Homework 11

CS41, Spring 2023 — Due Sunday, April 23

1. Let SIMPLE be the problem of determining, given an unweighted undirected graph G and a positive integer k, whether there is a simple path of length $\geq k$ in G. Prove that this problem is NP-complete.

2. Let's say that a set S of vertices in an unweighted undirected graph is *4-spaced* if all vertices in the set are at distance at least 4 from each other. That is, for all $u, v \in S$ with $u \neq v$, we have $d(u, v) \geq 4$.

   Let 4SPACE be the problem of determining, given an undirected graph G with positive integer vertex weights and a positive integer k, whether there is a 4-spaced set of vertices in G with total weight $\geq k$. Prove that this problem is NP-complete.

3. Let BIG-VC be a restricted version of the vertex cover problem where k is required to be $\geq |V|/5$. More formally, given an undirected graph $G = (V, E)$ and a positive integer k, BIG-VC(G, k) is true if and only if $k \geq |V|/5$ and there is some subset $S \subseteq V$ such that every edge in E includes at least one vertex from S. Prove that this problem is NP-complete.