Astronomía Avanzada I (Semester 1 2025)

# Stellar Variability (I)

## Introduction to Stellar Variability & Pulsation

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# Motivation



Although the Greeks considered *the heavens* as unchanging, careful observers have known otherwise:

Hipparchos observed a new star in Scorpius in 134 BC, which inspired him to create the **first stellar catalog** so that new stars could more easily be recognised in the future.

# Motivation

**What are variable stars?**

A star is a variable star if we can measure its brightness changing over time.

There are many **types** of variable stars, regarding how those brightness changes are caused by things happening inside, on the surface of, or around that star.

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A star is a variable star if we can measure its brightness changing over time.

There are many **types** of variable stars, regarding how those brightness changes are caused by things happening inside, on the surface of, or around that star.

From supernovae to stars with exoplanets, variable stars are of high interest in astronomical research.

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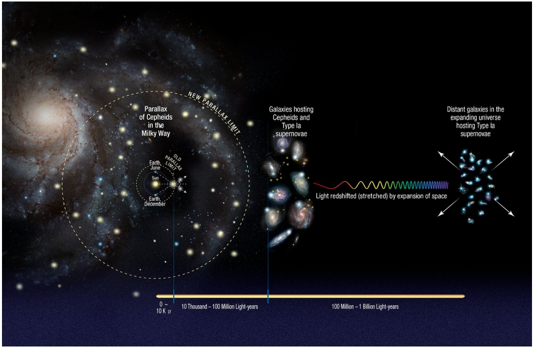
Research on variable stars **provides information about stellar properties**, such as mass, radius, luminosity, temperature, internal and external structure, composition, and evolution.



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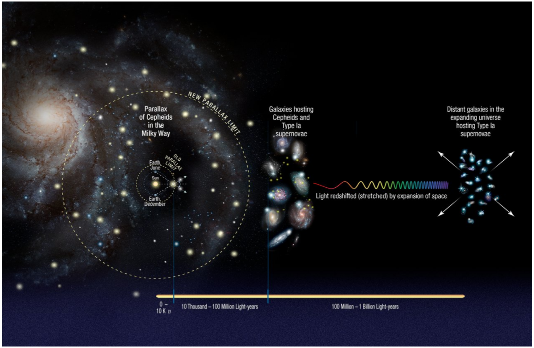
In addition, variable stars provide **distance information** (keyword: *distance ladder*) in our galactic neighborhood.



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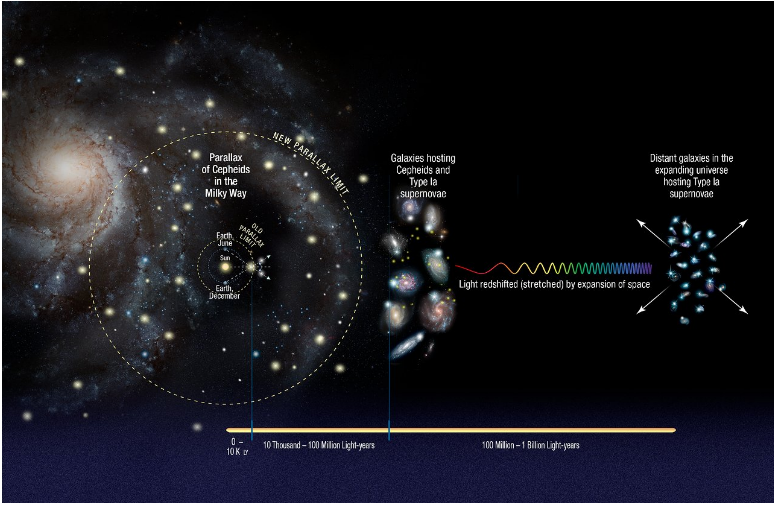
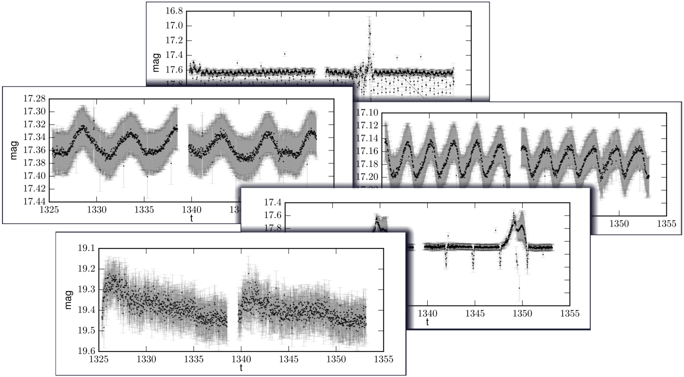



Credit: NASA, ESA, A. Feild (STScI), and A. Riess (STScI/JHU)

# Variable Stars

Variable stars are stars showing a change in brightness.




A selection of variable star light curves from the TESS survey.

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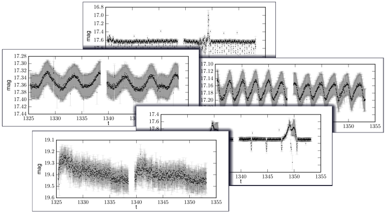

few parts per million **change in luminosity** factor 1000

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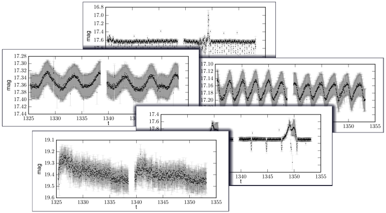

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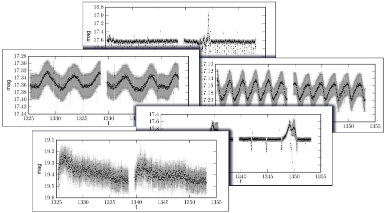

| few parts per million | **change in luminosity** | factor 1000 |
|---|---|---|
| seconds | **temporal baseline** | centuries |
| periodic | **signal shape** | aperiodic/ random |

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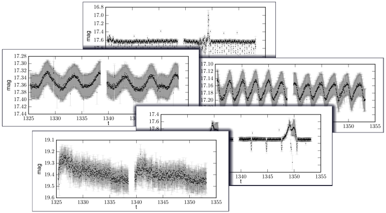


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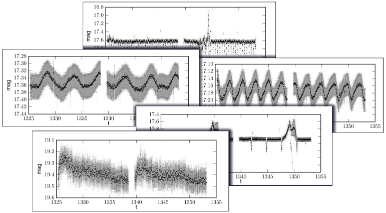

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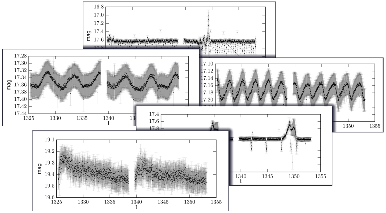
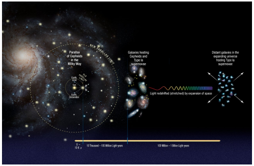


variations provide important and often unique information about the nature and evolution of stars

and the galaxies that host them

and our universe in general



# Variable Stars

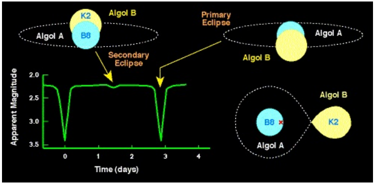
Variable stars are stars showing a change in brightness.

**Intrinsic Variables**

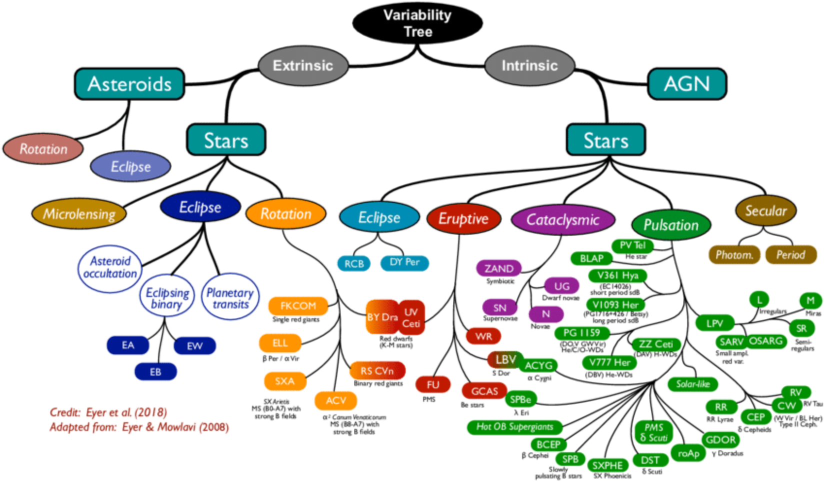
Stars whose energy output actually varies (pulsating stars, erupting or explosive stars)

**Extrinsic Variables**

Stars that only appear to vary due to geometric/ external effects (eclipses in binary systems, etc.)



# Variability Tree




Credit: Eyer et al. (2018)
Adapted from: Eyer & Mowlavi (2008)

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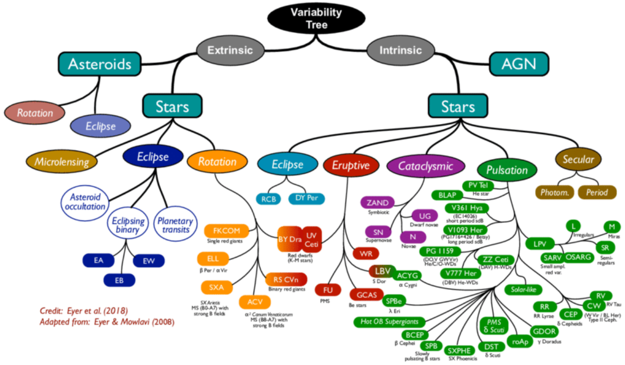



many astronomical sources vary - describe, classify and select astronomical sources by their variability

# Discovery of Variable Stars



There is a group of stars which change their brightness in a regular manner: **periodic variable stars**

**historically:** they were much harder to detect than (super-)nova

1596: David Fabricius noted that the star *o* Ceti (now known as Mira) was sometimes visible, sometimes not

1638: Johannes Holwarda found a visibility cycle of 11 months for Mira

1700s: William Herschel discovered the variability of $\alpha$ Herculis and 44i Bootis

1850s: $\sim$18 periodic variable stars known

1890: establishment of the Variable Star Section of the British Astronomical Association (BAAVSS)

1911: founding of the American Association of Variable Star Observers

# Discovery of Periodic Variable Stars

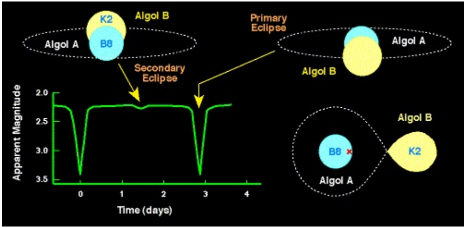


**causes of variability?**

John Goodricke and Edward Pigott: proposed the theory that Algol's variability might be caused by eclipses of the star by a planetary companion

we know today:

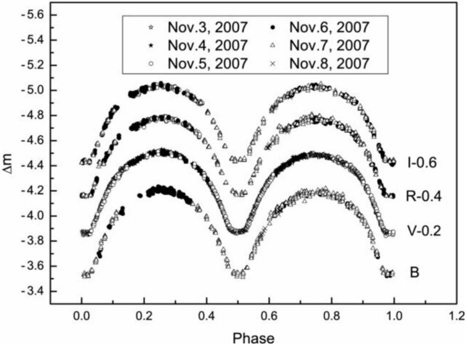
Algol is a three-star system, consisting of $\beta$ Persei (Per) A, $\beta$ Per B and $\beta$ Per C. They regularly pass in front of each other, causing eclipses. This is an **eclipsing binary star**. The condition for this to happen is that the plane of the binary is (almost) edge-on to us.



# Discovery of Periodic Variable Stars



Sometimes the two stars are so close that their outer atmospheres merge: these are known as **contact binaries**.




The light curve of the contact binary AE Phoenicis. The fact that there is no part of the orbit where the light curve is flat shows that the stars are never separate but are actually in contact. Credit: He at al. (2009)

# (Early) Classification of Variable Stars



systematic observation of variable stars reveiled **differences in their light curves**

Pigott (1780s): variable stars: nova, long-period variables, short-period variables

# (Early) Classification of Variable Stars



systematic observation of variable stars reveiled **differences in their light curves**

Pigott (1780s): variable stars: nova, long-period variables, short-period variables

Pickering (1880s): a more detailed scheme:

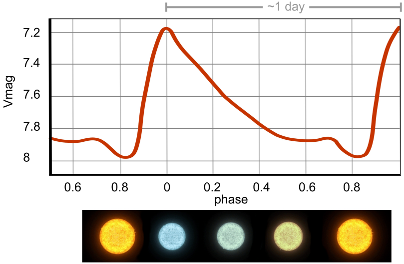
- (Ia) normal novae: now known to be nearby ones in our own galaxy;
- (Ib) novae in nebulae: now known to be supernovae in other galaxies;
- (IIa) long-period variables: cool, large-amplitude pulsating variables;
- (IIb) U Geminorum stars: dwarf novae;
- (IIc) R Coronae Borealis stars: stars which suddenly and unpredictably decline in brightness;
- (III) irregular variables: a motley collection;
- (IVa) short-period variables such as Cepheids and RR Lyrae stars;
- (IVb) Beta Lyrae type eclipsing variables; and
- (V) Algol type eclipsing variables.

# Pulsating Variable Stars

**underlying physics of variability:**

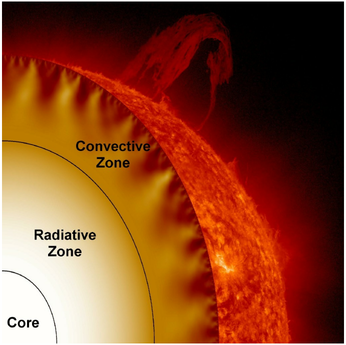
idea: (at least some) periodic variability might be caused by pulsations (A. Ritter 1873)

Observational studies by Harlow Shapley and others around 1915, and the concurrent theoretical studies by Eddington, established the pulsational nature of the Cepheids, cluster type variables (RR Lyrae stars), and long-period variables.



# Cause of Pulsation

Energy from a stellar core - regardless if it is a pulsating star or not - cannot reach the surface directly: instead it is transported by **radiation**, where atoms absorb and then re-emit photons; or by **convection**, where hot material rises and colder material sinks, just like a boiling pot.




credit: NASA/ Marshall Solar Physics

# Cause of Pulsation



**cause of pulsation:**
Lack of hydrostatic equilibrium beneath the surface drives the pulsation cycle with expansion and contraction of the outer layers of a star and subsequent change in brightness:

hydrostatic equilibrium:

acceleration

mass within a sphere of radius *r*

pressure gradient

$$\rho \frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -G \frac{M_r \rho}{r^2} - \frac{\mathrm{d}P}{\mathrm{d}r}$$

For pulsating variables of most types:

$$P\,\rho^{1/2} = \mathrm{const}$$

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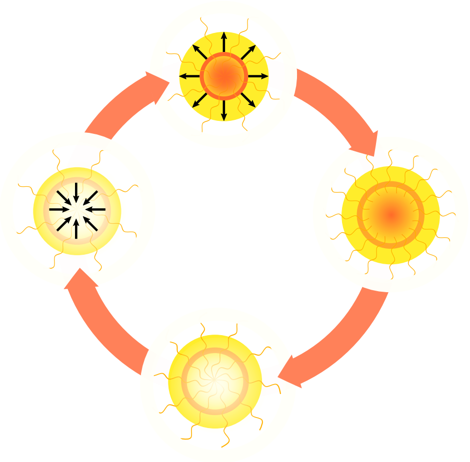

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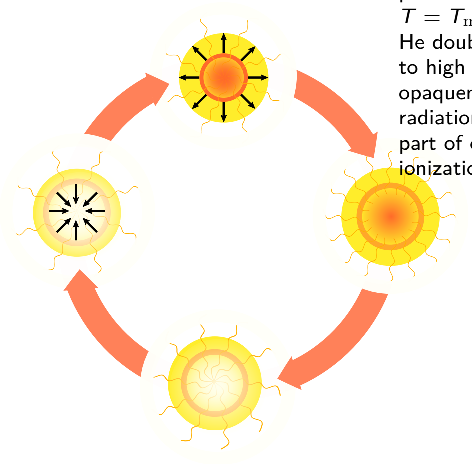

point of greatest compression:
$T = T_{\max}$
He doubly ionized (HeIII) due to high $T$
opaqueness of HeIII causes radiation absorption (dimmest part of cycle) ⇒ increase of ionization, $T$ and pressure.

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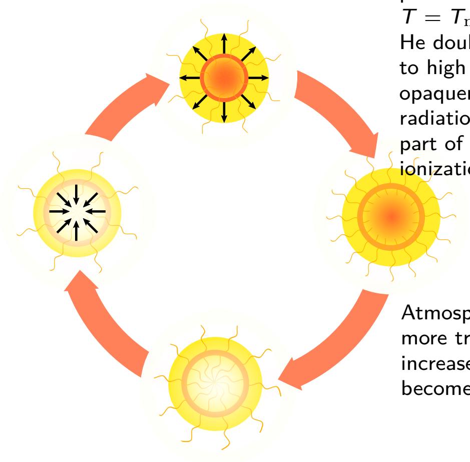

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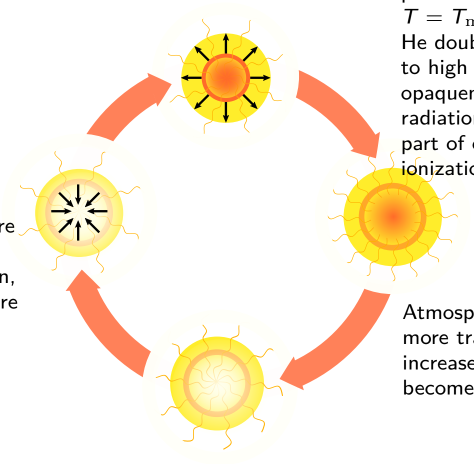

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Atmosphere falls back, contraction, temperature rises.

# Cause of Pulsation



**cause of pulsation:**
Lack of hydrostatic equilibrium beneath the surface drives the pulsation cycle with expansion and contraction of the outer layers of a star and subsequent change in brightness:

Before it reaches equilibrium, HeII becomes HeIII and starts cycle over again.

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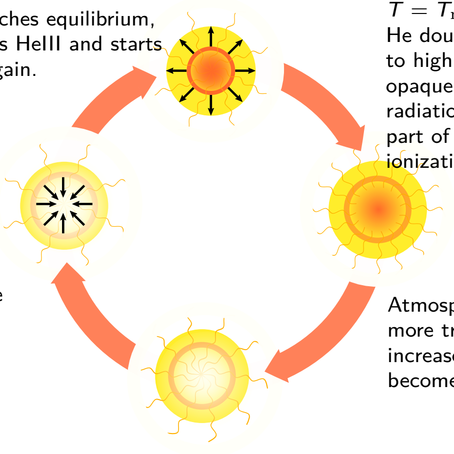

Atmosphere falls back, contraction, temperature rises.

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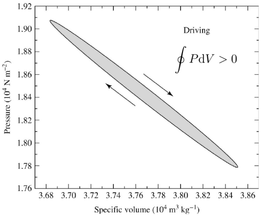
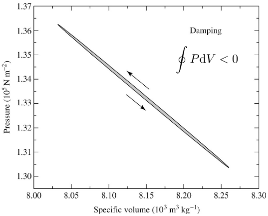
**What Drives Stellar Pulsation:**

Nuclear mechanism: $\epsilon$ mechanism
Insignificant to drive pulsation: displacement, $\delta r/R$

Piston layer in stellar interior: $\kappa$ and $\gamma$ mechanisms
Opacity must increase with compression (partial ionization zones). The $\kappa$ mechanism is the driving mechanism behind the changes in luminosity of many types of pulsating variable stars.

Kramers law: $\kappa \propto \rho/T^{3.5}$
Lower $T \Rightarrow$ heat exchanges with adjacent stellar layers ($\gamma$ mechanism)

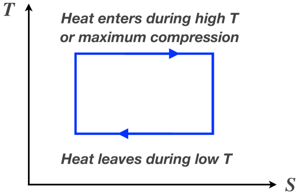


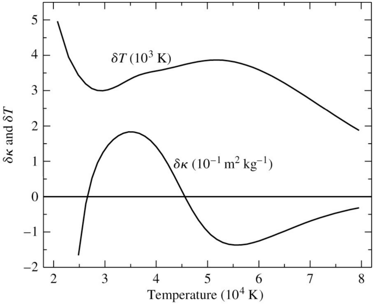



# Modeling Stellar Pulsation



**Carnot cycle in heat absorbing**



# Modeling Stellar Pulsation



The **radial oscillations** of a pulsating star are the result of density waves resonating in the star's interior. The **pulsation period** can be approximated by the time it would take for a sound wave to cross the diameter of the model star.

Using the hydrostatic equilibrium

$$\begin{aligned}\frac{\mathrm{d}P}{\mathrm{d}r} &= -\frac{GM_r\rho}{r^2} \\ &= -\frac{G(\frac{4}{3}\pi r^3\rho)\rho}{r^2} = -\frac{4}{3}\pi G\rho^2 r,\end{aligned}$$

and integrating with the boundary condition that $P = 0$ at the surface $r = R$, we have

$$P(r) = \frac{2}{3}\pi G\rho^2(R^2 - r^2)$$

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The **pulsation period** is then

$$p \sim 2\int_0^R \frac{\mathrm{d}r}{v_s} \sim 2\int_0^R \frac{\mathrm{d}r}{\sqrt{\frac{2}{3}\gamma\pi G\rho(R^2 - r^2)}}$$

# Modeling Stellar Pulsation

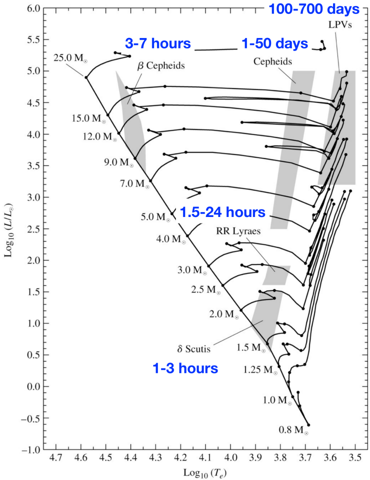


Since

$$\int_0^R (R^2 - r^2)^{-1/2}\, \mathrm{d}r = \frac{\pi}{2},$$

we get the **period-mean density relation**

$$p \sim \sqrt{\frac{2\pi}{2\gamma g \rho}} \propto \rho^{-1/2}$$



# Modeling Stellar Pulsation



The equation of motion can be written as

$$\rho \frac{\mathrm{d}^2 r}{\mathrm{d}t^2} = -G\frac{M_r \rho}{r^2} - \frac{\mathrm{d}P}{\mathrm{d}r}$$

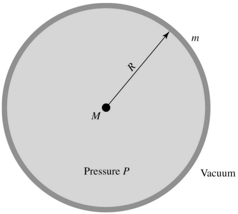
which is nonlinear and has to be solved numerically.

An alternative is to linearize the differential equation.

# Modeling Stellar Pulsation



Consider a one-zone model of a pulsating star, consisting of a central point mass $M$ and a single spherical shell of mass $m$ and radius $R$.




We have

$$m\frac{\mathrm{d}^2 R}{\mathrm{d}t^2} = -\frac{GMm}{R^2} + 4\pi R^2 P$$

At the equilibrium state, we have

$$\frac{GMm}{R_0^2} = 4\pi R_0^2 P_0$$

# Modeling Stellar Pulsation



Now let $R = R_0 + \delta R$ and $P = P_0 + \delta P$. With the first-order approximation

$$\frac{1}{(R_0 + \delta R)^2} \sim \frac{1}{R_0^2}\left(1 - 2\frac{\delta R}{R_0}\right)$$

we obtain

$$m\frac{\mathrm{d}^2(\delta R)}{\mathrm{d}t^2} = -\frac{GMm}{R_0^2} + \frac{2GMm}{R_0^3}\delta R + 4\pi R_0 2P_0 + 8\pi R_0 P_0 \delta R + 4\pi R_0^2 \delta P,$$

where $\mathrm{d}^2 R_0/\mathrm{d}t^2 = 0$ at the equilibrium state.

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where $\mathrm{d}^2 R_0/\mathrm{d}t^2 = 0$ at the equilibrium state.

Eliminating equal terms given by the equilibrium state, we have

$$m\frac{\mathrm{d}^2(\delta R)}{\mathrm{d}t^2} = \frac{2GMm}{R_0^3}\delta r + 8\pi R_0 P_0 \delta R + 4\pi R_0^2 \delta P.$$

Assume that the oscillations are adiabatic, that is, $PV^\gamma = \mathrm{const} \propto PR^{3\gamma}$. The linearized version of this expression is then

$$\frac{\delta P}{P_0} = -3\gamma\frac{\delta R}{R_0}$$

# Modeling Stellar Pulsation

Substituting $\delta P$ with $\delta R$ and rearranging the equation of motion, we have

$$\frac{\mathrm{d}^2(\delta R)}{\mathrm{d}t^2} = -(3\gamma - 4)\frac{GM}{R_0^3}\delta R.$$

If $\gamma > 4/3$, the above equation is for simple harmonic motion with the solution $\delta R = A\sin(\omega t)$, where $A$ is the amplitude and $\omega$ is the frequency:

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Finally, the **pulsation period of the one-zone model** is just

$$p = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{4}{3}\pi G\rho_0(3\gamma - 4)}}$$

where $\rho_0 = M/\frac{4}{3}\pi R_0^3$. Recall that for an ideal monoatomic gas, $\gamma = 5/3$. In an adiabatic process, monoatomic gases have an idealised $\gamma$-factor $(C_p/C_v)$ of $5/3$, as opposed to $7/5$ for ideal diatomic gases where rotation (but not vibration) also contributes. $C_p$ is the molar heat capacity at constant pressure, $C_v$ is the molar heat capacity at constant volume.

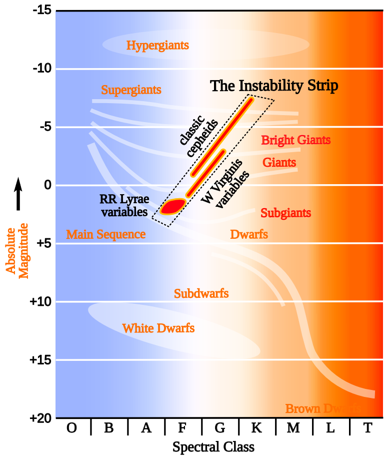
# Modeling Stellar Pulsation

In reality, the $\kappa$ mechanism involves **heat exchanges**, that is, the piston layer is not adiabatic.

In the case of nonadiabatic oscillations, the time dependence of the pulsation is taken to be $\delta R \propto \exp(i\sigma t)$, where $\sigma = \omega + i\kappa$. In this expression, $\omega$ is the usual pulsation frequency, while $\kappa$ is a stability coefficient that characterizes the time for the growth or decay of the oscillation.

# Cause of Pulsation

This is called **radial mode** pulsation. It is found in large-amplitude pulsating variables in the HR-diagram *instability strip*: Cepheids, Miras and RR Lyrae stars.



# Cause of Pulsation

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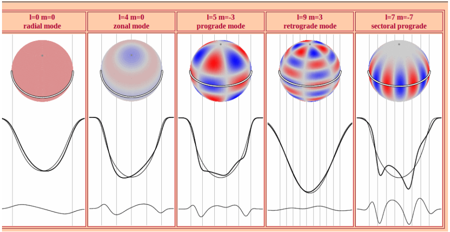
There are stars whose pulsation is **non-radial**: a star changes shape, but not volume. Non-radial pulsation leads to smaller amplitudes of variation. Some stars – $\beta$ Cephei, $\delta$ Scuti stars and to a small amount also RR Lyrae stars – pulsate in both radial and non-radial modes.




models of stars with non-radial pulsations (copyright Coen Schrijvers)
http://staff.not.iac.es/ jht/science/

# Types of Pulsating Variable Stars

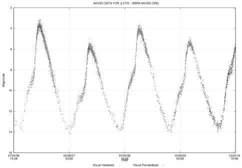
| Type | Range of Periods | Population Type | Mode |
|---|---|---|---|
| Long-Period Variables | 100 - 700 days | I, II | R |
| Classical Cepheids | 1 - 50 days | I, II | R |
| W Virginis | 2 - 45 days | II | R |
| RR Lyrae | 1.5 - 24 hours | II | R (NR) |
| $\delta$ Scuti | 1 - 3 hours | II | R, NR |
| $\beta$ Cephei | 3-7 hours | I | R, NR |
| ZZ Ceti stars | 100 - 1000 sec | I | NR |

# Types of Pulsating Variable Stars



**Miras**

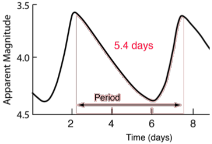
- red giants on the asymptotic giant branch (AGB), will expel their outer envelopes as planetary nebulae and become white dwarfs within a few million years
- pulsation periods longer than 100 days
- amplitudes greater than one magnitude in infrared and 2.5 magnitude at visual wavelengths



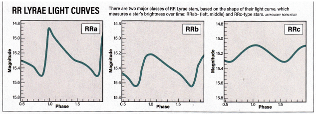
# Types of Pulsating Variable Stars



### Cepheids




- brightness enables us to observe them in other galaxies in our Local Group (such as the Magellanic Clouds, M31 and M33)
- period-luminosity relation makes them important standard candles $\Rightarrow$ distance ladder




### RR Lyrae stars

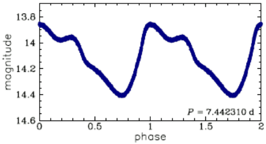
- numerous in the Milky Way halo (globular clusters), thus once called *cluster variables*
- less bright than Cepheids, shorter period ($\sim$1 day)
- period-luminosity relation and their age makes them important tracers of the old Milky Way halo substructure

# Cepheids

## Classical (Type I) Cepheids

bright yellow, highly luminous, supergiant pulsating variables

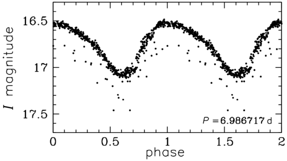
- amplitudes: $\sim$0.01 - 2 $\mathrm{mag}_V$
- periods: 1 - 135 days
- variability is strictly regular
- spectral type: F at maximum light, G to K at minimum light; the longer the period, the later the spectral type




## Population II (Type II) Cepheids

similar light curve than Type I, but different evolutional history

- older, low mass stars
- important fossils of the first generation of stars in our galaxy




Soszyński et al. (2018), OGLE data

# RR Lyrae stars

prototype: RR Lyrae (variability discovered by Williamina Fleming, $\sim$ 1900); RR Lyrae stars are old helium-burning variable stars of spectral type A5 to F5 with 0.5 $M_\odot$

**RRab**
asymmetrical light curves with steep ascend

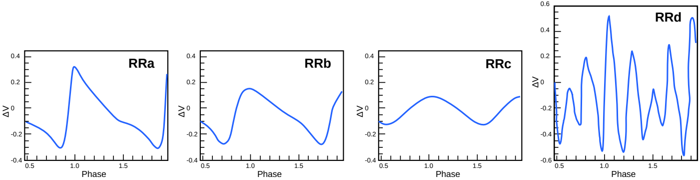
- periods: 0.3 - 1.2 days
- amplitudes: 0.5 - 2 $\mathrm{mag}_V$

**RRc**
nearly symmetrical light curves

- periods: 0.2 - 0.5 days
- amplitudes: $< 0.8$ $\mathrm{mag}_V$

**RRd**
double-mode RR Lyrae stars, fundamental and first overtone

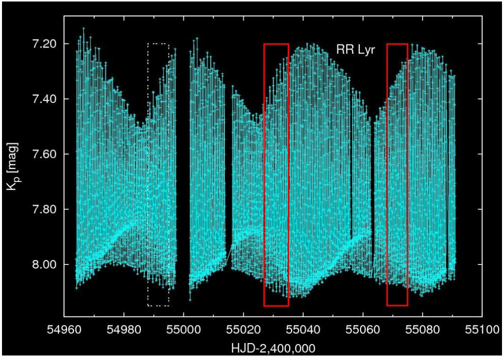
- fundamental period: 0.5 days
- period ratio: 0.74 days



# RR Lyrae stars

discovery by Harlow Shapley and Richard Prager (1916) independently:

RR Lyrae (the prototype's) light curve is modulated in amplitude and shape




observation of a RR Lyrae star with Blazhko effect from the Kepler survey

# RR Lyrae stars

period-amplitude-shape modulation is today known as the **Blazhko effect**

its explanation remains one of the enduring mysteries in astrophysics to this day

# RRLyrae stars



period-amplitude-shape modulation is today known as the **Blazhko effect**
its explanation remains one of the enduring mysteries in astrophysics to this day

two promising **theories** for explaining the Blazhko effect:

(i) resonance between the radial fundamental period of pulsation, and a non-radial period; or

(ii) a deformation or splitting of the radial period by a magnetic field in the star

# Pulsating Stars as Distance Estimators



The **distance modulus** equation alone is not enough:

$$d = 10^{(m - M + 5)}/5 \text{ parsec}$$

# Pulsating Stars as Distance Estimators

The **distance modulus** equation alone is not enough:

$$d = 10^{(m - M + 5)}/5 \text{ parsec}$$

?

Pulsating stars are a powerful tool for determining distances in astronomy, because the period of pulsation is correlated with the luminosity of the star, and this relation can be calibrated

⇒ the **period-luminosity(-metallicity) relation**

# Pulsating Stars as Distance Estimators

The best-known relation between period and absolute magnitude is the direct proportionality law for **Classical Cepheid variables** (Henrietta Swan Leavitt (1908)).

⇨ foundation for scaling **galactic and extragalactic distances**

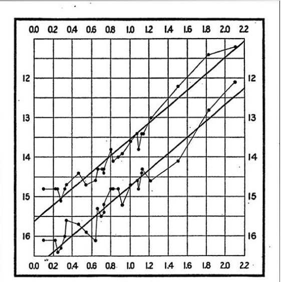



FIG. 2.

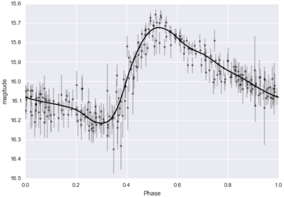
Plot from Leavitt's 1912 paper. The horizontal axis is the logarithm of the Cepheid's period, and the vertical axis is its apparent magnitude.

# Pulsating Stars as Distance Estimators



Cepheid and RR Lyrae stars are variable stars with the period being directly related to their true (absolute) brightness.

basic concept:




- measure apparent mean brightness $m$
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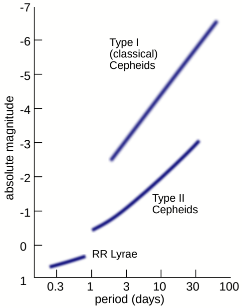


- measure apparent mean brightness $m$
- measure period $P$
- using **period-luminosity relation**, get absolute brightness $M$
- solve for distance using **distance modulus** equation $d = 10^{(m-M+5)}/5$ parsec where 1 parsec $= 3.086^{16}$ m $= 3.26156$ lyr

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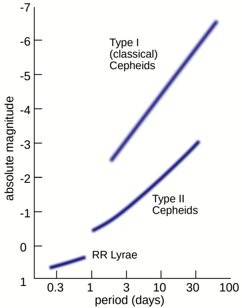


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$\Rightarrow$ allow us to create *3D maps* of structures within and beyond our Milky Way

# The Period-Luminosity(-Metallicity) Relation



Globular clusters have only little depth - we can treat all the stars in a cluster as being at $\sim$ the same distance from Earth

color-magnitude diagram of stars in a globular cluster (M3):

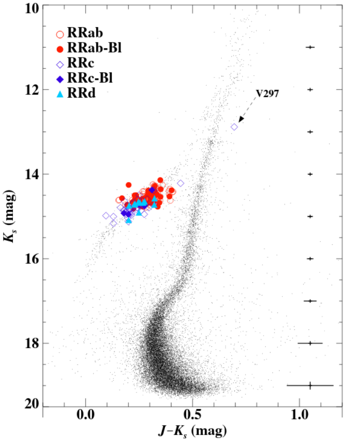


Figure 3 taken from Bhardwaj et al., AJ 160, 220 (2020)

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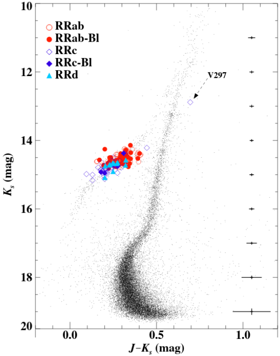


all RR Lyrae stars have $\sim$ the same apparent magnitude

$\Rightarrow$ as the distance must be $\sim$ the same, they also have the same absolute magnitude

if we know the absolute magnitude, we can compute the distance to each star from the distance modulus

# The Period-Luminosity(-Metallicity) Relation



A closer look:

for each RR Lyrae star in the cluster, plot the apparent magnitude as function of its period

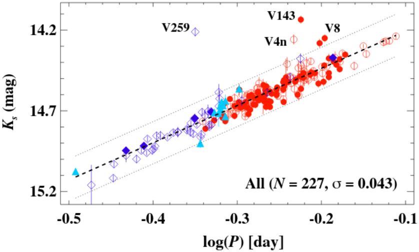


Figure 10 (slightly modified) taken from Bhardwaj et al., AJ 160, 220 (2020)

⇒ slight **trend**: stars with longer periods are a bit brighter

# The Period-Luminosity(-Metallicity) Relation



To put everything together:

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metallicity is the abundance of elements present in star that are heavier than hydrogen and helium

?

For $d(m, P, Z)$, we need to **calibrate** the Period-Luminosity(-Metallicity) Relation.

# The Period-Luminosity(-Metallicity) Relation



**calibrate** the Period-Luminosity(-Metallicity) Relation:

The following methods can be used to determine absolute magnitudes, e.g.:

- Statistical study of the motions of field RR Lyrae stars: statistical parallax. This gives values of $M_V$ ranging from $+0.9$ for short-period, high-metallicity stars, to $+0.5$ for longer-period, lower-metallicity stars. As a statistical method, it must be applied to a large sample of stars, which might not be homogeneous.

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- Fitting of the main sequence of globular clusters containing RR Lyrae stars to a standard main sequence determined for nearby Population II stars with known distances; there are, however, very few of these. This method gives a mean of about $+0.4$ for the RR Lyrae stars in several clusters.

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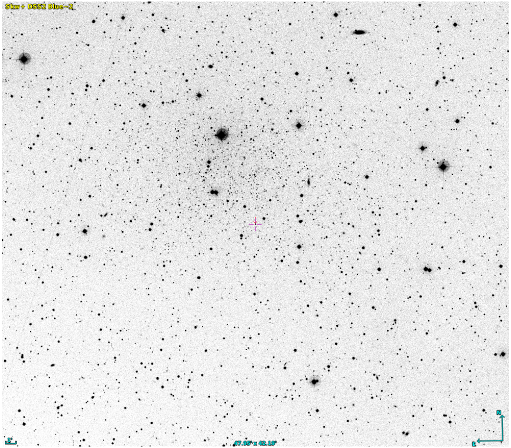
- The Baade-Wesselink method (infer distance from measurement of change in radius (from velocity) and angular diameter) has been applied to some of the brightest RR Lyrae stars; it gives an absolute magnitude of about $+0.5$.

# The Period-Luminosity(-Metallicity) Relation



## Distance to Draco dwarf spheroidal (dSph) galaxy

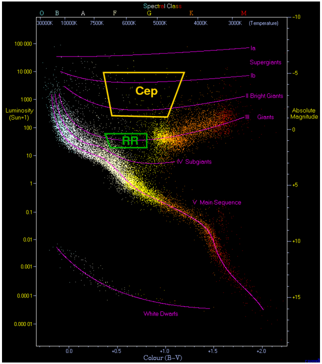


# The Period-Luminosity(-Metallicity) Relation



RR Lyrae stars allow for calculating the distance to globular clusters and dwarf galaxies from measuring their average apparent magnitude, and using the known absolute magnitude.

**problem:** RR Lyrae stars aren't really luminous. Take a look again at this HR diagram:



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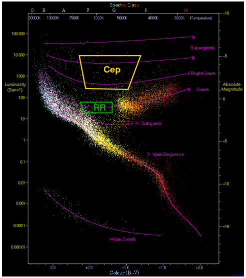


A typical RR Lyrae has a luminosity $\sim 50 L_\odot \Rightarrow$ not powerful enough for us to see these stars in distant galaxies.

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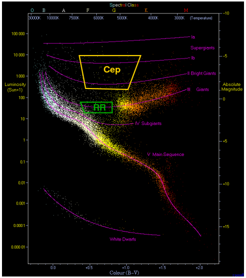
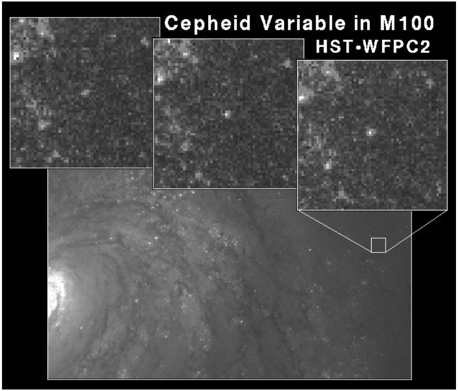



We can use RR Lyrae to measure distances only to the very closest galaxies - members of our own Local Group.
A better choice for **larger distances: Cepheids** which are 100 times more luminous than RR Lyrae stars.

# The Period-Luminosity(-Metallicity) Relation

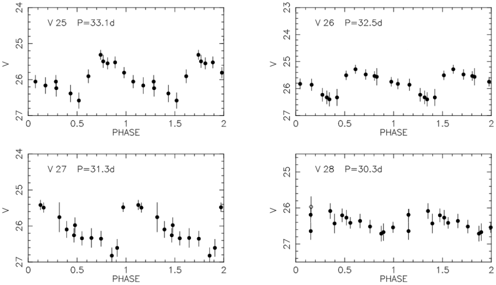


**example:** The Distance to M100



**example:** The Distance to M100

We determine the period and apparent magnitude of a few Cepheid light curves in this galaxy.



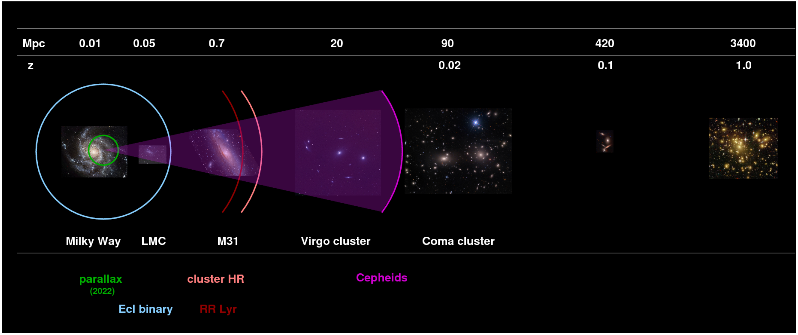

1. Pick one variable star. Measure its average apparent magnitude, $m_v$.
2. Use its period to determine its absolute magnitude $M_V$.
3. Compute the distance modulus $(m_V - M_V)$.
4. Compute the distance to this galaxy, in Mpc.

**Answer:** You should get a distance of $\sim 17$ Mpc.

# The Period-Luminosity(-Metallicity) Relation

As Cepheids are more luminous than RR Lyrae, with them we can measure distances farther into the depths of space.

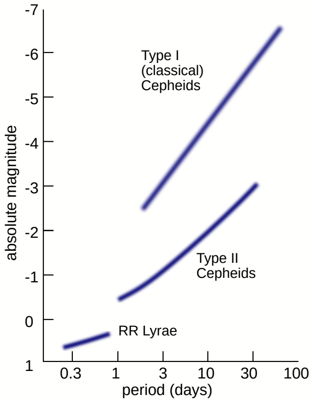
In particular, Cepheids allow to measure the distance to the nearest galaxy cluster, the Virgo Cluster.



# The Period-Luminosity(-Metallicity) Relation



Period-luminosity relations are known for several types of pulsating variable stars: type I Cepheids, type II Cepheids, RR Lyrae variables, Mira variables, and other long-period variable stars.



# The Period-Luminosity(-Metallicity) Relation



**Population I Cepheids:**

Population I Cepheids are stars with relatively high metallicity (like our Sun) and thus are *second generation* stars, found in the disk of our galaxy. Classical Cepheids (also known as Population I Cepheids, type I Cepheids, or Delta Cepheid variables) undergo pulsations with very regular periods on the order of days to months.

The following relationship between a Population I Cepheid's period $P$ and its mean absolute magnitude $M_{\mathrm{v}}$ was established from Hubble Space Telescope trigonometric parallaxes for 10 nearby Cepheids (Thomas et al. (2007), Benedict et al. (2002)):

$$M_{\mathrm{v}} = (-2.43 \pm 0.12)\,(\log_{10} P - 1) - (4.05 \pm 0.02)$$

# The Period-Luminosity(-Metallicity) Relation



**Population II Cepheids:**

Population II Cepheids are stars found in globular clusters that are low-metallicity "first generation" stars. Type II Cepheids are fainter than their classical Cepheid counterparts for a given period by about 1.6 magnitudes (about 4 times less luminous).

Period-Luminosity Relations of Population II Cepheids (McNamara 1995):

$P < 10$ days: $M_v = -1.61 \log_{10} P - 0.05$,

$P > 10$ days: $M_v = -4.17 \log_{10} P + 3.06$

The steep slope for stars with $P > 10$ days may be due to an increase of mass with the period of pulsation.

# The Period-Luminosity(-Metallicity) Relation



**RR Lyrae Stars:**

The PLZ relation is given as (e.g. Catelan2004, Sollima2006):

$$M_\lambda = \alpha_\lambda \log_{10}(P/P_{\mathrm{ref}}) + \beta_\lambda([\mathrm{Fe/H}] - [\mathrm{Fe/H}]_{\mathrm{ref}}) + M_{\mathrm{ref},\lambda} + \epsilon$$

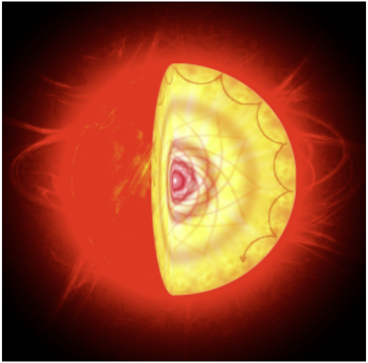
where $\lambda$ denotes the bandpass, $P$ is the period of pulsation, $M_{\mathrm{ref}}$ is the absolute magnitude at a reference period and metallicity, and $\alpha$, $\beta$ describe the dependence of the absolute magnitude on period and metallicity.

The $\epsilon$ is a standard normal random variable centered on 0 and with a standard deviation of the uncertainty in $M_\lambda$ in order to model the intrinsic scatter in the absolute magnitude convolved with unaccounted measurement uncertainties.

# Further Pulsation Analysis

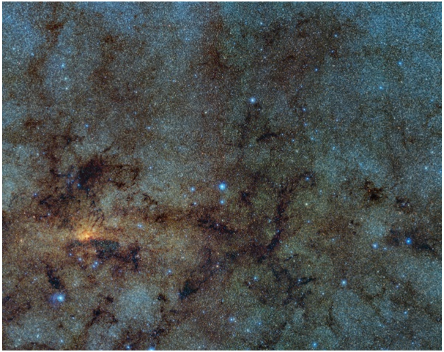
Pulsation is useful for **probing the interiors of stars**, and testing models of stellar structure and evolution: **Asteroseismology**.

Stars have many resonant modes and frequencies, and the path of sound waves passing through a star depends on the speed of sound, which in turn depends on local temperature and chemical composition.



# Further Pulsation Analysis

As pulsating stars allow for calculating distances, we can use them to **estimate interstellar reddening** (from dust).



This IR view of the Milky Way's crowded center (taken as part of the ESO's VISTA Variables in the Via Lactea Survey) reveils numerous RR Lyrae stars in our galaxy's bulge, hinting that it is old and may have been built up as primordial star clusters merged over time.

# Outlook

So far, we saw periodic variability in pulsating stars as an example for variability caused by stellar atmospheres.

Up to two-thirds of all stars have at least one companion, i.e. are in binary or multiple star systems. When this is the case, their atmospheres might interact.

In the next lecture, we will see how the interaction of stellar atmospheres can influence variability.