COMP6212 (2016/17): Computational Finance Assignment 3 Report

Ningning Liu (ln1u16@soton.ac.uk)
MSc Data Science
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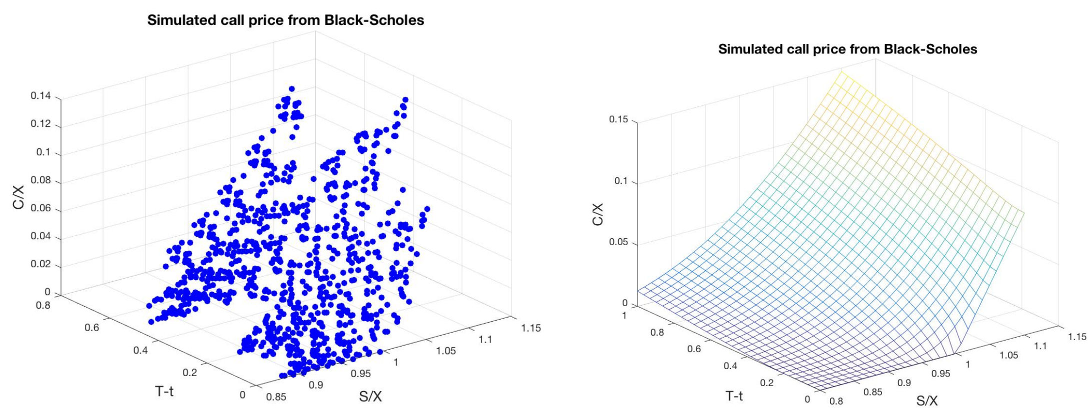
This report was done to summarize the work for COMP6212 assignment 3.

Task 1: Simulate call option prices from Black-Scholes formula

I have simulated five call option prices from Black-Scholes formula. The input parameters are taken from Assignment 2. So we have five strike prices ($X_1 = 2925, X_2 = 3025, X_3 = 3125, X_4 = 3225, X_5 = 3325$), interest rate ($r$= 0.06), volatilities ($\sigma$) and the stock price ($S$). The call option prices are normalized by associated strike prices and plotted against the time to maturity ($T - t$) and normalized stock prices in Fig 1.



(a) (b)

Figure 1*: Call option prices are generated from Black-Scholes formula and normalized by strike prices. Each call prices are plotted against normalized stock prices and time to maturity. (a) gives 3D scatter plot and (b) shows the surface plot. The sampled points are denser when time is closed to maturity. Because there are always options expired currently and on a later time.*

Fig 1 shows the scatters are denser when time closed to maturity as there are always options expired currently and a later time due to CBOE strategy mentioned in [1].

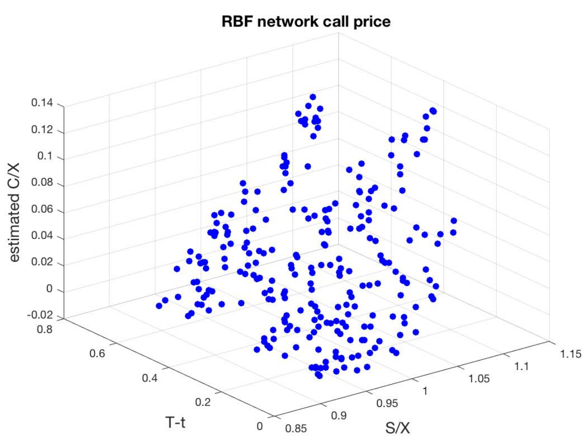
Task 2: Construct learning network and evaluate performance

In this task, a Radius Basis Function (RBF) model is trained and tested based on the dataset simulated from Task 1. Suppose data vector $\boldsymbol{x} = [S/X \quad (T-t)]^T$ and the RBF model is given by

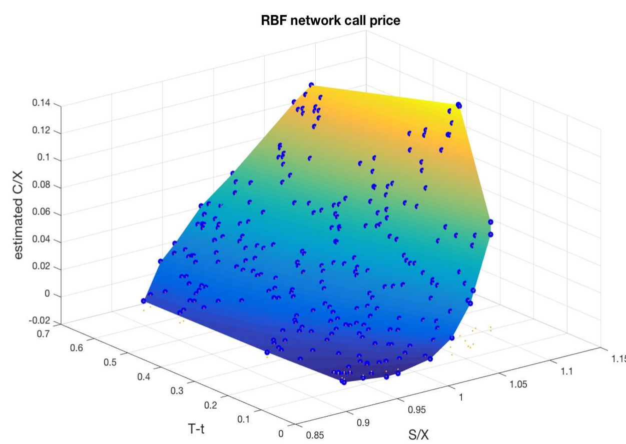
$$\frac{C}{X} = \sum_{j=4}^{J} \lambda_j \emptyset_j(\boldsymbol{x}) + \boldsymbol{w}^T \boldsymbol{x} + w_0 \qquad (\mathbf{1})$$

where the nonlinear RBF function $\emptyset_j(\boldsymbol{x}) = \left[(\boldsymbol{x} - \boldsymbol{m}_j)^T \boldsymbol{\Sigma}_j (\boldsymbol{x} - \boldsymbol{m}_j) + b_j\right]^{1/2}$, the bias term $b_j = 0$ and $J = 4$. So `fitgmdist` is used to extract the parameters $\boldsymbol{m}_j$ and $\boldsymbol{\Sigma}_j$ for four centers in each $\boldsymbol{x}$. So the data is localized into four clusters within its own means and covariance. Next, I map the data into seven columns matrix A and solve the equation by least squares. Let the true normalized call prices denoted by y. The weight **W** of model ($\lambda_j, j = 1,2,3,4;\ \boldsymbol{w}\ and\ w_0$) can be given by pseudo inverse $\widehat{\boldsymbol{W}} = (A^T A)^{-1} A^T y$. In Matlab, we calculate estimated weights as $\widehat{\boldsymbol{W}} = A \backslash y$. Take call option with strike price 2925 as an example. The resulting $\widehat{\boldsymbol{W}} = [3.78\ \ -1.36\ \ 0.77\ \ 0\ \ -0.001\ \ 0.004\ \ 0.003]^{\boldsymbol{T}}$. So the estimated weights are substituted into RBF model, then it gives equation **(2)**.

$$
\begin{aligned}
\frac{\widehat{C}}{X} = \quad & 3.78 \times \sqrt{\begin{bmatrix} \frac{S}{X} - 1.041 \\ T - t - 0.05 \end{bmatrix}^T \begin{bmatrix} 0.0002 & -0.0004 \\ -0.004 & 0.7302 \end{bmatrix} \begin{bmatrix} \frac{S}{X} - 1.041 \\ T - t - 0.05 \end{bmatrix}} \\
& -1.36 \times \sqrt{\begin{bmatrix} \frac{S}{X} - 1.032 \\ T - t - 0.529 \end{bmatrix}^T \begin{bmatrix} 0.284 & 0.3792 \\ 0.3792 & 0.7302 \end{bmatrix} \begin{bmatrix} \frac{S}{X} - 1.032 \\ T - t - 0.529 \end{bmatrix}} \qquad \textbf{(2)} \\
& +0.77 \times \sqrt{\begin{bmatrix} \frac{S}{X} - 1.079 \\ T - t - 0.358 \end{bmatrix}^T \begin{bmatrix} 0.0003 & -0.0011 \\ -0.0011 & 0.0176 \end{bmatrix} \begin{bmatrix} \frac{S}{X} - 1.079 \\ T - t - 0.358 \end{bmatrix}} \\
& -0.001 \times \frac{S}{X} - 0.004 \times (T - t) + 0.003
\end{aligned}
$$



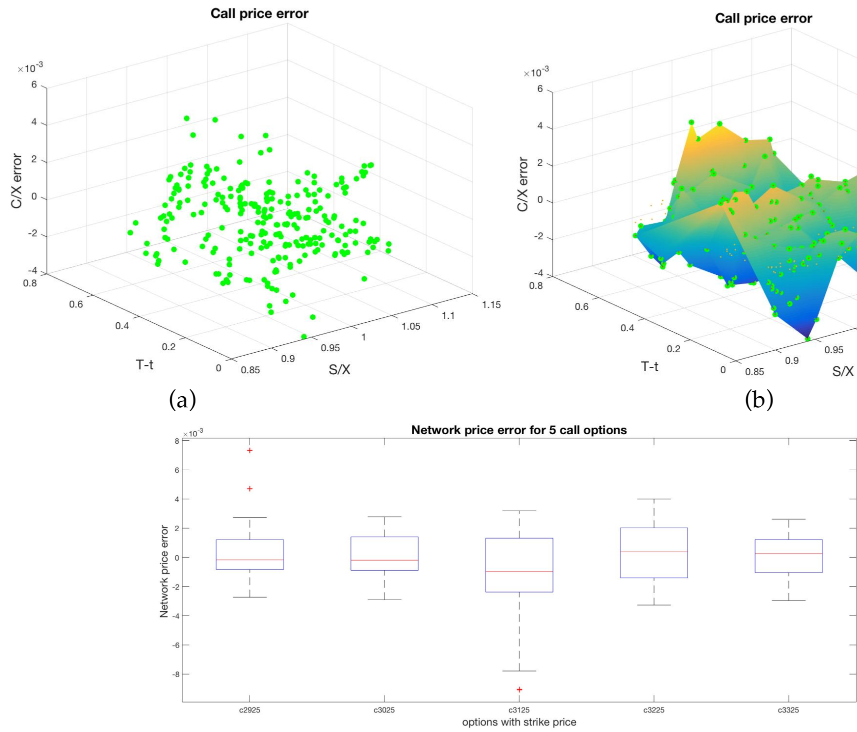
(a)



(b)

***Figure 2**: (a) gives 3D scatter plot of estimated call prices from RBF model whereas (b) generates the surface plot for the same data points.*

After training phase, a test set is used to test the RBF model and estimated call prices from RBF model is plotted in Fig 2. We can see that the scatters in Fig 2 have the similar pattern as those in Fig 1 that the points are denser when approaching maturity. Fig 3 gives the plot of call price error, i.e. $\widehat{C/X} - C/X$. Fig 3 (a) and (b) gives overview all price errors. Without considerations of outliers in Fig 3 (c), the price errors over five call options are consistently small ranging from 0.004 to -0.004. So, the estimated prices from RBF model are closed to the true price.

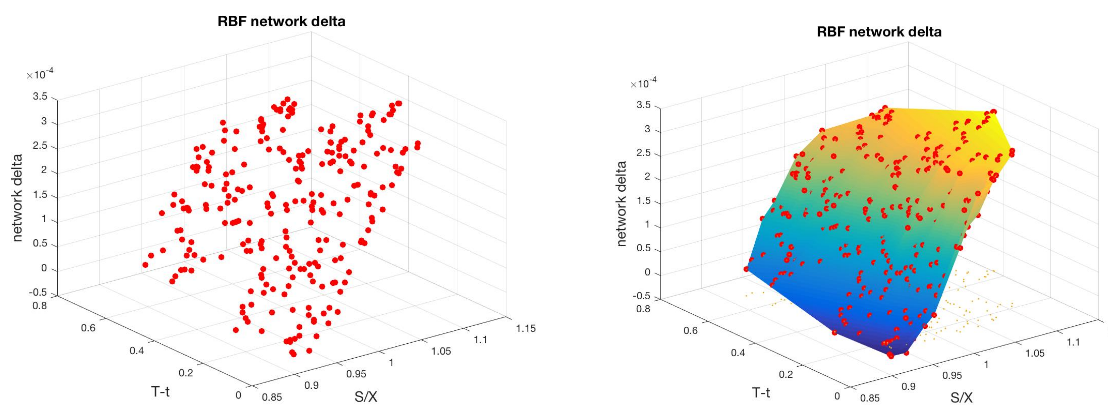


(a) (b)

(c)

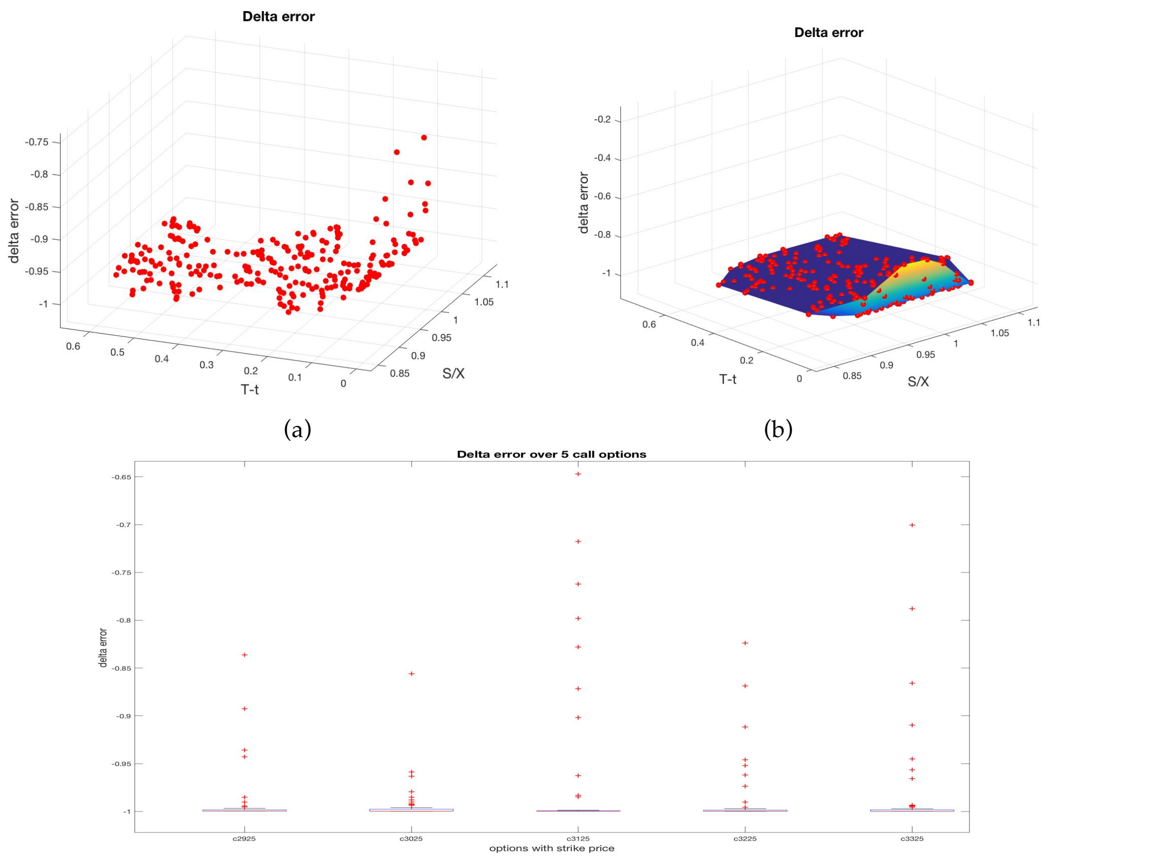
***Figure 3**: (a) gives 3D scatter plot of call price error $\widehat{C/X}$-C/X whereas (b) generates the surface plot for the same data points. (c) gives boxplot of price errors over five different call options. The errors are consistently small over different options. So estimated call prices from RBF model are very closed to simulated call prices.*

Next, I further compute the delta $\frac{\widehat{\partial c}}{\partial s}$ from RBF model. Usually, delta represents sensitivity of option price relative to the change of stock price. the Using symbolic in Matlab, delta is derived directly from equation **(1)**. So delta for five options are computed and plotted in Fig 4. The associated delta errors $\frac{\widehat{\partial c}}{\partial s}$ - $\frac{\partial c}{\partial s}$ are plotted in Fig 5. The delta errors over five options are ranging from -0.95 to -0.8 with some large fluctuations when approaching to expiry. In general, RBF model can extract delta from data and approximate the true delta with relatively small error.



(a) (b)

***Figure 4**: (a) gives 3D scatter plot of delta extracted RBF models whereas (b) generates the surface plot for the same data points.*



(a) (b) (c)

***Figure** 5: (a) gives 3D scatter plot of delta error $\frac{\widehat{\partial c}}{\partial s}$ - $\frac{\partial c}{\partial s}$ whereas (b) generates the surface plot for the same data point. (c) gives boxplot of delta error over five different call options. In (a) and (b), there are some rise when time closed to maturity. This also is reflected in (c) as there are several outliers in each call option.*

Discussion

RBF model is a nonparametric learning model that can approximate option prices and reliably extract delta from it. It provides another alternative to estimated Black-Scholes equation. RBF model has a few advantages over conventional estimation from Black-Scholes. Initially, the data are classified into four clusters with its own mean and covariance. Nonlinear RBF localized the data via Mahalanobis distance. So any data outside clusters will be zero. Next, weights of four clusters are obtained through solving least square equations. Thus, the optimized weights can make clusters to fit data. Secondly, price error and delta error are relatively small. RBF model can be good approximation for pricing options. Lastly, RBF model requires no volatility estimation compared to Black-Scholes formula. So RBF model can provide more accurate and efficient approach.

Reference:

[1] J. Hutchinson, A. Lo, and T. Poggio, “A nonparametric approach to pricing and hedging derivative securities via learning networks,” The Journal of Finance, Vol. 49, no. 3, pp 851- 889, 1994.