Math 323 - Final Review - Spring 25

1.) Find an equation of the plane containing the two parallel lines given below.

$\ell_1 : x = 1 + 3t,\ \ y = 4 - t,\ \ z = -3 + 2t$
$\ell_2 : x = 6t,\ \ y = 1 - 2t,\ \ z = -1 + 4t$

2.) Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

$x = 4t^4 + 1,\ \ y = t^8 + t^6,\ \ z = t^5 + 5\ ;\ \ (5, 2, 4)$

3.) Find the first partial derivatives of each function.

(a.) $f(x, y) = \dfrac{xy^2}{x^2 + y}$

(b.) $f(x, y, z) = xy\tan^{-1}(y^2 z)$

4.) Find the directional derivative of the function at the given point in the direction of the vector v.

$f(x, y) = 2\sqrt{x} - y^2,\ \ (1, 2),\ \ v =< 3, -4 >$

5.) Find an equation of the tangent plane to the given surface at the specified point.

$x^2 - 2y^2 + z^2 + yz = 2,\ \ (2, 1, -1)$

6.) Find and classify the critical points of each function.

(a.) $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$

(b.) $f(x, y) = 3xy - x^2y - xy^2$

7.) Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint(s). Give all points where these extreme values occur.

(a.) $f(x, y) = x^2y;\ x^2 + 2y^2 = 6$

(b.) $f(x, y, z) = 2x + 6y + 10z;\ x^2 + y^2 + z^2 = 35$

8.) Evaluate.

(a.) $\int\int_D (x+y)\, dA$, D is the region bounded by $y = x^2$ and $x = y^2$

(b.) $\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy$

9.) Evaluate.

(a.) $\displaystyle\int\int\int_E 6xy\, dV$, where E lies under the plane $z = 1 + x + y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.

(b.) $\displaystyle\int\int\int_E x\, dV$, where E is bounded by the paraboloid $x = 4y^2 + 4z^2$ and the plane $x = 4$.

(c.) $\displaystyle\int\int\int_E xyz\, dV$, where E is the solid tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(1,1,0)$, and $(1,0,1)$.

10.) Evaluate.

(a.) $\displaystyle\int\int\int_H (9 - x^2 - y^2)\, dV$

H is the solid hemisphere $x^2 + y^2 + z^2 \leq 9$, $z \geq 0$.

(b.) $\displaystyle\int\int\int_E x^2\, dV$

E is bounded by the xz-plane and the hemispheres $y = \sqrt{9 - x^2 - z^2}$ and $y = \sqrt{16 - x^2 - z^2}$.

(c.) $\displaystyle\int_0^1 \int_{-\sqrt{1-y^2}}^0 \int_0^{\sqrt{1-x^2-y^2}} x\, dz\, dx\, dy$

11.) The given vector field $\mathbf{F}$ is conservative. Find a function f such that $\mathbf{F} = \nabla f$.

(a.) $\mathbf{F}(x,y) = (xy\cos(xy) + \sin(xy))\,\mathbf{i} + x^2\cos(xy)\,\mathbf{j}$

(b.) $\mathbf{F}(x,y,z) = <\ 2x + y - 2xz\ ,\ x + z^3 + 1\ ,\ 3yz^2 - x^2 - 2z\ >$

12.) Evaluate.

(a.) $\displaystyle\int_C xyz^2\,ds$

C is the line segment from $(-1,5,0)$ to $(1,6,4)$.

(b.) $\displaystyle\int_C x^5y^4\,ds$

C is the right half of the circle $x^2+y^2=4$.

(c.) $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$

$\mathbf{F}(x,y,z)=\sin(x)\,\mathbf{i}+\cos(y)\,\mathbf{j}+xz\,\mathbf{k}$.

C is given by $\mathbf{r}(t)=t^3\,\mathbf{i}-t^2\,\mathbf{j}+t\,\mathbf{k}\,,\ 0\le t\le 1$.

(d.) $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$

$\mathbf{F}(x,y,z)=<\ 2x+y-2xz\,,\ x+z^3+1\,,\ 3yz^2-x^2-2z\ >$.

C is any path from $(1,0,-2)$ to $(0,2,3)$. (see 11.)(b.))

(e.) $\displaystyle\oint_C (xy+e^{\sqrt{x}})\,dx+(x^2y^3-\sin(y^2))\,dy$

C is the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$.

(f.) $\displaystyle\oint_C y^3\,dx-x^3\,dy$

C is the circle $x^2+y^2=4$.

(g.) $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$

$\mathbf{F}(x,y)=(\sqrt{x}+y^3)\,\mathbf{i}+(x^2+\sqrt{y})\,\mathbf{j}$.

C consists of the arc of the curve $y=\sin(x)$ from $(0,0)$ to $(\pi,0)$ and the line segment from $(\pi,0)$ to $(0,0)$.

13.) Find an equation of the tangent plane to the given parametric surface at the specified point.

$x = u + 2v, \quad y = 3u - v, \quad z = v^2 \;\; ; \;\; (-3, 5, 4)$

14.) Evaluate.

(a.) $\displaystyle\int\int_S x^6 \, dS$

S is the unit sphere $x^2 + y^2 + z^2 = 1$.

(b.) $\displaystyle\int\int_S \mathbf{F} \cdot d\mathbf{S}$

$\mathbf{F}(x, y, z) = x\,\mathbf{i} - z\,\mathbf{j} + y\,\mathbf{k}$.

S is the part of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant, with orientation toward the origin.

(c.) $\displaystyle\int\int_S \mathbf{F} \cdot d\mathbf{S}$

$\mathbf{F}(x, y, z) = y\,\mathbf{i} + x\,\mathbf{j} + xyz\,\mathbf{k}$.

S is the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the square $[0, 1] \times [0, 1]$, and has upward orientation.

(d.) $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$

$\mathbf{F}(x, y, z) = xy\,\mathbf{i} + 2z\,\mathbf{j} + 3y\,\mathbf{k}$.

C is the curve of intersection of the plane $x + z = 5$ and the cylinder $x^2 + y^2 = 9$.
C is oriented counterclockwise as viewed from above.

(e.) $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$

$\mathbf{F}(x, y, z) = (x + y^2)\,\mathbf{i} + (y + z^2)\,\mathbf{j} + (z + x^2)\,\mathbf{k}$.

C is the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.
C is oriented counterclockwise as viewed from above.

(f.) $\displaystyle\int\int_S \mathbf{F}\cdot d\mathbf{S}$

$\mathbf{F}(x,y,z) = (\cos(z) + xy^2)\,\mathbf{i} + (xe^{-z})\,\mathbf{j} + (\sin(y) + x^2 z)\,\mathbf{k}$.
S is the surface of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$.
S has outward orientation.

(g.) $\displaystyle\int\int_S \mathbf{F}\cdot d\mathbf{S}$

$\mathbf{F}(x,y,z) = (y^4 + z^4)\,\mathbf{i} + (x^3 + y^3 + z^3)\,\mathbf{j} + (3x^2 z + 3xy^2)\,\mathbf{k}$.
S is the unit sphere $x^2 + y^2 + z^2 = 1$ with outward orientation.