Math 323 - Review 1 - Spring 25

1.) Determine whether the given vectors are orthogonal, parallel, or neither.

(a.) $\mathbf{u} =< 1, -3, 7 >$ $\mathbf{v} =< -6, 5, 3 >$

(b.) $\mathbf{u} =< 3, 5, -2 >$ $\mathbf{v} =< 2, -1, 4 >$

(c.) $\mathbf{u} =< 8, -4, 12 >$ $\mathbf{v} =< -6, 3, -9 >$

2.) Find an equation of the plane through the points $(2, -1, 4)$, $(3, 2, -1)$, and $(1, 3, 5)$.

3.) Find the intersection point of the given line and the given plane:

$\ell : x = 3 + t,\ \ y = 1 + 2t,\ \ z = 2 - 3t$
$P : x - y + 2z = 13$

4.) Let the line ℓ_0 be defined by $x = 1 - t,\ y = 2 + 3t,\ z = 1 - 2t$. Determine if each given line is parallel to, skew to, or intersects ℓ_0.

$\ell_1 : x = 3 + 2t,\ \ y = 1 - 6t,\ \ z = 4t$
$\ell_2 : x = 2 + t,\ \ y = 5 - 2t,\ \ z = 1 + 3t$
$\ell_3 : x = 6 + 2t,\ \ y = 1 + t,\ \ z = 5 + t$

5.) Find an equation of the plane containing the two parallel lines given below.

$\ell_1 : x = 1 + 3t,\ \ y = 4 - t,\ \ z = -3 + 2t$
$\ell_2 : x = 6t,\ \ y = 1 - 2t,\ \ z = -1 + 4t$

6.) Find parametric equations for the line of intersection of the planes $2x - y + 3z = 5$ and $3x - 2y - z = 7$.

7.) Find parametric equations for the line that passes through the point p and intersects the line ℓ orthogonally.

$p(-2, 1, -10)$
$\ell : x = -1 + 2t,\ \ y = 3 - t,\ \ z = 2 + 5t$

8.) Let $\mathbf{r}(t) =< e^{t\sin(3t)}, \dfrac{5t-1}{\sqrt{3t^2+2}}, \sec^3(\sqrt{1+t^2}) >$. Find $\mathbf{r}'(t)$.

9.) Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

$x = 4t^4 + 1,\ \ y = t^8 + t^6,\ \ z = t^5 + 5\ ;\ \ (5, 2, 4)$

10.) Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) =< t^2,\, e^t,\, \sin(t) >$ and $\mathbf{r}(0) =< 1, 3, 2 >$.

11.) Evaluate $\displaystyle\int \left(\frac{e^{2t}}{e^t+1}\,\mathbf{i} + \frac{e^t}{e^{2t}+1}\,\mathbf{j} + \frac{e^t}{e^{2t}-1}\,\mathbf{k} \right) dt.$

12.) Find the length of the curve.

(a.) $\mathbf{r}(t) =< t^2,\, 9t,\, 4t^{3/2} >,\ 1 \le t \le 4$

(b.) $\mathbf{r}(t) =< t,\, \ln(t),\, 2\sqrt{2}\,t >,\ 1 \le t \le 2$

13.) The position of a particle is given by $\mathbf{r}(t) =< \frac{1}{2}t,\, \ln(1-t^2),\, -\frac{\sqrt{3}}{2}t >$ for $0 \le t \le \frac{1}{2}$.

(a.) Show the speed of the particle during this time interval is given by $s(t) = \dfrac{1+t^2}{1-t^2}$.

(b.) Find the total distance traveled by the particle during this time interval.

14.) A projectile is fired with an initial speed of 100 m/s and angle of elevation 60°. Find (a.) the range of the projectile and (b.) the maximum height reached by the projectile.

15.) The acceleration, initial velocity, and initial position of a particle traveling through space are given below. The particle's trajectory intersects the surface $x^2 - xz - y = 0$ exactly twice. Find these two intersection points.

$\mathbf{a}(t) =< 2,\, 12t+12,\, 2 >,\ \ \mathbf{v}(0) =< 3,\, 9,\, 1 >,\ \ \mathbf{r}(0) =< 1,\, -2,\, 0 >$

16.) Find a unit vector making an angle of 30° with $\mathbf{i}$ and making equal angles with $\mathbf{j}$ and $\mathbf{k}$.

17.) The *twisted cubic* given by $\mathbf{r}(t) =< t^3, t^2, t >$ intersects the plane $2x + 9y - 11z = 60$ three times. One of these intersection points is $(-27, 9, -3)$. Find the other two.

18.) Let the sphere $\mathcal{S}$ and the line L be given as below. Exactly how close does the line L get to the sphere $\mathcal{S}$?

$\mathcal{S} : x^2 + y^2 + z^2 - 2x + 6y - 4z + 5 = 0$
$L : x = -1 + t,\ \ y = -3 - 2t,\ \ z = 10 + 7t$

19.) The lines given below are skew. Find parametric equations for the line which intersects them both orthogonally.

$\ell_1 : x = 6 + 2t,\ \ y = -1 + t,\ \ z = -5 - 2t$
$\ell_2 : x = 8 - 5t,\ \ y = -1 + 2t,\ \ z = 3 - t$