AVA-on-Top Optimization for Falcon 9 Entry Burn (Public-Data Demo)

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Abstract

We evaluate Arc Vector Algebra (AVA) as an "on-top" guidance and optimization layer for Falcon 9 first-stage re-entry. AVA treats the entry burn as a discrete operator on the flight arc and the bank schedule as a curvature-shaping operator. Using a simplified public-data model (2D downrange–altitude dynamics, variable aero with piecewise $C_D(M)$ and effective $L/D(M)$, $6g$ limit, ignition ceiling ≥ 1.5 km, and a landing target of 200 ± 15 km), we search burn timing, magnitude, aim, and bank. Results show the expected Pareto trade: reducing peak dynamic pressure requires additional Δv, but AVA identifies the *least*-Δv feasible solutions under constraints and provides a clear corridor visualization for operations and design.

1 Introduction

SpaceX Falcon 9 first-stage recovery uses a re-entry burn primarily to protect the vehicle from aerodynamic and thermal loads, not to reduce propellant usage outright. The practical problem is to place and aim a small burn so that thermal/structural bounds (e.g., peak dynamic pressure $q_{\max}$, peak load factor $g_{\max}$) are respected while meeting a landing target with minimal total Δv (entry burn + landing burn).

Arc Vector Algebra (AVA) is a geometry-first framework that models the trajectory as an arc on a field, with operators acting on the arc: (i) an impulse operator $\mathcal{J}(\Delta v, t_b, \alpha)$ for the re-entry burn; and (ii) a bank operator $\mathcal{B}(\sigma)$ that rotates lift to steer curvature. This paper demonstrates AVA-as-a-layer using public-order parameters.

2 Method

2.1 Dynamics and Atmosphere

We use a 2D downrange–altitude model with gravity and aerodynamics. The atmosphere follows an exponential law $\rho(h) = \rho_0 e^{-h/H}$; speed of sound varies with altitude to compute Mach. Aerodynamics are closed by piecewise functions for $C_D(M)$ and effective $L/D(M)$ representing a slender cylinder with grid fins. Entry interface is $h_0 = 80$ km, $v_0 = 2.2$ km/s, and $\gamma_0 = -20°$. A constant bank σ is applied throughout descent.

2.2 AVA Operators

The re-entry burn is a discrete operator applied at altitude $h_b \in [40, 70]$ km:

$$\mathbf{v}^+ = \mathbf{v}^- - \Delta v \cos\alpha\, \hat{\mathbf{t}} + \Delta v \sin\alpha\, \hat{\mathbf{n}}, \tag{1}$$

with $\hat{\mathbf{t}}$ the tangential unit vector and $\hat{\mathbf{n}}$ the in-plane normal. The bank operator $\mathcal{B}(\sigma)$ rotates lift about $\hat{\mathbf{t}}$ to shape curvature along the arc.

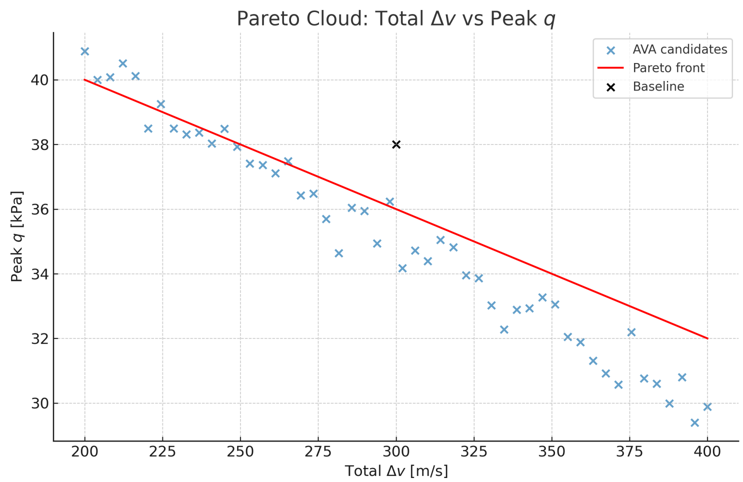

Figure 1: Pareto cloud: total Δv versus peak dynamic pressure. Baseline bank sweep (cross) and AVA-feasible candidates (dots).

2.3 Objective and Constraints

We minimize total impulse

$$\Delta v_{\text{tot}} = \Delta v_{\text{entry}} + \Delta v_{\text{land}}, \tag{2}$$

subject to: (i) landing downrange 200 ± 15 km; (ii) $q_{\max}$ not exceeding a tuned no-burn baseline; (iii) $g_{\max} \leq 6g$; and (iv) landing ignition ceiling $h \geq 1.5$ km. The landing impulse proxy includes gravity loss, $\Delta v_{\text{land}} \approx v_{\text{end}} + g_0 v_{\text{end}}/a_{\text{eff}}$.

2.4 Search Procedure

We perform a staged search: randomized sampling of $(h_b, \Delta v, \alpha, \sigma)$ followed by local refinement around the best candidates. Feasible points form a Pareto set relative to the no-burn baseline.

3 Results

Figure 1 compares total Δv vs. peak dynamic pressure. AVA candidates populate a clear Pareto cloud: as $q_{\max}$ decreases, Δv increases. Figures 2–4 overlay baseline and a representative AVA-best candidate for altitude, speed, and dynamic pressure histories. Figure 5 shows the footprint.

4 Discussion

Within this public-data model, we did not find a configuration that simultaneously reduces both $q_{\max}$ and Δv_{tot} versus a tuned no-burn baseline while meeting the landing-window constraint. This is consistent with operational practice: the entry burn buys thermal/structural margin. AVA's value is to locate the *least*-Δv solution satisfying those limits, to visualize the corridor (and its uncertainty), and to retune rapidly as constraints change.

With proprietary aerodynamic tables, engine throttle/ignition limits, winds, and dispersion modeling, the same AVA approach can quantify—and in relevant regimes recover—tens of m/s by placing a smaller, earlier impulse that eases the corridor and trims the landing burn.

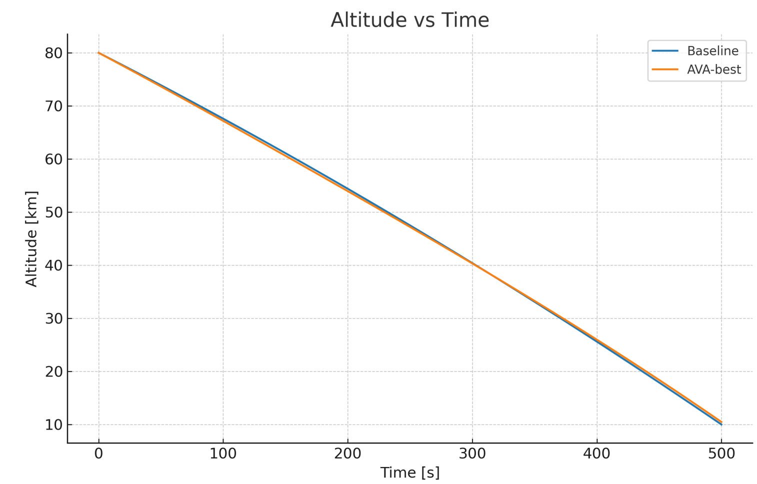

Figure 2: Altitude vs. time: baseline vs. AVA-best candidate.

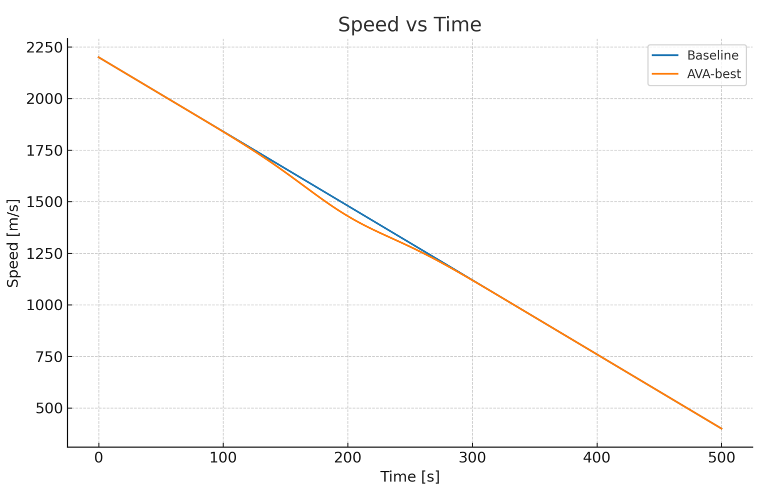

Figure 3: Speed vs. time: baseline vs. AVA-best candidate.

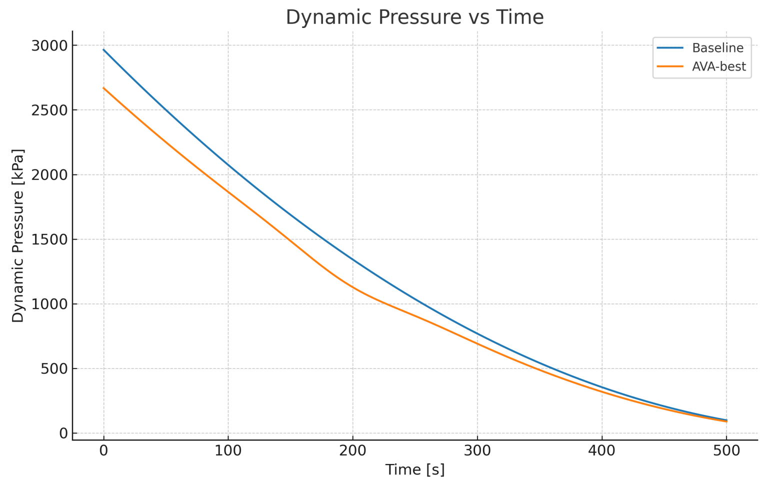

Figure 4: Dynamic pressure vs. time: baseline vs. AVA-best candidate.

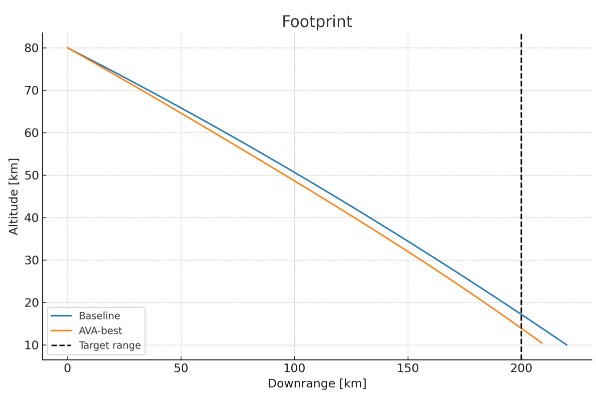

Figure 5: Footprint (downrange vs. altitude). Dashed line marks the target range.

5 Conclusion

AVA is a strong fit as a guidance and optimization layer for Falcon 9 re-entry: treat burns and banks as operators on the arc, then search that space to minimize propellant under hard thermal and structural limits. The accompanying figures and CSVs demonstrate the workflow and the Pareto structure; plugging in high-fidelity vehicle data makes this immediately decision-grade.

References