Imperial College London

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PhD week 18-Weekly summary

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In this week, I was looking at:

- Polynomial chaos expansion
- Paul-Remo Wagner's PhD thesis
- Bruno Sudret's PhD thesis

Terms encountered:

- Why need probabilistic measure
- Hilbert space-square integrable
- Method of weight residuals (Collocation-dirac,subdomain-1,least square method-R(x))
- Galerkin method-error function orthogonal to shape function
- PCA for reducing the dimension for input variables;PCE describe the relation between input and output
- Intrusive method vs non-intrusive method (intrusive not applicable, need to modify the FEM)

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Did:

$$Y^{PCE} = \sum_{\boldsymbol{\alpha} \in A} y_{\alpha} \Psi_{\alpha}(X) \tag{1}$$

- Construct the basis
- Compute the coefficient
- check the accuracy
- post process

Construct PCE polynomial basis

$$\Psi_\alpha(x) \overset{def}{=} \sum_{i=1}^{M} \Psi_{\alpha_i}^{(i)}(x_i) \tag{2}$$

$\alpha = \{\alpha_1, \ldots, \alpha_M\}, \alpha \in \mathbb{N}^M$

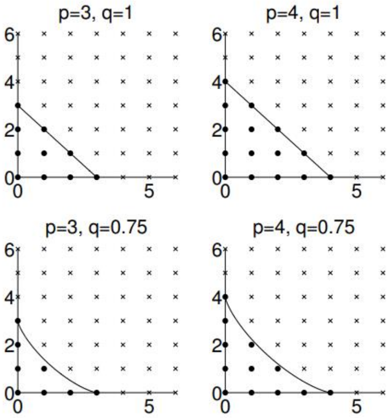
Note: Need to truncate to finite numbers: (1)Restriction of maximum interaction (2) Hyperbolic truncation

$$A^{M,p,q} = \{\alpha \in A^{M,p} : ||\alpha||_q \leq p\} \tag{3}$$

$$||\alpha||_q = \left(\sum_{i=1}^{M} \alpha_i^q\right)^{1/q}$$

If X is mutually dependent, use probabilistic transform into independent + copula

Construct PCE polynomial basis



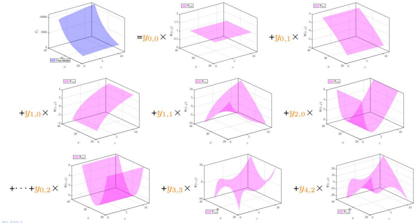


Figure 2: PCE 90% error band and predictive distribution

Compute the coefficient

- Projection Projection methods use the orthogonality of the basis functions to compute the coefficients by numerical integration (quadrature)

$$y_\alpha = E[\Psi_\alpha(X) \cdot M(X)] \tag{4}$$

- Regression/ sparse solution/ ordinary least square/least-square minimization/compressive sensing/stochastic collocation

check the accuracy

- Generalisation error (no good, overfitting)

$$\epsilon_{Gen} \stackrel{def}{=} E\left[(M(X) - M_{PCE}(X))^2\right] \tag{5}$$

- Leave-one-out (okay)

$$\epsilon_{\mathrm{LOO}} = \frac{1}{N}\sum_{i=1}^{N}\left(M(x^{(i)}) - M_{PCE}^{\sim i}(x^{(i)})\right)^2 \tag{6}$$

Post process

Mean of the PCE:

$$E[Y] =< M, 1 >_{L^2_{f_{\mathbf{X}}}}, \quad f_X = a_0 \tag{7}$$

Variance of PCE:

$$\mathrm{Var}[Y] = E[Y^2] - E[Y]^2 =< M, M >_{L^2_{f_{\mathbf{X}}}} - < M, 1 >_{L^2_{f_{\mathbf{X}}}} \approx \sum_{\alpha \in A \setminus \{0\}} a_\alpha^2 \tag{8}$$

which equals to Bruno's said on UQlab:

$Y_i = \sum_{\alpha \in \mathcal{A}} a_\alpha \Psi_\alpha(\mathbf{x}) \quad ; \quad Y_j = \sum_{\alpha \in \mathcal{A}} b_\alpha \Psi_\alpha(\mathbf{x})$

$\mathrm{Cov}\left[Y_i, Y_j\right] = \sum_{\alpha \in \mathcal{A} \setminus 0} a_\alpha b_\alpha$

Havenot done or understood:

- "This possibly correlated Gaussian mismatch model should be properly propagated when you sample the predictive distributions" Bruno said. Which i dont understand now
- the calculation procedure PCE works
- a lot of the rest in their PhD thesis.