Linear regression workbook

This workbook will walk you through a linear regression example. It will provide familiarity with Jupyter Notebook and Python. Please print (to pdf) a completed version of this workbook for submission with HW #1.

ECE 239AS, Winter Quarter 2019, Prof. J.C. Kao, TAs M. Kleinman and A. Wickstrom and K. Liang and W. Chuang

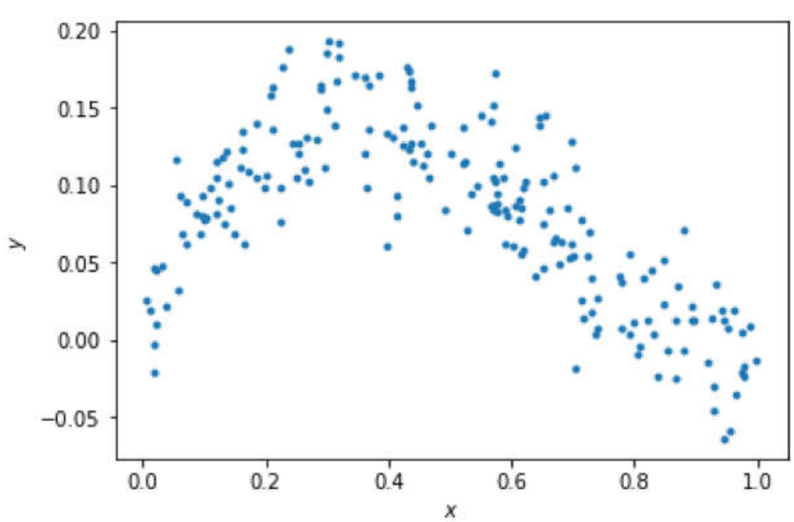
In [109]:
```
import numpy as np
import matplotlib.pyplot as plt

#allows matlab plots to be generated in line
%matplotlib inline
```

Data generation

For any example, we first have to generate some appropriate data to use. The following cell generates data according to the model: $y = x - 2x^2 + x^3 + \epsilon$

In [121]:
```
np.random.seed(0)   # Sets the random seed.
num_train = 200     # Number of training data points

# Generate the training data
x = np.random.uniform(low=0, high=1, size=(num_train,))
y = x - 2*x**2 + x**3 + np.random.normal(loc=0, scale=0.03, size=(num_train,))
f = plt.figure()
ax = f.gca()
ax.plot(x, y, '.')
ax.set_xlabel('$x$')
ax.set_ylabel('$y$')
```

Out[121]: `Text(0,0.5,'$y$')`



QUESTIONS:

Write your answers in the markdown cell below this one:

(1) What is the generating distribution of x?

(2) What is the distribution of the additive noise ϵ?

ANSWERS:

(1) uniform distribution between 0 and 1

(2) normal distribution with mean=0 and std=0.03

Fitting data to the model (5 points)

Here, we'll do linear regression to fit the parameters of a model $y = ax + b$.

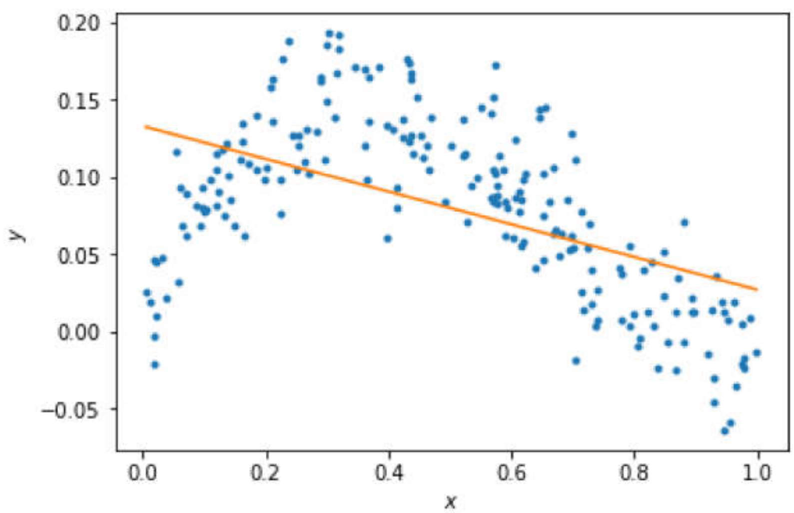
In [122]:

```
# xhat = (x, 1)
xhat = np.vstack((x, np.ones_like(x)))

# =================== #
# START YOUR CODE HERE #
# =================== #
# GOAL: create a variable theta; theta is a numpy array whose elements are [a, b]

#least square

#define X,Y that are used in least square method
X = xhat.T
Y = y.T
theta = np.dot(np.dot(np.linalg.inv(np.dot(X.T,X)),X.T),Y)
print("theta for linear regression is ",theta)


# ================== #
# END YOUR CODE HERE #
# ================== #
```

```
theta for linear regression is  [-0.10599633  0.13315817]
```

In [123]:
```
# Plot the data and your model fit.
f = plt.figure()
ax = f.gca()
ax.plot(x, y, '.')
ax.set_xlabel('$x$')
ax.set_ylabel('$y$')

# Plot the regression line
xs = np.linspace(min(x), max(x),50)
xs = np.vstack((xs, np.ones_like(xs)))
plt.plot(xs[0,:], theta.dot(xs))
```

Out[123]: [<matplotlib.lines.Line2D at 0x188f7011f98>]



QUESTIONS

(1) Does the linear model under- or overfit the data?

(2) How to change the model to improve the fitting?

ANSWERS

(1) Underfit

(2) use a more complex model, use a polynomial regression model with n>1 instead of a linear model

Fitting data to the model (10 points)

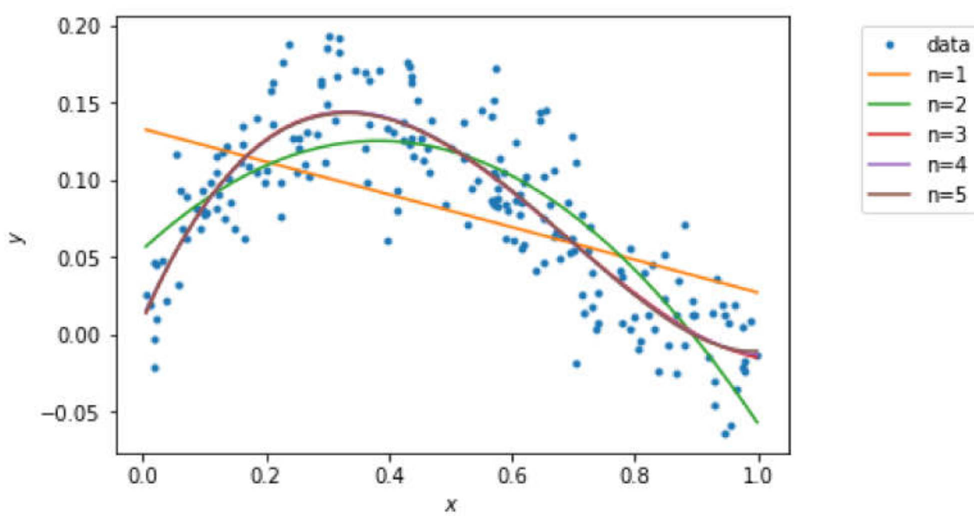
Here, we'll now do regression to polynomial models of orders 1 to 5. Note, the order 1 model is the linear model you prior fit.

In [124]:

```
N = 5
xhats = []
thetas = []

# =================== #
# START YOUR CODE HERE #
# =================== #

# GOAL: create a variable thetas.
# thetas is a list, where theta[i] are the model parameters for the polynomial fit of o
rder i+1.
#   i.e., thetas[0] is equivalent to theta above.
#   i.e., thetas[1] should be a length 3 np.array with the coefficients of the x^2, x,
 and 1 respectively.
#   ... etc.

# xhats = [x^m x^(m-1) ... x^2 x 1]
xhats = np.vstack((x,np.ones_like(x)))
for i in range(N):
    if i==0: # when i=0, xhats don't change
        pass
    else:   # when i>0, need to add one row
        xhats = np.vstack((x**(i+1),xhats))
        #print(xhats)
    X = xhats.T
    Y = y.T
    theta = np.dot(np.dot(np.linalg.inv(np.dot(X.T,X)),X.T),Y)
    thetas.append(theta)
#print(thetas)
print(X.shape)

pass

# ================== #
# END YOUR CODE HERE #
# ================== #
```

```
(200, 6)
```

In [125]:
```
# Plot the data
f = plt.figure()
ax = f.gca()
ax.plot(x, y, '.')
ax.set_xlabel('$x$')
ax.set_ylabel('$y$')

# Plot the regression lines
plot_xs = []
for i in np.arange(N):
    if i == 0:
        plot_x = np.vstack((np.linspace(min(x), max(x),50), np.ones(50)))
    else:
        plot_x = np.vstack((plot_x[-2]**(i+1), plot_x))
    plot_xs.append(plot_x)

for i in np.arange(N):
    ax.plot(plot_xs[i][-2,:], thetas[i].dot(plot_xs[i]))

labels = ['data']
[labels.append('n={}'.format(i+1)) for i in np.arange(N)]
bbox_to_anchor=(1.3, 1)
lgd = ax.legend(labels, bbox_to_anchor=bbox_to_anchor)
```



Calculating the training error (10 points)

Here, we'll now calculate the training error of polynomial models of orders 1 to 5.

In [126]:

```
training_errors = []

# ==================== #
# START YOUR CODE HERE #
# ==================== #

# GOAL: create a variable training_errors, a list of 5 elements,
# where training_errors[i] are the training loss for the polynomial fit of order i+1.

#print(X[:,4:6].shape)
for i in range(5):
    err = np.sum(np.square(Y - np.dot(X[:,(4-i):6],thetas[i])))
    training_errors.append(err)
print("training errors for polynomial regression are:\n ",training_errors)
best_N_loc = np.argmin(training_errors)
N_list = range(5)
best_N = N_list[best_N_loc]+1
print("\nthe best N for polynomial regression is ",best_N)

# ================== #
# END YOUR CODE HERE #
# ================== #

print ('Training errors are: \n', training_errors)
```

```
training errors for polynomial regression are:
  [0.4759922176725402, 0.2184984441853706, 0.16339207602210742, 0.16330707470593958,
 0.1632295839105059]

the best N for polynomial regression is  5
Training errors are:
 [0.4759922176725402, 0.2184984441853706, 0.16339207602210742, 0.16330707470593958,
 0.1632295839105059]
```

QUESTIONS

(1) What polynomial has the best training error?

(2) Why is this expected?

ANSWERS

(1) 5

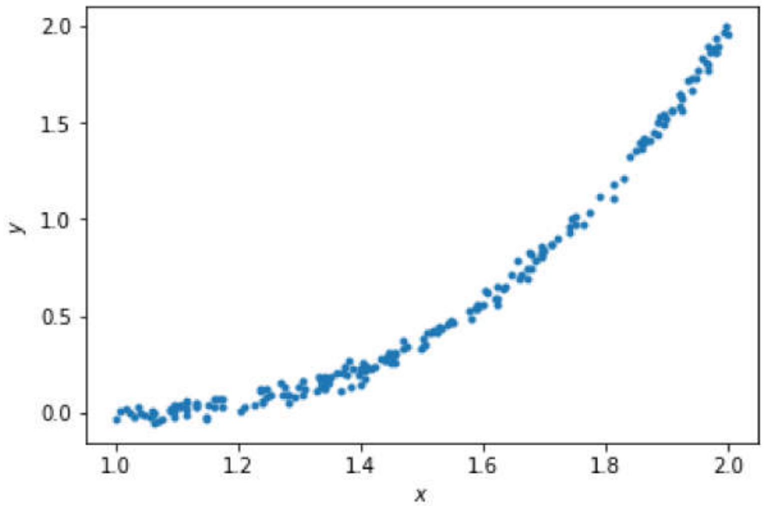
(2) Because n=5 overfits the training data since it's the most complex it can capture the noise

Generating new samples and testing error (5 points)

Here, we'll now generate new samples and calculate testing error of polynomial models of orders 1 to 5.

In [127]:
```
x = np.random.uniform(low=1, high=2, size=(num_train,))
y = x - 2*x**2 + x**3 + np.random.normal(loc=0, scale=0.03, size=(num_train,))
f = plt.figure()
ax = f.gca()
ax.plot(x, y, '.')
ax.set_xlabel('$x$')
ax.set_ylabel('$y$')
```

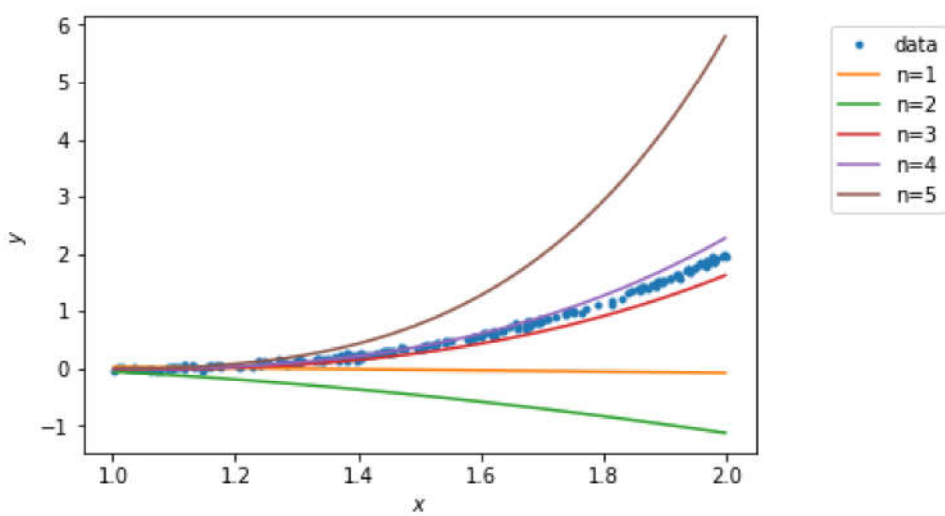
Out[127]: Text(0,0.5,'y')



In [128]:
```
xhats = []
for i in np.arange(N):
    if i == 0:
        xhat = np.vstack((x, np.ones_like(x)))
        plot_x = np.vstack((np.linspace(min(x), max(x),50), np.ones(50)))
    else:
        xhat = np.vstack((x**(i+1), xhat))
        plot_x = np.vstack((plot_x[-2]**(i+1), plot_x))

    xhats.append(xhat)
```

In [129]:

```
# Plot the data
f = plt.figure()
ax = f.gca()
ax.plot(x, y, '.')
ax.set_xlabel('$x$')
ax.set_ylabel('$y$')

# Plot the regression lines
plot_xs = []
for i in np.arange(N):
    if i == 0:
        plot_x = np.vstack((np.linspace(min(x), max(x),50), np.ones(50)))
    else:
        plot_x = np.vstack((plot_x[-2]**(i+1), plot_x))
    plot_xs.append(plot_x)

for i in np.arange(N):
    ax.plot(plot_xs[i][-2,:], thetas[i].dot(plot_xs[i]))

labels = ['data']
[labels.append('n={}'.format(i+1)) for i in np.arange(N)]
bbox_to_anchor=(1.3, 1)
lgd = ax.legend(labels, bbox_to_anchor=bbox_to_anchor)
```



In [130]:
```python
testing_errors = []

# ==================== #
# START YOUR CODE HERE #
# ==================== #

# GOAL: create a variable testing_errors, a list of 5 elements,
# where testing_errors[i] are the testing loss for the polynomial fit of order i+1.
for i in range(5):
    testing_err = np.sum(np.square(y.T - np.dot(xhats[i].T,thetas[i])))
    testing_errors.append(testing_err)



# ================== #
# END YOUR CODE HERE #
# ================== #

print ('Testing errors are: \n', testing_errors)
```

```
Testing errors are: 
 [161.72330369101172, 426.38384890115907, 6.25139421682068, 2.374153041858252, 429.82
04358482615]
```

QUESTIONS

(1) What polynomial has the best testing error?

(2) Why polynomial models of orders 5 does not generalize well?

ANSWERS

(1) 4.

(2) Because it overfits the training data and it is too specific for x in [0,1] so that it cannot predict unseen datas well.