Homework 3: Props 2.3, 2.5, 2.6, 2.8

> **Prop. 2.3.**
> $1 \in \mathbb{N}$

Proof.
Assume $1 \notin \mathbb{N}$. By prop. 2.2, and the fact that $1 \neq 0$, we know $-1 \in \mathbb{N}$.
According Axiom 2.1(ii), $1 \in \mathbb{N}$, because $(-1)(-1) = 1$.
This forms a contradiction with our assumption that $1 \notin \mathbb{N}$.
Therefore, $1 \in \mathbb{N}$, because $1 \neq 0$ and $-1 \notin \mathbb{N}$.

> **Prop. 2.5.**
> For each $n \in \mathbb{N}$ there exists $m \in \mathbb{N}$ such that $m > n$

Proof.
From the definition of $m > n$, we know $m - n \in \mathbb{N}$, or $m + (-n) \in \mathbb{N}$.
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> **Prop. 2.6.**
> $(m, n \in \mathbb{Z})\ m \leq n \leq m \Rightarrow m = n$

Proof.
Assume $m \neq n$.
From the definition of $m \leq n$, we know $m < n$ or $m = n$, but $m = n$ has been assumed false.
We know $n - m \in \mathbb{N}$, by definition of $m < n$.
From $m \leq n \leq m$, we know $n - m \in \mathbb{N}$ and $m - n \in \mathbb{N}$
By Axiom 2.1(i),
$(n - m) + (m - n) \in \mathbb{N}$
$\Rightarrow (n - n) + (-m + n) \in \mathbb{N}$
$\Rightarrow 0 + 0 \in \mathbb{N}$
$\Rightarrow 0 \in \mathbb{N}$.
However, by Axiom 2.1(iii), $0 \notin \mathbb{N}$.
Assuming the law of excluded middle, by contradiction, $m = n$.

> **Prop. 2.8.**
> $(m, n \in \mathbb{Z})$ Exactly one of the following is true: $m < n$, $m = n$, $m > n$

Proof.

First prove $m < n$ and $m > n$ cannot both be true.

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