Practice Problems

Thursday, 23 February 2023 12:46 PM

Q1. $T(n) = 8T(\frac{n}{2}) + n^2$

Sol^n $T(\frac{n}{2}) = 8T(\frac{n}{4}) + \frac{n^2}{4}$

$T(\frac{n}{4}) = 8T(\frac{n}{8}) + \frac{n^2}{16}$

$\therefore T(n) = 8(8(8T(\frac{n}{8}) + \frac{n^2}{16}) + \frac{n^2}{4}) + n^2$

$= 8^k T(\frac{n}{2^k}) + n^2 + 2n^2 + 4n^2 + \dots$

$n = 2^k \quad \therefore k = \log_2 n$

$= 8^{\log_2 n} T(1) + n^2(2^k - 1)$

$= n^3 T(1) + n^2 \cdot n + n^2$

$= O(n^3)$ //

Q2. $T(n) = 2T(\frac{n}{2}) + n\log n$

Ans. $T(\frac{n}{2}) = 2T(\frac{n}{4}) + \frac{n}{2}\log\frac{n}{2}$

$T(\frac{n}{4}) = 2T(\frac{n}{8}) + \frac{n}{4}\log\frac{n}{4}$

$\therefore T(n) = 2(2(2T(\frac{n}{8}) + \frac{n}{4}\log\frac{n}{4}) + \frac{n}{2}\log\frac{n}{2}) + n\log n$

$= 2^k T(\frac{n}{2^k}) + n[\log n + \log\frac{n}{2} + \dots]$

$n = 2^k \quad \therefore k = \log n$

$= nT(1) + n[\log n \cdot \frac{n}{2} \cdot \frac{n}{4} \cdot \dots 4 \cdot 2]$

$= n + n(\log 2 + \log 4 + \log 8 \dots)$

$= n + n(1 + 2 + 3 + \dots + \log n)$

$= n + n\frac{(\log n)(\log n + 1)}{2}$

$\Rightarrow O(n(\log n)^2)$ //

Q3. $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}$

$\therefore T(n) = 2(2(2T(\frac{n}{2^3}) + \frac{n}{4\log\frac{n}{4}}) + \frac{n}{2\log\frac{n}{2}}) + \frac{n}{\log n}$

$= 2^k T(\frac{n}{2^k}) + n(\frac{1}{\log n} + \frac{1}{\log\frac{n}{2}} + \dots)$

$n = 2^k$

$= nT(1) + n(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{\log n})$

$= n + n\log(\log n)$ ← $1 + \frac{1}{2} + \frac{1}{3} \dots$

$\therefore T(n) = O(n\log(\log n))$ //

Q4. $T(n) = 2T(\sqrt{n}) + C$

$T(\sqrt{n}) = 2T(n^{1/4}) + C$

$T(n^{1/4}) = 2T(n^{1/8}) + C$

$\therefore T(n) = 2(2(2T(n^{1/8}))) + C + 2C + 4C$

$= 2^k T(n^{1/2^k}) + 2^k - 1$

$n^{1/2^k} = C \qquad \therefore \log n = 2^k$

$\therefore \frac{1}{2^k}\log n = C \qquad \therefore \log(\log n) = k$.

$\therefore T(n) = \log n \cdot 1 + \log n - 1$

$\therefore T(n) = O(\log n)$ //

Q5. $T(n) = 2T(\sqrt{n}) + \log n$

$T(n^{1/2}) = 2T(n^{1/4}) + \log n^{1/2}$

$T(n^{1/4}) = 2T(n^{1/8}) + \log n^{1/4}$

$\therefore T(n) = 2(2(2T(n^{1/2^k}) + \frac{1}{4}\log n) + \frac{1}{2}\log n) + \log n$

$= 2^k T(n^{1/2^k}) + k \cdot \log n$

$n^{1/2^k} = 1$

$\therefore k = \log(\log n)$

$\therefore T(n) = \log n \cdot T(1) + \log(\log n) \cdot \log n$

$\therefore T(n) = O(\log n \cdot \log(\log n))$ //

Q6. $T(n) = \sqrt{2}\,T(\frac{n}{2}) + \sqrt{n}$

$T(\frac{n}{2}) = \sqrt{2}\,T(\frac{n}{4}) + \sqrt{\frac{n}{2}}$

$T(\frac{n}{4}) = \sqrt{2}(T(\frac{n}{8})) + \sqrt{\frac{n}{4}}$

$\therefore T(n) = \sqrt{2}(\sqrt{2}(\sqrt{2}T(\frac{n}{2^k}) + \sqrt{\frac{n}{4}}) + \sqrt{\frac{n}{2}}) + \sqrt{n}$

$= (\sqrt{2})^k T(\frac{n}{2^k}) + \sqrt{n} \cdot k$

$2^k = n \quad \therefore k = \log n$

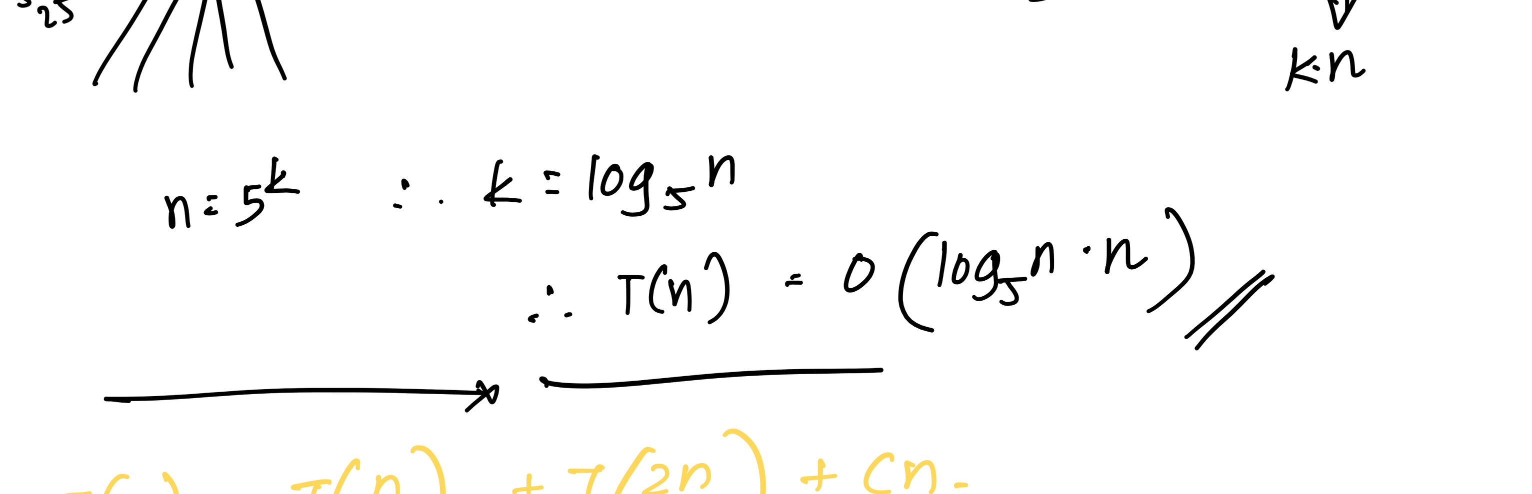
$= (\sqrt{2})^{\log n} T(1) + \sqrt{n} \cdot \log n$

$= \sqrt{n} + \sqrt{n}\log n$

$\therefore T(n) = O(\sqrt{n}\log n)$ //

★ Recursive Tree Method

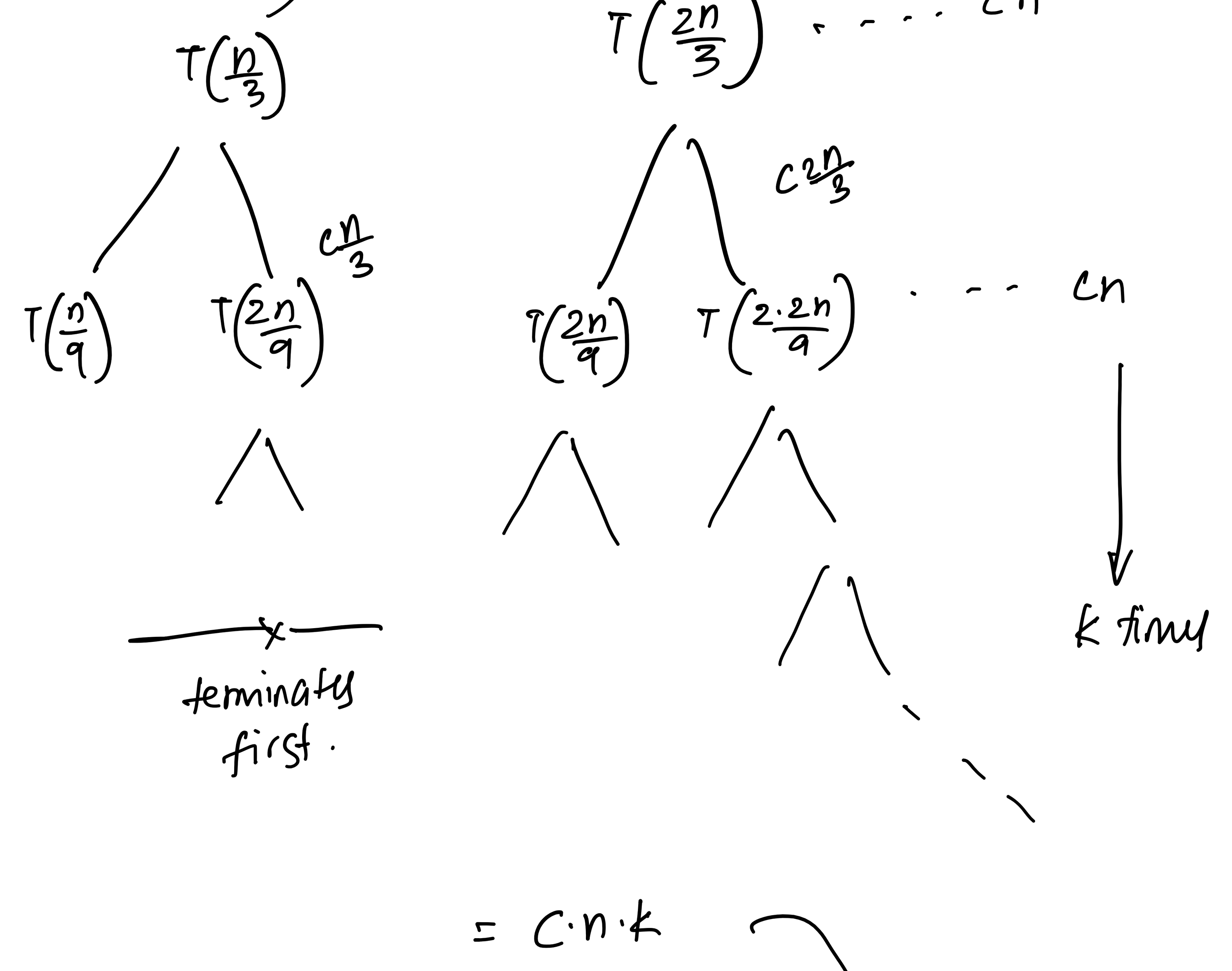
Q1. $T(n) = 5T(\frac{n}{5}) + n$



$n = 5^k \quad \therefore k = \log_5 n$

$\therefore T(n) = O(\log_5 n \cdot n)$ //

Q2. $T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + cn$.



$= c \cdot n \cdot k$ → $= O(\log_{3/2} n \times n)$ //

$\frac{n}{(3/2)^k} = 1$

$\therefore k = \log_{3/2} n$

Q3. $T(n) = T(\frac{n}{10}) + T(\frac{9n}{10}) + cn$

Similar as above

$= O(n\log_{10/9} n)$ //