Let L be a language specified by a regular expression $r = (a \cup ab)^*$. The task is to find a regular expression for $\overline{L}$, the complement of L. The alphabet Σ is $\{a, b\}$.

Method

We design an NFA for r, convert the NFA to a DFA, flip the accepting and non-accepting states to take the complement and then generate the regular expression of this DFA.

Obtaining the NFA

An NFA for the given example is:

$$N_1 = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_0\})$$

where δ is defined as :

	ε	a	b
q_0	$\{q_1\}$		
q_1		$\{q_2\}$	
q_2	$\{q_0\}$		$\{q_3\}$
q_3	$\{q_0\}$		

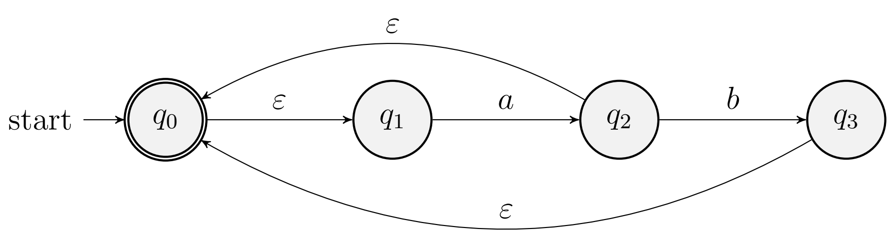

Figure 1: NFA for R

Now observe the ε closures of all states in fig 1:

$$\begin{aligned}\varepsilon(q_0) &= \{q_0, q_1\}\\ \varepsilon(q_1) &= \{q_1\}\\ \varepsilon(q_2) &= \{q_0, q_1, q_2\}\\ \varepsilon(q_3) &= \{q_0, q_1, q_3\}\end{aligned} \tag{1}$$

Constructing the DFA

We obtain the DFA by the subset construction method. The states are

$$\begin{aligned} d_0 &= \{q_0, q_1\}, \\ d_1 &= \{q_0, q_1, q_2\}, \\ d_2 &= \{q_0, q_1, q_3\} \\ d_3 &= \emptyset \end{aligned} \tag{2}$$

$$D_1 = \big(\{d_0, d_1, d_2, d_3\}, \{a, b\}, \delta, d_0, \{d_0, d_1, d_2\}\big)$$

where δ is defined as:

	a	b
d_0	d_1	d_3
d_1	d_1	d_2
d_2	d_1	d_3
d_3	d_3	d_3

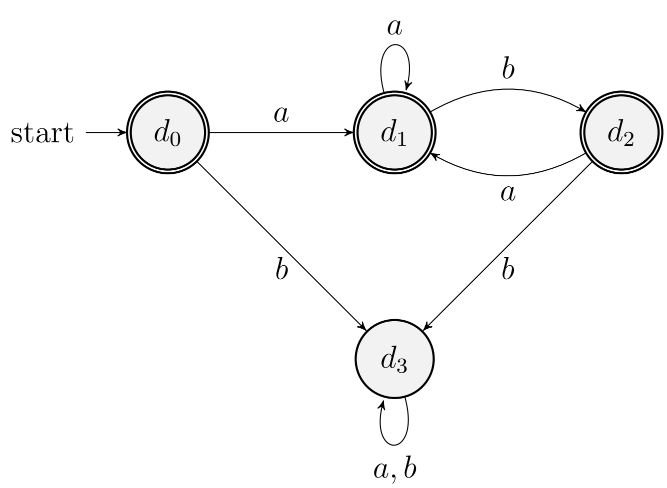

Figure 2: DFA D_1

We can find the complement of the DFA drawn above by flipping the accepting to non-accepting states and vice-versa. This gives us the DFA in Fig 3

$$D_1' = \big(\{d_0', d_1', d_2', d_3'\}, \{a, b\}, \delta, d_0', \{d_3'\}\big)$$

where δ is defined as:

	a	b
d_0'	d_1'	d_3'
d_1'	d_1'	d_2'
d_2'	d_1'	d_3'
d_3'	d_3'	d_3'

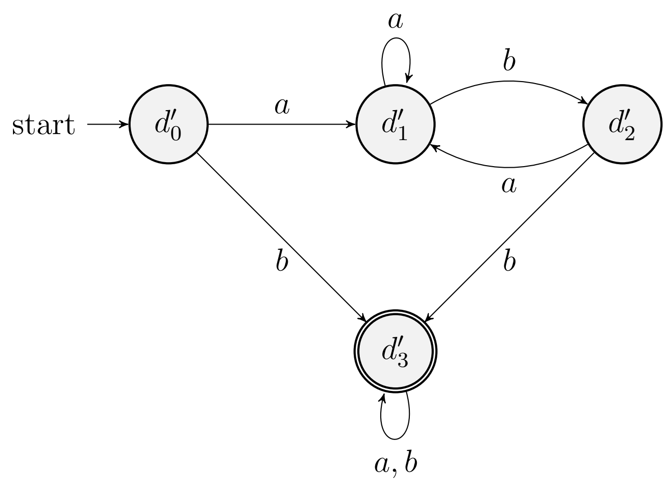

Figure 3: Complemented DFA D'_1

Obtaining the RegEx

From the DFA in Fig:3, we can obtain the following Equations

$$X_1 = aX_2 \cup bX_4 \tag{3}$$

$$X_2 = aX_2 \cup bX_3 \tag{4}$$

$$X_3 = aX_2 \cup bX_4 \tag{5}$$

$$X_4 = (a \cup b)X_4 \cup \varepsilon \tag{6}$$

We need to find the regular expression for X_1.
Applying Arden's lemma on equation 6, we obtain :

$$X_4 = (a \cup b)^* \tag{7}$$

By Arden's lemma, eq:5 and the Distributive Law we can transform eq:4:

$$\begin{aligned} X_2 &= aX_2 \cup baX_2 \cup bbX_4 \\ X_2 &= (a \cup ba)^* bbX_4 \end{aligned} \tag{8}$$

Now substituting for X_2 and X_4 we obtain:

$$X_1 = a(a \cup ba)^* bb(a \cup b)^* \ \cup \ b(a \cup b)^* \tag{9}$$

Hence the regular expression for $\overline{L}$ is $a(a \cup ba)^* bb(a \cup b)^* \ \cup \ b(a \cup b)^*$