Math 323 - Review 2 - Spring 25

1.) Find the first partial derivatives of each function.

(a.) $f(x, y) = x^5y^3 + 3x^2y^4$

(b.) $f(x, y) = \dfrac{xy^2}{x^2 + y}$

(c.) $f(x, y, z) = xy\tan^{-1}(y^2z)$

(d.) $f(x, y, z, t) = e^{yz+t}\cos(xz)$

2.) Find all second partial derivatives of $f(x, y) = \sin^3(2x + 3y)$.

3.) Let $w = \dfrac{xyz^2}{\sqrt{2x - 3y}}$. Find $\dfrac{\partial^4 w}{\partial y \partial x \partial z^2}$.

4.) Use the Chain Rule to find dw/dt.

$w = \ln(\sqrt{x^2 + y^2 + z^2}),\ x = \sin(t),\ y = \cos(t),\ z = \tan(t)$

5.) Let $w = x^2\sin(y) + ye^{xy}$, where $x = s + 2t$ and $y = st$. Use the Chain Rule to find $\partial w/\partial s$ and $\partial w/\partial t$.

6.) Find the directional derivative of the function at the given point in the direction of the vector v.

$f(x, y) = 2\sqrt{x} - y^2, \quad (1, 2), \quad v =< 3, -4 >$

7.) Find the maximum rate of change of f at the given point and the direction in which it occurs.

$f(x, y, z) = \tan(x + 2y + 3z),\ (-5, 1, 1)$

8.) Find an equation of the tangent plane to the given surface at the specified point.

$x^2 - 2y^2 + z^2 + yz = 2,\ (2, 1, -1)$

9.) Define $g(x, y, z) = z^{xyz}$, $z > 0$. Find $\nabla g(x, y, z)$.

10.) Assume $f : \mathbb{R}^2 \to \mathbb{R}$ is differentiable, and suppose the directional derivative of f at $O(1,-3)$ in the direction towards the point $P(4,-1)$ is $4/\sqrt{13}$ and the directional derivative of f at $O(1,-3)$ in the direction towards the point $Q(3,2)$ is $-1/\sqrt{29}$. Find the directional derivative of f at $O(1,-3)$ in the direction towards the point $S(5,-2)$.

11.) Let $\mathcal{C}$ be the curve of intersection of the ellipsoid $x^2 + 2y^2 + 3z^2 = 39$ and the plane $3x + y - 7z = 0$. Find parametric equations for the tangent line to $\mathcal{C}$ at the point $(5,-1,2)$.

12.) Find the two points on the hyperboloid $x^2 + 4y^2 - z^2 = 4$ where the tangent plane is parallel to the plane $2x + 2y + z = 5$.

13.) Find and classify the critical points of each function.

(a.) $f(x,y) = x^2 - xy + y^2 + 9x - 6y + 10$

(b.) $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$

(c.) $f(x,y) = 3xy - x^2y - xy^2$

(d.) $f(x,y) = x^4 + y^4 - 4xy + 2$

(e.) $f(x,y) = x^2y + xy^2 + x + y + 1$

14.) Find the absolute maximum and minimum values of f on the set D. Give all points where these extreme values occur.

(a.) $f(x,y) = 3 + xy - x - 2y$, D is the triangular region with vertices $(1,0)$,$(5,0)$, and $(1,4)$

(b.) $f(x,y) = x^2 + y^2 + x^2y + 4$, $D = \{\, (x,y) \in \mathbb{R}^2 \,:\, |x| \le 1, |y| \le 1\}$

15.) Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint(s). Give all points where these extreme values occur.

(a.) $f(x,y) = x^2y;\ x^2 + 2y^2 = 6$

(b.) $f(x,y,z) = 2x + 6y + 10z;\ x^2 + y^2 + z^2 = 35$

(c.) $f(x,y,z) = x + 2y;\ x + y + z = 1,\ y^2 + z^2 = 4$

16.) Evaluate.

(a.) $\int_0^1 \int_0^1 xy\sqrt{x^2+y^2}\, dy\, dx$

(b.) $\int\int_D \dfrac{x}{1+xy}\, dA,\ \ D = [0,1] \times [0,1]$

(c.) $\int\int_D x\cos(y)\, dA,\ \ D$ is bounded by $y = 0,\ \ y = x^2$, and $x = 1$

(d.) $\int\int_D y^3\, dA,\ \ D$ is the triangular region with vertices $(0,2)$,$(1,1)$, and $(3,2)$

(e.) $\int\int_D (x+y)\, dA,\ \ D$ is the region bounded by $y = x^2$ and $x = y^2$

17.) Evaluate.

(a.) $\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy$

(b.) $\int_0^4 \int_{\sqrt{x}}^2 \dfrac{1}{y^3+1}\, dy\, dx$

18.) Evaluate $\displaystyle\int\int_D \cos(x^2+y^2)\, dA$, where D is the region that lies above the x-axis within the circle $x^2+y^2=9$.

19.) Use a double integral to find the area of one loop of the rose $r = \sin(2\theta)$.

20.) A rectangular box must be built to have a volume of exactly 6480 cubic feet. The material used for the top and bottom faces of the box costs \$5 per square foot, the material used for the front and back faces costs \$3 per square foot, and the material used for the left and right faces costs \$2 per square foot. Find the total cost of materials for the cheapest box that can be built according to these specifications.

21.) Given that a certain pentagon has the shape of a rectangle surmounted by an upright isosceles triangle, and that the perimeter of this pentagon is exactly 1, what is the greatest area that this pentagon could possibly have?