DIGITAL COMMUNICATION: Mathematical Analysis, Constellation diagram and Error probability derivation for BPSK

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CONTENTS

DIGITAL COMMUNICATION TECHNIQUES

BASEBAND TRANSMISSION

- A data stream in the type of Pulse Amplitude Modulation(PAM) signal is directly transmitted over a low pass channel.
- Wired communication

PASSBAND TRANSMISSION

- The incoming data stream is modulated onto a carrier(usually sinusoidal) signal with fixed frequency limits imposed by a passband channel of interest. .
- Wireless Communication.

DIGITAL MODULATION TECHNIQUES

- In Digital Communication, the modulation corresponds to switching or keying amplitude, frequency or phase of a carrier wave, according to digital data.
- There are many types of digital modulation techniques and we can even use a combination of these techniques as well.

  1. Amplitude Shift Keying(ASK)
  2. Frequency Shift Keying(FSK)
  3. Phase Shift Keying(PSK)

- There is M-ary signaling scheme, by which we may send one of M possible signals, during each signaling duration T.

BINARY PHASE SHIFT KEYING(BPSK)

- In binary signaling, the modulator produces one of 2 distinct signals in response to 1 bit of source data at a time.
- It corresponds to switching or keying phase of a carrier wave(sinusoidal), according to digital data.
- Binary Phase Shift Keying (**BPSK**) is a two phase modulation scheme, where the 0's and 1's in a binary message are represented by two different phase states in the carrier signal.
- It used in coherent systems.
- PSK technique is widely used for wireless LANs, bio-metric, contactless operations, along with RFID and Bluetooth communications.

MATHEMATICAL ANALYSIS OF BPSK

- In coherent BPSK systems, the pair of signals, $s_1(t)$ and $s_2(t)$ used to represent binary symbols 1 and 0, respectively, is defined by

$$1 \longrightarrow s_1(t)=(2E_b/T_b)^{1/2}\cos(2\pi f_c t)$$

$$0 \longrightarrow s_2(t)=(2E_b/T_b)^{1/2}\cos(2\pi f_c t+\pi)= -\left((2E_b/T_b)^{1/2}\cos(2\pi f_c t)\right)$$

where $0 \le t < T_b$; T_b is the *bit duration,* E_b is the *transmitted signal energy per bit* and f_c is the *carrier frequency* chosen to be n_c/T_b for some fixed integer or $f_c >> 1/T_b$

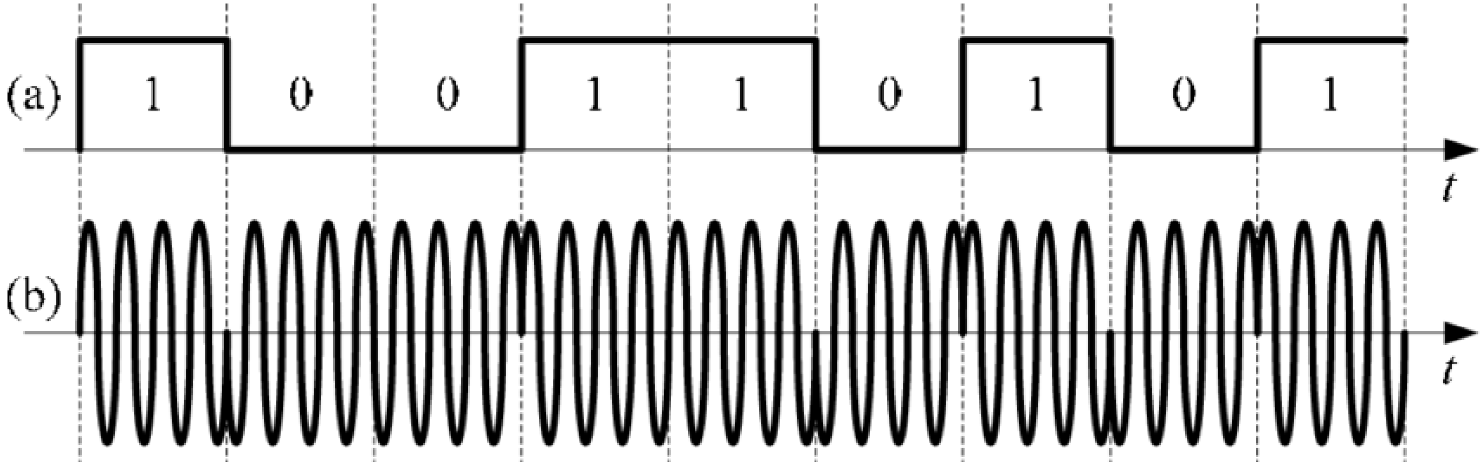


Figure 1
(a) binary signal
(b) BPSK modulated signal

SIGNAL SPACE REPRESENTATION

- Basis function, $\alpha_1(t) = s_1(t)/E_b^{1/2}$

- From the pair of signals from the previous equations, it is clear that in case of bpsk, the only one basis function of unit energy is, namely,

$$\alpha_1(t) = (2/T_b)^{1/2}\cos(2\pi f_c t) \qquad 0 \le t < T_b$$

Therefore $s_1(t)$ and $s_2(t)$ can be rewritten as,

$$s_1(t) = \alpha_1(t)E_b^{1/2} \qquad 0 \le t < T_b$$

$$s_2(t) = -\alpha_1(t)E_b^{1/2} \qquad 0 \le t < T_b$$

CONSTELLATION DIAGRAM OF BPSK

- A coherent BPSK system is characterized by a signal space that is one-dimensional (i.e. N=1), and has two message points (i.e. M =2). The coordinates of the message point are:

$$s_{21} = {}_0\int^{Tb} s_1(t)\alpha_1(t)dt = E_b^{1/2}$$

$$s_{11} = {}_0\int^{Tb} s_2(t)\alpha_1(t)dt = -E_b^{1/2}$$

Distance between 2 message points:

$D_{12} = 2E_b^{1/2}$

It has minimum average energy.

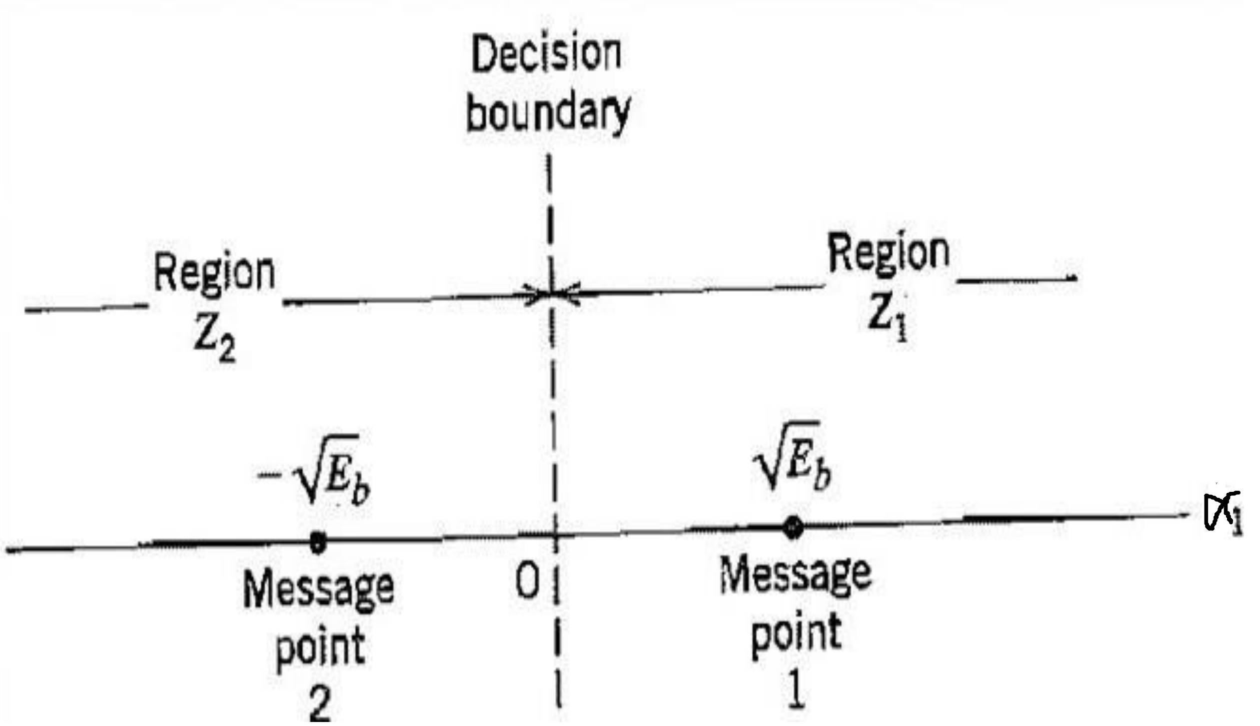


Figure 2:Signal-space diagram for coherent BPSK system

RULES FOR MAKING A DECISION

- To realize a rule for making a decision in favor of 1 or 0, we partition the signal space as in figure 2, into two regions

  1. The sets of points closest to message point 1 at $E_b^{1/2}$
  2. The sets of points closest to message point 0 at $-E_b^{1/2}$

- To accomplish this, a line (*Optimum Decision Boundary*)through the mid-point of the line joining the two message signals is constructed and the 2 appropriate regions Z_1 and Z_2 are marked.
- Decision rule:

  1. If signal $s_1(t)$ (or binary 1) was transmitted if the received signal point r falls, in region Z_1($r>0$)
  2. else signal $s_2(t)$(or binary 0) was transmitted ($r \leq 0$).

ERROR PROBABILITY OF BPSK

- Two kinds of erroneous situations can happen here:

1. Signal $s_2(t)$ is transmitted but the noise adds up in such a way that it falls in the region Z_1 and the receiver decides in favour of $s_1(t)$.

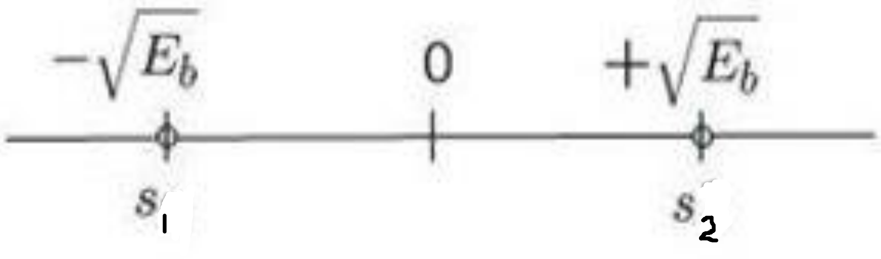
2. Signal $s_1(t)$ is transmitted but the noise adds up in such a way that it falls in the region Z_2 and the receiver decides in favor of $s_2(t)$.

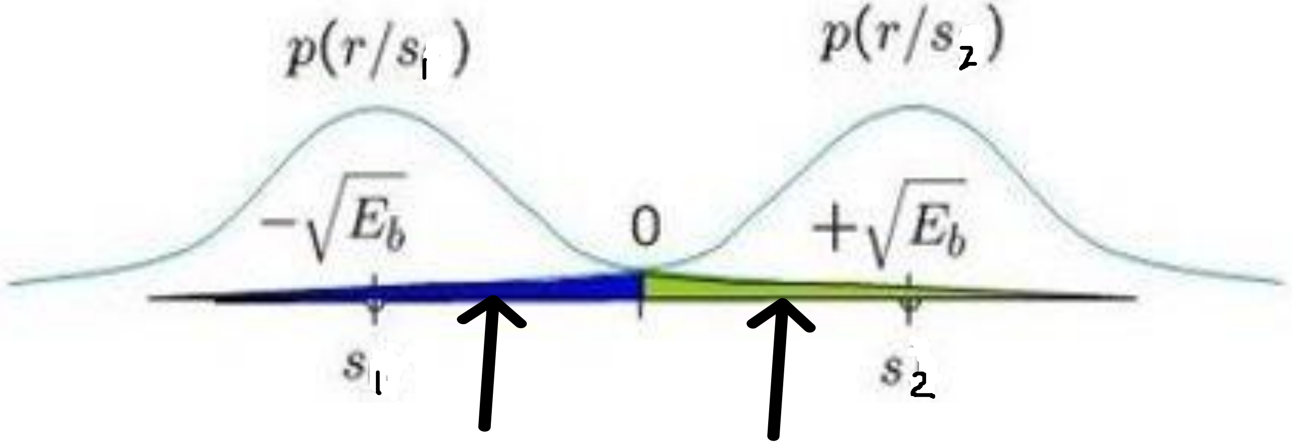
- Since, the values of the noise follows the Gaussian probability distribution function, the conditional probability of the receiver deciding in favor of given that is transmitted is

$$P(e|s_1) = P(r<0|s_1) = {}_{-\infty}\int^{0}((I/(\pi N_0)^{1/2})*\exp\{-((r-E_b^{1/2})^2/N_0\} = Q((2E_b/N_0)^{½})$$

- Due to symmetry, $P(e|s_2) = P(r>0|s_2) = Q((2E_b/N_0)^{½})$

ERROR PROBABILITY OF BPSK





$P(e|s_2)$ $P(e|s_1)$

ERROR PROBABILITY OF BPSK

- Since the signals $s_1(t)$ and $s_2(t)$ and are equally likely to be transmitted,

  $P(s_1)=P(s_2)=0.5$

Therefore, the average probability of error is

$$P_e = P(s_1)\times P(e|s_1)+P(s_2)\times P(e|s_2)$$
$$= 0.5\times P(e|s_1)+0.5\times P(e|s_2)$$
$$= Q((2E_b/N_0)^{½})$$

$\Rightarrow P_e$ depends on E_b/N_0

Q is *Q-function,* the *tail distribution function*.

This ratio is normally called *bit energy to noise density ratio (or SNR/bit).*

THANKS FOR WATCHING!