# PCE-Bayesian Inference-Excavation deflection

## 1 - INITIALIZE UQLAB

```
clearvars;
uqlab;
```

```
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C:\NY2023\D_document\UQLab_Rel2.0.0\LICENSE.

To request special permissions, please contact:
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Useful commands to get started with UQLab:
uqlab -doc           - Access the available documentation
uqlab -help          - Additional help on how to get started with UQLab
uq_citation help   - Information on how to cite UQLab in publications
uqlab -license       - Display UQLab license information
```

## 2 - RETRIEVE DATA SETS

X = input elastic modulus $E$ and loading position $\delta$ to begin PCE

```
X = xlsread("excavation_data",2);

Y = xlsread("excavation_data.xlsx",3);
```

## 3 - INPUT MODEL/PRIOR DISTRIBUTION OF THE MODEL PARAMETERS

PCE requires a choice of polynomial basis, a probabilistic input model needs to be defined. Specify the marginals of the probabilistic input model:

```
%Young's modulus
InputOpt.Marginals(1).Name = 'Elastic modulus';
InputOpt.Marginals(1).Type = 'Gaussian';
InputOpt.Marginals(1).Moments = [210000000,30000000];

%loading position
InputOpt.Marginals(2).Name = 'Loading position';
InputOpt.Marginals(2).Type = 'Gaussian';
InputOpt.Marginals(2).Moments = [0,1];

%Create an INPUT object based on the specified marginals:
myInput = uq_createInput(InputOpt);
```

## 4 - SURROGATE MODEL/POLYNOMIAL CHAOS EXPANSION (PCE) METAMODEL

Select PCE as the metamodeling tool:

```
MetaOpts.Type = 'Metamodel';
MetaOpts.MetaType = 'PCE';
MetaOpts.Method = 'LARS';
MetaOpts.TruncOptions.qNorm = 0.75;
```

Loop to the surrogate model; Use FEA data

```
MetaOpts.ExpDesign.X = X;
MetaOpts.ExpDesign.Y = Y;
```

Set the maximum polynomial degree to 5:

```
MetaOpts.Degree = 2:10;
```

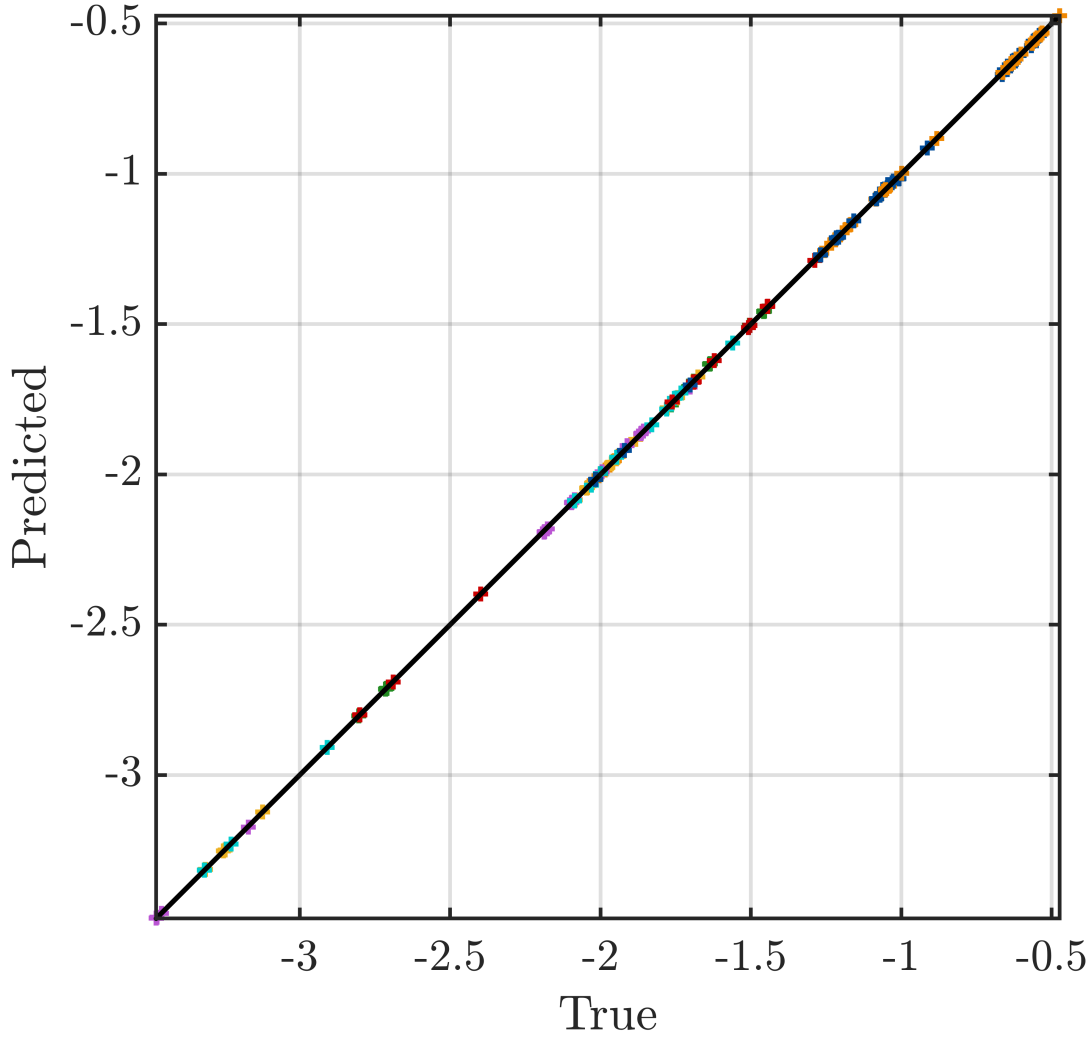
Create the metamodel object and add it to UQLab:

```
mySurrogateModel  = uq_createModel(MetaOpts);
```

```
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 11 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 5 and qNorm 0.75 for output variable 1
Final LOO error estimate: 3.042282e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 10 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 12 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 13 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 13 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 7 and qNorm 0.75 for output variable 2
Final LOO error estimate: 1.093585e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 10 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 5 and qNorm 0.75 for output variable 3
Final LOO error estimate: 5.376560e-04
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 10 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 11 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 12 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 13 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 6 and qNorm 0.75 for output variable 4
Final LOO error estimate: 1.352357e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 11 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 12 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 13 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 13 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 4 and qNorm 0.75 for output variable 5
Final LOO error estimate: 1.221514e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 10 was set to 0 to prevent crashes.
警告: Warning: numerical instability!! Gamma for LAR iteration 13 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 5 and qNorm 0.75 for output variable 6
Final LOO error estimate: 2.801315e-03
```

```
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
警告: Warning: numerical instability!! Gamma for LAR iteration 13 was set to 0 to prevent crashes.
The estimation of PCE coefficients converged at polynomial degree 5 and qNorm 0.75 for output variable 7
Final LOO error estimate: 5.925357e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 4 and qNorm 0.75 for output variable 8
Final LOO error estimate: 2.236863e-03
---                 Calculation finished!                                 ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 4 and qNorm 0.75 for output variable 9
Final LOO error estimate: 2.066843e-03
---                 Calculation finished!                                 ---
```

```
%Plot the ture vs. predicted values
YPCE = uq_evalModel(mySurrogateModel,X);
uq_plot(Y,YPCE,'+');
hold on;
uq_plot([min(Y) max(Y)], [min(Y) max(Y)], 'k');
axis equal;
axis([min(Y(:)) max(Y(:)) min(Y(:)) max(Y(:))]);
xlabel('True');ylabel('Predicted');
hold off;
```



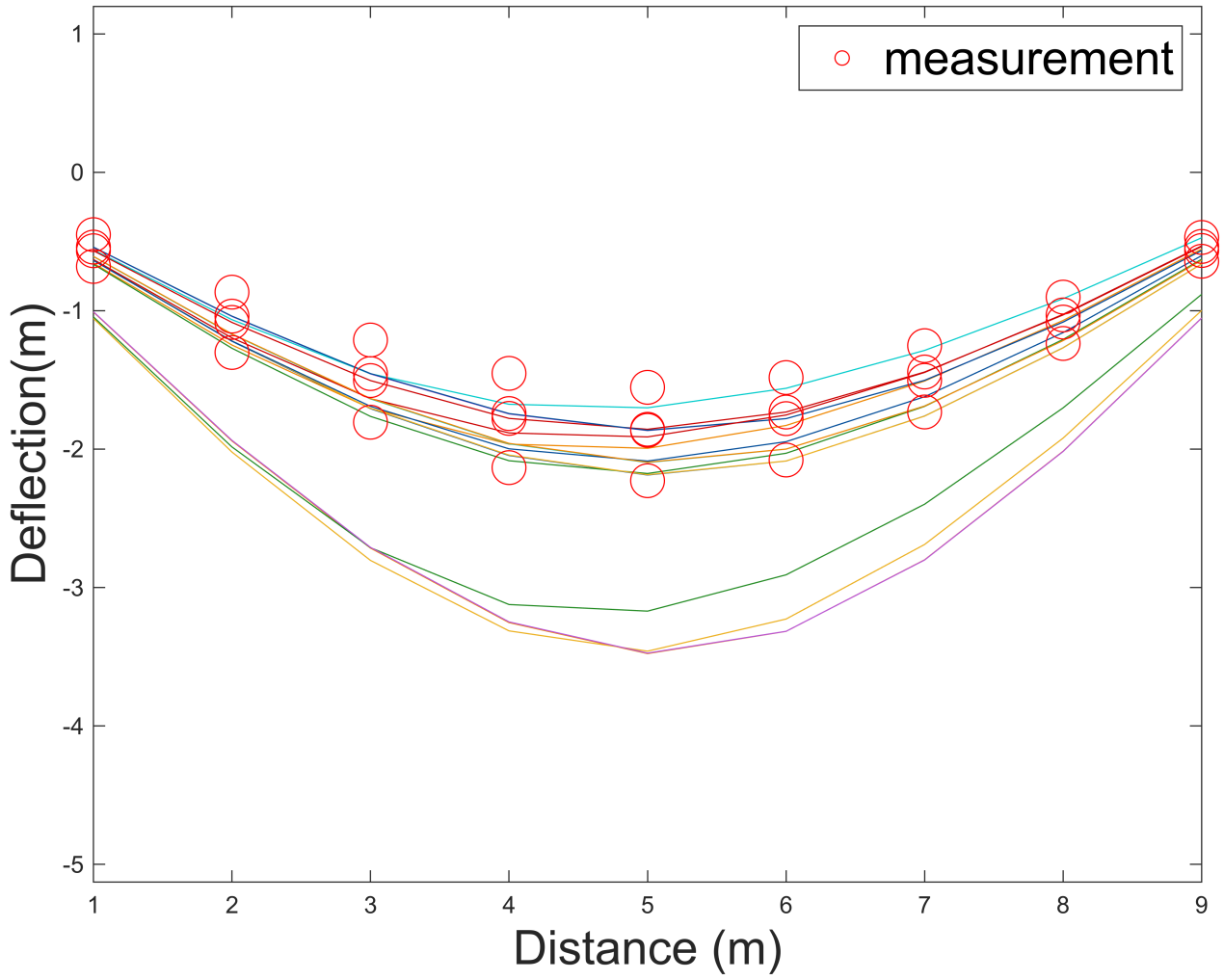

## 5 - MEASUREMENT DATA

```
myData.y =  xlsread("excavation_data.xlsx",4)';
myData.Name = 'middleDeflection';
position = 1:9
```

```
position = 1×9
     1     2     3     4     5     6     7     8     9
```

```
for i = 1:size(X,1)

    p = plot(position',Y(i,:)');
    hold on

end
axis equal;
xlabel('Distance (m)','FontSize',20);ylabel('Deflection(m)',"FontSize",20);
q = scatter(position,myData.y,200,'r','o');
```

```
legend(q(1),'measurement','FontSize',20) ;
hold off;
```




## 6 - BAYESIAN ANALYSIS

```
%The options of the Bayesian inversion analysis are specified with the following
structure:BayesOpts.Type = 'Inversion';
BayesOpts.Type = 'Inversion';
BayesOpts.Data = myData;

%Run the Bayesian inversion analysis:
myBayesianAnalysis = uq_createAnalysis(BayesOpts);
```

```
The discrepancy was not specified,
using unknown i.i.d. Gaussian discrepancy...
The solver was not specified, using MCMC
The sampler was not specified, using affine invariant ensemble sampler
Starting AIES...

|##############################| 100.00%
```

```
Finished AIES!
```

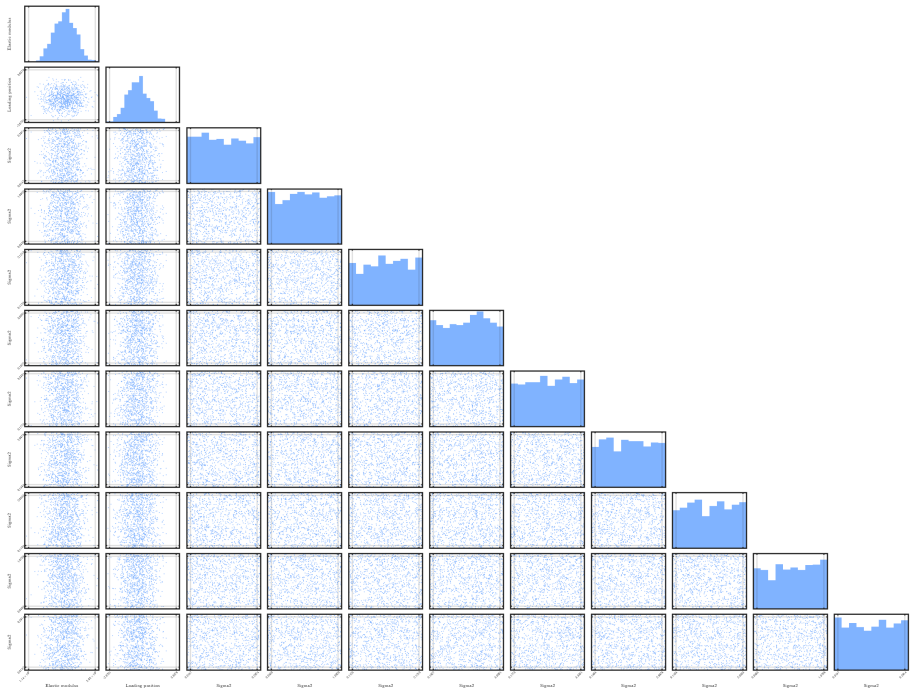
```
%print and display
uq_postProcessInversion(myBayesianAnalysis,'priorPredictive',1000);
uq_print(myBayesianAnalysis);
```

```
%---------------------- Inversion output ----------------------%
   Number of calibrated model parameters:         2
   Number of non-calibrated model parameters:     0

   Number of calibrated discrepancy parameters:   9

%------------------- Data and Discrepancy
%  Data-/Discrepancy group 1:
   Number of independent observations:            4

   Discrepancy:
      Type:                                       Gaussian
      Discrepancy family:                         Row
      Discrepancy parameters known:               No

   Associated outputs:
      Model 1:
         Output dimensions:                       1
                                                  to
                                                  9


%------------------- Solver
   Solution method:                               MCMC

   Algorithm:                                     AIES
   Duration (HH:MM:SS):                           00:01:08
   Number of sample points:                       3.00e+04

%------------------- Posterior Marginals
-------------------------------------------------------------------------
| Parameter        | Mean    | Std     | (0.025-0.97) Quant. | Type        |
-------------------------------------------------------------------------
| Elastic modulus  | 2.3e+08 | 1.6e+07 | (2e+08 - 2.7e+08)   | Model       |
| Loading position | -0.19   | 1.1     | (-2.2 - 1.6)        | Model       |
| Sigma2           | 0.14    | 0.095   | (0.0076 - 0.3)      | Discrepancy |
| Sigma2           | 0.52    | 0.36    | (0.03 - 1.1)        | Discrepancy |
| Sigma2           | 1       | 0.69    | (0.052 - 2.2)       | Discrepancy |
| Sigma2           | 1.3     | 0.9     | (0.091 -  3)        | Discrepancy |
| Sigma2           | 1.4     | 1       | (0.062 - 3.4)       | Discrepancy |
| Sigma2           | 1.4     | 0.93    | (0.067 - 3.1)       | Discrepancy |
| Sigma2           | 1.1     | 0.68    | (0.05 - 2.2)        | Discrepancy |
| Sigma2           | 0.48    | 0.33    | (0.018 - 1.1)       | Discrepancy |
| Sigma2           | 0.13    | 0.097   | (0.0055 - 0.3)      | Discrepancy |
-------------------------------------------------------------------------

%------------------- Point estimate
-----------------------------------------------
| Parameter        | Mean    | Parameter Type |
-----------------------------------------------
| Elastic modulus  | 2.3e+08 | Model          |
| Loading position | -0.19   | Model          |
| Sigma2           | 0.14    | Discrepancy    |
| Sigma2           | 0.52    | Discrepancy    |
| Sigma2           | 1       | Discrepancy    |
| Sigma2           | 1.3     | Discrepancy    |
```

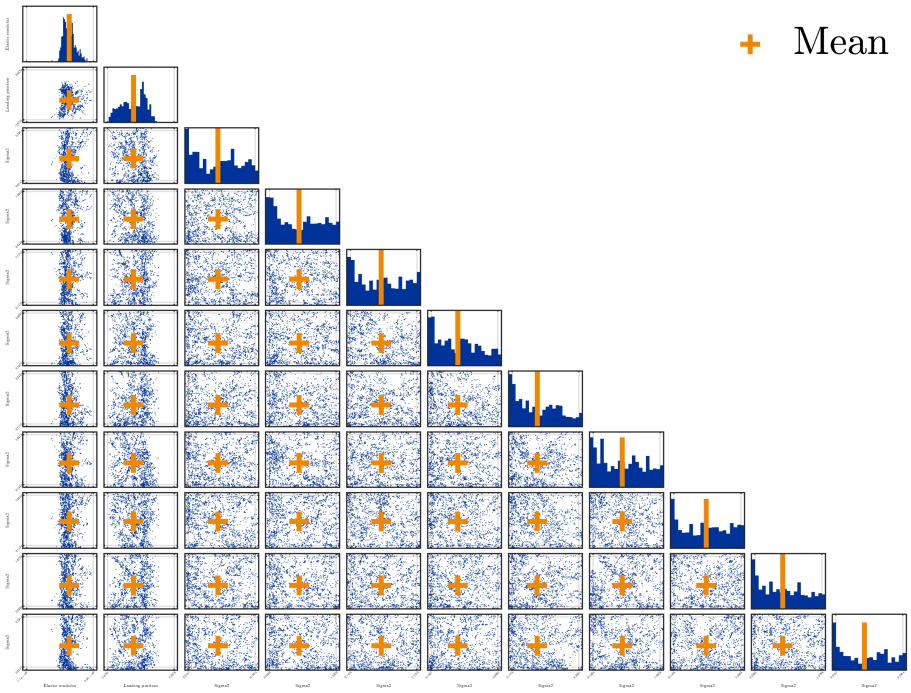
```
| Sigma2           | 1.4     | Discrepancy    |
| Sigma2           | 1.4     | Discrepancy    |
| Sigma2           | 1.1     | Discrepancy    |
| Sigma2           | 0.48    | Discrepancy    |
| Sigma2           | 0.13    | Discrepancy    |
-------------------------------------------------

%------------------- Correlation matrix (model parameters)
-------------------------------------------------------------
|                  |  Elastic modulus    Loading position  |
-------------------------------------------------------------
| Elastic modulus  |  1                  0.12              |
| Loading position |  0.12               1                 |
-------------------------------------------------------------

%------------------- Correlation matrix, 6 most important (discrepancy parameters)
-------------------------------------------------------------------------
|        |  Sigma2    Sigma2    Sigma2    Sigma2    Sigma2    Sigma2  |
-------------------------------------------------------------------------
| Sigma2 |  1         -0.016    0.1       -0.15     0.041     -0.23   |
| Sigma2 |  -0.016    1         -0.25     0.065     -0.18     -0.02   |
| Sigma2 |  0.1       -0.25     1         -0.079    -0.049    0.0074  |
| Sigma2 |  -0.15     0.065     -0.079    1         -0.22     -0.15   |
| Sigma2 |  0.041     -0.18     -0.049    -0.22     1         0.078   |
| Sigma2 |  -0.23     -0.02     0.0074    -0.15     0.078     1       |
-------------------------------------------------------------------------
```

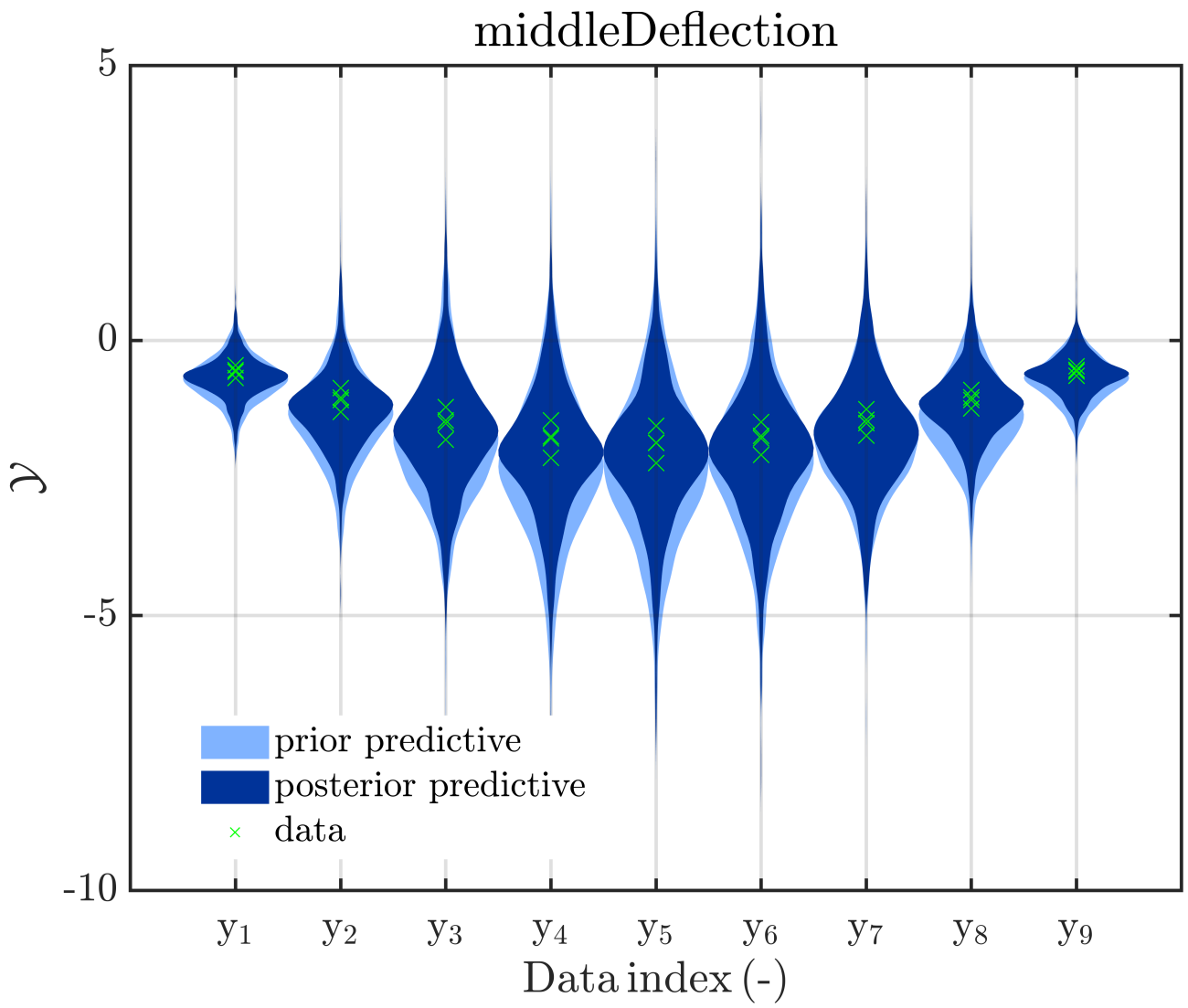
```
uq_display(myBayesianAnalysis);
```

# Prior Sample



Posterior Sample




middleDeflection
5
0
-5
-10
y
prior predictive
posterior predictive
data
y1
y2
y3
y4
y5
y6
y7
y8
y9
Data index (-)