3.3.12

$E(X) = 10$

$sd = 5$

(a) M: $P(X \geq 20) \leq \frac{10}{20} = \frac{1}{2}$

C: $P(X \geq 20) \leq \frac{1}{4}$ smallest upper

$\| $

$10 + k \cdot 5$

$k = 2$

(b) $X \sim bin(n, p)$ is not possible

$np = 10$ $sd = \sqrt{np(1-p)}$

$np(1-p) = 25 \rightarrow 1-p = \frac{25}{10} = 2.5$

3.3.16

(a)

x	-2	-1	0	3
$p(X=x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$E(X) = \frac{-2-1+0+3}{4} = 0$

$Var(X) = \frac{(-2)^2 + (-1)^2 + 0 + 9}{4} - 0^2 = \frac{7}{2}$

(b) $P(S_{100} \geq 25)$

sum of 100 draws is ~ normal

$\approx 1 - \Phi\left(\frac{25 - \frac{1}{2} - 0}{\sqrt{350}}\right)$

$= 1 - \Phi\left(\frac{24.5}{18.71}\right)$

$= 0.0952$

4.1.4

$$f(x) = cx^2(1-x)^2$$

(a) $\because \int_0^1 cx^2(1-x)^2 dx = 1$

$$c\int_0^1 (x^2 - 2x^3 + x^4) dx = 1$$

$$c\left(\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5}\right)\Big|_0^1 = 1$$

$$c = 30$$

(b) $E(X) = \int_0^1 30x^3(1-x)^2 dx$

$$= 30\int_0^1 (x^3 - 2x^4 + x^5) dx$$

$$= 30 \cdot \left(\frac{x^4}{4} - \frac{2x^5}{5} + \frac{x^6}{6}\right)\Big|_0^1$$

$$= 30 \cdot \left(\frac{1}{4} - \frac{2}{5} + \frac{1}{6}\right) = \frac{1}{2}$$

(c) $E(X^2) = \int_0^1 30x^4(1-x)^2 dx$

$$= 30\int_0^1 (x^4 - 2x^5 + x^6) dx$$

$$= 30\left(\frac{1}{5} - \frac{1}{3} + \frac{1}{7}\right)$$

$$= \frac{2}{7}$$

4.1.7

$x \sim N(\mu, \sigma^2)$ $\begin{cases} \mu = 12 \times 5 + 10 = 70 \text{ inches} \\ \sigma^2 = \end{cases}$

$P(x > 72) = 0.1$

$P(x \le 72) = 1 - 0.1 = 0.9$

$P(x > 74) = P\left(\frac{x-70}{\sigma} > \frac{74-70}{\sigma}\right)$

$= P(Z > 2.56)$

$= 1 - \Phi(2.56) = 0.0052$

$\mu = 100 \times 0.0052 = 0.52$

$1 - (e^{-\mu} + e^{-\mu}\mu) = 1 - e^{-.52}(1 + 0.52) = 0.096$

4.1.8 $m = 12, \quad SD = 1.1$

(a) $N(12, 1.1^2)$

$P(11.8 < x < 12.2)$

$= \Phi\left(\frac{12.2-12}{1.1}\right) - \Phi\left(\frac{11.8-12}{1.1}\right)$

(b) $\Phi\left(\frac{12.2-12}{0.11}\right) - \Phi\left(\frac{11.8-12}{0.11}\right)$ $\sigma = \frac{1.1}{\sqrt{100}} = 0.11$

According to CLT, if a sample is sufficiently large, the mean will be normal, so it's not necessary for single measurement to be normal.

4.1.12

(a) X is $[-2, 2]$

$$f(x)dx = P(X \in dx) = \frac{2 \times (2-|x|)dx}{4 \times \frac{1}{2} \times 2 \times 2} = \frac{1}{4}(2-|x|)dx$$

$$f(x) = \begin{cases} \frac{(2-|x|)}{4} & \text{for } -2 \le x \le 2 \\ 0 & \text{else} \end{cases}$$

(b) X is $[-2, 1]$

$$f(x)dx = P(X \in dx) = \frac{(2+x)dx}{\frac{1}{2} \times 3 \times 2} = \frac{1}{3}(2+x)dx$$

$$f(x) = \begin{cases} \frac{1}{3}(2+x)dx & \text{for } -2 \le x \le 1 \\ 0 & \text{else} \end{cases}$$

(c)

$$f(x) = \begin{cases} \text{linear} & x = [-1, 0] \\ \text{constant} & x = [0, 1] \\ \text{linear} & x = [1, 2] \end{cases}$$

area $= 2h$

$h = \frac{1}{2}$

4.2.4

$X \sim Exp(\frac{1}{10})$

(a) $P(X > 20) = e^{-\frac{1}{10} \cdot 20} = e^{-2}$

(b) $\frac{1}{2} = P(X > m) = e^{-m\lambda}$

$$m = \frac{\log 2}{\frac{1}{10}} = 6.93$$

(C) $var(X) = \frac{1}{(\frac{1}{10})^2} = \frac{1}{\frac{1}{100}} = 100$

$sd(X) = \sqrt{var(X)} = 10$

(d) W = average lifetime of 100...

$P(W > 11) = 1 - P(W \le 11)$

$= 1 - (1 - e^{-\frac{1}{10} \cdot 11})$

$= 0.158$

(e) $P(X_1 + X_2 > 22)$

$= P(N < 2)$

$= e^{-2.2} + 2.2e^{-2.2}$

$22\lambda = 22 \times 0.1 = 2.2$

$N \sim Pois(2.2)$

4.2.8

$$f(t) = \frac{1}{3} f_{y_1}(t) + \frac{2}{3} f_{y_2}(t) \quad (t > 0)$$

(a) $$P(X \geq 200) = \frac{1}{3} e^{-\frac{200}{100}} + \frac{2}{3} e^{-\frac{200}{200}}$$

$$= \frac{1}{3} e^{-2} + \frac{2}{3} e^{-1}$$

(b) $$E(X) = \frac{1}{3} E(y_1) + \frac{2}{3} E(y_2)$$

$$= \frac{1}{3} \times 100 + \frac{2}{3} \times 200$$

$$= \frac{500}{3}$$

(c) $$E(X^2) = \frac{1}{3} E(y_1^2) + \frac{2}{3} E(y_2^2)$$

$$= \frac{1}{3} \times 2 \times 100^2 + \frac{2}{3} \times 2 \times 200^2$$

$$= 6 \times 100^2 + \frac{4}{3} \times 200^2 = 60000$$

$$Var(X) = E(X^2) - E(X)^2$$

$$= 60000 - \frac{2500}{9}$$

$$= \frac{29000}{9}$$

4.rev.2

$$f(x) = c(x+x^2) \quad \text{for } 0<x<1$$

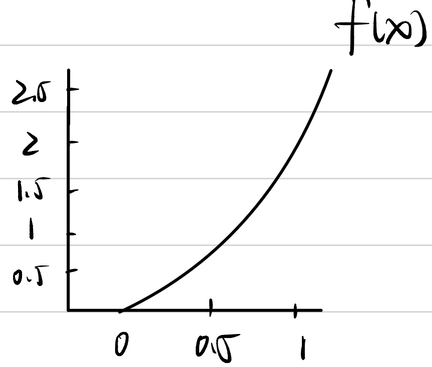
$$\int_0^1 (x+x^2)\,dx = \frac{5}{6}$$

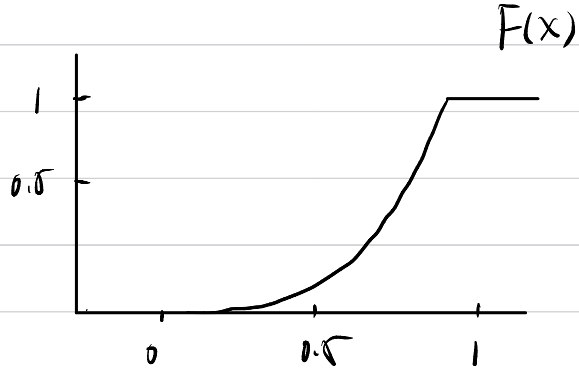
$$\therefore \quad c = \frac{5}{6}$$

$$c\int_{-\infty}^{x} (u+u^2)\,du \quad \begin{cases} 0 & \text{for } x<0 \\ \frac{6}{5}\left(\frac{x^2}{2}+\frac{x^3}{3}\right) & \text{for } 0 \le x \le 1 \\ 1 & \text{for } x>1 \end{cases}$$

$$E(x) = \frac{7}{10}$$

$$sd(x) = \frac{\sqrt{5}}{10}$$





4.rev.14

(a) $T_r \sim \text{Gamma}(4,3)$

$$P(T_4>2) = \sum_{k=0}^{3} e^{-6}\frac{6^k}{k!}$$

$$= e^{-6}(1+6+18+36)$$

(b) $\because$ Poison processes are memoryless

$T_1,\ T_2-T_1,\ T_3-T_2$ are all indepent Exp(λ)

$$P(T_1<1,\ T_2-T_1<1,\ T_3-T_2<1) = P(T_1<1)^3$$

$$= (1-e^{-3})^3$$

(c) $X \sim \text{unif}(0,4)$

$$N_{\text{first 2 min}} = \text{binom}\left(10, \frac{1}{2}\right)$$