Iniciación en la Investigación (Semester 1 2025)

Lecture 2: The Blazhko effect (II)

Nina Hernitschek
Centro de Astronomía CITEVA
Universidad de Antofagasta

May 16, 2025

Possible explanations for the Blazhko effect

We have seen how the light curves from RR Lyrae stars with Blazhko effect look like, and what are possible methods to detect them.

What are possible **explanations** for the observed light-curve modulations?

Possible explanations for the Blazhko effect

Several models were proposed to explain the Blazhko phenomenon. Some of them are rather ideas, as they lack rigorous physical description.

Possible explanations for the Blazhko effect

Several models were proposed to explain the Blazhko phenomenon. Some of them are rather ideas, as they lack rigorous physical description.

Testing of the models faces a basic difficulty: we still lack realistic, 3D models that could correctly describe turbulent convection present in the envelopes of RR Lyrae stars and could describe the nonlinear interaction between the radial modes, and radial and nonradial modes.

Possible explanations for the Blazhko effect

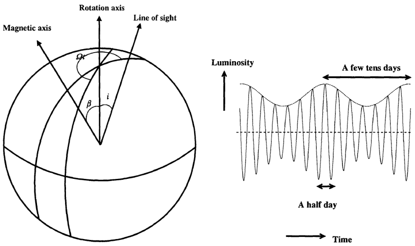
Nevertheless, some models could be ruled out on observational and/or theoretical grounds.
Here, a a few of those models is given.

For extensive review of some models, see Kovács (2009).

Possible explanations for the Blazhko effect

Magnetic oblique rotator/pulsator model (Shibahashi 2000)

Assumption: RR Lyrae stars have substantial magnetic fields, and their magnetic symmetry axis is inclined to the rotation axis of the star. The model is inspired by the observation that the modulation periods are of the same order as the rotation periods of those stars.
An rigid rotating star with angular frequency Ω and a dipole magnetic field is assumed, its symmetry axis passes through the stellar axis.

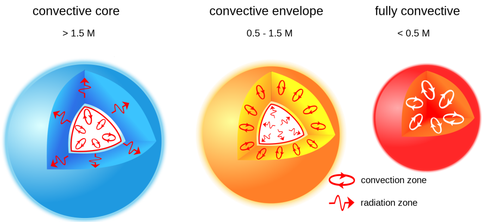


This model requires strong dipole magnetic field of 1 kG which is not detected (e.g. Kolenberg & Bagnulo 2009). Also, it predicts strictly periodic modulation, despite observed modulation can be strongly irregular.

Possible explanations for the Blazhko effect

Resonant nonradial rotator/pulsator model (Nowakowski & Dziembowski 2001)

Explains modulation as due to rotational splitting of nonradial modes that are in 1:1 resonance with the fundamental mode. The model predicts strictly periodic modulation which is, as from observations, clearly not the case.



Possible explanations for the Blazhko effect

Magnetic field variability (Stothers 2006)

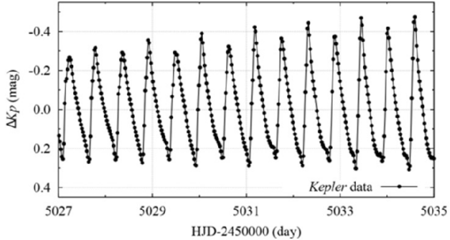
Explains modulations by a variable magnetic field affecting the efficiency of envelope convection and causes the modulation of pulsation. In this interpretation of the modulating Blazhko effect in RR Lyrae stars, turbulent convection inside the hydrogen and helium ionization zones becomes cyclically weakened and strengthened owing to the presence of a transient magnetic field that is generated in situ by either a turbulent or a rotational dynamo mechanism.

The idea lacks rigorous elaboration. For recent critique of this idea see Smolec et al. (2011) and Molnár et al. (2012a) who test variable turbulent convection as the cause of the Blazhko effect (the Stothers model).

Possible explanations for the Blazhko effect

9:2 radial mode resonance model (Buchler & Kolláth 2011)

The 9:2 radial mode resonance model explains period doubling in RR Lyrae stars by proposing that a resonance between the fundamental mode and the ninth radial overtone causes the observed phenomenon. This resonance shifts the period of the overtone by up to 10%, leading to the characteristic alternation of pulsation cycles with higher and lower amplitudes. Hydrodynamic models have successfully reproduced this period doubling and identified the 9:2 resonance as the underlying cause.



Period doubling in the light curve of RR Lyr, seen by Kepler. Source: Smolec (2016)

Possible explanations for the Blazhko effect

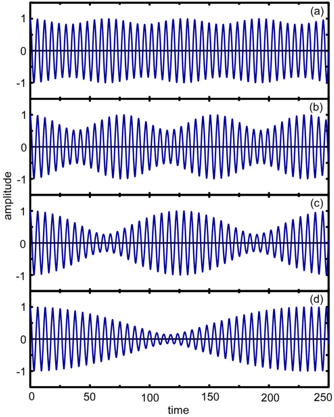
9:2 radial mode resonance model (Buchler & Kolláth 2011)

The 9:2 radial mode resonance model explains period doubling in RR Lyrae stars by proposing that a resonance between the fundamental mode and the ninth radial overtone causes the observed phenomenon. This resonance shifts the period of the overtone by up to 10%, leading to the characteristic alternation of pulsation cycles with higher and lower amplitudes. Hydrodynamic models have successfully reproduced this period doubling and identified the 9:2 resonance as the underlying cause.

This conclusion is based on the analysis of amplitude equations. The resulting modulation may be quasi-periodic or chaotic. There are no realistic hydrodynamic calculations confirming the hypothesis.

Possible explanations for the Blazhko effect

Nonresonant radial-nonradial mode interaction (Cox 2013, Bryant 2015b)
Assumes excitation of two modes, radial and nonradial, of nearly the same frequency, that are not phase-locked.



Normal mode displacement x for different coupling strengths C, where phase-locking would occur for $C \gtrsim 0.05$. Source: Bryant et al. (2016)

Simulating the Blazhko effect

We can (quite easily) simulate light curve shapes, also such that show an amplitude or period modulation.

Simulating the Blazhko effect

We can (quite easily) simulate light curve shapes, also such that show an amplitude or period modulation.

However: A hydrodynamic simulation that automatically generates the Blazhko effect would be important.

Convective Hydrocodes for Radial Stellar Pulsation

The "Warsaw code" refers to computational models and codes developed in Poland, particularly by researchers at the Nicolaus Copernicus Astronomical Center.

Convective Hydrocodes for Radial Stellar Pulsation

The ”Warsaw code” refers to computational models and codes developed in Poland, particularly by researchers at the Nicolaus Copernicus Astronomical Center.

The codes are used to simulate the internal processes of stars, including their convection, radiation, and pulsation dynamics. They can be used to predict the periods, amplitudes, and light curves of RR Lyrae stars.

Convective Hydrocodes for Radial Stellar Pulsation

The "Warsaw code" refers to computational models and codes developed in Poland, particularly by researchers at the Nicolaus Copernicus Astronomical Center.

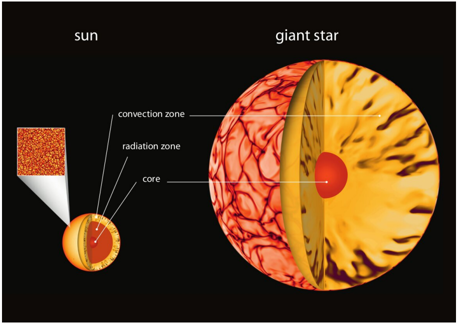
The codes are used to simulate the internal processes of stars, including their convection, radiation, and pulsation dynamics. They can be used to predict the periods, amplitudes, and light curves of RR Lyrae stars.

The predictions from the Warsaw codes are compared with actual observations of RR Lyrae stars, allowing researchers to refine their models and understand the physical processes driving these stellar pulsations.

See e.g. Smolec & Moskalik (2008).

Convective Hydrocodes for Radial Stellar Pulsation

Turbulent convection is an important physical process acting in many types of stars.



Turbulent convection is a **three-dimensional process** transporting energy through many length-scales: from macroscopic eddy cells to microscopic molecular scales, were energy is dissipated.

Convective Hydrocodes for Radial Stellar Pulsation

Stellar convection is complicated to model due to the turbulence.

However, many essential features of convection may be described with simple **one-dimensional models**.

Convective Hydrocodes for Radial Stellar Pulsation

In pulsating variable stars, large-scale motions of gas interact with smaller-scale turbulent motions and time-dependent models are necessary to describe the coupling between them.

In the code, all quantities are decomposed into mean and fluctuating parts, for example for velocity and temperature we have $\mathbf{U} = \bar{\mathbf{U}} + \mathbf{U}'$ and $\mathbf{T} = \bar{\mathbf{T}} + \mathbf{T}'$ respectively.

Convective Hydrocodes for Radial Stellar Pulsation

The convective hydrocodes implement the time-dependent turbulent convection model.

In the following, U stands for the fluid velocity, which is the time derivative of the radius R:

$$U = \frac{\mathrm{d}R}{\mathrm{d}t}$$

M_R is the mass enclosed in radius R, V is the specific volume (which is inverse of density ρ). E and P are energy and pressure of the gas including radiation, where E_t and P_t are turbulent energy and turbulent pressure. The viscous energy and momentum transfer rates are denoted by E_q and U_q. F_r, F_c and F_t are radiative, convective and turbulent flux, respectively. The C term describes the coupling between gas energy and turbulent energy equations.

Convective Hydrocodes for Radial Stellar Pulsation

Momentum and energy equations are then given by

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial}{\partial R}(P + P_t) + U_q - \frac{GM_R}{R^2}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} + P\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial[R^2(F_r + F_c)]}{R^2\partial R} - C$$

$$\frac{\mathrm{d}e_t}{\mathrm{d}t} + P_t\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial(R^2 F_t)}{\partial R} + E_q + C$$

Convective Hydrocodes for Radial Stellar Pulsation

Momentum and energy equations are then given by

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial}{\partial R}(P + P_t) + U_q - \frac{GM_R}{R^2}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} + P\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial[R^2(F_r + F_c)]}{R^2\partial R} - C$$

$$\frac{\mathrm{d}e_t}{\mathrm{d}t} + P_t\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial(R^2 F_t)}{\partial R} + E_q + C$$

The sum of the last two equations form the total energy equation:

$$\frac{\mathrm{d}(E + e_t)}{\mathrm{d}t} + (P + P_t)\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial[R^2(F_r + F_c + F_t)]}{R^2\partial R} + E_q$$

Convective Hydrocodes for Radial Stellar Pulsation

Momentum and energy equations are then given by

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial}{\partial R}(P + P_t) + U_q - \frac{GM_R}{R^2}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} + P\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial[R^2(F_r + F_c)]}{R^2\partial R} - C$$

$$\frac{\mathrm{d}e_t}{\mathrm{d}t} + P_t\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial(R^2 F_t)}{\partial R} + E_q + C$$

The sum of the last two equations form the total energy equation:

$$\frac{\mathrm{d}(E + e_t)}{\mathrm{d}t} + (P + P_t)\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{1}{\rho}\frac{\partial[R^2(F_r + F_c + F_t)]}{R^2\partial R} + E_q$$

A more detailed description can be found in Smolec & Moskalik (2008).

Convective Hydrocodes for Radial Stellar Pulsation

For the radiative transfer one may use time-dependent, detailed treatment (Vienna code), or simpler models, like diffusion approximation (Florida-Budapest code).

The first approach has higher accuracy and leads to better physical description of the model's structure, especially in the outer parts.

However, in classical pulsator's models, time dependent treatment gives essentially the same results as simple diffusion approximation (Kovács and Kanbur 1998, Feuchtinger, Buchler and Kolláth 2000).

Convective Hydrocodes for Radial Stellar Pulsation

Thus, the **diffusion approximation** is used in the hydrocode, which has the advantage of very low numerical costs.

In this approximation, the radiative flux F_r is given by

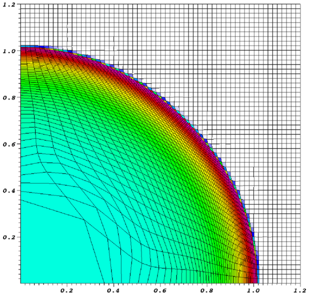
$$F_r = -\frac{4\sigma}{3} 4\pi R^2 \frac{1}{\kappa} \frac{\partial T^4}{\partial M}$$

Radiation pressure P_r and radiative energy E_r are included in P and E togther with gas contibution: $P = P_g + P_r$, $E = E_g + E_r$. We have $P_r = aT^4/3$ and $E_r = aT^4/\rho$ where a is the radiation constant.

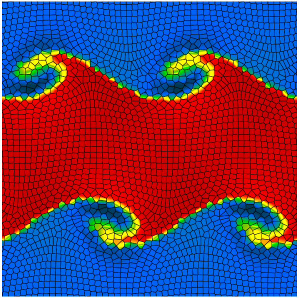
Pressure, energy as well as other thermodynamic quantities are calculated as function of T and V from the equation-of-state (EOS).

Convective Hydrocodes for Radial Stellar Pulsation

In pulsation hydrocodes, hydrodynamic equations are **discretized on a mesh**, which is either Lagrangian (fixed mass zones) or adaptive.



Lagrangian mesh



AREPO adaptive mesh

Convective Hydrocodes for Radial Stellar Pulsation

The more sophisticated adaptive grid is very useful in resolving the narrow features present in classical pulsator's models such as shocks or hydrogen partial ionization regions (PIR). These are not very well resolved by Lagrangian mesh.

Convective Hydrocodes for Radial Stellar Pulsation

The more sophisticated adaptive grid is very useful in resolving the narrow features present in classical pulsator's models such as shocks or hydrogen partial ionization regions (PIR). These are not very well resolved by Lagrangian mesh.

However, the use of adaptive mesh in purely radiative models just smoothed the light curves, not changing their overall shape (Buchler 1998). In convective models, hydrogen PIR is widened by the convective motions and therefore, is numerically less troublesome.

Convective Hydrocodes for Radial Stellar Pulsation

The more sophisticated adaptive grid is very useful in resolving the narrow features present in classical pulsator's models such as shocks or hydrogen partial ionization regions (PIR). These are not very well resolved by Lagrangian mesh.

However, the use of adaptive mesh in purely radiative models just smoothed the light curves, not changing their overall shape (Buchler 1998). In convective models, hydrogen PIR is widened by the convective motions and therefore, is numerically less troublesome.

Therefore, the "Warsaw code" uses the Lagrangian mesh.

Convective Hydrocodes for Radial Stellar Pulsation

In classical pulsators, inner parts of the star do not participate in the oscillations.

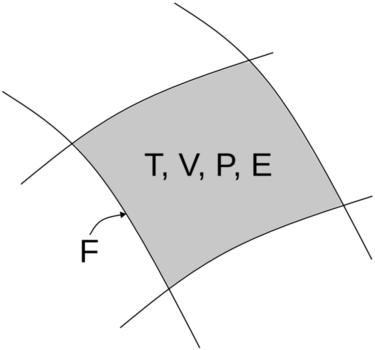
Thus, only the outer parts of the star, so-called **envelope**, are modeled. Rotation and magnetic fields are neglected.

The model is specified by:

- mass M
- luminosity L
- effective temperature T_{eff}
- chemical composition X and Z
- mixing-length parameter α, a free parameter that represents the efficiency of convection.

Convective Hydrocodes for Radial Stellar Pulsation

The model is divided into N mass zones according to the Lagrangian mesh. All quantities are defined either at the zones (thermodynamic quantities: T, V, P, E) or in between, at so-called interfaces (the fluxes).



Convective Hydrocodes for Radial Stellar Pulsation

The static model is constructed in **two steps**.

In the first step, an initial model is constructed without turbulent pressure and turbulent flux ($\alpha_p = \alpha_t = 0$), with turbulent energy and coupling term defined at the interfaces.

Convective Hydrocodes for Radial Stellar Pulsation

The static model is constructed in **two steps**.

In the first step, an initial model is constructed without turbulent pressure and turbulent flux ($\alpha_p = \alpha_t = 0$), with turbulent energy and coupling term defined at the interfaces.

This initial model is constructed by integration of the static equilibrium equations from the surface inward.

Convective Hydrocodes for Radial Stellar Pulsation

The static model is constructed in **two steps**.

In the first step, an initial model is constructed without turbulent pressure and turbulent flux ($\alpha_p = \alpha_t = 0$), with turbulent energy and coupling term defined at the interfaces.

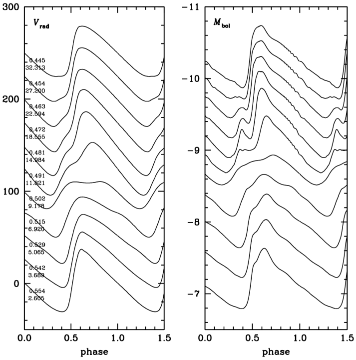
This initial model is constructed by integration of the static equilibrium equations from the surface inward.

The final model, with turbulent pressure and turbulent flux included and with turbulent energy and coupling term defined at the zones, is constructed through the **multivariate Newton-Rhapson iterations**.

Newton's method (also called Newton-Raphson) is an iterative method for finding the roots of a differentiable function f, which are solutions to the equation $f(x) = 0$.

Convective Hydrocodes for Radial Stellar Pulsation

For **testing the code**, Smolec & Moskalik (2008) use Galactic chemical composition, $X = 0.7$ and $Z = 0.02$, and mass-luminosity relation: $\log(L/L_{\odot}) = 3.56 \log(M/M_{\odot}) + 0.7328$ (Szabó et al. 2007).



Full amplitude radial velocity curves (left panel) and bolometric light curves (right panel). Model masses are increasing by 0.5 $M_{\odot}$, starting from 4.0 $M_{\odot}$ at the bottom of the figures up to 9 $M_{\odot}$ at the top. Source: Smolec & Moskalik (2008), Fig. 5.

Concerning ϕ_{21} the model curves are shifted toward shorter periods in comparison to observations. This is connected with the location of the $2\omega_0 = \omega_2$ resonance center.

Convective Hydrocodes for Radial Stellar Pulsation

Period-amplitude modulations, period doubling etc. can be found in some simulations, e.g.:

Smolec, R.; Survey of non-linear hydrodynamic models of type-II Cepheids; MNRAS, 456, 3475 (2016); PDF, `arXiv:1512.01550`

Smolec, R., Moskalik, P.; Chaos in hydrodynamic BL Herculis models; MNRAS, 441, 101 (2014); PDF, `arXiv:1403.4937`

Convective Hydrocodes for Radial Stellar Pulsation

The possible models are constrained by **observational facts** and by the laws of physics.

According to Zoltán Kolláth (2018)[1], the major observational constraints that one should take into consideration are the following:

- The amplitude of the modulation is comparable to the amplitude of the pulsation mode itself. Most of the mechanisms in models are weak in generating large amplitude modulations.
- The incidence rate of modulated stars are high, about 50% (see e.g. Jurcsik et al., 2009; Prudil & Skarka, 2017). The models should also answer what is the selection process between modulated and stable pulsations.
- The observed time scale of the modulation is an important empirical parameter of the Blazhko effect - all modulation scales can be handled by the theory. In addition, multiple modulation scales present another restriction for most of the models.

[1] `https://www.pta.edu.pl/pliki/proc/vol6/v6p137.pdf`

Convective Hydrocodes for Radial Stellar Pulsation

- Structure of the side lobes, asymmetries, complex frequency displacements excludes lots of the mechanism.
- Extra (Additional) modes or peaks in the Fourier spectra can be a feature independent from the Blazhko effect, but the pulsation models should provide some clues on the multi-mode properties of RR Lyrae stars.
- Anomaly of the infrared and the radius modulation strength is a very recent constraint (Jurcsik & Hajdu, 2017; Jurcsik et al., 2018) on theory. It is also a dichotomy between radius variation and the light curve or it can be simply treated as the dependence of modulation amplitude on the depth of the stellar atmosphere.
- The occurrence of period doubling in Blazhko modulated light curves present another possible constraint on models. Note that it appears with high amplitude only at specific phases of the Blazhko cycle.
- Finally any idea that is not compatible with the known physics of stellar pulsation should be ruled out.