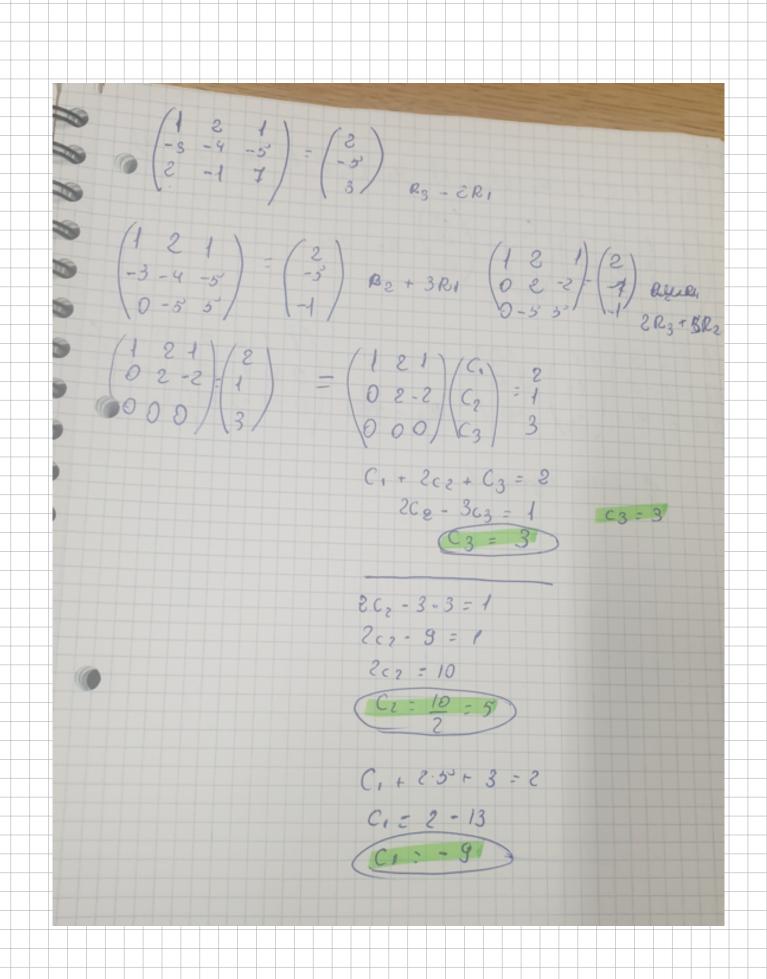


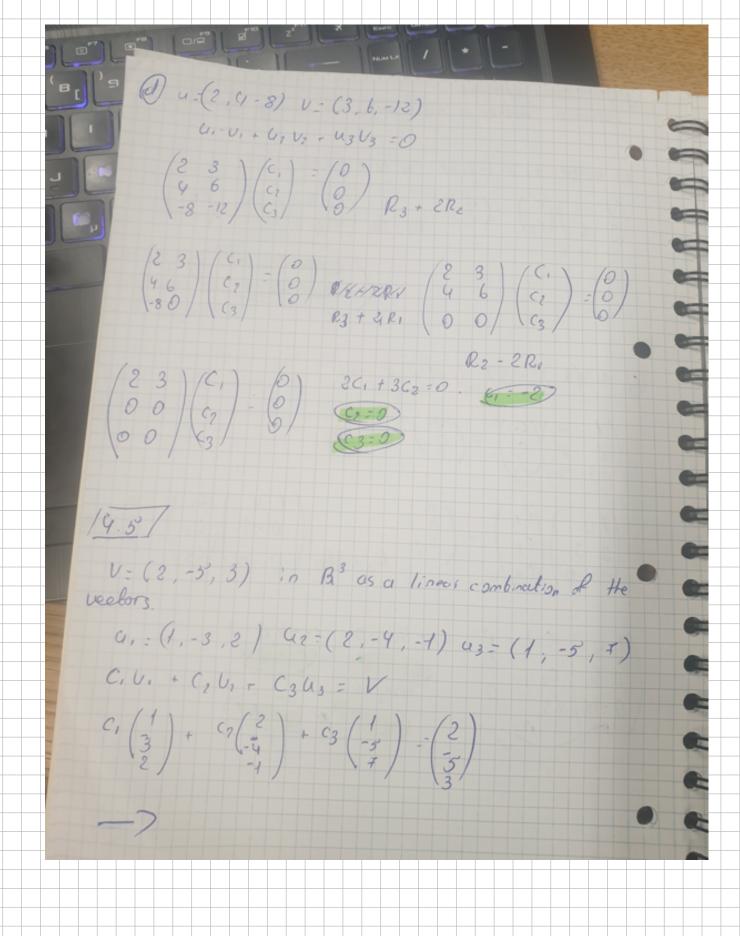
$$U = (1-3) \qquad U = (-7,6)$$

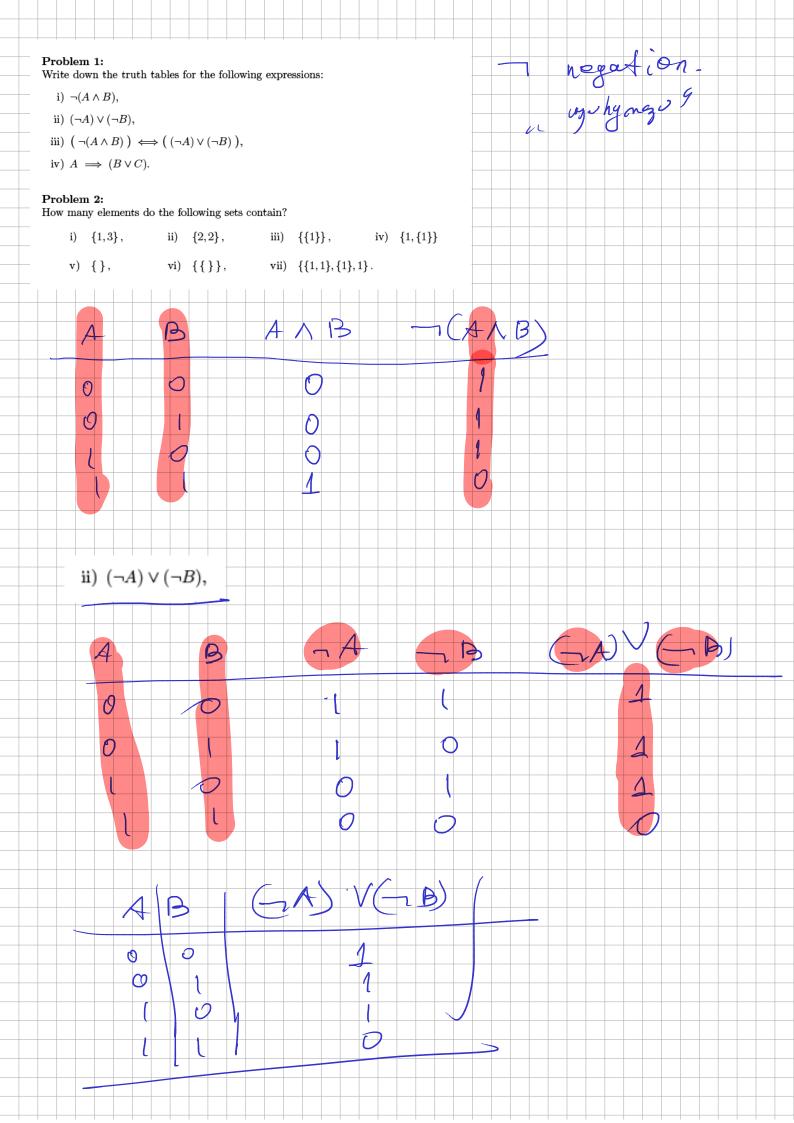
$$C_1 = 2C_2 \qquad C_1 \qquad U + C_2 \qquad U = 0$$

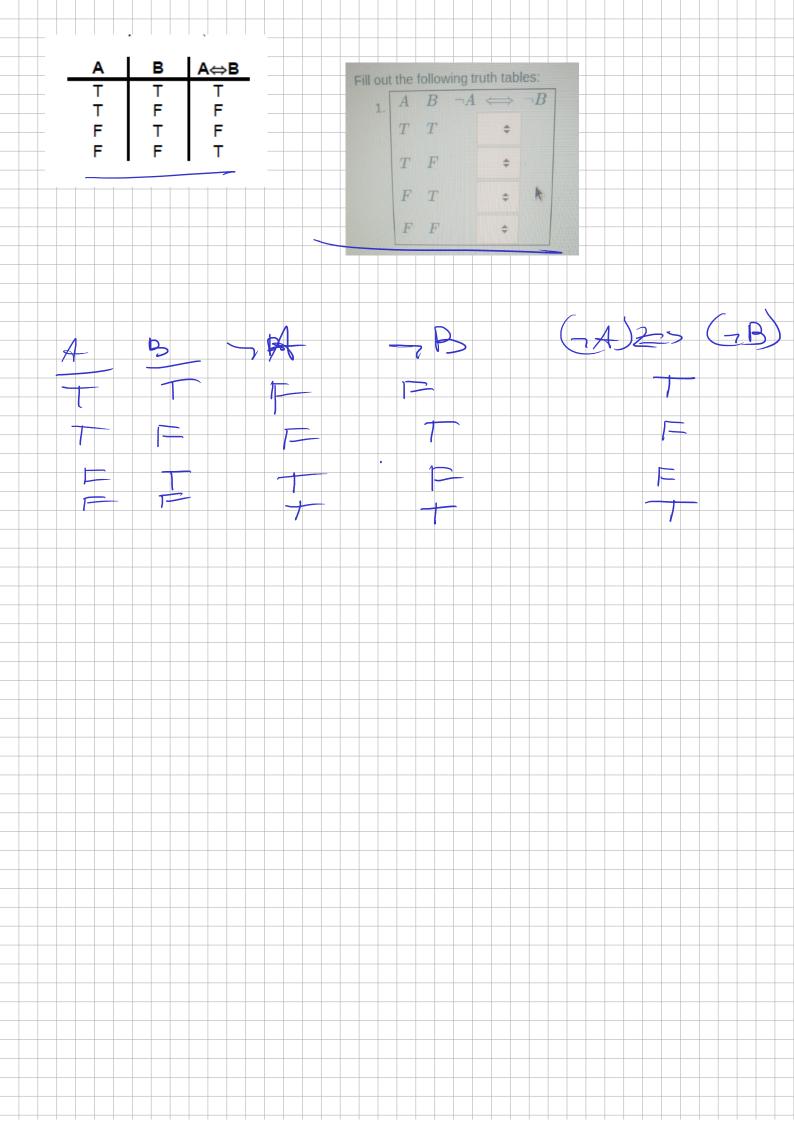
$$C_1 \neq 0 \qquad C_2 \neq 0$$

are inicarry dependent, where C13 C2 15 VM/8 (c) u = (1, 2, -3), v = (4, 5, -6)c, u + C2 - 0 C, +4C2 =0 | C, t4.0 = 0 0 = 3 = 0 0 = 0 0 = 0u, ot las - Indep-









## **Basis and Dimension 4.24.** Determine whether or not each of the following form a basis of $\mathbb{R}^3$ : (a) (1, 1, 1), (1, 0, 1);(c) (1,1,1), (1,2,3), (2,-1,1);(b) (1,2,3), (1,3,5), (1,0,1), (2,3,0); (d) (1, 1, 2), (1, 2, 5), (5, 3, 4). (a and b) No, because a basis of $\mathbb{R}^3$ must contain exactly three elements because dim $\mathbb{R}^3 = 3$ . (c) The three vectors form a basis if and only if they are linearly independent. Thus, form the matrix whose rows are the given vectors, and row reduce the matrix to echelon form: $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 5 \end{bmatrix}$ The echelon matrix has no zero rows; hence, the three vectors are linearly independent, and so they do form a basis of $\mathbb{R}^3$ . $\alpha$ (1,1,1), (1,2,3), (2,-1,1);u, u<sub>2</sub> u<sub>3</sub>, 2. U. + y. U2 + 2. U3 = 2 = -(+6=5 Lon U1 2 42 , U2

Extend  $\{u_1 = (1, 1, 1, 1), u_2 = (2, 2, 3, 4)\}$  to a basis of  $\mathbb{R}^4$ . First form the matrix with rows  $u_1$  and  $u_2$ , and reduce to echelon form:  $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 3 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ Then  $w_1=(1,1,1,1)$  and  $w_2=(0,0,1,2)$  span the same set of vectors as spanned by  $u_1$  and  $u_2$ . Let  $u_3=(0,1,0,0)$  and  $u_4=(0,0,0,1)$ . Then  $w_1,\,u_3,\,w_2,\,u_4$  form a matrix in echelon form. Thus, they are linearly independent, and they form a basis of  $\mathbf{R}^4$ . Hence,  $u_1,\,u_2,\,u_3,\,u_4$  also form a basis of  $\mathbf{R}^4$ . Ø  $\mathcal{O}$ Ø