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Random Field and Compressing Sampling

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In the last week, my work is mainly foucsed on two parts:

- Part 1: Maltab coding to realize the random field and back-generate CPT data
- Part 2: Read the paper (Wang 2017[1]) on the Bayesian compressive sampling

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Random field generation steps:

- Step 1: Latin Hypercube Sampling G_1
- Step 2: Obtain the autocorrelated covariance matrix C
- Step 3: Get the Gaussian field $G_2 = LG_1$
- Step 4: Change the G_2 considering the SoF into non-Gaussian (Log-normal) G_3

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Step 1: Latin Hypercube Sampling G_1 $(\mu = 0, \sigma = 1)$

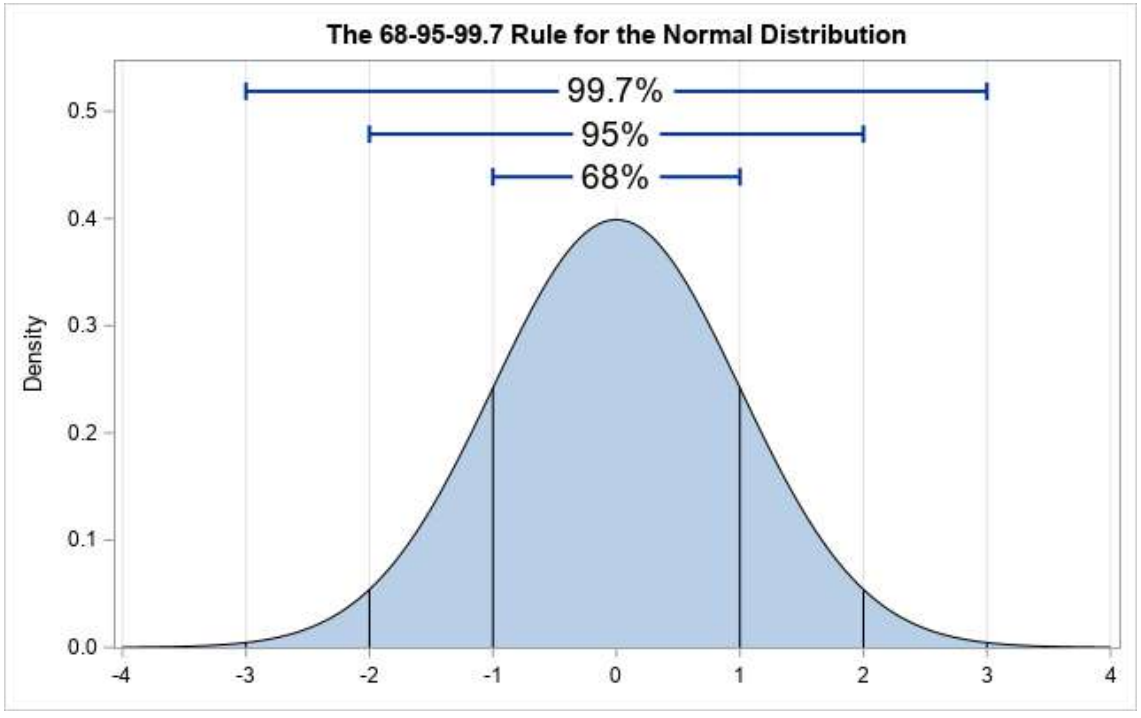

Figure 1: Standard Normal distribution

[1]Note: the elements is independent with each other.

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Step 2:Autocorrelated covariance matrix C into lower triangular matrix L

- Calcuate matrix C

$$C\left(\tau_x, \tau_x\right) = \exp\left(-\frac{2\tau_x}{\delta_h} - \frac{2\tau_y}{\delta_v}\right) \tag{1}$$

- Cholesky decomposition into L

$$A = LL^T \tag{2}$$

Step 3: Get the Gaussian field $G_2 = LG_1$

- G_2 is still Gaussian random field, but linked to the spatial correlation length SoF ($\mu = 0, \sigma = 1$)

- At the moment, the element in G_2 is depended

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Step 4: Normal G_2 into non-Gaussian (Log-normal) G_3

Premise: Testing data has shown shear strength follows the log-normal distribution. G_3 can be obtained through these equations below:

- $$\sigma_{lns_u} = \sqrt{ln(1 + COV_{s_u}^2)} \tag{3}$$
- $$\mu_{lns_u} = ln\mu_{s_u} - \frac{1}{2}\sigma_{lns_u}^2 \tag{4}$$
- $$G_3 = \exp(\mu_{lns_u} + \sigma_{lns_u} G_2) \tag{5}$$

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Compressing sampling

Definition: A theory that can reconstruct the soil property profile from sparse measurement data

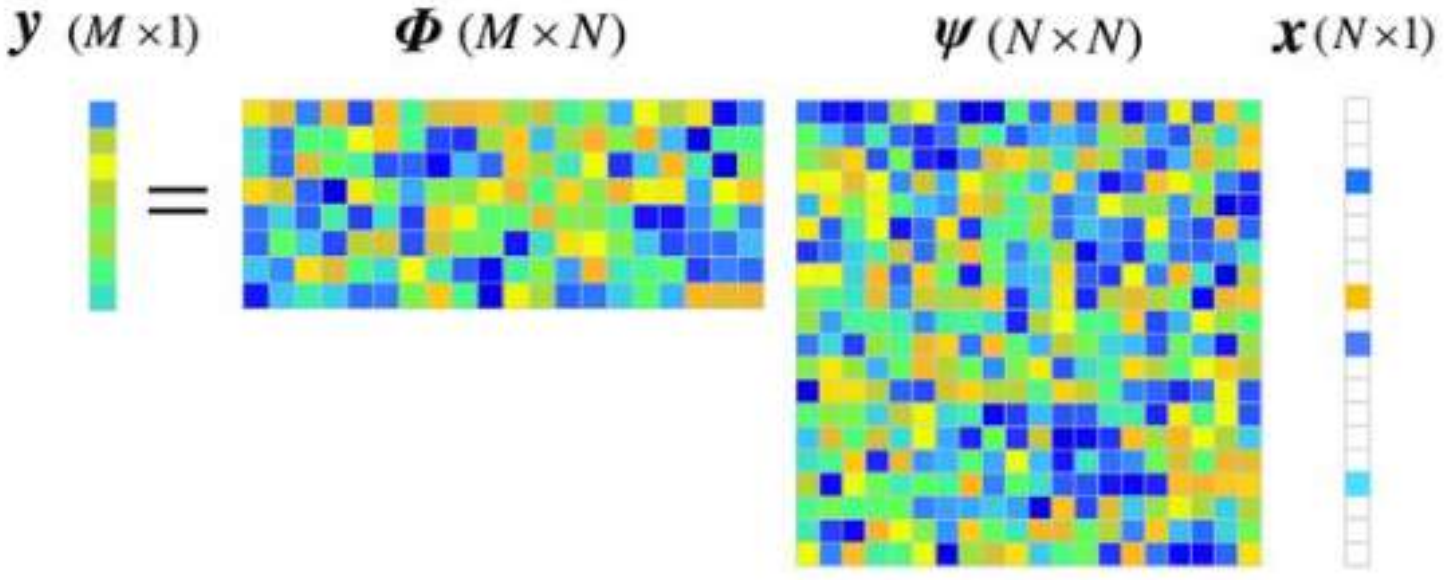

Figure 2: Compressing sampling

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What I know so far

- The symbols in the figure, basis functions, wavelet
- Discrete Wavelet Transfrom (DWT) is a good way to get the pre-specified basis functions
- DWT method is the context I need to study

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Reference

[1] Yu Wang and Tengyuan Zhao. "Statistical interpretation of soil property profiles from sparse data using Bayesian compressive sampling". In: *Géotechnique* 67.6 (2017), pp. 523–536.