A CLEAN TITLE

LUCAS KERBS

A Fun Subtitle

February 2022 – LucasThesis v1

Ohana means family.
Family means nobody gets left behind, or forgotten.

— Lilo & Stitch

Dedicated to the loving memory of Rudolf Miede.
1939 – 2005

ABSTRACT

Short summary of the contents in English. . . a great guide by Kent Beck how to write good abstracts can be found here:

https://plg.uwaterloo.ca/~migod/research/beckOOPSLA.html

ZUSAMMENFASSUNG

Kurze Zusammenfassung des Inhaltes in deutscher Sprache. . .

We have seen that computer programming is an art,
because it applies accumulated knowledge to the world,
because it requires skill and ingenuity, and especially
because it produces objects of beauty.

— Donald E. Knuth [1]

ACKNOWLEDGMENTS

Put your acknowledgments here.

Many thanks to everybody who already sent me a postcard!

Regarding the typography and other help, many thanks go to Marco Kuhlmann, Philipp Lehman, Lothar Schlesier, Jim Young, Lorenzo Pantieri and Enrico Gregorio[1], Jörg Sommer, Joachim Köstler, Daniel Gottschlag, Denis Aydin, Paride Legovini, Steffen Prochnow, Nicolas Repp, Hinrich Harms, Roland Winkler, Jörg Weber, Henri Menke, Claus Lahiri, Clemens Niederberger, Stefano Bragaglia, Jörn Hees, Scott Lowe, Dave Howcroft, José M. Alcaide, David Carlisle, Ulrike Fischer, Hugues de Lassus, Csaba Hajdu, Dave Howcroft, and the whole LaTeX-community for support, ideas and some great software.

Regarding LyX: The LyX port was intially done by *Nicholas Mariette* in March 2009 and continued by *Ivo Pletikosić* in 2011. Thank you very much for your work and for the contributions to the original style.

1 Members of GuIT (Gruppo Italiano Utilizzatori di TeX e LaTeX)

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Part I

PART I: *NAME*

1 INTRODUCTION

1.1 FUNCTIONAL CALCULUS

We will start our consideration with extending single input functions to spaces of Matrices. Since matrices form a ring, we can take any $f \in \mathbb{C}[x]$ and compute $f(A)$ for some $A \in M_n(\mathbb{C})$. If we require that A is self-adjoint—and hence diagonalizable as $A = U\Lambda U^*$—then it is a standard result that:

$$\begin{aligned} f(A) &= a_n A^n + \cdots + a_1 A + a_0 \\ &= a_n \left(U\Lambda U^*\right)^n + \cdots + a_1 U\Lambda U^* + a_0 \\ &= a_n U\Lambda^n U^* + \cdots + a_1 U\Lambda U^* + a_0 \\ &= U\left(a_n\Lambda^n + \cdots + a_1\Lambda + a_0\right)U^* \\ &= U\left(f(\Lambda)\right)U^* \end{aligned}$$

Further, since Λ is diagonal and f is a polynomial,

$$f\left(\begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}\right) = \begin{bmatrix} f(\lambda_1) & & \\ & \ddots & \\ & & f(\lambda_n) \end{bmatrix}$$

Therefore, given a self-adjoint matrix A and a polynomial $f \in \mathbb{C}[x]$

$$f(A) = Uf(\Lambda)U^* = U\ \mathrm{diag}\{f(\lambda_1), \ldots, f(\lambda_n)\}\ U^*$$

With this in mind, we can extend a function $g : [a, b] \to \mathbb{C}$ to a function on self adjoint (normal?) matrices with their spectrum in $[a, b]$. Let A be such a matrix (diagonalized by the unitary matrix U), and define

$$g(A) = U \begin{bmatrix} g(\lambda_1) & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} U^*$$

Thus, for each $n \in \mathbb{N}$, g induces a function on the self-adjoint $n \times n$ matrices with spectrum in $[a, b]$. We can extend many of the function theoretic properties (e. g.convexity, monotonicity, derivatives, etc.) to these matrix values functions.

The natural ordering explain why natural? on self-adjoint matrices is called the **Loewner Order**:

Definition i.1 (Loewner Ordering)**.** *For like size self-adjoint matrices, we say that $A \preceq B$ if $B - A$ is positive semidefinite and $A \prec B$ is $B - A$ is positive definite*

Part II

THE SHOWCASE

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Part III

APPENDIX