

# Perceptron Learning

BY

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# Perceptron-Learning

- ▶ The input vectors are allowed to be either binary or bipolar. However, the outputs must be in bipolar form
- ▶ The bias  $w_0$  is adjustable but the threshold  $\theta$  used in the activation function is fixed
- ▶ Learning takes place only when the computed output does not match the target output.
- ▶ The threshold  $\theta$  of the activation function may be interpreted as a separating band of width  $2\theta$  between the region of positive response and negative response.
- ▶ The band separating the regions of positive response from that of the negative response is defined by the pair of lines

$$w_0x_0 + w_1x_1 + w_2x_2 = \theta$$

$$w_0x_0 + w_1x_1 + w_2x_2 = -\theta$$

# Algorithm

## Procedure Perceptron-Learning

- Step 1.** Initialize all weights,  $w_0, \dots, w_m$ .
- Step 2.** Set learning rate  $\eta$  such that  $0 < \eta \leq 1$ , and threshold  $\theta$ .
- Step 3.** For each training pair  $s : t$  do Steps 4-8.
- Step 4.** Activate the input units,  $x_i = s_i$ , for  $i = 0, \dots, m$ .
- Step 5.** Compute the net input to the output unit

$$y\_in = \sum_{i=0}^m w_i x_i$$

- Step 6.** Compute the activation of the output unit using the function

$$y\_out = \begin{cases} 1, & \text{if } y\_in > \theta \\ 0, & \text{if } -\theta \leq y\_in \leq \theta \\ -1, & \text{if } y\_in < -\theta \end{cases}$$

- Step 7.** If there is an error, i.e.,  $y\_out \neq t$ , then adjust the weights as follows

$$w_i \text{ (new)} = w_i \text{ (old)} + \eta \times t \times x_i$$

If, however, no error has occurred, the weights are kept unchanged.

- Step 8.** If there were no error, i.e.,  $y\_out = t$ , for the entire set of training pairs, then stop. Otherwise go to Step 3.

## Perceptron Training Rule

- Weights modified for each example
- Update Rule:

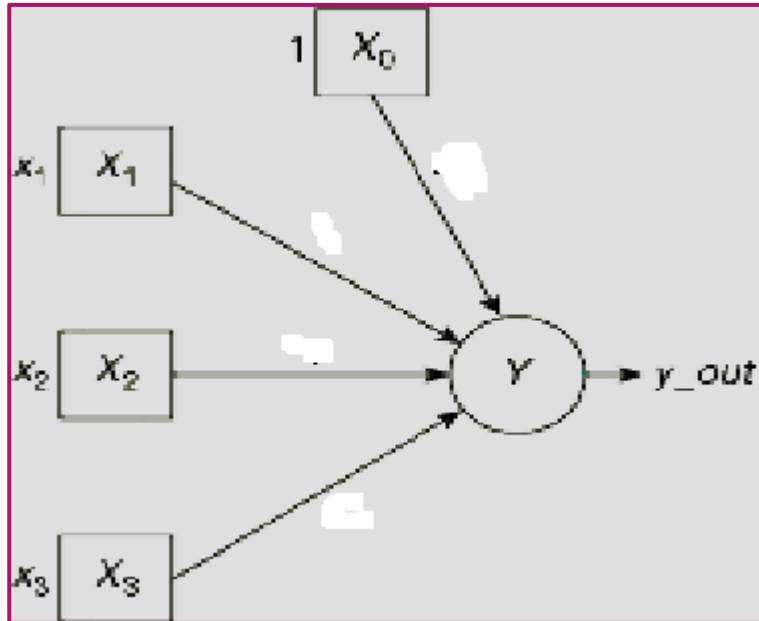
$$w_i \leftarrow w_i + \Delta w_i$$

where

$$\Delta w_i = \eta(t - o)x_i$$

learning rate    target value    perceptron output    input value

# Sample Demonstration



Input pattern			Target output ( $t$ )
$x_1$	$x_2$	$x_3$	
-1	1	1	-1
1	-1	1	-1
1	1	-1	-1
1	1	1	1

the initial weights are all kept at 0 and the learning rate is set to  $\eta = 1$ . Both the inputs and the outputs are presented in bipolar form.

#	Input				Net input	Out- put	Tar- get	Weight adjustments				Weights			
	$x_0$	$x_1$	$x_2$	$x_3$				$\Delta w_0$	$\Delta w_1$	$\Delta w_2$	$\Delta w_3$	$w_0$	$w_1$	$w_2$	$w_3$
0												0	0	0	0
1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1
2	1	-1	1	1	2	1	-1	-1	1	-1	-1	0	2	0	0
3	1	1	-1	1	2	1	-1	-1	-1	1	-1	-1	1	1	-1
4	1	1	1	-1	2	1	-1	-1	-1	-1	1	-2	0	0	0

Epoch #1

#	Input				Net input	Out- put	Tar- get	Weight adjustments				Weights			
	$x_0$	$x_1$	$x_2$	$x_3$				$y_{in}$	$y_{out}$	$t$	$\Delta w_0$	$\Delta w_1$	$\Delta w_2$	$\Delta w_3$	$w_0$
0												-2	0	0	0
1	1	1	1	1	-2	-1	1	1	1	1	1	-1	1	1	1
2	1	-1	1	1	0	0	-1	-1	1	-1	-1	-2	2	0	0
3	1	1	-1	1	0	0	-1	-1	-1	1	-1	-3	1	1	-1
4	1	1	1	-1	0	0	-1	-1	-1	-1	1	-4	0	0	0
Epoch #2															

#	Input				Net input	Out-put	Tar-get	Weight adjustments				Weights			
	$x_0$	$x_1$	$x_2$	$x_3$	$y_{in}$	$y_{out}$	$t$	$\Delta w_0$	$\Delta w_1$	$\Delta w_2$	$\Delta w_3$	$w_0$	$w_1$	$w_2$	$w_3$
0												-4	0	0	0
1	1	1	1	1	-4	-1	1	1	1	1	1	-3	1	1	1
2	1	-1	1	1	-2	-1	-1	0	0	0	0	-3	1	1	1
3	1	1	-1	1	-2	-1	-1	0	0	0	0	-3	1	1	1
4	1	1	1	-1	-2	-1	-1	0	0	0	0	-3	1	1	1

Epoch #3

#	Input				Net input	Out-put	Tar-get	Weight adjustments				Weights			
	$x_0$	$x_1$	$x_2$	$x_3$	$y_{in}$	$y_{out}$	$t$	$\Delta w_0$	$\Delta w_1$	$\Delta w_2$	$\Delta w_3$	$w_0$	$w_1$	$w_2$	$w_3$
0												-3	1	1	1
1	1	1	1	1	0	0	1	1	1	1	1	-2	2	2	2
2	1	-1	1	1	0	0	-1	-1	1	-1	-1	-3	3	1	1
3	1	1	-1	1	0	0	-1	-1	-1	1	-1	-4	2	2	0
4	1	1	1	-1	0	0	-1	-1	-1	-1	1	-5	1	1	1

Epoch #4



#	Input				Net input	Out-put	Tar-get	Weight adjustments				Weights			
	$x_0$	$x_1$	$x_2$	$x_3$	$y_{in}$	$y_{out}$	$t$	$\Delta w_0$	$\Delta w_1$	$\Delta w_2$	$\Delta w_3$	$w_0$	$w_1$	$w_2$	$w_3$
0												-5	1	1	1
1	1	1	1	1	-2	-1	1	1	1	1	1	-4	2	2	2
2	1	-1	1	1	-2	-1	-1	-1	1	-1	-1	-4	2	2	2
3	1	1	-1	1	-2	-1	-1	-1	-1	1	-1	-4	2	2	2
4	1	1	1	-1	-2	-1	-1	-1	-1	-1	1	-4	2	2	2

Epoch #5

#	Input				Net input	Out- put	Tar- get	Weight adjustments				Weights				
	$x_0$	$x_1$	$x_2$	$x_3$				$y_{in}$	$y_{out}$	$t$	$\Delta w_0$	$\Delta w_1$	$\Delta w_2$	$\Delta w_3$	$w_0$	$w_1$
0													-4	2	2	2
1	1	1	1	1	2	1	1	0	0	0	0	-4	2	2	2	2
2	1	-1	1	1	-2	-1	-1	0	0	0	0	-4	2	2	2	2
3	1	1	-1	1	-2	-1	-1	0	0	0	0	-4	2	2	2	2
4	1	1	1	-1	-2	-1	-1	0	0	0	0	-4	2	2	2	2
Epoch #6																

It will be Continued....