Homework 12

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1. To solve the house building problem form HW 9 using MIP, we create a binary variable $x_{i,j}$ for every possible site at any one (i, j) spot on the n x n grid. We set the constraint for rows for all i is $\sum_{j=1}^{n} x_{i,j} \leq 1$ and the constraint for each column for all j is $\sum_{i=1}^{n} x_{i,j} \leq 1$ to ensure that every house built will have an unobstructed view. For all (i, j) where the site is unsuitable, we add the constraint $x_{i,j} \leq 0$. Our objective function is to maximize the number of houses using $\textsc{max} \sum_{v \in V} x_{i,j}$.

2. To solve CLIQUE-AND-IND from Lab 11 using MIP, we create a binary variable x_v for every vertex $v \in V$ to determine if any one vertex $v \in V$ is of the independent set. For all $u, v \in E$ where $u \neq v$, we set the constraint $y_u + y_v \leq 1$. We also create a binary variable y_v for every vertex $v \in V$ to determine if any v is part of a clique. For all $u, v \notin E$ where $u \neq v$, we set the constraint $x_u + x_v \leq 1$. We also set the constraint that $\sum_{v \in V} x_v \geq k$ to ensure that all we have a clique of size at least k. We set our objective function as $\textsc{max} \sum_{v \in V} y_v \geq k$ to ensure that our independent set of also of size at least k.

3. To solve DOUBLE-SAT from Lab 11 using MIP, we create the binary variables t_i and $f_i \in \{0, 1\}$ for each literal in the statement to keep track of one true/false assignment that satisfies the statement. We also create the variables t'_i and $f'_i \in \{0, 1\}$ for each literal in the statement that keeps track of another true/false assignment that satisfies the statement. We set the constraints $t_i + f_i \leq 1$ and $t'_i + f'_i \leq 1$ to ensure that each literal is only given one assignment of true or false.

   Then, we create the binary variable $d_i \in \{0, 1\}$ that checks if each literal i has a different value in both assignments. We set the constraint that $\sum d_i \geq 1$ to ensure there is least one different between the assignments. Let $g(\ell)$ and $g'(\ell)$ be a function that takes a literal ℓ such that

$$g(\ell) = \begin{cases} t_i & \ell = i \\ f_i & \ell = \bar{i} \end{cases} \qquad \text{and} \qquad g'(\ell) = \begin{cases} t'_i & \ell = i \\ f'_i & \ell = \bar{i} \end{cases}$$

   For each literal ℓ in each clause c_j in the statement, we set the constraint $\sum_{\ell \in c_j} g(\ell) \geq 1$ and $\sum_{\ell \in c_j} g'(\ell) \geq 1$. We will set a trivial objective function of maximizing 0 with target 0.

4. To solve 4SPACE from HW 11 using MIP, we create a binary variable $x_i = \{0, 1\}$ for each vertex $v \in V$. Let d be a function that determines the distance between any two vertices $u, v \in V$ where $u \neq v$. We define S as the set of all (u, v) pairs where $d(u, v) \geq 4$. We set the constraint $x_u + x_v \leq 1$ where $u, v \notin S$ and $u \neq v$ and set our objective function as $\textsc{max} \sum_{v \in V} x_i \geq k$.