Iniciación en la Investigación (Semester 1 2025)

Lecture 3: Detecting the Blazhko Effect

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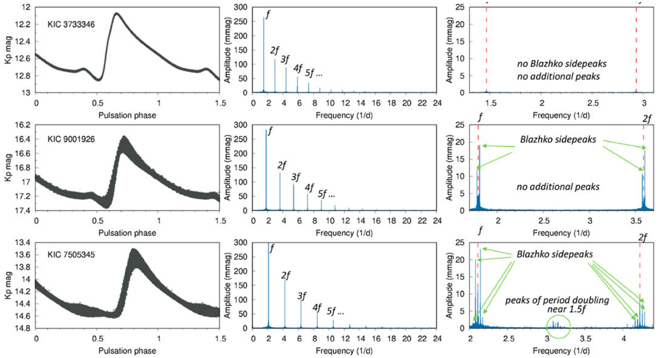
Motivation

With short cadences, survey such as Kepler enable for a detailed light curve analysis.



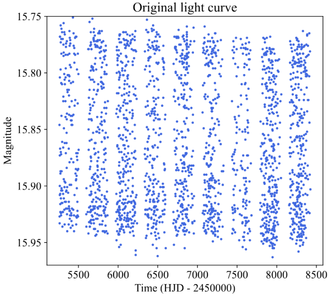
Examples of non-Blazhko, Blazhko, and period doubled Blazhko RRab stars of the Kepler field. Panels show folded light curves, Fourier spectra, and zooms of residual spectra, respectively. Source: Benkö et al. (2014).

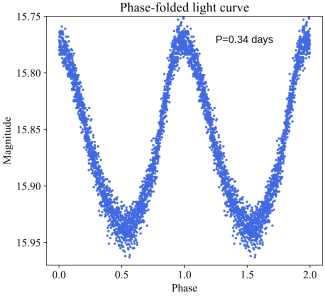
Motivation

In the following, we will see several methods for a detailed light-curve analysis to detect and describe the Blazhko effect.

Phase-Folding

Idea: Instead of showing the time on the x axis, show the phase.





The result is known as **phase diagram** or a **folded light-curve**.

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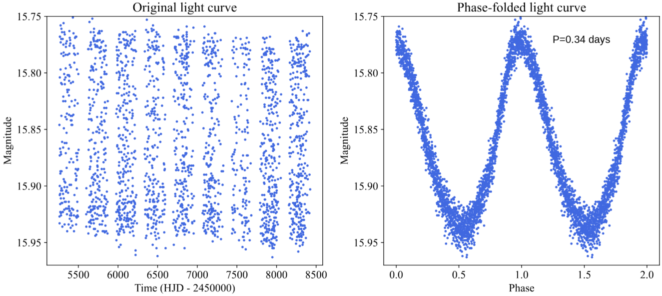

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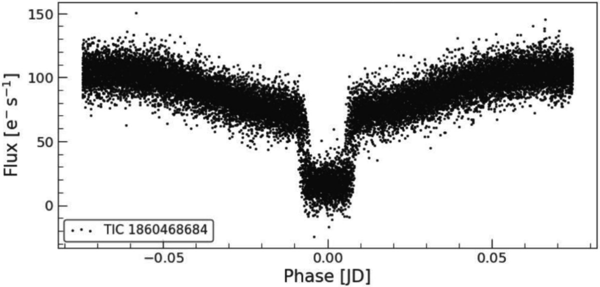
It can be made for any kind of periodic signal, not only pulsating variable stars like RR Lyrae.

Phase-Folding



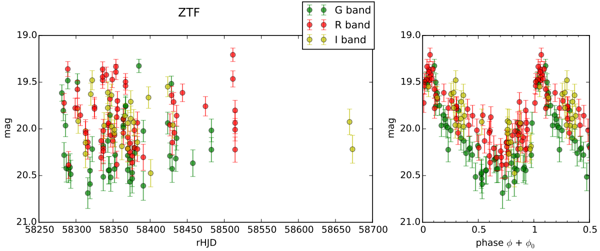
Phase-folded light curve of a possible binary system.

Phase-Folding

Detailed Light Curve Analysis

The **phase** is the fraction of the cycle, from zero to one whole period. Periodic features such as peaks and dips will appear at the same phase point along the horizontal axis if the correct period is used.

```
phase = (time % period)/period + phaseoffset
```



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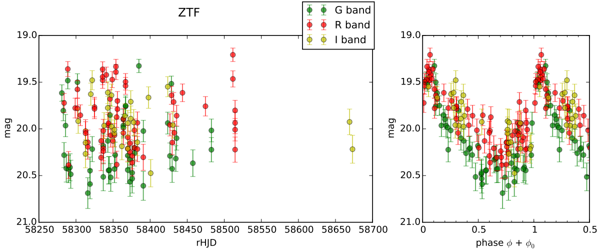
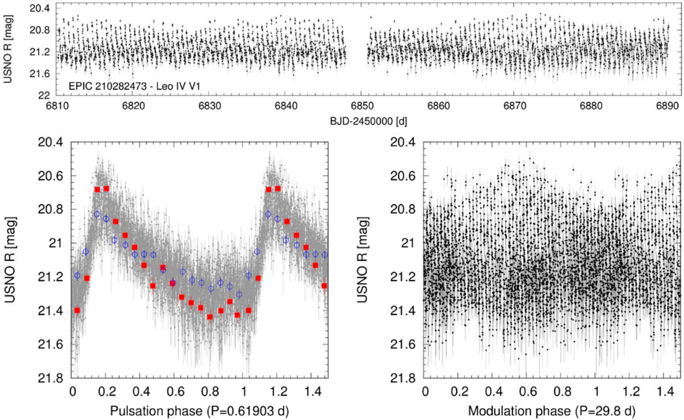


This process helps to enhance the signal-to-noise ratio and reveal the underlying periodic pattern, particularly useful for detecting faint or noisy signals.

necessary for this: determining the period!

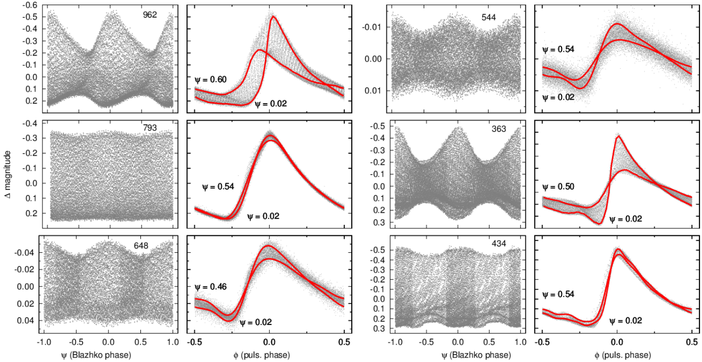
Phase-Folding



Upper panel: The light curve of the faintest Blazhko star from the K2 mission, EPIC 210282473. Lower left panel: phase curve folded by the pulsation period, 2-day binned data from the maximum-amplitude phase (red) and minimum-amplitude phase (blue). Lower right panel: phase curve folded with the modulation period. Source: Molnár et al. (2015).

Phase-Folding

Detailed Light Curve Analysis



Phase-folded CoRoT Blazhko light curves. Left: folded by the Blazhko-period, right: folded by the pulsational period. From Szabó et al. (2014)

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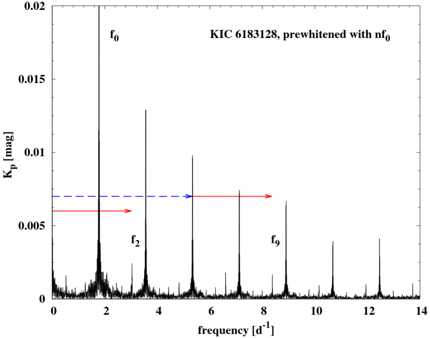
The Phase Dispersion Minimization technique (Stellingwerf 1978) consists of folding the data at candidate periods, together with a binning analysis of the variance at each candidate periods. It is a least squares fit, as in the periodogram, but to a mean curve that is determined by the data, rather than a sine wave.

Fourier Analysis

Fourier analysis can be used for frequency (period) detection, but also to find significant peaks beside the pulsation frequency.

Such were found in RR Lyrae data from the CoRoT and Kepler space telescopes.



Fourier spectrum of Kepler data of V354 Lyr (KIC 6183128). Note the higher amplitude $nf_0 + f_2$ peaks at 6.6 and 8.4. The latter may indicate the presence of a resonant f_9 mode. Solid and dashed arrows visualize the possible $3f_0 + f_2 = f_9$ resonance. (Molnár et al. 2012)

Fourier Analysis

How to search for multiperiodicity

As a first step, the lightcurve is fitted with the Fourier sum of the form

$$m_I(t) = \langle m_I \rangle + \sum_k A_k \cos(2\pi k f_o t + \phi_k) \quad (1)$$

The **pulsation frequency** $f_0 = 1/P_0$ is also adjusted in the fitting process.

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The **pulsation frequency** $f_0 = 1/P_0$ is also adjusted in the fitting process.

If instrumental systematics are present in the data, they can be added to Eq. (1) e.g. by a cosine term.

Fourier Analysis

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1. The fit Eq. (1) is subtracted from the data. The Fourier power spectrum of the residuals is then computed in order to reveal a secondary frequency, if present.

2. The fitting equation Eq. (1) is supplemented with an additional cosine term with frequency f_1. The data is then fitted for different trial values of f_1, keeping fixed the primary frequency f_0 and the number of its harmonics, but recalculating their amplitudes and phases for each trial. A secondary frequency, if present in the data, should reduce the dispersion of the fit significantly.

Fourier Analysis

How to search for multiperiodicity

As a third step, the Fourier fit with two identified frequencies and their linear combinations is performed. To this effect, we fit the lightcurve with the following equation:

$$m_I(t) = \langle m_I \rangle + \sum_{k,n} A_{k,n} \cos\left[2\pi(kf_0 + nf_1)t + \phi_{kn}\right] \tag{2}$$

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The search for additional periodicities is then **repeated**, again using both methods: Fourier transform of residuals (method 1) and minimization of dispersion with respect to a new trial frequency f_2 (method 2). The process is stopped when no significant new terms appear.

O-C Method

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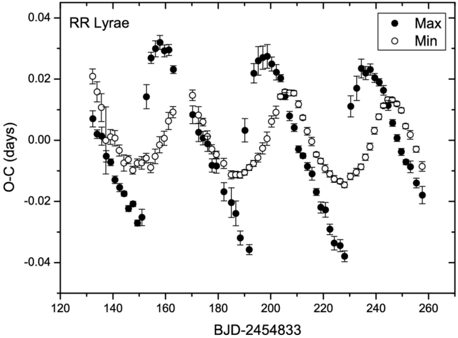
More precisely, the methods involves **comparing observed times of maximum light** with times predicted by a given period and phase offset. Deviations between these observed and calculated times, known as O-C variations, can reveal information about the star's pulsational properties and also potential binary nature.

O-C Method

In **observed-minus-calculated (O-C) diagrams**, long-term deviations from a light curve with constant period are plotted against time.
As the name suggests, O-C diagrams measures this difference between the calculated or modelled (C) phase assuming a constant period, and the observed (O) phase. To calculate accurate residuals, accurate periods are necessary.



O - C diagram of the star RR Lyr for the pulsation maxima and minima. Source: Li & Qian (2014)

O-C Method

Caution must however be taken, as different effects other than the Blazhko effect can lead to O-C variations.

We see this in the following example on how to separate causes of O-C variations.

O-C Method

Separating the causes of O-C variations in an RR Lyrae star with the Blazhko effect

A complete example on light curve analysis from Sylla et al. (2024).

Binarity in pulsating stars can be revealed through well-timed photometric data over sufficiently long time bases because of the light travel time effect (LTTE) on the pulsations, which manifests as variation in the timings of maximum light in the O-C (observed minus calculated) diagram. However, O-C variations can also have other causes, such as the Blazhko effect or a sudden or gradual change in the main pulsation period.

The paper by Sylla et al. (2024) aims to disentangling the Blazhko effect and period changes from the potential LTTE on V1109 Cas, an RR Lyrae star suspected to be part of a binary system.

O-C Method

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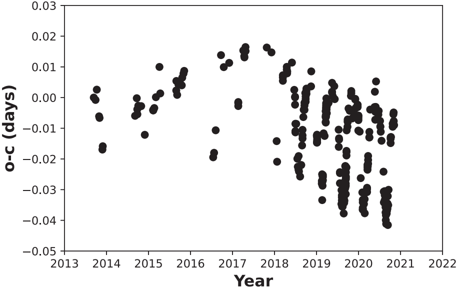
Methods: Fourier analysis to model the pulsation modulated by the Blazhko effect. From the fit to the observed light curves, a refined O-C diagram without the scatter caused by Blazhko modulation was constructed.

Results: If the remaining O-C variation is due to a period break, refining the O-C diagram can almost entirely remove the trends. If interpreting the variation as an LTTE, different possible configurations can be considered.

O-C Method

Data

The star V1109 Cas has an apparent visual magnitude ranging from 12.67 to 13.47. The potential binarity of this star was revealed by its O-C variations from the GEOS RR Lyrae Survey (GRRS; Le Borgne et al. 2018).



Original O-C diagram of the suspected binary RR Lyrae star V1109 Cas from the GEOS database, based on a mean period of $P_0 = 0.436196000$ days. Source: Sylla et al. (2024)

O-C Method

Methodology

Calculation of the O-C diagram is done in the following way:
For a given pulsation period P, one can index each time of maximum by an integer E, counting the number of maxima that have occurred since an initial heliocentric Julian date, HJD_0. for regular pulsation (i.e.: no modulation of period or amplitude), the time C_E of the Eth maximum is:

$$C_E = HJD_0 + E \times P. \tag{1}$$

For an observation recorded at time HJD, the first step consists in determining the count E, the number of pulsation cycles since HJD_0:

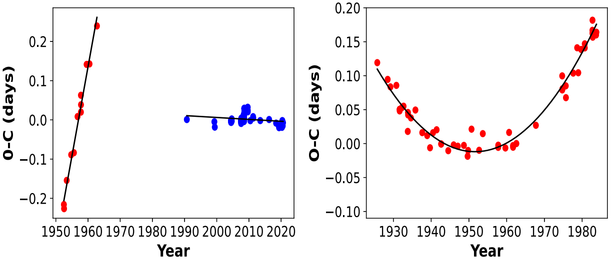
$$E = \lfloor (HJD - HJD_0)/P \rceil , \tag{2}$$

where the $\lfloor \rceil$ symbol stands for *round to the nearest integer*. For a given E, the date of the observed maximum is OE and can be compared to the corresponding time CE calculated using Eq. (1). A set of $OE - CE$ is named $O - C$. If the star is pulsating regularly and the correct period P is known, all $O - C$ sets should be equal to zero.

O-C Method

Methodology

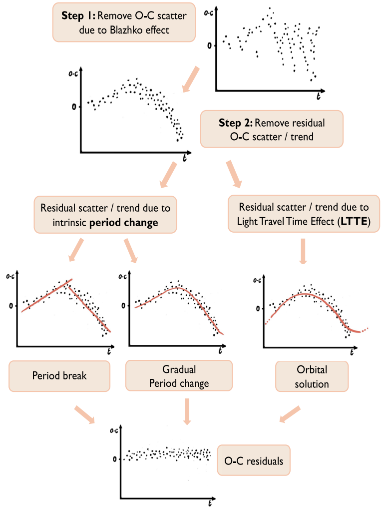
The Blazhko effect leads to magnitude changes at maximum brightness, whereas the induced magnitude changes at the minimum brightness are much smaller.



O-C diagrams showing a period break with a sequence of linear fits for AV Dra (left) and parabolic behaviour with a parabolic fit for BN Aqr (right).

O-C Method

Schematic overview of the methodology to determine refined O-C diagrams for the suspected binary RR Lyrae star V1109 Cas.



O-C Method

Fourier analysis to account for the Blazhko modulation

A Fourier fit for a Blazhko-modulated star encompasses the primary frequency, its harmonics, and often symmetrical multiplet patterns around these frequencies, indicative of the Blazhko effect.
We are following the approach outlined by Kolenberg et al. (2011):

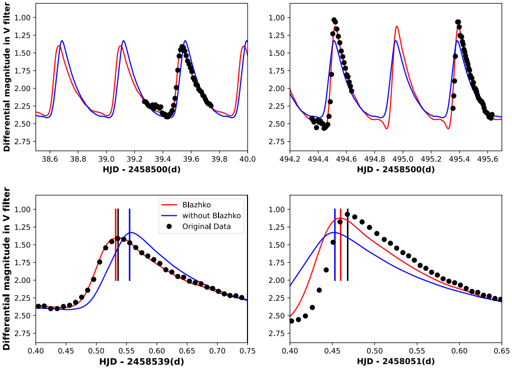
$$
\begin{aligned}
m(t) = & A_0 + \sum_{i=1}^{n} [A_i \sin(2\pi(kf_0 t + \phi_i)) \\
& + A_i^{+} \sin(2\pi((kf_0 + f_B)t + \phi_i^{+})) \\
& + A_i^{-} \sin(2\pi((kf_0 - f_B)t + \phi_i^{-}))] \\
& + B_0 \sin(2\pi(f_B t + \phi_B))
\end{aligned}
$$

O-C Method

Detailed Light Curve Analysis

Refined O-C diagrams

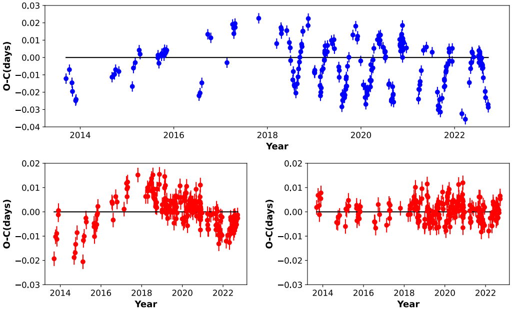
Refined O-C diagrams are made based on a constant mean pulsation period modulated due to the Blazhko effect.



A subset of the photometric data and the best fits without (blue) and with Blazhko effect (red). Differences in O-C values of the order of several minutes are found. Source: Sylla et al. (2024), Figure 5.

O-C Method

Refined O-C diagrams



O-C diagrams extracted from the Blazhko effect with a mean period (bottom left), and with a period break (bottom right). In the bottom left panel, we see a remaining variation that could be due to an orbit or an intrinsic period change. In the top panel, the blue color represents the case where no Blazhko effect is extracted.
Source: Sylla et al. (2024), Figure 6.

O-C Method

Results from refined O-C diagrams

The analysis reveals that a period break is a likely explanation for the O-C variations in the RR Lyrae star V1109 Cas.

Without considering the Blazhko effect, the O-C σ is equal to 0.013 days (18 minutes). Subsequently, after subtracting the Blazhko effect, considering a mean pulsation period, the standard deviation amounts to 0.008 days (11 minutes). Taking into account a period break reduces the standard deviation to 0.006 days (8 minutes), as illustrated in Figure 6.

The residual O-C dispersion can be due to various factors, including insufficiently modeled period changes or modulations. However, we do not see a clear remaining trend in the residual O-C diagram, which would be indicative of orbital motion. Thus, taking into account a period break removes the potential orbital signature from the O-C diagram.

Wavelet Transform

The Fourier transform is a special case of the wavelet transform. The **continuous wavelet transformation** of a function of time $x(t)$ is defined as (Grossman et al. 1989):

$$W(\omega, \tau; x(t)) = \omega^{1/2} \int x(t) f^*(w(t-\tau))\mathrm{d}t \quad (1)$$

$$= \omega^{-1/2} \int x(\omega^{-1} z + \tau) f^*(z)\mathrm{d}z \quad (2)$$

where f^* is the complex conjugate of f, and the function $f(z)$ is the **wavelet kernel**.

The transformation depends on two parameters, the scale factor ω and the time shift τ.

By chosing a wavelet kernel which is concentrated near $z = 0$, we explore the behavior of $x(t)$ near $t = \tau$.

Weighted Wavelet-Z (WWZ)

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The **discrete wavelet transform** (and also the discrete Fourier transformation) can be applied to an observed time series consisting of N data values $x(t_\alpha)$, taken at a discrete set of N times $\{t_\alpha, \alpha = 1, 2, ..., N\}$.

The **discrete Fourier transform (DFT)** is

$$D(\omega; x(t)) = N^{-1} \sum_{\alpha=1}^{N} x(t_\alpha) e^{i\omega t_\alpha} \quad (3)$$

The discrete wavelet transform (DWT) is

$$W(\omega, \tau; x(t)) = \sqrt{\omega} \sum_{\alpha=1}^{N} x(t_\alpha) f^*(\omega(t_\alpha - \tau)) \quad (4)$$

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The WWZ algorithm projects the data onto a set of sine and cosine trial functions (the waveform); a test sine with fixed frequency is fitted to the data using the Gaussian wavelet window function

$$W(\omega, \tau) = \exp[-c\omega^2(t-\tau)^2] \tag{5}$$

as the weighting function of the data, where ω is the test frequency, τ is the center of the test window, and c is a tuning constant used to adjust the width of the window. The algorithm was designed with irregularly sampled data in mind and is analogous to date-compensated Fourier methods.

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WWZ analysis is available, for example, in the following Python library: `https://www.astro.princeton.edu/~jhartman/vartools.html` (see also the paper Hartman & Barkos 2016)

Summary

Methods for detecting period-amplitude modulations such as the Blazhko effect typically involve phase-folding.

First indicators of the Blazhko effect can often be seen from a phase-folded light curve.

A detailed analysis is possible with applying methods such as a series of O-C diagrams calculated, or Weighted Wavelet-Z.