Equivalent Circuit Model

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1 Thevenin Model

1.1 $I > 0$ for charge

$$
\begin{cases} \dot{x} = Ax + Bu & \text{(1a)} \\ y = \mathrm{OCV}(\mathrm{SoC}) + V_1 + V_2 + R_0 I & \text{(1b)} \end{cases}
$$

$$
A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{R_1 C_1} & 0 \\ 0 & 0 & -\frac{1}{R_2 C_2} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{3600 Q_c} \\ \frac{1}{C_1} \\ \frac{1}{C_2} \end{bmatrix}, \quad x = \begin{bmatrix} \mathrm{SoC} \\ V_1 \\ V_2 \end{bmatrix}, \quad u = I
$$

1.2 $I > 0$ for discharge

$$
\begin{cases} \dot{x} = Ax + Bu & \text{(2a)} \\ y = \mathrm{OCV}(\mathrm{SoC}) - V_1 - V_2 - R_0 I & \text{(2b)} \end{cases}
$$

$$
A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{R_1 C_1} & 0 \\ 0 & 0 & -\frac{1}{R_2 C_2} \end{bmatrix}, \quad B = \begin{bmatrix} -\frac{1}{3600 Q_c} \\ \frac{1}{C_1} \\ \frac{1}{C_2} \end{bmatrix}, \quad x = \begin{bmatrix} \mathrm{SoC} \\ V_1 \\ V_2 \end{bmatrix}, \quad u = I
$$

2 Nonlinear Double-Capacitor Model

2.1 $I > 0$ for charge

$$
\begin{cases} \dot{x} = Ax + Bu & \text{(3a)} \\ y = h(V_s) + V_1 + R_0 I & \text{(3b)} \end{cases}
$$

$$
A = \begin{bmatrix} -\frac{1}{C_b(R_b+R_s)} & \frac{1}{C_b(R_b+R_s)} & 0 \\ \frac{1}{C_s(R_b+R_s)} & -\frac{1}{C_s(R_b+R_s)} & 0 \\ 0 & 0 & -\frac{1}{R_1 C_1} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{R_s}{C_b(R_b+R_s)} \\ \frac{R_b}{C_s(R_b+R_s)} \\ \frac{1}{C_1} \end{bmatrix}, \quad x = \begin{bmatrix} V_b \\ V_s \\ V_1 \end{bmatrix}, \quad u = I
$$

2.2 $I > 0$ for discharge

$$
\begin{cases}
\dot{x} = Ax + Bu & \text{(4a)} \\
y = h(V_s) - V_1 - R_0 I & \text{(4b)}
\end{cases}
$$

$$
A = \begin{bmatrix} -\frac{1}{C_b(R_b+R_s)} & \frac{1}{C_b(R_b+R_s)} & 0 \\ \frac{1}{C_s(R_b+R_s)} & -\frac{1}{C_s(R_b+R_s)} & 0 \\ 0 & 0 & -\frac{1}{R_1 C_1} \end{bmatrix}, \quad B = \begin{bmatrix} -\frac{R_s}{C_b(R_b+R_s)} \\ -\frac{R_b}{C_s(R_b+R_s)} \\ \frac{1}{C_1} \end{bmatrix}, \quad x = \begin{bmatrix} V_b \\ V_s \\ V_1 \end{bmatrix}, \quad u = I
$$