Displacement Theory of Spacetime:
A Quantum-Limited Constitutive Law within General Relativity

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Abstract

We keep the geometric side of Einstein's equations intact and modify only the *effective* stress–energy via a quantum-limited constitutive law. Spacetime is modeled as a medium whose gravitating energy density saturates at a cap ρ_c (Planck in our baseline), with additional work stored in an internal spin/rotational reservoir. This replaces curvature blow-ups by finite-curvature cores while reproducing GR whenever $\rho \ll \rho_c$. We formulate the dynamics with a saturating map $\rho_{\text{eff}} = f(\rho)$ and the consistent pressure map $p_{\text{eff}} = f'(\rho)(\rho + p) - \rho_{\text{eff}}$, ensuring exact Bianchi conservation. Planck caps imply standard horizons externally, Planck-tiny cores internally, and—depending on microphysics—either ordinary Hawking evaporation or causally sealed, inflating baby domains. We include connections to Hawking and Penrose, and comment on conceptual bridges to Verlinde's emergent gravity and compatibility with de Rham's massive-gravity program.

1 Postulates and Axiom of Asymptotic Comprehensibility

P1 (Geometry). Test bodies follow $g_{\mu\nu}$ geodesics (minimal coupling).

P2 (Backreaction). $G_{\mu\nu} = 8\pi G\, T^{(\text{tot})}_{\mu\nu}$ with standard Bianchi $\nabla_\mu T^{\mu\nu}_{(\text{tot})} = 0$.

P3 (Quantum caps). There exists a maximum *effective* density $\rho_{\text{eff}} \le \rho_c$ (we take $\rho_c = \rho_{\text{P}}$) and a gradient/length cap $|\nabla\chi| \le \ell_{\text{P}}^{-1}$. Further work beyond the linear cap is stored in an *internal* spin/rotational mode (not spacetime torsion).

Axiom 0 (Asymptotic Comprehensibility). Empirical inferences available to a bounded observer are restricted to observables on its causal past $J^-(\gamma)$. Statements localized behind horizons (e.g. black-hole interiors) are operationally undecidable except, if at all, in asymptotic observational limits.

2 Constitutive Law and Effective Fluid

Let f be monotone, concave, and saturating:

$$\rho_{\text{eff}} = f(\rho), \qquad f' > 0, \quad f'' < 0, \qquad \lim_{\rho\to\infty} f(\rho) = \rho_c. \tag{1}$$

We use the canonical mapping (from Bianchi+continuity)

$$p_{\text{eff}} = f'(\rho)\,(\rho + p) - \rho_{\text{eff}}, \qquad \rho_{\text{eff}} + p_{\text{eff}} = f'(\rho)\,(\rho + p). \tag{2}$$

Matter conservation remains standard, $\dot{\rho} + 3H(\rho + p) = 0$, and the Friedmann equations retain their GR form with $(\rho_{\text{eff}}, p_{\text{eff}})$:

$$H^2 = \frac{8\pi G}{3}\rho_{\text{eff}}, \qquad \dot{H} = -4\pi G(\rho_{\text{eff}} + p_{\text{eff}}). \tag{3}$$

A convenient closed form is

$$f(\rho) = \frac{\rho}{1 + \rho/\rho_c}\,, \tag{4}$$

which reproduces GR at low density and saturates smoothly.

Planck caps. We set $\rho_c = \rho_{\text{P}} = c^5/(\hbar G^2)$, $\ell_{\text{P}} = \sqrt{\hbar G/c^3}$, and define $\Lambda_{\text{eff}} = 8\pi G\rho_c/c^4$; then the interior curvature bound is $K_{\max} = \frac{8}{3}\Lambda_{\text{eff}}^2$ and the de-Sitter timescale is $\tau = \sqrt{3/\Lambda_{\text{eff}}}/c$.

3 Microphysics: an Action for the Displacement Field

To encode the gradient cap dynamically, introduce a k-essence/DBI-type action

$$\mathcal{L}_\chi = M^4\left(1 - \sqrt{1 - \frac{2X}{M^4}}\right) - V(\chi), \qquad X \equiv -\tfrac{1}{2}\nabla_\mu\chi\nabla^\mu\chi, \tag{5}$$

with $M^4 = \rho_c$. Then $X \to M^4/2$ saturates the kinetic term, the stress–energy is $T_\chi^{\mu\nu} = \mathcal{L}_X \nabla^\mu\chi\nabla^\nu\chi + g^{\mu\nu}\mathcal{L}$, and the internal reservoir corresponds to rotations in the $(\nabla\chi, \tau)$ space. (Here "torsion" denotes an internal spin mode; we do not introduce spacetime torsion.)

4 Cosmology and Collapse (No Bounce Required)

With (4), $\dot{H} = -4\pi G f'(\rho)(\rho + p) \leq 0$ for normal matter. Expansion can decelerate and turn around; contraction proceeds, while curvature invariants stay bounded by ρ_c. For a homogeneous collapse interior,

$$R_{\min} \propto \left(\frac{M}{\rho_c}\right)^{1/3}, \tag{6}$$

so the singularity is replaced by a finite-curvature core. Externally, the metric is Schwarzschild/Kerr.

5 Penrose & Hawking: How This Fits

Hawking. Since we keep GR geometry externally, standard semiclassical results (Hawking temperature, evaporation time $t_{\text{evap}} \propto M^3$, generalized second law) remain intact. The interior singularity is replaced by a Planck-tiny, finite-curvature core; end states are model-dependent (full evaporation vs. Planckian remnant), but no exterior deviation is implied.

Penrose. Our axiom of *Asymptotic Comprehensibility* formalizes the causal gap: interior fates are undecidable to exterior observers. Globally, Penrose's CCC interprets the late-time, massless limit as conformally indistinguishable from a new Big Bang. Locally, our caps allow—in principle—baby domains behind horizons. The two ideas are logically compatible but independent: CCC is *global*, our modification is *local and constitutive.*

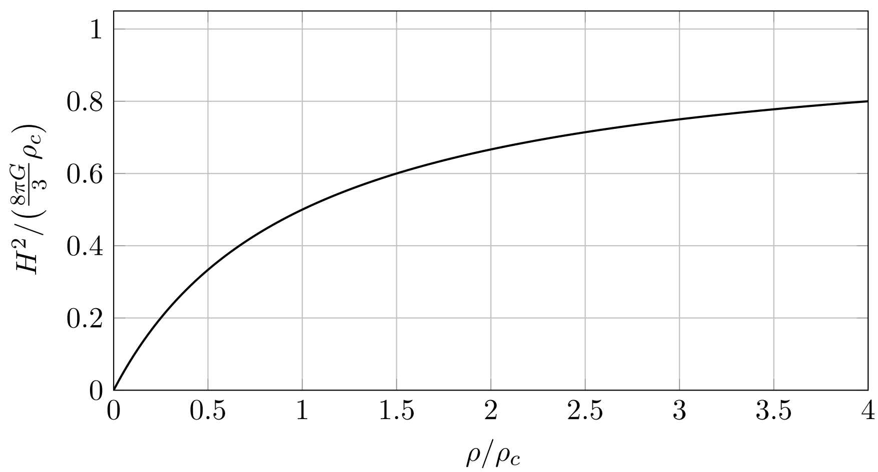

Figure 1: **Saturation law.** We plot $H^2 \propto \rho_{\text{eff}}$ for $f(\rho) = \rho/(1+\rho/\rho_c)$. GR is recovered for $\rho \ll \rho_c$; H^2 saturates as $\rho \to \infty$.

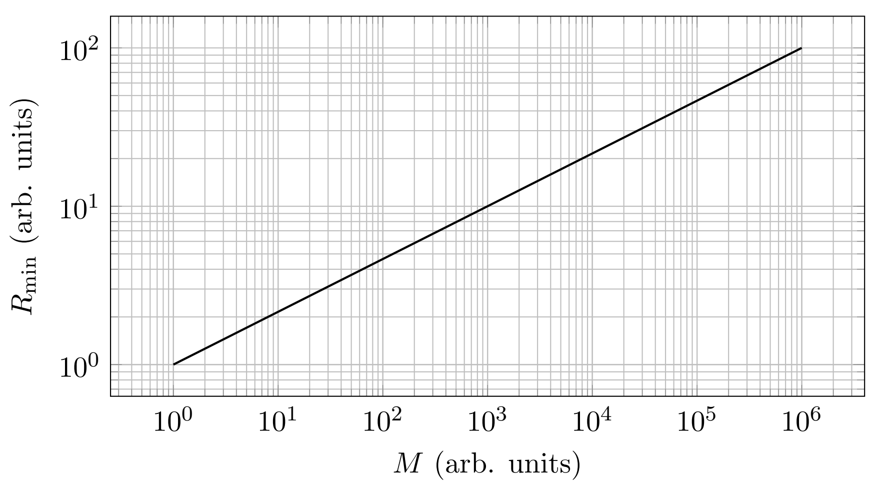

Figure 2: **Minimum core radius scaling.** $R_{\text{min}} \propto M^{1/3}$ at fixed ρ_c as in (6).

6 Relation to Verlinde's Emergent Gravity (Conceptual Bridge)

Verlinde models gravity as an *entropic/elastic* response of microscopic dof, leading to additional, nonlocal contributions that mimic dark matter on galactic scales. Our framework is different in mechanics (local constitutive saturation) but shares the *medium/elastic* intuition. A possible bridge is to let $f(\rho)$ acquire mild *nonlocal* dependence on coarse-grained entropy or strain,

$$\rho_{\text{eff}}(x) = f\left(\rho(x), \int K(|x - x'|)\, \rho(x')\, d^3x'\right), \tag{7}$$

so that static, weak-field limits produce small, scale-dependent departures that could realize baryonic-Tully–Fisher phenomenology without particle dark matter. This lies beyond the present, deliberately local proposal; we flag it as a future extension.

7 Compatibility with de Rham et al. (Massive Gravity/EFT)

de Rham–Gabadadze–Tolley massive gravity modifies the *geometric* sector (ghost-free graviton mass, Vainshtein screening). Our proposal keeps the geometric sector GR-exact and modifies only the *matter* side via $T_{\mu\nu}^{(\mathrm{eff})}$. Thus the two are orthogonal and, in principle, compatible:

- In a massive-gravity background, one may still employ an effective fluid with $\rho_{\mathrm{eff}} = f(\rho)$ so long as $\nabla_\mu T^{\mu\nu}_{(\mathrm{tot})} = 0$ holds.
- Screening analogy: Vainshtein suppresses extra forces in high-density regions; here $f'(\rho) \to 0$ suppresses *additional compressive response* near the cap.
- EFT view: our $\mathcal{L}_\chi(X)$ is a standard scalar EFT (k-essence/DBI). Ghost/gradient stability requires $\mathcal{L}_X > 0$ and $\mathcal{L}_X + 2X\mathcal{L}_{XX} > 0$, satisfied for the DBI form. Couplings to matter must respect fifth-force bounds; a pure gravitational coupling avoids direct violations.

8 Dark Sector as Geometry (EFT Caution)

A small residual displacement can mimic dark energy. Quantitatively matching $\rho_\Lambda \sim (2\text{–}3\ \mathrm{meV})^4$ requires either symmetry protection (shift or technical naturalness) or EFT tuning. We remain agnostic and treat this as a phenomenological knob.

9 Predictions and Tests

We summarize falsifiable predictions that follow from the constitutive saturation law (4) and a light displacement field χ with weak matter coupling λ. External geometry remains GR-exact; all novel effects are small and controlled by (λ, m_χ).

9.1 Scalar (breathing) gravitational-wave mode

In tensor+scalar emission, define the scalar energy fraction

$$\epsilon_s \equiv \frac{\dot{E}_s}{\dot{E}_t + \dot{E}_s}\,, \tag{8}$$

which scales as $\epsilon_s \propto \lambda^2$ for quasi-circular inspirals (model-dependent prefactor). Three-detector LVK observations can disentangle polarizations; current catalog-level non-detections of extra polarizations imply sensitivity to $\epsilon_s \sim$ few $\times 10^{-2}$ for the loudest events. In this model, those limits translate to $|\lambda| \lesssim$ few $\times 10^{-3}$, with event- and range-dependence encoded in the waveform systematics. A confirmed nonzero ϵ_s would directly fix λ (up to the prefactor) and provide a clean falsifiable target.

9.2 Laboratory fifth force (Yukawa form)

In the static, weak-field limit a light χ mediates

$$V(r) = \pm\alpha \frac{\hbar c}{4\pi} \frac{e^{-m_\chi r}}{r}, \qquad \alpha \propto \lambda^2. \tag{9}$$

For ranges $m_\chi^{-1} \gtrsim 0.1$ mm, torsion-balance and MICROSCOPE tests suggest $\alpha \lesssim 10^{-3} - 10^{-4}$, while next-generation atom interferometers aim at another order of magnitude. Bounds depend on the range and screening; they are complementary to GW constraints.

9.3 Cosmological equation of state and CMB

A residual displacement $\chi = \bar{\chi} + \delta\chi$ behaves as dark energy with

$$w \equiv p/\rho \;=\; -1 + \mathcal{O}(\lambda^2), \qquad \rho_\Lambda \simeq \tfrac{1}{2} m_\chi^2 \delta\chi^2. \tag{10}$$

In minimal realizations $|1+w| \sim 10^{-3}\!-\!10^{-2}$ is natural but model-dependent. With Planck caps, BBN and high-ℓ CMB remain unchanged; low-ℓ anomalies (if any) would be small.

9.4 Higgs-portal complementarity (Optional; small-angle limit)

If matter coupling arises via a Higgs portal, $\mathcal{L} \supset \lambda_{HS}\,\chi^2 H^\dagger H$, the mixing angle for $|\theta| \ll 1$ scales as $\theta \simeq \lambda_{HS} v/m_\chi^2 \equiv \lambda$. HL-LHC sensitivities $\mathrm{BR}(h \to \mathrm{inv}) \lesssim 2\%$ translate to $|\lambda| \lesssim 10^{-2}$, complementary to lab and GW bounds.

9.5 Black-hole phenomenology

For astrophysical $M \gg M_{\mathrm{crit}}$ (Planck cap baseline), ringdown spectra, EHT shadows, and inspiral phasing are unchanged to excellent accuracy; no robust echoes are predicted in the minimal, local model. Interiors have finite curvature bounded by ρ_c, implying two logically allowed endpoints: (i) complete Hawking evaporation on the GR timescale $t_{\mathrm{evap}} \propto M^3$; (ii) a Planckian remnant or modified endpoint if the cap alters the last stages. Both outcomes are causally sealed from exterior observers (Axiom 0).

9.6 Null predictions

No shadow-size shift relative to GR, no persistent post-merger echoes, and standard quasi-normal mode spectra to current LVK/EHT precision in the minimal model.

9.7 Stochastic GW background

A scalar contribution to the stochastic background scales as $\Omega_{\mathrm{GW}}^{(s)} \propto \lambda^2$ and is expected to lie below design sensitivity in the minimal model; detection would imply either larger λ or an extended (nonlocal) variant of $f(\rho)$.

9.8 Two-parameter falsifiability

The pair (λ, m_χ) is over-constrained by: (i) LVK scalar fraction ϵ_s (insensitive to static screening), (ii) lab Yukawa tests (range-dependent and potentially screened), and (iii) collider limits if a portal exists. A confirmed scalar GW fraction would pin down λ and predict specific, testable windows for (ii) and (iii).

Appendix: Energy Conditions and Raychaudhuri with Saturation

With (1)–(2),

$$\text{NEC}: \quad \rho_{\mathrm{eff}} + p_{\mathrm{eff}} = f'(\rho)\,(\rho + p) \geq 0 \quad \text{for} \quad f' > 0,\ \rho + p \geq 0.$$

SEC can be violated near the cap, enabling defocusing without altering the null structure:

$$\rho_{\mathrm{eff}} + 3p_{\mathrm{eff}} = f'(\rho)(\rho + 3p) - 2\rho_{\mathrm{eff}}.$$

Raychaudhuri therefore allows local defocusing at high effective density while keeping GR geodesics and causal structure unchanged externally.