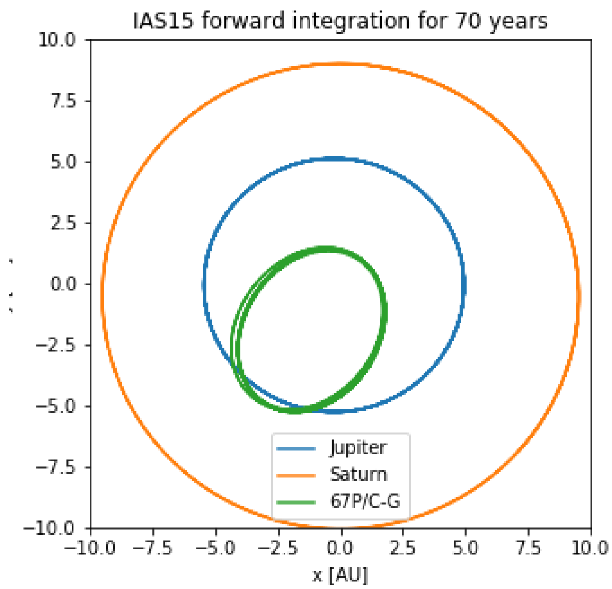
(c) *Look at the leapfrog and symplectic epicyclic integrator code. Make a flow chart for how these functions work.*





2. Hello Comets

(a) *Set up and run the 67P/C-G example problem following the documentation. Include a plot of the 67P/C-G comet and two planet orbit evolved over 70 years in your solution set.*



(b) *Experiment with the timestep and integrator choice. How does the result change with different integrators? What integrator(s) do you think work best for this problem? How does the result change as a function of timestep? E.g., you can evaluate this by looking at energy conservation and the change in orbits.*

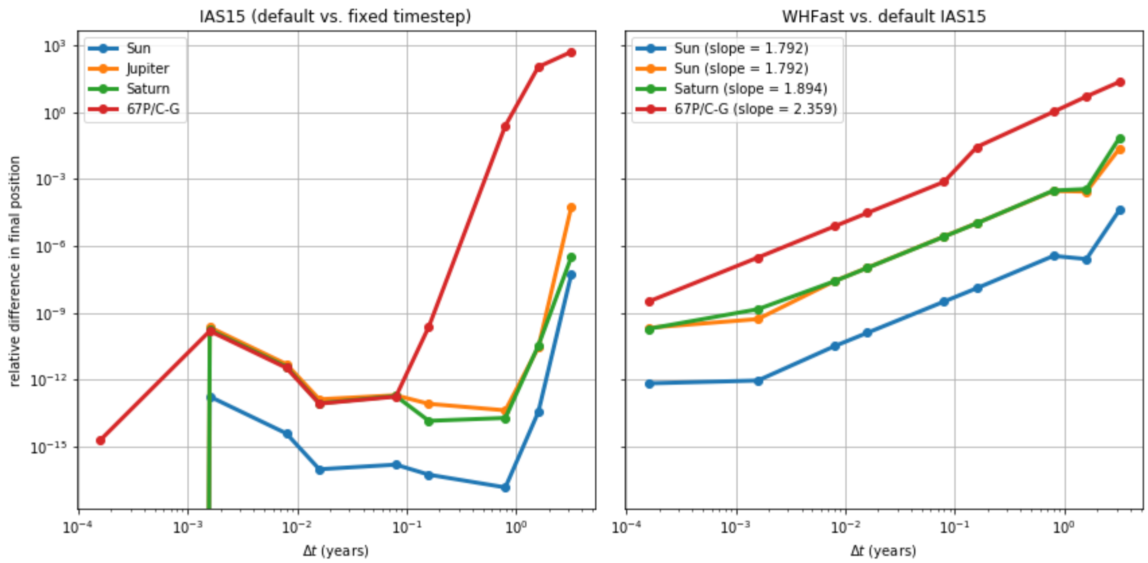


Figure 1: Comparison of: fixed-Δt timestep IAS15 versus default adaptive timestepping IAS15 (left plot); and fixed-Δt timestep WHFast versus default adaptive timestepping IAS15 (right plot), by comparing relative difference in final particle positions.

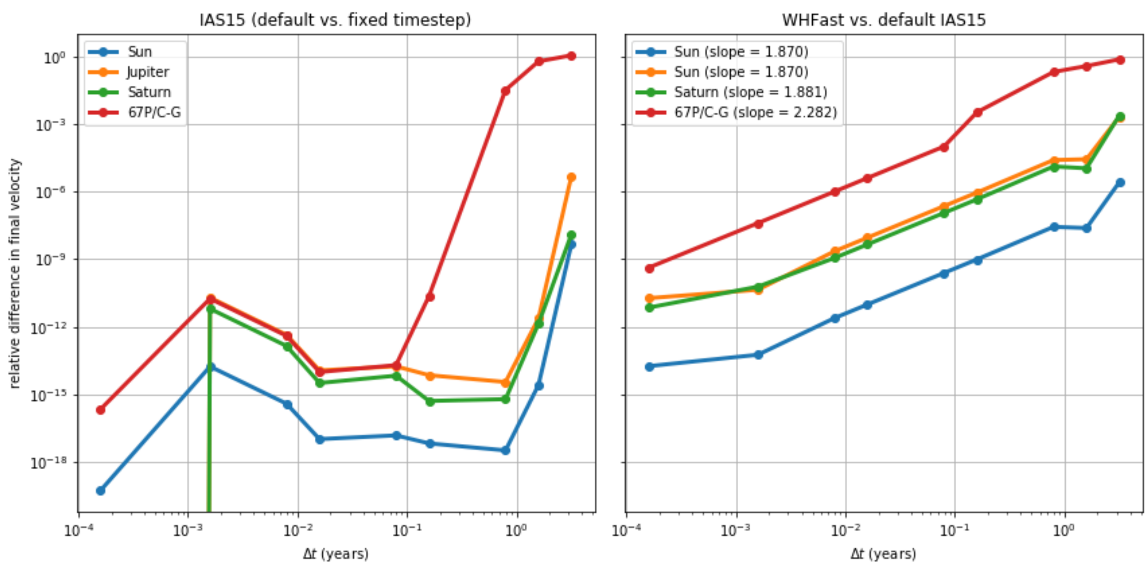


Figure 2: Comparison of: fixed-Δt timestep IAS15 versus default adaptive timestepping IAS15 (left plot); and fixed-Δt timestep WHFast versus default adaptive timestepping IAS15 (right plot), by comparing relative difference in final particle positions.

IAS15 with fixed timestep size does not appear linear in order. WHFast appears to be roughly second-order in Δt. Since the Solar System has the Sun as a dominant central body, WHFast should be an acceptable option; however, IAS15 is much higher-order and is probably the default choice for good reasons. With fixed timestep size, IAS15 shows strong improvement as the size step initially decreases; the interesting behavior at small Δt might be due to machine precision limits.a

(c) *How does the orbit of 67P/C-G change when you add the rest of the Solar System planets? Which is the largest source of error - the parameter choices such as the integrator and/or timestep, or the missing bodies?*

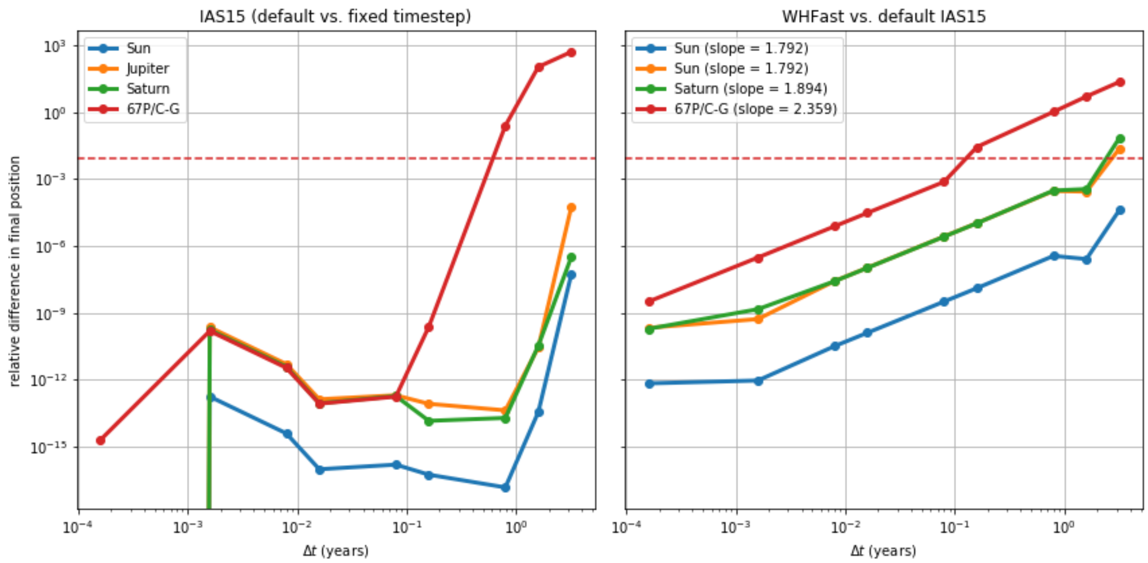


Figure 3: Same as Figure 1, but with a dashed horizontal line showing the relative difference in 67P/C-G's position after adding the other Solar System planets and integrating forward for 70 years using the default IAS15 integrator.

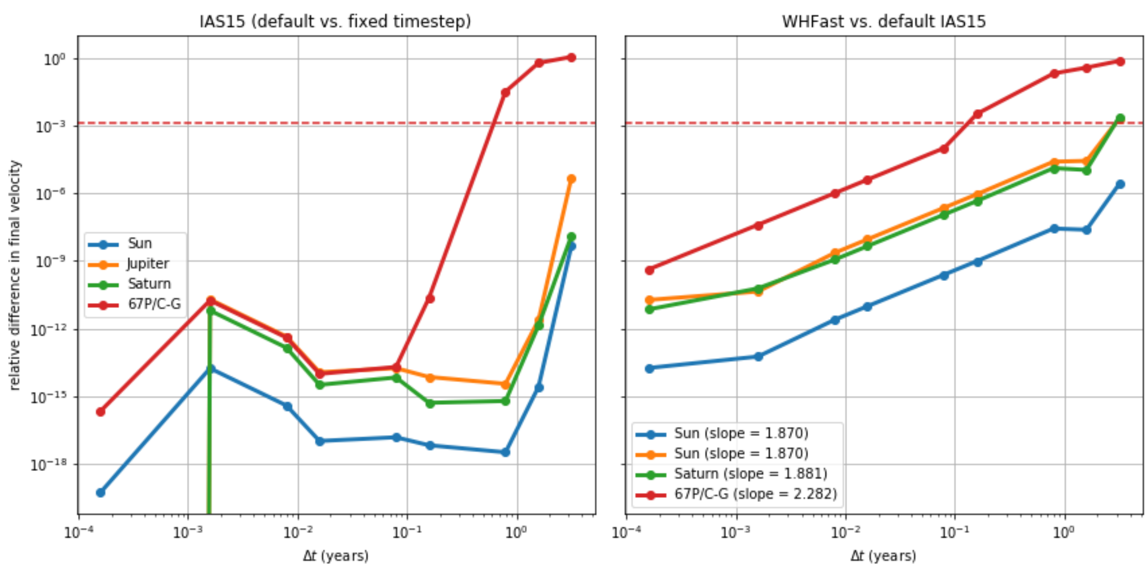


Figure 4: Same as Figure 2, but with a dashed horizontal line showing the relative difference in 67P/C-G's velocity after adding the other Solar System planets and integrating forward for 70 years using the default IAS15 integrator.

The influence of the other Solar System planets on the orbit of 67P/C-G quickly becomes more important than the difference due to integrator choice or timestep size even for relatively large timestep choices.

(d) *If you completed part 3a in HW 1, compare the Rebound Sun/Earth orbit with your result from HW1. What do you notice?*

The Rebound Sun/Earth simulation gives a relative difference of 1.415×10^{-4} in the radial position of Earth after one orbit. First-order Euler does not achieve this even with a stepsize of $\Delta t \sim 10^{-5}$ years; fourth-order Runge-Kutta achieves this with a stepsize between $\Delta t \sim 10^{-1} - 10^{-2}$ years.

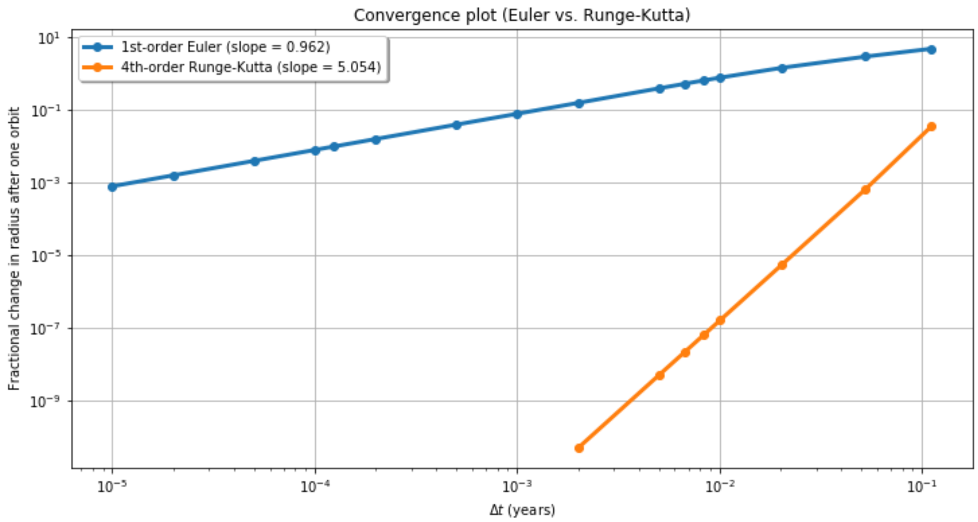


Figure 5: Convergence plots from HW 1.

3. Finding Planet X (actually Planet Nine) *There is a long history of speculation that another undiscovered planet might exist outside our Solar system. Originally this planet was known as "Planet X." A more recent 2016 study has hypothesized anew about the presence of a large planet beyond the Kuiper belt. Set up a Solar system with all the planets and Pluto. Suppose Planet Nine has a semi-major axis of 460 AU and eccentricity of 0.4. How massive would it need to be to perturb Pluto's orbit by 10 percent after 10 years of evolution? Given your answer, why do you think we haven't found Planet Nine yet?*

Testing different masses for Planet Nine, the semimajor axis of Pluto's orbit changes by more than 10% when $m = 52\ M_{\odot}$. This is extremely massive, and does not seem likely.