Analog Ising chain simulation with transmons

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Introduction

Currently, the applications of medium-scale systems composed of superconducting qubits are mostly limited to testing basic principles of quantum computation, demonstrating those as proof-of-concept designs and developing scalable software and hardware interfaces to them. Although this is useful in terms of encouraging future developments in the domain, an alternative approach exists to exploit the built-in quantum properties of such devices to experiment with fundamental physical models. There already were[**?**][**?**][**?**] some successful attempts to use small arrays of superconducting qubits to observe inherently quantum analog behaviour of these systems, and this work is aimed to continue those studies. We are developing a chip to experimentally simulate crystal structure, many-body localization and heat transport properties with a chain of XX-coupled transmons.

Study concept

The Hamiltonian of five transmons interacting only with the nearest neighbors can be written as the sum of the Hamiltonian of the transmons without and with the interaction:

$$\hat{H}_{full} = \sum_{i=1}^{5} \hat{1}_1 \otimes \hat{1}_2 \otimes \ldots \otimes \hat{H}_{i_0} \otimes \ldots \otimes \hat{1}_5 + \sum_{i=1}^{4} \frac{e^2 M^{-1}_{i,j=i+1}}{2} \hat{1}_1 \otimes \hat{1}_2 \otimes \ldots \otimes \hat{n}_i \otimes \hat{n}_j \otimes \ldots \otimes \hat{1}_5,$$

Here $\hat{H}_{i_0}$ - single transmon Hamiltonian, and $M^{-1}_{i,j=i+1}$ - inverse capacity matrix element. Ising Hamiltonian can be written as

$$H = -\frac{1}{2}\sum_i h_i \sigma_{z,i} + \hbar \sum_i J_{x,i}\sigma_{x,i}\sigma_{x,i+1},$$

where $J_{y,i}, J_{z,i} = 0$. Since two above-described Hamiltonians have a similar structure, it becomes possible to carry out analog modeling of the spin chain.
At the same time we can write Hamiltonian of five-resonator chain using classical mechanics:

$$\hat{H}_{cl} = \sum_{i=0}^{4} \left(\frac{\varphi_i^2}{2L_i} + \frac{q_i^2 M^{-1}_{i,i}}{2} \right) + \sum_{i=0}^{3} M^{-1}_{i,i+1} q_i q_{i+1}.$$

Design

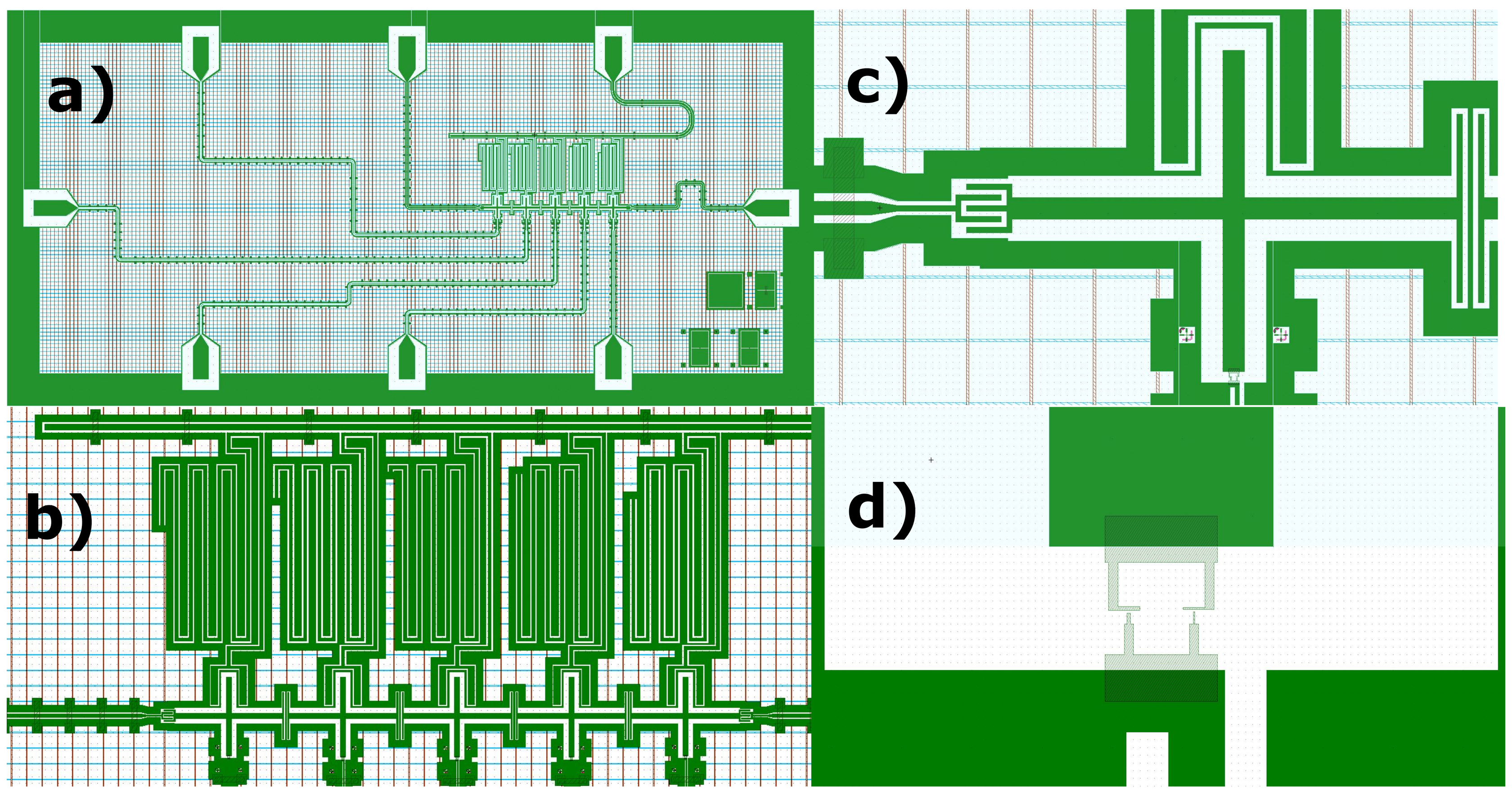


Fig. 1: (a) Sample design: 5-transmon chain with XX-coupling; 5 resonators and flux-bies lines; 2 microwave drive lines(for 1st and 5th qubits). **(b) (c)(d)** Zoomed areas with resonators, microwave antenna and SQUID respectively.

ANSYS Maxwell capacitances simulation

To control the interaction with the nearest neighbors mainly and relaxation in microwave drive antenna, the calculation of the capacitancies of the design was carried out.

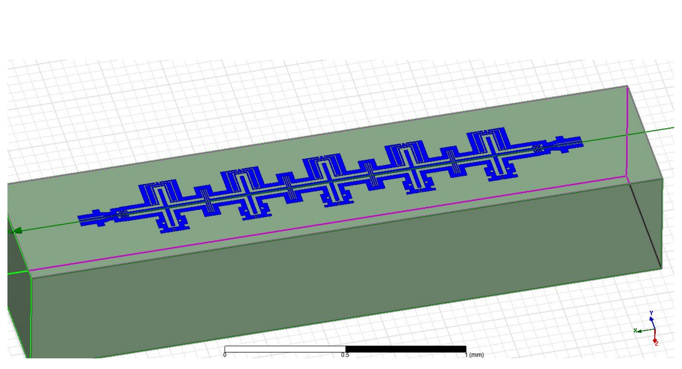
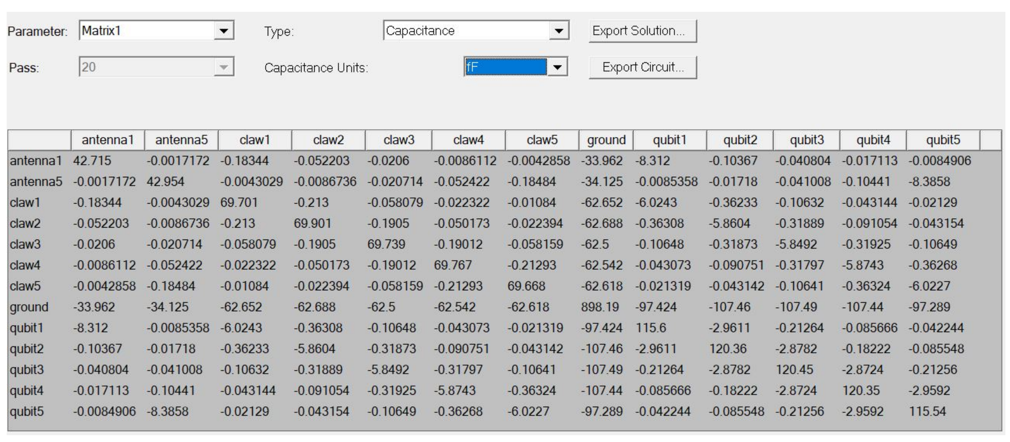


Fig. 2: Calculation of capacitancies by the finite element method.



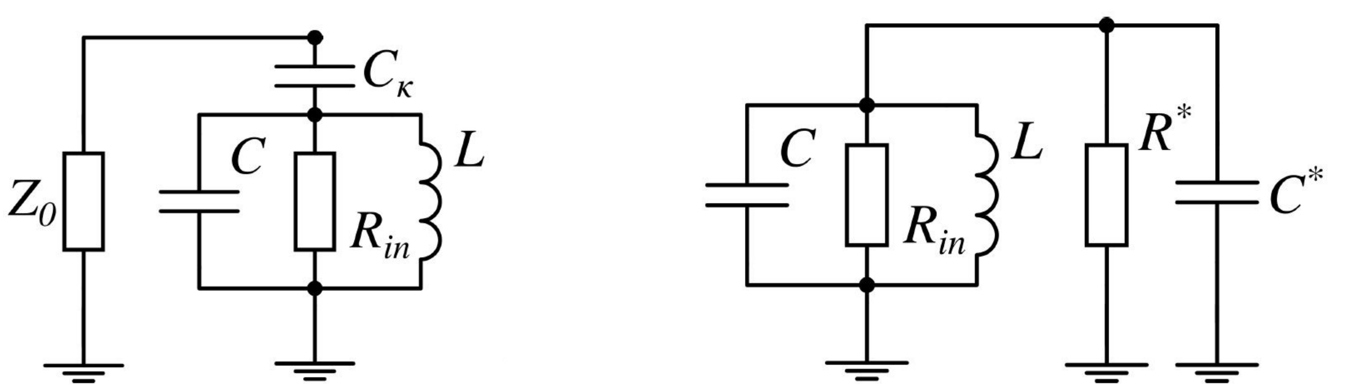
Parameter: Matrix1 | Type: Capacitance | Export Solution...
Pass: 20 | Capacitance Units: fF | Export Circuit...

	antenna1	antenna5	claw1	claw2	claw3	claw4	claw5	ground	qubit1	qubit2	qubit3	qubit4	qubit5
antenna1	42.715	-0.0017172	-0.18344	-0.052203	-0.0206	-0.0086112	-0.0042858	-33.962	-8.312	-0.10367	-0.040804	-0.017113	-0.0084906
antenna5	-0.0017172	42.954	-0.0043029	-0.0086736	-0.020714	-0.052422	-0.18484	-34.125	-0.0085358	-0.01718	-0.041008	-0.10441	-8.3858
claw1	-0.18344	-0.0043029	69.701	-0.213	-0.058079	-0.022322	-0.01084	-62.652	-6.0243	-0.36233	-0.10632	-0.043144	-0.02129
claw2	-0.052203	-0.0086736	-0.213	69.901	-0.1905	-0.050173	-0.022394	-62.688	-0.36308	-5.8604	-0.31889	-0.091054	-0.043154
claw3	-0.0206	-0.020714	-0.058079	-0.1905	69.739	-0.19012	-0.058159	-62.5	-0.10648	-0.31873	-5.8492	-0.31925	-0.10649
claw4	-0.0086112	-0.052422	-0.022322	-0.050173	-0.19012	69.767	-0.21293	-62.542	-0.043073	-0.090751	-0.31797	-5.8743	-0.36268
claw5	-0.0042858	-0.18484	-0.01084	-0.022394	-0.058159	-0.21293	69.668	-62.618	-0.021319	-0.043142	-0.10641	-0.36324	-6.0227
ground	-33.962	-34.125	-62.652	-62.688	-62.5	-62.542	-62.618	898.19	-97.424	-107.46	-107.49	-107.44	-97.289
qubit1	-8.312	-0.0085358	-6.0243	-0.36308	-0.10648	-0.043073	-0.021319	-97.424	115.6	-2.9611	-0.21264	-0.085666	-0.042244
qubit2	-0.10367	-0.01718	-0.36233	-5.8604	-0.31873	-0.090751	-0.043142	-107.46	-2.9611	120.36	-2.8782	-0.18222	-0.085548
qubit3	-0.040804	-0.041008	-0.10632	-0.31889	-5.8492	-0.31797	-0.10641	-107.49	-0.21264	-2.8782	120.45	-2.8724	-0.21256
qubit4	-0.017113	-0.10441	-0.043144	-0.091054	-0.31925	-5.8743	-0.36324	-107.44	-0.085666	-0.18222	-2.8724	120.35	-2.9592
qubit5	-0.0084906	-8.3858	-0.02129	-0.043154	-0.10649	-0.36268	-6.0227	-97.289	-0.042244	-0.085548	-0.21256	-2.9592	115.54

Fig. 3: The table with obtained capacitancies.

Relaxation in antenna

To make relaxation rate of chain in transmission line predominant we made sufficient capacitance between them and calculated this rate using formula $Im(\omega) = \frac{1}{2R^*C}$, where $R^* = \frac{1+\omega^2 C_k^2 Z_0^2}{\omega^2 C_k^2 Z_0}$ and $C^* = \frac{C_k}{1+\omega^2 C_k^2 Z_0^2} \simeq C_k$

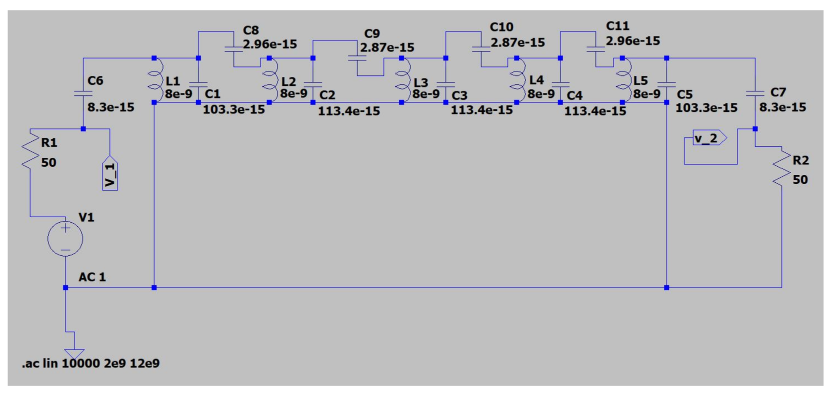


(a) Real world circuit configuration. (b) Norton equivalent of (a).

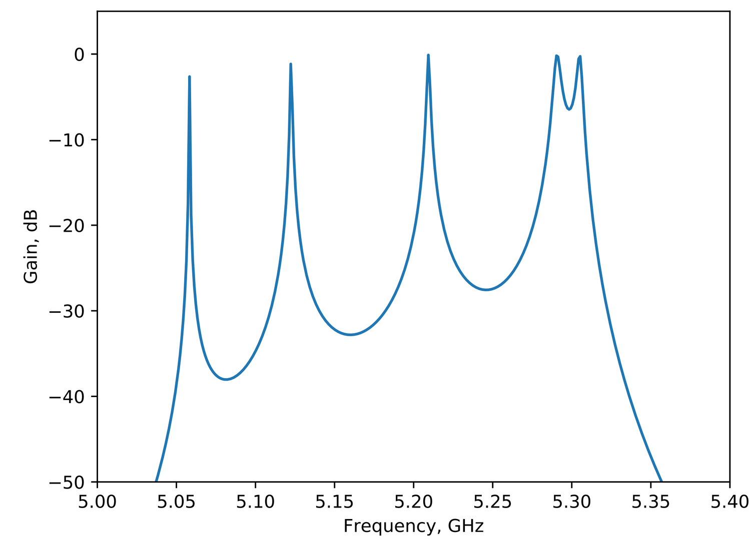
Fig. 4: Equivalent circuit for a $\lambda/4$ TLR(transmission line resonator), capacitively coupled to the transmission line near resonance.

Dynamics simulation. Classical approach

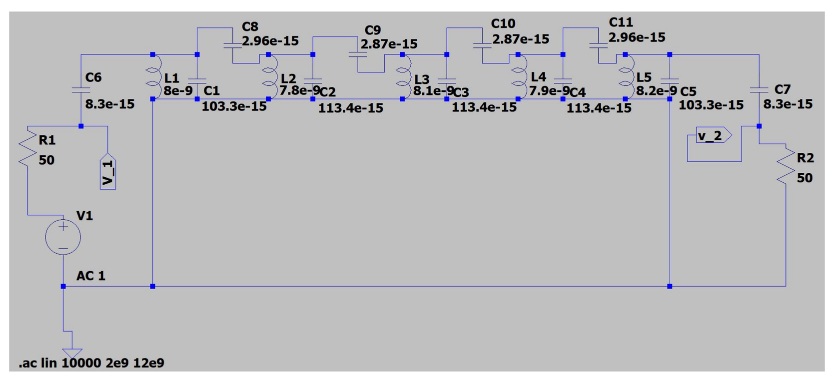
We made calculation of chain resonant frequencies in LTspice program without and with inductance spread:



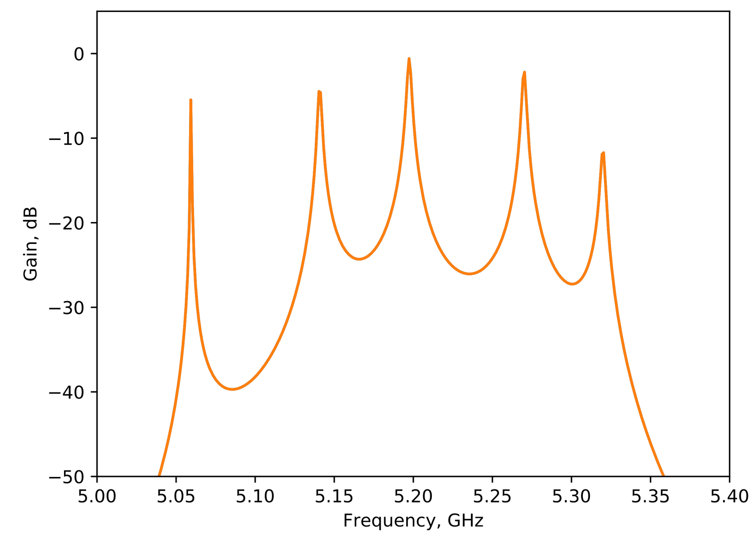
(a): Electrical circut scheme in LTspice with equal inductancies



(b): Classical mode frequencies in GHz: {5.059, 5.124, 5.21, 5.291, 5.306}

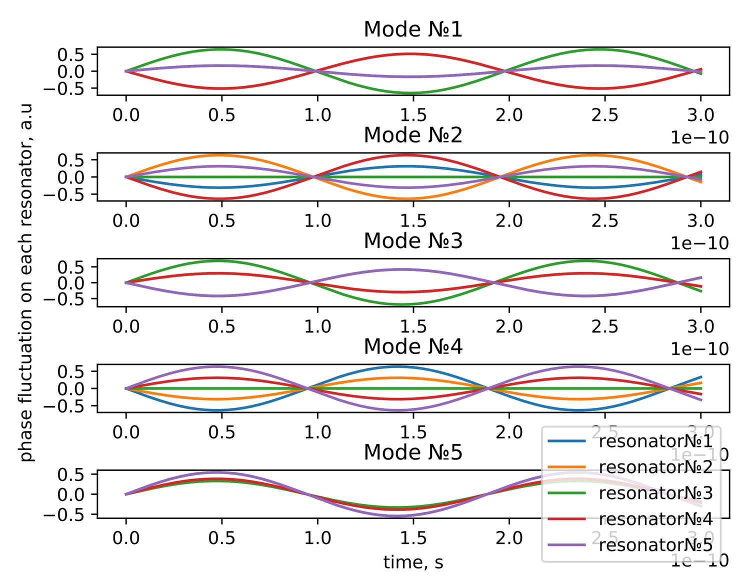


(c): Electrical circut scheme in LTspice with different inductancies

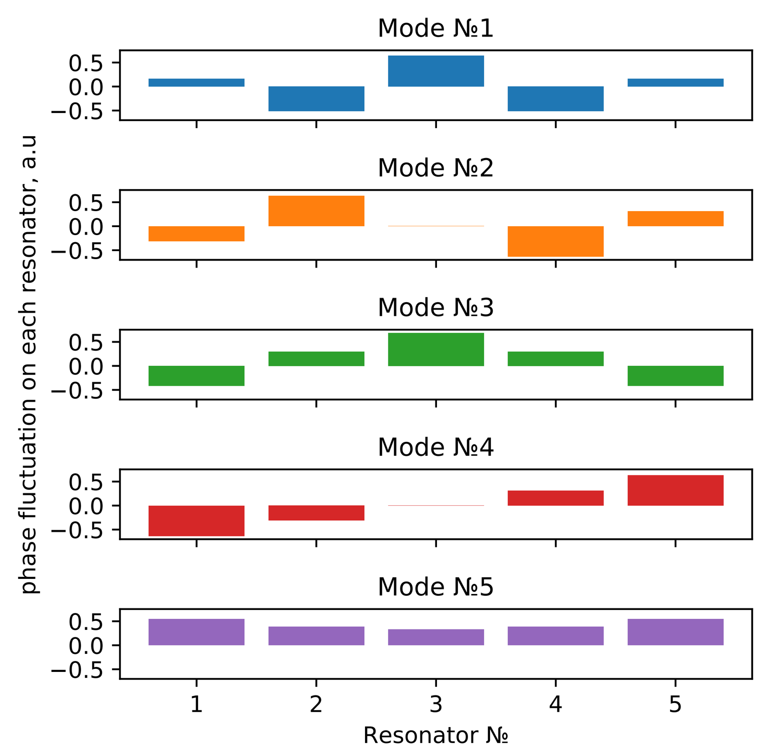


(d): Classical mode frequencies in GHz: {5.06, 5.142, 5.198, 5.271, 5.321}

Using secular equation we calculated classical fluctuations:



(e): Classical modes.



(f): Fluctuation distribution at different modes at 0.05ns. Here we can see amplitudes and relative phases of fluctuations.

Dynamics simulation. Quantum approach

Calculated quantum chain frequencies in GHz:{4.893,4.957,5.041,5.116,5.129}. Using Lindblad Master Equation Solver in QuTiP we can simulate chain dynamics by applying drive with one of modes frequency (see Fig.5). $H_{drive} = f(t)\hat{n}_1 \otimes \hat{1} \otimes \hat{1} \otimes \hat{1} \otimes \hat{1}$, where $f(t) = \hbar f \theta(t - begin)\theta(begin + t_{exc} - t)\sin(\omega_{drive} t + \varphi)$

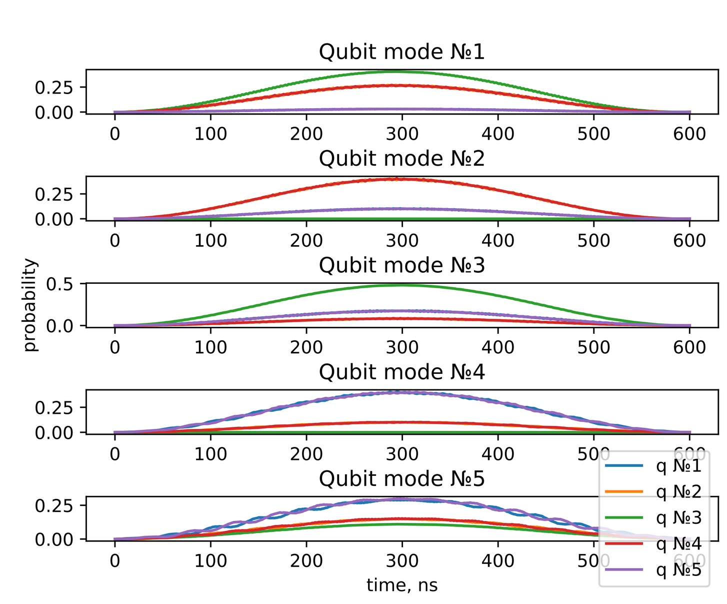


Fig. 5: Chain dynamics. Probability means squared projection on $|00..i..00>$ state for each qubit. 2nd and 4th lines merge into one as well as 1st and 5th.

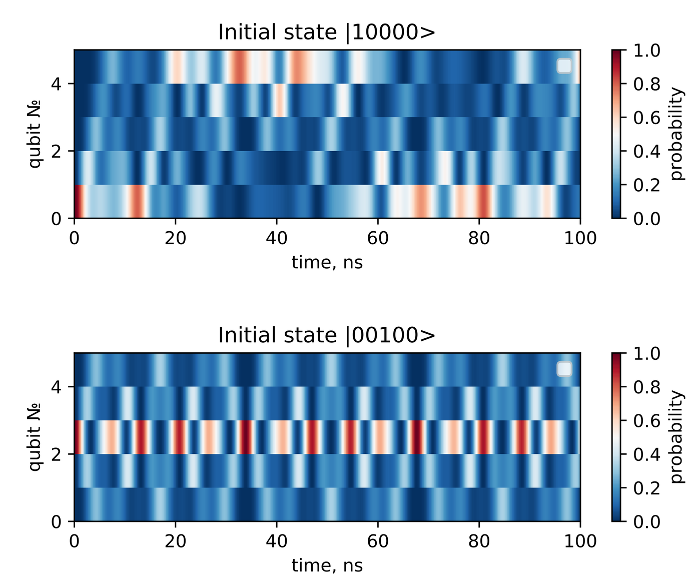


Fig. 6: The time evolution of excitation distribution after the initially localized state by placing one microwave photon into 1st or 3rd qubit. Probability means squared projection on $|00..i..00>$ state for each qubit.

Conclusion

In this paper, we designed and investigated a chain of five transmons. Through modeling, it is shown that this design is suitable for simulating the Ising model. We are looking forward to the opportunity of making an experiment.

Referencies