Quantitative Finance Project

Abstract

This study investigates portfolio risk, return distribution assumptions, derivative pricing, and hedging performance for major technology stocks using a multi-faceted quantitative approach. We construct minimum- and maximum-volatility portfolios to assess risk trade-offs, and test whether daily log returns follow a normal distribution through statistical tests and visual diagnostics. We apply the Black–Scholes model to examine European call and put option prices and their deltas, and employ Monte Carlo simulations—including a Heston stochastic volatility model—to evaluate delta hedging strategies under time-varying volatility conditions.

Major findings include pronounced differences in volatility and return profiles between the optimized portfolios and clear evidence that stock log returns deviate from normality, exhibiting skewness and heavy tails. Under stochastic volatility conditions, delta hedging produced a wider profit-and-loss distribution, underscoring the limitations of the constant-volatility assumption and the importance of accounting for volatility dynamics. This integrated analysis enhances understanding of risk assessment, option pricing, and hedging effectiveness in realistic market conditions. Future research can extend these approaches by exploring more complex stochastic models or broader asset classes to further refine risk management and hedging strategies.

Introduction

The primary objective of this project is to explore core topics in quantitative finance—portfolio risk, return distribution analysis, option pricing, and hedging performance—through empirical data and modeling techniques. By applying both classical and modern tools, the project aims to provide insights into the assumptions, limitations, and practical applications of financial models in real-world markets.

Research Motivation and Significance

Understanding how risk is distributed across assets is central to investment decision-making. By comparing high-risk versus low-risk portfolios, we illustrate the fundamental trade-off between volatility and potential return, helping investors design strategies aligned with their risk tolerance. Testing the normality of log returns is equally important, as many models—including Black–Scholes—rely on this assumption; deviations from normality highlight model risk and the need for more robust approaches. The Black–Scholes model itself remains a cornerstone in option pricing because of its analytical elegance, intuitive interpretation, and widespread use in financial markets, making it a natural starting point before considering more advanced stochastic models. Finally, delta hedging—though theoretically effective under constant volatility—often fails in practice when market volatility changes over time. This motivates the use of stochastic volatility models, such as the Heston model, to evaluate hedge performance under more realistic conditions.

Data and Methodological Framework

The analysis is based on daily historical price data of four leading technology stocks—Apple (AAPL), Microsoft (MSFT), Amazon (AMZN), and Meta (META)—collected from Yahoo Finance over the period July 2024 to July 2025. The project is organized into four parts:

1. Portfolio Risk Analysis: Constructing minimum- and maximum-volatility portfolios to compare risk-return profiles.
2. Return Distribution Tests: Evaluating whether stock log returns follow a normal distribution using statistical tests and graphical tools.
3. Option Pricing with Black–Scholes: Implementing the Black–Scholes model to study option prices and sensitivity under different conditions.
4. Delta Hedging under Stochastic Volatility: Simulating stock paths using both random volatility assignments and the Heston model to assess the robustness of hedging strategies.

This framework connects foundational financial theories with computational techniques, highlighting both the insights they provide and their practical limitations.

This project is made up of four parts, each exploring a different idea in quantitative finance through hands-on Python work. It begins with selecting several well-known tech stocks and analyzing their portfolio risks by looking at standard deviations and covariances. The second part investigates whether the log returns of these stocks follow a normal distribution, using both statistical tests and visual tools to assess the assumption. In the third section, the Black-Scholes model is applied to study how the prices of call and put options react to changes in time and the underlying asset's price. Lastly, the project simulates stock paths with changing volatility to examine how delta-hedging strategies perform when the constant-volatility assumption no longer holds. Each part ties together data analysis and quantitative methods to better understand risk, pricing, and financial modeling in practice.

Part I:

Objective:

This section analyzes the risks associated with a selected group of technology stocks and constructs minimum and maximum volatility portfolios using Python.

Data Source

We selected four major technology stocks: Apple (AAPL), Microsoft (MSFT), Amazon (AMZN), and Meta Platforms (META).

Daily historical price data was downloaded from Yahoo Finance and covers the year from July 2024 to July 2025.

Methodology

We calculated the daily logarithmic return for each stock using adjusted closing prices.

The standard deviation of returns was used to measure individual stock volatility.

We calculated the covariance matrix to assess the relationships between stock returns.

Using these values, we formulated two portfolio optimization problems:

Minimize total portfolio volatility

Maximize total portfolio volatility

To meet each investment objective, we employed optimization techniques under portfolio constraints: no short selling, asset weights must sum to one, and each individual stock must comprise between 10% and 35% of the total portfolio.

Results

The optimal weights for the minimum-volatility portfolio are:

AAPL: Weight = 0.3500

MSFT: Weight = 0.1637

AMZN: Weight = 0.1363

META: Weight = 0.3500

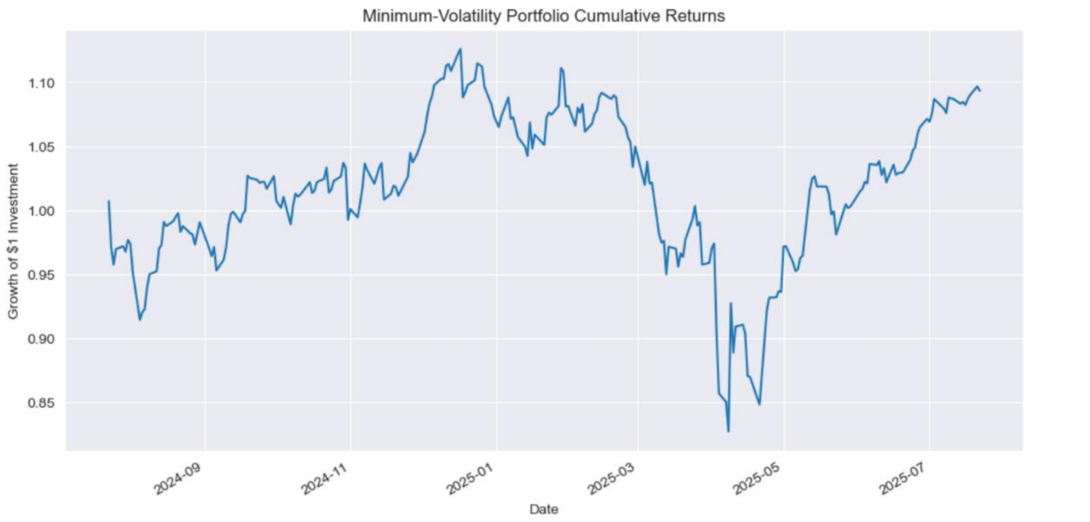

Figure 1 Cumulative Returns of the Minimum-Volatility Portfolio (2024 - 2025)

This chart shows the cumulative growth of $1 invested in the minimum volatility portfolio over a one-year period. Despite a modest drawdown in the middle of the time frame, the portfolio steadily recovered, ultimately achieving a positive cumulative return, consistent with the goal of preserving capital while limiting volatility.

The optimal weights for the maximum-volatility portfolio are:

AAPL: Weight = 0.2000

MSFT: Weight = 0.3500

AMZN: Weight = 0.3500

META: Weight = 0.1000

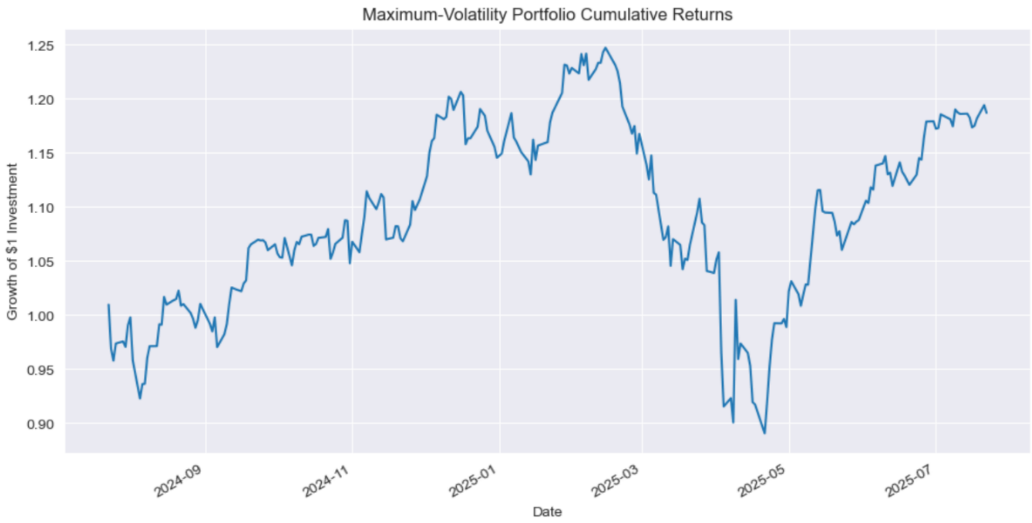

Figure 2 Cumulative Returns of the Maximum-Volatility Portfolio (2024 - 2025)

In contrast, the maximum volatility portfolio experienced greater fluctuations, both upward and downward. While its ultimate return was higher than the minimum volatility portfolio, it also faced greater drawdowns, reflecting the trade-off between risk and return in portfolio construction.

Sharpe Ratio Analysis

To evaluate performance beyond volatility, we calculated the Sharpe Ratio:

$$\text{Sharpe Ratio} = \frac{E[R_p] - R_f}{\sigma p}$$

When $R_f = 0$

- The minimum-volatility portfolio achieved a lower Sharpe Ratio, reflecting its stability but limited upside.
- The maximum-volatility portfolio, while riskier, recorded a higher Sharpe Ratio, indicating stronger return potential per unit of risk.

This highlights the importance of considering both absolute volatility and risk-adjusted returns when comparing portfolios.

The heat map

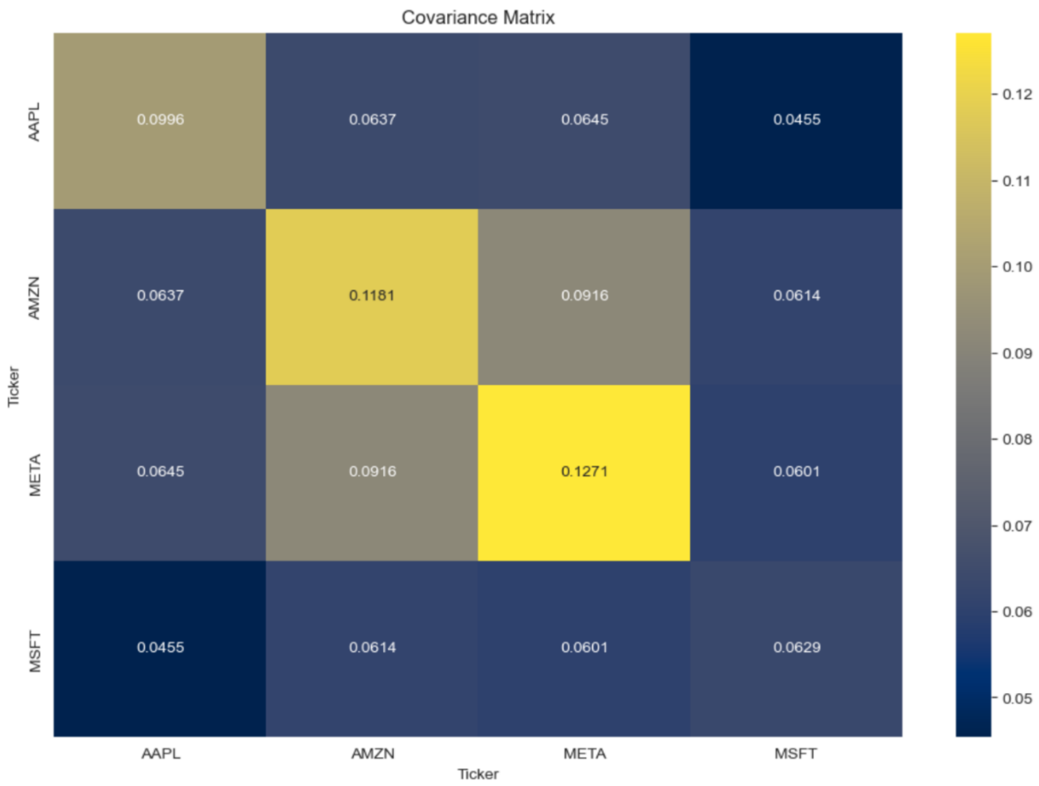

Figure 3 Covariance Matrix of AAPL, MSFT, AMZN, and META

The heat map above illustrates the covariance between the daily log returns of four selected stocks. Brighter colors, like yellow, represent a stronger relationship in how the returns move together. For instance, META shows the highest individual volatility and shares relatively high covariance with both AMZN and AAPL, suggesting its returns tend to move more closely in line with those stocks.

Part II:

Objective:

In this section, I explore whether the daily logarithmic returns of four major technology stocks (AAPL, MSFT, AMZN, and META) follow a normal distribution, as often assumed in financial models. I also analyze how this assumption holds when using different treatments, such as removing extreme values or analyzing annualized returns.

Theoretical Background

In the Black–Scholes framework, the underlying asset price is modeled as a geometric

Brownian motion:

$$\log\left(\frac{S_t}{S_0}\right) \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)t, \sigma^2 t\right)$$

This implies that log returns should follow a normal distribution. Testing this assumption is important because deviations (e.g., skewness, excess kurtosis) can indicate model risk and challenge the validity of Black–Scholes–based pricing.

Methodology

1. Visual Inspection: Daily Returns

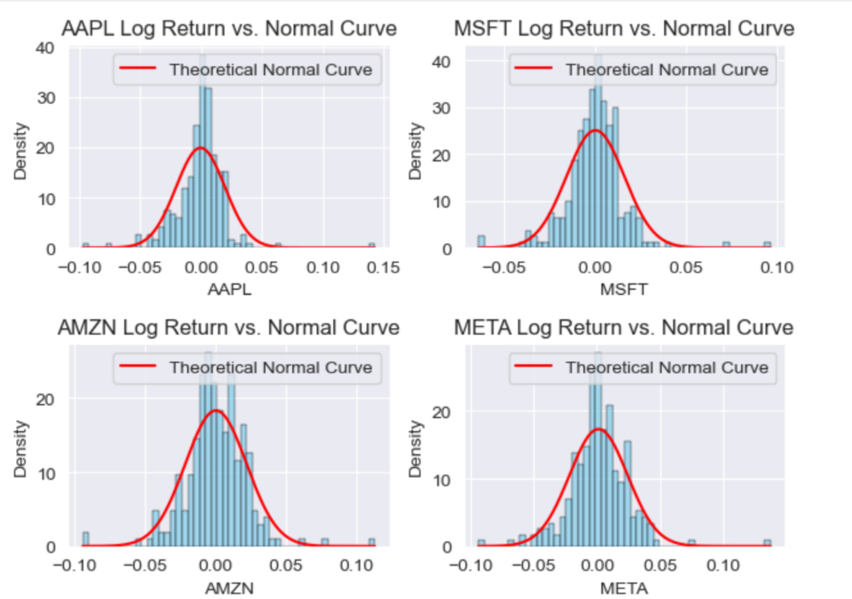

Figure 4 Histograms of Daily Log Returns with Theoretical Normal Curves (AAPL, MSFT, AMZN, META)

- AAPL and MSFT exhibit sharp peaks and heavy tails compared to the normal curve.
- AMZN and META appear closer to normality, though still with some deviations.
- Overall, daily returns deviate from strict normality, particularly due to skewness and kurtosis.

2.Effect of Removing Extremes

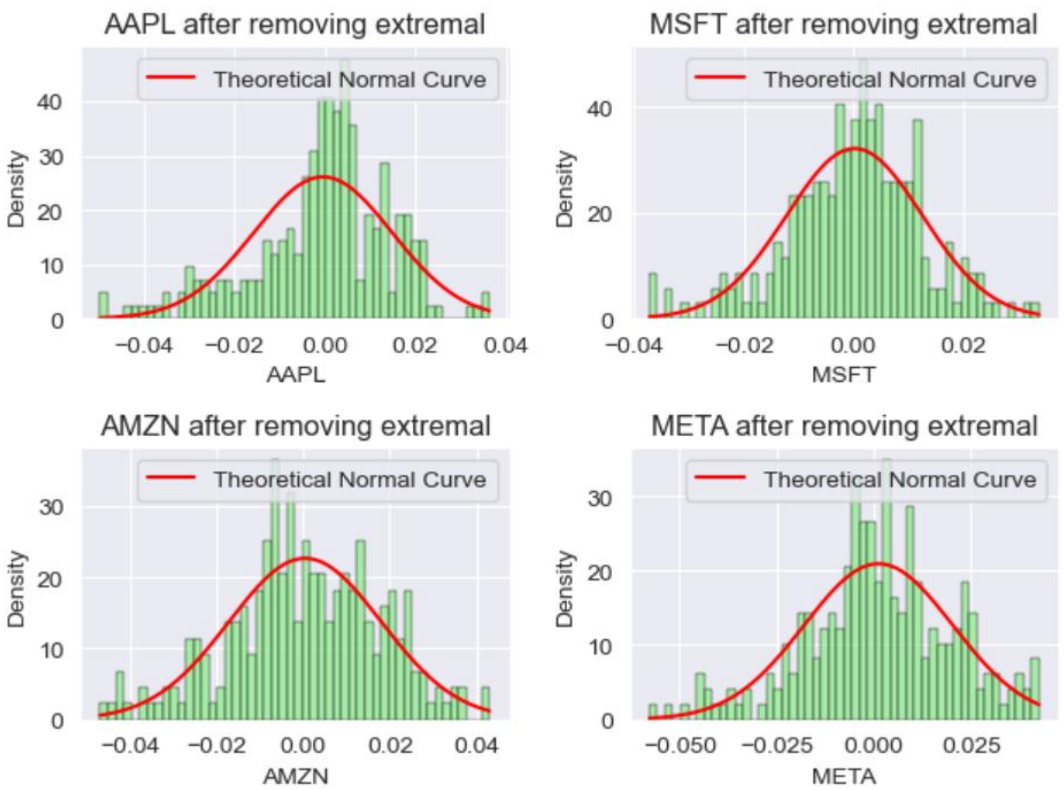

Figure 5 Histograms of Log Returns after Removing Extremes (AAPL, MSFT, AMZN, META)

- After trimming extreme values, all four stocks show distributions more consistent with the normal curve.
- Skewness and kurtosis are reduced significantly, confirming that outliers drive much of the deviation from normality.

2. Portfolio Returns

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Logarithmic return of the portfolio Mean=0.0009, Standard Deviation=0.0209
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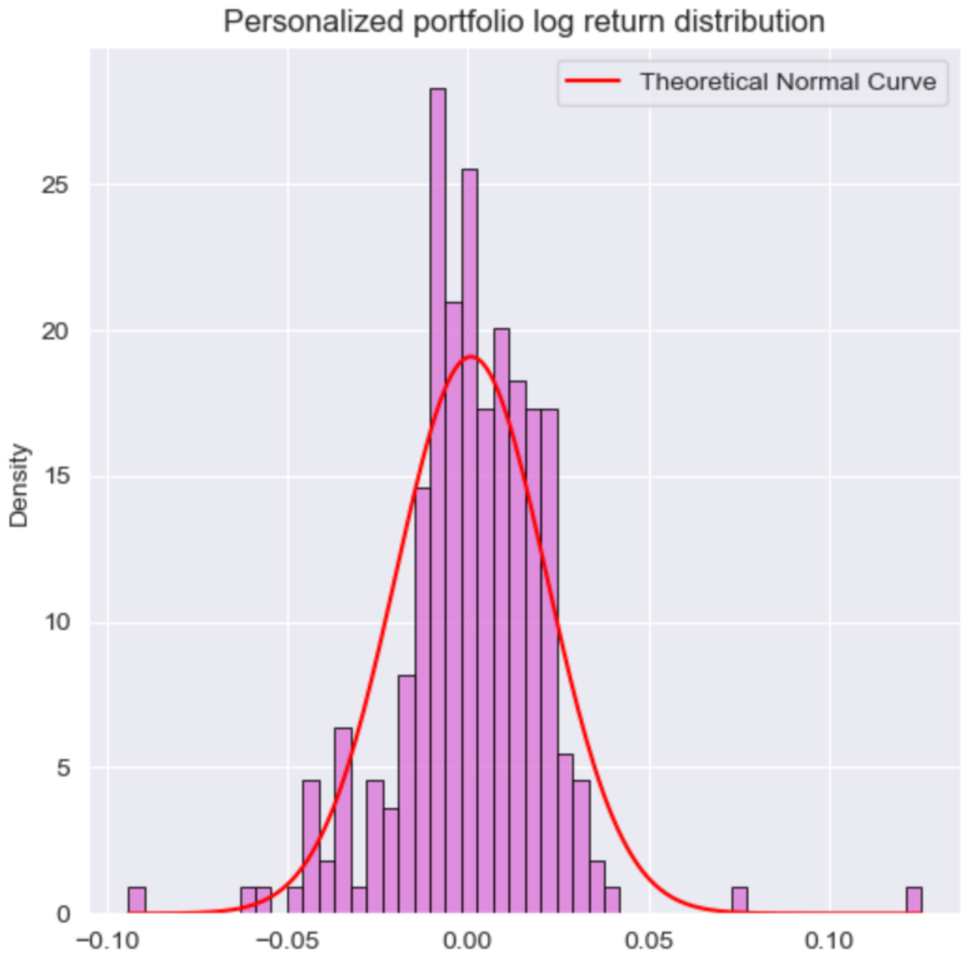

Figure 6 Log Return Distribution of the Personalized Portfolio vs. Normal Curve

- The personalized portfolio shows returns more concentrated around the mean, though still with fat tails.
- Diversification reduces idiosyncratic risk but does not fully eliminate non-normality.

3. Q–Q Plot Analysis

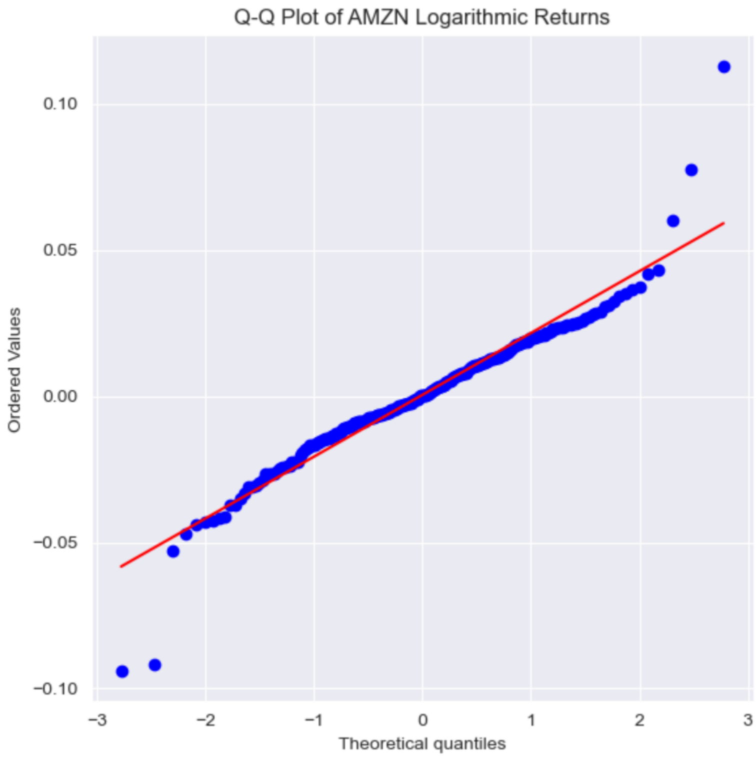

Figure 7 Q－Q Plot of AMZN Daily Log Returns\

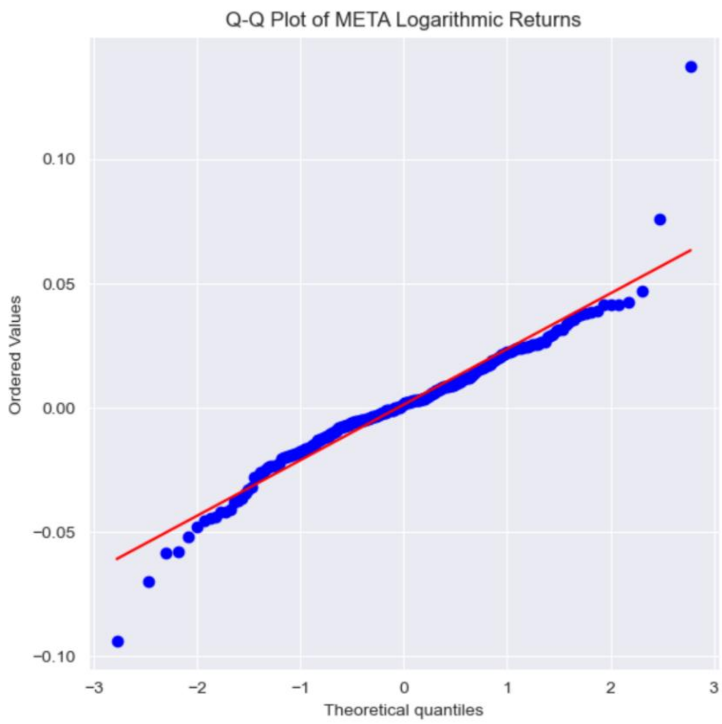

Figure 8 Q－Q Plot of META Daily Log Returns

- Points mostly follow the 45° line in the middle range, suggesting approximate normality.

- Deviations at the tails (both left and right) show that AMZN returns still exhibit fat tails relative to the normal distribution.
- This tail behavior is consistent with empirical finance literature.

Interpretation

- Daily returns generally reject the normality assumption.
- Outliers and tail events (e.g., sudden market moves) are the main source of deviation.
- Aggregated returns (e.g., monthly) would likely appear more normal, consistent with the Central Limit Theorem.
- These findings imply that the Black–Scholes assumption of normal log returns is too simplistic, especially at high frequency, and may understate the probability of extreme events.

Part III

Objective:

In this section, I will use the Black-Scholes model to calculate and visualize the price and delta of European call and put options. The goal is to understand how option prices and their sensitivity respond to changes in time to expiration and the spot price of the underlying asset.

Theoretical Framework

The Black–Scholes closed-form solution for a European call option is given by:

$$C = S_0\Phi(d_1) - Ke^{-rt}\Phi(d_2)$$

With

$$d_1 = \frac{\mathrm{In}\left(\frac{S_0}{K}\right)+\left(r+\frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}},\; d_2 = d_1 - \sigma\sqrt{t}$$

where S0S_0S0 is the current stock price, KKK the strike price, rrr the risk-free rate, ttt the time to maturity, and σ\sigmaσ the volatility of returns.

This formulation shows that option value depends jointly on the underlying price, time, interest rate, and volatility.

Option Greeks

To understand sensitivities, we introduce option Greeks, which are derivatives of the option value with respect to model parameters:

- Delta (Δ): measures sensitivity to the underlying price.
- Vega (ν): measures sensitivity to volatility.
- Theta (Θ): measures sensitivity to time decay.

Intuition and Visualization

- Delta can be interpreted geometrically as the slope of the option value curve with respect to the spot price. In graphical terms, delta shows how much the option price will change for a small change in the stock price.
- Vega captures how much the option price increases when volatility rises. Options become more valuable in uncertain markets because higher volatility increases the probability of finishing in-the-money.
- Theta reflects time decay: as expiration approaches, the option loses time value, leading to a negative impact on option price.

Results

1. **By applying the Black–Scholes formulas in Python, we generated three visualizations:**

Call Option Price vs. Time to Expiration

Figure 9 Call Option Value Across Time to Expiration

Demonstrates how longer maturities increase option value due to higher time value.

2. Call Option Price and Delta vs. Spot Price

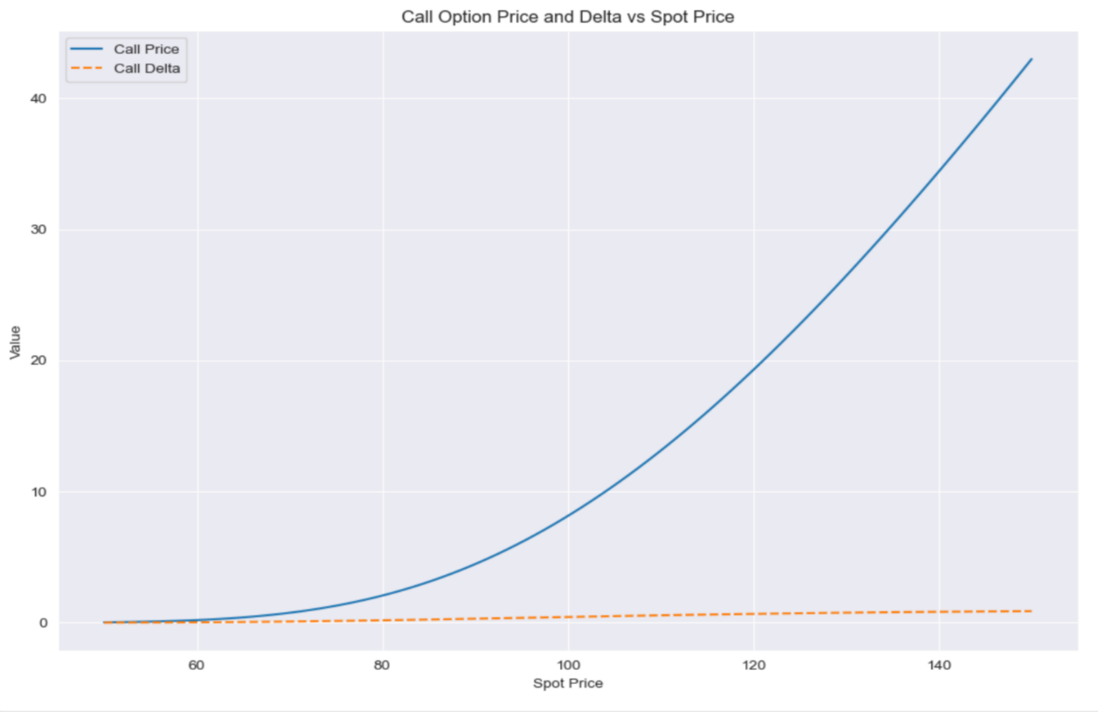

Figure 10 Call Option Price and Delta Sensitivity to Spot Price

Shows that delta increases as the option becomes deeper in-the-money, approaching 1 as the option behaves like the underlying asset.

3. Put Option Price and Delta vs. Spot Price

Figure 11 Put Option Price and Delta Sensitivity to Spot Price

Illustrates that put delta is negative, reflecting the inverse relationship between put value and stock price.

These analyses highlight how the Black–Scholes model not only provides option values but also tools for risk management through the Greeks.

Part IV

Objective

In this section, I will examine the performance of delta hedging strategies when stock volatility is non-constant. I will use two different models to generate non-constant volatility: one using randomly assigned volatility levels and the other using the Heston stochastic volatility model. The goal is to analyze how the profit distribution changes under more realistic market behavior.

Methodology

1. Delta Hedging Strategy

- At each step, we compute the option's delta:

$$\Delta_t = \Phi(d_1)$$

- The hedge is rebalanced daily by adjusting the underlying position to remain delta-neutral.

2. Sources of Hedging Error
   Even with rebalancing, profits do not converge to zero because of:
   - Discrete hedging frequency: trading is not continuous.
   - Stochastic volatility: realized volatility deviates from the model assumption.
   - Numerical error: discretization and simulation noise.
3. Models of Non-Constant Volatility
   - Random Volatility Model: volatility is drawn from discrete levels (0.2, 0.3, 0.45) with predefined probabilities.
   - Heston Model: volatility follows a mean-reverting stochastic process, producing volatility clustering and more realistic dynamics.
4. Comparison Framework
   - Constant σ hedging: benchmark case using Black–Scholes assumptions.
   - Non-constant σ hedging: simulations using random volatility and the Heston model.

Results

Cumulative Hedging P&L

Figure 12 Cumulative Hedging P&L over Time

- Under constant volatility, cumulative P&L fluctuates narrowly around zero.
- Under non-constant volatility, deviations are larger, reflecting increased hedging error.

Profit Distribution

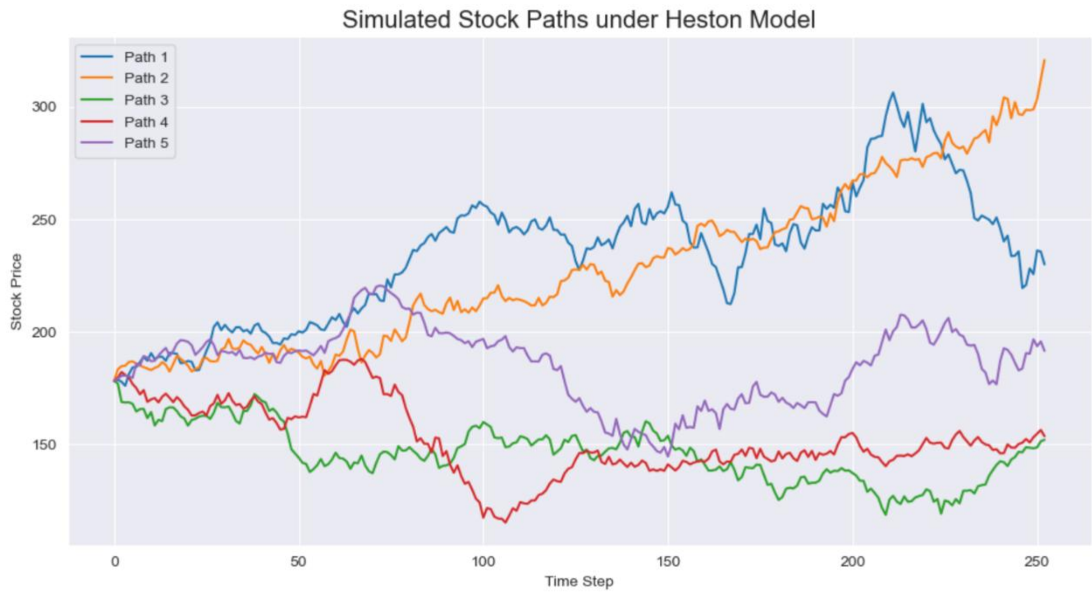

Figure 13 Histogram of Hedging Profits with 95% Confidence Interval

- Constant volatility hedges produce a tighter profit distribution.
- Stochastic volatility widens the distribution and creates fat tails, showing greater risk of extreme outcomes.

Constant vs. Non-Constant Volatility Comparison

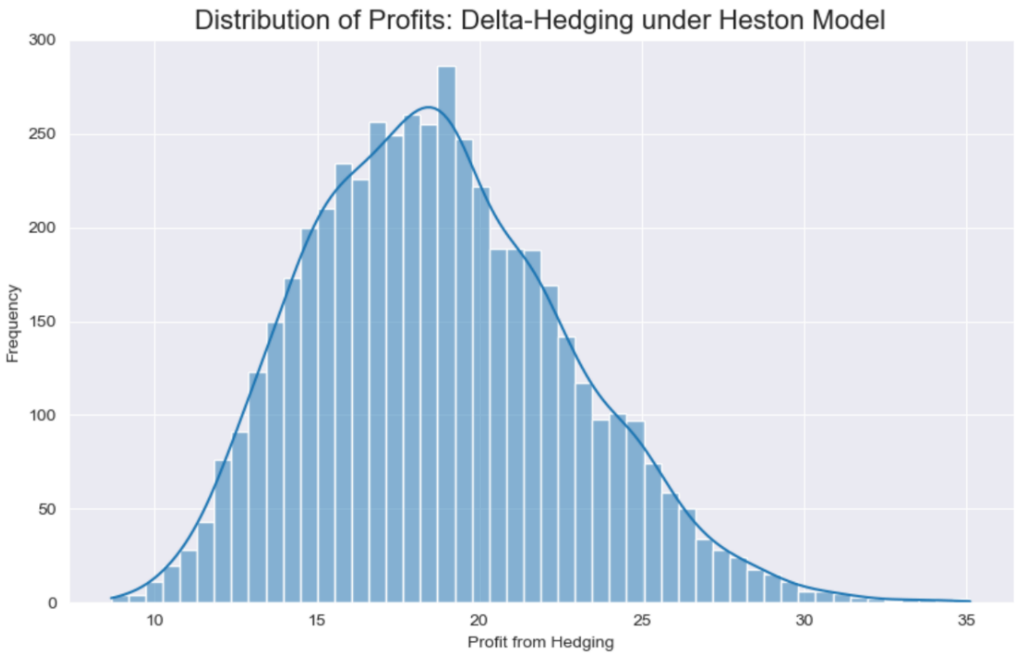

Figure 14 Hedging P&L: Constant vs. Non-Constant Volatility

- The comparison clearly shows that constant σ assumptions underestimate risk.
- Non-constant volatility leads to greater dispersion and larger tail risks in hedging profits.

Conclusion and Discussion

Which Model is Closer to Reality?

Among the models explored, the Heston stochastic volatility model provides a more realistic representation of market behavior compared to the constant volatility assumption in Black–Scholes. It captures volatility clustering and fat tails observed in real financial data, making it better suited for practical applications.

Strengths and Weaknesses of Portfolio Construction Methods

The minimum-variance portfolio provides stability and reduces drawdowns but may underperform in terms of return, especially in bullish markets. The maximum-volatility portfolio, while capable of achieving higher returns, exposes investors to greater downside risk. This trade-off underscores the importance of balancing return objectives with risk tolerance in portfolio design.

Stability of Hedging Strategies in Real Markets

- Constant σ delta hedging works well in theory but fails under stochastic volatility.
- Non-constant σ hedging (using Heston or custom volatility) produces wider P&L distributions, reflecting the limitations of simple delta hedging.
- Incorporating Gamma and Vega hedging (or sigma-hedging) offers more stability in realistic market environments by addressing higher-order risks.

Summary Table

Topic	Method/Model	Strengths	Weaknesses	Closer to Reality
Portfolio Construction	Minimum-variance portfolio	Stable, low drawdowns	May reduce potential returns	✓ (for risk-averse investors)
	Maximum-volatility portfolio	Higher upside potential	High drawdowns, unstable	✗
Return Distribution	Normality assumption (Black–Scholes)	Analytical simplicity	Ignores fat tails, skewness	✗
	Empirical log returns (tests)	Captures real-world deviations	Harder to model	✓
Option Pricing	Black–Scholes	Elegant closed-form solution, widely used	Unrealistic volatility assumption	✗
	Heston stochastic volatility	Captures clustering and dynamics	More complex, simulation-based	✓
Hedging	Delta hedging (constant σ)	Theoretically perfect under assumptions	Fails under stochastic volatility	✗

	Delta hedging (non-constant σ)	More realistic performance assessment	Wider profit distribution	✓
	Gamma/Vega hedging	Addresses higher-order risks	More complex, costly to implement	✓ (ideal but harder)