Capacitance of the metal-oxide-semiconductor (MOS) structure

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Si based MOS is the key

As a material

Si is abundant in nature

↓

High quality native oxide (SiO_2)

↓

Appropriate mechanical strength

Market

Microelectronics market

↓

80% is dominated by CMOS

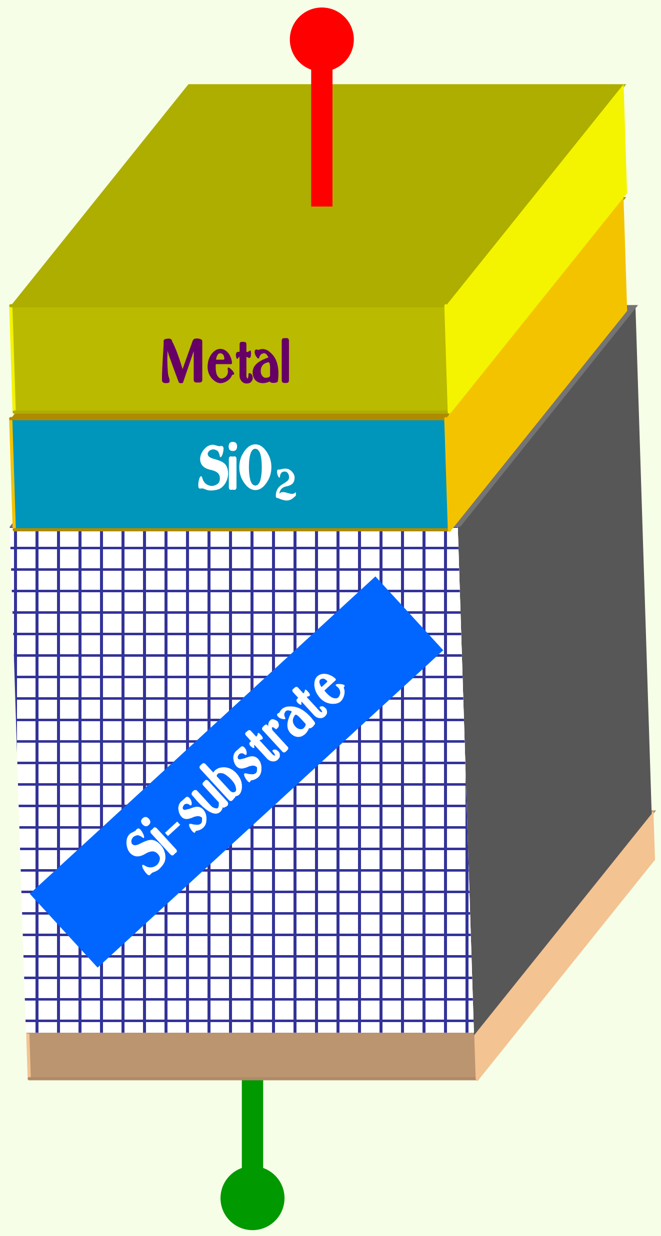
↓

97% is covered by Si

Applications:

Memory and Storages; Sensors; Diagnostics; Almost in all types of electronic circuits; Filters; Etc. ....

Metal-Oxide-Semiconductor (MOS) structure

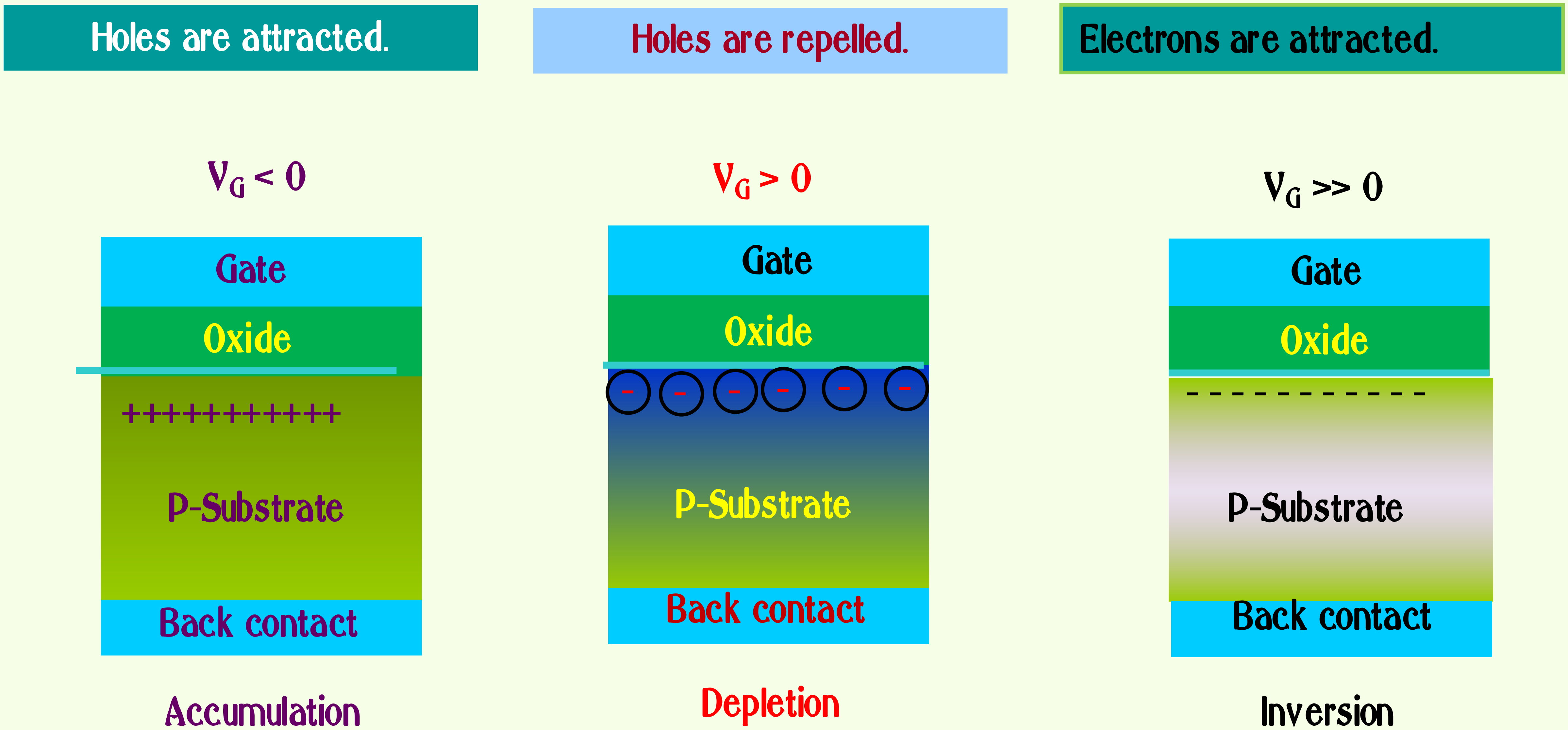


MOS structure

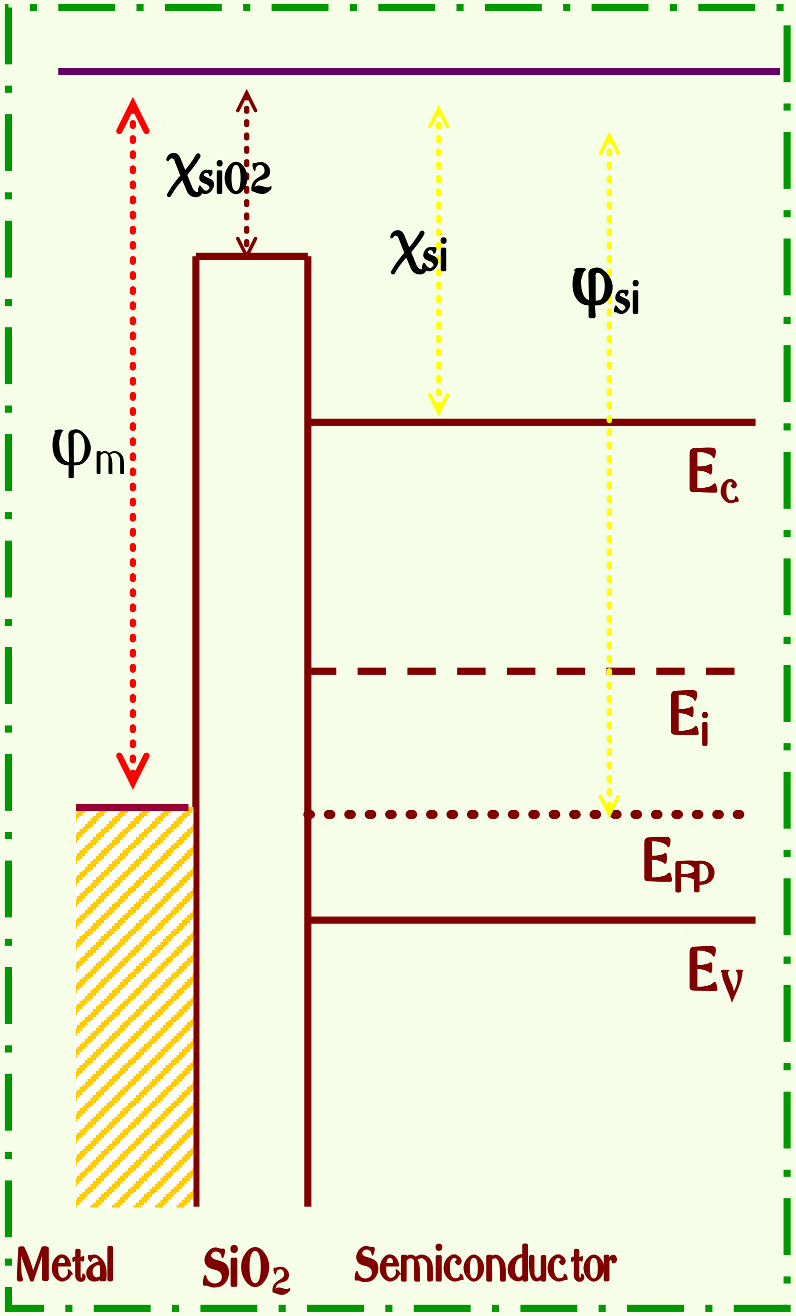
Schematic view:

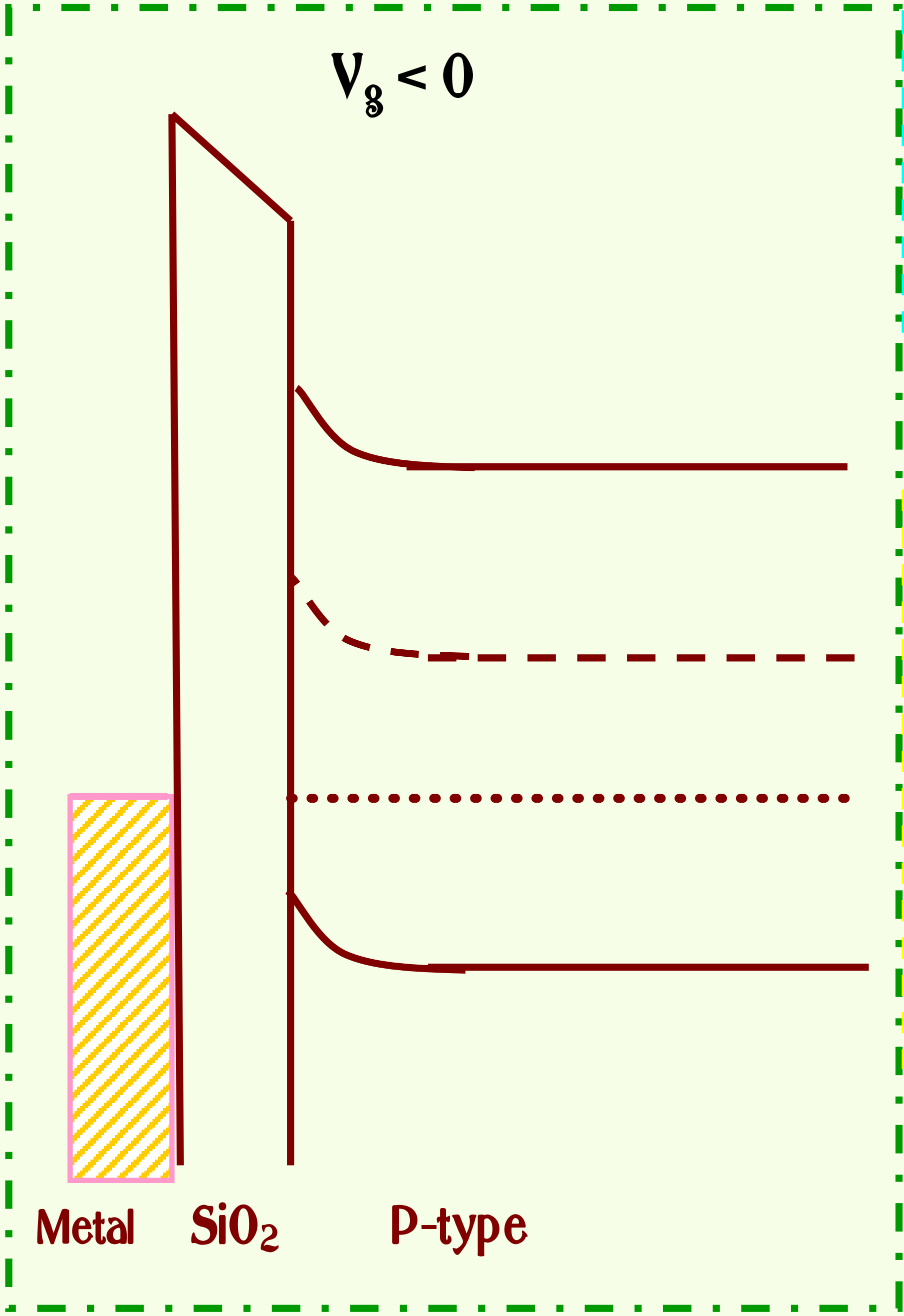
- A Si substrate (p- or n-type)
- SiO_2 is grown on it (@800 - 900^{0}C)
- Ohmic contacts are taken from the top and bottom
- Sometimes, A Poly-Si layer is grown on SiO_2, on which gate ohmic contact is taken.

Distribution of charges: different biases



Band diagram in MOS





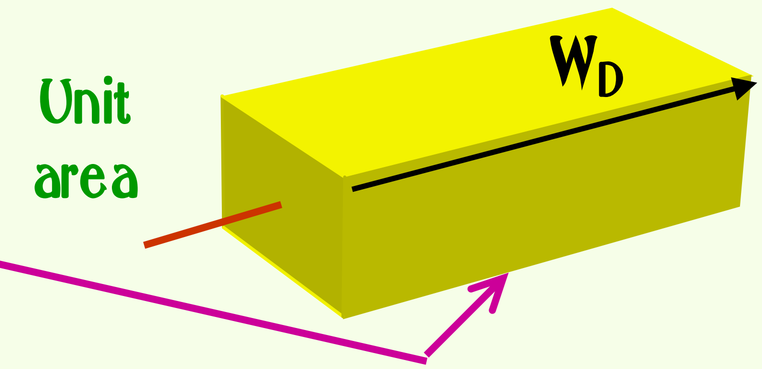
When the band bends downward, $\psi_s > 0$; and when it bends upward, $\psi_s < 0$.

- For $\psi_s<0$, accumulation of holes (band bends upward) for p-type substrate.
- For $\psi_s > 0$, accumulation of electrons for n-type substrate.

Graphical variation of potential, field, charge

- For charge neutrality of the system, it is required that,

$$Q_M = -(Q_n + \boxed{qN_A W_D}) = -Q_s$$



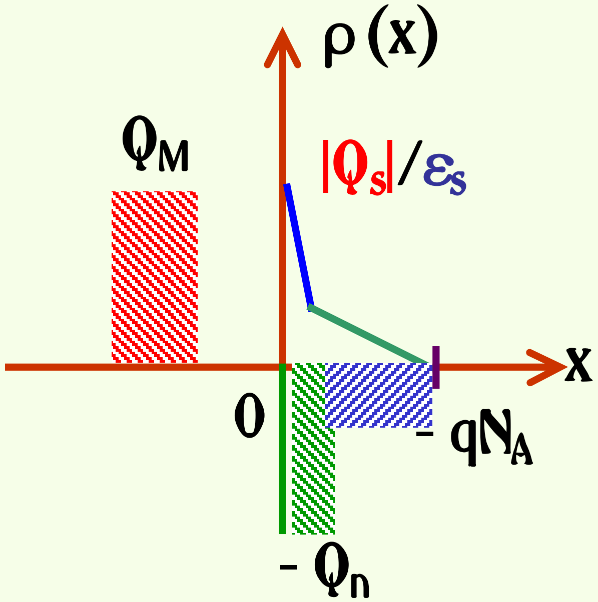
where

- Q_M: charges/unit area on the metal,
- Q_n: electrons/unit area near the surface (inversion region)
- qN_AW_D: the ionized acceptors/unit area in the space-charge region with depletion width
- Q_s: total charges/unit area in the semiconductor.
- Clearly, in the absence of any work-function difference, the applied voltage will partly appear across the insulator and partly across the semiconductor. Thus,

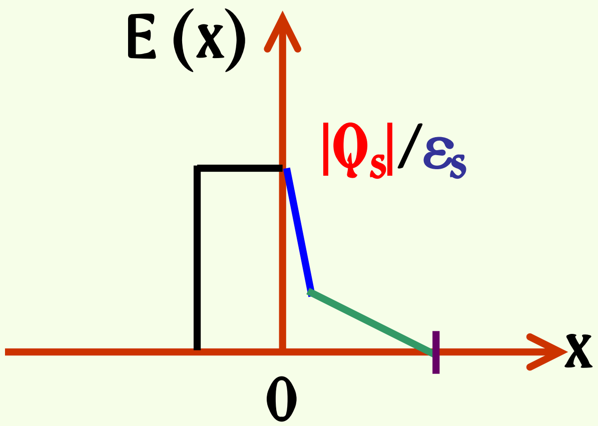
$$V = V_{ox} + \psi_s$$

where V_i is the potential across the insulator and is given by

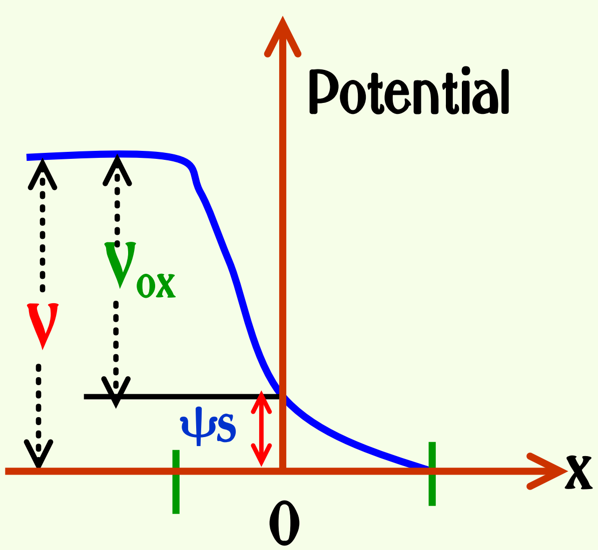
Graphical variation of potential, field, charge

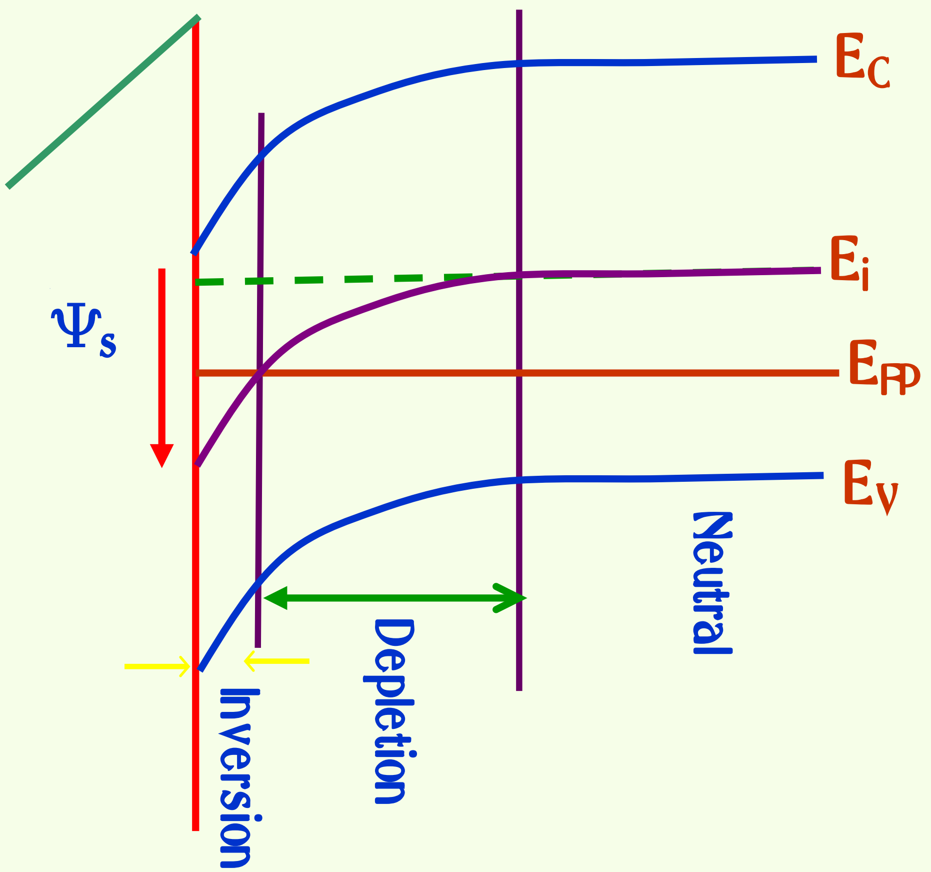


Charge distribution.



Electric field distribution

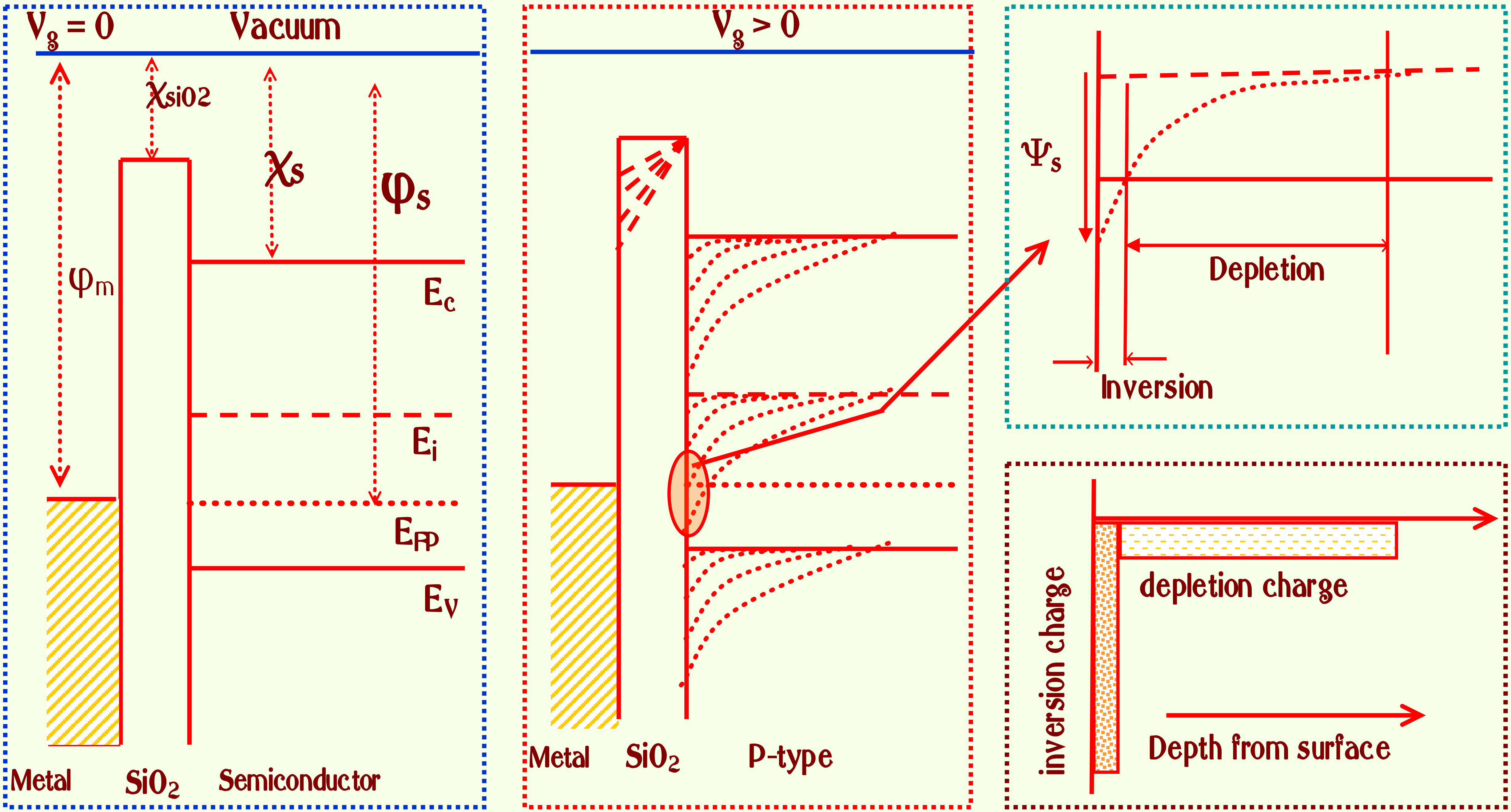




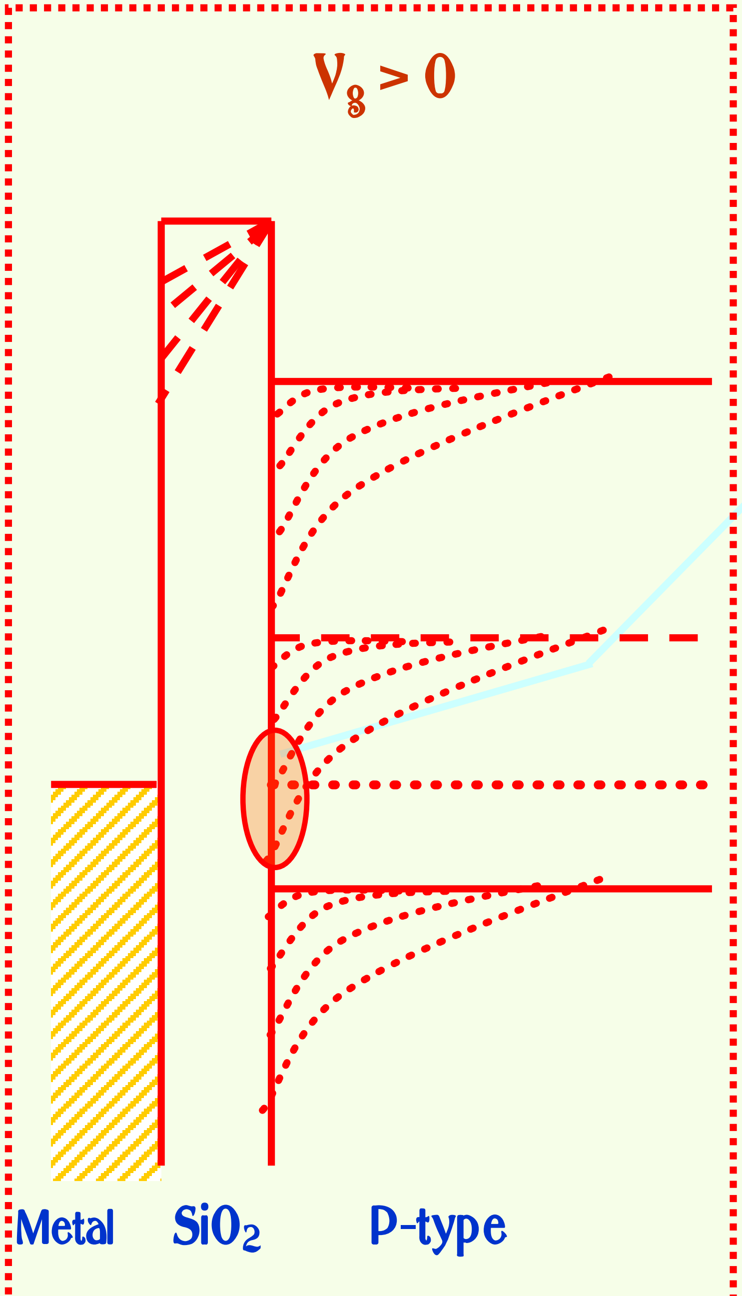
Band diagram

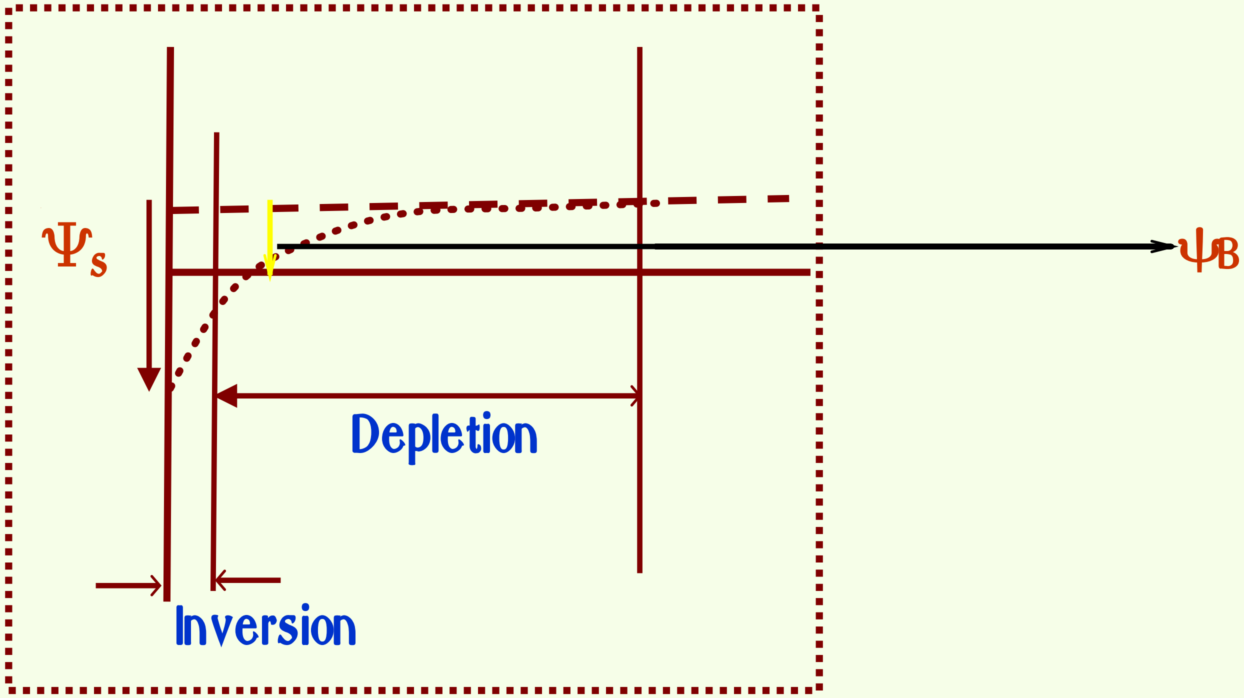
The electric field and potential can be obtained by I[st] and 2[nd] integrations of Poisson's equation, respectively..

How does an inversion layer form?



Band diagram at inversion condition





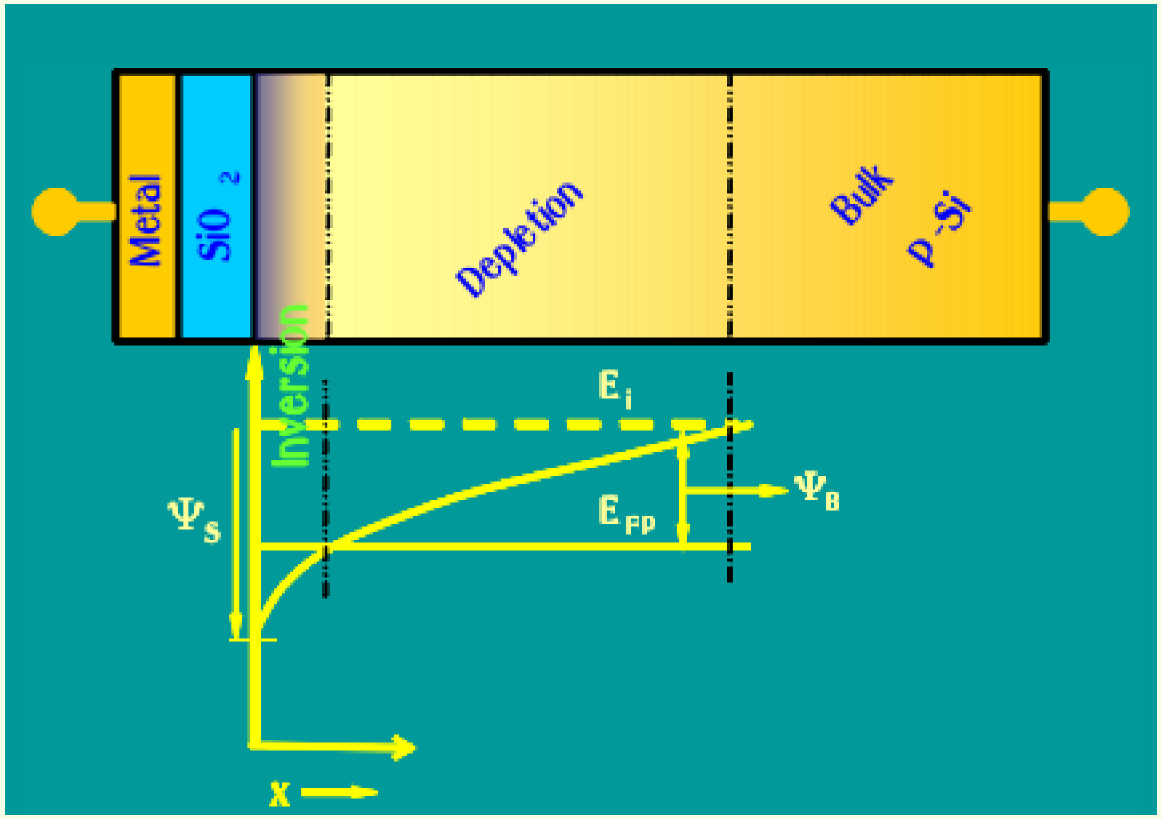
$\psi_s=0$	Flat-band condition
$\psi_B>\psi_s>0$	Depletion of holes (bands bending downward)
$\psi_s=\psi_B$	Fermi-level at mid-gap, $E_F=E_i(0)$, $n_p(0)=p_p(0)=n_i$
$2\psi_B>\psi_s>\psi_B$	Weak inversion (electron enhancement)
$\psi_s>2\psi_B$	Strong inversion [$n_p(0)=p_{p0+}$ or N_A

Derivation: space charge

Redistribution of carriers due to the application of bias:

$$n_p(x) = n_{po} e^{\frac{q\psi(x)}{kT}} = n_{po} e^{\beta\psi(x)}$$

$$p_p(x) = n_{po} e^{-\frac{q\psi(x)}{kT}} = n_{po} e^{-\beta\psi(x)}$$



Where ψ_s is '+' when the band is bent downward, n_{po} and p_{po} are the equilibrium densities of electrons and holes, respectively, in the bulk of the semiconductor, and

$$\beta = q/kT$$

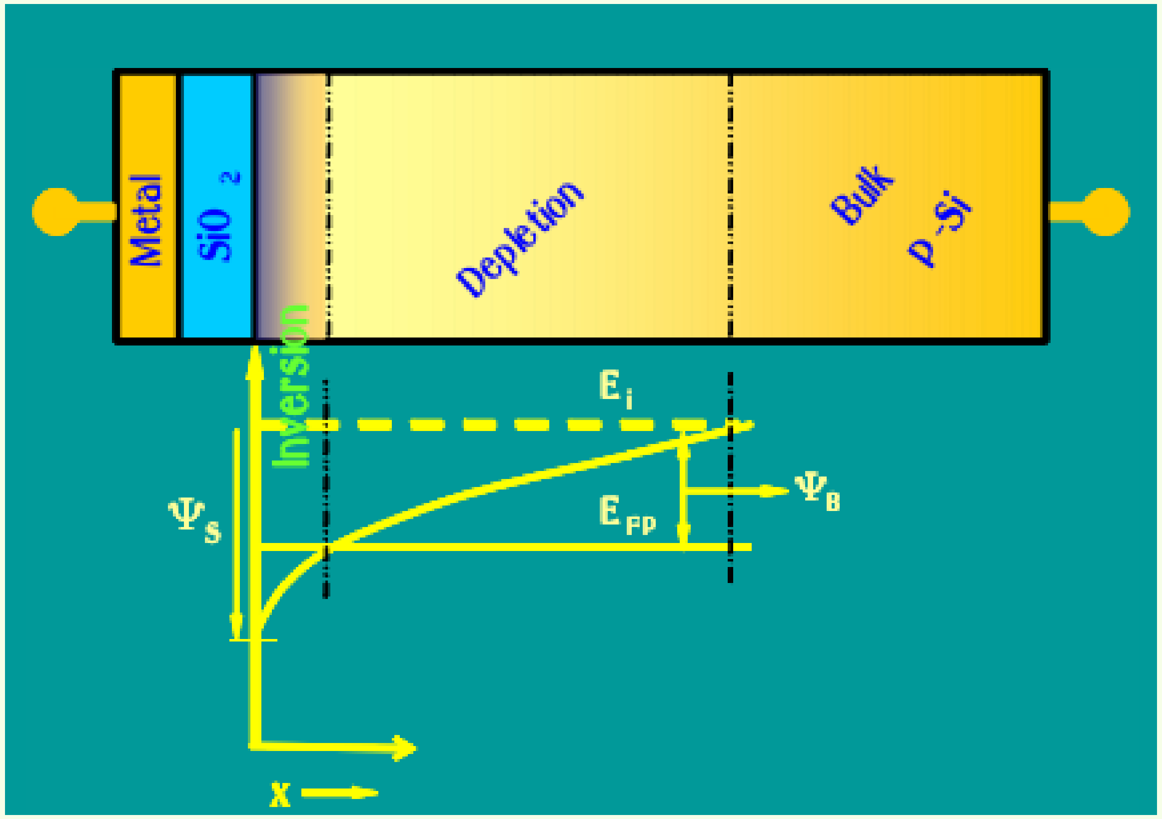
At the surface the densities are:

$$n_p(0) = n_{po} e^{\beta\psi_s}$$

$$p_p(0) = p_{po} e^{-\beta\psi(x)}$$

Derivation: space charge

From the previous discussions and with the help of the above equations, the following of surface potential can be distinguished:



$\psi_s < 0$: Accumulation of holes (band bends downward)

$\psi_s = 0$: Flat-band voltage

$\psi_{BP} > \psi_s > 0$: Depletion of holes (band bends downward)

$\psi_s = \psi_{BP}$: Fermi-level at midgap, $E_F = E_i(0); n_p(0) = p_p(0) = n_i$.

$2\psi_{BP} > \psi_s > \psi_{BP}$: Weak inversion

$\psi_s > 2\psi_{BP}$: Strong inversion

Derivation: space charge

The potential $\psi(x)$ as a function of distance can be obtained by using the one-dimensional Poisson's equation:

$$\frac{d^2\psi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon_s}$$

Where $\rho(x)$ is the total space-charge density given by:

$$\rho(x) = q\left(N_D^+ - N_A^- + p_p - n_p\right)$$

N_D^+ : ionized donors density

N_A^- : ionized acceptor density

Now, in the bulk of the semiconductor, far from the surface, charge neutrality must exist.

Therefore at: $\psi_p(\infty) = 0$, we have $\rho(x) = 0$ and

Derivation: space charge

$$N_D^+ - N_A^- + p_{po} - n_{po} = 0; \text{ or, } N_D^+ - N_A^- = n_{po} - p_{po}$$

Now, $n_p(0) = n_{po} e^{\beta\psi_s}$ and, $p_p(0) = p_{po} e^{\beta\psi(x)}$

$$\therefore \frac{d^2\psi(x)}{dx^2} = -\frac{q}{\varepsilon_s}\left(n_{po} - p_{po} + p_p - n_p\right)$$

$$\therefore \frac{d^2\psi(x)}{dx^2} = \frac{q}{\varepsilon_s}\left(p_{po} - p_p + n_p - n_{po}\right)$$

$$\therefore \frac{d^2\psi(x)}{dx^2} = \frac{q}{\varepsilon_s}\left(p_{po} - p_{po}e^{-\beta\psi_s} + n_{po}e^{\beta\psi_s} - n_{po}\right)$$

$$\therefore \frac{d^2\psi(x)}{dx^2} = \frac{q}{\varepsilon_s}\left[\left(p_{po}(1 - e^{-\beta\psi_s}) + n_{po}(e^{\beta\psi_s} - 1)\right)\right] = \frac{qp_{po}}{\varepsilon_s}\left[\frac{n_{po}}{p_{po}}(e^{\beta\psi_s} - 1) + (1 - e^{-\beta\psi_s})\right]$$

Derivation: space charge

$$\text{or, } \frac{d}{d\psi}\left(\frac{d\psi}{dx}\right).\left(\frac{d\psi}{dx}\right) = \frac{qp_{po}}{\varepsilon_s}\left[\frac{n_{po}}{p_{po}}(e^{\beta\psi_s}-1)+(1-e^{-\beta\psi_s})\right]$$

$$\text{or, } \int_0^{\frac{d\psi}{dx}}\left(\frac{d\psi}{dx}\right).d\left(\frac{d\psi}{dx}\right) = \frac{qp_{po}}{\varepsilon_s}\int_0^{\psi}\left[\frac{n_{po}}{p_{po}}(e^{\beta\psi_s}-1)+(1-e^{-\beta\psi_s})\right]d\psi$$

$$\text{or, } \frac{1}{2}.\left(\frac{d\psi}{dx}\right)^2 = \frac{qp_{po}}{\varepsilon_s}\left[\frac{n_{po}}{p_{po}}\left(\frac{1}{\beta}e^{\beta\psi_s}-\psi\right)+\left(\psi-\frac{1}{\beta}e^{-\beta\psi_s}\right)\right]_0^{\psi}$$

$$\text{or, } \left(\frac{d\psi}{dx}\right)^2 = \frac{2qp_{po}}{\beta\varepsilon_s}\left[\frac{n_{po}}{p_{po}}\left(e^{\beta\psi_s}-\beta\psi-1\right)+\left(e^{-\beta\psi_s}+\beta\psi-1\right)\right]$$

$$\text{or, } \frac{d\psi}{dx} = \sqrt{\frac{2p_{po}kT}{\varepsilon_s}}.\sqrt{\frac{n_{po}}{p_{po}}\left(e^{\beta\psi_s}-\beta\psi-1\right)+\left(e^{-\beta\psi_s}+\beta\psi-1\right)}$$

Derivation: space charge

Therefore, the surface field of the MOS structure, $E_s = -\frac{d\psi}{dx}$

From the surface field, we can deduce the total space charge per unit area by applying Gauss's law:

$$Q_s = -E_s \varepsilon_s = \varepsilon_s . \sqrt{\frac{2 p_{po} kT}{\varepsilon_s}} . \sqrt{\frac{n_{po}}{p_{po}} \left(e^{\beta\psi_s} - \beta\psi - 1\right) + \left(e^{-\beta\psi_s} + \beta\psi - 1\right)}$$

$$\text{or, } Q_s = \sqrt{2\varepsilon_s p_{po} kT} . \sqrt{\frac{n_{po}}{p_{po}} \left(e^{\beta\psi_s} - \beta\psi - 1\right) + \left(e^{-\beta\psi_s} + \beta\psi - 1\right)}$$

$$\text{or, } Q_s = \sqrt{2\varepsilon_s p_{po} kT} . F\left(\beta\psi, \frac{n_{po}}{p_{po}}\right)$$

$$\text{where, } F\left(\beta\psi, \frac{n_{po}}{p_{po}}\right) = \sqrt{\frac{n_{po}}{p_{po}} \left(e^{\beta\psi_s} - \beta\psi - 1\right) + \left(e^{-\beta\psi_s} + \beta\psi - 1\right)}$$

Derivation: space charge

A typical variation of space charge density Qs will be as follows:

i). ψ is negative: Q_s is positive $\rightarrow$ corresponds to accumulation and function F is dominated by the first term:

$$\Longrightarrow Q_s \propto e^{\frac{q|\psi_s|}{2kT}}$$

ii). $\psi = 0$: $Q_s = 0$ $\rightarrow$ corresponds to flatband condition.

iii). For $2\psi_B > \psi_S > 0$: Q_s is negative and we get depletion and weak inversion condition. The function F is now dominated by the second term.

$$\Longrightarrow Q_s \propto \sqrt{\psi_s}$$

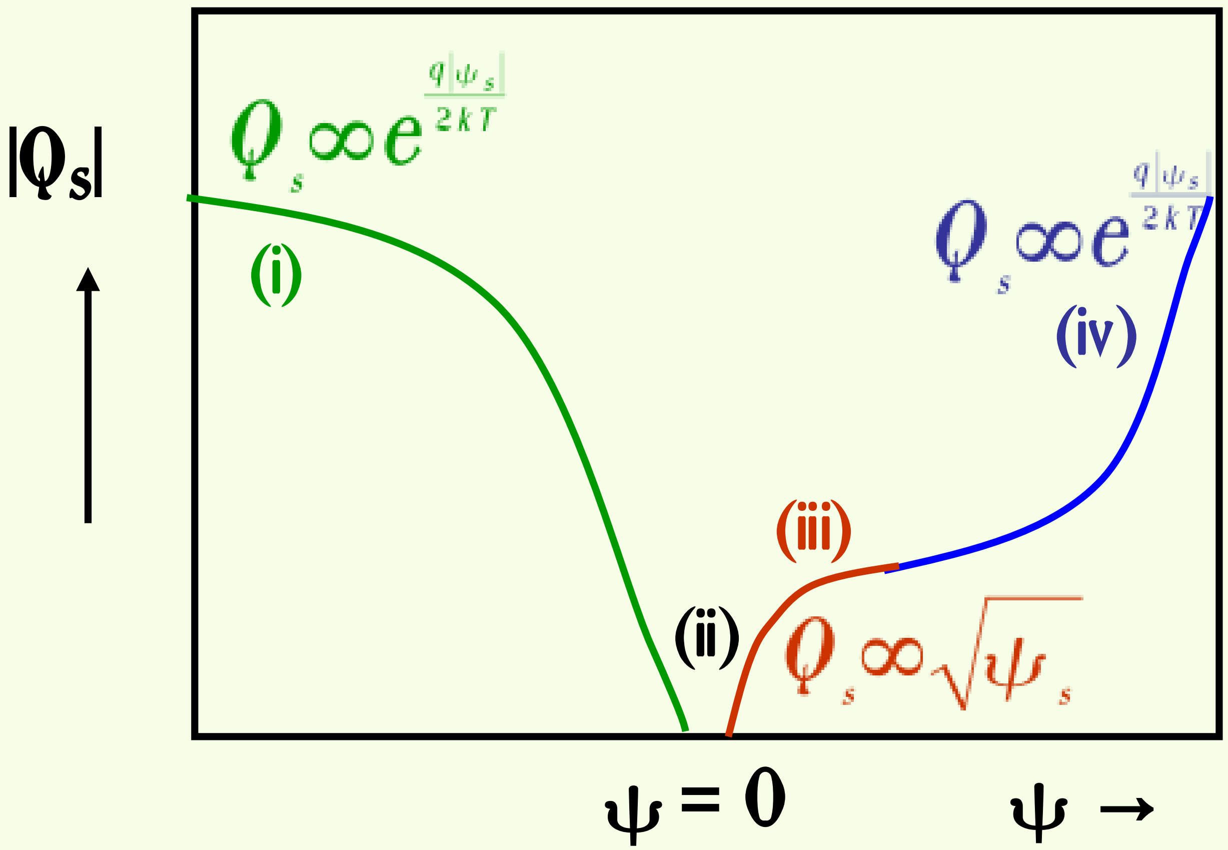
iv). For $\psi_S > 2\psi_B$, we will have strong inversion condition with F dominated by the fourth term.

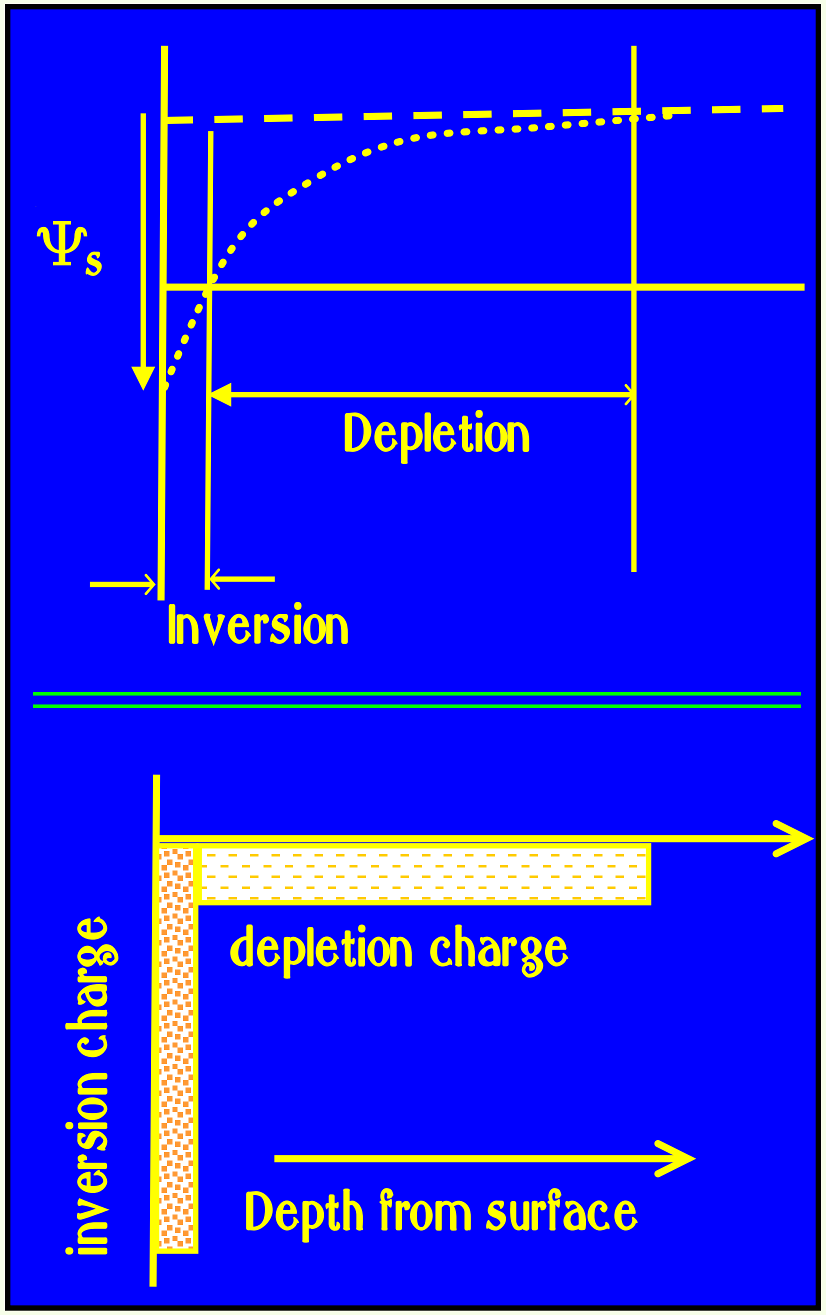
$$\Longrightarrow Q_s \propto e^{\frac{q|\psi_s|}{2kT}}$$

Derivation: space charge

- Also note that this strong inversion begins at a surface potential,

$$\psi_s(\text{strong inversion}) \approx 2\psi_p = \frac{2kT}{q}\ln\left(\frac{N_A}{n_i}\right)$$

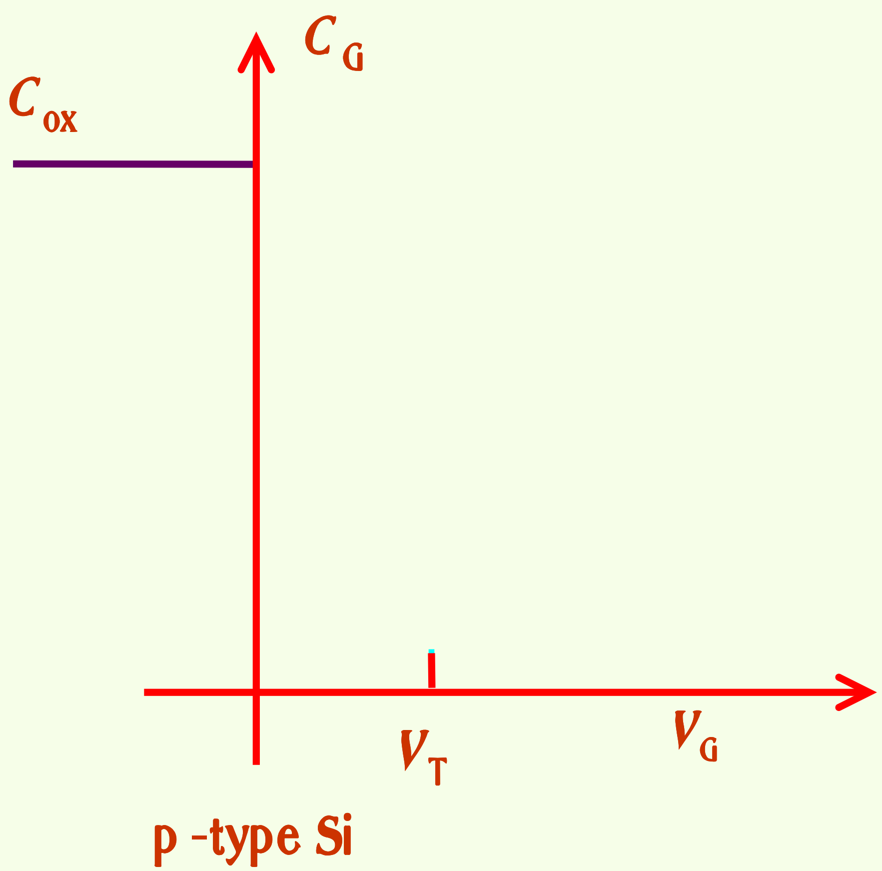


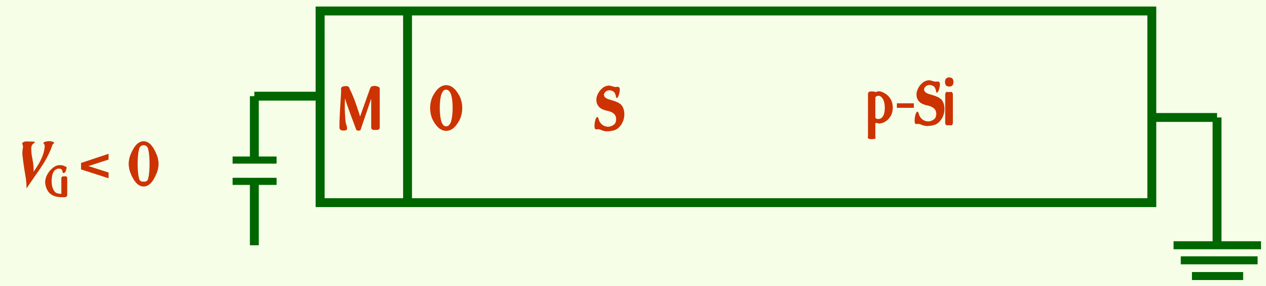


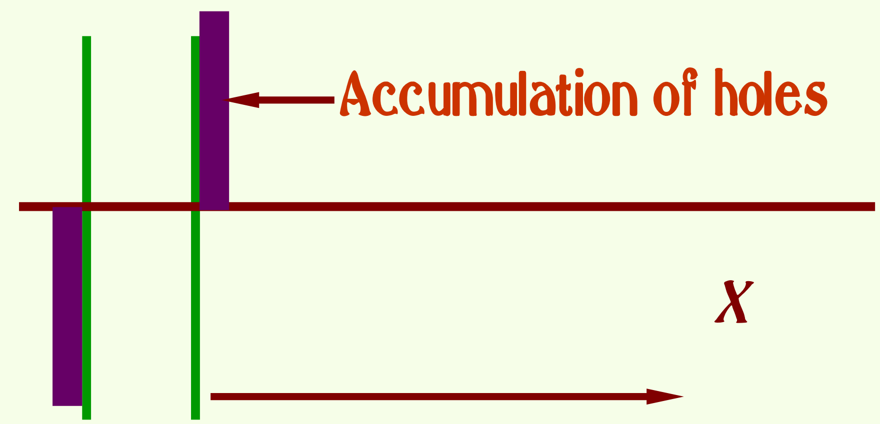
MOS - capacitor under accumulation

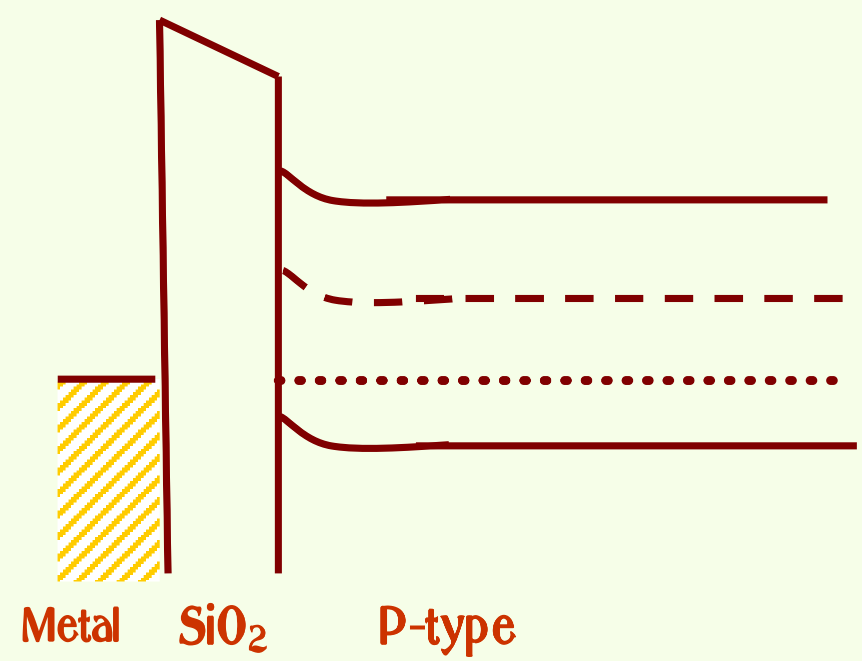
$V_G < 0;\ C_G = C_{ox}$

$$\text{where, } C_{ox} = \frac{\varepsilon_{ox} A}{T_{ox}}$$







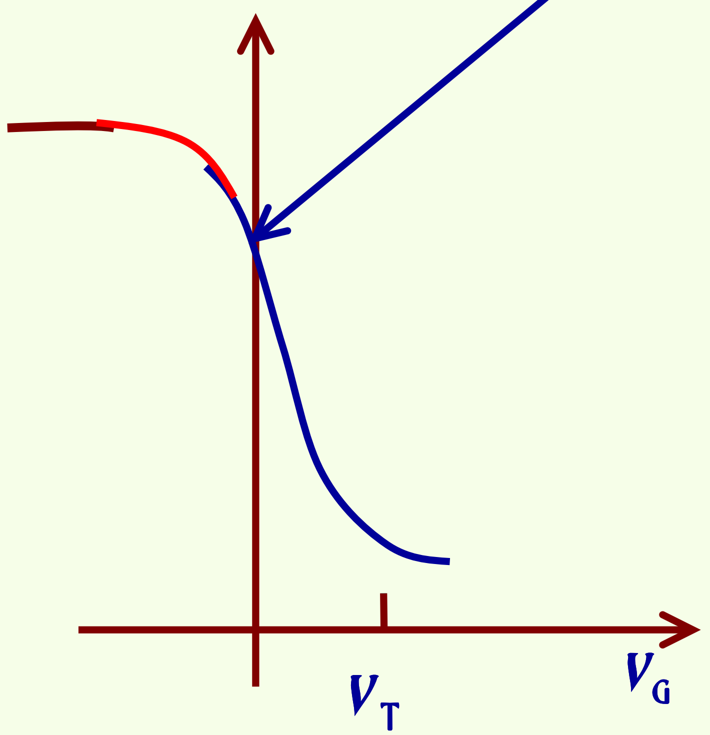


MOS capacitor under depletion

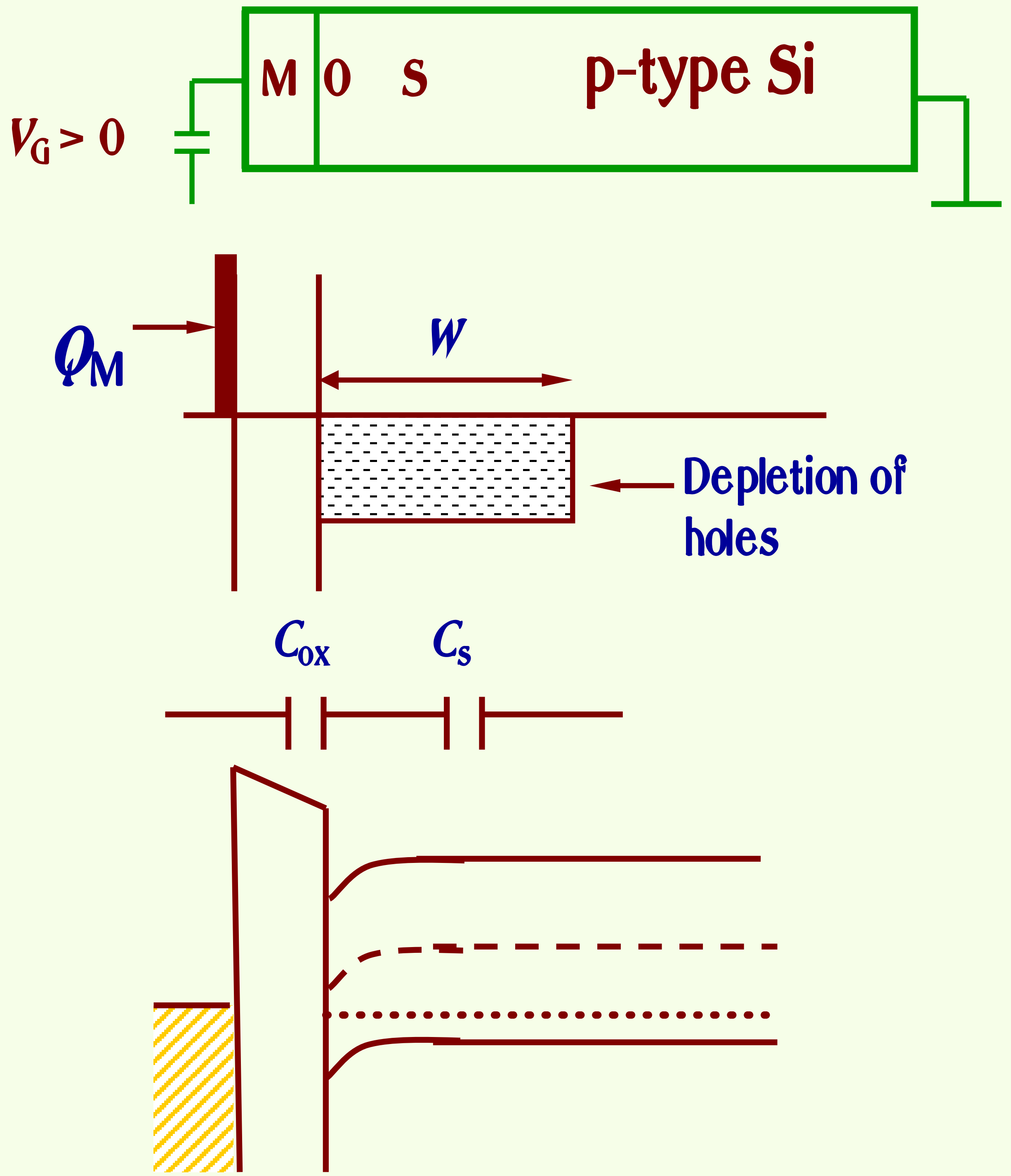
- Depletion condition: $V_G > 0$

$$C_{ox} = \frac{\varepsilon_{ox} A}{T_{ox}} \qquad C_S = \frac{\varepsilon_S A}{W}$$

$$\frac{1}{C_G} = \frac{1}{C_{ox}} + \frac{1}{C_S} \Rightarrow C_G = \frac{C_{ox} C_S}{C_{ox} + C_S}$$

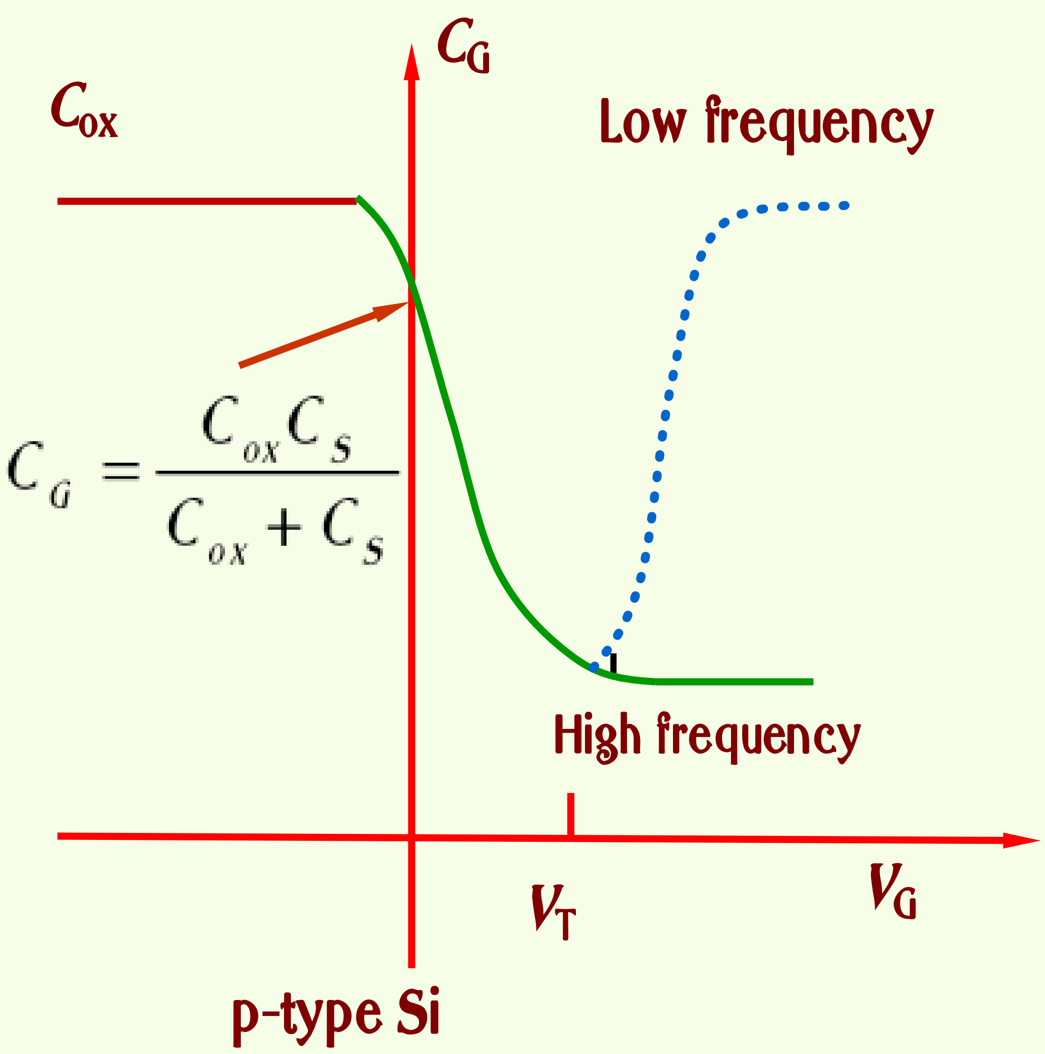


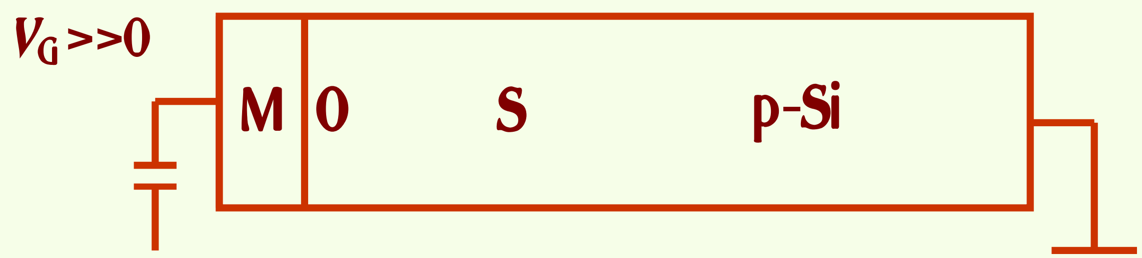
p -type Si

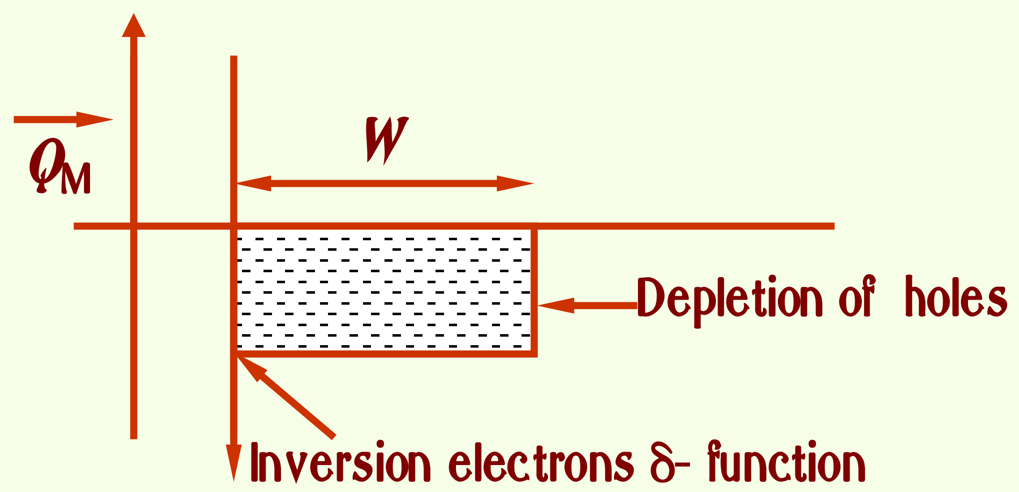


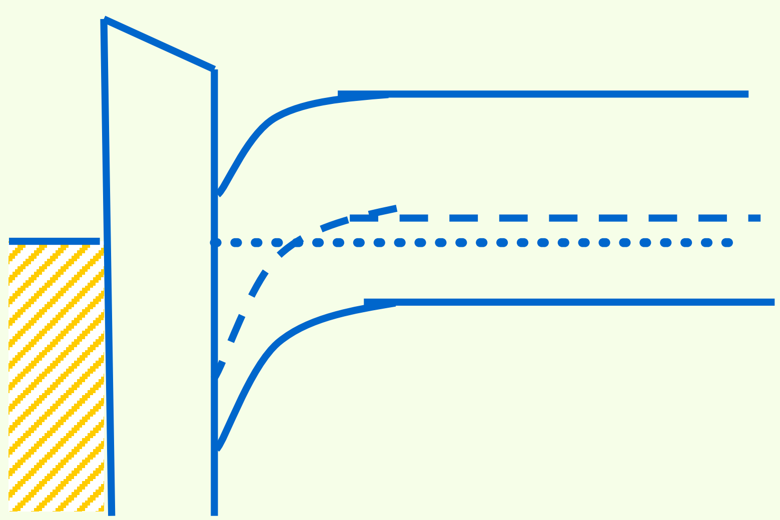
MOS capacitor under inversion

- Inversion condition $\psi_s = 2\Psi_B$









Graphical variation of potential, field, charge

$$V_{ox} = E_{ox}T_{ox} = \frac{|Q_s|T_{ox}}{\varepsilon_{ox}} = \frac{|Q_s|}{C_{ox}}$$

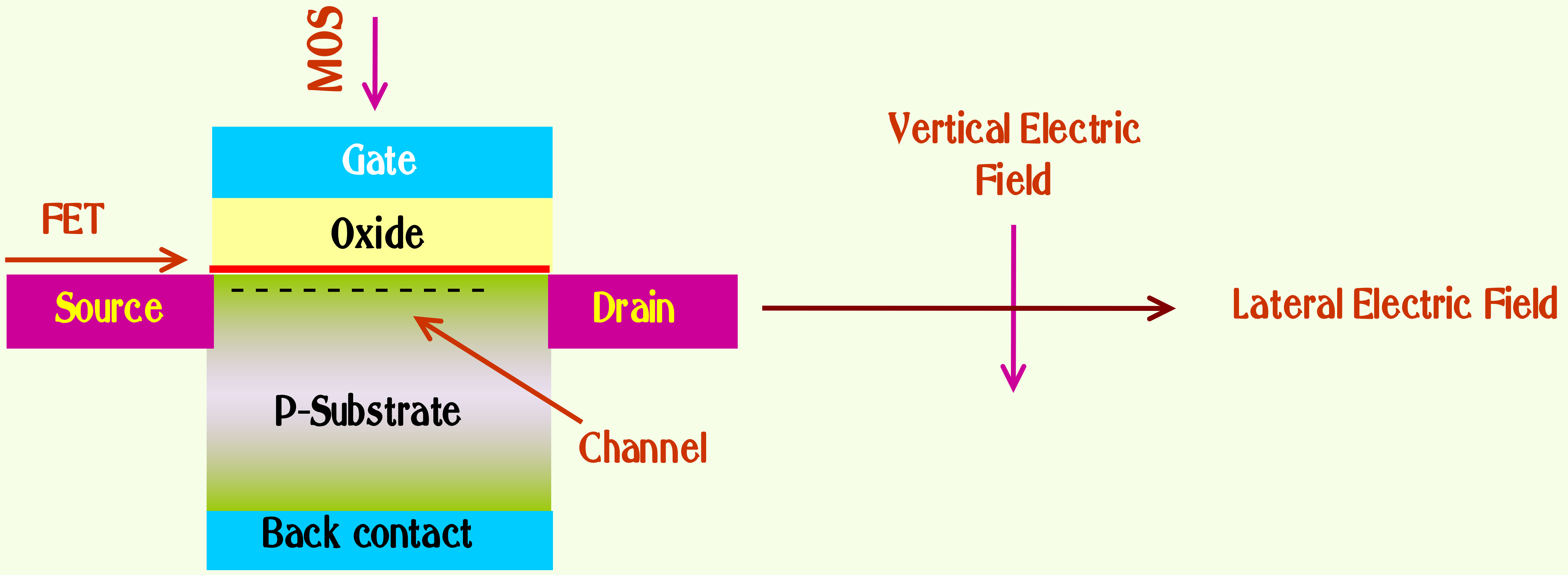
- The total capacitance C of the system is a series combination of the insulator capacitance

$$C_{ox} = \frac{\varepsilon_{ox}}{T_{ox}}$$

and the semiconductor depletion-layer capacitance C_D:

$$C = \frac{C_{ox}C_s}{C_{ox} + C_s}$$

MOSFET



Two orthogonal electric fields work together to initiate the operation of a MOSFET. Vertical field applied from the gate creates a channel for the carriers and lateral electric field drags the carriers from source to the drain, leading to generate a current along the channel.