

Consider a Gaussian profile

$$\rho(r) = e^{-\alpha r^2}. \quad (1)$$

The total mass is

$$M = 4\pi \int_0^\infty r^2 \rho(r) \, dr = \left(\frac{\pi}{\alpha}\right)^{3/2}. \quad (2)$$

The gravitational potential at a radius  $a$  from the origin can be written as

$$\begin{aligned} \Phi(a) &= \Phi_{\text{in}} + \Phi_{\text{out}} \\ &= -\frac{GM_{\text{in}}}{a} - 4\pi G \int_a^\infty r' \rho(r') \, dr' \\ &= -4\pi G \left\{ \frac{1}{a} \int_0^a r'^2 \rho(r') \, dr' + \int_a^\infty r' \rho(r') \, dr' \right\} \\ &= -4\pi G \left\{ \frac{1}{a} \left[ -\frac{ae^{-\alpha a^2}}{2\alpha} + \frac{\sqrt{\pi} \cdot \text{erf}(a\sqrt{\alpha})}{4\alpha^{3/2}} \right] + \frac{e^{-\alpha a^2}}{2\alpha} \right\} \\ &= -4\pi G \left( \frac{\sqrt{\pi} \cdot \text{erf}(a\sqrt{\alpha})}{4a\alpha^{3/2}} \right). \end{aligned} \quad (3)$$