PCE-Bayesian Inference-Excavation deflection

1 - INITIALIZE UQLAB

```
clearvars;
uqlab;
```

```
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F:\abaqustemp\UQLab_Rel2.0.0\LICENSE.

To request special permissions, please contact:
  - Stefano Marelli (marelli@ibk.baug.ethz.ch).

Useful commands to get started with UQLab:
uqlab -doc           - Access the available documentation
uqlab -help          - Additional help on how to get started with UQLab
uq_citation help   - Information on how to cite UQLab in publications
uqlab -license       - Display UQLab license information
```

2 - RETRIEVE DATA SETS

X = input elastic modulus *E* and loading position *δ* to begin PCE

```
X = xlsread("excavation_data",2);

Y = xlsread("excavation_data.xlsx",3);
```

3 - INPUT MODEL/PRIOR DISTRIBUTION OF THE MODEL PARAMETERS

PCE requires a choice of polynomial basis, a probabilistic input model needs to be defined. Specify the marginals of the probabilistic input model:

```
%Young's modulus
InputOpt.Marginals(1).Name = 'Elastic modulus';
InputOpt.Marginals(1).Type = 'Gaussian';
InputOpt.Marginals(1).Moments = [210000000,30000000];

%loading position
InputOpt.Marginals(2).Name = 'Loading position';
InputOpt.Marginals(2).Type = 'Gaussian';
InputOpt.Marginals(2).Moments = [0,1];

%Create an INPUT object based on the specified marginals:
myInput = uq_createInput(InputOpt);
```

4 - SURROGATE MODEL/POLYNOMIAL CHAOS EXPANSION (PCE) METAMODEL

Select PCE as the metamodeling tool:

```
MetaOpts.Type = 'Metamodel';
```

```
MetaOpts.MetaType = 'PCE';
MetaOpts.Method = 'LARS';
MetaOpts.TruncOptions.qNorm = 0.75;
```

Loop to the surrogate model; Use FEA data

```
MetaOpts.ExpDesign.X = X;
MetaOpts.ExpDesign.Y = Y;
```

Set the maximum polynomial degree to 5:

```
MetaOpts.Degree = 2:10;
```

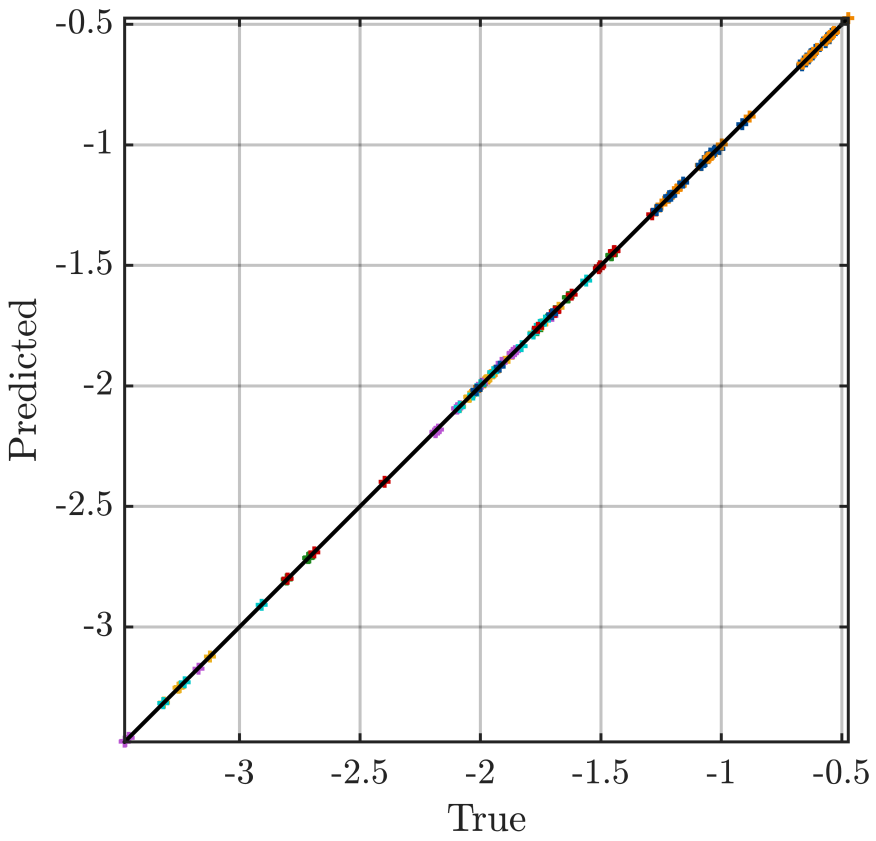
Create the metamodel object and add it to UQLab:

```
mySurrogateModel  = uq_createModel(MetaOpts);
```

```
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 10 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 12 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 5 and qNorm 0.75 for output variable 1
Final LOO error estimate: 3.042282e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 10 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 11 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 7 and qNorm 0.75 for output variable 2
Final LOO error estimate: 1.093585e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 10 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 11 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 12 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 13 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 5 and qNorm 0.75 for output variable 3
Final LOO error estimate: 5.376560e-04
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 10 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 11 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 12 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 13 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 13 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 6 and qNorm 0.75 for output variable 4
Final LOO error estimate: 1.352357e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 10 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 11 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 4 and qNorm 0.75 for output variable 5
Final LOO error estimate: 1.221514e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 11 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 12 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 5 and qNorm 0.75 for output variable 6
Final LOO error estimate: 2.801315e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 11 was set to 0 to prevent crashes.
```

```
警告: Warning: numerical instability!! Gamma for LAR iteration 12 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 5 and qNorm 0.75 for output variable 7
Final LOO error estimate: 5.925357e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 11 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 12 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 4 and qNorm 0.75 for output variable 8
Final LOO error estimate: 2.236863e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 10 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 11 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 12 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 13 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 4 and qNorm 0.75 for output variable 9
Final LOO error estimate: 2.066843e-03
---                 Calculation finished!                                 ---
```

```
%Plot the ture vs. predicted values
YPCE = uq_evalModel(mySurrogateModel,X);
uq_plot(Y,YPCE,'+');
hold on;
uq_plot([min(Y) max(Y)], [min(Y) max(Y)], 'k');
axis equal;
axis([min(Y(:)) max(Y(:)) min(Y(:)) max(Y(:))]);
xlabel('True');ylabel('Predicted');
hold off;
```



5 - MEASUREMENT DATA

```
measurement =  xlsread("excavation_data.xlsx",4)';
myData.y = 0.3*randn(size(measurement)).*measurement + measurement+0.5;
myData.Name = 'middleDeflection';
position = 1:9
```

```
position = 1×9
     1     2     3     4     5     6     7     8     9
```

6 - BAYESIAN ANALYSIS

```
%The options of the Bayesian inversion analysis are specified with the following structure:Baye
BayesOpts.Type = 'Inversion';
BayesOpts.Data = myData;

%Run the Bayesian inversion analysis:
myBayesianAnalysis = uq_createAnalysis(BayesOpts);
```

```
The discrepancy was not specified,
using unknown i.i.d. Gaussian discrepancy...
The solver was not specified, using MCMC
The sampler was not specified, using affine invariant ensemble sampler
Starting AIES...

|##############################| 100.00%

Finished AIES!
```

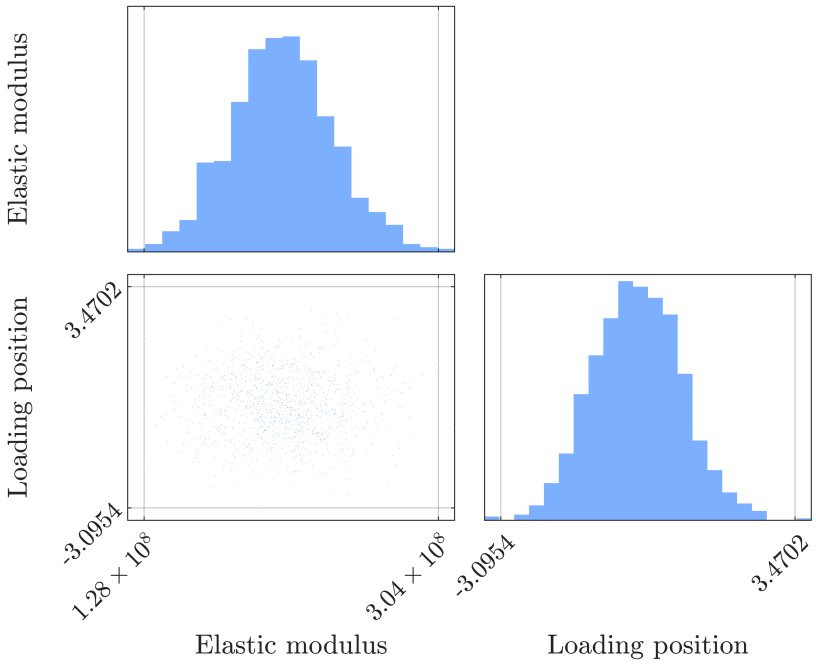
```
%print and display

uq_print(myBayesianAnalysis);
```

```
%----------------------- Inversion output -----------------------%
   Number of calibrated model parameters:         2
   Number of non-calibrated model parameters:     0

   Number of calibrated discrepancy parameters:   9

%------------------- Data and Discrepancy
%  Data-/Discrepancy group 1:
   Number of independent observations:            1

   Discrepancy:
      Type:                                       Gaussian
      Discrepancy family:                         Row
      Discrepancy parameters known:               No

   Associated outputs:
      Model 1:
         Output dimensions:                       1
                                                  to
                                                  9


%------------------- Solver
   Solution method:                               MCMC
```

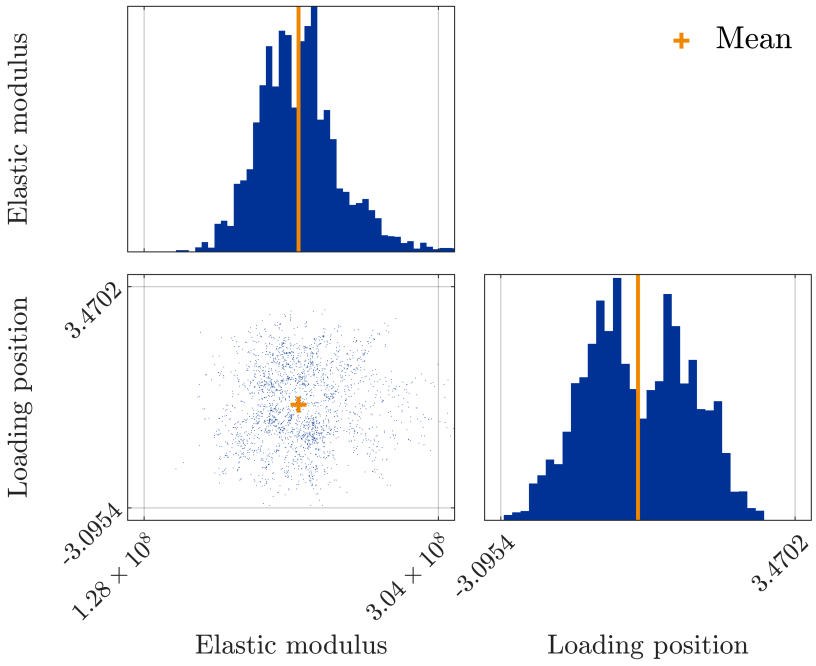
```
   Algorithm:                                AIES
   Duration (HH:MM:SS):                      00:07:03
   Number of sample points:                  3.00e+04

%------------------- Posterior Marginals
-------------------------------------------------------------------------------
| Parameter        | Mean    | Std     | (0.025-0.97) Quant. | Type        |
-------------------------------------------------------------------------------
| Elastic modulus  | 2.2e+08 | 2.4e+07 | (1.8e+08 - 2.7e+08) | Model       |
| Loading position | -0.035  | 1.2     | (-2.2 -  2)         | Model       |
| Sigma2           | 0.24    | 0.17    | (0.0054 - 0.53)     | Discrepancy |
| Sigma2           | 1.2     | 0.65    | (0.076 - 2.1)       | Discrepancy |
| Sigma2           | 1.3     | 0.97    | (0.031 - 3.2)       | Discrepancy |
| Sigma2           | 4.4     | 2.5     | (0.41 -  9)         | Discrepancy |
| Sigma2           | 0.93    | 0.5     | (0.15 - 1.8)        | Discrepancy |
| Sigma2           | 0.79    | 0.38    | (0.15 - 1.4)        | Discrepancy |
| Sigma2           | 4.7     | 2.5     | (0.72 - 8.9)        | Discrepancy |
| Sigma2           | 4.4     | 2.2     | (0.79 - 8.5)        | Discrepancy |
| Sigma2           | 0.11    | 0.06    | (0.0083 - 0.2)      | Discrepancy |
-------------------------------------------------------------------------------

%------------------- Point estimate
-----------------------------------------------
| Parameter        | Mean    | Parameter Type |
-----------------------------------------------
| Elastic modulus  | 2.2e+08 | Model          |
| Loading position | -0.035  | Model          |
| Sigma2           | 0.24    | Discrepancy    |
| Sigma2           | 1.2     | Discrepancy    |
| Sigma2           | 1.3     | Discrepancy    |
| Sigma2           | 4.4     | Discrepancy    |
| Sigma2           | 0.93    | Discrepancy    |
| Sigma2           | 0.79    | Discrepancy    |
| Sigma2           | 4.7     | Discrepancy    |
| Sigma2           | 4.4     | Discrepancy    |
| Sigma2           | 0.11    | Discrepancy    |
-----------------------------------------------

%------------------- Correlation matrix (model parameters)
-------------------------------------------------------------
|                  |  Elastic modulus    Loading position  |
-------------------------------------------------------------
| Elastic modulus  |  1                  0.065             |
| Loading position |  0.065              1                 |
-------------------------------------------------------------

%------------------- Correlation matrix, 6 most important (discrepancy parameters)
-------------------------------------------------------------------------------
|        |  Sigma2    Sigma2     Sigma2    Sigma2     Sigma2    Sigma2  |
-------------------------------------------------------------------------------
| Sigma2 |  1         0.039      0.17      0.055      -0.084    0.0025  |
| Sigma2 |  0.039     1          0.033     -0.0037    -0.12     0.07    |
| Sigma2 |  0.17      0.033      1         -0.045     -0.063    0.066   |
| Sigma2 |  0.055     -0.0037    -0.045    1          -0.15     0.027   |
| Sigma2 |  -0.084    -0.12      -0.063    -0.15      1         -0.15   |
| Sigma2 |  0.0025    0.07       0.066     0.027      -0.15     1       |
-------------------------------------------------------------------------------
```

```
%

%%%plot the mean and 95% error
uq_display_uq_inversion_MCMC(myBayesianAnalysis,'scatterplot',[1 2]);
```

Prior Sample

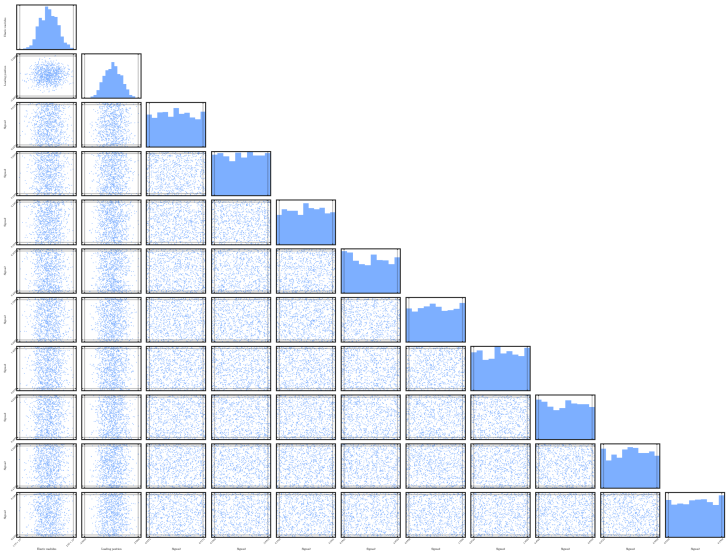


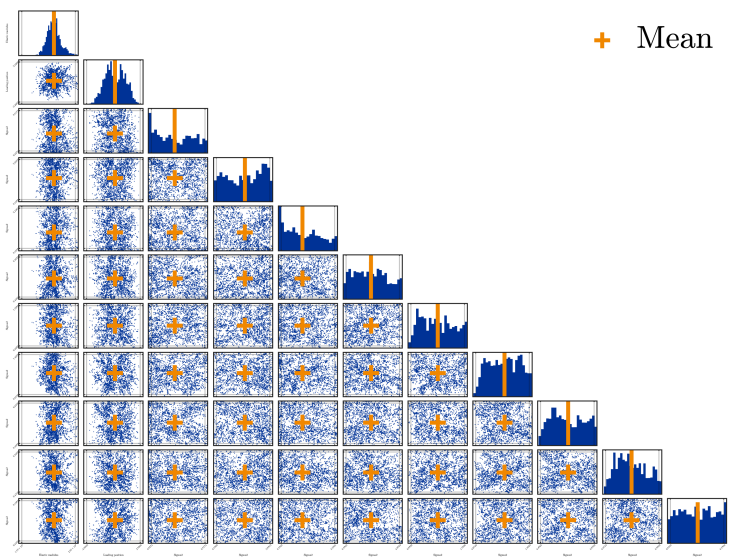
Posterior Sample



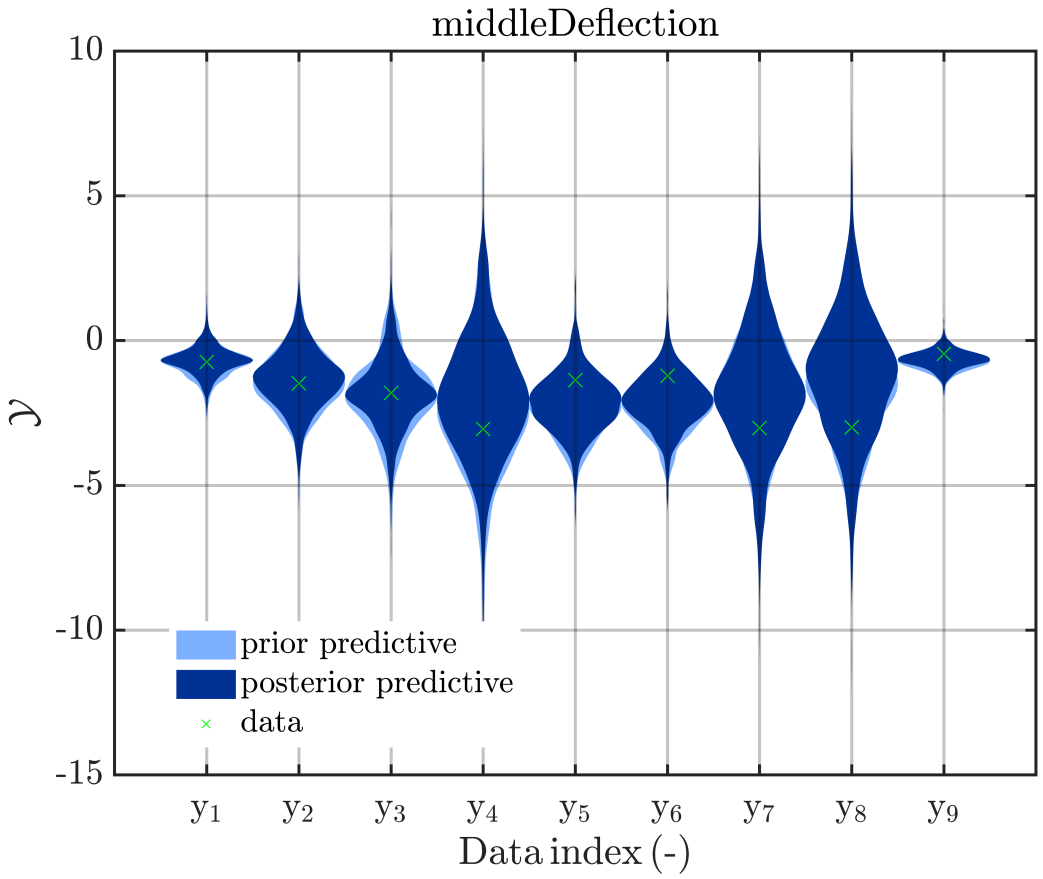
```
uq_postProcessInversion(myBayesianAnalysis,'priorPredictive',1000);


uq_display(myBayesianAnalysis);
```

Prior Sample






```
hold off;
```




```
%Get the lower bound of predictive posterior


for i = 1:size(position,2)
    p(i) = fitdist(myBayesianAnalysis.Results.PostProc.PostPredSample.Sample(:,i),'kernel');
end
Line_space= -10:0.1:5;

LB_deflection = min(myBayesianAnalysis.Results.PostProc.PostPredSample.Sample)';
%Get the upper bound of predictive posterior
UB_deflection = max(myBayesianAnalysis.Results.PostProc.PostPredSample.Sample)';
%Get the mean bound of predictive posterior
mean_deflection = mean(myBayesianAnalysis.Results.PostProc.PostPredSample.Sample)';

X_dense = linspace(position(1),  position(end), 1000);
LB_deflection_interp = spline(position,LB_deflection,X_dense);
UB_deflection_interp = spline(position,UB_deflection,X_dense);

Y_new = [LB_deflection_interp;UB_deflection_interp]';

plot(X_dense',Y_new);hold on;
plot(position',mean_deflection);Y_True = measurement';
plot(position',Y_True,'r','Linewidth',2,'MarkerSize',8);
scatter(position,myData.y,200,'r','o');
```

```
legend('Lower Bound','Upper Bound','Mean','Ground Truth','Measurement');
xlabel('Distance (m)','FontSize',20);ylabel('Deflection(m)',"FontSize",20);
```