Uncertainty quantification (UQ) in geotechnical engineering

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Presentation Overview

1. UQ and uncertainty types
2. UQ framework
3. How to consider uncertainties into UQ framework
4. Applications

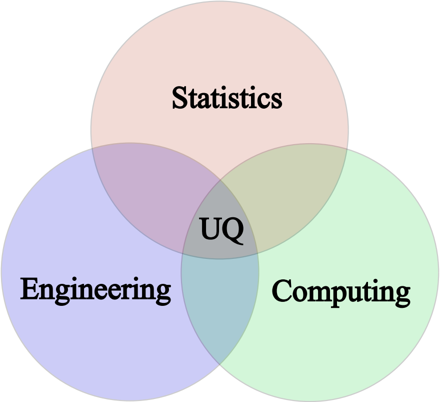
What is Uncertainty Quantification (UQ)?

All models are wrong, but some are useful.
–George E.P. Box

How accurate are the models?
When are probabilistic models useful in engineering problems?

UQ provides a framework answering these questions and making model useful.

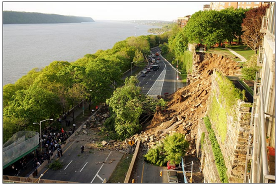
What is Uncertainty Quantification (UQ)?



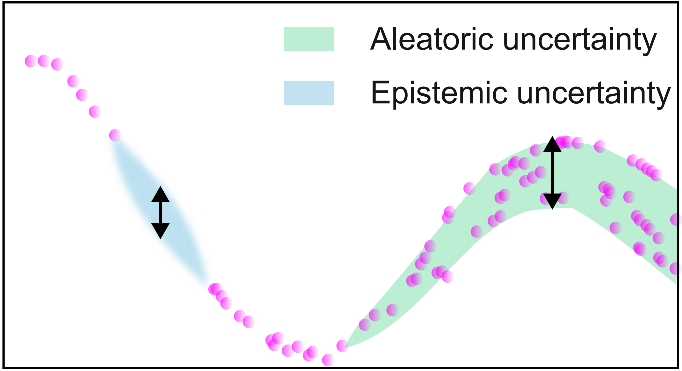
UQ is the science of quantitative characterization and reduction of uncertainties in both computational and real world applications[1]

[1] Victor E. Saouma and M. Amin Hariri-Ardebili. “Uncertainty Quantification”. In: *Aging, Shaking, and Cracking of Infrastructures: From Mechanics to Concrete Dams and Nuclear Structures*. Cham: Springer International Publishing, 2021, pp. 423–454.

Uncertain events in geotechnics

[1] capitalgeotechnical



Aleatoric uncertainty vs Epistemic uncertainty

Distinguish:

Aleatoric uncertainty

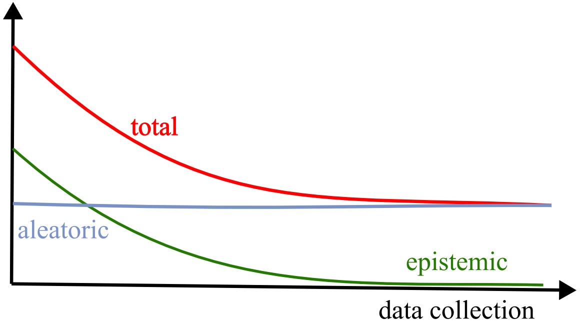
statistical variability, inherently random effects (**irreducible**)

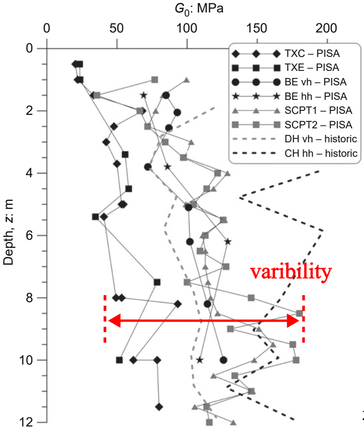
Epistemic uncertainty

model uncertainty, a lack of knowledge (**reducible**)

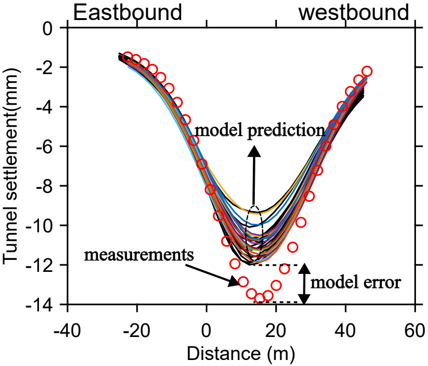
Components of uncertainty :

Total uncertainty $\approx$ aleatoric uncertainty $+$ epistemic uncertainty



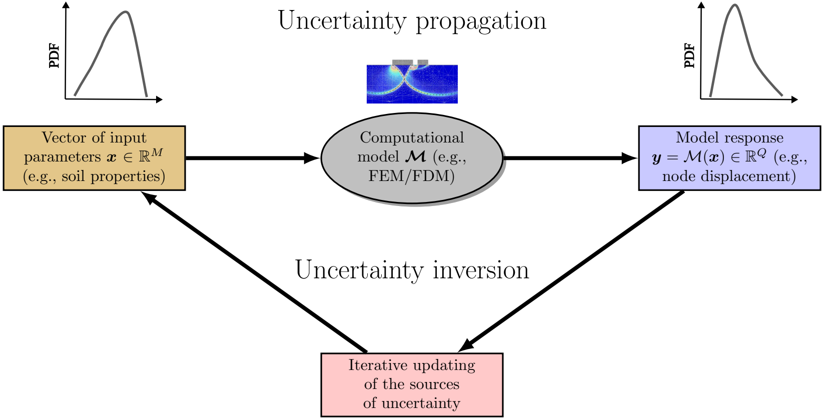


[2]



[2] Lidija Zdravković et al. "Ground characterisation for PISA pile testing and analysis". In: *Géotechnique* 70.11 (2020), pp. 945–960.

UQ framework and components

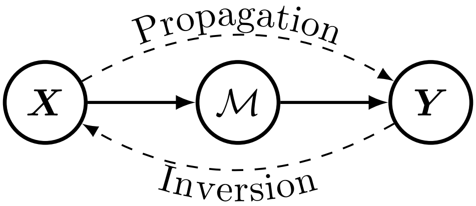


[3]

[3] Bruno Sudret. "Uncertainty quantification in the simulation of complex systems". In: *1st International Conference on Infrastructure Resilience*. ETH Zurich. 2018.

UQ framework and components

Two UQ types of problems:



Four components:

1. Model assessment
2. Uncertainty propagation
3. Model calibration
4. Sensitivity analysis

Uncertainty quantification for engineering problems

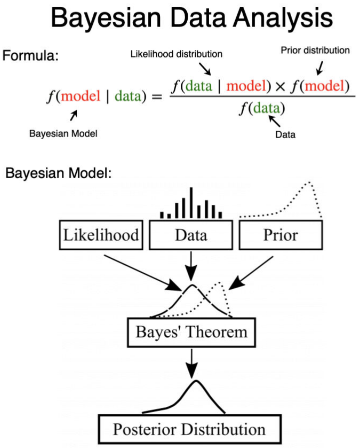
Research topics

- Modelling uncertainty
- Surrogate models (low dimensions/fidelity; high dimensions/fidelity)
- Stochastic inverse problem
- active learning
- Random field
- ...

Choice for the model calibration:

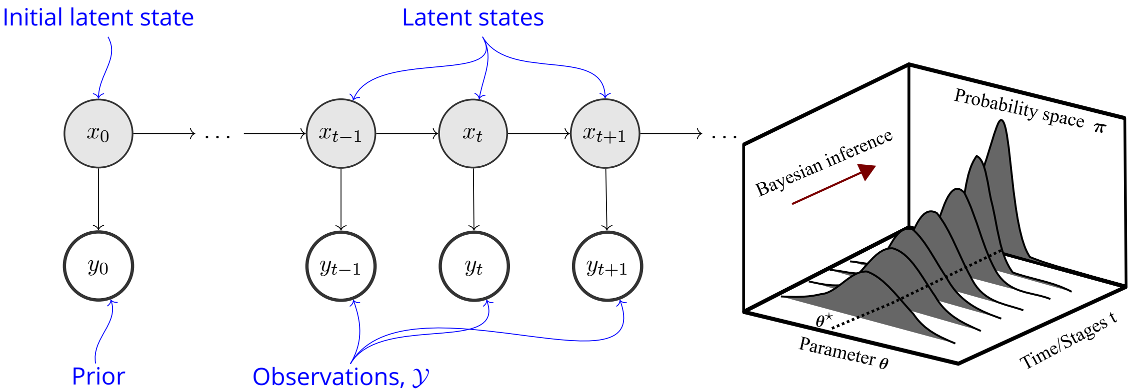
- **Lack** or **no** data
- **Substantial** volume data
- **Combination** of two above: Bayesian methods

$$\pi(\boldsymbol{x}|\mathcal{Y}) = \frac{\mathcal{L}(\boldsymbol{x}|\mathcal{Y}) \cdot \pi(\boldsymbol{x})}{\pi(\mathcal{Y})}$$



From Matt Dancho

Sequential model calibration



Connecting reality with black-box simulator

$$y \leftrightarrow \mathcal{M}(x)$$

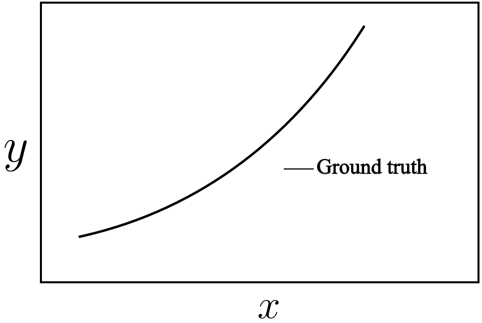
Challenges:

- Measurement data contains noise
- Simulator predictions have model errors
- With a surrogate, it will introduce new error

These uncertainties/errors above will hinder the calibration process

Connecting reality with black-box simulator

Consider a synthetic problem with no observation error:



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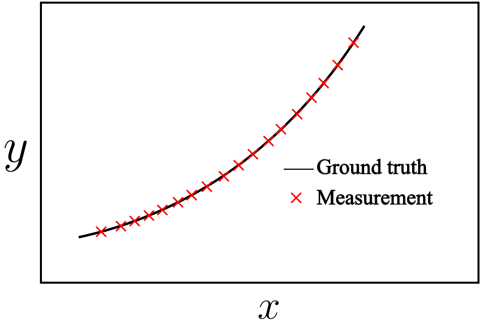
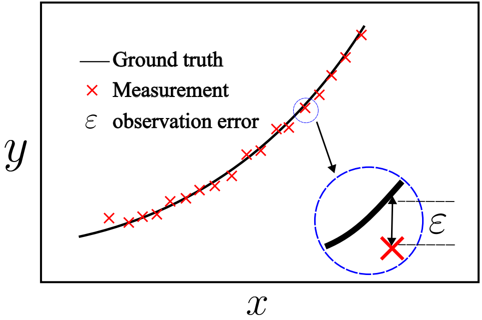


Connecting reality with black-box simulator

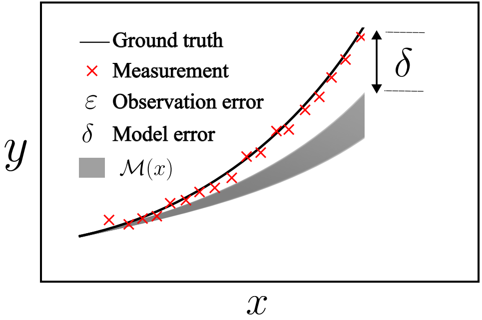
observation error ε



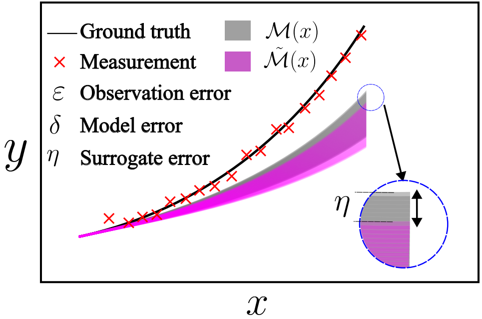
[2] observation error may contain systematic and instrument

Connecting reality with black-box simulator

Model discrepancy δ

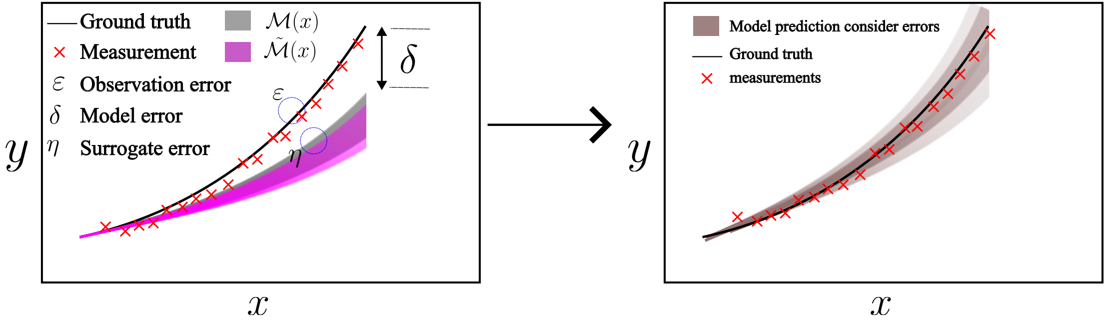


Surrogate error η



Connecting reality with black-box simulator

$$y = \tilde{\mathcal{M}}(x) + \eta + \delta + \varepsilon$$



$$\text{Real physics } \varepsilon \xrightarrow{-\delta} \text{Computational Model } \mathcal{M}(x) \xrightarrow{-\eta} \text{Surrogate } \tilde{\mathcal{M}}(x)$$

Uncertainties/errors

- observation error ε
- Model discrepancy δ
- Surrogate error η

Real physics:

$$\begin{aligned} \boldsymbol{y} &= \mathcal{M}(\boldsymbol{x}) + \delta + \boldsymbol{\varepsilon} \\ &= \tilde{\mathcal{M}}(\boldsymbol{x}) + \eta + \delta + \boldsymbol{\varepsilon} \end{aligned}$$

Question: where should the uncertainties go to?

$$\pi(\boldsymbol{x}|\mathcal{Y}) = \frac{\mathcal{L}(\boldsymbol{x}|\mathcal{Y}) \cdot \pi(\boldsymbol{x})}{\pi(\mathcal{Y})} = \frac{\mathcal{L}(\boldsymbol{x}|\mathcal{Y}) \cdot \pi(\boldsymbol{x})}{\int_{\mathcal{D}_{\boldsymbol{X}}} \pi(\boldsymbol{x})\pi(\mathcal{Y}|\boldsymbol{x})\mathrm{d}\boldsymbol{x}}$$

Incorporating uncertainties into the $\mathcal{L}(x|\mathcal{Y})$

If only consider observation error ε

$$\boldsymbol{y} = \mathcal{M}(\boldsymbol{x}) + \boldsymbol{\varepsilon};\ \boldsymbol{\varepsilon} \in \mathcal{N}(\boldsymbol{\varepsilon}|\mathbf{0}, \boldsymbol{\Sigma})$$

$$\boldsymbol{y} - \mathcal{M}(\boldsymbol{x}) = \boldsymbol{\varepsilon};\ \boldsymbol{\varepsilon} \in \mathcal{N}(\boldsymbol{\varepsilon}|\mathbf{0}, \boldsymbol{\Sigma})$$

$$\begin{aligned}\mathcal{L}(\boldsymbol{x}|\mathcal{Y}) =& N(\boldsymbol{y}|\mathcal{M}(\boldsymbol{x}), \boldsymbol{\Sigma}) \\ =& \frac{1}{\sqrt{(2\pi)\det(\boldsymbol{\Sigma})}} \exp\left(-\frac{1}{2}\left(\boldsymbol{y} - \mathcal{M}(\boldsymbol{x})\right)^{\mathsf{T}} \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{y} - \mathcal{M}(\boldsymbol{x})\right)\right)\end{aligned}$$

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Incorporate η and δ into $\mathcal{L}$:

$$Q(x, \boldsymbol{b}, \boldsymbol{c}) = \mathcal{M}(x) + \eta(x, \boldsymbol{b}) + \delta(x, \boldsymbol{c});$$

$$\begin{aligned}\mathcal{L}(\boldsymbol{x}|\mathcal{Y}) =& N(\boldsymbol{y}|Q(x, \boldsymbol{b}, \boldsymbol{c}), \boldsymbol{\Sigma}) \\ =& \frac{1}{\sqrt{(2\pi)\det(\boldsymbol{\Sigma})}} \exp\left(-\frac{1}{2}\left(\boldsymbol{y} - Q(x, \boldsymbol{b}, \boldsymbol{c})\right)^{\mathsf{T}} \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{y} - Q(x, \boldsymbol{b}, \boldsymbol{c})\right)\right)\end{aligned}$$

Bayesian inference results

Difficulty with calculating evidence $\pi(\mathcal{Y})$

$$\pi(\boldsymbol{x}|\mathcal{Y}) = \frac{\mathcal{L}(\boldsymbol{x}|\mathcal{Y}) \cdot \pi(\boldsymbol{x})}{\pi(\mathcal{Y})}$$

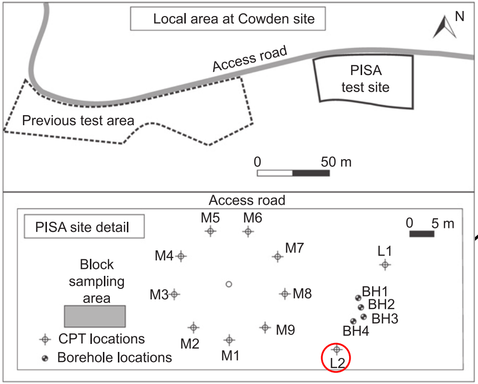
Computational methods:

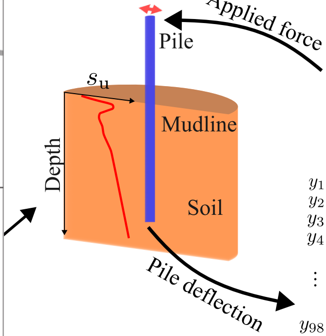
$$\pi(\boldsymbol{x}|\mathcal{Y}) \approx \mathcal{L}(\boldsymbol{x}|\mathcal{Y}) \cdot \pi(\boldsymbol{x})$$

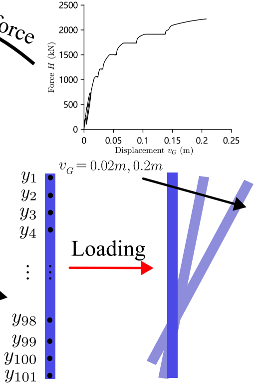
Samples from the posterior can be obtained through *Sampling methods* or *Optimization methods*: *MCMC*, *variational methods*,...

Application: inverse analysis of PISA pile

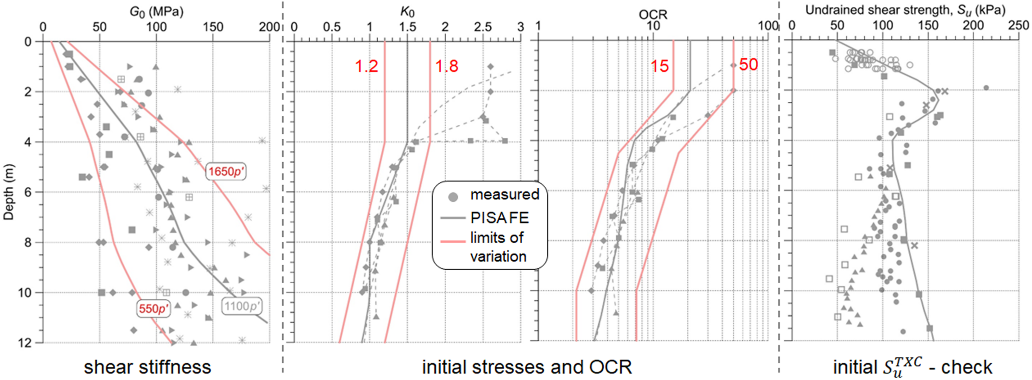
PISA pile loading:







Range of Parameters



3

[3] Lidija Zdravković. "Geotechnical Engineering for a Sustainable Society". In: *Rankine lecture* (2024).

Application: inverse analysis of PISA pile

In total, three parameters will be modelled in as random variables. These are (1) K_0, (2) OCR and (3) G_{max}. Engineering judgement is applied in selecting the values for the upper and lower bounds of each of the three parameters and for the shape of the K_0 and OCR distribution with depth. In order to explore the design space most effectively, uniform distributions were assumed for all three parameters.

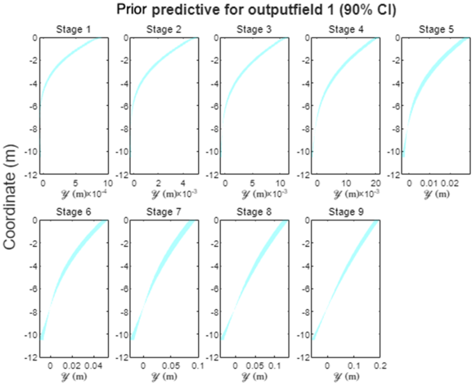
Table: Characteristic soil properties

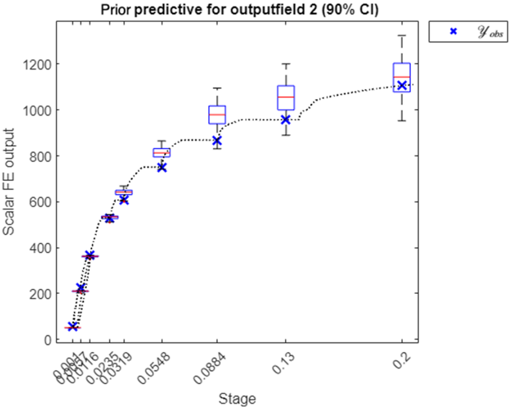
Parameter	Lower bound	Upper bound
K_0	1.35	1.8
OCR	15	50
G_{max}	55E+3	165E+3

[3]

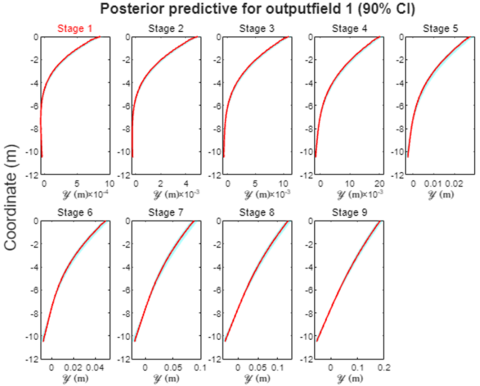
[3] Lidija Zdravković. "Geotechnical Engineering for a Sustainable Society". In: *Rankine lecture* (2024).

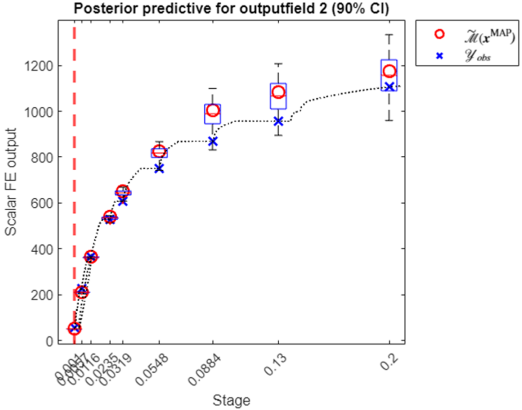
Prior predictive





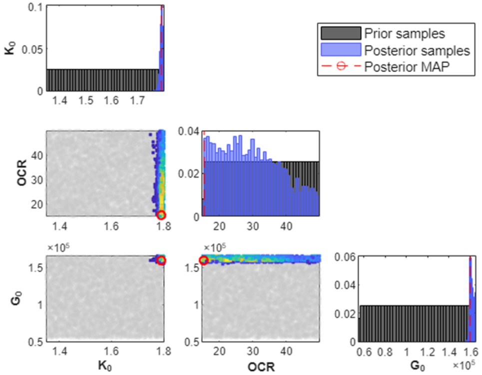
Posterior predictive after stage one

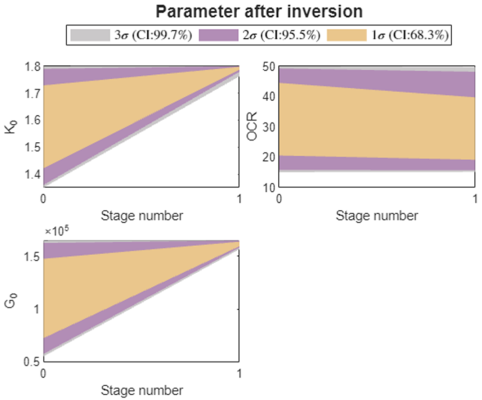




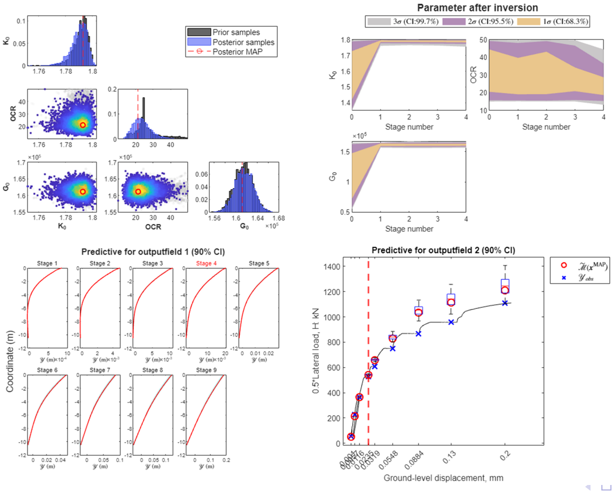
Application: inverse analysis of PISA pile

Parameters updating after stage one

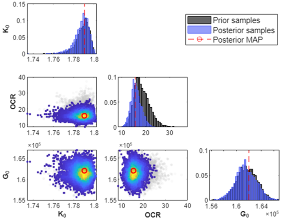


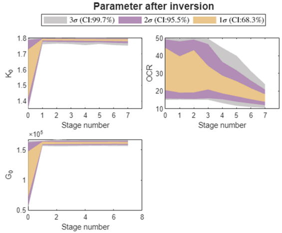


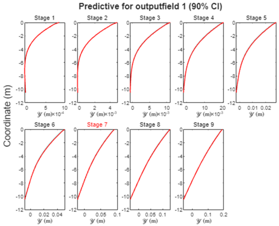
After stage four

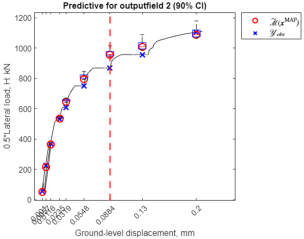


After stage seven

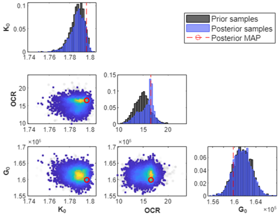




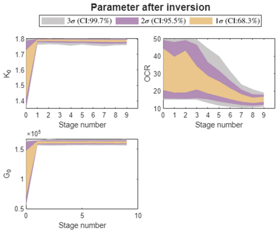




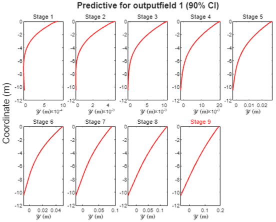
Finished!



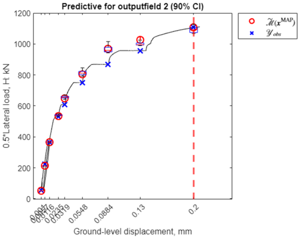
Prior samples
Posterior samples
Posterior MAP
K0
OCR
G0



Parameter after inversion
3σ (CI:99.7%)
2σ (CI:95.5%)
1σ (CI:68.3%)
Stage number



Predictive for outputfield 1 (90% CI)
Stage 1
Stage 2
Stage 3
Stage 4
Stage 5
Stage 6
Stage 7
Stage 8
Stage 9
Coordinate (m)



Predictive for outputfield 2 (90% CI)
0.5*Lateral load, H, kN
Ground-level displacement, mm

Summary:

1. Careful with aleatoric and epistemic types of uncertainties
2. Four UQ components are equally important
3. Consider different types of uncertainty into Bayesian inference required linked with likelihood-(currently doing)

References I

- Saouma, Victor E. and M. Amin Hariri-Ardebili. “Uncertainty Quantification”. In: *Aging, Shaking, and Cracking of Infrastructures: From Mechanics to Concrete Dams and Nuclear Structures*. Cham: Springer International Publishing, 2021, pp. 423–454.
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Thank you!

Questions? Comments?