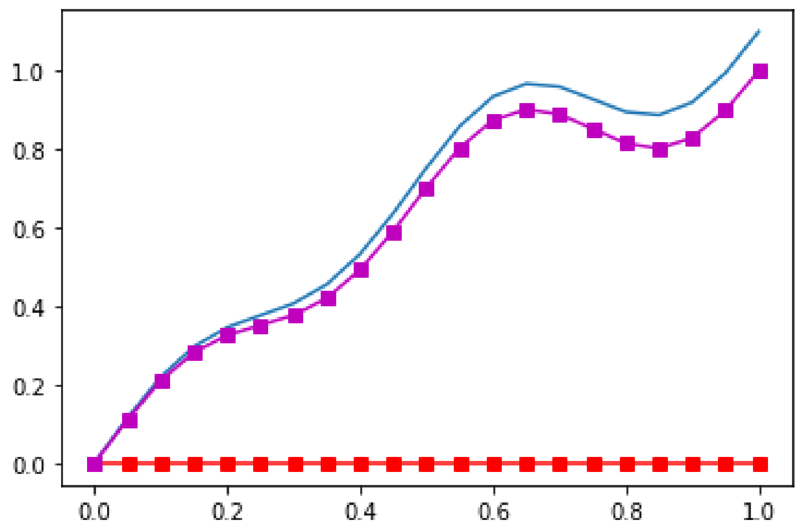
Coding project :

a. The analytical solution is included in the final page of this pdf.

b. For the mesh size of 0.05, with K = 2, the numerical solution for u(x) was computed using Gauss Order 3. The results are shown below:

Node coordinates	Nodal Solutions
0	0
0.05	0.110854
0.1	0.209824
0.15	0.282319
0.2	0.326129
0.25	0.351646
0.3	0.377113
0.35	0.420794
0.4	0.493138
0.45	0.592001
0.5	0.702848
0.55	0.803891
0.6	0.874273
0.65	0.902155
0.7	0.889636
0.75	0.852575
0.8	0.815316
0.85	0.802275
0.9	0.829488
0.95	0.89928
1	1



The sum of error computed using the exact solution and the numerical solution for this case for the given solution was 1.055292 .

c) For the different k values, the sum of error as a function of mesh size have been reported below:

	Sum of Absolute Errors				
Mesh Size	**K = 2**	**K = 4**	**K = 8**	**K = 16**	**K = 32**
0.01	5.07580053	5.025649	4.942662	4.781237	4.463883
0.025	2.060597036	2.040671	2.007523	1.943311	2.020504
0.04	1.305756683	1.294434	1.273786	1.236529	1.691531
0.05	1.055292	1.045692	1.029252	1.086744	11.36046
0.075	0.703755506	0.697467	0.687471	0.725085	1.36156
0.5	0.151654712	0.138726	4.349043	1.082095	135.3575

a) Analytical solution for the problem is as follows:

The analytical solution can be given by:-

$$\frac{\partial^2 u}{\partial x^2} = K^2 \cos\left(\frac{\pi K x}{L}\right) + 5(1-K^2)\sin\left(\frac{2\pi L K x}{L}\right)$$

Integrating;

$$\frac{\partial u}{\partial x} = \left[\frac{K^2 L}{\pi K}\sin\left(\frac{\pi K x}{L}\right) - 5(1-K^2)\cos\left(\frac{2\pi L K x}{L}\right) \times \frac{L}{2\pi L K} + C_1\right]$$

Integrating again;

$$u = \left[-\frac{K^2 L^2}{\pi K^2}\cos\left(\frac{\pi K x}{L}\right) - 5(1-K^2)\sin\left(\frac{2\pi K x}{L}\right) \times \frac{L^2}{2\pi K^2} + C_1 x + C_2\right]$$

At

~~At~~ $L=1$;

$$\therefore u = \frac{-L}{\pi^2}\cos(\pi K x) - 5(1-K^2)\sin\left(\frac{2\pi K x}{L}\right)\frac{L^2}{2\pi K^2} + C_1 x + C_2.$$

Using $L=1$; applying Dirichlet conditions, at $x=0$ & $x=1$;

we get

$\cancel{C_1 x = \left(1 + \frac{1}{\pi^2}\cos(\pi K)\right) x}$ $\cancel{C_2 =} \frac{1}{\pi^2}$

$ET \Rightarrow$

$c_1 x = \left(1 + \frac{1}{\pi^2}\cos(\pi K)\right)x \qquad c_2 = \frac{1}{\pi^2}$

$\therefore$ ~~An~~ Analytical solution:-

$$\frac{-1}{\pi^2} x\cos(K\pi x) - \frac{5(1-K^2)}{4\pi^2 K^2}\sin(2K\pi x) + \left(1 + \frac{1}{\pi^2}\cos(\pi K)\right)x + \frac{1}{\pi^2}$$