Astronomía Avanzada I (Semester 1 2024)

Stellar Atmospheres (5)

Spectral Lines

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Recap

LTE = Maxwell + Boltzmann + Saha

The **Boltzmann equation** describes the degree of excitation of an atom or ion, e.g. $N(\mathrm{H}_{n=2})/N(\mathrm{H}_{n=1})$.

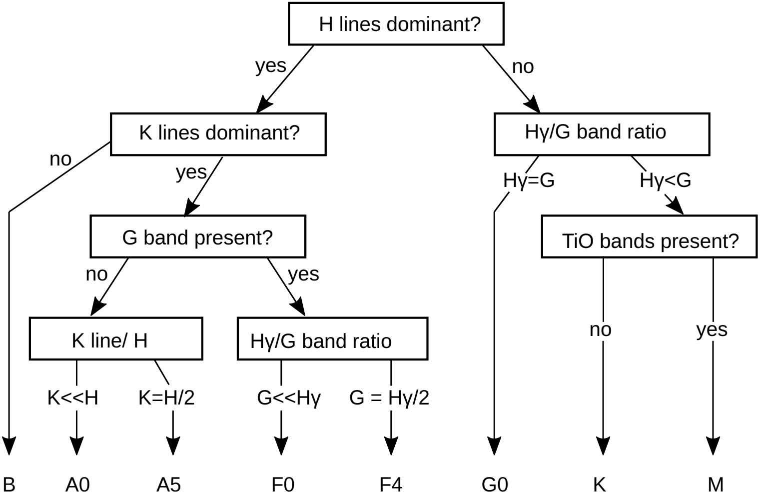
The **Saha equation** describes the degree of ionization of successive ions, e.g. $N(\mathrm{He}^+)/N(\mathrm{He}^0)$ or $N(\mathrm{He}^{2+})/N(\mathrm{He}^+)$.

The **Partition function** is the weighted sum of the number of ways an atom or ion can arrange its electrons with the same energy.

Ionization is an extremely energy-consuming process. Ionization happens within a very small temperature interval.

Saha-Boltzmann explains the spectral type (or temperature) dependence of lines in stellar atmospheres, e.g. strongest Balmer series at spectral type A and strong CaII lines in Solar-type stars.

Recap: Spectral Classes



Recap: Stellar Opacity

The removal of energy from a beam of photons as it passes through matter is happens by line absorption (bound-bound), photoelectric absorption (bound-free), inverse Bremsstrahlung (free-free) and photon scattering.

Stimulated emission acts as negative opacity by creating photons that add to the beam.

Stellar atmospheres are predominantly H (90 %), whilst He makes up almost all the rest. These two elements provide most of the opacity over most wavelength for the majority of the hot stars.

Metallicity

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Iron-to-Hydrogen ratio for the Sun:

$$[\mathrm{Fe/H}]_\odot = \log_{10}\left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right) \simeq -4.33$$

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For other stars:
The abundance ratio is the common logarithm of the ratio of a star's iron abundance compared to that of the Sun and is calculated thus:

$$[\mathrm{Fe}/\mathrm{H}] \equiv \log_{10}\left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_\star - \log_{10}\left(\frac{N_{\mathrm{Fe}}}{N_{\mathrm{H}}}\right)_\odot$$

where N_{Fe} and N_{H} are the number of iron and H atoms per unit of volume, respectively.
So if a star has $[\mathrm{Fe}/\mathrm{H}] = 0$, it has the same iron abundance as the Sun, for $[\mathrm{Fe}/\mathrm{H}] = -1$, it has one tenth the solar value.

Metallicity

Observations:

- Globular clusters are generally metal-poor
- Disk stars span a range of metallicities
- Open clusters are generally more metal-rich

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What can we conclude from this?

In the 1940s, Walter Baade introduced the concept of stellar populations:

Population I
metal rich $[\mathrm{Fe/H}] > -1$
disk stars
open clusters

Population II
metal poor $[\mathrm{Fe/H}] < -1$
halo stars
globular clusters

Metallicity

One might expect the metallicity of a star to be related to **when** it formed: very old stars formed before many SNe exploded, thus formed from gas containing mostly H and He.

Recently, there has been interest in the very first generation of stars that formed after the Big Bang. Those would have $Z = 0$. They are called population III. The halo star with the lowest metallicity currently known, has $[\mathrm{Fe/H}] \approx -5$, and might well be one of the first stars to have formed in the Milky Way.

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However: Stars in old GCs, for example, typically have [Fe/H]~ -1 (a bit below the solar value).

$\Rightarrow$ Populations do not refer to age, but to chemical composition.

The evolution of Z within the Milky Way, or within galaxies in general, is called their **chemical evolution**.

Metallicity

Note that metallicity also correlates with color:

metal poor makes stars bluer, metal rich makes stars redder

Why?

Metallicity

Stellar Atmospheres (5)

Recap

Chemical Composition

Spectral Lines

Summary

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Line blanketing: lots of metals (particularly Fe) in the atmospheres of stars absorb preferentially blue light, so the star looks a bit redder.

Opacity: more metals absorb energy from the interior of the star, making red giants "swell up" even more, and give them cooler (redder) temperatures.

Chemical Composition of Population I

The stellar atmosphere is a mixture of chemical elements, present as atoms, ions or molecules.

Abundances are given as given as mass fractions β_k. They add up to 1: X+Z+Y=1

Abundances for **Population I stars**:

$$\beta_{\mathrm{H}} = 0.71 \longrightarrow \text{X}$$

$$\beta_{\mathrm{He}} = 0.28 \longrightarrow \text{Y}$$

$$\left.\begin{array}{l} \beta_{\mathrm{C}} = 0.004 \\ \beta_{\mathrm{N}} = 0.001 \\ \beta_{\mathrm{O}} = 0.009 \\ \quad\vdots \\ \beta_{\mathrm{Fe}} = 0.001 \\ \quad\vdots \end{array}\right\} \text{Z (''metals'')}$$

Chemical Composition of Population II

Population II stars have:

$\beta_{\mathrm{H}} = \beta_{\mathrm{H}}^{\odot}$
$\beta_{\mathrm{He}} = \beta_{\mathrm{He}}^{\odot}$
$\beta_{\mathrm{Z}} = 0.1 - 0.00001\beta_{\mathrm{Z}}^{\odot}$

Chemically peculiar stars, e.g. He stars have:

$\beta_{\mathrm{H}} \leq 0.002 \ll \beta_{\mathrm{H}}^{\odot}$
$\beta_{\mathrm{He}} = 0.64 \gg \beta_{\mathrm{He}}^{\odot}$
$\beta_{\mathrm{C}} = 0.029 \gg \beta_{\mathrm{C}}^{\odot}$
$\beta_{\mathrm{N}} = 0.003 \gg \beta_{\mathrm{N}}^{\odot}$
$\beta_{\mathrm{O}} = 0.002 \gg \beta_{\mathrm{O}}^{\odot}$

Other **chemically peculiar stars**, e.g. PG1159 stars have:

$\beta_{\mathrm{H}} \leq 0.05 \ll \beta_{\mathrm{H}}^{\odot}$
$\beta_{\mathrm{He}} = 0.25 \gg \beta_{\mathrm{He}}^{\odot}$
$\beta_{\mathrm{C}} = 0.55 \gg \beta_{\mathrm{C}}^{\odot}$
$\beta_{\mathrm{N}} < 0.02$
$\beta_{\mathrm{O}} = 0.15 \gg \beta_{\mathrm{O}}^{\odot}$

Line Formation

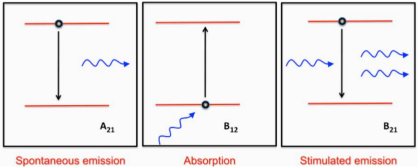
We obtained earlier that the emergent flux from the stellar surface is π times the source function at an optical depth of 2/3:

$$F_\lambda(0) = \pi S_\lambda(\tau_\lambda = 2/3) = \pi B_\lambda(T(\tau_\lambda = 2/3))$$

In spectral lines, the opacity is much larger, thus we see much higher layers at these wavelengths. These layers have a lower temperature and so B_λ is smaller, leading to a smaller F_λ in the line than F_c, the continuum flux in the neighborhood of the line.

Line Formation

There are 3 basic kinds of line processes associated with **bound-bound** transitions of atoms or ions:



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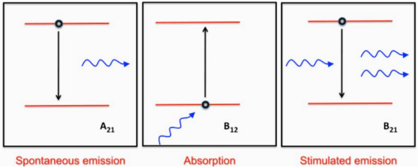

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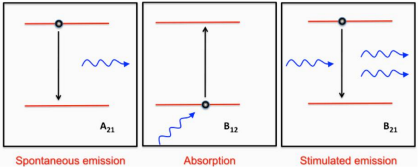

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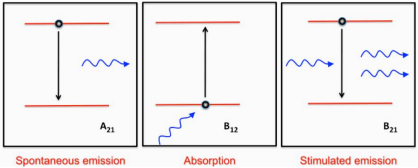

Spontaneous Emission, in which an electron spontaneously decays to a lower level, emitting the energy difference as a photon.

Absorption, in which the absorbed photon induces a bound electron to go into a higher energy level.

Stimulated Emission, in which an incoming photon induces an electron in a higher energy level to decay to a lower level, emitting a second photon that is nearly identical in energy (and even phase) to the original photon.

Line Formation

The probability that the atom will emit or absorb a quantum of energy is described by **Einstein coefficients**, written as A_{ji}, B_{ij}, and B_{ji}.

Einstein coefficients give the probability that a particle spontaneously emits a photon (A_{ji}), the probability to absorb a photon (B_{ji}), and the probability to emit a photon under the influence of another incoming photon, the so-called induced or stimulated emission (B_{ij}). Einstein's coefficients are valid for all radiation fields.

Line Formation - Emission

Consider an upper level, j, and a lower level, i, separated by an energy $h\nu_0$. The probability that the atom will **spontaneously emit** its quantum of energy within a time $\mathrm{d}t$ and in a solid angle $\mathrm{d}\omega$ is $A_{ji}\,\mathrm{d}t\,\mathrm{d}\omega$.

The proportionality constant, A_{ji}, is the Einstein probability coefficient for spontaneous emission per second.
Spontaneous emission occurs independently of the radiation field. The emission is isotropic.

For atomic line radiation,

$$\varepsilon = \frac{h\nu}{4\pi} n_j A_{ji}$$

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example: For Hα, $A_{32} = 4.4 \times 10^7\,\mathrm{s}^{-1}$. If at a time $t_0 = 0$ there are $N_i(0)$ atoms in level j, then at time t we have $N_j(t) = N_u(0)\exp(-A_{ji}t)$. The lifetime is $1/A_{ji}$.

Line Formation - Absorption

Consider an upper level, j, and a lower level, i, separated by an energy $h\nu_0$.

Photons with energies close to $h\nu_0$ will be **absorbed**, causing transitions from levels i to j.

The probability per unit time for this process will evidently be proportional to the mean intensity J_ν at the frequency ν_0.
The transition probability of absorption per unit time is the Einstein coefficient B_{ij}.

Line Formation - Stimulated Emission

Planck's law does not follow from considering only spontaneous emission and absorption. One also include stimulated emission, which like absorption is proportional to the mean intensity J.

The system goes from an upper level j to a lower level i, stimulated by the presence of a radiation field ($h\nu$ corresponding to the energy difference between levels u and l). The transition probability of stimulated emission per unit time is the Einstein coefficient B_{ji}.

Stimulated emission occurs into the same state (frequency, direction, polarization) as the photon that stimulated the emission.
The process of stimulated emission is sometimes referred to as a process of **negative absorption**.

Relation between Einstein Coefficients

The Einstein coefficients are not independent. To find a relation between them, let's assume strict Thermodynamic Equilibrium (TE), and, for simplicity, adopt a 2-level approximation.

In TE, each process is in equilibrium with its inverse, i.e., within one line there is no netto destruction or creation of photons (detailed balance): Transitions $1 \to 2$ equal to $2 \to 1$ and $0 = A_{21} n_2 + B_{21} n_2 \rho(\nu) - B_{12} n_1 \rho(\nu)$.

In the following, we have n_1, n_2: number density of electrons in levels 1, 2. Along with detailed balancing, at temperature T we may use our knowledge of the equilibrium energy distribution of the atoms, as stated in the Maxwell-Boltzmann distribution, and the equilibrium distribution of the photons, as stated in Planck's law of black body radiation to derive universal relationships between the Einstein coefficients.

Relation between Einstein Coefficients

From Boltzmann distribution we have for the numbers:

$$n_1/n_2 = g_1/g_2 e^{h\nu_{21}/kT}$$

$$n_1 B_{12} J_\nu = n_2 A_{21} + n_2 B_{21} J_\nu, \qquad J_\nu = \frac{A_{21}/B_{21}}{(\frac{n_1}{n_2})(\frac{B_{12}}{B_{21}}) - 1}$$

From this we get

$$B_\nu(T) = \frac{A_{21}}{B_{21}} \left(\frac{g_1 B_{12}}{g_2 B_{21}} e^{\frac{h\nu_{21}}{kT}} - 1 \right)^{-1}$$

We compare this with the Planck blackbody radiation:

$$B_\nu(T) = \frac{2h\nu_{21}^3}{c^2} \left(e^{\frac{h\nu_{21}}{kT}} - 1 \right)^{-1}$$

We finally find:

$$\frac{A_{21}}{B_{21}} = \frac{2h\nu_{21}^3}{c^2} \Rightarrow A_{21} = B_{21} \frac{2h\nu_{21}^3}{c^2}, \quad \frac{g_1 B_{12}}{g_2 B_{21}} = 1 \Rightarrow g_1 B_{12} = g_2 B_{21}$$

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If one of the Einstein coefficients is known, the other two can be calculated.

Relation between Einstein Coefficients

Important: The Einstein's coefficients are atomic constants. Although the above relations were derived under the conditions of TE, these relations hold in any non-TE state.

Total amount of absorbed photons per unit time at a given frequency is

$$n_1 B_{12} J_\nu - n_2 B_{21} J_\nu = n_1 B_{12} J_\nu \left(1 - \frac{n_2 B_{21}}{n_1 B_{12}}\right) = n_1 B_{12} J_\nu \left(1 - \frac{g_1 n_2}{g_2 n_1}\right)$$

Thus, to take into account negative absorption (stimulated emission), one must multiply the number of absorbed photons, assuming LTE (Bolzmann), by

$$(1 - e^{-h\nu_{12}/kT})$$

Lifetime of an Atom in the Excited State

In the absence of collisions and of any other transitions than the ul one, the mean lifetime of particles in state u is the **lifetime** $1/A_{ul}$.

If at a time $t_0 = 0$ there are $N_j(0)$ atoms in level j, then at time t the population is

$$N_j(t) = N_j(0) \exp -A_{ji} t$$

Typical value of A_{ji} is $10^7 - 10^8 \,\mathrm{s}^{-1}$. For H$\alpha$, $A_{21} = 4.4 \times 10^7 \,\mathrm{s}^{-1}$, so lifetime is $\sim 10^{-8} \,\mathrm{s}$.

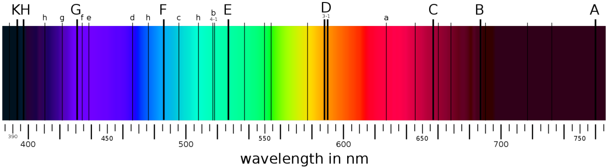
However, not all transitions are allowed, some are **forbidden**.

In practice, strictly forbidden means very low probability of occurrence.

Nomenclature

Fraunhofer lines:

Strong absorption lines in the visible part of the spectrum often have a unique Fraunhofer line designation, such as K for a line at 393.366 nm from singly ionized Ca^+. Some of the Fraunhofer "lines" are blends of multiple lines from several different species. The lines are named after German physicist Joseph von Fraunhofer, who observed them in 1814.



Fraunhofer lines in the Solar spectrum

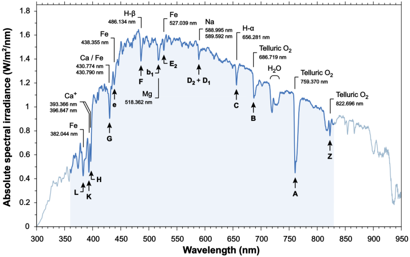
Fraunhofer lines:

current usage:

The Fraunhofer C, F, G', and h lines correspond to the α, β, γ, and δ lines of the Balmer series of emission lines of the H atom. The Fraunhofer letters are now rarely used for those lines.

The D1 and D2 lines form a pair known as the *sodium doublet*, the centre wavelength of which (589.29 nm) is given the letter "D". The D1 and D2 lines correspond to the fine-structure splitting of the excited states.

The Fraunhofer H and K letters are also still used for the CaII doublet.



Nomenclature

Ions:

The lines are designated according to the level of ionization by adding a Roman numeral to the designation of the chemical element. Neutral atoms are denoted with the Roman numeral I, singly ionized atoms with II, and so on, so that, for example:

CuII: copper ion with $+1$ charge, Cu^{1+}

FeIII: iron ion with $+2$ charge, Fe^{2+}

More detailed designations usually include the line wavelength and may include a multiplet number (for atomic lines) or band designation (for molecular lines). Many spectral lines of atomic hydrogen also have designations within their respective series, such as the Lyman series or Balmer series.

Line Depth

Stellar Atmospheres (5)

Recap

Chemical Composition

Spectral Lines

Summary

We now take a look at specific **features** of spectral lines.

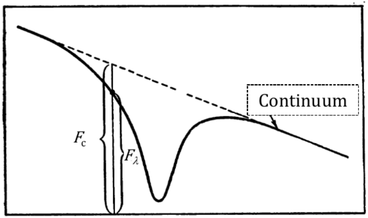
The relative intensity r_λ:

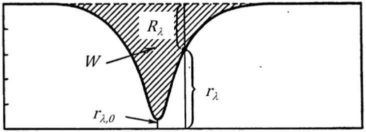
$$r_\lambda = \frac{F_\lambda}{F_c}$$

The line depth R_λ:

$$R_\lambda = \frac{F_c - F_\lambda}{F_c} = 1 - \frac{F_\lambda}{F_c}$$

The largest line depth is the central line depth $R_{\lambda,0}$.



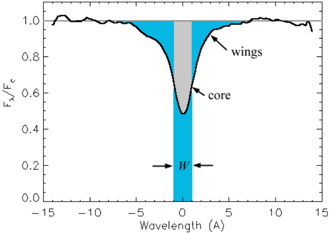


Equivalent Width

The equivalent width, W, is a measure of the strength (width) of a spectral line:

$$W = \int \frac{F_c - F_\lambda}{F_c} \, \mathrm{d}\lambda$$

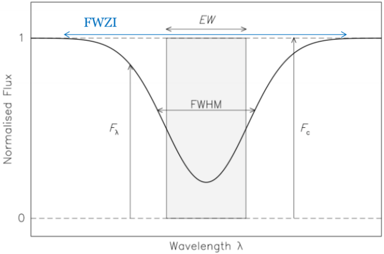
Equivalent widths (EW) are measured in Å. The larger the value of W, the stronger the line.
The equivalent width of absorption lines is positive, emission lines have negative equivalent widths.



Shown in the figure is the equivalent width W of an absorption line. The integral is shown as the total area inside the absorption line. Then a same-area rectangle, from the continuum to the 0 flux line, is created.

FWHM

Another measure of the width of a spectral line is the full width at half-maximum (FWHM), $(\Delta\lambda)_{1/2}$, which is the change in wavelength from one side of the line to the other where the depth equals $(F_c - F_\lambda)/(F_c - F_{\lambda 0}) = 1/2$.
The FWHM is used to describe how broadened the spectral feature is: the higher the value, the more broadened the line.

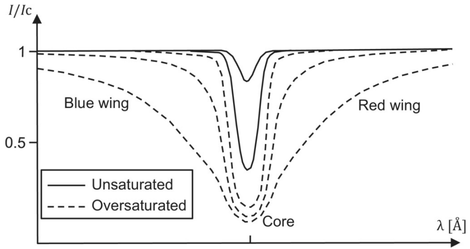


Shown in the figure above is the FWHM of an emission line (the same idea for absorption lines). Get the peak (maximum) value of the emission line, and draw a line at the half point. The width of the spectral feature at this flux value is the FWHM. Similar is the full width at zero intensity (FWZI).

Line Core and the Wings

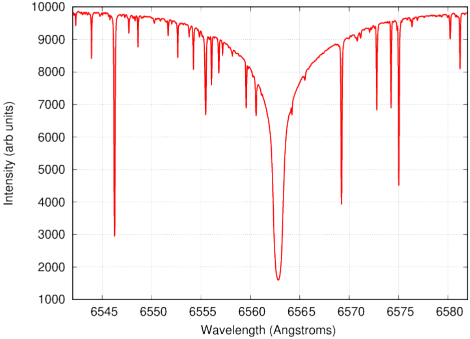
We denote optically thick lines as those in which the line core is saturated, i.e. reaching zero intensity (and optical thin ones as the opposite). In reality, zero intensity is only reached for lines in non-LTE.

The region close to the center of the spectal line is referred to as the **line core**, whilst the **wings** merge into the local continuum.



Example: Lines in the Solar Spectrum

Strong spectral lines in the Solar spectrum typically have equivalent widths $W_\lambda \sim 1$Å, such as the NaI D lines in the yellow. In other stars, line equivalent widths can reach tens or even hundreds of Å.



close-up of Solar Hα line

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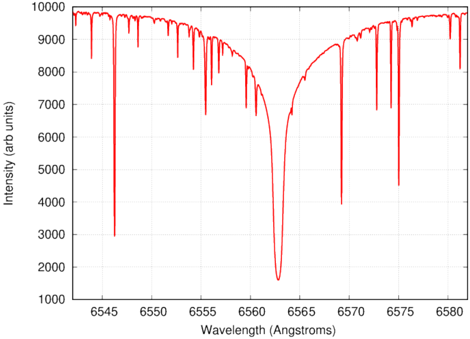

close-up of Solar Hα line

The **shape** of an absorption (or emission) line depends on several factors.

Line Broadening

All the spectral lines are not monochromatic but have a finite width and a particular profile:
In each spectral line, photons of different frequencies (but close to central frequency ν_0) can be absorbed.
Width and shape of a line depend directly in atomic transitions and plasma environment.

Different processes that cause the **broadening of spectral lines** exist.
Each of the meachanisms produces its **distinctive line profile**.

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The three main processes are:

Natural broadening: Each individual energy state has a finite lifetime.

Doppler broadening: Thermal motions of the gas particles.

Pressure broadening: Collisions between neutral atoms or close encounters with ions.

Natural Broadening

Energy levels are not infinitely sharp.
An unavoidable source of broadening is due to the **Heisenberg uncertainty principle**:
A decaying state j does not have a perfectly defined energy E_j, but rather a superposition of states spread around E_j.
The longer the atom is in a state ($\mathrm{d}t$ is high), the more precisely its energy can be measured ($\mathrm{d}E$ is low). A large transition probability leads to a short life in the state ($\mathrm{d}t$ low) and a large energy uncertainty ($\mathrm{d}E$ high).

$$E_\gamma = \frac{hc}{\lambda} \Rightarrow \Delta E_\gamma = \frac{hc}{\lambda^2}\Delta\lambda$$

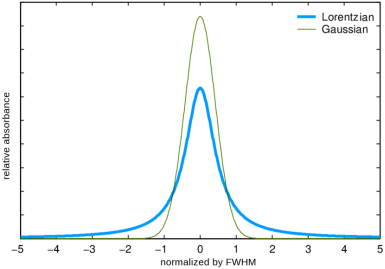
since $\Delta E \sim \frac{h}{2\pi\Delta t}$, the uncertainty in the photon's wavelength involving a transition between two energy levels is

$$\Delta\lambda \sim \frac{\lambda^2}{2\pi c}\left(\frac{1}{\Delta t_i} + \frac{1}{\Delta t_f}\right)$$

where Δt_i and Δt_f are the lifetimes in the initial and final states, respectively.

Natural Broadening

This **natural broadening** has the form of a Lorentzian function, also referred to as a damping profile (a.k.a. Lorentz profile), because the shape is characteristic of the radiation emitted by an electric charge undergoing damped simple harmonic oscillations.



A more careful calculation gives the FWHM for natural broadening as

$$(\Delta\lambda)_{1/2} = \frac{\lambda^2}{\pi c}\frac{1}{\Delta t_0}$$

where Δt_0 is the lifetime for the transition.

Thermal (Doppler) Broadening

The light emitting atoms in a stellar atmosphere are not at rest, but have a **thermal motion**, thus a Maxwellian velocity distribution.

Because the particles produce Doppler shifts, the line of sight velocities follow a distribution.

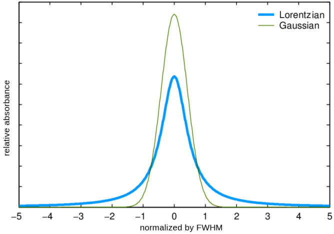
In LTE, atoms in a gas are moving randomly with a distribution of speeds described by the Maxwell-Boltzmann distribution function, with the most probable speed $v_{\mathrm{mp}} = \sqrt{2kT/m}$. The emissions are Doppler-shifted based on $\Delta\lambda/\lambda = \pm|v_r|/c$. Hence, the line width increased by Doppler broadening is

$$\Delta\lambda \sim \frac{2\lambda}{c}\sqrt{\frac{2kT}{m}}$$

Thermal (Doppler) Broadening

Stellar Atmospheres (5)

Doppler broadening has the form of a Gaussian function.



A more careful analysis gives the FWHM of the line for Doppler broadening:

$$(\Delta\lambda)_{1/2} = \frac{2\lambda}{c}\sqrt{\frac{2kT\ln 2}{m}}$$

Large-scale turbulent motions can contribute:

$$(\Delta\lambda)_{1/2} = \frac{2\lambda}{c}\sqrt{\frac{(2kT + v_{\mathrm{turb}})\ln 2}{m}}$$

Pressure Broadening

The orbitals of an atom can be perturbed in a collision with a neutral atom or by a close encounter involving the electric field of an ion. The effect is called **pressure broadening**.

Given a collision cross-section σ, the average time between collisions is

$$\Delta t \sim \frac{l}{v} = \frac{1}{n\sigma}\sqrt{\frac{m}{2kT}}.$$

Thus the line width due to pressure broadening is of the order of

$$\Delta\lambda = \frac{\lambda^2}{c}\frac{1}{\pi\Delta t_0} \sim \frac{\lambda^2}{c}\frac{n\sigma}{\pi}\sqrt{\frac{2kT}{m}}.$$

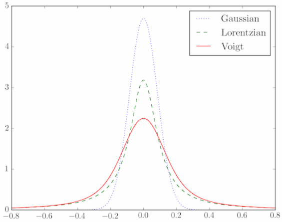
The general shape of the line due to pressure broadening is similar to that found for natural broadening: we find a Lorentzian profile.

Joint Effect of Line Broadening

We combine the effect of natural broadening, thermal broadening and pressure broadening:
The physical argument behind this is that the effects of broadening are decoupled, thus every point on a collision-broadened lineshape is further broadened by Doppler effects.

The resulting **Voigt profile** is a probability distribution given by a **convolution** of a Lorentz distribution and a Gaussian distribution:

$$\begin{aligned}\phi_V(\nu) &= \phi_D(\nu) * \phi_L(\nu)\\ &= \int_{-\infty}^{\infty} \phi_D(u)\phi_L(\nu - u)\,\mathrm{d}u\end{aligned}$$



The Voigt profile is commonly used in the modelling emission and absorption line shapes.

Summary

Quantitive spectral analyses - what can we learn?

Line position:
chemical composition, velocities, redshift

Shape of line profile:
temperature, density, abundance, rotation, turbulence, magnetic field

Temporal variation:
companion, surface structure (spots), pulsation