Transistor Modeling

By

Dr. ANUPAM KARMAKAR

Associate Professor & Head

Department of Electronic Science

University of Calcutta

<akelc@caluniv.ac, akarmakar.cu@gmail.com>

Introduction

- For the analysis of small-signal AC response of BJT amplifier the knowledge of modeling the transistor is important.
- The input signal will determine whether it's a small-signal (AC) or large signal (DC) analysis.
- The goal when modeling small-signal behavior is to make of a transistor that work for small-signal enough to "keep things linear" (i.e., not to distort too much).
- There are two models commonly used in the small-signal analysis:
  - (a) r_e model
  - (b) hybrid equivalent model

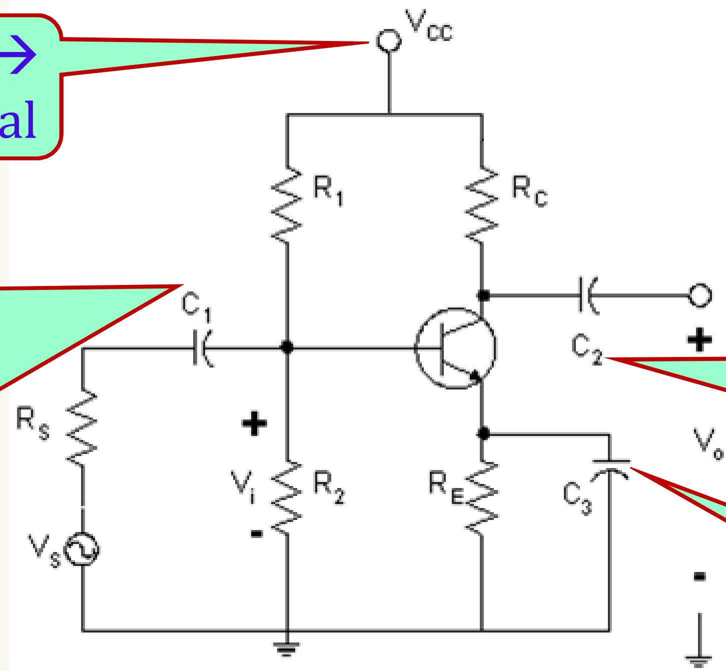
Disadvantages

r_e - model : Fails to account the output impedance level of device and feedback effect from output to input.

Hybrid equivalent model : Limited to specified operating condition in order to obtain accurate result.

CB-mode: Voltage-divider Configuration under AC analysis

DC supply → "0" potential

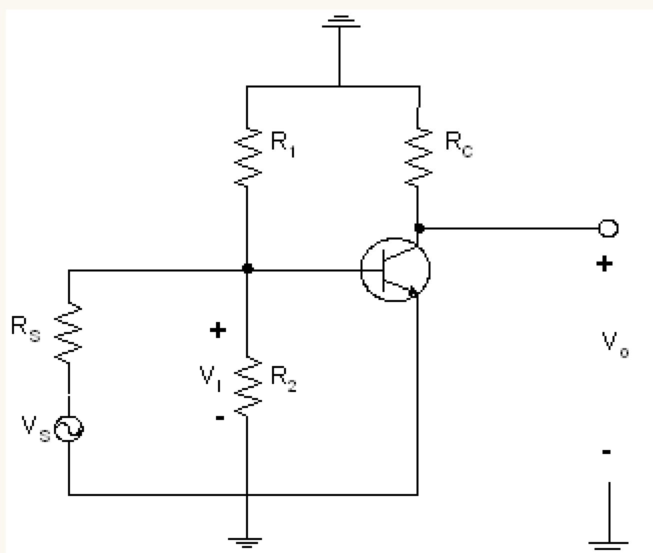


- I/p coupling capacitor → s/c
- Large values
- Block DC and pass AC signal

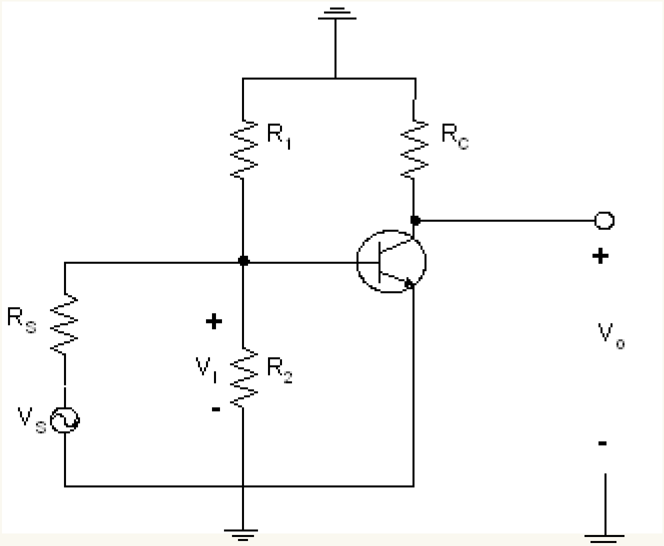
- O/p coupling capacitor → s/c
- Large values
- Block DC and pass AC signal

- Bypass capacitor → s/c
- Large values

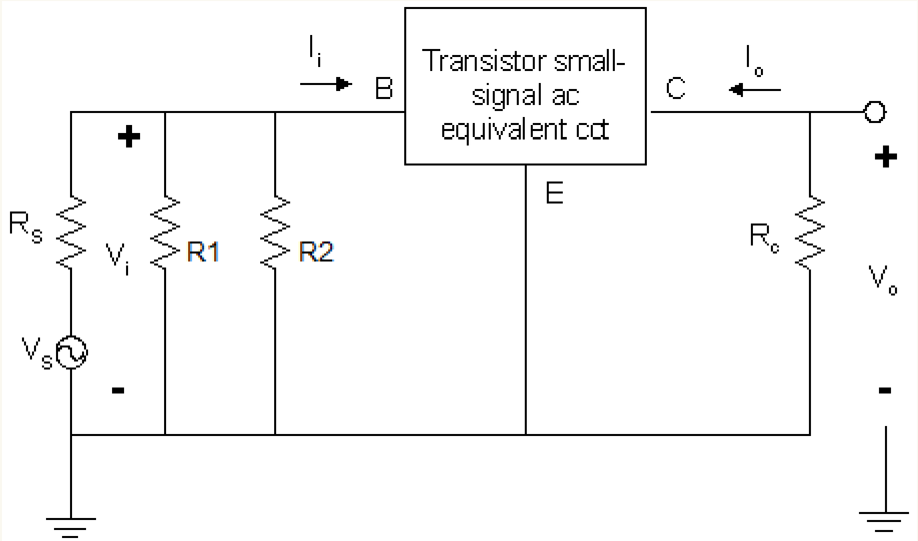
Redrawn voltage-divider configuration after removing dc supply and inserting s/c for the capacitors



Transistor Small-Signal AC Equivalent Circuit



Redrawn voltage-divider configuration after removing dc supply and inserting s/c for the capacitors



Redrawn for small-signal AC analysis

AC Analysis:

(1) Replace all DC sources by ground.
(2) Coupling and Bypass capacitors are short cct. The effect of there capacitors is to set a lower cut-off frequency for the cct.
(3) Inspect the cct. (replace BJTs with its small signal model : r_e or hybrid).
(4) Solve for voltage and current transfer function, i/o and o/p impedances.

The r_e – Transistor Model

- Small-signal r_e – model of transistor is simple.
- It employs a diode and controlled current source to depict the behavior of a transistor in the region of interest.
- Output characteristics reveal that BJT amplifiers are referred to as current-controlled devices.
- In the active region of operation of a transistor, emitter-base junction is forward biased and collector-base junction is reverse biased.

The r_e – Transistor Model

Common-Base Configuration:

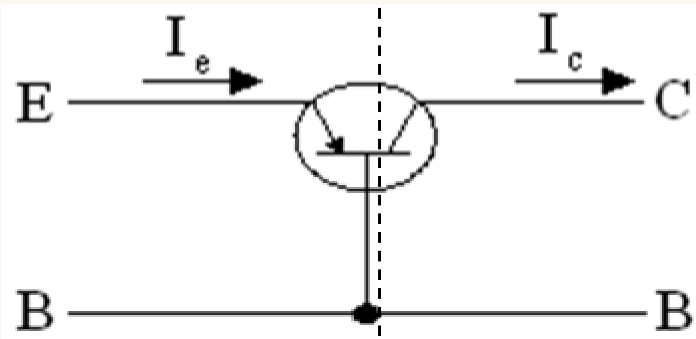


Fig. (b): CB *pnp* transistor.

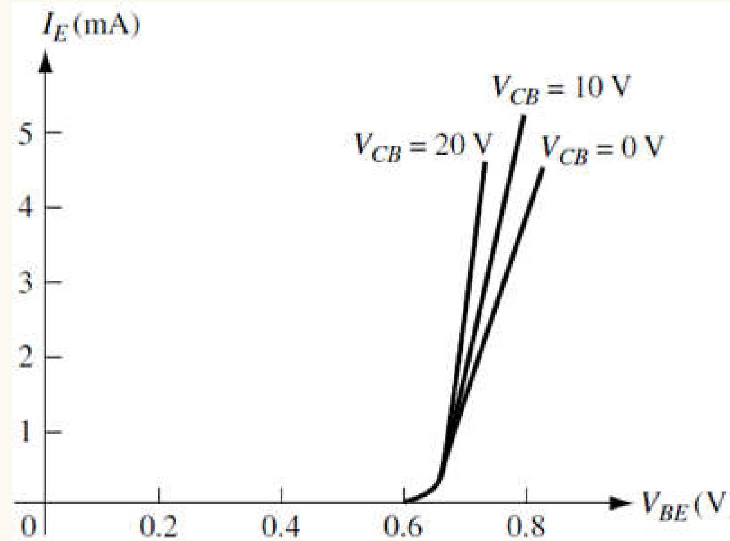


Fig. (b): CB *pnp* transistor input characteristics.

- The straight line-segment of input characteristics (Fig. b) of BJT in active region portrays that the forward biased emitter-base junction (input side) of the transistor resembles with that of a forward biased diode (Fig. c).

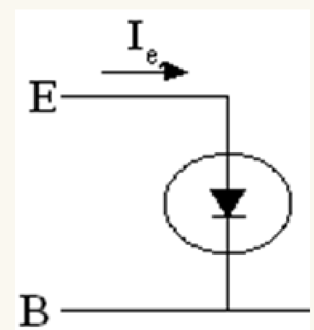


Fig. (c): Equivalent circuit of input side.

The r_e – Transistor Model: Common-Base Configuration

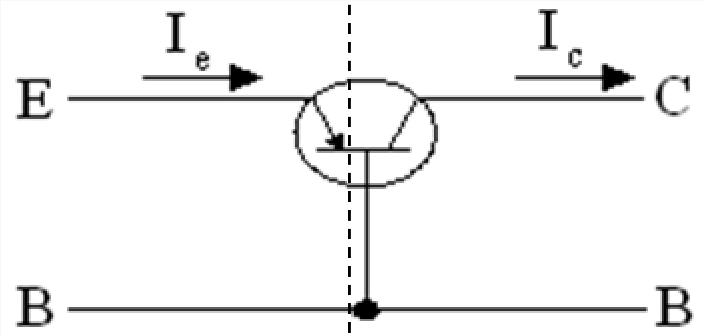


Fig. (a): CB *pnp* transistor.

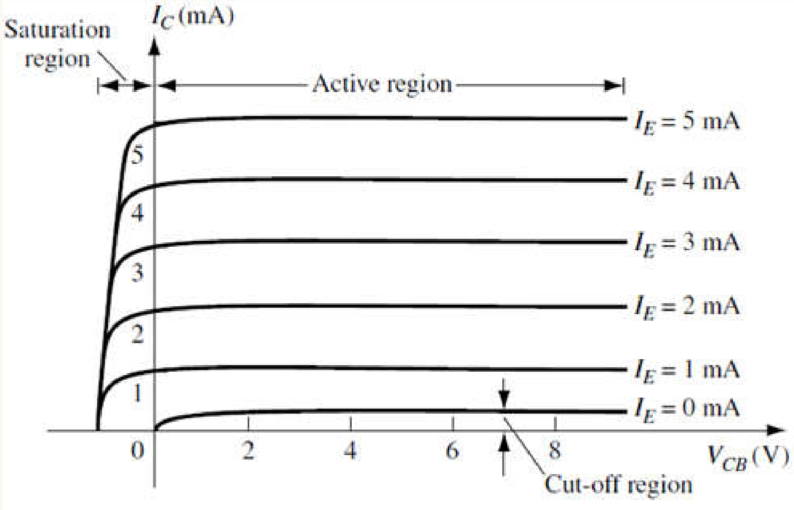


Fig. : CB *pnp* transistor output characteristics.

- The straight line-segment of output characteristics (Fig. b) of BJT in active region portrays that the collector-base junction (output side) of the transistor resemble with that of a constant current source (Fig. c).

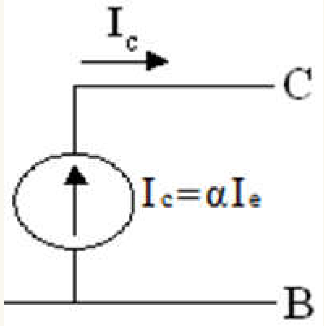
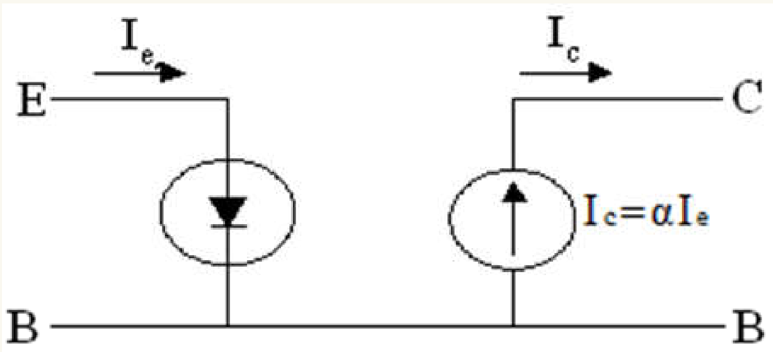


Fig. (c): Equivalent circuit of output side.

- The current source in Fig. (c) establishes the fact that the output current $I_c = \alpha I_e$, depends on the controlling current I_e , the emitter (input) current in the input side.

The r_e – Transistor Model: Common-Base Configuration

• Therefore the equivalence at the input-output terminals with the current-controlled source, providing a link between the two have been established as depicted in Fig. (a).



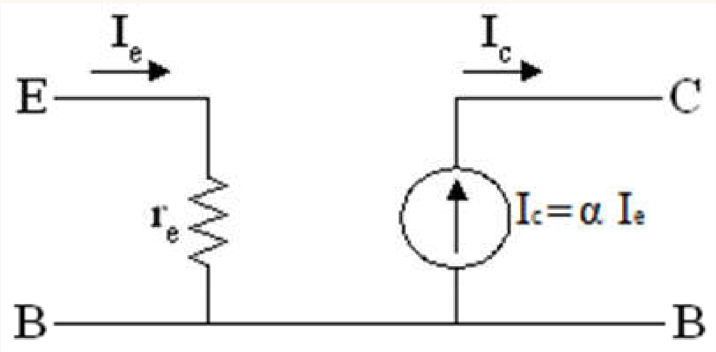


Fig. (a): $\boldsymbol{r_e}$ – model of transistor in CB configuration.

• Forward resistance of a diode is determined by $r_{ac} = \dfrac{26\text{ mV}}{I_d}\Omega$, where I_d is the diode current at the Q-point.

• In CB transistor configuration, the diode current is emitter current I_E , thus the notation of the diode resistance is r_e and its value is given by

$$r_e = \frac{26\text{ mV}}{I_E}\Omega$$

The r_e – Transistor Model

Common-Emitter Configuration:

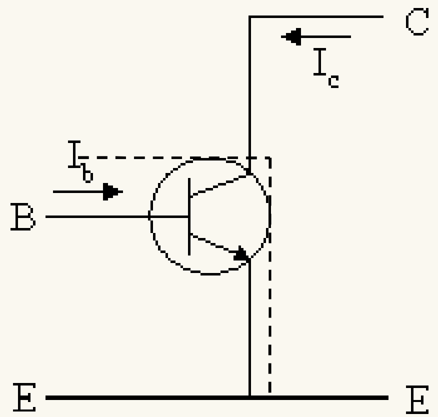


Fig. (a): CE *npn* transistor.

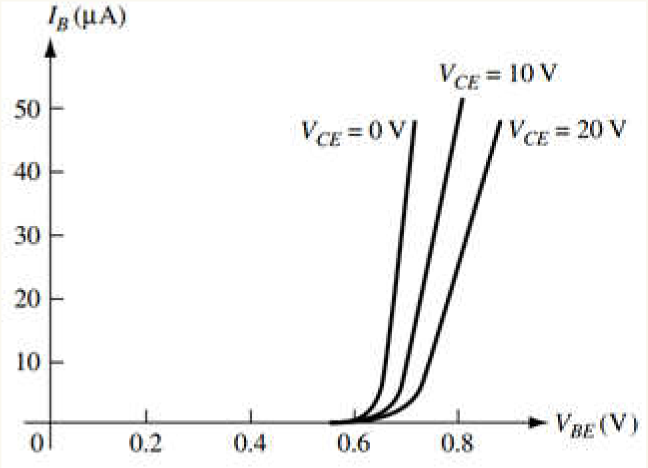


Fig. (b): CE *npn* transistor input characteristics.

- The straight line-segment of input characteristics (Fig. b) of BJT in active region portrays that the forward biased emitter-base junction (input side) of the transistor resembles with that of a forward biased diode (Fig. c).

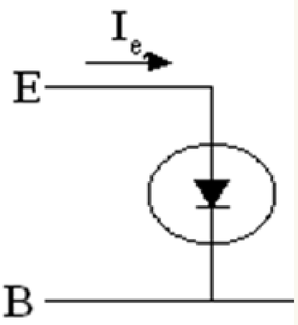


Fig. (c): Equivalent circuit of input side.

The r_e – Transistor Model

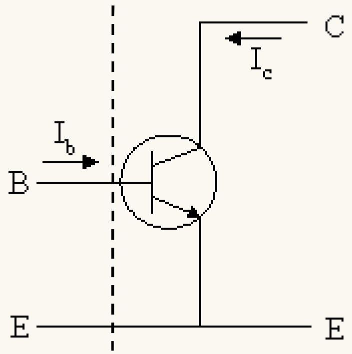


Fig. (a): CE *npn* transistor.

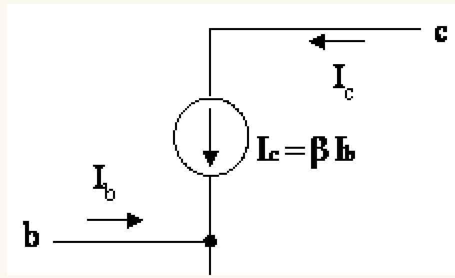


Fig. (c): Equivalent circuit of output side.

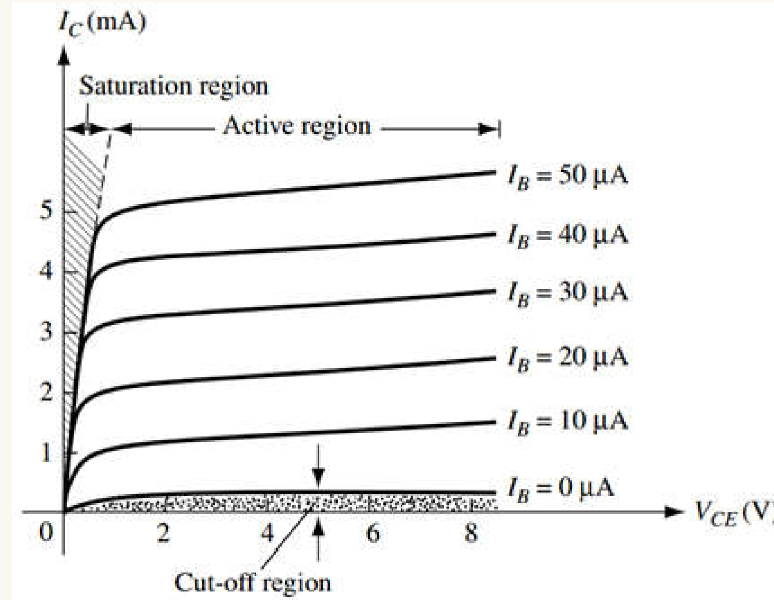


Fig. : CB *pnp* transistor output characteristics.

- The straight line-segment of output characteristics (Fig. b) of BJT in active region portrays that the collector-emitter terminals (output side) of the transistor resemble with that of a constant current source (Fig. c).

- The current source in Fig. (c) establishes the fact that the output current $I_C = \beta I_B$, depends on the controlling current I_B , the base (input) current in the input side.

The r_e – Transistor Model

- Therefore the equivalence at the input-output terminals with the current-controlled source, providing a link between the two has been established as depicted in Fig. (a).

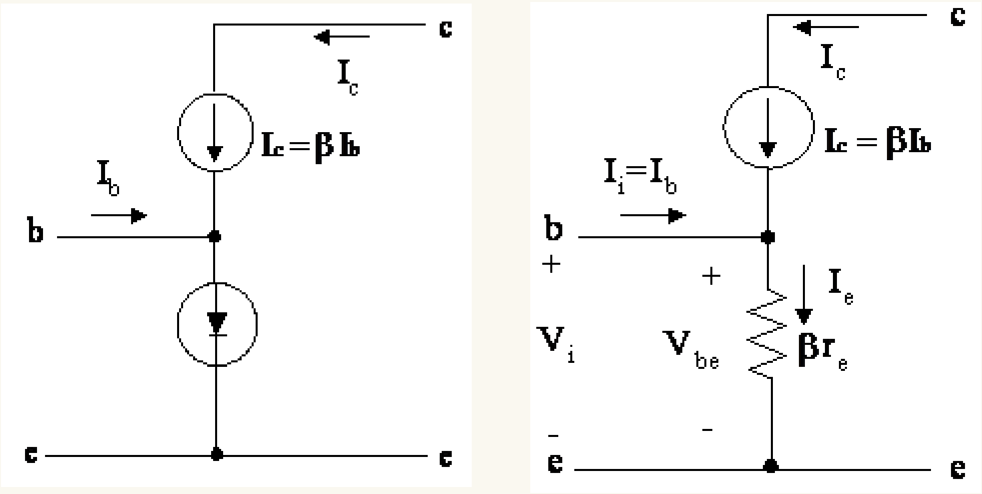


Fig. (a): r_e- model of transistor in CE configuration.

- In CE configuration, $I_C = \beta I_B$.

Hence, current through the diode is given by

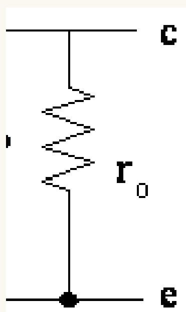
$$I_E = I_C + I_B = \beta I_B + I_B = (\beta + 1)I_B \approx \beta I_B \,.$$

- Thus, the ac resistance of the diode is given by

$$r_{ac} = \frac{26 \text{ mV}}{I_B} = \frac{26 \text{ mV}}{I_E/\beta} \qquad \text{or,} \; r_{ac} = \beta \frac{26mV}{I_E} = \beta r_e$$

The r_e – Transistor Model

- Close examination of output characteristics of CE transistor configuration reveals that the slope of the curves increases with increase in collector current. The steeper the slope, the less the level of output impedance (r_o) between the collector and emitter terminals as shown in Fig.



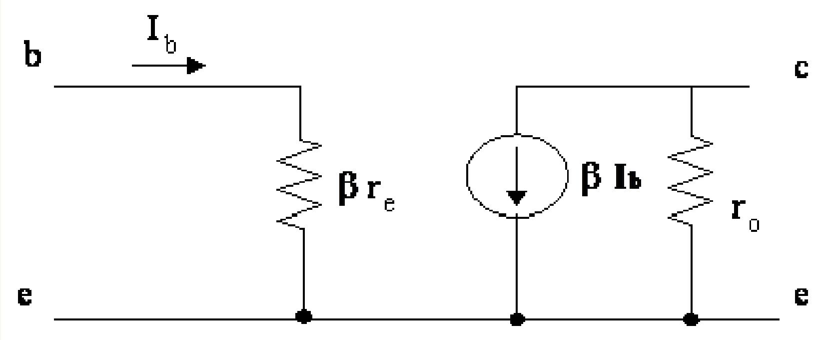


Fig. (a): r_e- model of transistor in CE configuration.

- Considering the fact that the resistance between the base and emitter terminal is βr_e , the collector current is βI_b , and the output resistance is r_o , the equivalent r_e – model of a transistor in CE configuration can be depicted as in Fig.

Forward Biased Diode Resistance

- Diode current is given by

$$I_d = I_S\left(e^{kT/V} - 1\right)$$

- where, I_S is reverse saturation current, k = Boltzmann's constant, V = voltage across the forward biased diode, T = temperature in K.
- Taking derivative w.r.t. the applied bias, results in

$$\frac{d(I_d)}{dV} = \frac{d}{dV}\left[I_S\left(e^{kT/V} - 1\right)\right]$$

or, $$\frac{dI_d}{dV} = \frac{k}{T} I_S e^{kT/V} = \frac{k}{T}(I_d + I_d) \cong \frac{k}{T} I_d$$

- At room temperature, T = 25 + 273 = 298 K, and then substituting k = 11,600, yields

$$\frac{dI_d}{dV} = \frac{11600}{298} I_d \cong 38.93 I_d$$

- Therefore, the diode ac resistance can be determined as

$$r_d = \frac{dV}{dI_d} = \frac{1}{38.93 I_d} = \frac{0.026}{I_d} = \frac{26\text{ mV}}{I_d}$$

Hybrid-Parameters Model

By

Dr. ANUPAM KARMAKAR

Associate Professor & Head

Department of Electronic Science

University of Calcutta

<akelc@caluniv.ac, akarmakar.cu@gmail.com>

Hybrid-Parameters Model

For the basic three terminal devices, there are two ports (pair of terminals), the input terminals at the left and output terminals at the right. Four variables are related by the equations:

$$V_i = f(I_i, V_o)$$
$$I_o = f(I_i, V_o)$$

$$V_i = h_{11}I_i + h_{12}V_o \qquad \dots \dots \quad (1a)$$
$$I_o = h_{21}I_i + h_{22}V_o \qquad \dots \dots \quad (1b)$$

The parameters relating the four variables are called *h-parameter* from the word 'hybrid', because of the mixture of variables (*V* and *I*) in each equation results in a 'hybrid' set of units of measurement for the *h*-parameters

h-Parameters

If in (1a), V_o is set to zero by short-circuiting the output terminals, then

$$h_{11} = h_i = \left.\frac{V_i}{I_i}\right|_{V_0 = 0} \quad \dots \quad (2a)$$

The ratio (2a) indicates that h_{11} is an impedance parameter to be measured ohm (Ω). Since h_{11} (h_i) is the ratio of *input* voltage to *input* current with output terminals *shorted*, so it is called ***short-circuit input impedance*** parameter.

If in (1a), I_i is set to zero by opening the input terminals, then

$$h_{12} = h_r = \left.\frac{V_i}{V_o}\right|_{I_i = 0} \quad \dots \quad (2b)$$

The ratio (2b) indicates that h_{12} is a unit less ratio of voltage parameter. Since h_{12} (h_r) is the ratio of *input* voltage to *output* voltage with input terminals *open*, so it is called ***open-circuit reverse voltage gain*** parameter.

h-Parameters

Since each term of (1a) has the unit of volt, let the *Kirchhoff's voltage law* is applied in reverse, resulting in the circuit of Fig.

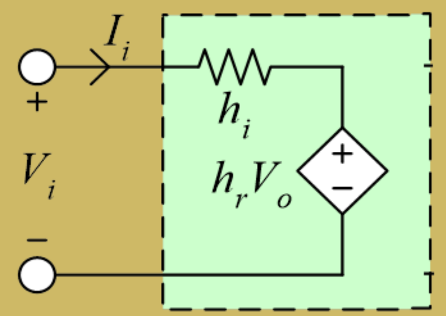


Fig. : *h*-parameter equivalent circuit of eq. (1a).

If in (1b), V_o is set to zero by short-circuiting the output terminals, then

$$h_{21} = h_f = \left.\frac{I_0}{I_i}\right|_{V_0 = 0} \quad \ldots \quad (2c)$$

The ratio (2c) indicates that h_{21} (h_f) is a unit less ratio of *output* current to *input* current with output terminals *shorted*, so it is called ***short-circuit forward current gain*** parameter.

h-Parameters

If in (1b), I_i is set to zero by opening the input terminals, then

$$h_{22} = h_o = \left.\frac{I_0}{V_o}\right|_{I_i = 0} \quad \ldots \quad (2d)$$

The ratio (2d) indicates that h_{12} is an admittance parameter to be measured mho (Ʊ) or siemen (S).

Since h_{22} (h_o)is the ratio of *output* current to *output* voltage with input terminals *open*, it is called ***open-circuit output admittance*** parameter.

Since each term of (1b) has the unit of current, let the *Kirchhoff's current law* is applied in reverse, resulting in the circuit of Fig.

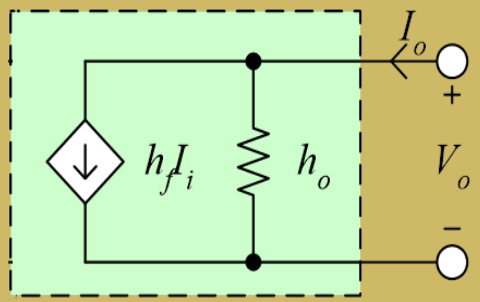


Fig. : *h*-parameter equivalent circuit of eq. (1b).

h-Parameter Model

Thus, the complete equivalent circuit for the basic three terminal linear devices is shown in Fig.

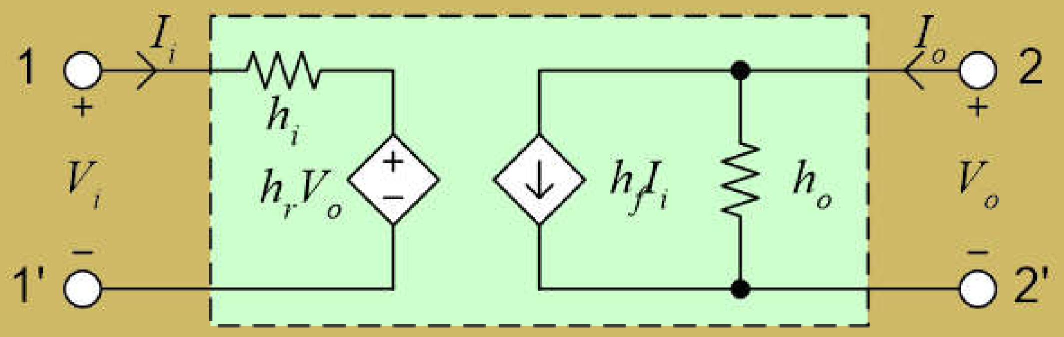
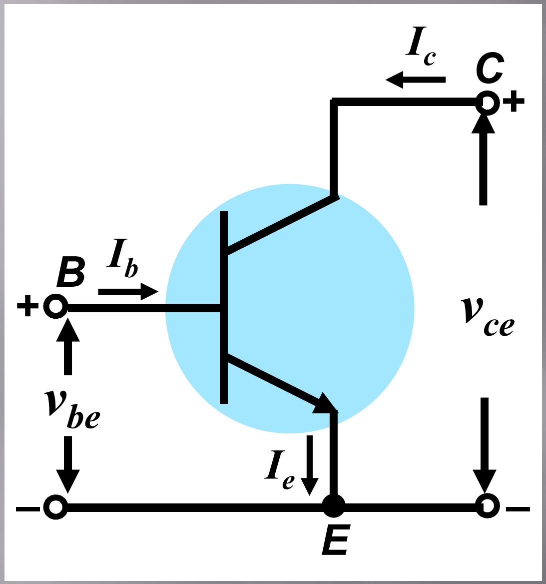


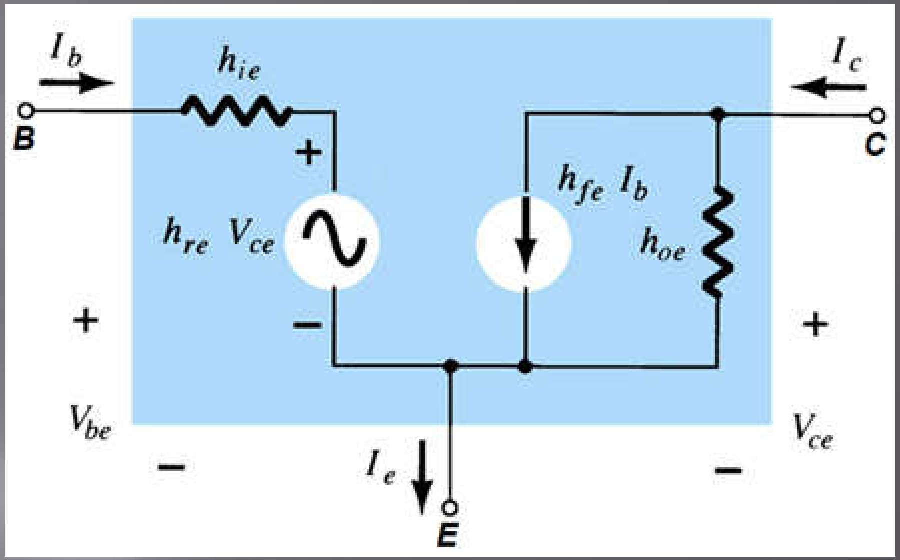
Fig. : Complete hybrid equivalent circuit.

$$V_i = h_{11}I_i + h_{12}V_o = h_iI_i + h_rV_o$$
$$I_o = h_{21}I_i + h_{22}V_o = h_fI_i + h_oV_o$$

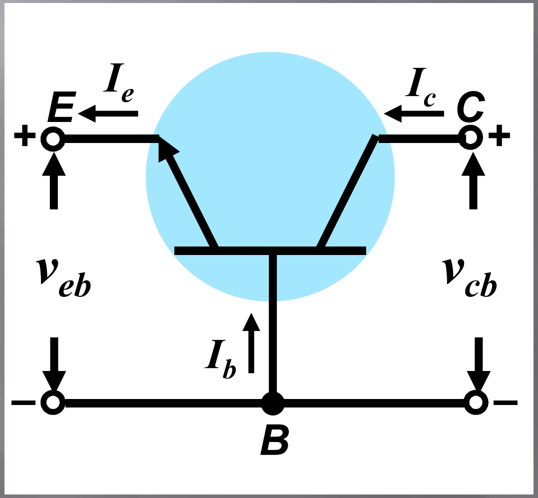
h_{11} = input impedance = h_i
h_{12} = reverse voltage gain = h_r
h_{21} = forward current gain = h_f
h_{22} = output admittance = h_o

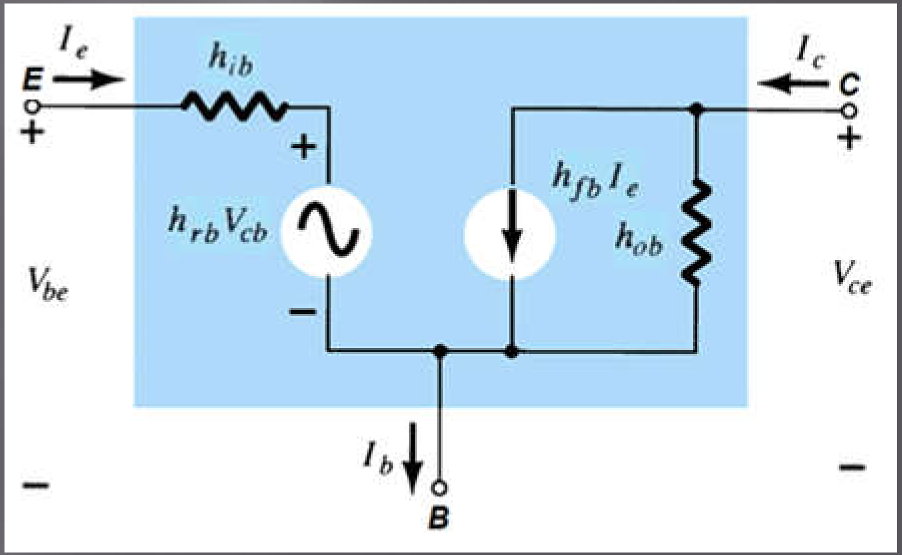
CE *h*-Parameter Equivalent Circuit



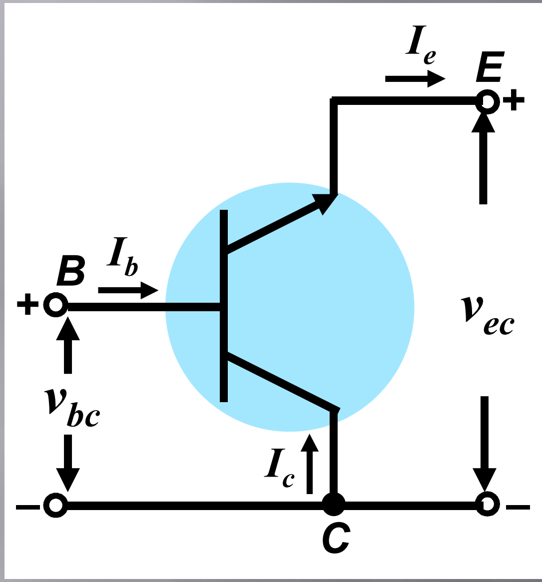


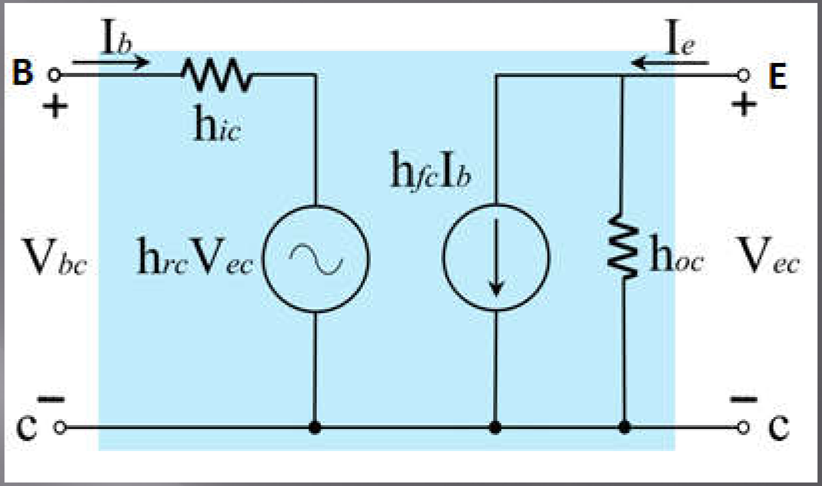
CB *h*-Parameter Equivalent Circuit





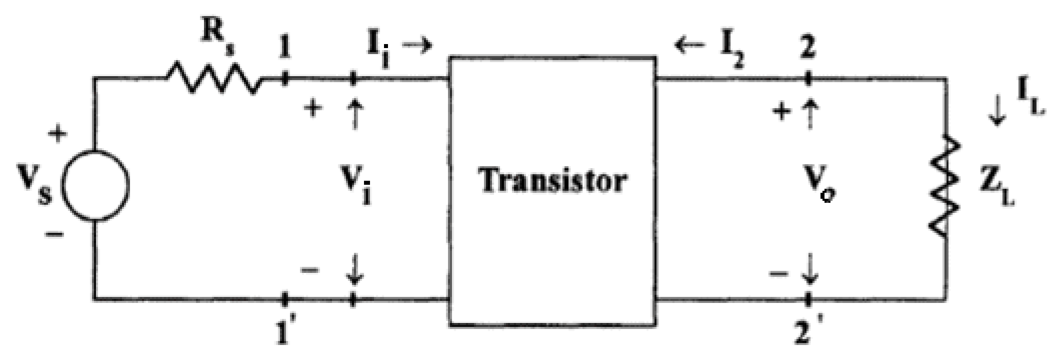
CC h-Parameter Equivalent Circuit



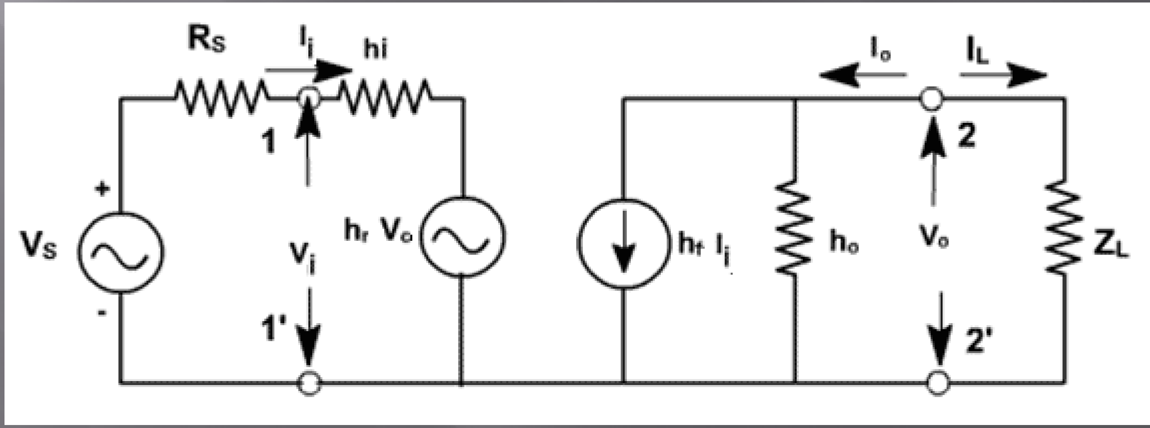


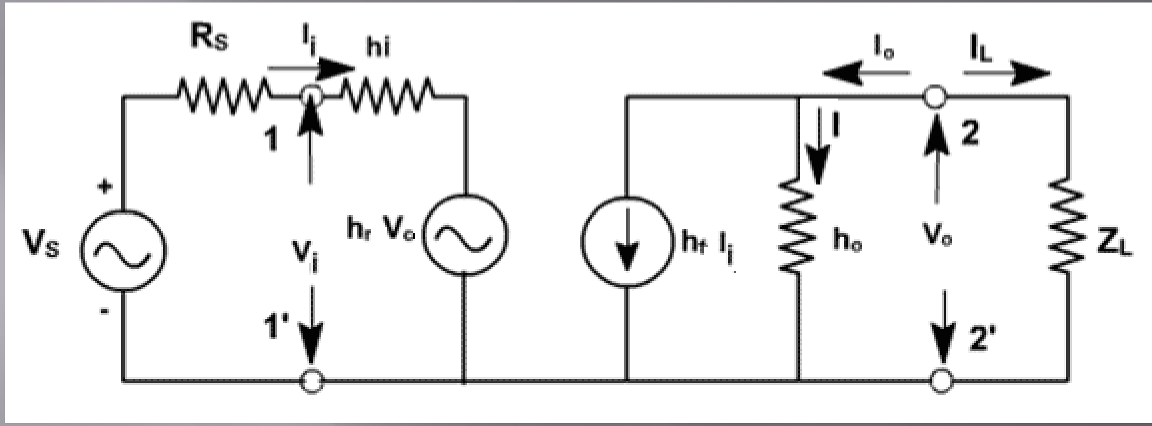
Analysis of Transistor Amplifier using *h*-Parameter Model

A basic amplifier circuit is shown in Fig. 1, where Z_L is the load resistance and R_S is the signal-source resistance.



Substituting the transistor with its *h*-parameter model, the above circuit is as shown in Fig. 2. It is assumed that *h*-parameters remain substantially constant over the operating range.





Current Gain or Amplification Factor, A_i:

For the transistor amplifier stage, current gain A_i is defined as the ratio of output current to input current and is given by:

$$A_i = \frac{I_L}{I_i} = \frac{-I_o}{I_i} \quad \dots \quad \dots \quad \text{(ia)}$$

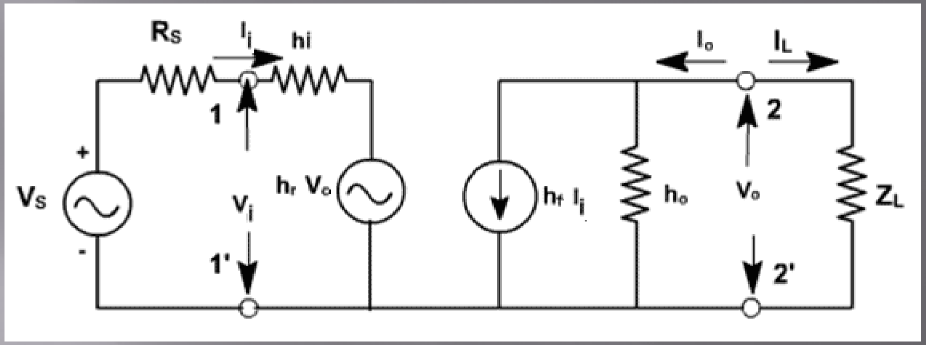
From the circuit of Fig. 2, applying *KCL* to the node 2 of the output loop, yields $I_o = h_f I_i + I = h_f I_i + V_o h_o \quad \dots \quad \dots \quad \text{(ib)}$

Substituting, $V_o = -I_o Z_L$, gives

$$I_o = h_f I_i - h_o Z_L I_o \quad \dots \quad \dots \text{(ic)}$$

so that,

$$A_i = \frac{-I_o}{I_i} = \frac{-h_f}{1 + h_o Z_L} \quad \dots \quad \dots \quad (1)$$



Voltage Gain or Amplification Factor, A_v:

Voltage gain A_v is defined as the ratio of output voltage to input voltage and is given by:

$$A_v = \frac{V_0}{V_i} \quad \ldots \quad \ldots \quad \text{(iia)}$$

From the circuit of Fig. 2, applying *KVL* to the input loop, results

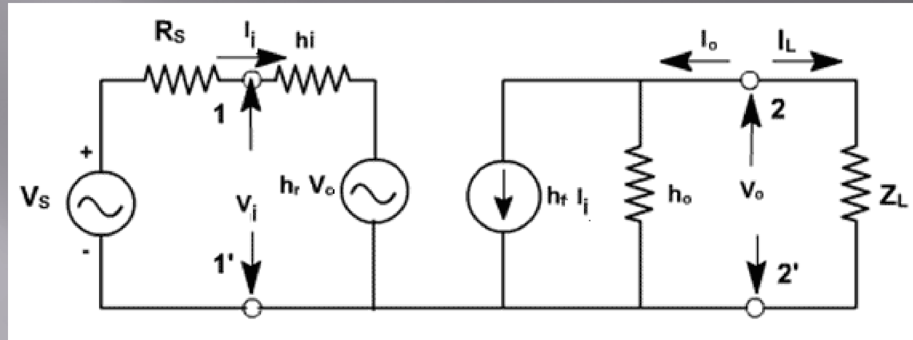
$$V_i = h_i I_i + h_r V_o \quad \ldots \quad \ldots \quad \text{(iib)}$$

From (1), $$I_i = \frac{(1+h_o Z_L)I_o}{h_f} = \frac{-(1+h_o Z_L)V_o}{h_f Z_L} \quad \ldots \quad \text{(iic)}$$ [putting, $I_o = -V_o/Z_L$]

Substituting (iic) in (iib), results $$V_i = \frac{-(1+h_o Z_L)h_i}{h_f Z_L} V_o + h_r V_o \quad \ldots \quad \ldots \quad \text{(iid)}$$

Solving for V_o/V_i, yields $$A_v = \frac{V_o}{V_i} = \frac{-h_o Z_L}{h_i + (h_i h_o - h_f h_r)Z_L} \quad \ldots \quad \ldots \quad (2)$$

Alternatively, $$A_v = \frac{V_o}{V_i} = \frac{-I_o Z_L}{V_i} = \frac{A_i I_i Z_L}{V_i} = \frac{A_i Z_L}{Z_i} \quad \ldots \quad \ldots \quad \text{(2a)}$$



Input Impedance, Z_i:

The amplifier input impedance Z_i is the impedance seen looking into the input terminals (1, 1′) and is given by:

$$Z_i = \frac{V_i}{I_i} \quad \dots \quad \dots \quad \text{(iiia)}$$

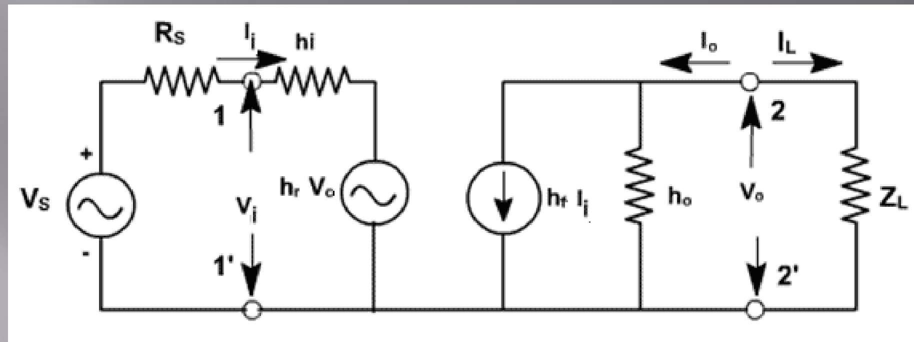
From the circuit of Fig. 2, applying *KVL* to the input loop, yields

$$V_i = h_i I_i + h_r V_o \quad \dots \quad \dots \quad \text{(iiib)}$$

Thus from (iiia), $Z_i = \dfrac{h_i I_i + h_r V_o}{I_i} = h_i + \dfrac{h_r V_o}{I_i} \quad \dots \quad \text{(iiic)}$

Using (1), $V_o = -I_o Z_L = A_i I_i Z_L$, and then substituting V_o in (iiic), results

$$Z_i = h_i + h_r A_i Z_L = h_i - \frac{h_r h_r Z_L}{1 + h_o Z_L} \quad \dots \quad \dots \quad (3)$$



Output Impedance, Z_o and Admittance Y_o:

By definition, the amplifier output impedance Z_o is obtained by setting the source voltage V_s to zero load impedance to infinity and driving the output terminals (2, 2′) from a generator voltage V_o. If the current drawn from the generator (V_o) is I_o, thus $Z_o = \dfrac{V_o}{I_o}$ … … (iva) with $V_s = 0$ and $Z_L = \infty$

From the circuit of Fig. 2, putting V_s = 0 and applying *KCL* to the input loop, yields $R_s I_i + h_i I_i + h_r V_o = 0$

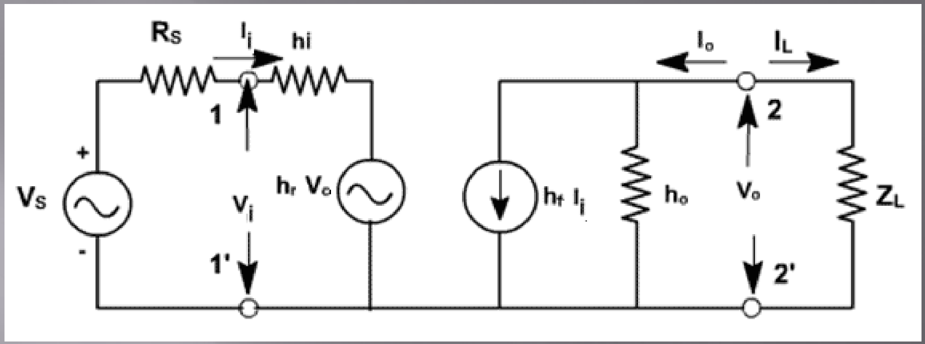
$$\text{or,} \quad I_i = \frac{-h_r V_o}{R_s + h_i} \quad \dots \quad \dots \quad \text{(ivb)}$$

From the output loop of the circuit of Fig. 2,

$$I_o = h_f I_i + h_o V_o = \frac{-h_f h_r V_o}{R_s + h_i} + V_o \quad \dots \quad \dots \quad \text{(ivc)}$$

Hence, $Z_o = \dfrac{V_o}{I_o} = \dfrac{1}{h_o - h_f h_r / (R_s + h_i)}$ … (4) $Y_o = \dfrac{1}{Z_o} = h_o - \dfrac{h_f h_r}{(R_s + h_i)}$ … (4a)

Note that output impedance/admittance is a function of source resistance.



Power Gain or Amplification Factor, A_p:

The average output power delivered to the load is given by:

$$P_L = V_L I_L = V_o I_o$$

The input power supplied by the source is:

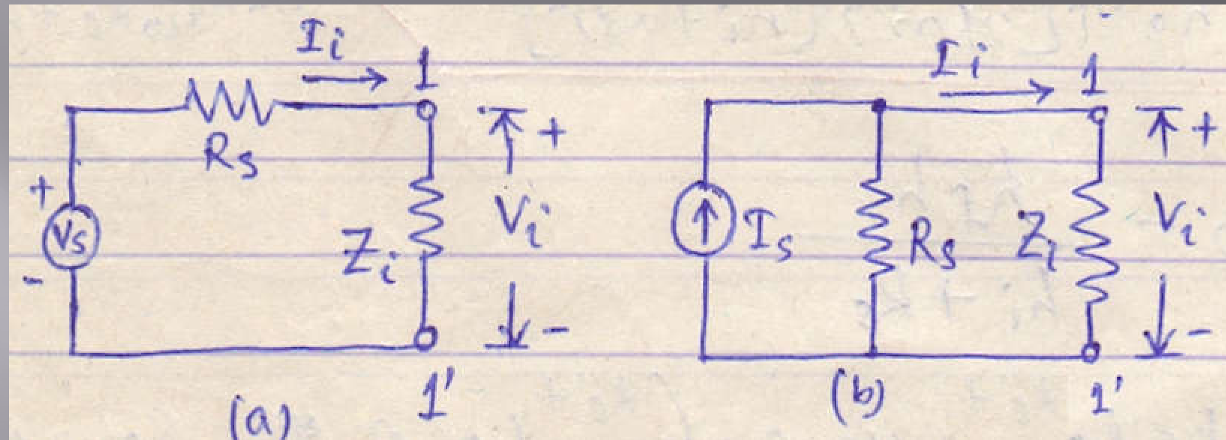
$$P_i = V_i I_i$$

Therefore, power gain A_p is given by:

$$A_p = \frac{V_o I_o}{V_i I_i} = A_v A_p$$

In terms of h-parameters, using (1) and (2)

$$A_p = \frac{h_f^2 Z_L}{(1 + h_o Z_L)[h_i + (h_i h_o - h_f h_r) Z_L]} \quad \dots \quad \dots (5)$$



Voltage Gain or Amplification Factor, A_{vs} (taking in account the Source Resistance R_s):

This overall voltage gain A_{vs} is defined by:

$$A_{vs} = \frac{V_0}{V_s} = \frac{V_0}{V_i}\frac{V_i}{V_s} = A_v \frac{V_i}{V_s} \quad \dots \quad \dots \quad \text{(i)}$$

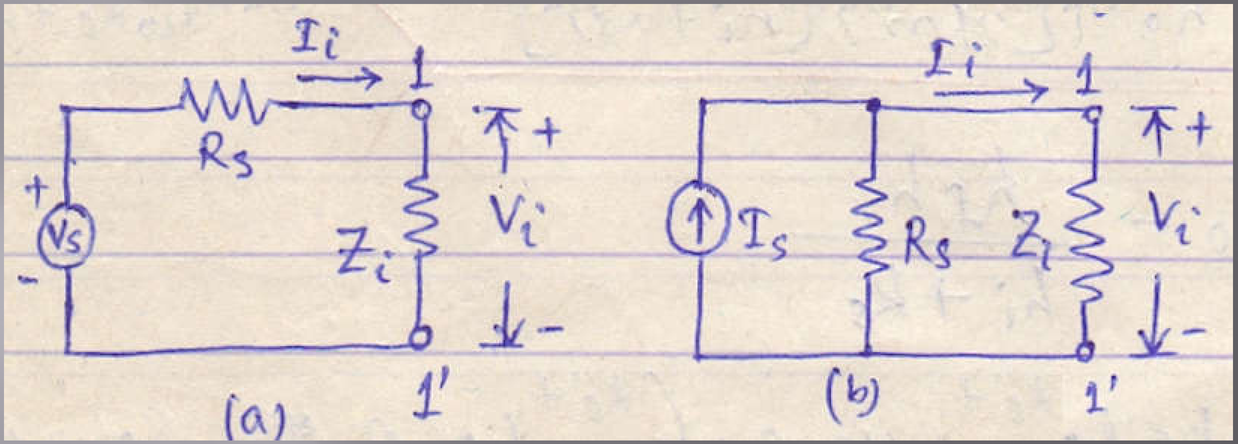
From the equivalent input circuit of the amplifier shown in Fig. 3(a):

$$V_i = Z_i I_i = Z_i \frac{V_s}{Z_i + R_s} \quad \dots \quad \dots \quad \text{(ii)}$$

Hence, $$A_{vs} = A_v \frac{V_i}{V_s} = A_v \frac{Z_i}{Z_i + R_s} = \frac{A_i Z_L}{Z_i} \frac{Z_i}{Z_i + R_s} = \frac{A_i Z_L}{Z_i + R_s} \quad \dots \quad (6)$$ [using (2a)]

For an ideal voltage source, $R_s = 0$, then $A_{vs} = A_v$.

In practice, $R_s \neq 0$, thus, A_{vs} is less than A_v.



Current Gain or Amplification Factor, A_{is} (taking in account the Source Resistance R_s):

Converting the voltage source into its equivalent current source as shown in Fig. 3(b), then this overall current gain A_{is} is defined by:

$$A_{is} = \frac{-I_0}{I_s} = \frac{-I_0}{I_i}\frac{I_i}{I_s} = A_i \frac{I_i}{I_s} \quad \dots \quad \dots \quad \text{(i)}$$

From Fig. 3(b):
$$I_i = \frac{V_i}{Z_i} = \frac{I_s(R_s // Z_i)}{Z_i} = \frac{I_s}{Z_i}\left(\frac{R_s Z_i}{Z_i + R_s}\right) = \frac{I_s R_s}{Z_i + R_s} \quad \dots \quad \dots \quad \text{(ii)}$$

Hence,
$$A_{is} = A_i \frac{I_i}{I_s} = A_i \frac{R_s}{Z_i + R_s} \quad \dots \quad \dots \quad (7)$$

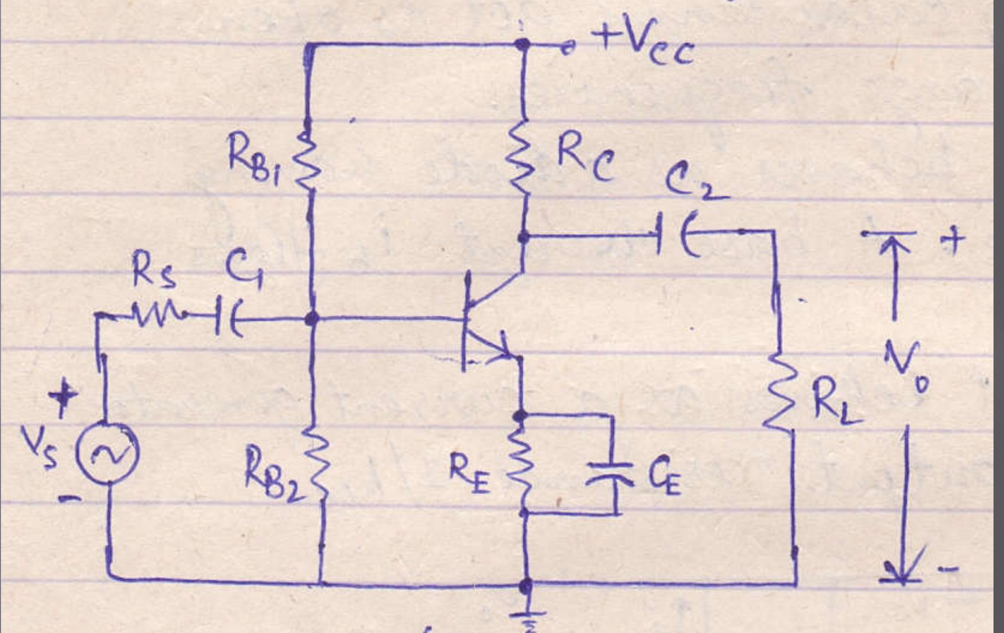
For an ideal current source, $R_s = \infty$, then $A_{is} = A_i$.

Thus, taking source resistance into account, the voltage and current gains are related by

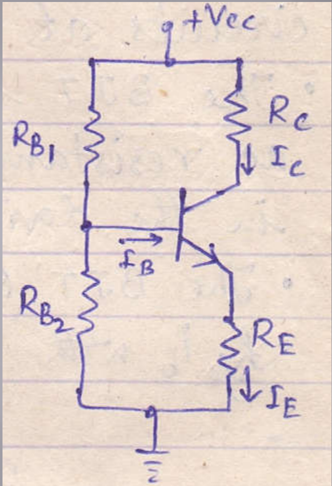
$$A_{vs} = A_{is} \frac{Z_L}{R_s} \quad \dots \quad \dots \quad (8)$$

Analysis of BJT CE Amplifier using *h*-Parameter Model

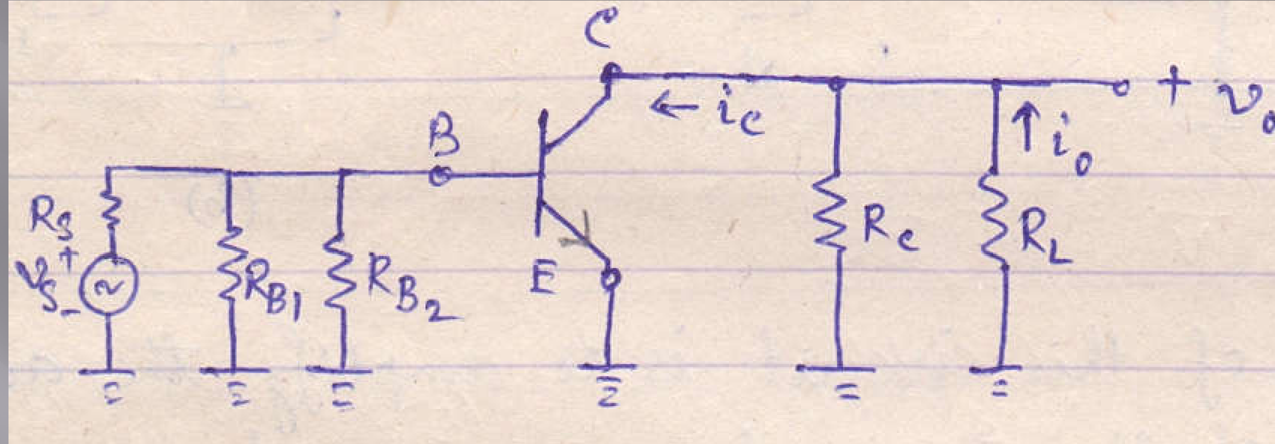
A BJT common-emitter ac amplifier is shown in Fig. (a). Resistor R_s is added to control the input current from the source V_s, and resistor R_L represents the load as seen by the amplifier



(a)



(b)



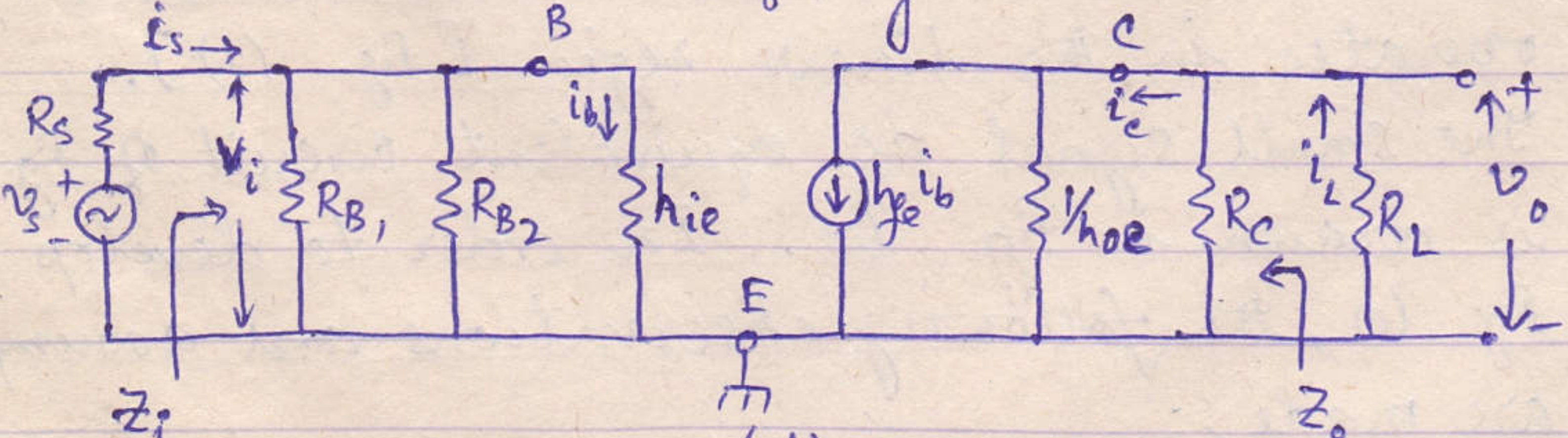
(c)

The purpose of this circuit is to amplify the ac input signal v_s, to produce an ac output signal v_o that is larger in amplitude. The capacitors act as short circuits for the mid-range frequencies. Resistors R_{B1}, R_{B2}, R_C, and R_E provide the dc bias so that the BJT operates in the linear region [Fig. (b)]. The small signal ac equivalent circuit of fig. (a) is drawn in fig. (c). In order to develop

In order to develop Fig. (c), the following observations and assumptions are made:

- Any node whose voltage is constant in time is considered to be ac ground. The resistance of all supplies is assumed negligible with respect to circuit parameters, so power supply nodes are ac ground.
- C_1, C_2, and C_E act as short circuit at mid-range frequencies.
- Device and wiring capacitances act as open circuits at midrange frequencies.
- The BJT input behaves as a diode having ac resistance h_{ie}. A base current i_b flows in the device.
- The BJT output behaves as a current generator $h_{fe} i_b$ with an output resistance $1/h_{oe}$

Substituting the approximate hybrid small-signal equivalent circuit for the transistor of fig. will result in the network of fig.



(d)

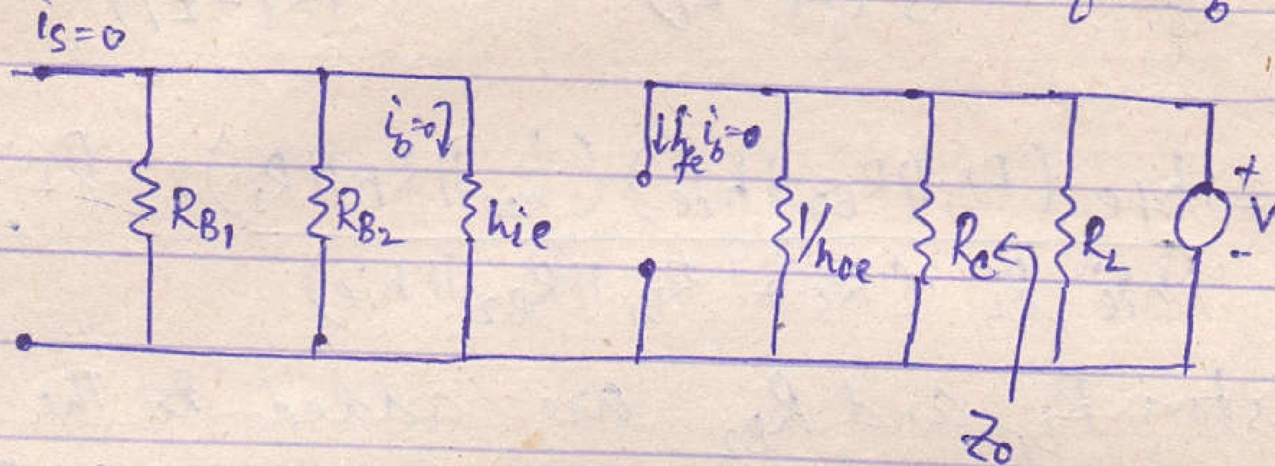
<u>Input impedance Z_i :</u>

The input impedance is given by

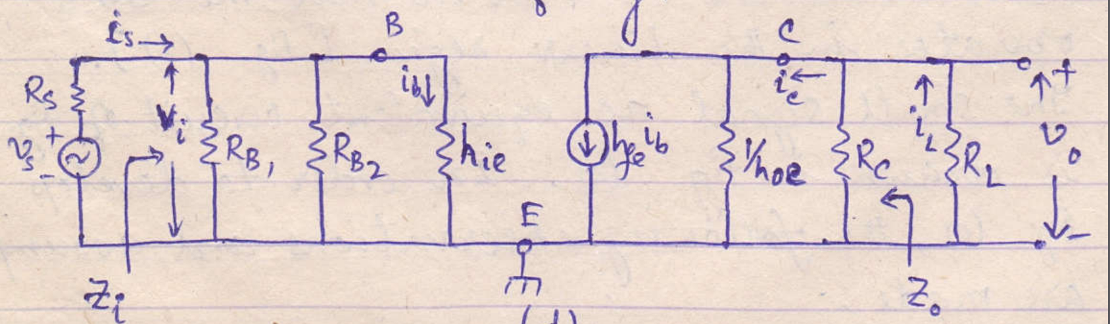
$$Z_i = R_{B_1} \parallel R_{B_2} \parallel h_{ie} \quad \cdots (1)$$

Output impedance Z_o:

When $v_s = 0$, then $i_s = 0$ and therefore $i_b = 0$ and $h_{fe} i_b = 0$.



Thus, $Z_o = \frac{1}{h_{oe}} \| R_c$ ... (2)



(d)

Current Gain A_i :

$$A_i = \frac{i_L}{i_s} = \frac{i_L}{i_b} \cdot \frac{i_b}{i_s}$$

from fig. $v_i = i_s (R_{B_1} \| R_{B_2} \| h_{ie}) = i_b h_{ie}$

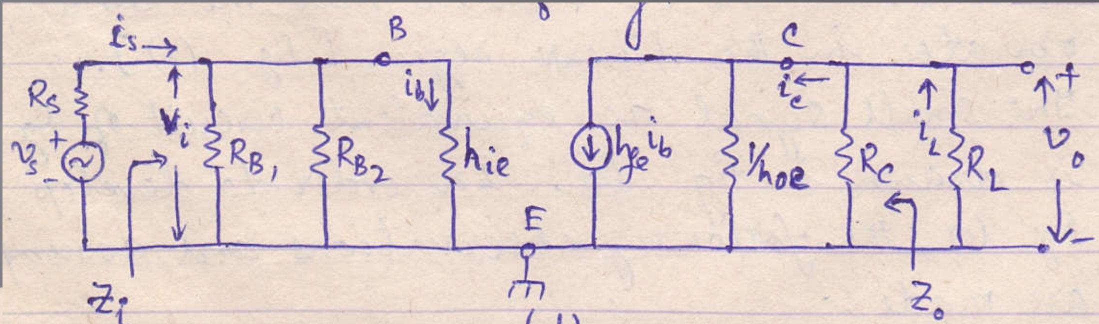
$$\therefore \quad \frac{i_b}{i_s} = \frac{R_{B_1} \| R_{B_2} \| h_{ie}}{h_{ie}}$$

and $\quad -v_o = i_L R_L = h_{fe} i_b (1/h_{oe} \| R_c \| R_L)$

$$\therefore \quad \frac{i_L}{i_b} = \frac{h_{fe} (\frac{1}{h_{oe}} \| R_c \| R_L)}{R_L}$$

So the current gain of the circuit becomes

$$A_i = h_{fe} \frac{(R_{B_1} \| R_{B_2} \| h_{ie})}{h_{ie}} \frac{(\frac{1}{h_{oe}} \| R_c \| R_L)}{R_L} \quad \ldots (3)$$



(d)

Voltage Gain A_v:

From fig.

$$v_o = -i_L R_L = -h_{fe} i_b \left(\frac{1}{h_{oe}} \,||\, R_C \,||\, R_L\right)$$

and $\quad v_s = i_s (R_s + Z_i)$

so the voltage gain of the circuit is given by

$$\therefore A_v = \frac{v_o}{v_s} = -\frac{i_L R_L}{i_s (R_s + Z_i)} = -A_i \frac{R_L}{(R_s + Z_i)}$$

$$\text{or, } A_v = -\frac{h_{fe} (R_{B_1} \,||\, R_{B_2} \,||\, h_{ie}) \left(\frac{1}{h_{oe}} \,||\, R_C \,||\, R_L\right) \cdot \cancel{R_L}}{h_{ie} \cancel{R_L} \,(R_s + R_{B_1} \,||\, R_{B_2} \,||\, h_{ie})} \quad \ldots (4)$$

Note: Resistors R_{B_1} and R_{B_2} are added to the amplifier to improve the thermal stability of the circuit. The following effects are associated with these resistors:

- The input impedance Z_i of the amplifier is reduced.
- The current gain A_i of the amplifier is reduced.
- The voltage gain of the amplifier is approximately unchaned for the case $R_s \ll (R_{B_1} \| R_{B_2} \| h_{ie})$. Otherwise, A_v also is reduced.

For a special case, if R_{B_1} and $R_{B_2} \gg h_{ie}$, and $1/h_{oe}$ and $R_L \gg R_C$, then

$$A_v = -h_{fe} \cdot \frac{R_C}{R_s + h_{ie}} \quad \ldots\ldots (5)$$

Example: A CE amplifier uses a transistor with $h_{ie} = 1100\,\Omega$, $h_{re} = 2.5\times10^{-4}$, $h_{fe} = 50$, and $h_{oe} = 25\times10^{-6}\,S$. If $R_L = 10k$ and $R_s = 1k$, find the various gains and the input and output impedances.

Solution:

$$A_I = -\frac{h_{fe}}{1 + h_{oe}R_L} = -\frac{50}{1+25\times10^{-6}\times10^{4}} = -40$$

$$R_i = h_{ie} + h_{re}A_I R_L = 1100 - 2.5\times10^{-4}\times40\times10^{4} = 1000\,\Omega = 1k$$

$$A_V = \frac{A_I R_L}{R_i} = -\frac{40\times10}{1} = -400$$

$$A_V = \frac{A_I R_L}{R_i} = -\frac{40\times10}{1} = -400$$

$$A_{Vs} = \frac{A_V R_i}{R_i + R_s} = \frac{-400\times1}{1+1} = -200$$

$$A_{Is} = \frac{A_I R_s}{R_i + R_s} = \frac{-40\times1}{1+1} = -20$$

$$Y_o = h_{oe} - \frac{h_{fe}h_{re}}{h_{ie}+R_s} = 25\times10^{-6} - \frac{50\times2.5\times10^{-4}}{1100+1000}$$

$$= 19\times10^{-6}\,S = 19\,\mu A/V$$

$$\text{or, } Z_o = \frac{1}{Y_o} = \frac{10^6}{19}\,\Omega = 52.6K$$

Example

~~The transistor~~ A CE amplifier uses a transistor with $h_{ie} = 1k\Omega$, $h_{re} = 5\times10^{-4}$, $h_{fe} = 100$, and $h_{oe} = 25\times10^{-6}$ S. The load resistance is $5k\Omega$ the source resistance is $1k\Omega$ ~~find the current amplification, and~~ find the various gains and the input and output impedances.

Solution:

$$A_I = -\frac{h_{fe}}{1+h_{oe}Z_L} = -\frac{100}{1+25\times10^{-6}\times5\times10^{3}} = -88.89$$

$$Z_i = h_{ie} + h_{re}A_I Z_L = 10^3 - 88.89\times5\times10^{-4}\times5\times10^{3} = 777.8\Omega$$

$$A_V = A_I\frac{Z_L}{Z_i} = -88.89\times\frac{5\times10^{3}}{777.8} = -$$

$$A_{VS} = \frac{A_V Z_i}{Z_i+R_s} = \frac{A_I Z_L}{Z_i+R_s} = -\frac{88.89\times5\times10^{3}}{10^3+777.8} = -250$$

$$Y_0 = h_{oe} - \frac{h_{fe}h_{re}}{h_{ie}+R_s} = 25\times10^{-6} - \frac{100\times5\times10^{-4}}{5\times10^{3}+1\times10^{3}}$$

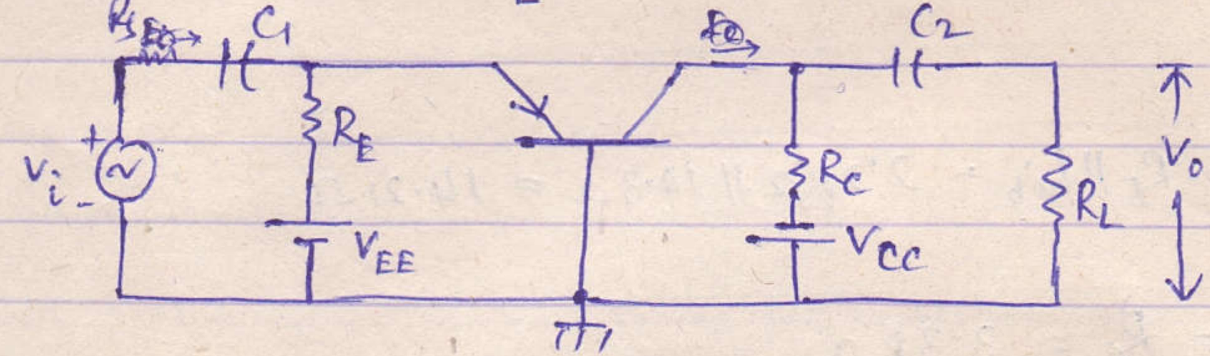
$\cancel{A_{IS} = \frac{A_I R_s}{}}$

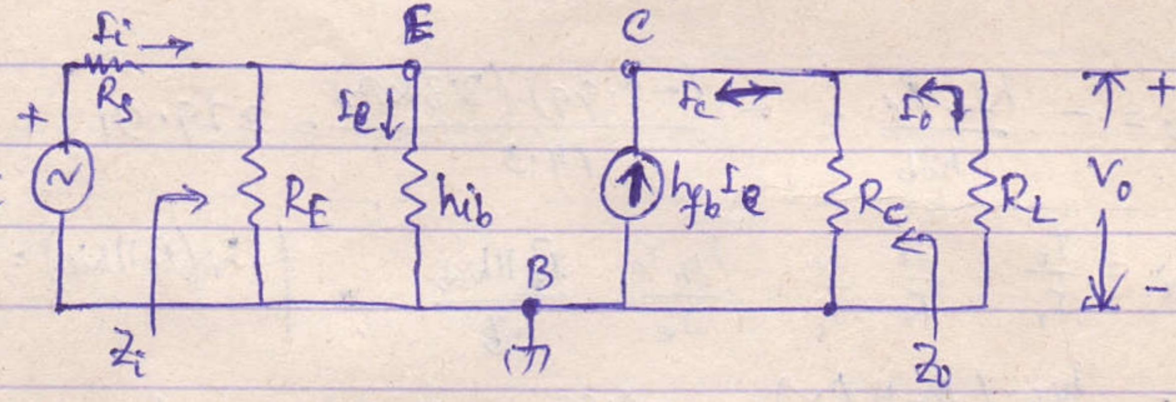
$$Z_0 = \frac{1}{Y_0} =$$

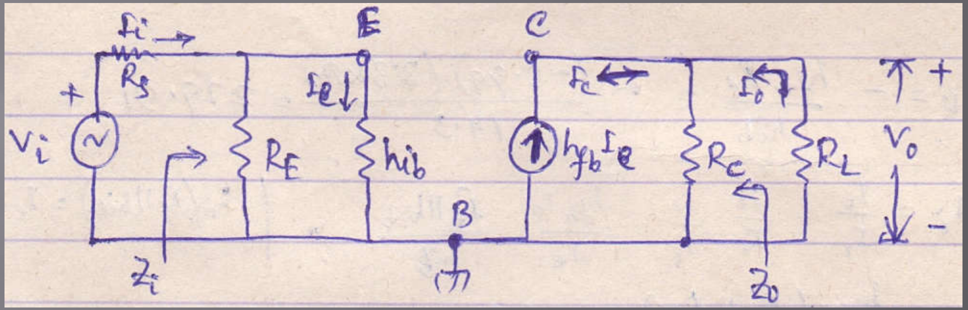
$$A_{P0} = A_{V0}A_{I0} =$$

BJT Common-Base Amplifier

A BJT common-base circuit, its dc portion, and its h-parameter (approximate) ac equivalent are shown in Fig. In this configuration, the output resistance $1/h_{ob}$ is usually sufficiently large to neglect its impact on $R_C \| R_L$.







Z_i : $Z_i = R_E \parallel h_{ib}$

Z_o : $Z_o = R_c$

A_v : $V_o = -I_o R_L = -h_{fe} I_e \,(R_c \parallel R_L)$

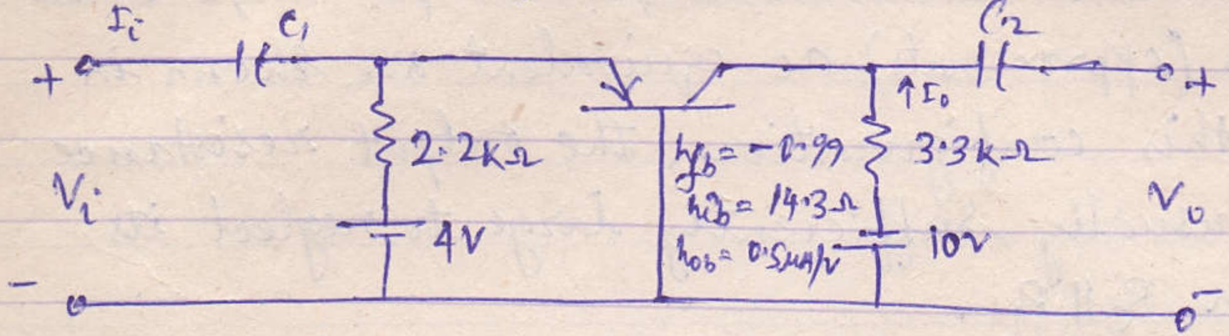
$V_i = I_i (R_s + Z_i) = \dfrac{h_{ib} I_e}{Z_i} (R_s + Z_i)$

$$\therefore \; A_v = \frac{V_o}{V_i} = \frac{-h_{fe}(R_c \parallel R_L)}{h_{ib}} \cdot \frac{Z_i}{(R_s + Z_i)} = -\frac{h_{fe}}{h_{ib}} \left(\frac{Z_i}{R_s + Z_i}\right)(R_c \parallel R_L)$$

A_I : $$A_I = \frac{I_o}{I_i} = \frac{I_o}{I_e} \cdot \frac{I_e}{I_i} = \frac{h_{fe}(R_c \parallel R_L)}{R_L} \cdot \frac{Z_i}{h_{ib}} = \frac{h_{fe}}{h_{ib}} \cdot Z_i \cdot \frac{(R_c \parallel R_L)}{R_L}$$

Example: For the network of Fig. determine:

(a) Z_i ; (b) Z_o ; (c) A_v ; (d) A_i.



Solution: (a) $Z_i = R_E \| h_{ib} = 2.2k\Omega \| 14.3\Omega = 14.21\,\Omega$

(b) $Z_o = R_C = 3.3k\Omega$

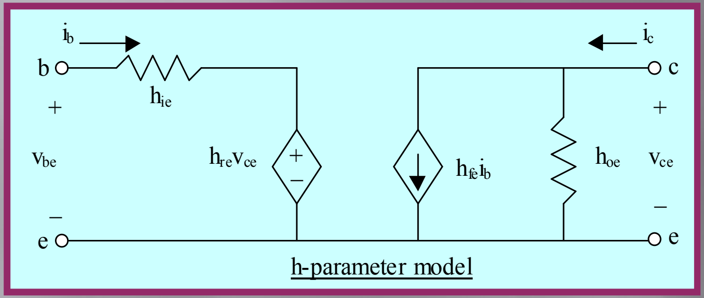
(c) $A_v = -\dfrac{h_{fb} R_C}{h_{ib}} = -\dfrac{(-0.99)(3.3k\Omega)}{14.3} = 229.91$

(d) $A_i = \dfrac{I_o}{I_i} = \dfrac{I_o}{I_e} \cdot \dfrac{I_e}{I_i} = \dfrac{h_{fb} I_e}{I_e} \cdot \dfrac{R_E \| h_{ib}}{h_{ib}}$ | $I_i (R_E \| h_{ie}) = I_e h_{ie}$

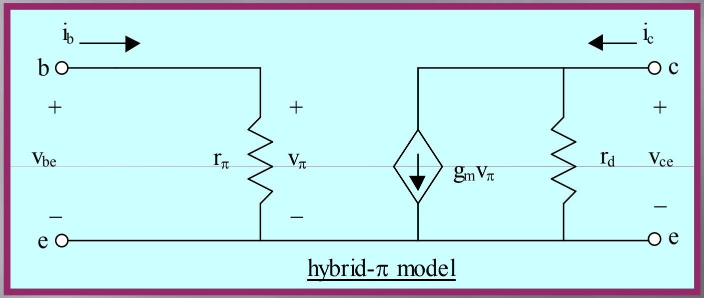
$= \dfrac{h_{fb}}{h_{ib}} (R_E \| R_{ib}) = \dfrac{-0.99}{14.3\Omega} \times 14.21\Omega = -1$

Three Small signal Models of CE Transistor

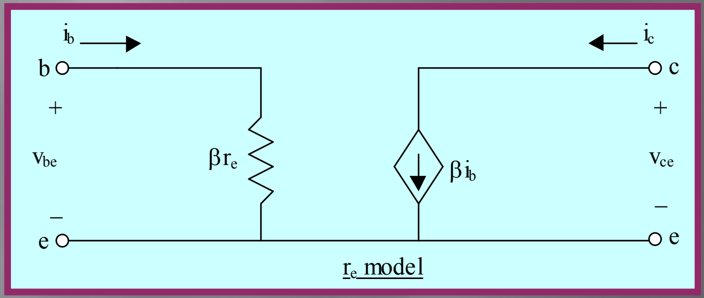
The Mid-frequency small-signal models



h-parameter model



hybrid-π model



r_e model

Alternate names:

$h_{fe} = \beta_{ac} = \beta_o = \beta$

$$g_m = \frac{38.92}{n}|I_C| \quad \text{(Note: Uses DC value of } I_C\text{)}$$

where n = 1 (typical, Si BJT)

$$\beta_o = h_{fe} \qquad r_d = \frac{1}{h_{oe}}$$

$$h_{re} = 0 \qquad r_\pi = h_{ie} = \frac{\beta_o}{g_m}$$

$$r_e = \frac{26\text{ mV}}{I_B} \quad \text{(Note: uses DC value of } I_B\text{)}$$

$$\beta_o = h_{fe}$$

$$\beta_o r_e = h_{ie}$$

$$h_{re} = 0$$

$$h_{oe} = 0, \quad \text{or use } r_d = \frac{1}{h_{oe}}$$

THANK YOU