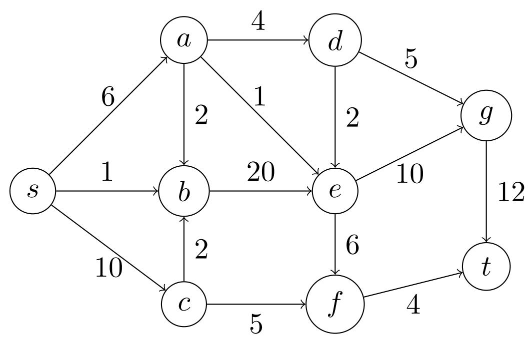
Homework 8

CS41, Spring 2023 — Due Sunday, April 2

1. Suppose you have a connected undirected graph $G = (V, E)$ with weighted edges and a designated subset $U \subseteq V$, and you want to find a spanning tree that is as light as possible under the constraint that the vertices of U must all be **leaves** of this spanning tree. Intuitively, you could think of this as building the cheapest possible rail network in which certain stations are required to be terminals. Design a $O(|E| \log |V|)$-time algorithm for this problem.

2. Find a maximum flow for the flow network below. You may express your flow in a picture or just by listing the flow on each edge.



3. Suppose that G is some flow network and that (L, R) is a min cut (i.e., an (s, t) cut of minimum capacity) in G. Prove or disprove: If we add 1 to the capacity of every edge in G, then (L, R) will still be a min cut.

4. Prove or disprove: In any flow network, the number of times the Ford-Fulkerson algorithm can augment the flow is at most the number of distinct s–t paths.