Homework 7

CS41, Spring 2023 — Due Sunday, March 26

1. Consider the alphabet $\Sigma = \{\texttt{a}, \texttt{b}, \texttt{c}, \texttt{d}, \texttt{e}, \texttt{f}\}$. Find values for $p_{\texttt{a}}$, $p_{\texttt{b}}$, $p_{\texttt{c}}$, $p_{\texttt{d}}$, $p_{\texttt{e}}$, and $p_{\texttt{f}}$ such that the Huffman code is unique—meaning that there are never any key "ties" when ExtractMin is performed—and the codeword for `c` is `00001`. List the codewords for all six characters.

2. Given an undirected connected graph $G = (V, E)$ (in adjacency list form) and a strictly positive weight function $w : E \to (0, \infty)$, you want to find the minimum total weight of a set $F \subseteq E$ such that the graph $(V, E \setminus F)$—that is, G with the edges in F removed—is acyclic. Give a greedy algorithm that solves this problem in $O(|E| \log |V|)$ time. Briefly explain your algorithm and its running time.

3. Prove or give a counterexample for each of the following statements. In each one, $G = (V, E)$ is a weighted, connected, undirected graph.

   (a) If $|E| \geq |V|$ and there is a unique heaviest edge, then there is an MST that does not contain that edge.

   (b) If u and v are two vertices and e is an edge on the unique shortest u–v path, then e is part of some MST.

   (c) If there is some edge e that is the unique lightest edge in some cycle in G, then e is part of every minimum spanning tree.

   (d) If there is some edge e has the (not necessarily unique) minimum weight in G, then e is part of some minimum spanning tree.