Imperial College London

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Scale of Fluctuation(SoF)

Ningxin Yang, PhD student

Imperial College London

Concept of SoF

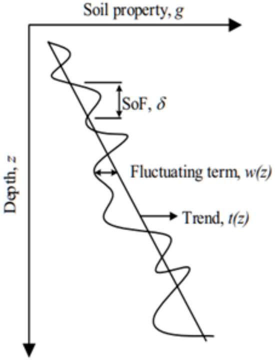

Figure 1: Illustration of the soil inherent variability[1]

The randnom field can be expressed as:

$$g\left(z\right) = t\left(z\right) + w\left(z\right) \qquad (1)$$

Some bullet point list for SoF:

- soil are rarely homogeneous
- $g\left(z\right)$ can be detrended into a stationary random function $w\left(z\right)$
- indicator of the estimated distance over a soil property
- measure for spatial variability of a soil property in a random field

Characteristics of SoF

1. For the SoF reported,horizontal SoF is generally larger than the vertical SoF.
2. Vertical SoF is relatively narrow, from 0.06 to 2.6 m
3. horizontal SoF for CPT is fairly broad compared with Vertical SoF, from 0.14 to 80 m.

Calculation of SoF-ACFM method

$$\hat{\rho}(\tau) = \frac{\sum_{i=1}^{n(\tau)} \left[w(z_i) - \bar{w}\right]\left[w(z_i + \tau) - \bar{w}\right]}{\left[n(\tau) - 1\right]\hat{\sigma}^2} \tag{2}$$

where $\hat{\rho}(\tau)$ is autocorrelation function, $\bar{w}$ and $\hat{\sigma}^2$ denote the sample mean and the sample variance of $w(z)$, $n(\tau)$ is number of lag distance τ

- Autocorrelation fitting method (ACFM) is to be one of the most widely used methods for estimating SoF.
- The main idea of ACFM is to fit theoretical models to the sample autocorrelation function $\hat{\rho}(\tau)$based on an ordinary least squares approach

Table 1: Theoretical autocorrelation models

Model	Autocorrelation fuction
Triangular	$\rho(\tau) = \begin{cases} 1 - \lvert\tau\rvert/\delta & \lvert\tau\rvert \leq \delta \\ 0 & \lvert\tau\rvert > \delta \end{cases}$
Exponential	$\rho(\tau) = \exp\left(-\frac{2\lvert\tau\rvert}{\delta}\right)$
Squared exponential	$\rho(\tau) = \exp\left(-\pi\left(\frac{\tau}{\delta}\right)^2\right)$
Cosine exponential	$\rho(\tau) = \cos\left(\frac{\tau}{\delta}\right)\exp\left(\frac{-\lvert\tau\rvert}{\delta}\right)$
Second-order Markov	$\rho(\tau) = \left(1 + \frac{4\lvert\tau\rvert}{\delta}\right)\exp\left(-\frac{4\lvert\tau\rvert}{\delta}\right)$

[1]Note: τ is the separation distance and δ is the scale of fluction.

Five common theoretical autocorrelation models are given above

Calculation of SoF-Fitting

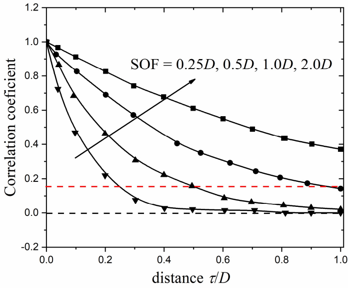

Figure 2: Illustration of the soil inherent variability followed [1]

- Get autocorrelation coefficients for varying τ
- Utilize the autocorrelation function to fit the coefficients
- Obtain the distance as the intersection with the redline

Reference

[1] X Nie et al. "Scale of fluctuation for geotechnical probabilistic analysis scale of fluctuation for geotechnical probabilistic analysis". In: *Proc. of the 5th International Symposium on Geotechnical Safety and Risk*. Vol. 1. 2015, pp. 834–40.