Problem 1)

$$\forall x\,[Happy(x) \Rightarrow [\exists y\,\exists z\,Loves(x,y) \wedge Loves(x,z)]]$$

$$\hookrightarrow \forall x\; \neg Happy(x) \vee [\exists y\,\exists z\,Loves(x,y) \wedge Loves(x,z)].$$

$$\hookrightarrow \forall x\,(\neg Happy(x) \vee \exists y\,Loves(x,y)) \wedge (\neg Happy(x) \vee \exists z\,Loves(x,z))$$

$$\hookrightarrow \forall x\,\exists y\,[\neg Happy(x) \vee Loves(x,y)] \wedge \exists z\,[\neg Happy(x) \vee Loves(x,z)]$$

$$\hookrightarrow \forall x\,\exists y\,\exists z\,[\neg Happy(x) \vee Loves(x,y)] \wedge [\neg Happy(x) \vee Loves(x,z)]$$

$$\forall x\,\exists y\,\exists z\,[Happy(x) \Rightarrow Loves(x,y)] \wedge [Happy(x) \Rightarrow Loves(x,z)]$$

$$\hookrightarrow \forall x\,\exists y\,\exists z\,[Happy(x) \Rightarrow Loves(x,y) \wedge Loves(x,z)]$$

Problem 1)

$\forall x\,[Happy(x) \Rightarrow [\exists y\, \exists z\, Loves(x,y) \wedge Loves(x,z)]]$

↳ $\forall x\; \neg Happy(x) \vee [\exists y\, \exists z\, Loves(x,y) \wedge Loves(x,z)]$

↳ $\forall x\, (\neg Happy(x) \vee \exists y\, Loves(x,y)) \wedge (\neg Happy(x) \vee \exists z\, Loves(x,z))$

↳ $\forall x\, \exists y\, [\neg Happy(x) \vee Loves(x,y)] \wedge \exists z\, [\neg Happy(x) \vee Loves(x,z)]$

↳ $\forall x\, \exists y\, \exists z\, [\neg Happy(x) \vee Loves(x,y)] \wedge [\neg Happy(x) \vee Loves(x,z)]$

$\forall x\, \exists y\, \exists z\, [Happy(x) \Rightarrow Loves(x,y)] \wedge [Happy(x) \Rightarrow Loves(x,z)]$

↳ $\forall x\, \exists y\, \exists z\, [Happy(x) \Rightarrow Loves(x,y) \wedge Loves(x,z)]$

Problem 2)

Dan has only One CellPhone

- Has(x, y) will be used as the relation
- Cellphone(x) will be the object cell phone

$\forall x, y\, [Has(Dan, Cellphone(x)) \wedge Has(Dan, Cellphone(y)) \Rightarrow x = y]$

Chapter 9 -
9.1, 9.2, 9.5.1,

Problem 3)

a) $(A \wedge B) \Rightarrow C$

→ eliminate the implication (figure 7.11)

$\neg(A \wedge B) \vee C$

→ De morgan

$(\neg A \vee \neg B) \vee C$

→ CNF

$\neg A \vee \neg B \vee C$

b) $D \Rightarrow E$

→ eliminate the implication (figure 7.11) (CNF)

$\neg D \vee E$

c) $E \Rightarrow A$

→ eliminate the implication (figure 7.11) (CNF)

$\neg E \vee A$

d) $D \Rightarrow G$

→ eliminate the implication (figure 7.11) (CNF)

$\neg D \vee G$

e) $G \Rightarrow B$

→ Eliminate the implication (figure 7.11) (CNF)

$\neg G \vee B$

f) D (already in CNF form)

proof is on next page/image

Problem 3 continued

The sentences we have so far after converting them into CNF:

a) $\neg A \vee \neg B \vee C$

b) $\neg D \vee E$

c) $\neg E \vee A$

d) $\neg D \vee G$

e) $\neg G \vee B$

f) D

Proof by resolution with refutation "C"

assume $\neg C$ we will call $\neg C$ as (x)

→ Unify (b) and (d): $(\neg D \vee E)\quad(\neg D \vee G)$
result → $(\neg D \vee E \vee G)$ we will call this (g)

→ Unify (g) and (f): $(\neg D \vee E \vee G)\quad D$
result → $E \vee G$ we will call this (h)

→ unify (h) and (e): $(E \vee G)\quad(\neg G \vee B)$
result → $E \vee B$ we will call this (i)

→ unify (i) and (c): $(E \vee B)\quad(\neg E \vee A)$
result → $B \vee A$ we will call this(j)

→ unify (j) and (a): $(B \vee A)\quad(\neg A \vee \neg B \vee C)$
result → C we will call this(k)

contradiction! C cannot exist as we assumed $\neg C$,
therefore with proof by refutation/contradiction, "C"

Problem 4)

a) Horses are faster than dogs.

→ All horses are faster than all dogs

→ $\forall x, y$ Faster (Horse(x), Dog(y))

b) There is a dog that is faster than every rabbit.

→ There exists a dog that is faster than all rabbits

→ $\exists x \ \forall y$ Faster(Dog(x), Rabbit(y))

c) Silver is a horse and Bunny is a rabbit. [Note that Silver and Bunny are names of the horse and rabbit]

Horse(Silver)

Rabbit(Bunny)

d) Faster is transitive

Faster(x,y) $\wedge$ Faster(y,z) $\Rightarrow$ Faster(x,z)

Proof by refutation found on next page/image

Problem 4)

a) Horses are faster than dogs.

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c) Silver is a horse and Bunny is a rabbit. [Note that Silver and Bunny are names of the horse and rabbit]

Horse (Silver)

Rabbit (Bunny)

d) Faster is transitive

Faster (x, y) $\wedge$ Faster (y, z) => Faster (x, z)

Convert to CNF

a) (removed universal) Faster (Horse(x), Dog(y)) already in CNF form

b) $\exists x \; \forall y$ Faster (Dog(x), Rabbit(y)) → Skolemize and removed universal quantification

Faster (Dog(F(x)), Rabbit (y))

c) Horse (Silver)

Rabbit (Bunny)

d) Faster(x, y) $\wedge$ Faster (y, z) => Faster (x, z)

$\neg$(Faster(x, y) $\wedge$ Faster(y, z)) $\vee$ Faster(x, z)

$\neg$Faster(x, y) $\vee$ $\neg$Faster(y, z) $\vee$ Faster (x, z)

Problem 4 continued

Sentences we have so far:

a) Faster(Horse(x), Dog(y))
e) Faster(Dog(F(x)), Rabbit(y))
c1) Horse(Silver)
c2) Rabbit(Bunny)
d) ¬Faster(x, y) ∨ ¬Faster(y, z) ∨ Faster(x, z)

Resolution with refutation "Silver is faster than Bunny":
=> Faster(Silver, Bunny)

Assume: ¬Faster(Silver, Bunny)
~~for (d), assume x = Horse(x), y = Dog(y), z = Rabbit~~
unify c1 and a: Horse(Silver) Faster(Horse(x), Dog(y))
result: Faster(Silver, Dog(y)) ← e
unify c2 and e: Rabbit(Bunny) Faster(Dog(F(x)), Rabbit(y))
result: Faster(Dog(F(x)), Bunny) ← f

unify e, f, d
¬Faster(Silver, Dog(y)) ∨ Faster(Dog(F(x)), Bunny)
∨ Faster(Silver, Bunny)

Faster(Silver, Bunny) contradicts w/ initial assumption
¬Faster(Silver, Bunny)
ergo Faster(Silver, Bunny)

Problem 5)

a) MRV heuristic used in Backtracking because it picks a variable that is most likely to cause a failure soon and then prunes the tree.

it avoids pointless searches through other variables.

LCV heuristic prefers the value that rules out the fewest choices for neighboring variables in the constraint graph.

b) forward checking process establishes arc consistency.

c) Any search algorithm as long as we don't stop once a solution is found, and we don't exit the algorithm.