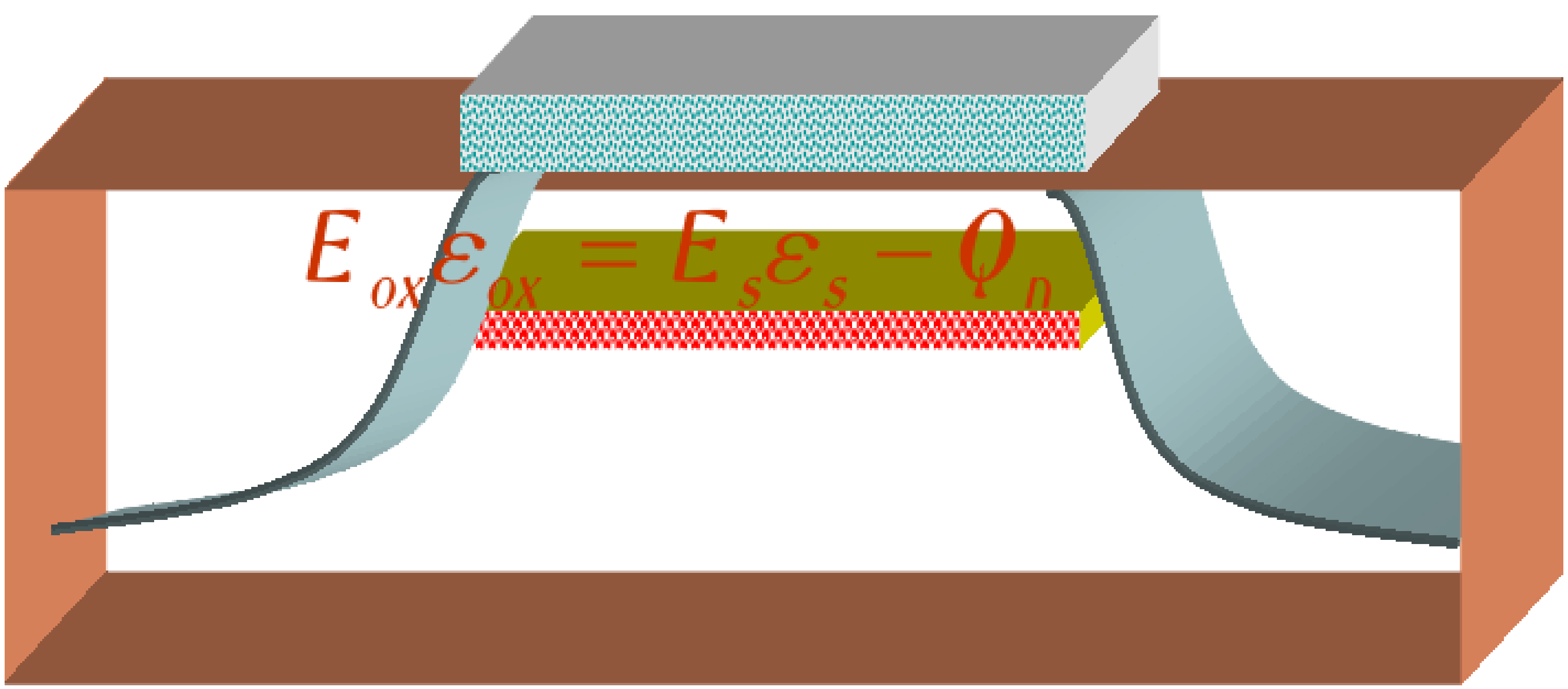
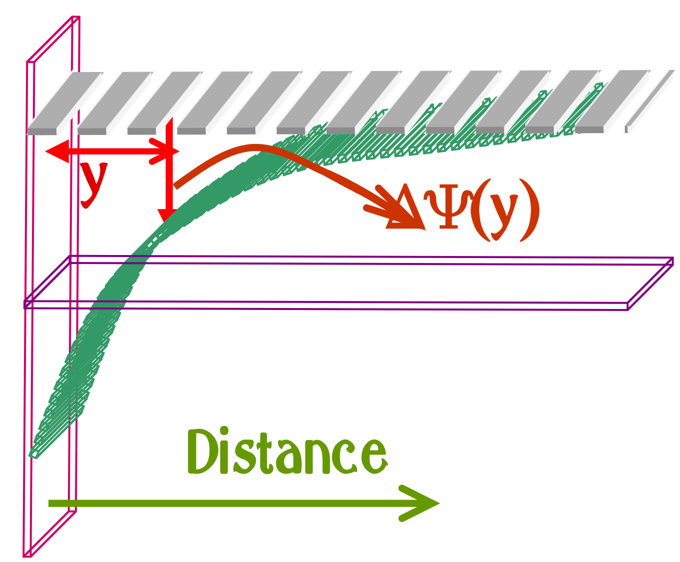
Derivation of current-voltage characteristics of metal-oxide-semiconductor field effect transistors (MOSFETs)

Transport in a MOSFET

Charge sheet model:

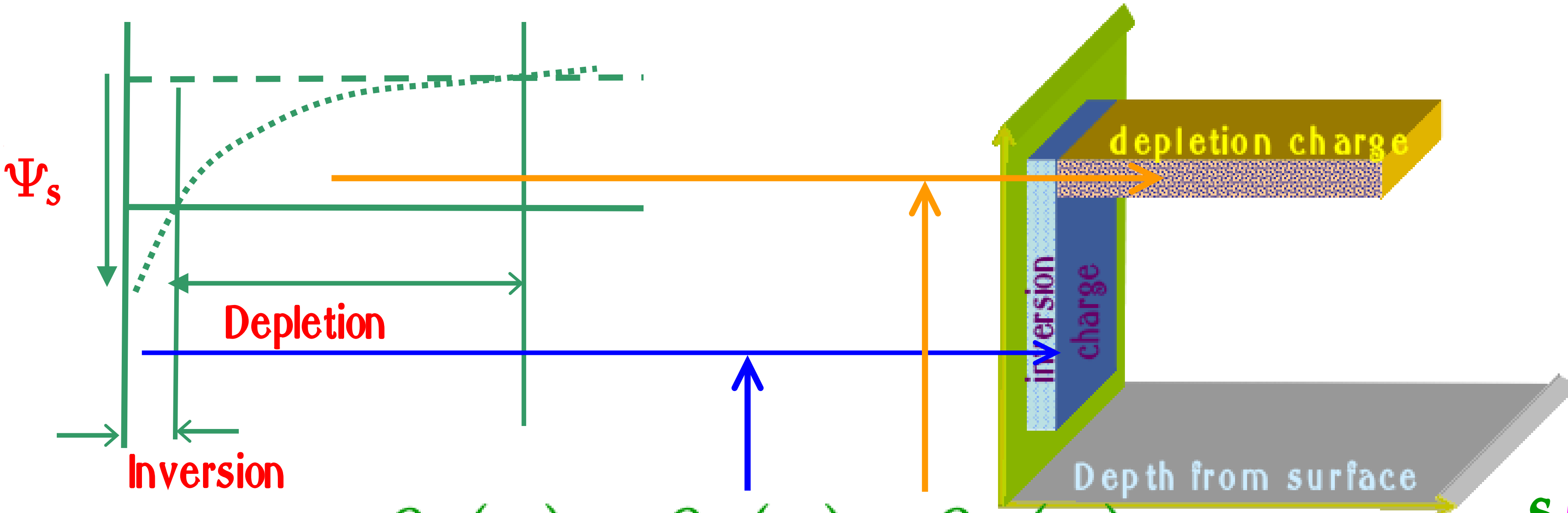
- channel is very thin, no voltage drop across it.
- vertical electric field is very high compared to lateral electric field.
- total charge at the metal side is equal to the net charge in the semiconductor side.

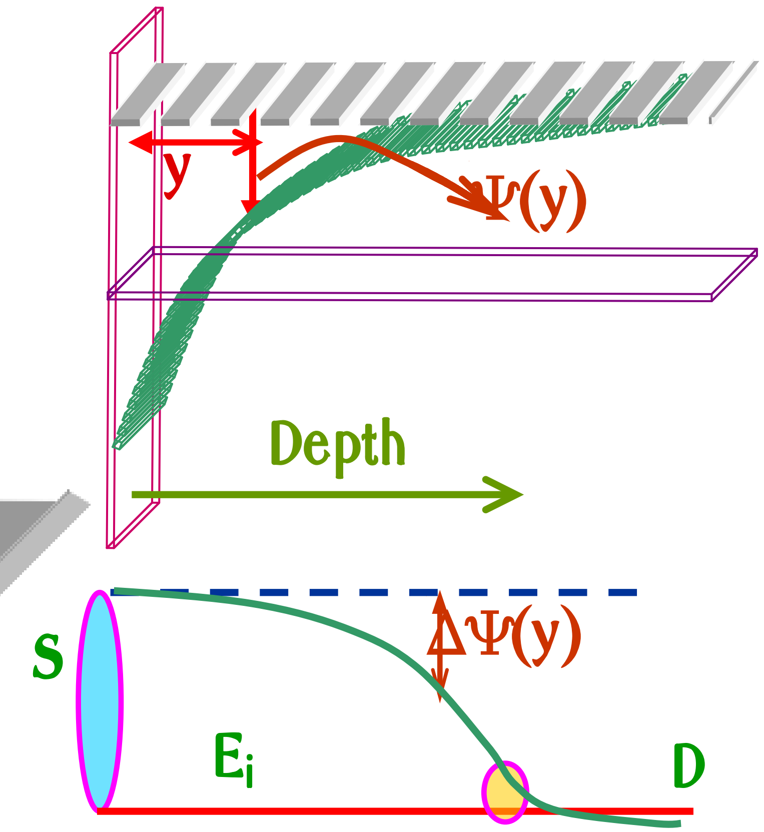




Charge balance on two sides of the 'charge sheet':

$$E_{ox}\varepsilon_{ox} = E_s\varepsilon_s - Q_n$$





$$Q_s(y) = Q_n(y) - Q_B(y)$$

$$E_{ox} = \frac{V_G - \Psi_s}{t_{ox}} = \frac{V_G - (2\Psi_B + \Delta\psi(y))}{t_{ox}}$$

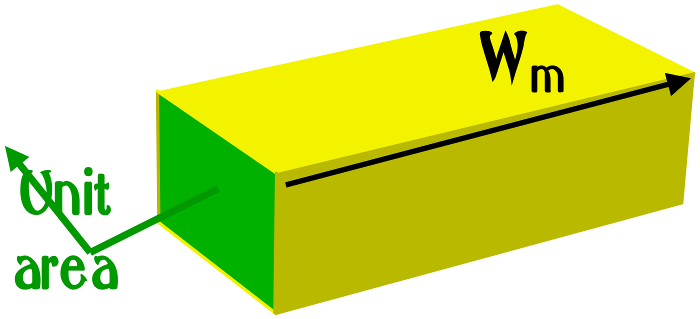
$$E_s = \sqrt{\frac{2q(2\Psi_B + \Delta\psi(y))}{\varepsilon_s}}$$

$$\begin{cases} Q_B = -qN_A W_m \\ = -\sqrt{2qN_A\varepsilon_s(V_D + 2\psi_B)} \end{cases}$$

The charge in the inversion layer is given by:

$$|Q_n(y)| = [V_G - \Delta\psi(y) - 2\Psi_B]C_{ox} - \sqrt{2\varepsilon_s qN_A(2\Psi_B + \Delta\psi(y))}$$

Charge in the depletion region:



W_m: Maximum depletion width.

N_A: Doping concentration/Cm^3

q: Electronic charge

$$Q_B = -qN_A W_m$$

$$Q_B = -qN_A W_m = -\sqrt{2qN_A \varepsilon_s (V_D + 2\psi_B)}$$

Theoretical: Current-Voltage Characteristics

We shall now derive the basic MOSFET characteristics under the following idealized conditions:

- the gate structure corresponds to an ideal MOS capacitor, i.e., there are no interface traps nor mobile oxide charges;
- only drift current is considered;
- doping in the channel is uniform;
- reverse leakage current is negligible; and
- transverse field ($\mathcal{E}_x$ in the x-direction) in the channel is much larger than the longitudinal field ($\mathcal{E}_y$, in the y-direction).

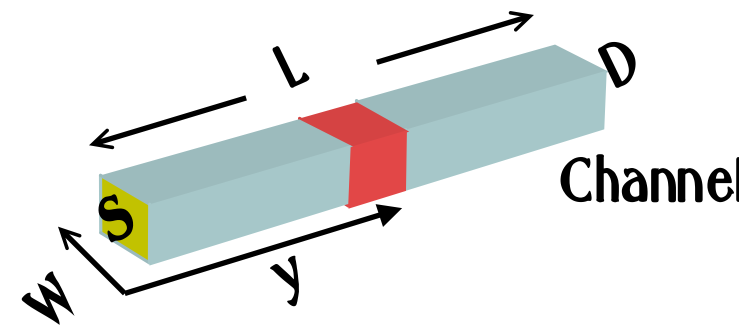
Derivation of Current-Voltage Characteristics

$$I_D(y) = W.|Q_n(y)|v(y)$$

Current per unit channel length through a cross-section at a point y from the source/ channel interface. W is width of the device.

$$\int_0^L I_D(y)dy = W\int_0^L |Q_n(y)|v(y).dy$$

$$I_D(y) = \frac{W}{L}\int_0^L |Q_n(y)|v(y).dy$$



$$v(y) = \mu E(y) = \mu.\frac{\Delta\psi(y)}{dy}$$

$$|Q_n(y)| = [V_G - \Delta\psi(y) - 2\Psi_B]C_{ox} - \sqrt{2\varepsilon_s qN_A(2\Psi_B + \Delta\psi(y))}$$

$$|Q_n(y)| = C_{ox}\left[(V_G - 2\Psi_B - \Delta\psi(y)) - \frac{\sqrt{2\varepsilon_s qN_A(2\Psi_B + \Delta\psi(y))}}{C_{ox}}\right]$$

Derivation of Current-Voltage Characteristics

- Thus the current can be represented by:

$$I_D = \frac{W}{L}\int_0^L C_{ox}\left[\left(V_G - 2\Psi_B - \Delta\psi(y)\right) - \frac{\sqrt{2\varepsilon_s qN_A(2\Psi_B + \Delta\psi(y))}}{C_{ox}}\right]\mu.\frac{\Delta\psi(y)}{dy}.dy$$

$$I_D = \frac{W.C_{ox}.\mu}{L}\int_0^{V_D}\left[\left(V_G - 2\Psi_B - \Delta\psi(y)\right) - \frac{\sqrt{2\varepsilon_s qN_A(2\Psi_B + \Delta\psi(y))}}{C_{ox}}\right]\Delta\psi(y)$$

$$I_D = \frac{W.C_{ox}.\mu}{L}\int_0^{V_D}\left[\left(V_G - 2\Psi_B - \Delta\psi(y)\right) - \frac{\sqrt{2\varepsilon_s qN_A}}{C_{ox}}.(2\Psi_B + \Delta\psi(y))^{\frac{1}{2}}\right]\Delta\psi(y)$$

Derivation of Current-Voltage Characteristics

$$I_D = \frac{W.C_{ox}.\mu}{L}\left[\int_0^{V_D}(V_G - 2\Psi_B - \Delta\psi(y))\Delta\psi(y) - \int_0^{V_D}\frac{\sqrt{2\varepsilon_s qN_A}}{C_{ox}}.(2\Psi_B + \Delta\psi(y))^{\frac{1}{2}}\Delta\psi(y)\right]$$

$$Int\_1 = \int_0^{V_D}(V_G - 2\Psi_B - \Delta\psi(y))\Delta\psi(y)$$

$$Int\_2 = \int_0^{V_D}\frac{\sqrt{2\varepsilon_s qN_A}}{C_{ox}}.(2\Psi_B + \Delta\psi(y))^{\frac{1}{2}}\Delta\psi(y)$$

$$Int\_1 = \left(V_G - 2\Psi_B - \frac{V_D}{2}\right).V_D$$

$$Int\_2 = \frac{2}{3}.\frac{\sqrt{2\varepsilon_s qN_A}}{C_{ox}}\left[(2\Psi_B + V_D)^{\frac{3}{2}} - (2\Psi_B)^{\frac{3}{2}}\right]$$

Derivation of Current-Voltage Characteristics

$$I_D = \frac{W.C_{ox}.\mu}{L}.(Int\_1 + Int\_2)$$

$$I_D = \frac{W}{L}.\mu_n.C_{ox}\left\{(V_G - V_{FB} - 2\Psi_B)V_D - \frac{2}{3}\frac{\sqrt{2\varepsilon_s qN_A}}{C_{ox}}\left[(V_D + 2\Psi_B)^{\frac{3}{2}} - (2\Psi_B)^{\frac{3}{2}}\right]\right\}$$

$$Now,\ (V_D + 2\Psi_B)^{\frac{3}{2}} - (2\Psi_B)^{\frac{3}{2}}$$

$$= (2\Psi_B)^{\frac{3}{2}}\left[\left(1 + \frac{V_D}{2\Psi_B}\right)^{\frac{3}{2}} - 1\right] = (2\Psi_B)^{\frac{3}{2}}\left[\left(1 + \frac{V_D}{2\Psi_B}\right).\left(1 + \frac{V_D}{2\Psi_B}\right)^{\frac{1}{2}} - 1\right]$$

$$= (2\Psi_B)^{\frac{3}{2}}\left[\left(1 + \frac{V_D}{2\Psi_B}\right).\left(1 + \frac{V_D}{2\Psi_B} + \frac{1}{2}.\frac{V_D^2}{2\Psi_B^2} + \ldots\ldots\right) - 1\right]$$

$$I_D = \frac{W}{L}.\mu_n.C_{ox}\left\{\left(V_G - 2\Psi_B - \frac{V_D}{2}\right)V_D - \frac{2}{3}\frac{\sqrt{2\varepsilon_s qN_A}}{C_{ox}}\left[3.\sqrt{\frac{\Psi_B}{2}}.V_D\right]\right\}$$

Derivation of Current-Voltage Characteristics

$$I_D = \frac{W}{L}.\mu.C_{ox}\left\{\left(V_G - 2\Psi_B - \frac{V_D}{2}\right)V_D - \frac{2}{3}\frac{\sqrt{2\varepsilon_s qN_A}}{C_{ox}}\left[3.\sqrt{\frac{\Psi_B}{2}}.V_D\right]\right\}$$

$$I_D = \frac{W}{L}.\mu.C_{ox}(V_G - V_{th})V_D$$

where

$$V_{th} = 2\Psi_B + \frac{\sqrt{2\varepsilon_s qN_A(2\Psi_B)}}{C_{ox}}$$

- **Threshold voltage (V_{th}):**

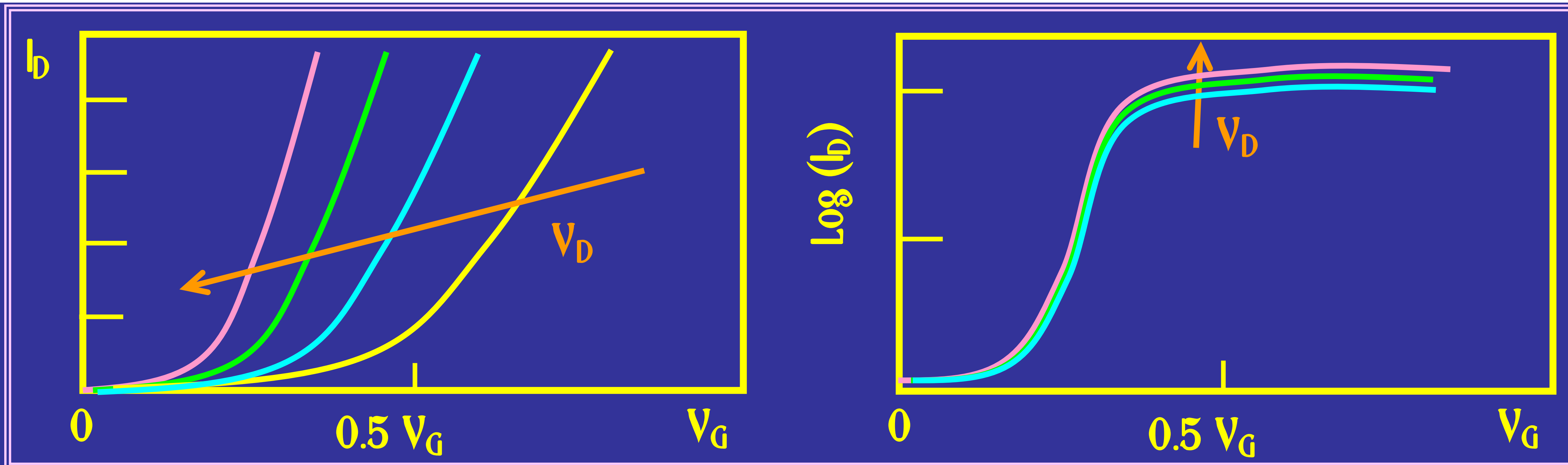
Minimum gate voltage required to create an inversion layer in a MOSFET.

<u>If flat-band voltage is considered</u>:

$$I_D = \frac{W}{L}.\mu_n.C_{ox}\left\{\left(V_G - V_{FB} - 2\Psi_B - \frac{V_D}{2}\right)V_D - \frac{2}{3}\frac{\sqrt{2\varepsilon_s qN_A}}{C_{ox}}\left[3.\sqrt{\frac{\Psi_B}{2}}.V_D\right]\right\}$$

$$V_{th} = V_{FB} + 2\Psi_B + \frac{\sqrt{2\varepsilon_s qN_A(2\Psi_B)}}{C_{ox}}$$

Theoretical: Current-Voltage Characteristics



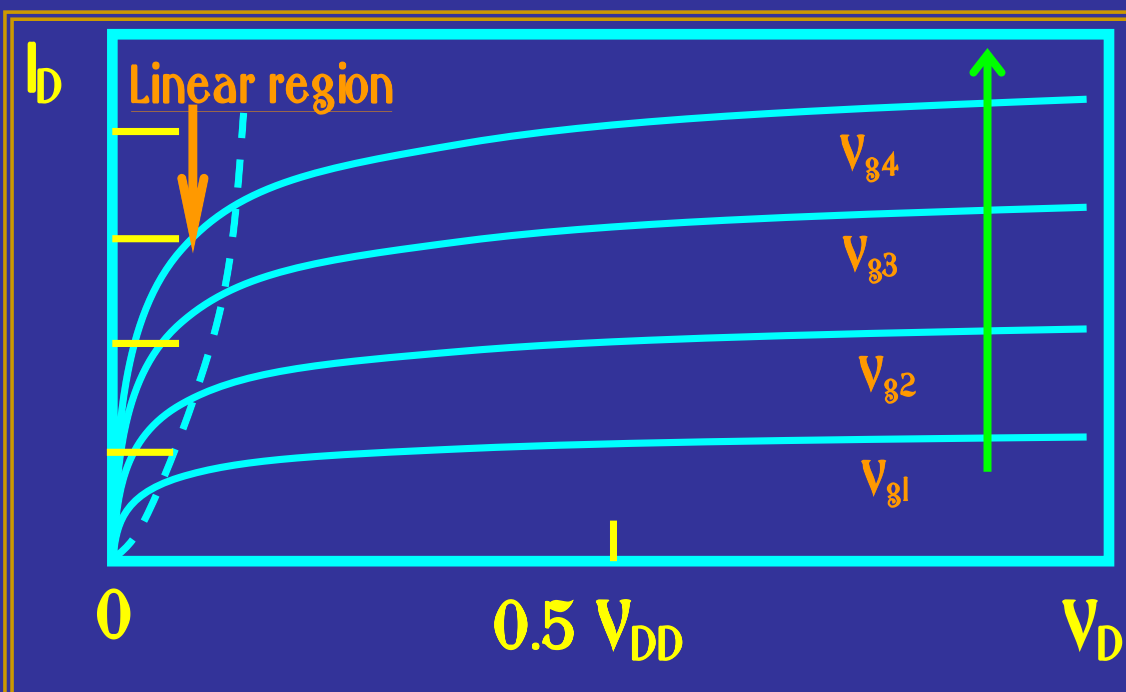
Transfer characteristics

Linear region:

$$I_D = \frac{\varepsilon_{ox}\varepsilon_0\mu}{t_{ox}}\frac{W}{L}.(V_G - V_{th})V_D$$

Saturation region:

$$I_D = \frac{\varepsilon_{ox}\varepsilon_0\mu}{t_{ox}}.\frac{W}{L}.\frac{(V_G - V_{th})^2}{2}$$



Output characteristics