Wind Power Forecast Analysis

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02418 Statistical Modelling: Theory and practice
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Overview

Analysis of Wind Power

The weekly returns for an ETF is analyzed and modeled.

Methods of analysis:

- Descriptive statistics
- Wind Power transformation
- Wind Direction: von Mises
- Wind Speed: log-normal
- Regression Model
- Residual analysis
- AR(1) Time-series model

Descriptive Statistics: Wind Power

Summary Statistics					
Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
2.5e-05	0.051	0.193	0.276	0.439	0.936

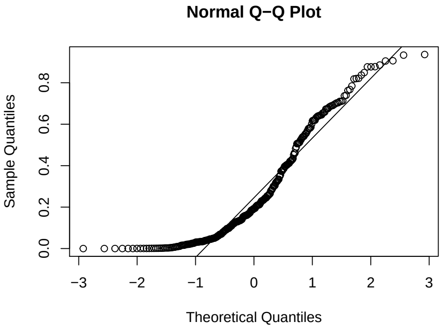

Figure: Wind power plotted to test normality

Wind Power Transformations

Transformation 1: $(\lambda > 0)$

$$y^{\lambda} = \frac{1}{\lambda} log(\frac{y^{\lambda}}{1 - y^{\lambda}}) \tag{1}$$

Transformation 2: $(\lambda \in (0, 1))$

$$y^{\lambda} = 2 log(\frac{y^{\lambda}}{(1 - y)^{1-\lambda}}) \tag{2}$$

Box-Cox Transformation

$$\begin{cases} y^{\lambda} = \frac{y^{\lambda} - 1}{\lambda} & \lambda \neq 0 \\ ln(y) & \lambda = 0 \end{cases} \tag{3}$$

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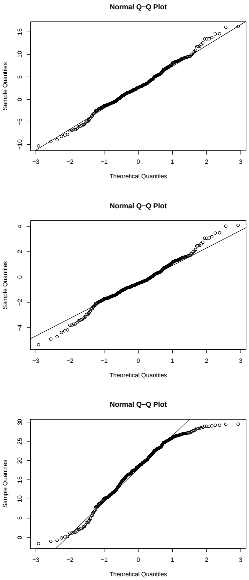
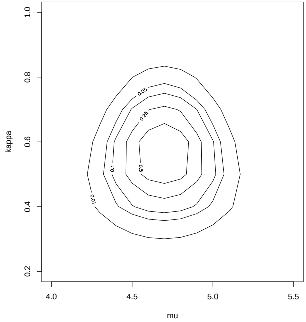
Lambda values = (T1 0.35, T2 0.25, BC 0.25)



Wind Direction

von Mises distribution is a continuous probability distribution on the circle.

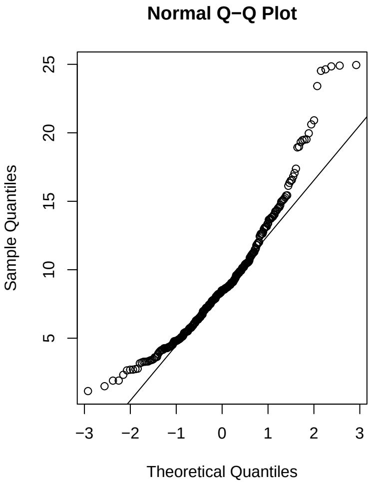


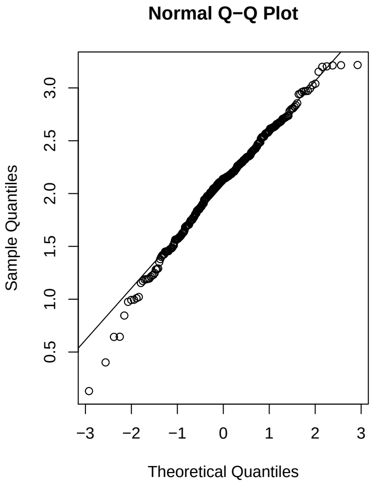
Parameters for WD		
Parameter	Value	95% CI
μ	4.696	[1.238, 8.154]
κ	0.564	[0.352, 0.776]

Conclusion

Wind direction according to this model is most frequently in the southwest direction (0 = north) but with low confidence.

Wind Speed





$\mu = 2.08$

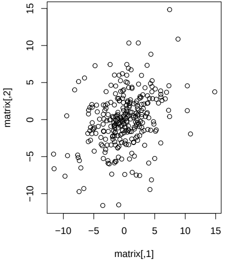
Regression models

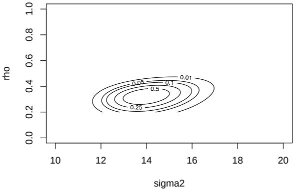
$$y^{\lambda} = \beta_0 + \beta_1 ws + \beta_2 ws^2 \tag{4}$$

Model	AIC
m1	1583.305
m2 (with ws log)	1585.400
m3 (ws^3 *term*)	1585.258
m4 (without ws^2)	1601.692
m5 (with wd)	1585.300
m6 (with wd using von misses)	1585.208

Regression Model Summary ($\lambda = 0.2$)				
	Value	95% CI	t	p
Intercept	-7.146	[-8.926, -5.366]	-7.869	7.49e-14
WS30	1.7048	[1.370, 2.039]	9.994	$<$ 2e-16
WS30^2	-0.0321	[-0.046, -0.018]	4.572	7.20e-06

Residual Analysis





Results

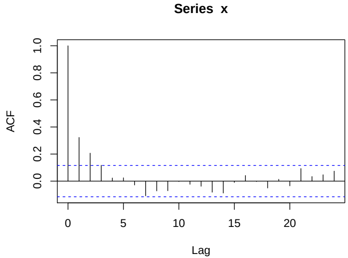
$\sigma^2 = 13.94$ $\rho = 0.32$
The information matrix was computed by numerical methods and compared with the algebraic form for the Fisher information:

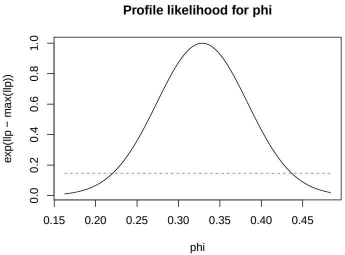
$$\begin{vmatrix} 1.477 & -7.429 \\ -7.429 & 395.142 \end{vmatrix}$$

$$e = \begin{bmatrix} e_1 & e_2 \\ \vdots & \vdots \\ e_{n-1} & e_n \end{bmatrix} \qquad \Sigma = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$$

AR(1) Time-series Model

$$x_t = \theta_0 + \phi_1 x_{t-1} + et \tag{5}$$





AR and Linear Model Combination

AR(1) model plus the linear model. Residuals are more normal, likelihood is less