Homework 3

You are allowed to work with a partner on this and all subsequent homework assignments. When working with a partner, you are expected to work as a team on all problems, not to divide up the problems. The free version of Overleaf supports collaborative editing in teams of two. Only one of you needs to submit on Gradescope, but that person needs to add the other partner to the group, as shown at the link below.

https://help.gradescope.com/article/m5qz2xsnjy-student-add-group-members

1. Prove by induction that the number of leaves in a binary tree is always one more than the number of nodes that have two children.

2. You are given an unsorted array $A[1..n]$ of n elements where $n = 3k$ for some positive integer k. Describe a linear-time deterministic algorithm that finds all the elements in the array which occur at least k times. Explain why your algorithm is correct.

3. Consider an $n \times n$ *grid graph* $G = (V, E)$, where n is a power of 2. That is, V is the set of all ordered pairs (x, y) where $x, y \in \{1, \ldots, n\}$, and E consists of horizontal edges $\{(x, y), (x+1, y)\}$ for all $1 \le x < n$ and vertical edges $\{(x, y), (x, y+1)\}$ for all $1 \le y < n$. Each vertex (x, y) has an integer *elevation* $e(x, y)$, and (x, y) is a *local minimum* if $e(x, y) \le e(x', y')$ for all neighbors (x', y') of (x, y).

   Suppose that you want to find a local minimum by taking a walk on this graph. You start at vertex $(1, 1)$, and in each round you can either move to a neighboring vertex or query the elevation of your current vertex. Design an algorithm that will always complete this task in $O(n)$ rounds, and explain why your algorithm is correct.