Math 323 - Review 2 Answers - Spring 25

1.)(a.) $f_x(x,y) = 5x^4y^3 + 6xy^4,\ f_y(x,y) = 3x^5y^2 + 12x^2y^3$

(b.) $f_x(x,y) = \frac{y^3 - x^2y^2}{(x^2+y)^2},\ f_y(x,y) = \frac{2x^3y + xy^2}{(x^2+y)^2}$

(c.) $f_x(x,y,z) = y\tan^{-1}(y^2z),\ f_y(x,y,z) = x\tan^{-1}(y^2z) + \frac{2xy^2z}{1+y^4z^2},\ f_z(x,y,z) = \frac{xy^3}{1+y^4z^2}$

(d.) $f_x(x,y,z,t) = -ze^{yz+t}\sin(xz),\ f_y(x,y,z,t) = ze^{yz+t}\cos(xz),$
$f_z(x,y,z,t) = e^{yz+t}(y\cos(xz) - x\sin(xz)),\ f_t(x,y,z,t) = e^{yz+t}\cos(xz)$

2.) $f_{xx}(x,y) = 24\sin(2x+3y)\cos^2(2x+3y) - 12\sin^3(2x+3y)$
$f_{yy}(x,y) = 54\sin(2x+3y)\cos^2(2x+3y) - 27\sin^3(2x+3y)$
$f_{xy}(x,y) = f_{yx}(x,y) = 36\sin(2x+3y)\cos^2(2x+3y) - 18\sin^3(2x+3y)$

3.) $\frac{\partial^4 w}{\partial y \partial x \partial z^2} = \frac{4x^2 - 21xy + 9y^2}{(2x-3y)^{5/2}}$

4.) $\frac{dw}{dt} = \frac{x\cos(t) - y\sin(t) + z\sec^2(t)}{x^2+y^2+z^2} = \tan(t)$

5.) $\partial w/\partial s = 2x\sin(y) + x^2t\cos(y) + y^2e^{xy} + xyte^{xy} + te^{xy}$
$\partial w/\partial t = 4x\sin(y) + x^2s\cos(y) + 2y^2e^{xy} + xyse^{xy} + se^{xy}$

6.) $19/5$

7.) $\sqrt{14}$; direction specified by $< 1, 2, 3 >$

8.) $4x - 5y - z = 4$

9.) $\nabla g(x,y,z) = z^{xyz} < yz\ln(z),\ xz\ln(z),\ xy\ln(z) + xy >$

10.) $7/\sqrt{17}$

11.) $x = 5 + 8t,\ y = -1 + 53t,\ z = 2 + 11t$

12.) $(2, \frac{1}{2}, -1)$ and $(-2, -\frac{1}{2}, 1)$

13.)(a.) local min. value of -11 at $(-4, 1)$

(b.) local max. value of $\frac{125}{27}$ at $(-\frac{5}{3}, 0)$
local min. value of 0 at $(0, 0)$
saddle points at $(-1, 2)$ and $(-1, -2)$

(c.) local max. value of 1 at $(1, 1)$
saddle points at $(0, 0)$, $(0, 3)$, and $(3, 0)$

(d.) local min. value of 0 at $(1, 1)$ and $(-1, -1)$
saddle point at $(0, 0)$

(e.) saddle points at $(-1, 1)$ and $(1, -1)$

14.)(a.) absolute max. value of 2 at $(1, 0)$ and $(3, 2)$
absolute min. value of -2 at $(1, 4)$ and $(5, 0)$

(b.) absolute max. value of 7 at $(1, 1)$ and $(-1, 1)$
absolute min. value of 4 at $(0, 0)$

15.)(a.) max. value of 4 at $(2, 1)$ and $(-2, 1)$
min. value of -4 at $(2, -1)$ and $(-2, -1)$

(b.) max. value of 70 at $(1, 3, 5)$
min. value of -70 at $(-1, -3, -5)$

(c.) max. value of $1 + 2\sqrt{2}$ at $(1, \sqrt{2}, -\sqrt{2})$
min. value of $1 - 2\sqrt{2}$ at $(1, -\sqrt{2}, \sqrt{2})$

16.)(a.) $\frac{2}{15}(2\sqrt{2} - 1)$

(b.) $2\ln(2) - 1$

(c.) $\frac{1}{2}(1 - \cos(1))$

(d.) $\frac{147}{20}$

(e.) $\frac{3}{10}$

17.)(a.) $\frac{1}{6}(e^9 - 1)$

(b.) $\frac{2}{3}\ln(3)$

18.) $\frac{\pi}{2}\sin(9)$

19.) $\frac{\pi}{8}$

20.) \$6480

21.) $\frac{2-\sqrt{3}}{4}$