
Homework 1

CS41, Spring 2023

Due Sunday, January 29

Complete this assignment using the homework LaTeX template on the course web page, and submit a PDF on Gradescope. This is a solo assignment, meaning that you should write your own solutions, without reference to anyone else's written materials, online or in person. Never search online for the solution to a homework problem.

1. For each of the following pairs of functions, indicate whether $f = o(g)$, $f = \omega(g)$, $f = \Theta(g)$, or none of these hold. You don't need to prove your answers.

$f(n)$	$g(n)$
(a) $n - 100$	(a) $n - 200$
(b) $n^{1/2}$	(b) $n^{2/3}$
(c) $100n + \log n$	(c) $n + (\log n)^2$
(d) $n \log n$	(d) $10n \log(10n)$
(e) $\log(2n)$	(e) $\log(3n)$
(f) $10 \log n$	(f) $\log(n^2)$
(g) $n2^n$	(g) 3^n
(h) 2^n	(h) 2^{n+1}
(i) $n^2 / \log n$	(i) $n \cdot (\log n)^2$
(j) $n^{1.01}$	(j) $n \cdot (\log n)^2$
(k) $n^{0.1}$	(k) $(\log n)^{10}$
(l) \sqrt{n}	(l) $(\log n)^3$
(m) $n^{1/2}$	(m) $5^{\log n}$
(n) $n^{1+(-1)^n}$	(n) n
(o) $n!$	(o) 2^n
(p) $(\log n)^{\log n}$	(p) $2^{(\log n)^2}$

2. For each of the following statements, either prove (using the formal definition of big O) that it is true for all functions f , g , and h from positive integers to positive reals, or give a counterexample.

- (a) If $f = O(g)$ and $g = O(h)$, then $f = O(h)$.
- (b) If $f = O(h)$ and $g = O(h)$, then $f + g = O(h)$.
- (c) If $f = O(h)$ and $g = O(h)$, then $f \cdot g = O(h)$.
- (d) If $f(n) = O(g(n))$, then $\log f(n) = O(\log g(n))$.
- (e) If $f(n) = O(g(n))$, then $2^{f(n)} \neq O(2^{g(n)})$.
- (f) $f(n) = O(f(n/2))$.