Apollo 11 TLI Snapshot: AVA Single–Burn vs Split Baseline

Public data at TLI (space–fixed)

From Apollo 11 postflight tables (space–fixed frame) near the TLI event:

- Parking orbit speed (pre–TLI): $v_i = 7{,}793.1\ m/s = 7.7931\ km/s$.
- Post–TLI speed: $v_f = 10{,}834.3\ m/s = 10.8343\ km/s$.
- Post–TLI flight–path angle: $E = +7.367°$.
- Inclinations and nodes (deg): initial $i_1 = 32.521°$, $\Omega_1 = 123.088°$; post–TLI $i_2 = 31.383°$, $\Omega_2 = 121.847°$.

Direction change between pre/post velocities

Plane tilt between the two orbital planes (angle ζ):

$$\cos\zeta = \cos i_1 \cos i_2 + \sin i_1 \sin i_2 \cos(\Delta\Omega), \qquad \Delta\Omega = \Omega_2 - \Omega_1 \approx -1.241°.$$

Numerically this gives

$$\zeta \approx 1.314°.$$

Including the upward tilt from the flight–path angle E, the net change between *velocity vectors* (small–angle combination) is

$$\phi \ = \ \arccos(\cos E\ \cos\zeta) \ \approx \ \arccos(\cos 7.367° \cos 1.314°) \ \approx \ 7.483°\,.$$

AVA single–burn (canted impulse)

Match the post–TLI velocity in one shot using the law of cosines:

$$\Delta v_{single} = \sqrt{v_i^2 + v_f^2 - 2\, v_i v_f \cos\phi}.$$

With $v_i = 7.7931$, $v_f = 10.8343$ (km/s) and $\phi = 7.483°$,

$$\Delta v_{single} \approx 3.2691\ km/s\ .$$

Split baseline (energy then pointing)

A conservative two–step comparator that separates speed change and direction change:

$$\Delta v_{split} = \underbrace{|v_f - v_i|}_{pure tangential} + \underbrace{2\, v_i\, \sin\frac{\phi}{2}}_{pure plane/pointing at lower speed}.$$

Numerically,

$$|v_f - v_i| = 10.8343 - 7.7931 = 3.0412\ km/s, \qquad 2v_i \sin(\phi/2) = 2(7.7931)\sin(3.7415^\circ) = 1.0171\ km/s$$

so

$$\Delta v_{split} \approx 4.0583\ km/s\ .$$

Result: savings of the one–shot AVA burn

$$Saving \;=\; \frac{\Delta v_{split} - \Delta v_{single}}{\Delta v_{split}} \times 100\% \;=\; \frac{4.0583 - 3.2691}{4.0583} \times 100\% \;\approx\; 19.45\%\,.$$

Finite–duration ("warm") correction (constant turn–rate)

If the thrust direction rotates at a constant rate through the same total angle $\Theta = \phi$, the delivered magnitude needs a small scale factor

$$\rho \;=\; \rho_* \,\frac{\Theta}{2\sin(\Theta/2)}.$$

With $\phi = 7.483^\circ$ ($\Theta = 0.1306$ rad),

$$\frac{\Theta}{2\sin(\Theta/2)} \approx 1.000711 \quad \Rightarrow \quad overhead \approx 0.071\%\ (negligible here).$$

Notes

- The single–burn figure shows the geometric advantage of coupling energy and plane change in one canted impulse. Apollo's real TLI guidance already coupled these; the split baseline is a conservative comparator.
- For high–fidelity reproduction (gravity losses during the finite burn, mass depletion, steering limits), use a finite–burn propagator; the geometry–driven advantage remains indicative.