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PhD week 8-Weekly summary

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In this week, I studied:

- Some basic knowledge on probabilistic graphic model (mainly focused on Koller Friedman on youtube)
- Videos given by Prof.Karen Willcox
- Read the particle filter code in Calibration for Youngs modulus scale factor and geometric
- Listened to some **10th NUMGE 2023** conference these days, but I didn't understand

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Probabilistic graphic model

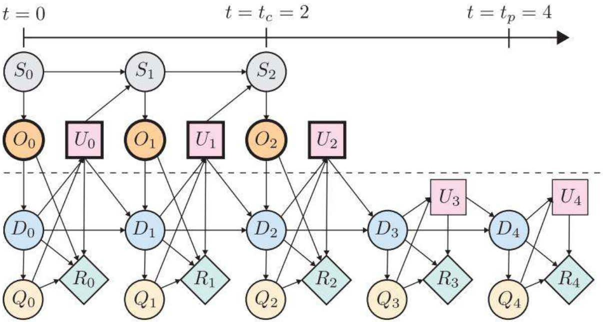

Figure 1: Digital Twin [1]

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Understanding the equations:

- Calibration and Assimilation

$$p(D_0, ..., D_{t_c}, Q_0, ..., Q_{t_c}, R_0, ..., R_{t_c} | o_0, ..., o_{t_c}, u_0, ..., u_{t_c}) = \prod_{t=0}^{t_c} [\phi_t^{update} \phi_t^{QoI} \phi_t^{evaluation}] \tag{1}$$

- Prediction:

$$p(D_0, ..., D_{t_p}, Q_0, ..., Q_{t_p}, R_0, ..., R_{t_p}, U_{t_c+1}, ..., U_{t_p} | o_0, ..., o_{t_c}, u_0, ..., u_{t_c}) \propto \prod_{t=0}^{t_p} [\phi_t^{dynamics} \phi_t^{QoI} \phi_t^{evaluation}] \prod_{t=0}^{t_c} \phi_t^{assimilation} \prod_{t=t_c+1}^{t_p} \phi_t^{control} \tag{2}$$

Calibration for geometric parameters

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} \tag{3}$$

$P(A)$: $\sim \mathcal{N}(mu_{prior}, sigma_{prior})$ $P(B|A)$:$\sim \mathcal{N}(mu_{likeli}, sigma_{likeli})$

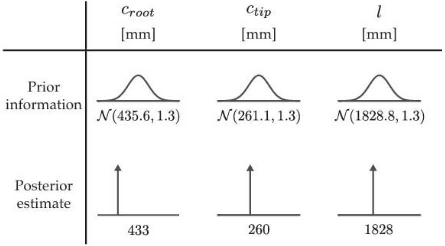

Figure 2: Geometric parameter calibration [1]

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Calibration for Youngs modulus scale factor e

$P(A)$: $\sim \mathcal{N}(mu_{prior}, sigma_{prior})$ $P(B|A)$:$\not\sim \mathcal{N}(mu_{likeli}, sigma_{likeli})$

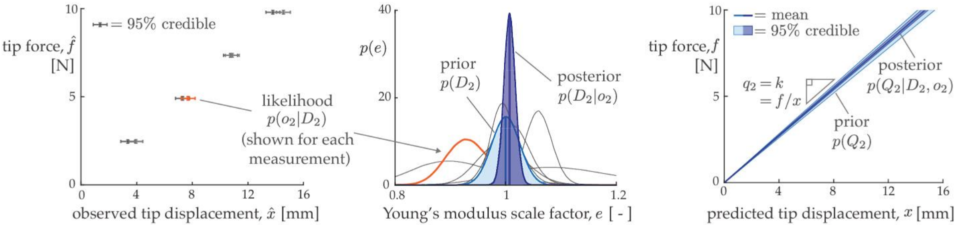

Figure 3: Youngs modulus scale factor e[1]

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Calibration for Youngs modulus scale factor e

$$\hat{F} = \hat{k}\hat{X} \tag{4}$$

$$k = 0.5752e + 0.108 \tag{5}$$

$\hat{F} \sim \mathcal{N}$; $\hat{X} \sim \mathcal{N}$; $\hat{F}/\hat{X} \nsim \mathcal{N}$

$p(k)$ is non-Gaussian; $p(e)$ is non-Gaussian

Kernel density estimation to set up likelihood functions

- Get 1000000 samples from $\hat{F}$ and $\hat{X}$
- Calculate $\hat{k}$, then convert $\hat{k}$ into $\hat{e}$ with Eq.4 and Eq.5
- Use fitdist function to get the PDF of $\hat{e}$

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Particle filtering:

- Step 1: Draw 10^6 samples from prior and set uniform initial weights
- Step 2: Put them in the likelihood function to get new weights (logspace for better numerics)
- Step 3: Normalize the weight sum into 1
- Step 4: Resampling and use kernel density estimation to get new PDF of posterior

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Reference

[1] Michael G Kapteyn, Jacob VR Pretorius and Karen E Willcox. "A probabilistic graphical model foundation for enabling predictive digital twins at scale". In: *Nature Computational Science* 1.5 (2021), pp. 337–347.