CISC-271 Assignment 4

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Problem #1: **Least-Squares Crossing Lines**

1) The focus point:

5.0844
5.1362
20.5131

dj:

```
result =

    0.1953    0.1943   -0.1590   -0.1192         0
    0.1890   -0.1472    0.1487   -0.1192    0.1063
    0.9624    0.9698    0.9760    0.9857    0.9943
```

2) Summarize your results:

Calculation of the focus point:

```
function [c] = Question1(P,S)

Upper = S - P;
distance = sqrt(sum((S-P).^2));

distance = [distance;distance;distance];
result=Upper./distance;
A = zeros(3);
B = zeros(3,1);
for i=1:5
    Dj = eye(3)-(result(:,i)*result(:,i)');
    D1 = Dj'*Dj;
    A = A + D1;
    D2 = Dj'*Dj*P(:,i);
    B = B + D2;
    c = A\B;
end
```

Upper calculates the upper part of this equation: $\vec{d}_j = \dfrac{\vec{x}_j - \vec{r}_j}{\|\vec{x}_j - \vec{r}_j\|}$ which is (SpatialPoints – PlanarPoints).

Distance calculates the lower part of this equation (||Xj – Rj||). It is calculated by:

$\sqrt{(x1-x2)^2 + (y1-y2)^2 + (z1-z2)^2}$

x1, y1, z1 : SpatialPoints

x2, y2, z2 : PlanarPoints

distance = [distance;distance;distance] was written so that distance would be a 3x5 matrix instead of a 1x5 matrix. This way when calculating dj, the matrix size would match.

$$D_j = I - [\vec{d}_j\, \vec{d}_j^{\,T}]$$

eye(3) is an identity matrix size 3x3, Dj calculates this equation: $D_j = I - [\vec{d}_j\, \vec{d}_j^{\,T}]$

$$\left[\sum_{j=1}^{m} \overbrace{D_j^T D_j}^{D1}\right] \hat{c} = \left[\sum_{j=1}^{m} \overbrace{D_j^T D_j \vec{r}_j}^{D2}\right]$$

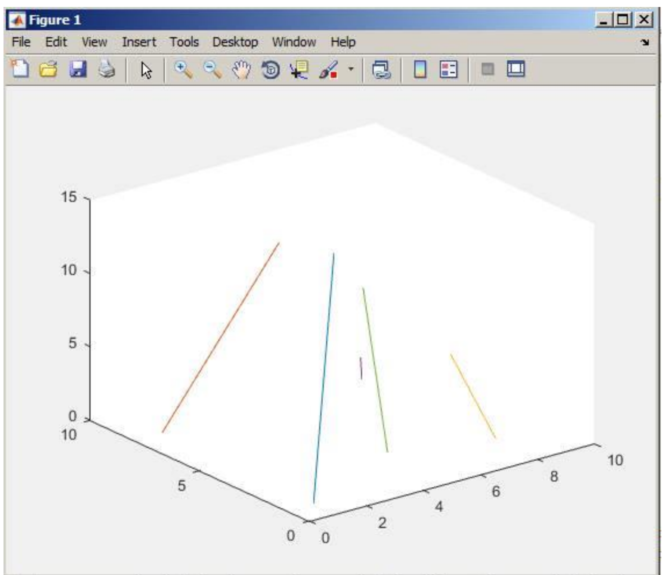
D1 calculates the left part of the equation each time (added onto A). D2 calculates the right part of the equation (added onto B).

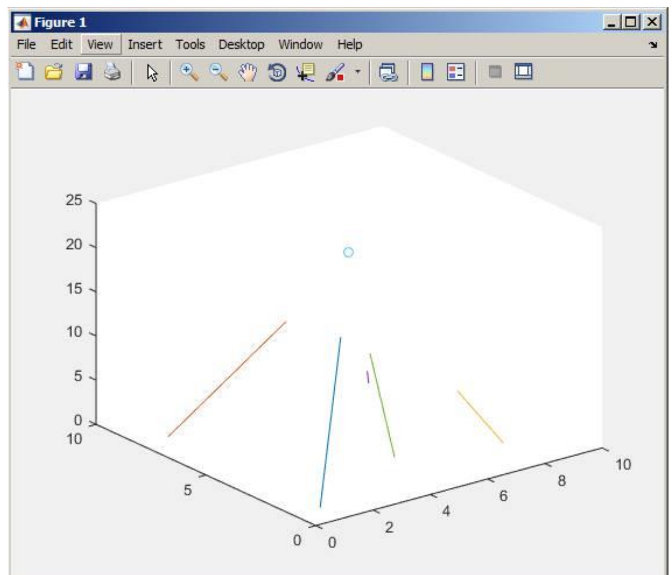
Both A and B are initialized as zero matrices with specified size.

The focus point is finally calculated as c=A\B.

<u>Results plotted:</u>

(The code for plotting the two graphs is written in “Part1.m”)





This graph plots data sets PlanarPoints and SpatialPoints’s relation as a line	This graph plots the focus point onto the previous graph

From the two graphs we can see that the focus point lies in the middle of these lines.

<u>Code:</u>

```
function Part1(A,B,c)
% A is PlanarPoints, B is SpatialPoints
% c is the focus point of A and B
for i=1:5
    pts=[A(:,i)';B(:,i)'];
    plot3(pts(:,1),pts(:,2),pts(:,3))
    hold on;
end
scatter3(c(1),c(2),c(3))
```

Problem #2: **Least-Squares Planar Registration**

a)

RMS error
0.1230
0.1056

t
-1.6111
-5.4188

R	
0.5183	-0.8552
0.8552	0.5183

b)

RMS error
0.1170
0.0959

t
-3.1415
1.6180

R	
-0.7105	0.7037
-0.7037	-0.7105

c)

RMS error
0.5511
0.3068

t
-3.3106
1.5086

R	
-0.7034	0.7108
-0.7108	-0.7034

d)

RMS error
NaN
NaN

t
NaN
NaN

R	
NaN	0
0	NaN

This example is hard to solve because the dataset dP contains identical points:

-1.1	1	-1.1
1.1	1	1.1

And dataset dQ has three separate points on a straight line:

1	0	-1
1	0	-1

There is no way to rotate a single point to two different positions.

<u>Code:</u>

```
% mean of data set P and Q
Pmean = mean(P,2);
Qmean = mean(Q,2);

% subtract mean from data
[n, m] = size(P);
A = P - Pmean*ones(1,m);
B = Q - Qmean*ones(1,m);

% convert to complex vectors
rowA = A(1,:) + i*A(2,:);
rowB = B(1,:) + i*B(2,:);

r=rowA*rowB';
r=r/norm(r);

%convert r to real
R1 = real(r);
%convert r to imaginary
R2 = imag(r);
```

```
R = [[R1, R2]; [-1 * R2, R1]];
t = Qmean - (R*Pmean);

% Qj=R*Pj + t
for j = 1:m
    Qj(:,j) = R*P(:,j)+t;
end

diff = Qj-Q;
rms = sqrt((sum(diff.^2,2))/size(P,2));
```