No Chain PCE -- Multiple point-Bayesian inference

1 - INITIALIZE UQLAB

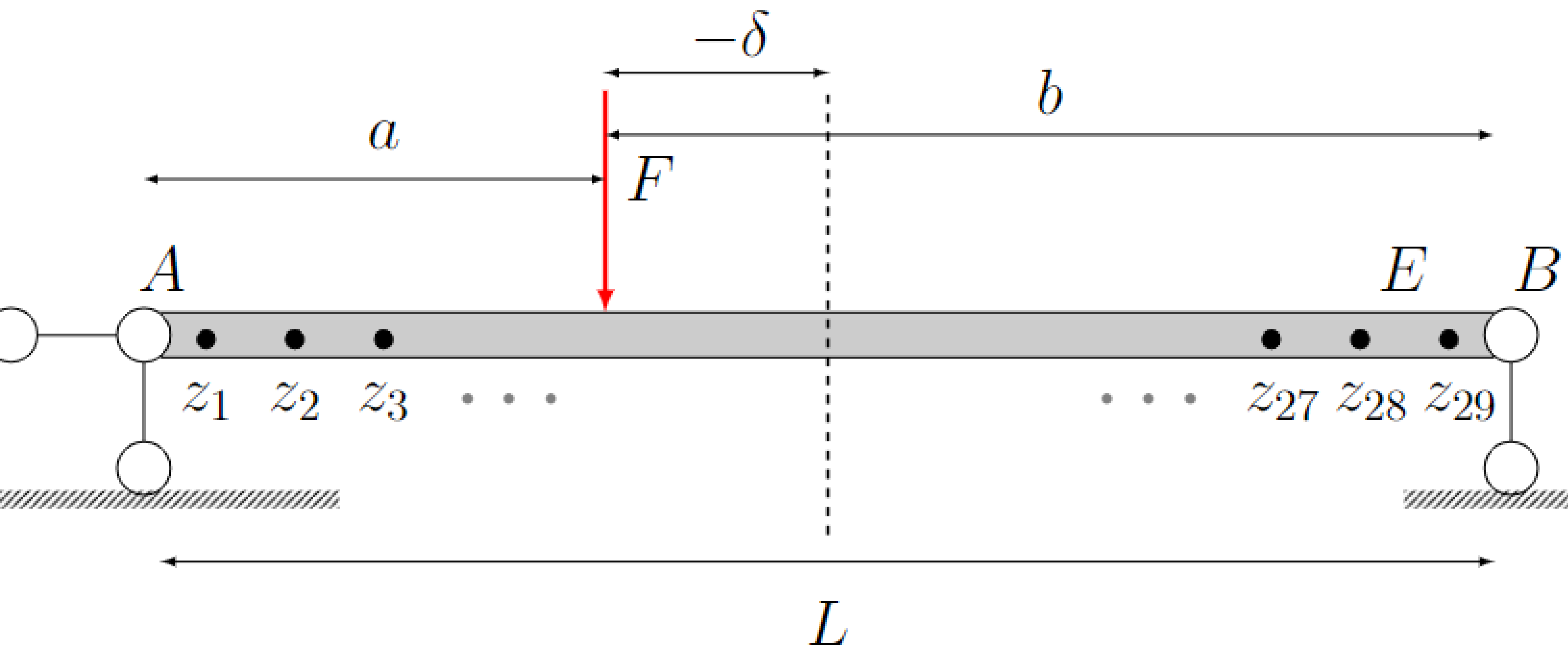
```
clc;clear all;close all;
clearvars
rng(100,'twister')
uqlab
```

```
Copyright 2013-2022, Stefano Marelli and Bruno Sudret, all rights reserved.
This is UQLab, version 2.0
UQLab is distributed under the BSD 3-clause open source license available at:
C:\NY2023\D_document\UQLab_Rel2.0.0\LICENSE.

To request special permissions, please contact:
  - Stefano Marelli (marelli@ibk.baug.ethz.ch).

Useful commands to get started with UQLab:
uqlab -doc          - Access the available documentation
uqlab -help         - Additional help on how to get started with UQLab
uq_citation help   - Information on how to cite UQLab in publications
uqlab -license      - Display UQLab license information
```

2 - COMPUTATIONAL MODEL



b_b = 0.15; % beam width (m)

b_h = 0.3; % beam height (m)

a % distance from the point A (m)

b % distance from the point B (m)

L = 30; % beam length (m)

F = 43000;% Concentrated force (N)

Computational model:

$$a = \frac{L}{2} - \delta ; b = \frac{L}{2} + \delta$$

$$\mathcal{M}(\vec{\theta}) = \frac{Fbz[(L^2 - b^2) - z^2]}{6LEI} \qquad z \leq a$$

$$\mathcal{M}(\vec{\theta}) = \frac{Fb[\frac{L}{b}(z-a)^3 + (L^2 - b^2)]}{6LEI} \qquad z > a$$

$$\vec{\theta} = [E, \delta, z]; \ \vec{z} = [z_1, z_2, ..., z_{28}, z_{29}];$$

E is elastic modulus; δ is the loading postion

$\vec{z}$ is the different measurement points along the beam;

$\mathcal{M}$ is the FE model; Y_i is the measurement data; N is the number of experiment expNum.

Create a MODEL from the function file:

```
%LHS sampling

% mean of LHS sampling for Gaussian distribution (E and delta)
mu_LHS = [25e9 0];

% sigma and Covariance martrix of LHS sampling for Gaussian distribution (E and 
delta)
sigma_LHS = [5e9 5].^2;
CovarianceMatrix_LHS = diag(sigma_LHS);

% LHS sampling
N = 10;
LHS_sample = lhsnorm(mu_LHS, CovarianceMatrix_LHS, N);    %
size(LHS_sample)
```

```
ans = 1×2
    10     2
```

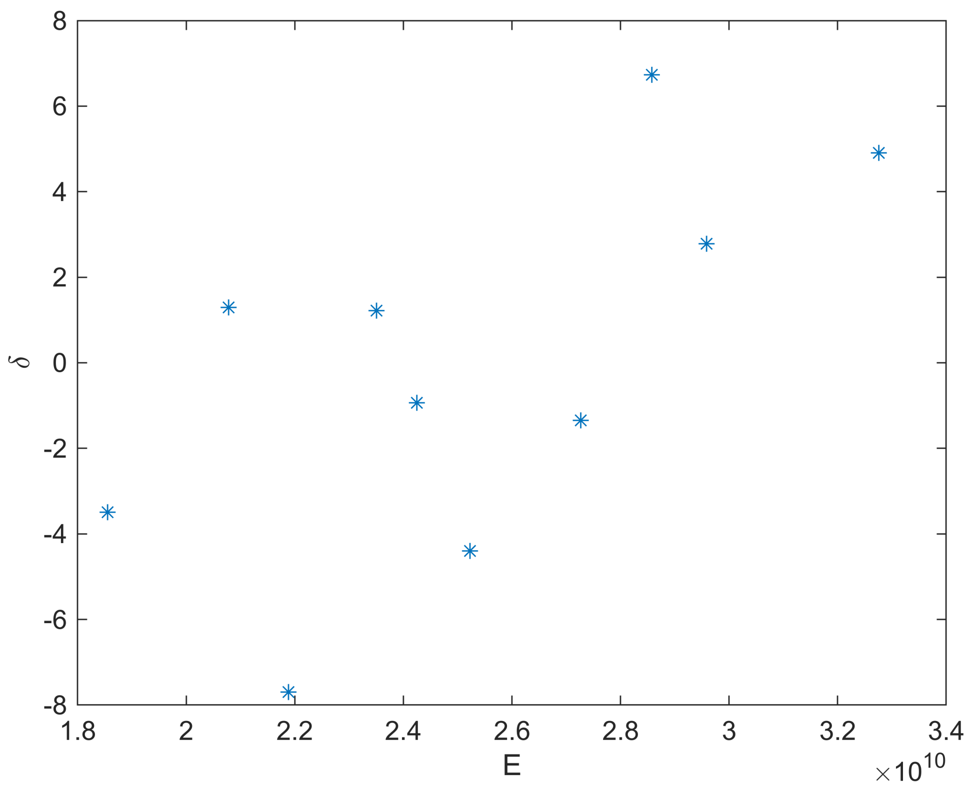
```
E = LHS_sample(:,1);
size(E)
```

```
ans = 1×2
    10     1
```

```
delta = LHS_sample(:,2);
size(delta)
```

```
ans = 1×2
    10     1
```

```
%plot LHS sampling
figure
plot(E,delta,'*')
xlabel('E');
ylabel('\delta');
```




```
%FE realizaiton
FE_deflection = Deflection(LHS_sample);
size(FE_deflection)
```

```
ans = 1×2
    10    29
```

```
FE_realization = [LHS_sample,FE_deflection];
size(FE_realization)
```

```
ans = 1×2
    10    31
```

3 - PROBABILISTIC INPUT MODEL

$E \sim \mathcal{u}(19e9, 31e9)$; $\delta \sim \mathcal{u}(-10, 10)$

```
% Young's modulus
InputOpts.Marginals(1).Type = 'Uniform';
```

```
minE = min(FE_realization(:,1));
size(minE)
```

```
ans = 1×2
     1     1
```

```
maxE = max(FE_realization(:,1));
size(maxE)
```

```
ans = 1×2
     1     1
```

```
InputOpts.Marginals(1).Parameters  = [minE maxE];


% Concentrated load loading position
InputOpts.Marginals(2).Type = 'Uniform';
mindelta = min(FE_realization(:,2));
size(mindelta)
```

```
ans = 1×2
     1     1
```

```
maxdelta = max(FE_realization(:,2));
size(maxdelta)
```

```
ans = 1×2
     1     1
```

```
InputOpts.Marginals(2).Parameters  = [mindelta maxdelta];


myInput = uq_createInput(InputOpts);
```

4 - POLYNOMIAL CHAOS EXPANSION (PCE) METAMODELS $\tilde{\mathcal{M}}(r)$

Calculate the polynomial chaos expansion (PCE) coefficients.

Select PCE as the metamodeling tool in UQLab:

```
metaopts.Type = 'Metamodel';
metaopts.MetaType = 'PCE';
```

Select the sparse-favouring least-square minimization LARS for the

PCE coefficients calculation strategy:

```
metaopts.Method = 'LARS';
```

Select the PCE options and create the PCE model:

```
metaopts.Degree = 1:20;
```

Experimental design

```
X = FE_realization(:,1:2);
Y = FE_realization(:,3:end);
metaopts.ExpDesign.X = X;
metaopts.ExpDesign.Y = Y;
```

Calculation

```
myPCE = uq_createModel(metaopts);
```

```
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 3 and qNorm 1.00 for output variable 1
Final LOO error estimate: 5.097821e-02
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 3 and qNorm 1.00 for output variable 2
Final LOO error estimate: 5.171531e-02
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 3 and qNorm 1.00 for output variable 3
Final LOO error estimate: 5.304908e-02
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 3 and qNorm 1.00 for output variable 4
Final LOO error estimate: 5.514977e-02
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 3 and qNorm 1.00 for output variable 5
Final LOO error estimate: 5.828222e-02
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 3 and qNorm 1.00 for output variable 6
Final LOO error estimate: 6.283652e-02
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 4 and qNorm 1.00 for output variable 7
Final LOO error estimate: 5.004127e-02
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 4 and qNorm 1.00 for output variable 8
Final LOO error estimate: 4.074612e-02
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 4 and qNorm 1.00 for output variable 9
Final LOO error estimate: 5.962551e-02
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 10
Final LOO error estimate: 6.140990e-02
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 3 and qNorm 1.00 for output variable 11
Final LOO error estimate: 2.257637e-02
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 3 and qNorm 1.00 for output variable 12
Final LOO error estimate: 2.401268e-03
---                 Calculation finished!                               ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 3 and qNorm 1.00 for output variable 13
```

```
Final LOO error estimate: 1.785492e-02
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 4 and qNorm 1.00 for output variable 14
Final LOO error estimate: 3.766679e-02
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 15
Final LOO error estimate: 4.064697e-02
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 16
Final LOO error estimate: 1.472465e-02
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 17
Final LOO error estimate: 4.537749e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 18
Final LOO error estimate: 3.491705e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 19
Final LOO error estimate: 2.300402e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 20
Final LOO error estimate: 6.097679e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 21
Final LOO error estimate: 7.811666e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 22
Final LOO error estimate: 7.488687e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 23
Final LOO error estimate: 7.032053e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 24
Final LOO error estimate: 6.656092e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 25
Final LOO error estimate: 6.352329e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 26
Final LOO error estimate: 6.113533e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 27
Final LOO error estimate: 5.933809e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 28
Final LOO error estimate: 5.808627e-03
---                 Calculation finished!                              ---
---   Calculating the PCE coefficients by regression.   ---
The estimation of PCE coefficients converged at polynomial degree 2 and qNorm 1.00 for output variable 29
```

```
Final LOO error estimate: 5.734784e-03
---                 Calculation finished!                               ---
```

Print a summary of the resulting PCE metamodel:

```
uq_print(myPCE, [1:29])
```

```
%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               3
   q-norm:                       1.00
   Size of full basis:           10
   Size of sparse basis:         7
   Full model evaluations:       10
   Leave-one-out error:          8.2639515e-03
   Modified leave-one-out error: 5.0978207e-02
   Mean value:                   0.2687
   Standard deviation:           0.0518
   Coef. of variation:            19.289%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               3
   q-norm:                       1.00
   Size of full basis:           10
   Size of sparse basis:         7
   Full model evaluations:       10
   Leave-one-out error:          8.3834417e-03
   Modified leave-one-out error: 5.1715311e-02
   Mean value:                   0.5348
   Standard deviation:           0.1031
   Coef. of variation:            19.282%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               3
   q-norm:                       1.00
   Size of full basis:           10
   Size of sparse basis:         7
   Full model evaluations:       10
   Leave-one-out error:          8.5996552e-03
   Modified leave-one-out error: 5.3049077e-02
   Mean value:                   0.7954
   Standard deviation:           0.1533
   Coef. of variation:            19.271%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               3
   q-norm:                       1.00
   Size of full basis:           10
   Size of sparse basis:         7
   Full model evaluations:       10
   Leave-one-out error:          8.9401929e-03
   Modified leave-one-out error: 5.5149767e-02
   Mean value:                   1.0479
   Standard deviation:           0.2018
   Coef. of variation:            19.260%
%--------------------------------------------------%
```

```
%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               3
   q-norm:                       1.00
   Size of full basis:           10
   Size of sparse basis:         7
   Full model evaluations:       10
   Leave-one-out error:          9.4479870e-03
   Modified leave-one-out error: 5.8282219e-02
   Mean value:                   1.2895
   Standard deviation:           0.2483
   Coef. of variation:            19.252%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               3
   q-norm:                       1.00
   Size of full basis:           10
   Size of sparse basis:         7
   Full model evaluations:       10
   Leave-one-out error:          1.0186273e-02
   Modified leave-one-out error: 6.2836517e-02
   Mean value:                   1.5176
   Standard deviation:           0.2922
   Coef. of variation:            19.251%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               4
   q-norm:                       1.00
   Size of full basis:           15
   Size of sparse basis:         5
   Full model evaluations:       10
   Leave-one-out error:          1.6090979e-02
   Modified leave-one-out error: 5.0041272e-02
   Mean value:                   1.7247
   Standard deviation:           0.3653
   Coef. of variation:            21.181%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               4
   q-norm:                       1.00
   Size of full basis:           15
   Size of sparse basis:         5
   Full model evaluations:       10
   Leave-one-out error:          1.3102083e-02
   Modified leave-one-out error: 4.0746117e-02
   Mean value:                   1.9187
   Standard deviation:           0.4004
   Coef. of variation:            20.868%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               4
   q-norm:                       1.00
   Size of full basis:           15
   Size of sparse basis:         5
   Full model evaluations:       10
   Leave-one-out error:          1.9172830e-02
```

```
   Modified leave-one-out error: 5.9625508e-02
   Mean value:                   2.0919
   Standard deviation:           0.4292
   Coef. of variation:            20.515%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         4
   Full model evaluations:       10
   Leave-one-out error:          2.7341381e-02
   Modified leave-one-out error: 6.1409905e-02
   Mean value:                   2.2362
   Standard deviation:           0.4502
   Coef. of variation:            20.134%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               3
   q-norm:                       1.00
   Size of full basis:           10
   Size of sparse basis:         9
   Full model evaluations:       10
   Leave-one-out error:          6.0412638e-05
   Modified leave-one-out error: 2.2576372e-02
   Mean value:                   2.3513
   Standard deviation:           0.4563
   Coef. of variation:            19.408%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               3
   q-norm:                       1.00
   Size of full basis:           10
   Size of sparse basis:         7
   Full model evaluations:       10
   Leave-one-out error:          3.3127076e-04
   Modified leave-one-out error: 2.4012681e-03
   Mean value:                   2.4596
   Standard deviation:           0.4823
   Coef. of variation:            19.608%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               3
   q-norm:                       1.00
   Size of full basis:           10
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          4.5123950e-03
   Modified leave-one-out error: 1.7854915e-02
   Mean value:                   2.5205
   Standard deviation:           0.4816
   Coef. of variation:            19.108%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
```

```
   Maximal degree:               4
   q-norm:                       1.00
   Size of full basis:           15
   Size of sparse basis:         7
   Full model evaluations:       10
   Leave-one-out error:          4.5213914e-03
   Modified leave-one-out error: 3.7666793e-02
   Mean value:                   2.5644
   Standard deviation:           0.4740
   Coef. of variation:            18.484%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         5
   Full model evaluations:       10
   Leave-one-out error:          1.2505598e-02
   Modified leave-one-out error: 4.0646969e-02
   Mean value:                   2.5745
   Standard deviation:           0.5347
   Coef. of variation:            20.770%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         5
   Full model evaluations:       10
   Leave-one-out error:          4.5302420e-03
   Modified leave-one-out error: 1.4724654e-02
   Mean value:                   2.5548
   Standard deviation:           0.5368
   Coef. of variation:            21.012%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         5
   Full model evaluations:       10
   Leave-one-out error:          1.3961008e-03
   Modified leave-one-out error: 4.5377492e-03
   Mean value:                   2.5063
   Standard deviation:           0.5316
   Coef. of variation:            21.212%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          2.5041512e-04
   Modified leave-one-out error: 3.4917055e-03
   Mean value:                   2.4551
```

```
   Standard deviation:           0.5020
   Coef. of variation:            20.445%
%------------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          1.6497823e-04
   Modified leave-one-out error: 2.3004018e-03
   Mean value:                   2.3550
   Standard deviation:           0.4817
   Coef. of variation:            20.454%
%------------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          4.3730807e-04
   Modified leave-one-out error: 6.0976789e-03
   Mean value:                   2.2280
   Standard deviation:           0.4570
   Coef. of variation:            20.513%
%------------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          5.6023035e-04
   Modified leave-one-out error: 7.8116665e-03
   Mean value:                   2.0761
   Standard deviation:           0.4282
   Coef. of variation:            20.626%
%------------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          5.3706715e-04
   Modified leave-one-out error: 7.4886866e-03
   Mean value:                   1.9017
   Standard deviation:           0.3948
   Coef. of variation:            20.763%
%------------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
```

```
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          5.0431874e-04
   Modified leave-one-out error: 7.0320535e-03
   Mean value:                   1.7074
   Standard deviation:           0.3567
   Coef. of variation:            20.893%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          4.7735582e-04
   Modified leave-one-out error: 6.6560915e-03
   Mean value:                   1.4958
   Standard deviation:           0.3143
   Coef. of variation:            21.009%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          4.5557087e-04
   Modified leave-one-out error: 6.3523294e-03
   Mean value:                   1.2693
   Standard deviation:           0.2679
   Coef. of variation:            21.110%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          4.3844505e-04
   Modified leave-one-out error: 6.1135326e-03
   Mean value:                   1.0303
   Standard deviation:           0.2184
   Coef. of variation:            21.194%
%--------------------------------------------------%

%------------ Polynomial chaos output ------------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          4.2555582e-04
   Modified leave-one-out error: 5.9338094e-03
   Mean value:                   0.7814
   Standard deviation:           0.1661
   Coef. of variation:            21.260%
```

```
%------------------------------------------------%

%----------- Polynomial chaos output -----------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          4.1657807e-04
   Modified leave-one-out error: 5.8086267e-03
   Mean value:                   0.5251
   Standard deviation:           0.1119
   Coef. of variation:            21.307%
%------------------------------------------------%

%----------- Polynomial chaos output -----------%
   Number of input variables:    2
   Maximal degree:               2
   q-norm:                       1.00
   Size of full basis:           6
   Size of sparse basis:         6
   Full model evaluations:       10
   Leave-one-out error:          4.1128230e-04
   Modified leave-one-out error: 5.7347842e-03
   Mean value:                   0.2638
   Standard deviation:           0.0563
   Coef. of variation:            21.336%
%------------------------------------------------%
```

Export the PCE strucuture

```
save myPCE
```

5 - Yval vs YPCE

Create a validation sample of size from the input model:

```
Xval =  FE_realization(:,1:2);
size(Xval)
```

```
ans = 1×2
    10     2
```

Evaluate the full model response at the validation sample points:

```
Yval =  FE_realization(:,3:end);
size(Yval)
```

```
ans = 1×2
    10    29
```

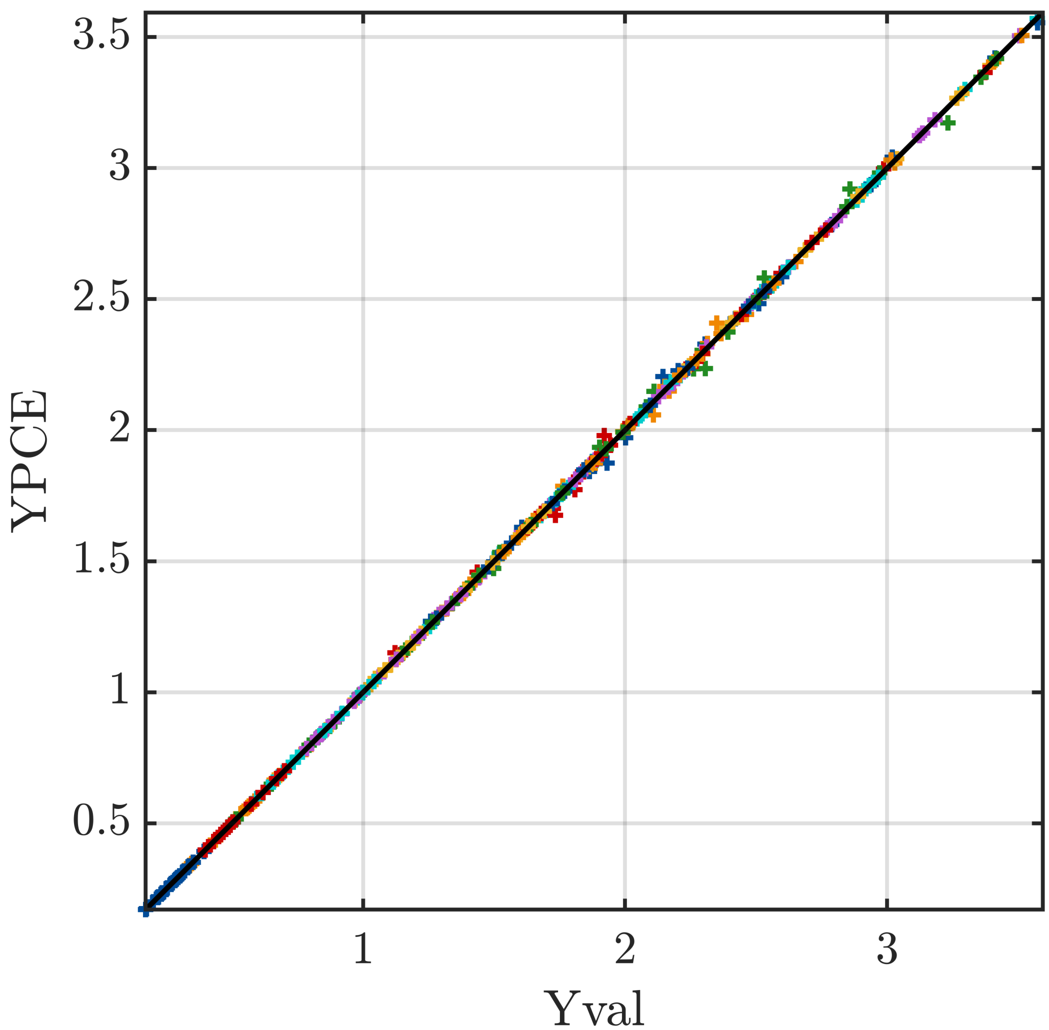
Responses for PCE

```
YPCE = uq_evalModel(myPCE,Xval);
size(YPCE)
```

```
ans = 1×2
    10    29
```

True vs predicted plot

```
close all;
uq_figure
uq_plot(Yval, YPCE, '+')
hold on
uq_plot([min(Yval,[],'all') max(Yval,[],'all')], [min(Yval,[],'all') max(Yval,
[],'all')], 'k')
hold off
axis equal
axis([min(Yval,[],'all') max(Yval,[],'all') min(Yval,[],'all') max(Yval,[],'all')])
xlabel('Yval');
ylabel('YPCE');
box on;
```



6 - Define the priors for E, δ and discrepancy σ

Note: priors for E and δ are different from input models above

By default, UQlab assumes an independent and identically distributed discrepancy

$\varepsilon \sim \mathcal{N}(0, \mu_y^2)$, with $\mu_y = \frac{1}{N}\sum_{i=1}^{N} y_i$

synthetic ground truth with 10% noise

E = 30e9Pa;δ = 4; noise = 3%

```
Measurement = GroundTruth(30e9,4,1,0.03);
size(Measurement)
```

```
ans = 1×2
     1    29
```

priors

```
%Priors on E , delta and sigma
PriorOpts.Marginals(1).Name = 'E';               % Young's modulus
PriorOpts.Marginals(1).Type = 'Gaussian';
PriorOpts.Marginals(1).Parameters = [25e9 5e9];   % (N/m^2)
PriorOpts.Marginals(1).Bounds = [10e9 35e9];

PriorOpts.Marginals(2).Name = 'delta';               % Concentrated load loading
position
PriorOpts.Marginals(2).Type = 'Gaussian';
PriorOpts.Marginals(2).Parameters = [0 5]; % (N/m)
PriorOpts.Marginals(2).Bounds = [-10  10];


% PriorOpts.Marginals(3).Name = 'sigma2'; % variance
% PriorOpts.Marginals(3).Type = 'Uniform';
% sigma2 = mean(Measurement(:,:),"all");
% PriorOpts.Marginals(3).Parameters = [0 sigma2^2];


myPriorDist = uq_createInput(PriorOpts);

SigmaOpts.Marginals(1).Name = 'Sigma2';
SigmaOpts.Marginals(1).Type = 'Uniform';
sigma2 = mean(Measurement(:,:),"all");
SigmaOpts.Marginals(1).Parameters = [0 sigma2.^2];

mySigmaDist = uq_createInput(SigmaOpts);
DiscrepancyOptsUnknownDisc.Type = 'Gaussian';
DiscrepancyOptsUnknownDisc.Prior = mySigmaDist;
```

7 - Define the custom-loglikelihood and measurement data for UQlab calculation

$$\ell\mathscr{L}(\vec{\theta},\epsilon \mid Y) = \prod_{i=1}^{N} \frac{1}{(2\pi)^{3/2}\det(\Sigma(\epsilon))^{1/2}} \exp\left(-\frac{1}{2}\left(Y_i - \mathscr{M}(\vec{\theta})\right)^T \Sigma(\epsilon)^{-1}\left(Y_i - \mathscr{M}(\vec{\theta})\right)\right)$$

```
myData.y = Measurement;
size(myData.y)
```

```
ans = 1×2
     1    29
```

```
myData.Name = 'Measurement on 29 points along the beam';
```

Loglikelihood still follows the Gaussian discrepancy criteria

```
myLogLikeli = @(params,y) myLogLikeli2(params,y);
```

8 - Solver options

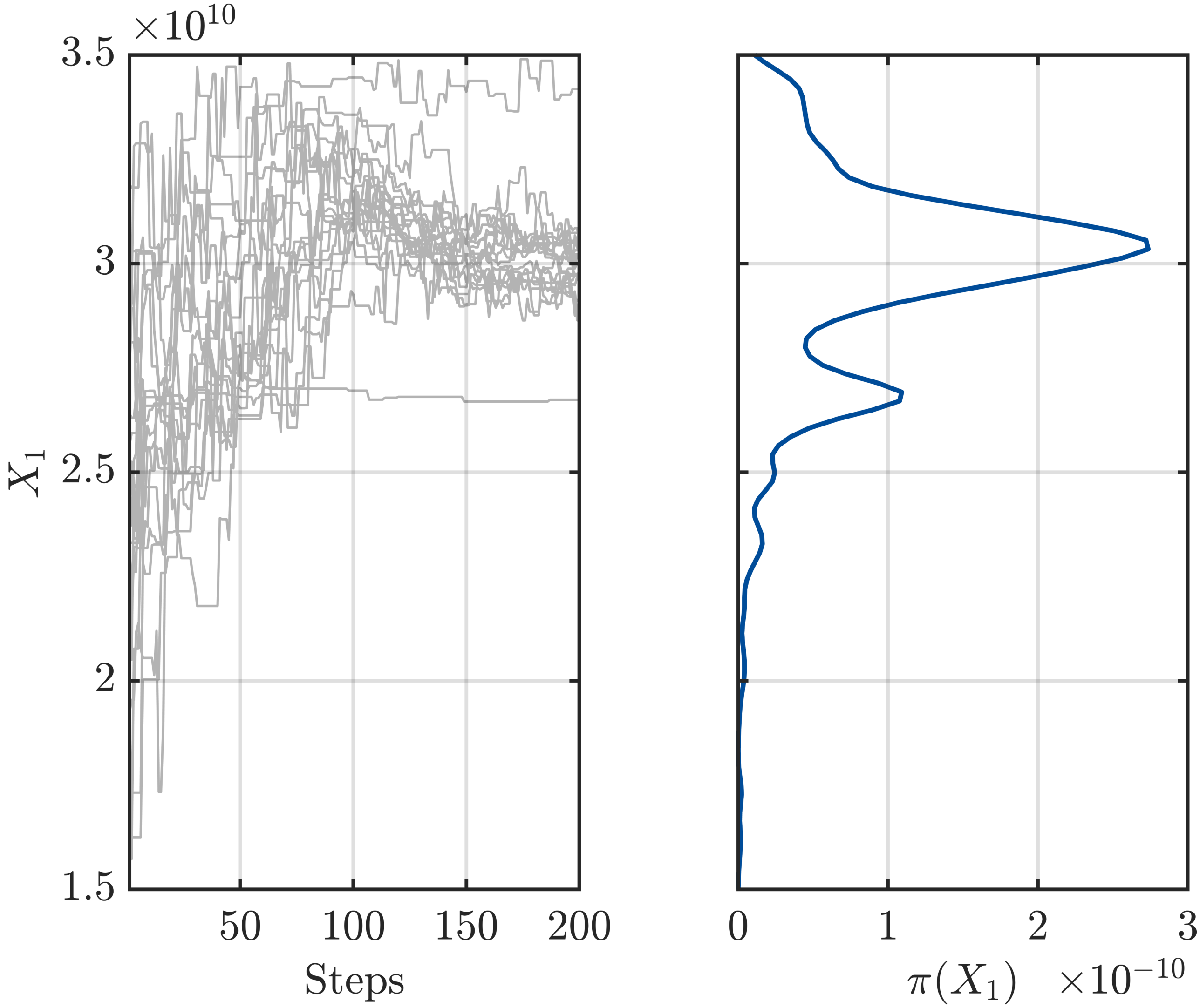
```
Solver.Type = 'MCMC';
Solver.MCMC.Visualize.Parameters = [1 2];
Solver.MCMC.Visualize.Interval = 10;
Solver.MCMC.Sampler = 'AIES';
Solver.MCMC.Steps = 200;
Solver.MCMC.NChains = 20;
Solver.MCMC.Proposal.PriorScale = 1e-3;
```

9 - Bayesian inference

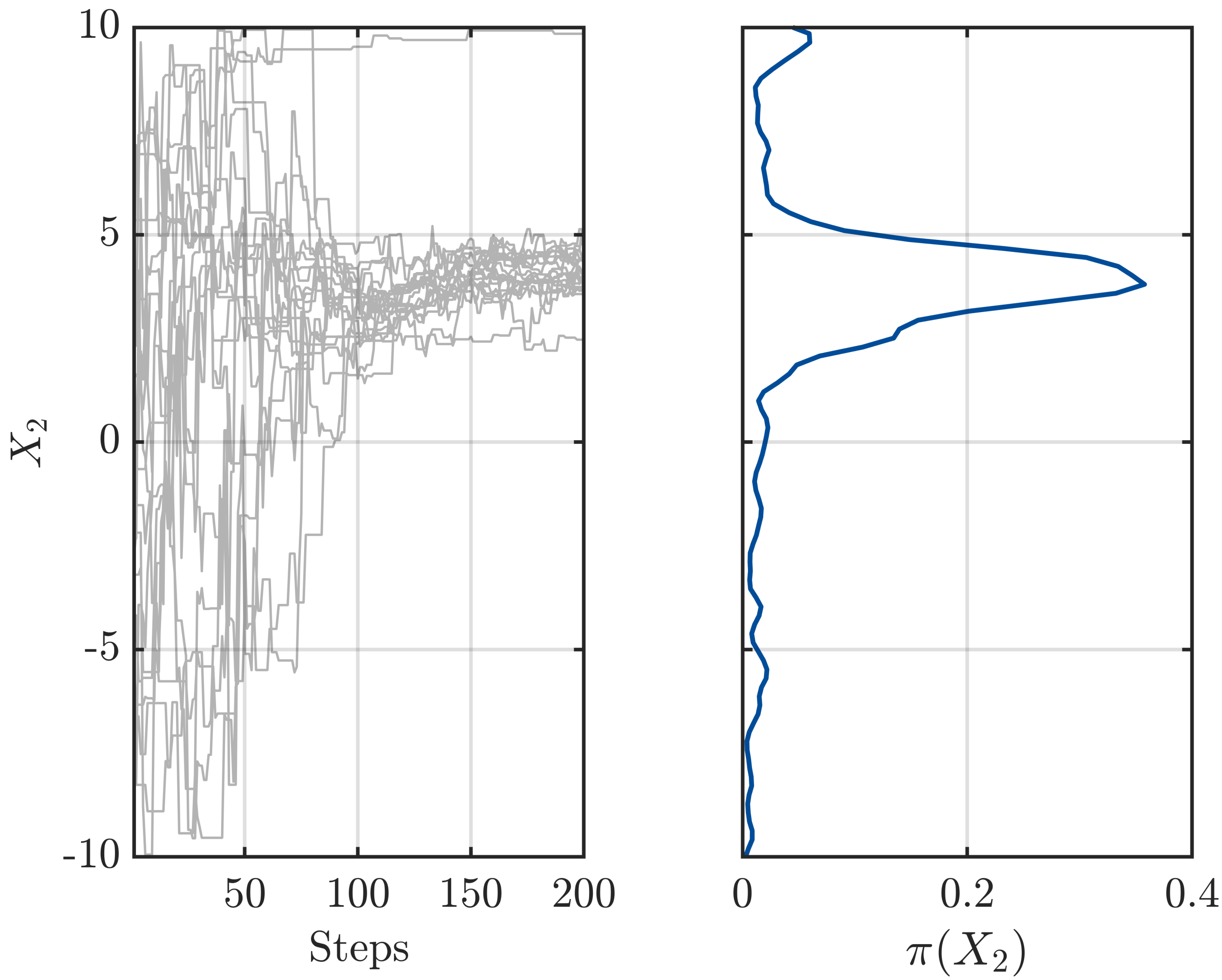
```
BayesOpts.Discrepancy = DiscrepancyOptsUnknownDisc;
BayesOpts.ForwardModel.Model = myPCE;
BayesOpts.Data = myData;
BayesOpts.Type = 'inversion';
BayesOpts.Solver = Solver;
BayesOpts.Prior = myPriorDist;
BayesAnalysis = uq_createAnalysis(BayesOpts);
```

```
Starting AIES...

|##                                |   5.00%|###                                      |  10.00%|#####                                    |  15
```



×10^10
3.5
3
2.5
2
1.5
X_1
50
100
150
200
Steps
0
1
2
3
π(X_1) ×10^-10




```
Finished AIES!
```

10 - Post-processing

Burn in 70%; badchain criteria $\delta > 10m$; confidence interval 90%; Point estimate - mean

```
badChainsIndex = squeeze(BayesAnalysis.Results.Sample(end,2,:) > 10);

uq_postProcessInversionMCMC(BayesAnalysis,'pointEstimate','mean','percentiles',
[0.05,0.95],'burnin',0.7,'badChains',badChainsIndex);


uq_print(BayesAnalysis);
```

```
%----------------------- Inversion output -----------------------%
   Number of calibrated model parameters:         2
   Number of non-calibrated model parameters:     0

   Number of calibrated discrepancy parameters:   1
```

```
%------------------- Data and Discrepancy
%  Data-/Discrepancy group 1:
   Number of independent observations:              1

   Discrepancy:
      Type:                                         Gaussian
      Discrepancy family:                           Scalar
      Discrepancy parameters known:                 No

   Associated outputs:
      Model 1:
         Output dimensions:                         1
                                                    to
                                                    29


%------------------- Solver
   Solution method:                                 MCMC

   Algorithm:                                       AIES
   Duration (HH:MM:SS):                             00:00:26
   Number of sample points:                         4.00e+03

%------------------- Posterior Marginals
---------------------------------------------------------------------
| Parameter | Mean    | Std     | (0.05-0.95) Quant.  | Type        |
---------------------------------------------------------------------
| E         | 3e+10   | 1.3e+09 | (2.8e+10 - 3.3e+10) | Model       |
| delta     | 4.3     | 1.4     | (2.7 - 7.4)         | Model       |
| Sigma2    | 0.0025  | 0.0025  | (0.0012 - 0.0086)   | Discrepancy |
---------------------------------------------------------------------

%------------------- Point estimate
---------------------------------------
| Parameter | mean    | Parameter Type |
---------------------------------------
| E         | 3e+10   | Model          |
| delta     | 4.3     | Model          |
| Sigma2    | 0.0025  | Discrepancy    |
---------------------------------------

%------------------- Correlation matrix (model parameters)
----------------------------
|       |  E        delta  |
----------------------------
| E     |  1        -0.85  |
| delta |  -0.85    1      |
----------------------------
```

```
%uq_display(BayesAnalysis);
```

11 - 90% error band on predictive posterior

90% confidence interval for E and δ

set 90% = 95%- 5%

```
uq_postProcessInversionMCMC(BayesAnalysis,'percentiles',[0.05,0.95]);
```

Obtained the lower bound and upper bound for E and δ

```
E_5_LowB = BayesAnalysis.Results.PostProc.Percentiles.Values(1,1);
size(E_5_LowB)
```

```
ans = 1×2
     1     1
```

```
E_95_UpperB = BayesAnalysis.Results.PostProc.Percentiles.Values(2,1);
size(E_95_UpperB)
```

```
ans = 1×2
     1     1
```

```
delta_5_LowB = BayesAnalysis.Results.PostProc.Percentiles.Values(1,2);
size(delta_5_LowB)
```

```
ans = 1×2
     1     1
```

```
Delta_95_UpperB = BayesAnalysis.Results.PostProc.Percentiles.Values(2,2);
size(Delta_95_UpperB)
```

```
ans = 1×2
     1     1
```

```
N_predict = 10
```

```
N_predict = 10
```

```
% sampling on E
E_90_sample_O = linspace(E_5_LowB,E_95_UpperB,N_predict)';
size(E_90_sample_O)
```

```
ans = 1×2
    10     1
```

```
%Shuffle the order
shuffledIndices = randperm(length(E_90_sample_O));
E_90_sample = E_90_sample_O(shuffledIndices);
size(E_90_sample)
```

```
ans = 1×2
    10     1
```

```
% sampling on delta
delta_90_sample_O = linspace(delta_5_LowB,Delta_95_UpperB,N_predict)';
size(delta_90_sample_O)
```

```
ans = 1×2
    10     1
```

```
%Shuffle the order
shuffledIndices = randperm(length(delta_90_sample_O));
delta_90_sample = delta_90_sample_O(shuffledIndices);
size(delta_90_sample)
```

```
ans = 1×2
    10     1
```

```
%plot the sampling on E and delta
figure
plot(E_90_sample,delta_90_sample,'*');
xlabel('E');
ylabel('\delta');

axis([min(E_90_sample,[],'all') max(E_90_sample,[],'all') min(delta_90_sample,
[],'all') max(delta_90_sample,[],'all')])
```

Predictive FE realization

```
Predict_sample = [E_90_sample,delta_90_sample];
size(Predict_sample)
```

```
ans = 1×2
    10     2
```

```
%Loop to get the predictive FE deflection

YPCE_Predict = [];

for i = 1:N_predict

    Xval_Predict = Predict_sample(i,:);

    Deflection = uq_evalModel(myPCE,Xval_Predict);
    YPCE_Predict = [YPCE_Predict;Deflection];


end

size(YPCE_Predict)
```

```
ans = 1×2
    10    29
```

Spline curve fitting to smooth the line for the 90CI

```
x = 1:29;%29 measurement position along the beam

for i = 1:size(YPCE_Predict,1)

    P = polyfit(x,YPCE_Predict(i,:),3);
    xi = 1:0.1:29;
    YPCE_Predict_Poly(i,:) = polyval(P,xi);
end
size(YPCE_Predict_Poly)
```

```
ans = 1×2
    10   281
```

Loop to fill the error band 90%CI

```
%loop to fill the error band
close all;
for i = 1:size(YPCE_Predict_Poly,1)-1
    hold on;
    fill([xi fliplr(xi)], [-YPCE_Predict_Poly(i,:) fliplr(-
YPCE_Predict_Poly(i+1,:))], 'cyan', 'FaceAlpha', 1,'EdgeColor','none');
end

 hold on;

 xlabel('Beam length \it{L} \rm(m)','FontSize',10);
 pbaspect([1 0.3 1]);
 ax = gca;
 ax.XAxisLocation = 'top';
 ylabel('Deflection (m)','FontSize',10);
 box on;
 set(ax,'FontSize',10);
 yticks('auto');
 ylim([-3.5 0])
```

scatter the measurement

```
x = 1:1:29;
size(x)
```

```
ans = 1×2
     1    29
```

```
for i = 1: size(myData.y,1)
    scatter(x,-myData.y(i,:),'black','x');
    hold on;
end
```

draw the mean value of 90%CI

```
plot(xi,-mean(YPCE_Predict_Poly),'red','LineWidth',1.5);
```

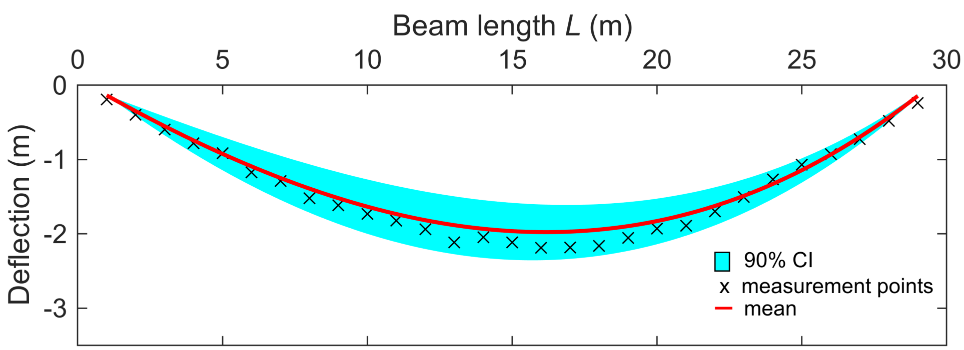
legend

```
rectangle('Position', [22, -2.5,0.5, 0.25], 'FaceColor', 'cyan');
text(23, -2.35, '90% CI', 'FontSize', 8);
text(22.15, -2.7, 'x  measurement points', 'FontSize', 8);

line([22,22.6],[-3,-3],'linestyle','-','color','red','LineWidth',1.0);
text(23, -3, 'mean', 'FontSize', 8);
```

Plot the predictive individual lines

```
hold off;
```




```
% beam position

x_beam = 1:1:29
```

```
x_beam = 1×29
     1     2     3     4     5     6     7     8     9    10    11    12    13 ···
```

```
for i = 1:size(YPCE_Predict,1)

    plot(x_beam,-YPCE_Predict(i,:));
```

```
    hold on;
end
 xlabel('Beam length \it{L} \rm(m)','FontSize',10);
 pbaspect([1 0.3 1]);
 ax = gca;
 ax.XAxisLocation = 'top';
 ylabel('Predictive deflection (m)','FontSize',10);
 box on;
 set(ax,'FontSize',10);
 yticks('auto');
```