

Let D be an free set with anchoring set B . We say a tracial free function $f : B \rightarrow \mathbb{C}$ is a global germ if it analytically continues along every path in D . Since every tracial free function is a free functions¹, the Universal Monodromy Theorem tells us that f has a unique analytic continuation to all of D . Pick an $X \in B$ and $Y \in D$ and let γ_1 be any path taking X to itself, γ_X be the constant path, and γ_2 take X to Y . Since f has a unique continuation to all of D , $f(\gamma_2\gamma_1) = f(\gamma_2\gamma_X)$ since we identify both with $f(Y)$. Therefore, γ_1 is trace equivalent to γ_X —and π_1^{tr} is trivial.

The above argument works perfectly for showing that π_1^{free} is trivial—but says nothing about π_1^{tr} .

¹THIS IS THE ISSUE!