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DAA Tutorial - 2

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Q.1) Solve the recursion relation

a] $T(n) = 2T\left(\frac{n}{2}\right) + n^2$

Ans. By Master's Theorem

$T(n) = aT\left(\frac{n}{b}\right) + \Theta(n^k \log^p n)$

$\therefore$ $a = 2$, $b = 2$
$k = 2$, $p = 1$

Here $a < b^k \Leftarrow 2 < 2^2$

This is case 3:
i) $p \geq 0$

$\therefore$ Complexity $= \Theta(n^2)$

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b] $T(n) = 2T(n-1) + 1$

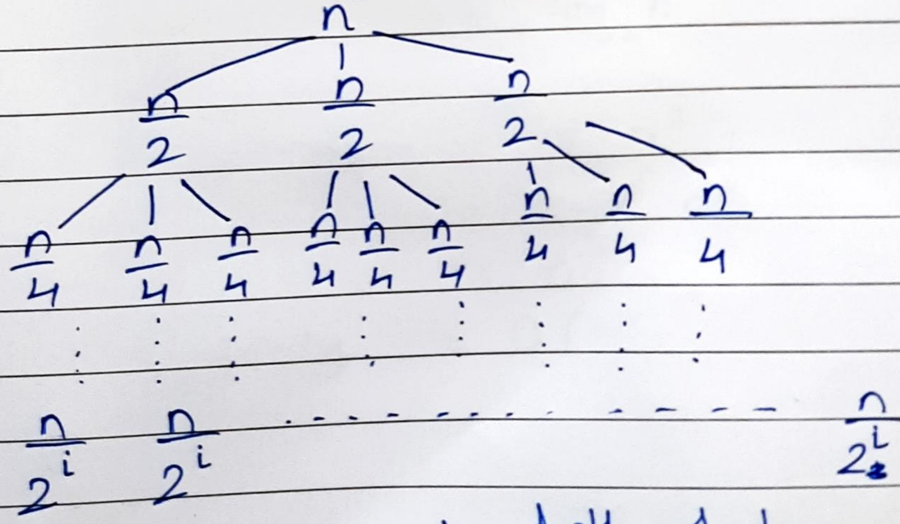
$\therefore T(n) = 2(2T(n-2) + 1) + 1$
$= 4T(n-2) + 2 + 1$
$= 4(2T(n-3) + 1) + 2 + 1$
$= 8T(n-3) + 4 + 2 + 1$
$\vdots$

$\therefore T(n) = 2^3 T(n-3) + 2^3 - 1$

$\therefore T(n) = 2^k T(n-k) + 2^k - 1$

$\therefore$ complexity $= O(2^n)$

c] $T(n) = 3T\left(\frac{n}{2}\right) + n$



i = depth of tree

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Depth of tree

$$\frac{n}{2^i} = 1$$

$$\therefore 2^i = n$$

$$\therefore i = \log_2 n$$

No. of leaves $= 3^i$

$$= 3^{\log_2 n}$$

$$= n^{\log_2 3}$$

Total cost $= n \log_2 n$

$\therefore$ Complexity $= \theta(n \log n)$

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d] $T(n) = 8T\left(\frac{n}{2}\right) + n^2$

$$n^2$$

Level 1 $\left(\frac{n}{2}\right)^2 \quad \left(\frac{n}{2}\right)^2 \quad \left(\frac{n}{2}\right)^2 \quad \left(\frac{n}{2}\right)^2 \quad \left(\frac{n}{2}\right)^2 \quad \left(\frac{n}{2}\right)^2 \quad \left(\frac{n}{2}\right)^2 \quad \left(\frac{n}{2}\right)^2$

$$= \frac{8n^2}{4} = 2n^2$$

Level 2 $\left(\frac{n}{4}\right)^2 \quad \cdots\cdots \quad \left(\frac{n}{4}\right)^2 \quad = 64\frac{n^2}{\cancel{8}16} = 4n^2$

$\vdots \qquad\qquad \vdots$

$\left(\frac{n}{2^i}\right)^2 \quad \cdots\cdots \quad \left(\frac{n}{2i}\right)^2 \quad = 2^i n^2$

No. of levels : $\frac{n}{2^i} = 1$

$\therefore n = 2^i$

$\therefore i = \log_2 n$

$\therefore$ Final leaves $= 2^i n^2$

$= 2^{\log_2 n} n^2$

$= n^3$

$\therefore$ Complexity $= O(n^3)$

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Q.2) True or False

$f(n) = 2^{\log n}$ $g(n) = n^{\log n}$ $\log n \log 2 \mid \log n \log n$ $\therefore \log n \log 2 < \log n \log n$ $\therefore \log 2 < \log n$ $\therefore f(n) < g(n)$	$f(n) = n^{\sqrt{2} \log n}$ $g(n) = 2^{\sqrt{n}}$ $\sqrt{2} \log n . \log n \mid \sqrt{n} \log 2$ $\sqrt{2} (\log n)^2 \mid \sqrt{n} . 1$ $\therefore f(n) < g(n)$
$\frac{4^n}{2^n} \mid \Theta(2^n)$ $\frac{4^n}{2^n} = \frac{2^{2n}}{2^n}$ $= 2^{2n-n}$ $= 2^n = \Theta(2^n)$	$n^2 2^{3 \log n} \mid \Theta(n^5)$ $n^2 2^{3 \log n} = n^2 2^{\log n^3}$ $= n^2 . n^3$ $= n^5$ $= \Theta(n^5)$

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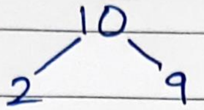
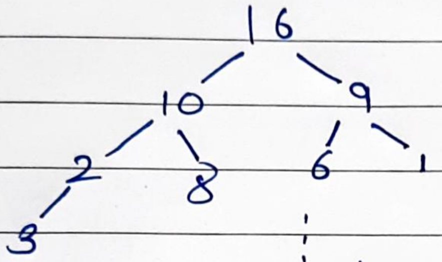
Q.3)



heapify



heapify