1 QuickSort

See the source code file `quickSort.cpp` and the tests given in `tests.cpp`.

2 Big-O Proofs

Problem 1. Show that $5n^3 + n^2 + 4$ is $O(n^3)$.

$$\exists\ c = 10,\ n_0 = 1 \text{ such that } \forall\ n \geq n_0,\ f(n) \leq c(n^3).$$

We know that the following inequalities hold:

$5n^3 \leq 5n^3$,

$n^2 \leq n^3$,

$4 \leq 4n^3$.

Adding up the values on the left-hand side and the right-hand side of the above inequalities, we get the following inequality:

$5n^3 + n^2 + 4 \leq 5n^3 + n^3 + 4n^3$.

Hence, $5n^3 + n^2 + 4 \leq 10n^3$, and we have proved that $5n^3 + n^2 + 4$ is $O(n^3)$.

Problem 2. Show that $2n^4 - 3n^2 + n$ is $O(n^4)$.

$$\exists\ c = 6,\ n_0 = 1 \text{ such that } \forall\ n \geq n_0,\ f(n) \leq c(n^4).$$

We know that the following inequalities hold:

$2n^4 \leq 2n^4$,

$-3n^2 \leq 3n^4$,

$n \leq n^4$.

Adding up the values on the left-hand side and the right-hand side of the above inequalities, we get the following inequality:
$2n^4 - 3n^2 + n \leq 2n^4 + 3n^4 + n^4$.

Hence, $2n^4 - 3n^2 + n \leq 6n^4$, and we have proved that $2n^4 - 3n^2 + n$ is $O(n^4)$.

3 Mystery Functions

Function A is $O(n)$ because the for loop goes through $\frac{n}{2}$ times.

Function B is $O(n^2)$ because it is a nested for loop that has a runtime of $n * n$, which is n^2.

Function C is $O(n * \log n)$ because the for loop outside goes n times, and the while loop's condition means that we go through $\log_2 n$ times.

Function D is $O(n^4)$ because it is a nested for loop that has a runtime of $n^2 * n^2$, which is n^4.

Function E is $O(4n)$ because it is a nested for loop that has a runtime of $n * 4$.

Function F is $O(n^3)$ because the for loop goes through n^3 times.

- ***The mystery functions A-E in sorted order from fastest to slowest are***:

  A E C B F D

- ***Overall Matching Results***:

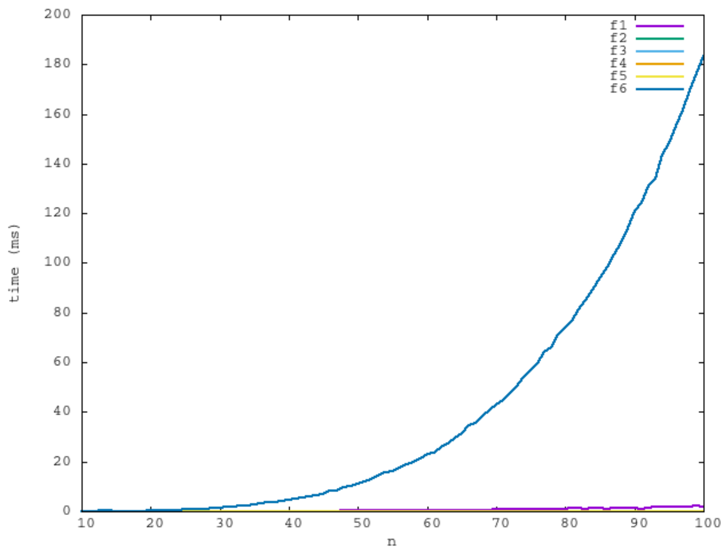
  A matches 3, B matches 5, C matches 4, D matches 6, E matches 2, F matches 1.

- ***Explanation***:

  fnD matches mystery function 6.

  Reason: when we first graph all 6 functions altogether on one graph within $10 \leq n \leq 100$, it is obvious that function 6 is the slowest. And according to our sorted order above, function D is the slowest, so these two match.

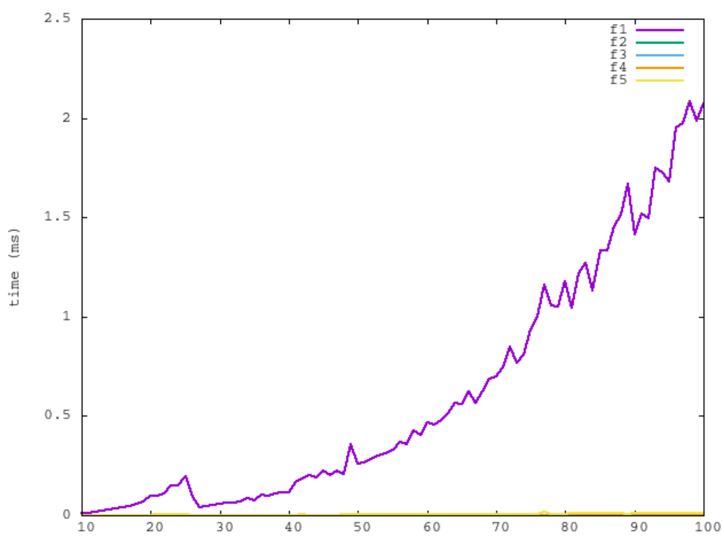
  Graph to support the claim:



fnF matches mystery function 1.

Reason: now we know that function 6 corresponds to fnD, so we can do the comparison between function 1-5. If we graph these 5 functions altogether on one graph within $10 \leq n \leq 100$, it is obvious that function 1 is the slowest amongst the five functions, which corresponds to function F.

Graph to support the claim:



fnB matches mystery function 5.

Reason: now we are left with functions 2-5. Putting these 4 functions altogether on one graph within $200 \leq n \leq 500$, it is obvious that function 5 is the slowest amongst these four, and hence function 5 corresponds to function B according to the sorting order above.

fnC matches mystery function 4.

Reason: since function 1, 5, 6 have already found the correct matches. We are then left with functions 2, 3, 4. Graphing these altogether on one graph within $800 \leq n \leq 1000$, it is obvious that function 4 is the slowest, which correponds to function C according to the sorting order above.

fnE matches mystery function 2.

Reason: now we are only left with function 2 and function 3. Graphing these two altogether within $1500 \leq n \leq 3000$, we see that function 2 is slower compared to function 3, so 3 corresponds to function E.

fnA matches mystery function 3.

Reason: we are left with only one option - function 3, which is the fastest and hence corresponds to function A.