PCE VS RSM

This example is with known exact solution. Compared with fitting between PCE and RSM

1 - INITIALIZE UQLAB

```
clc;clear all;close all;
clearvars
rng(100,'twister')
uqlab
```

```
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C:\NY2023\D_document\UQLab_Rel2.0.0\LICENSE.

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Useful commands to get started with UQLab:
uqlab -doc           - Access the available documentation
uqlab -help          - Additional help on how to get started with UQLab
uq_citation help    - Information on how to cite UQLab in publications
uqlab -license       - Display UQLab license information
```
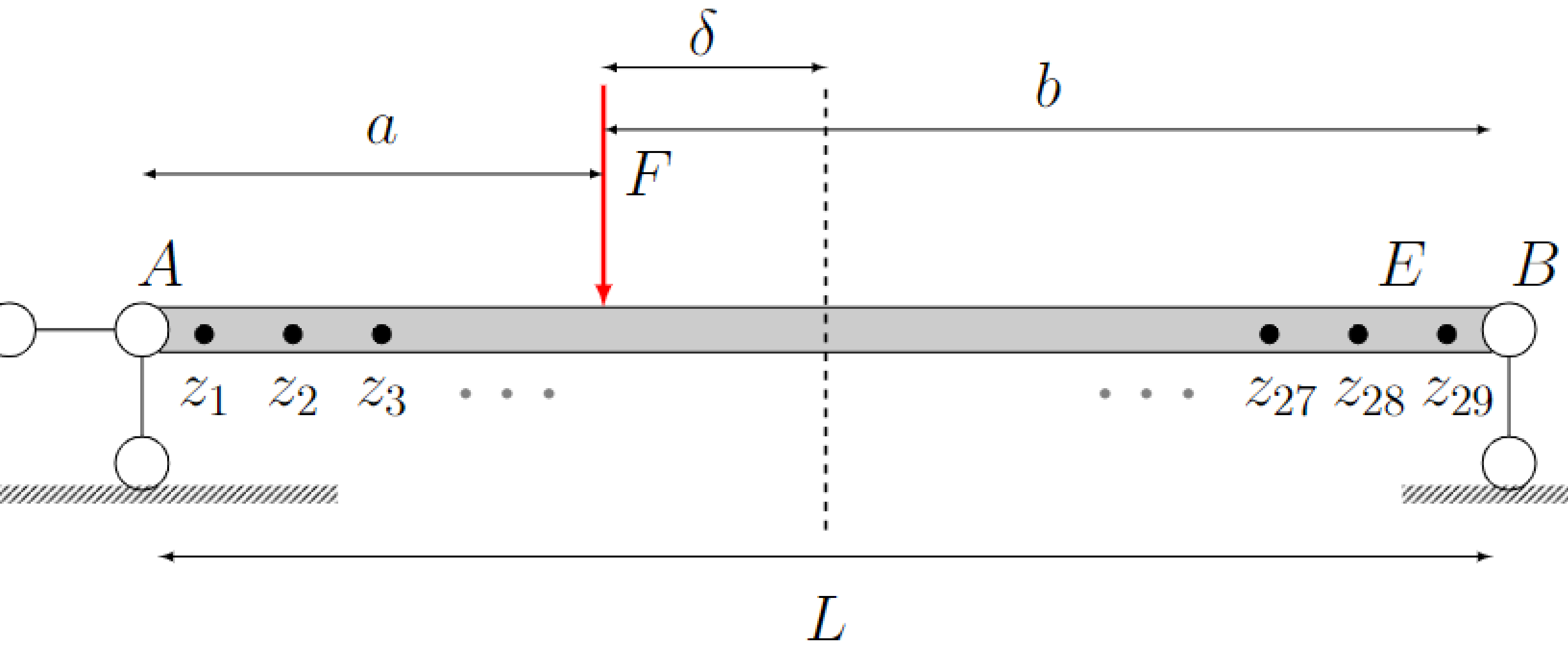

2 - COMPUTATIONAL MODEL



b_b = 0.15; % beam width (m)

b_h = 0.3; % beam height (m)

a % distance from the point A (m)

b % distance from the point B (m)

L = 30; % beam length (m)

F = 43000;% Concentrated force (N)

Computational model:

$$a = \frac{L}{2} + \delta; b = \frac{L}{2} - \delta$$

$$\mathscr{M}(E, \delta, z) = \frac{Fbz[(L^2 - b^2) - z^2]}{6LEI} \qquad z \leq a$$

$$\mathscr{M}(E, \delta, z) = \frac{Fb[\frac{L}{b}(z-a)^3 + (L^2 - b^2)]}{6LEI} \qquad z > a$$

Calculate the Mean-square-error data for surrogate

$$r(\vec{\theta}, \vec{z}) = \frac{1}{N}\sum_{i=1}^{N}(Y_i - \mathscr{M}(\vec{\theta}, \vec{z}))^2$$

$$\vec{\theta} = [E, \delta]; \vec{z} = [z_1, z_2, ..., z_{28}, z_{29}];$$

r is residual; $\vec{x}$ is parameters of interests; E is elastic modulus; δ is the loading postion

$\vec{z}$ is the different measurement points along the beam;

$\mathscr{M}$ is the FE model; Y_i is the measurement data; N is the number of experiment expNum.

Create a MODEL from the function file:

```
ModelOpts.mFile = 'Analytical_Beam_Solution';

myModel = uq_createModel(ModelOpts);
```

3 - PROBABILISTIC INPUT MODEL

$E \sim \mathcal{u}(19e9, 31e9)$; $\delta \sim \mathcal{u}(-10, 10)$

```
% Young's modulus
InputOpts.Marginals(1).Type = 'Uniform';
InputOpts.Marginals(1).Parameters  = [10e9 30e9];
%InputOpts.Marginals(1).Bounds = [10e9 35e9];

% Concentrated load loading position
InputOpts.Marginals(2).Type = 'Uniform';
InputOpts.Marginals(2).Parameters  = [-15 15];
%InputOpts.Marginals(2).Bounds = [-15 15];

myInput = uq_createInput(InputOpts);
```

4 - POLYNOMIAL CHAOS EXPANSION (PCE) METAMODELS $\widetilde{\mathcal{M}}(r)$

Calculate the polynomial chaos expansion (PCE) coefficients.

Select PCE as the metamodeling tool in UQLab:

```
metaopts.Type = 'Metamodel';
metaopts.MetaType = 'PCE';
```

Select the sparse-favouring least-square minimization LARS for the

PCE coefficients calculation strategy:

```
metaopts.Method = 'LARS';
```

Select the PCE options and create the PCE model:

```
metaopts.Degree = 1:20;
```

Experimental design

```
metaopts.ExpDesign.NSamples = 150;
metaopts.ExpDesign.Sampling = 'LHS';
```

Assign the beam function model as the full computational model of the PCE metamodel:

```
metaopts.FullModel = myModel;
```

Calculation

```
myPCE = uq_createModel(metaopts);
```

```
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 12 and qNorm 1.00 for output variable 1
Final LOO error estimate: 1.176465e-06
---                 Calculation finished!                               ---
```

5 - Results

Print a summary of the resulting PCE metamodel:

```
uq_print(myPCE)
```

```
%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               12
   q-norm:                       1.00
   Size of full basis:           91
   Size of sparse basis:         78
   Full model evaluations:       150
   Leave-one-out error:          3.7794039e-08
   Modified leave-one-out error: 1.1764647e-06
   Mean value:                   1.2002
```

```
    Standard deviation:          1.4708
    Coef. of variation:          122.544%
%------------------------------------------------%
```

Create a validation sample of size from the input model:

```
Xval = uq_getSample(1e4);
```

Evaluate the full model response at the validation sample points:

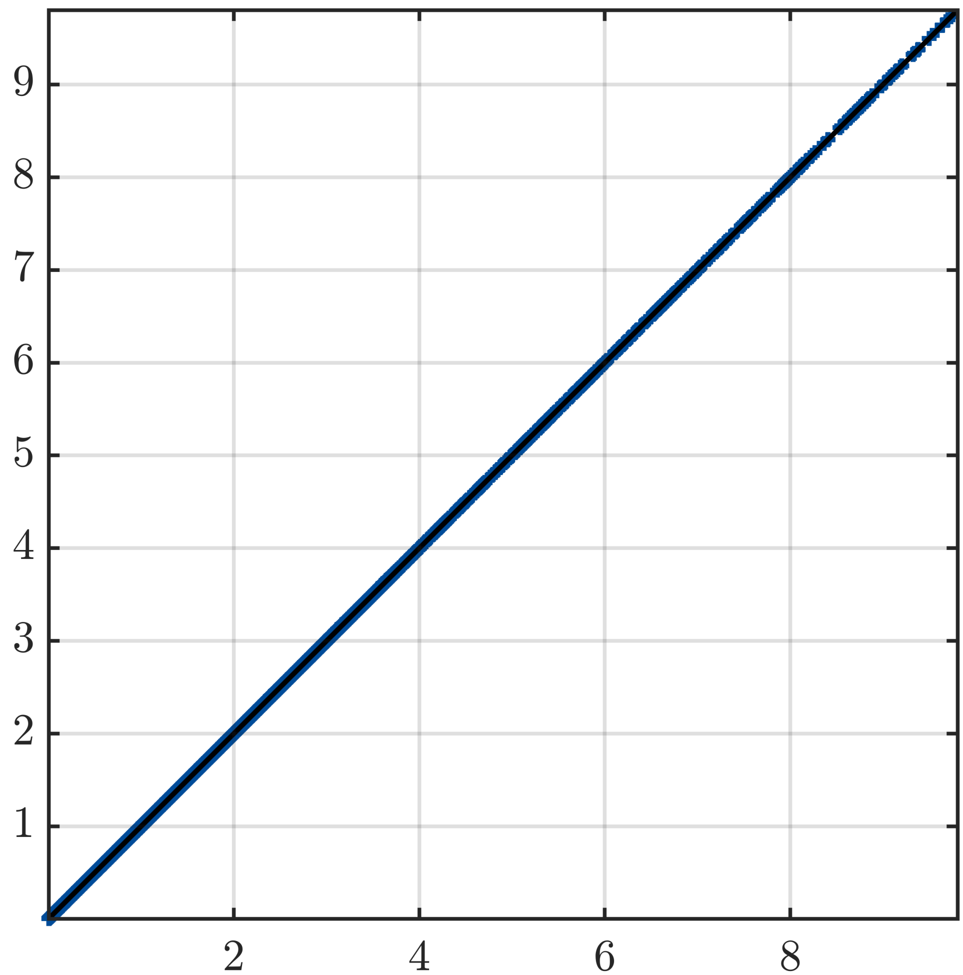
```
Yval = uq_evalModel(myModel,Xval);
```

Responses for PCE

```
YPCE = uq_evalModel(myPCE,Xval);
```

True vs predicted plot

```
close all;
uq_figure
uq_plot(Yval, YPCE, '+')
hold on
uq_plot([min(Yval) max(Yval)], [min(Yval) max(Yval)], 'k')
hold off
axis equal
axis([min(Yval) max(Yval) min(Yval) max(Yval)])
```



Export the PCE strucuture

```
save myPCE
```

6 - Define the priors for E, δ and discrepancy σ

By default, UQlab assumes an independent and identically distributed discrepancy

$\varepsilon \sim \mathcal{N}(0, \mu_y^2)$, with $\mu_y = \frac{1}{N} \sum_{i=1}^{N} y_i$

synthetic ground truth

E = 25e9;δ = -3;noise = 0;

```
Measurement = Deflection(25e9,-3,0);
```

priors

```
%Priors on E and p
PriorOpts.Marginals(1).Name = 'E';                % Young's modulus
PriorOpts.Marginals(1).Type = 'Gaussian';
PriorOpts.Marginals(1).Moments = [25e9 5e9];   % (N/m^2)
PriorOpts.Marginals(1).Bounds = [10e9 35e9];

PriorOpts.Marginals(2).Name = 'delta';                % Concentrated load loading
position
PriorOpts.Marginals(2).Type = 'Gaussian';
PriorOpts.Marginals(2).Moments = [0 5]; % (N/m)
PriorOpts.Marginals(2).Bounds = [-10  10];


PriorOpts.Marginals(3).Name = 'sigma2'; % variance
PriorOpts.Marginals(3).Type = 'Uniform';
sigma2 = mean(Measurement(:,:),"all");
PriorOpts.Marginals(3).Parameters = [0 sigma2^2];


myPriorDist = uq_createInput(PriorOpts);
```

7 - Define the custom-loglikelihood and measurement data for UQlab calculation

$$\log \mathscr{L}(\Theta) = \sum_{i=1}^{N} \left(-\frac{1}{2} \left(r_i(\vec{x}, \vec{z}) \right)^T \Sigma(\epsilon)^{-1} \left(r_i(\vec{x}, \vec{z}) \right) \right) - \frac{3N}{2} \log(2\pi) - \frac{N}{2} \log\left(\det(\Sigma(\epsilon))\right)$$

Monitored data Y_i for beam deflection have been used in PCE surrogate model. No measurements available for Bayesian inference: Pass zeros to measurement Y_i just for calculation, which equals:

$$r(\vec{x}, \vec{z}) \sim (0 - r(\vec{x}, \vec{z})) \sim (Y_i - \mathscr{M}(\vec{x}, \vec{z}))$$

$r(\vec{x}, \vec{z})$ dim 248X1 $\sim$ $(Y_i - \mathscr{M}(\vec{x}, \vec{z}))$ dim 248X29

Also, measurement data for Y_{10X29} changed into Y_{10X1}

```
y = zeros(1,1);
myData.y = y;
myData.Name = 'Zeros vector measurement for Bayesian inference';
```

Loglikelihood still follows the Gaussian discrepancy criteria

```
myLogLikeli = @(params,y) myLogLikeli2(params,y);
```

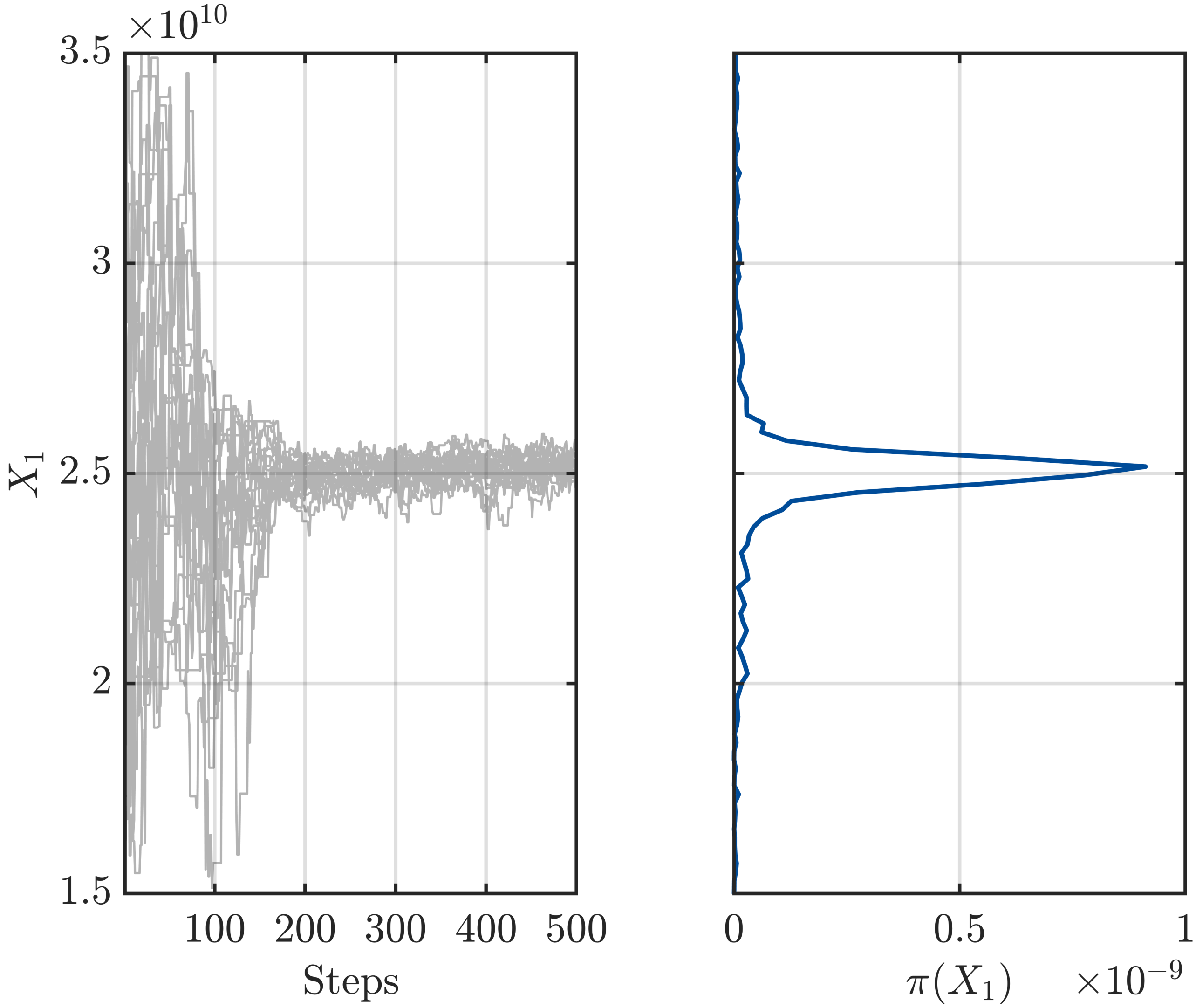
8 - Solver options

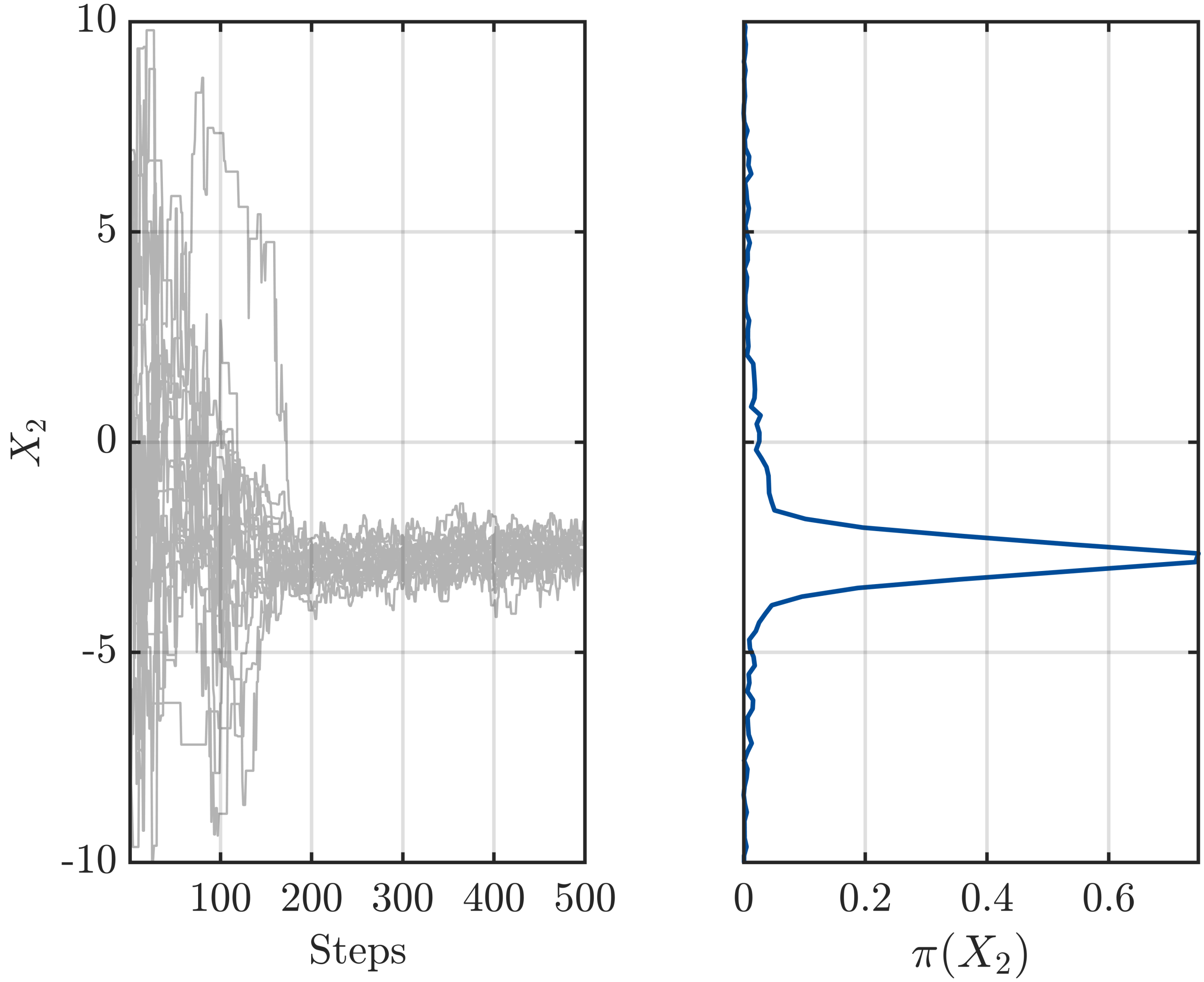
```
Solver.Type = 'MCMC';
Solver.MCMC.Visualize.Parameters = [1 2];
Solver.MCMC.Visualize.Interval = 10;
Solver.MCMC.Sampler = 'AIES';
Solver.MCMC.Steps = 500;
Solver.MCMC.NChains = 20;
Solver.MCMC.Proposal.PriorScale = 1e-3;
```

9 - Bayesian inference

```
BayesOpts.Data = myData;
BayesOpts.LogLikelihood = myLogLikeli;
BayesOpts.Type = 'inversion';
BayesOpts.Solver = Solver;
BayesAnalysis = uq_createAnalysis(BayesOpts);
```

```
Starting AIES...

|#                              |   1.80%|#                                       |   3.60%|##                                      |   6
```


$\times 10^{10}$
3.5
3
2.5
2
1.5
X_1
100
200
300
400
500
Steps
0
0.5
1
$\pi(X_1)$
$\times 10^{-9}$




```
Finished AIES!
```