ELECTRIC MACHINERY and POWER SYSTEM
FUNDAMENTALS

Stephen J. Chapman

Instructor's Manual

to accompany

Chapman

Electric Machinery and Power System Fundamentals

First Edition

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Instructor's Manual to accompany *Electric Machinery and Power System Fundamentals*, First Edition

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PREFACE

TO THE INSTRUCTOR

This Instructor's Manual is intended to accompany the third edition of *Electric Machinery and Power System Fundamentals*. To make this manual easier to use, it has been made self-contained. Both the original problem statement and the problem solution are given for each problem in the book. This structure should make it easier to copy pages from the manual for posting after problems have been assigned.

Many of the problems in Chapters 2, 5, 6, and 9 require that a student read one or more values from a magnetization curve. The required curves are given within the textbook, but they are shown with relatively few vertical and horizontal lines so that they will not appear too cluttered. Electronic copies of the corresponding open-circuit characteristics, short-circuit characteristics, and magnetization curves as also supplied with the book. They are supplied in two forms, as MATLAB MAT-files and as ASCII text files. Students can use these files for electronic solutions to homework problems. The ASCII files are supplied so that the information can be used with non-MATLAB software.

The solutions in this manual have been checked carefully, but inevitably some errors will have slipped through. If you locate errors which you would like to see corrected, please feel free to contact me at the address shown below, or at my email address schapman@tpgi.com.au. I greatly appreciate your input! My physical and email addresses may change from time to time, but my contact details will always be available at the book's Web site, which is http://www.mhhe.com/engcs/electrical/chapman/.

Thank you.

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Chapter 1: *Mechanical and Electromagnetic Fundamentals*

1-1. A motor's shaft is spinning at a speed of 1800 r/min. What is the shaft speed in radians per second?

SOLUTION The speed in radians per second is

$$\omega = \left(1800 \text{ r/min}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right) = 188.5 \text{ rad/s}$$

1-2. A flywheel with a moment of inertia of 4 kg · m² is initially at rest. If a torque of 5 N · m (counterclockwise) is suddenly applied to the flywheel, what will be the speed of the flywheel after 5 s? Express that speed in both radians per second and revolutions per minute.
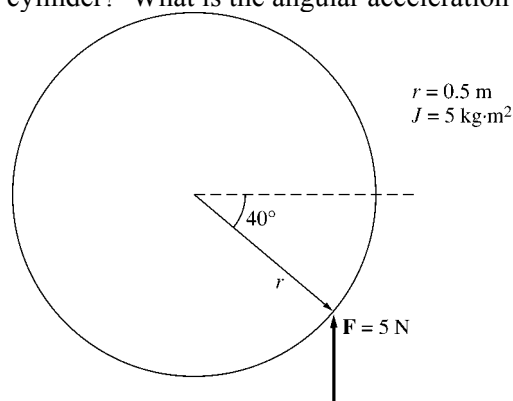
SOLUTION The speed in radians per second is:

$$\omega = \alpha \, t = \left(\frac{\tau}{J}\right) t = \frac{5 \text{ N} \cdot \text{m}}{4 \text{ kg} \cdot \text{m}^2}\left(5 \text{ s}\right) = 6.25 \text{ rad/s}$$

The speed in revolutions per minute is:

$$n = \left(6.25 \text{ rad/s}\right)\left(\frac{1 \text{ r}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 59.7 \text{ r/min}$$

1-3. A force of 5 N is applied to a cylinder, as shown in Figure P1-1. What are the magnitude and direction of the torque produced on the cylinder? What is the angular acceleration α of the cylinder?

SOLUTION The magnitude and the direction of the torque on this cylinder is:

$$\tau_{\text{ind}} = rF \sin \theta, \text{ CCW}$$
$$\tau_{\text{ind}} = \left(0.5 \text{ m}\right)\left(5 \text{ kg} \cdot \text{m}^2\right)\sin 40° = 1.607 \text{ N} \cdot \text{m}, \text{ CCW}$$

1-4. A motor is supplying 70 N · m of torque to its load. If the motor's shaft is turning at 1500 r/min, what is the mechanical power supplied to the load in watts? In horsepower?

SOLUTION The mechanical power supplied to the load is

$$P = \tau\omega = \left(70 \text{ N} \cdot \text{m}\right)\left(1500 \text{ r/min}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right) = 11,000 \text{ W}$$

$$P = \left(11,000 \text{ W}\right)\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = 14.7 \text{ hp}$$

1-5. A ferromagnetic core is shown in Figure P1-2. The depth of the core is 5 cm. The other dimensions of the core are as shown in the figure. Find the value of the current that will produce a flux of 0.003 Wb.

With this current, what is the flux density at the top of the core? What is the flux density at the right side of the core? Assume that the relative permeability of the core is 1000.
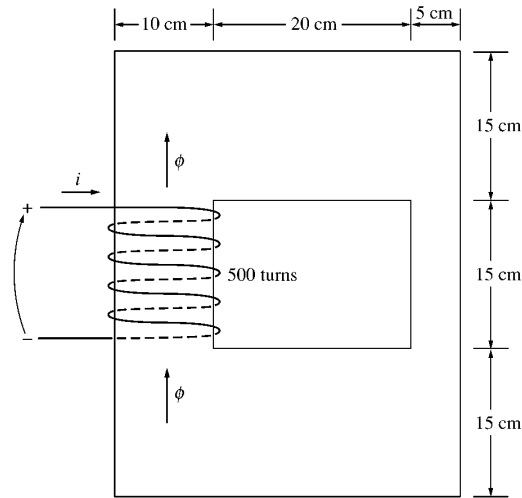
SOLUTION There are three regions in this core. The top and bottom form one region, the left side forms a second region, and the right side forms a third region. If we assume that the mean path length of the flux is in the center of each leg of the core, and if we ignore spreading at the corners of the core, then the path lengths are $l_1 = 2(27.5 \text{ cm}) = 55 \text{ cm}$, $l_2 = 30 \text{ cm}$, and $l_3 = 30 \text{ cm}$. The reluctances of these regions are:

$$\mathcal{R}_1 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.55 \text{ m}}{(1000)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.15 \text{ m})} = 58.36 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.30 \text{ m}}{(1000)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.10 \text{ m})} = 47.75 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.30 \text{ m}}{(1000)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.05 \text{ m})} = 95.49 \text{ kA} \cdot \text{t/Wb}$$

The total reluctance is thus

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 = 58.36 + 47.75 + 95.49 = 201.6 \text{ kA} \cdot \text{t/Wb}$$

and the magnetomotive force required to produce a flux of 0.003 Wb is

$$\mathcal{F} = \phi \, \mathcal{R} = (0.003 \text{ Wb})(201.6 \text{ kA} \cdot \text{t/Wb}) = 605 \text{ A} \cdot \text{t}$$

and the required current is

$$i = \frac{\mathcal{F}}{N} = \frac{605 \text{ A} \cdot \text{t}}{500 \text{ t}} = 1.21 \text{ A}$$

The flux density on the top of the core is

$$B = \frac{\phi}{A} = \frac{0.003 \text{ Wb}}{(0.15 \text{ m})(0.05 \text{ m})} = 0.4 \text{ T}$$

The flux density on the right side of the core is

$$B = \frac{\phi}{A} = \frac{0.003 \text{ Wb}}{(0.05 \text{ m})(0.05 \text{ m})} = 1.2 \text{ T}$$

1-6. A ferromagnetic core with a relative permeability of 2000 is shown in Figure P1-3. The dimensions are as shown in the diagram, and the depth of the core is 7 cm. The air gaps on the left and right sides of the

core are 0.050 and 0.070 cm, respectively. Because of fringing effects, the effective area of the air gaps is 5 percent larger than their physical size. If there are 300 turns in the coil wrapped around the center leg of the core and if the current in the coil is 1.0 A, what is the flux in each of the left, center, and right legs of the core? What is the flux density in each air gap?
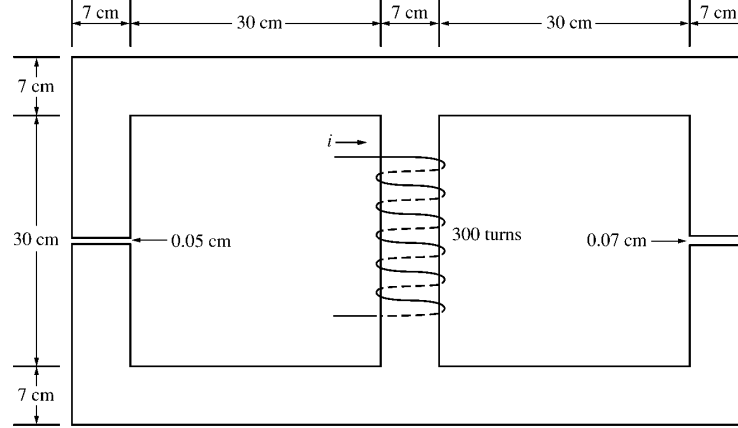
SOLUTION This core can be divided up into five regions. Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the left-hand air gap, $\mathcal{R}_3$ be the reluctance of the right-hand portion of the core, $\mathcal{R}_4$ be the reluctance of the right-hand air gap, and $\mathcal{R}_5$ be the reluctance of the center leg of the core. Then the total reluctance of the core is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.11\,\text{m}}{(2000)(4\pi \times 10^{-7}\ \text{H/m})(0.07\,\text{m})(0.07\,\text{m})} = 90.1\,\text{kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_0 A_2} = \frac{0.0005\,\text{m}}{(4\pi \times 10^{-7}\ \text{H/m})(0.07\,\text{m})(0.07\,\text{m})(1.05)} = 77.3\,\text{kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_r \mu_0 A_3} = \frac{1.11\,\text{m}}{(2000)(4\pi \times 10^{-7}\ \text{H/m})(0.07\,\text{m})(0.07\,\text{m})} = 90.1\,\text{kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_4 = \frac{l_4}{\mu_0 A_4} = \frac{0.0007\,\text{m}}{(4\pi \times 10^{-7}\ \text{H/m})(0.07\,\text{m})(0.07\,\text{m})(1.05)} = 108.3\,\text{kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_5 = \frac{l_5}{\mu_r \mu_0 A_5} = \frac{0.37\,\text{m}}{(2000)(4\pi \times 10^{-7}\ \text{H/m})(0.07\,\text{m})(0.07\,\text{m})} = 30.0\,\text{kA} \cdot \text{t/Wb}$$

The total reluctance is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_5 + \frac{(\mathcal{R}_1 + \mathcal{R}_2)(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} = 30.0 + \frac{(90.1 + 77.3)(90.1 + 108.3)}{90.1 + 77.3 + 90.1 + 108.3} = 120.8\,\text{kA} \cdot \text{t/Wb}$$

The total flux in the core is equal to the flux in the center leg:

$$\phi_{\text{center}} = \phi_{\text{TOT}} = \frac{\mathcal{F}}{\mathcal{R}_{\text{TOT}}} = \frac{(300\,\text{t})(1.0\,\text{A})}{120.8\,\text{kA} \cdot \text{t/Wb}} = 0.00248\,\text{Wb}$$

The fluxes in the left and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{\text{left}} = \frac{(\mathcal{R}_3 + \mathcal{R}_4)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{(90.1 + 108.3)}{90.1 + 77.3 + 90.1 + 108.3}(0.00248\ \text{Wb}) = 0.00135\ \text{Wb}$$

$$\phi_{\text{right}} = \frac{(\mathcal{R}_1 + \mathcal{R}_2)}{\mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{(90.1 + 77.3)}{90.1 + 77.3 + 90.1 + 108.3}(0.00248\ \text{Wb}) = 0.00113\ \text{Wb}$$

The flux density in the air gaps can be determined from the equation $\phi = BA$:

$$B_{\text{left}} = \frac{\phi_{\text{left}}}{A_{\text{eff}}} = \frac{0.00135\ \text{Wb}}{(0.07\ \text{cm})(0.07\ \text{cm})(1.05)} = 0.262\ \text{T}$$

$$B_{\text{right}} = \frac{\phi_{\text{right}}}{A_{\text{eff}}} = \frac{0.00113\ \text{Wb}}{(0.07\ \text{cm})(0.07\ \text{cm})(1.05)} = 0.220\ \text{T}$$

1-7.   A two-legged core is shown in Figure P1-4. The winding on the left leg of the core (N_1) has 600 turns, and the winding on the right (N_2) has 200 turns. The coils are wound in the directions shown in the figure. If the dimensions are as shown, then what flux would be produced by currents $i_1 = 0.5$ A and $i_2 = 1.00$ A? Assume $\mu_r = 1000$ and constant.

core depth = 15 cm
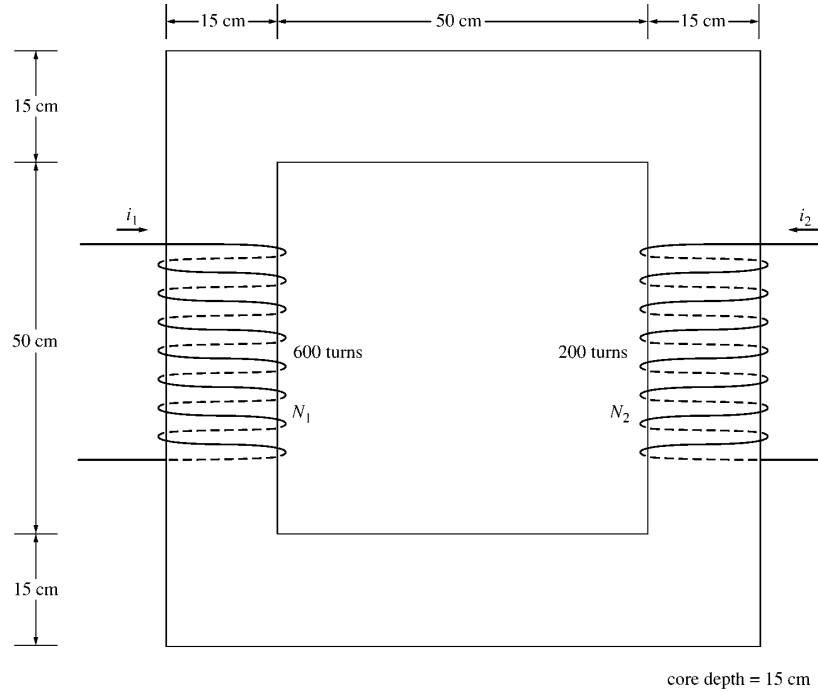
SOLUTION   The two coils on this core are would so that their magnetomotive forces are additive, so the total magnetomotive force on this core is

$$\mathcal{F}_{\text{TOT}} = N_1 i_1 + N_2 i_2 = (600\ \text{t})(0.5\ \text{A}) + (200\ \text{t})(1.0\ \text{A}) = 500\ \text{A}\cdot\text{t}$$

The total reluctance in the core is

$$\mathcal{R}_{\text{TOT}} = \frac{l}{\mu_r \mu_0 A} = \frac{2.60\ \text{m}}{(1000)(4\pi \times 10^{-7}\ \text{H/m})(0.15\ \text{m})(0.15\ \text{m})} = 92.0\ \text{kA}\cdot\text{t/Wb}$$

and the flux in the core is:

$$\phi = \frac{\mathcal{F}_{\text{TOT}}}{\mathcal{R}_{\text{TOT}}} = \frac{500\ \text{A}\cdot\text{t}}{92.0\ \text{kA}\cdot\text{t/Wb}} = 0.0054\ \text{Wb}$$

1-8. A core with three legs is shown in Figure P1-5. Its depth is 5 cm, and there are 200 turns on the leftmost leg. The relative permeability of the core can be assumed to be 1500 and constant. What flux exists in each of the three legs of the core? What is the flux density in each of the legs? Assume a 4% increase in the effective area of the air gap due to fringing effects.
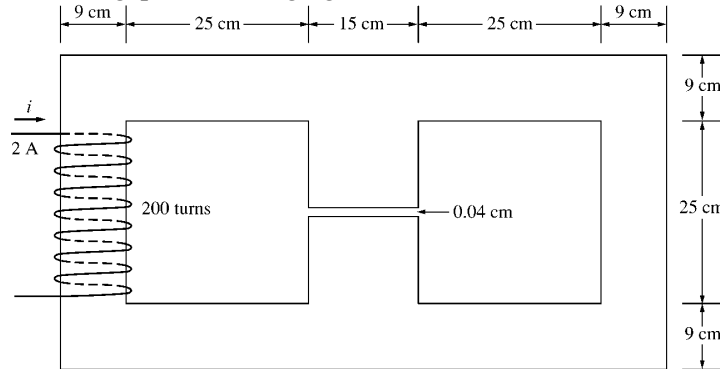
SOLUTION This core can be divided up into four regions. Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the center leg of the core, $\mathcal{R}_3$ be the reluctance of the center air gap, and $\mathcal{R}_4$ be the reluctance of the right-hand portion of the core. Then the total reluctance of the core is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_1 + \frac{(\mathcal{R}_2 + \mathcal{R}_3)\mathcal{R}_4}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.08 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.09 \text{ m})(0.05 \text{ m})} = 127.3 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_r \mu_0 A_2} = \frac{0.34 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.15 \text{ m})(0.05 \text{ m})} = 24.0 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_0 A_3} = \frac{0.0004 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.15 \text{ m})(0.05 \text{ m})(1.04)} = 40.8 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_4 = \frac{l_4}{\mu_r \mu_0 A_4} = \frac{1.08 \text{ m}}{(1500)(4\pi \times 10^{-7} \text{ H/m})(0.09 \text{ m})(0.05 \text{ m})} = 127.3 \text{ kA} \cdot \text{t/Wb}$$

The total reluctance is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_1 + \frac{(\mathcal{R}_2 + \mathcal{R}_3)\mathcal{R}_4}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} = 127.3 + \frac{(24.0 + 40.8)127.3}{24.0 + 40.8 + 127.3} = 170.2 \text{ kA} \cdot \text{t/Wb}$$

The total flux in the core is equal to the flux in the left leg:

$$\phi_{\text{left}} = \phi_{\text{TOT}} = \frac{\mathcal{F}}{\mathcal{R}_{\text{TOT}}} = \frac{(200 \text{ t})(2.0 \text{ A})}{170.2 \text{ kA} \cdot \text{t/Wb}} = 0.00235 \text{ Wb}$$

The fluxes in the center and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{\text{center}} = \frac{\mathcal{R}_4}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{127.3}{24.0 + 40.8 + 127.3}(0.00235 \text{ Wb}) = 0.00156 \text{ Wb}$$

$$\phi_{\text{right}} = \frac{\mathcal{R}_2 + \mathcal{R}_3}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{24.0 + 40.8}{24.0 + 40.8 + 127.3}(0.00235 \text{ Wb}) = 0.00079 \text{ Wb}$$

The flux density in the legs can be determined from the equation $\phi = BA$:

$$B_{\text{left}} = \frac{\phi_{\text{left}}}{A} = \frac{0.00235 \text{ Wb}}{(0.09 \text{ cm})(0.05 \text{ cm})} = 0.522 \text{ T}$$

$$B_{\text{center}} = \frac{\phi_{\text{center}}}{A} = \frac{0.00156 \text{ Wb}}{(0.15 \text{ cm})(0.05 \text{ cm})} = 0.208 \text{ T}$$

$$B_{\text{right}} = \frac{\phi_{\text{left}}}{A} = \frac{0.00079 \text{ Wb}}{(0.09 \text{ cm})(0.05 \text{ cm})} = 0.176 \text{ T}$$

1-9. A wire is shown in Figure P1-6 which is carrying 2.0 A in the presence of a magnetic field. Calculate the magnitude and direction of the force induced on the wire.
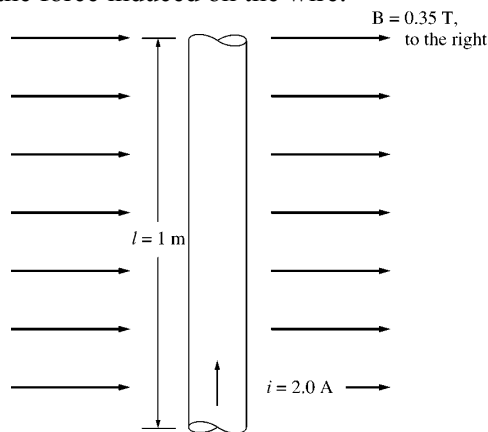
SOLUTION The force on this wire can be calculated from the equation

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B}) = ilB = (2 \text{ A})(1 \text{ m})(0.35 \text{ T}) = 0.7 \text{ N, into the page}$$

1-10. The wire shown in Figure P1-7 is moving in the presence of a magnetic field. With the information given in the figure, determine the magnitude and direction of the induced voltage in the wire.
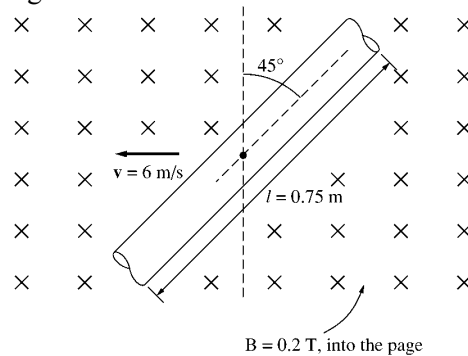
SOLUTION The induced voltage on this wire can be calculated from the equation shown below. The voltage on the wire is positive downward because the vector quantity $\mathbf{v} \times \mathbf{B}$ points downward.

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = vBl \ \cos \ 45° = (6 \text{ m/s})(0.2 \text{ T})(0.75 \text{ m}) \ \cos \ 45° = 0.636 \text{ V, positive down}$$

1-11. Repeat Problem 1-10 for the wire in Figure P1-8.
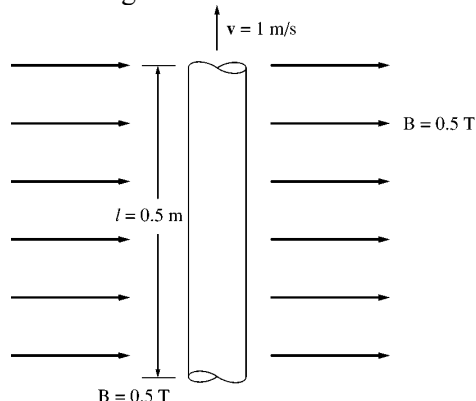
SOLUTION The induced voltage on this wire can be calculated from the equation shown below. The total voltage is zero, because the vector quantity $\mathbf{v} \times \mathbf{B}$ points into the page, while the wire runs in the plane of the page.

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = vBl \cos\ 90° = (1\,\text{m/s})(0.5\,\text{T})(0.5\,\text{m}) \cos\ 90° = 0\ \text{V}$$

1-12. The core shown in Figure P1-4 is made of a steel whose magnetization curve is shown in Figure P1-9. Repeat Problem 1-7, but this time do *not* assume a constant value of μ_r. How much flux is produced in the core by the currents specified? What is the relative permeability of this core under these conditions? Was the assumption in Problem 1-7 that the relative permeability was equal to 1000 a good assumption for these conditions? Is it a good assumption in general?
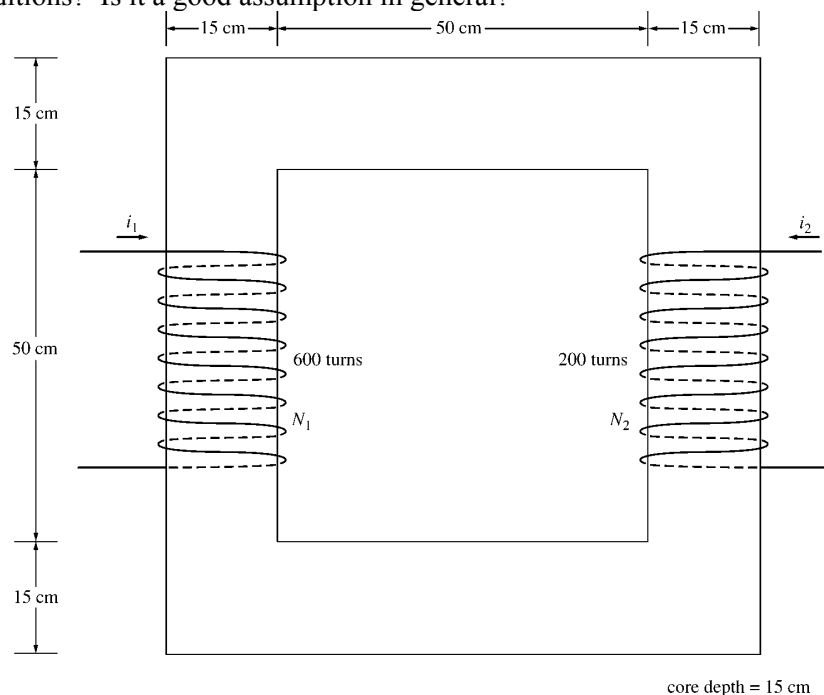
SOLUTION The magnetization curve for this core is shown below:
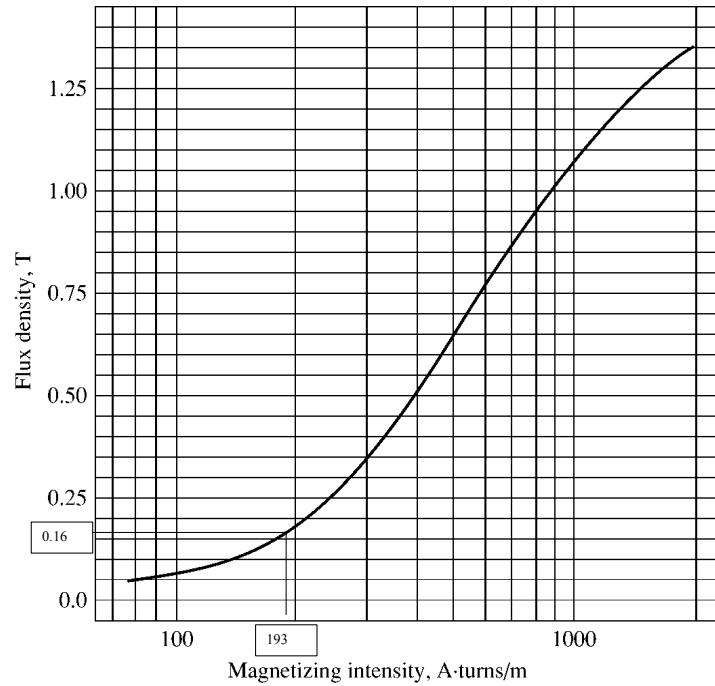
The two coils on this core are wound so that their magnetomotive forces are additive, so the total magnetomotive force on this core is

$$\mathcal{F}_{TOT} = N_1 i_1 + N_2 i_2 = (600 \text{ t})(0.5 \text{ A}) + (200 \text{ t})(1.0 \text{ A}) = 500 \text{ A} \cdot \text{t}$$

Therefore, the magnetizing intensity H is

$$H = \frac{\mathcal{F}}{l_c} = \frac{500 \text{ A} \cdot \text{t}}{2.60 \text{ m}} = 193 \text{ A} \cdot \text{t/m}$$

From the magnetization curve,

$$B = 0.16 \text{ T}$$

and the total flux in the core is

$$\phi_{TOT} = BA = (0.16 \text{ T})(0.15 \text{ m})(0.15 \text{ m}) = 0.0036 \text{ Wb}$$

The relative permeability of the core can be found from the reluctance as follows:

$$\mathcal{R} = \frac{\mathcal{F}_{TOT}}{\phi_{TOT}} = \frac{l}{\mu_r \mu_0 A}$$

Solving for μ_r yields

$$\mu_r = \frac{\phi_{TOT}}{\mathcal{F}_{TOT}} \frac{l}{\mu_0 A} = \frac{(0.0036 \text{ Wb})(2.6 \text{ m})}{(500 \text{ A} \cdot \text{t})(4\pi \times 10^{-7} \text{ H/m})(0.15 \text{ m})(0.15 \text{ m})} = 662$$

The assumption that $\mu_r = 1000$ is not very good here. It is not very good in general.

1-13. A core with three legs is shown in Figure P1-10. Its depth is 8 cm, and there are 400 turns on the center leg. The remaining dimensions are shown in the figure. The core is composed of a steel having the magnetization curve shown in Figure 1-10c. Answer the following questions about this core:

(a) What current is required to produce a flux density of 0.5 T in the central leg of the core?

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(b) What current is required to produce a flux density of 1.0 T in the central leg of the core? Is it twice the current in part *(a)*?

(c) What are the reluctances of the central and right legs of the core under the conditions in part *(a)*?

(d) What are the reluctances of the central and right legs of the core under the conditions in part *(b)*?

(e) What conclusion can you make about reluctances in real magnetic cores?
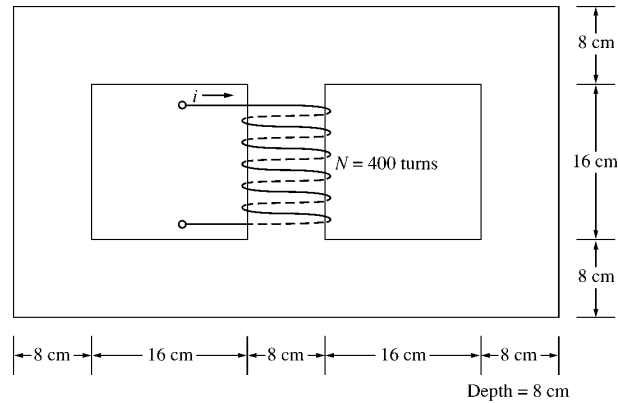
SOLUTION The magnetization curve for this core is shown below:
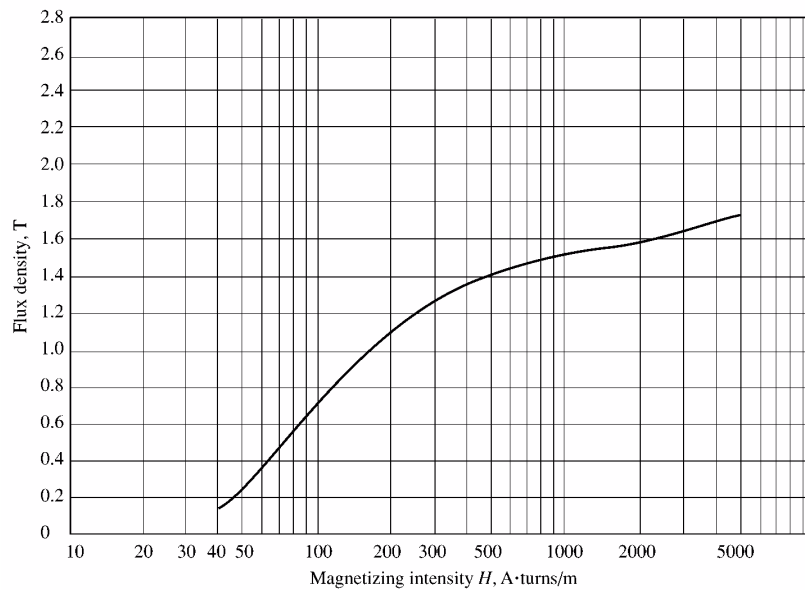
(a)   A flux density of 0.5 T in the central core corresponds to a total flux of

$$\phi_{TOT} = BA = (0.5 \text{ T})(0.08 \text{ m})(0.08 \text{ m}) = 0.0032 \text{ Wb}$$

By symmetry, the flux in each of the two outer legs must be $\phi_1 = \phi_2 = 0.0016 \text{ Wb}$, and the flux density in the other legs must be

$$B_1 = B_2 = \frac{0.0016 \text{ Wb}}{(0.08 \text{ m})(0.08 \text{ m})} = 0.25 \text{ T}$$

The magnetizing intensity H required to produce a flux density of 0.25 T can be found from Figure 1-10c. It is 50 A·t/m. Similarly, the magnetizing intensity H required to produce a flux density of 0.50 T is 70 A·t/m. Therefore, the total MMF needed is

$$\mathcal{F}_{TOT} = H_{center} \, l_{center} + H_{outer} \, l_{outer}$$

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$$\mathcal{F}_{TOT} = (70 \text{ A} \cdot \text{t/m})(0.24 \text{ m}) + (50 \text{ A} \cdot \text{t/m})(0.72 \text{ m}) = 52.8 \text{ A} \cdot \text{t}$$

and the required current is

$$i = \frac{\mathcal{F}_{TOT}}{N} = \frac{52.8 \text{ A} \cdot \text{t}}{400 \text{ t}} = 0.13 \text{ A}$$

(b) A flux density of 1.0 T in the central core corresponds to a total flux of

$$\phi_{TOT} = BA = (1.0 \text{ T})(0.08 \text{ m})(0.08 \text{ m}) = 0.0064 \text{ Wb}$$

By symmetry, the flux in each of the two outer legs must be $\phi_1 = \phi_2 = 0.0032 \text{ Wb}$, and the flux density in the other legs must be

$$B_1 = B_2 = \frac{0.0032 \text{ Wb}}{(0.08 \text{ m})(0.08 \text{ m})} = 0.50 \text{ T}$$

The magnetizing intensity *H* required to produce a flux density of 0.50 T can be found from Figure 1-10*c*. It is 70 A·t/m. Similarly, the magnetizing intensity *H* required to produce a flux density of 1.00 T is about 160 A·t/m. Therefore, the total MMF needed is

$$\mathcal{F}_{TOT} = H_{center} I_{center} + H_{outer} I_{outer}$$
$$\mathcal{F}_{TOT} = (160 \text{ A} \cdot \text{t/m})(0.24 \text{ m}) + (70 \text{ A} \cdot \text{t/m})(0.72 \text{ m}) = 88.8 \text{ A} \cdot \text{t}$$

and the required current is

$$i = \frac{\phi_{TOT}}{N} = \frac{88.8 \text{ A} \cdot \text{t}}{400 \text{ t}} = 0.22 \text{ A}$$

(c) The reluctance of the central leg of the core under the conditions of part *(a)* is:

$$R_{cent} = \frac{\mathcal{F}_{TOT}}{\phi_{TOT}} = \frac{(70 \text{ A} \cdot \text{t/m})(0.24 \text{ m})}{0.0032 \text{ Wb}} = 5.25 \text{ kA} \cdot \text{t/Wb}$$

The reluctance of the right leg of the core under the conditions of part *(a)* is:

$$R_{right} = \frac{\mathcal{F}_{TOT}}{\phi_{TOT}} = \frac{(50 \text{ A} \cdot \text{t/m})(0.72 \text{ m})}{0.0016 \text{ Wb}} = 22.5 \text{ kA} \cdot \text{t/Wb}$$

(d) The reluctance of the central leg of the core under the conditions of part *(b)* is:

$$R_{cent} = \frac{\mathcal{F}_{TOT}}{\phi_{TOT}} = \frac{(160 \text{ A} \cdot \text{t/m})(0.24 \text{ m})}{0.0064 \text{ Wb}} = 6.0 \text{ kA} \cdot \text{t/Wb}$$

The reluctance of the right leg of the core under the conditions of part *(b)* is:

$$R_{right} = \frac{\mathcal{F}_{TOT}}{\phi_{TOT}} = \frac{(70 \text{ A} \cdot \text{t/m})(0.72 \text{ m})}{0.0032 \text{ Wb}} = 15.75 \text{ kA} \cdot \text{t/Wb}$$

(e) The reluctances in real magnetic cores are not constant.

1-14. A two-legged magnetic core with an air gap is shown in Figure P1-11. The depth of the core is 5 cm, the length of the air gap in the core is 0.07 cm, and the number of turns on the coil is 500. The magnetization curve of the core material is shown in Figure P1-9. Assume a 5 percent increase in effective air-gap area to account for fringing. How much current is required to produce an air-gap flux density of 0.5 T? What are the flux densities of the four sides of the core at that current? What is the total flux present in the air gap?

Depth = 5 cm
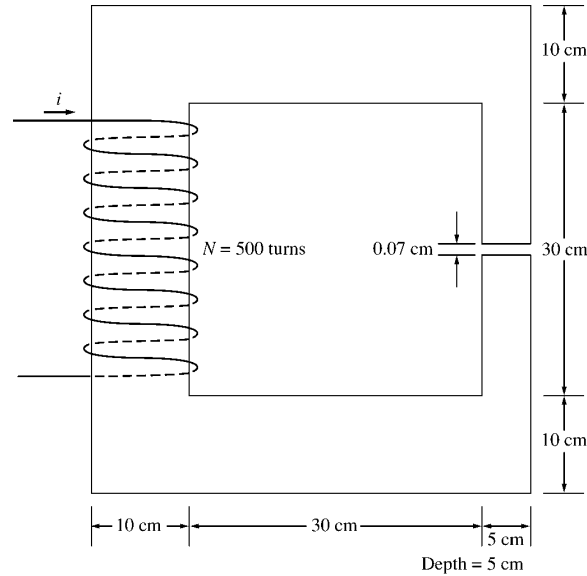
SOLUTION The magnetization curve for this core is shown below:
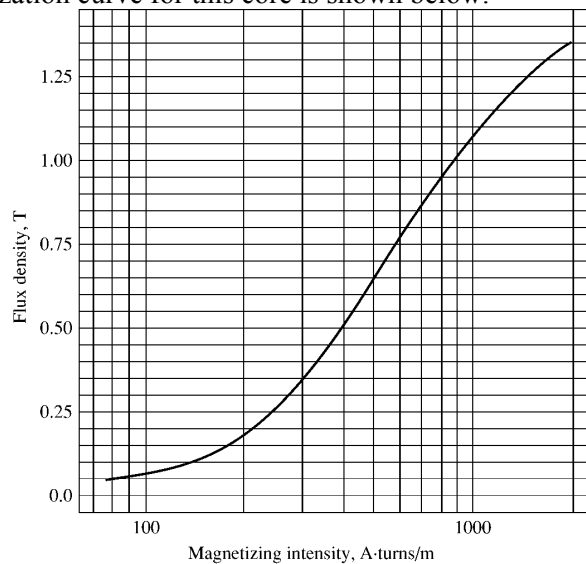
An air-gap flux density of 0.5 T requires a total flux of

$$\phi = BA_{\text{eff}} = (0.5\,\text{T})(0.05\,\text{m})(0.05\,\text{m})(1.05) = 0.00131\,\text{Wb}$$

This flux requires a flux density in the right-hand leg of

$$B_{\text{right}} = \frac{\phi}{A} = \frac{0.00131\,\text{Wb}}{(0.05\,\text{m})(0.05\,\text{m})} = 0.524\,\text{T}$$

The flux density in the other three legs of the core is

$$B_{\text{top}} = B_{\text{left}} = B_{\text{bottom}} = \frac{\phi}{A} = \frac{0.00131\,\text{Wb}}{(0.10\,\text{m})(0.05\,\text{m})} = 0.262\,\text{T}$$

The magnetizing intensity required to produce a flux density of 0.5 T in the air gap can be found from the equation $B_{\text{ag}} = \mu_o H_{\text{ag}}$:

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$$H_{ag} = \frac{B_{ag}}{\mu_0} = \frac{0.5\,\text{T}}{4\pi \times 10^{-7}\,\text{H/m}} = 398\,\text{kA} \cdot \text{t/m}$$

The magnetizing intensity required to produce a flux density of 0.524 T in the right-hand leg of the core can be found from Figure P1-9 to be

$$H_{right} = 410\,\text{A} \cdot \text{t/m}$$

The magnetizing intensity required to produce a flux density of 0.262 T in the right-hand leg of the core can be found from Figure P1-9 to be

$$H_{top} = H_{left} = H_{bottom} = 240\,\text{A} \cdot \text{t/m}$$

The total MMF required to produce the flux is

$$\mathcal{F}_{\text{TOT}} = H_{ag}\, l_{ag} + H_{right}\, l_{right} + H_{top}\, l_{top} + H_{left}\, l_{left} + H_{bottom}\, l_{bottom}$$
$$\mathcal{F}_{\text{TOT}} = (398\,\text{kA} \cdot \text{t/m})(0.0007\,\text{m}) + (410\,\text{A} \cdot \text{t/m})(0.40\,\text{m}) + 3(240\,\text{A} \cdot \text{t/m})(0.40\,\text{m})$$
$$\mathcal{F}_{\text{TOT}} = 278.6 + 164 + 288 = 731\,\text{A} \cdot \text{t}$$

and the required current is

$$i = \frac{\mathcal{F}_{\text{TOT}}}{N} = \frac{731\,\text{A} \cdot \text{t}}{500\,\text{t}} = 1.46\,\text{A}$$

The flux densities in the four sides of the core and the total flux present in the air gap were calculated above.

1-15. A transformer core with an effective mean path length of 10 in has a 300-turn coil wrapped around one leg. Its cross-sectional area is 0.25 in^2, and its magnetization curve is shown in Figure 1-10c. If current of 0.25 A is flowing in the coil, what is the total flux in the core? What is the flux density?
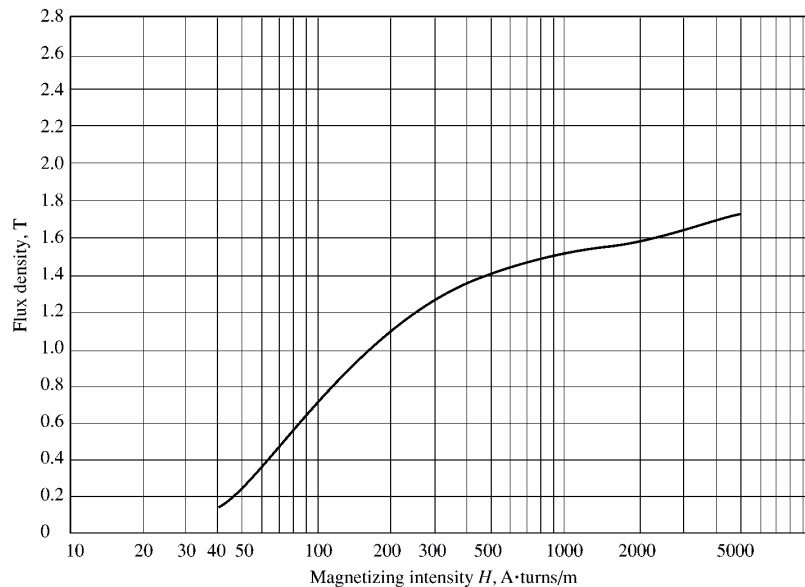
SOLUTION The magnetizing intensity applied to this core is

$$H = \frac{\mathcal{F}}{l_c} = \frac{Ni}{l_c} = \frac{(300\,\text{t})(0.25\,\text{A})}{(10\,\text{in})(0.0254\,\text{m/in})} = 295\,\text{A} \cdot \text{t/m}$$

From the magnetization curve, the flux density in the core is

$$B = 1.27 \text{ T}$$

The total flux in the core is

$$\phi = BA = (1.27 \text{ T})(0.25 \text{ in}^2)\left(\frac{0.0254 \text{ m}}{1 \text{ in}}\right)^2 = 0.000205 \text{ Wb}$$

1-16. The core shown in Figure P1-2 has the flux ϕ shown in Figure P1-12. Sketch the voltage present at the terminals of the coil.
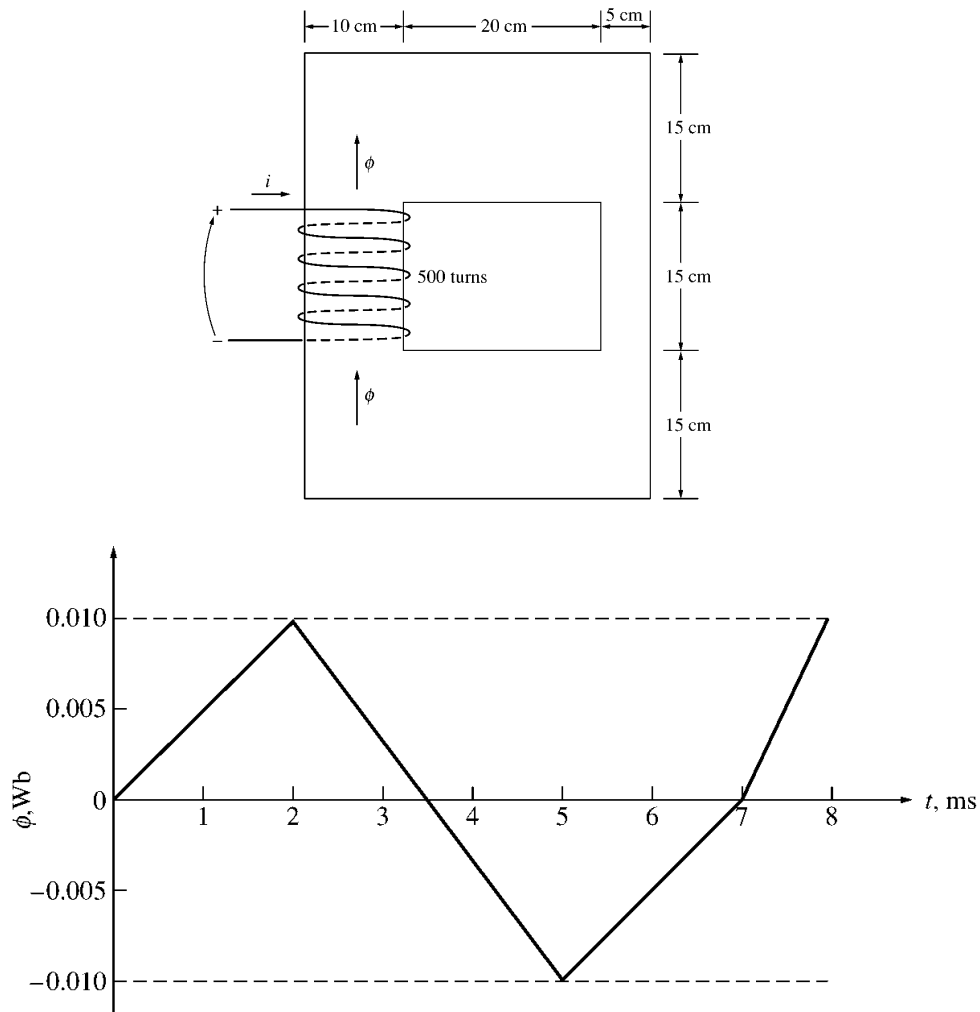
SOLUTION By Lenz' Law, an increasing flux in the direction shown on the core will produce a voltage that tends to oppose the increase. This voltage will be the same polarity as the direction shown on the core, so it will be positive. The induced voltage in the core is given by the equation

$$e_{\text{ind}} = N\frac{d\phi}{dt}$$

so the voltage in the windings will be

Time	$N\dfrac{d\phi}{dt}$	e_{ind}
$0 < t < 2$ s	$(500\,t)\dfrac{0.010\,\text{Wb}}{2\,\text{s}}$	2.50 V
$2 < t < 5$ s	$(500\,t)\dfrac{-0.020\,\text{Wb}}{3\,\text{s}}$	-3.33 V
$5 < t < 7$ s	$(500\,t)\dfrac{0.010\,\text{Wb}}{2\,\text{s}}$	2.50 V
$7 < t < 8$ s	$(500\,t)\dfrac{0.010\,\text{Wb}}{1\,\text{s}}$	5.00 V

The resulting voltage is plotted below:
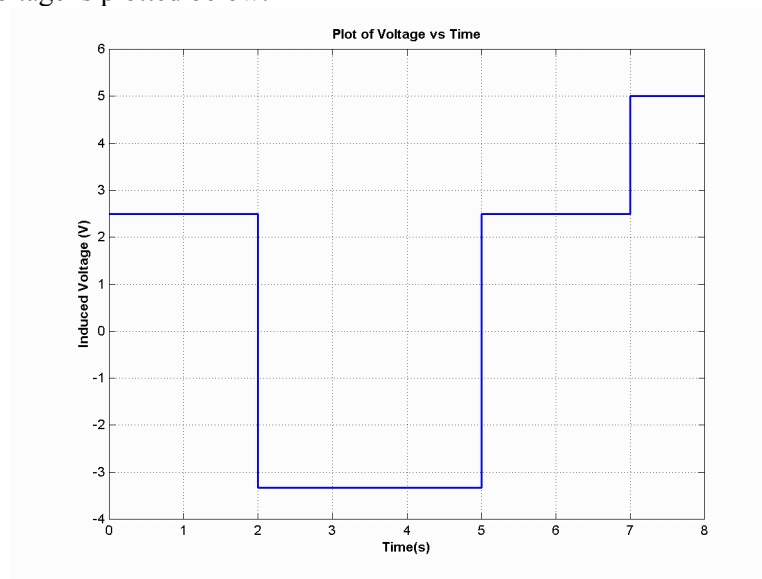
1-17. Figure P1-13 shows the core of a simple dc motor. The magnetization curve for the metal in this core is given by Figure 1-10*c* and *d*. Assume that the cross-sectional area of each air gap is 18 cm^2 and that the width of each air gap is 0.05 cm. The effective diameter of the rotor core is 4 cm.
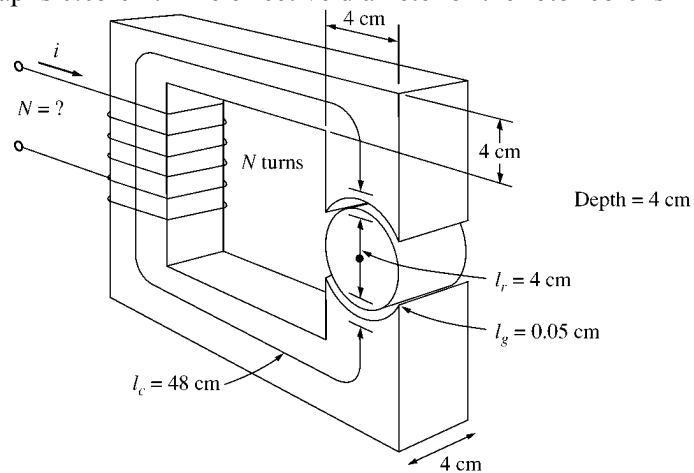
SOLUTION The magnetization curve for this core is shown below:
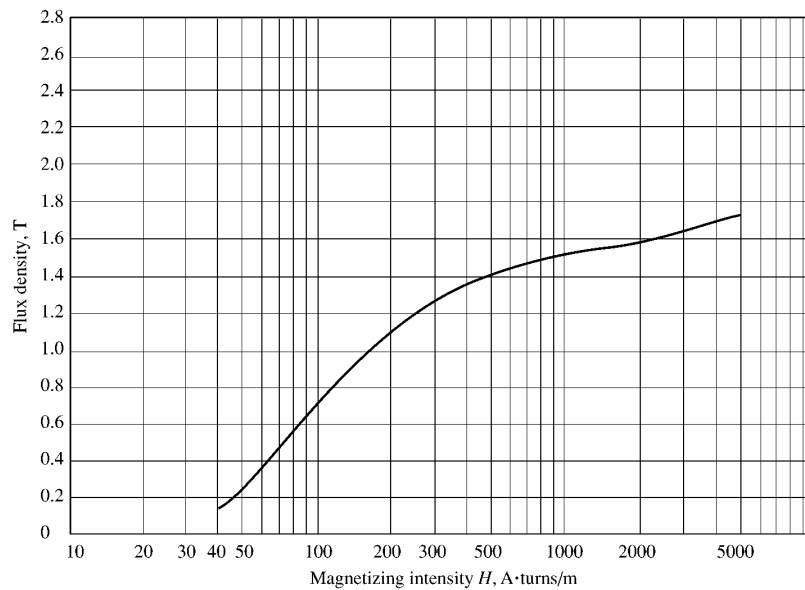
The relative permeability of this core is shown below:
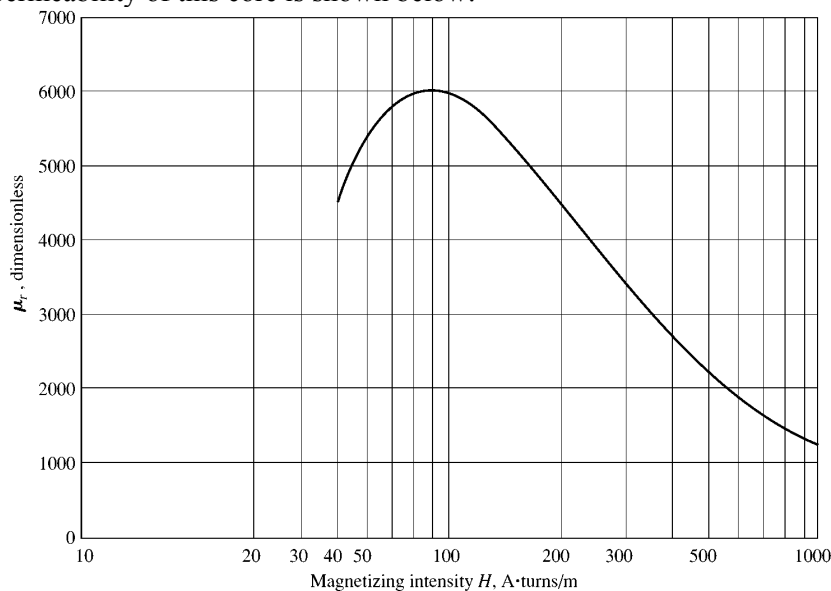
> **Note:** This is a design problem, and the answer presented here is not unique. Other values could be selected for the flux density in part *(a)*, and other numbers of turns could be selected in part *(c)*. These other answers are also correct if the proper steps were followed, and if the choices were reasonable.

SOLUTION

(a) From Figure 1-10*c*, a reasonable maximum flux density would be about 1.2 T. Notice that the saturation effects become significant for higher flux densities.

(b) At a flux density of 1.2 T, the total flux in the core would be

$$\phi = BA = (1.2 \text{ T})(0.04 \text{ m})(0.04 \text{ m}) = 0.00192 \text{ Wb}$$

(c) The total reluctance of the core is:

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_{\text{stator}} + \mathcal{R}_{\text{air gap 1}} + \mathcal{R}_{\text{rotor}} + \mathcal{R}_{\text{air gap 2}}$$

At a flux density of 1.2 T, the relative permeability μ_r of the stator is 3800, so the stator reluctance is

$$\mathcal{R}_{\text{stator}} = \frac{l_{\text{stator}}}{\mu_{\text{stator}} A_{\text{stator}}} = \frac{0.48 \text{ m}}{(3800)(4\pi \times 10^{-7} \text{ H/m})(0.04 \text{ m})(0.04 \text{ m})} = 62.8 \text{ kA} \cdot \text{t/Wb}$$

At a flux density of 1.2 T, the relative permeability μ_r of the rotor is 3800, so the rotor reluctance is

$$\mathcal{R}_{\text{rotor}} = \frac{l_{\text{rotor}}}{\mu_{\text{stator}} A_{\text{rotor}}} = \frac{0.04 \text{ m}}{(3800)(4\pi \times 10^{-7} \text{ H/m})(0.04 \text{ m})(0.04 \text{ m})} = 5.2 \text{ kA} \cdot \text{t/Wb}$$

The reluctance of both air gap 1 and air gap 2 is

$$\mathcal{R}_{\text{air gap 1}} = \mathcal{R}_{\text{air gap 2}} = \frac{l_{\text{air gap}}}{\mu_{\text{air gap}} A_{\text{air gap}}} = \frac{0.0005 \text{ m}}{(4\pi \times 10^{-7} \text{ H/m})(0.0018 \text{ m}^2)} = 221 \text{ kA} \cdot \text{t/Wb}$$

Therefore, the total reluctance of the core is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_{\text{stator}} + \mathcal{R}_{\text{air gap 1}} + \mathcal{R}_{\text{rotor}} + \mathcal{R}_{\text{air gap 2}}$$
$$\mathcal{R}_{\text{TOT}} = 62.8 + 221 + 5.2 + 221 = 510 \text{ kA} \cdot \text{t/Wb}$$

The required MMF is

$$\mathcal{F}_{\text{TOT}} = \phi \mathcal{R}_{\text{TOT}} = (0.00192 \text{ Wb})(510 \text{ kA} \cdot \text{t/Wb}) = 979 \text{ A} \cdot \text{t}$$

Since $\mathcal{F} = Ni$, and the current is limited to 1 A, one possible choice for the number of turns is $N = 1000$.

1-18. Assume that the voltage applied to a load is $\mathbf{V} = 208\angle -30° \text{ V}$ and the current flowing through the load is $\mathbf{I} = 5\angle 15° \text{ A}$.

(a) Calculate the complex power **S** consumed by this load.

(b) Is this load inductive or capacitive?

(c) Calculate the power factor of this load?

(d) Calculate the reactive power consumed or supplied by this load. Does the load consume reactive power from the source or supply it to the source?

SOLUTION

(a) The complex power S consumed by this load is

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = (208\angle -30° \text{ V})(5\angle 15° \text{ A})^* = (208\angle -30° \text{ V})(5\angle -15° \text{ A})$$
$$\mathbf{S} = 1040\angle -45° \text{ VA}$$

(b) This is a capacitive load.

(c) The power factor of this load is
$$\text{PF} = \cos(-45°) = 0.707 \text{ leading}$$

(d) This load supplies reactive power to the source. The reactive power of the load is
$$Q = VI\sin\theta = (208 \text{ V})(5 \text{ A})\sin(-45°) = -735 \text{ var}$$

1-19. Figure P1-14 shows a simple single-phase ac power system with three loads. The impedances of these three loads are

$$\mathbf{Z}_1 = 5\angle 30° \ \Omega \qquad \mathbf{Z}_2 = 5\angle 45° \ \Omega \qquad \mathbf{Z}_3 = 5\angle -90° \ \Omega$$

Answer the following questions about this power system.

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(a) Assume that the switch shown in the figure is open, and calculate the current **I**, the power factor, and the real, reactive, and apparent power being supplied by the source.

(b) Assume that the switch shown in the figure is closed, and calculate the current **I**, the power factor, and the real, reactive, and apparent power being supplied by the source.

(c) What happened to the current flowing from the source when the switch closed? Why?

$$\mathbf{V} = 120\angle 0° \text{ V}$$

SOLUTION
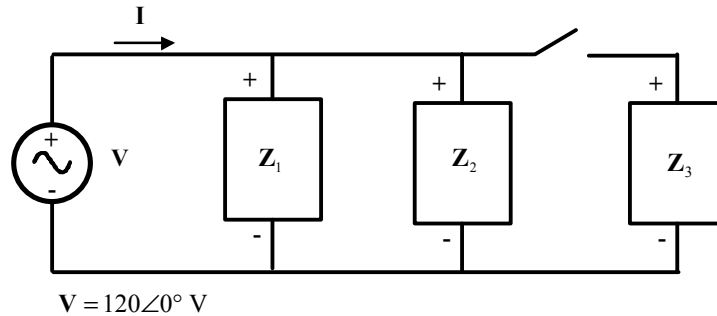
(a) With the switch open, only loads 1 and 2 are connected to the source. The current $\mathbf{I}_1$ in Load 1 is

$$\mathbf{I}_1 = \frac{120\angle 0° \text{ V}}{5\angle 30° \text{ A}} = 24\angle -30° \text{ A}$$

The current $\mathbf{I}_2$ in Load 2 is

$$\mathbf{I}_2 = \frac{120\angle 0° \text{ V}}{5\angle 45° \text{ A}} = 24\angle -45° \text{ A}$$

Therefore the total current from the source is

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 24\angle -30° \text{ A} + 24\angle -45° \text{ A} = 47.59\angle -37.5° \text{ A}$$

The power factor supplied by the source is

$$\text{PF} = \cos\theta = \cos(-37.5°) = 0.793 \text{ lagging}$$

The real, reactive, and apparent power supplied by the source are

$$P = VI\cos\theta = (120 \text{ V})(47.59 \text{ A})\cos(-37.5°) = 4531 \text{ W}$$
$$Q = VI\cos\theta = (120 \text{ V})(47.59 \text{ A})\sin(-37.5°) = -3477 \text{ var}$$
$$S = VI = (120 \text{ V})(47.59 \text{ A}) = 5711 \text{ VA}$$

(b) With the switch open, all three loads are connected to the source. The current in Loads 1 and 2 is the same as before. The current $\mathbf{I}_3$ in Load 3 is

$$\mathbf{I}_3 = \frac{120\angle 0° \text{ V}}{5\angle -90° \text{ A}} = 24\angle 90° \text{ A}$$

Therefore the total current from the source is

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 = 24\angle -30° \text{ A} + 24\angle -45° \text{ A} + 24\angle 90° \text{ A} = 38.08\angle -7.5° \text{ A}$$

The power factor supplied by the source is

$$\text{PF} = \cos\theta = \cos(-7.5°) = 0.991 \text{ lagging}$$

The real, reactive, and apparent power supplied by the source are

$$P = VI\cos\theta = (120 \text{ V})(38.08 \text{ A})\cos(-7.5°) = 4531 \text{ W}$$
$$Q = VI\cos\theta = (120 \text{ V})(38.08 \text{ A})\sin(-7.5°) = -596 \text{ var}$$

$$S = VI = (120 \text{ V})(38.08 \text{ A}) = 4570 \text{ VA}$$

(c) The current flowing *decreased* when the switch closed, because most of the reactive power being consumed by Loads 1 and 2 is being supplied by Load 3. Since less reactive power has to be supplied by the source, the total current flow decireases.

1-20. Demonstrate that Equation (1-50) can be derived from Equation (1-49) using simple trigonometric identities:

$$p(t) = v(t)\, i(t) = 2VI \cos \omega t \cos(\omega t - \theta) \tag{1-49}$$

$$p(t) = VI \cos\theta (1 + \cos 2\omega t) + VI \sin\theta \sin 2\omega t \tag{1-50}$$

SOLUTION

The first step is to apply the following identity from Appendix D:

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

The result is

$$p(t) = v(t)\, i(t) = 2VI \cos \omega t \cos(\omega t - \theta) \tag{1-49}$$

$$p(t) = 2VI \left\{ \frac{1}{2}\left[\cos(\omega t - \omega t + \theta) + \cos(\omega t + \omega t - \theta)\right] \right\}$$

$$p(t) = VI \left[\cos\theta + \cos(2\omega t - \theta)\right]$$

Now we must apply the angle addition identity from Appendix D to the second term:

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

The result is

$$p(t) = VI \left[\cos\theta + \cos 2\omega t \cos\theta + \sin 2\omega t \sin\theta\right]$$

Collecting terms yields the final result:

$$p(t) = VI \cos\theta (1 + \cos 2\omega t) + VI \sin\theta \sin 2\omega t$$

Chapter 2: *Three-Phase Circuits*

2-1.   Three impedances of $4 + j3\ \Omega$ are Δ-connected and tied to a three-phase 208-V power line. Find I_ϕ, I_L, P, Q, S, and the power factor of this load.

SOLUTION
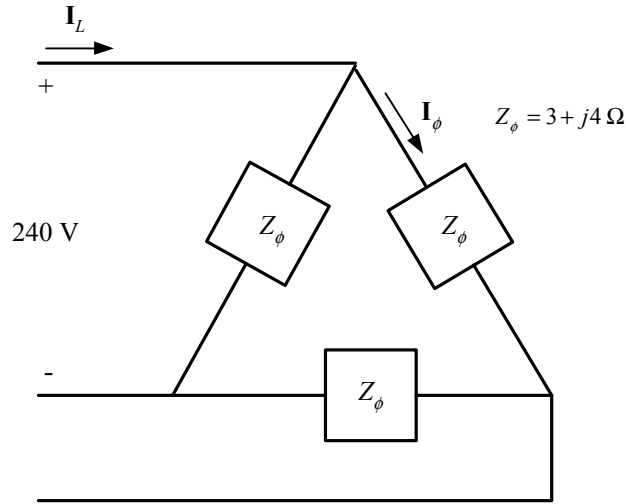
Here, $V_L = V_\phi = 208\ \text{V}$, and $Z_\phi = 4 + j3\ \Omega = 5\angle 36.87°\ \Omega$, so

$$I_\phi = \frac{V_\phi}{Z_\phi} = \frac{208\ \text{V}}{5\ \Omega} = 41.6\ \text{A}$$

$$I_L = \sqrt{3}I_\phi = \sqrt{3}\left(41.6\ \text{A}\right) = 72.05\ \text{A}$$

$$P = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{\left(208\ \text{V}\right)^2}{5\ \Omega}\cos\ 36.87° = 20.77\ \text{kW}$$

$$Q = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{\left(208\ \text{V}\right)^2}{5\ \Omega}\sin\ 36.87° = 15.58\ \text{kvar}$$

$$S = \sqrt{P^2 + Q^2} = 25.96\ \text{kVA}$$

$$\text{PF} = \cos\theta = 0.8\ \text{lagging}$$

2-2.   Figure P2-1 shows a three-phase power system with two loads. The Δ-connected generator is producing a line voltage of 480 V, and the line impedance is $0.09 + j0.16\ \Omega$. Load 1 is Y-connected, with a phase impedance of $2.5\angle 36.87°\ \Omega$ and load 2 is Δ-connected, with a phase impedance of $5\angle\text{-}20°\ \Omega$.

Generator   Load 1   Load 2

$$\mathbf{Z}_{\phi 1} = 2.5\angle 36.87° \, \Omega$$
$$\mathbf{Z}_{\phi 2} = 5\angle -20° \, \Omega$$
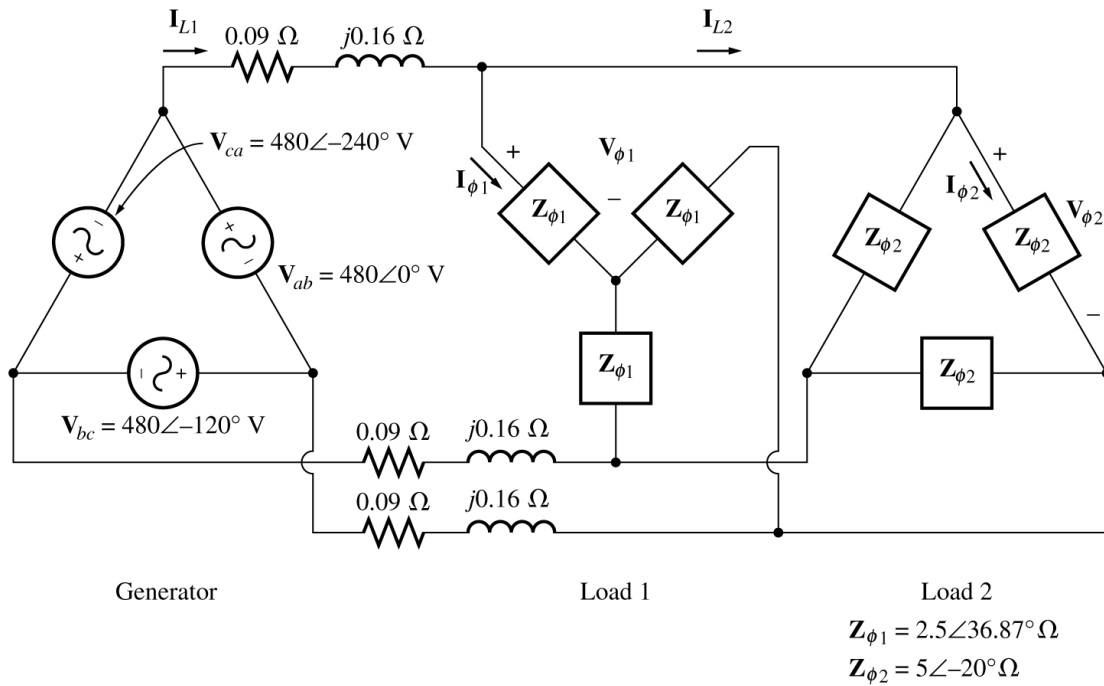
(a) What is the line voltage of the two loads?

(b) What is the voltage drop on the transmission lines?

(c) Find the real and reactive powers supplied to each load.

(d) Find the real and reactive power losses in the transmission line.

(e) Find the real power, reactive power, and power factor supplied by the generator.

SOLUTION To solve this problem, first convert the two deltas to equivalent wyes, and get the per-phase equivalent circuit.
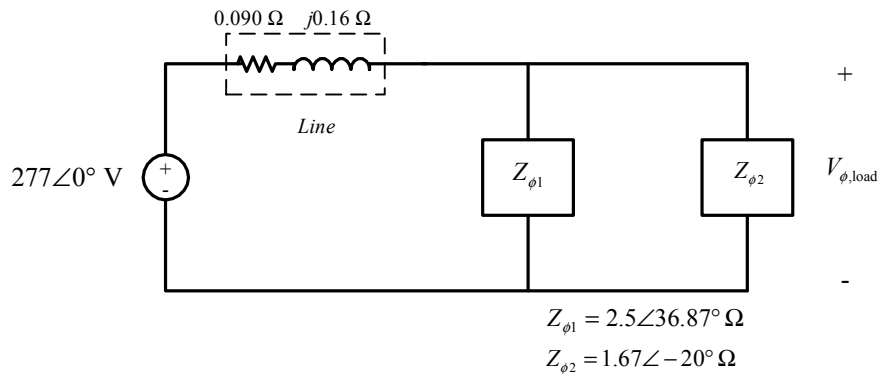
$$Z_{\phi 1} = 2.5\angle 36.87° \, \Omega$$
$$Z_{\phi 2} = 1.67\angle -20° \, \Omega$$

(a)   The phase voltage of the equivalent Y-loads can be found by nodal analysis.

$$\frac{\mathbf{V}_{\phi,\text{load}} - 277\angle 0° \text{ V}}{0.09 + j0.16 \, \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{2.5\angle 36.87° \, \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{1.67\angle -20° \, \Omega} = 0$$

$$\left(5.443\angle -60.6°\right)\left(\mathbf{V}_{\phi,\text{load}} - 277\angle 0° \text{ V}\right) + \left(0.4\angle -36.87°\right)\mathbf{V}_{\phi,\text{load}} + \left(0.6\angle 20°\right)\mathbf{V}_{\phi,\text{load}} = 0$$

$$\left(5.955\angle -53.34°\right)\mathbf{V}_{\phi,\text{load}} = 1508\angle -60.6°$$

$$\mathbf{V}_{\phi,\text{load}} = 253.2\angle -7.3° \text{ V}$$

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Therefore, the line voltage at the loads is $V_L \sqrt{3} \, V_\phi = 439$ V.

(b)   The voltage drop in the transmission lines is

$$\Delta \mathbf{V}_{line} = \mathbf{V}_{\phi,gen} - \mathbf{V}_{\phi,load} = 277\angle 0° \text{ V} - 253.2\angle -7.3° = 41.3\angle 52° \text{ V}$$

(c)   The real and reactive power of each load is

$$P_1 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(253.2 \text{ V})^2}{2.5 \,\Omega}\cos 36.87° = 61.6 \text{ kW}$$

$$Q_1 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(253.2 \text{ V})^2}{2.5 \,\Omega}\sin 36.87° = 46.2 \text{ kvar}$$

$$P_2 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(253.2 \text{ V})^2}{1.67 \,\Omega}\cos(-20°) = 108.4 \text{ kW}$$

$$Q_2 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(253.2 \text{ V})^2}{1.67 \,\Omega}\sin(-20°) = -39.5 \text{ kvar}$$

(d)   The line current is

$$\mathbf{I}_{line} = \frac{\Delta \mathbf{V}_{line}}{Z_{line}} = \frac{41.3\angle 52° \text{ V}}{0.09 + j0.16 \,\Omega} = 225\angle -8.6° \text{A}$$

Therefore, the loses in the transmission line are

$$P_{line} = 3I_{line}^2 R_{line} = 3(225 \text{ A})^2(0.09 \,\Omega) = 13.7 \text{ kW}$$

$$Q_{line} = 3I_{line}^2 X_{line} = 3(225 \text{ A})^2(0.16 \,\Omega) = 24.3 \text{ kvar}$$

(e)   The real and reactive power supplied by the generator is

$$P_{gen} = P_{line} + P_1 + P_2 = 13.7 \text{ kW} + 61.6 \text{ kW} + 108.4 \text{ kW} = 183.7 \text{ kW}$$

$$Q_{gen} = Q_{line} + Q_1 + Q_2 = 24.3 \text{ kvar} + 46.2 \text{ kvar} - 39.5 \text{ kvar} = 31 \text{ kvar}$$

The power factor of the generator is

$$\text{PF} = \cos\left[\tan^{-1}\frac{Q_{gen}}{P_{gen}}\right] = \cos\left[\tan^{-1}\frac{31 \text{ kvar}}{183.7 \text{ kW}}\right] = 0.986 \text{ lagging}$$

2-3.   The figure shown below shows a one-line diagram of a simple power system containing a single 480 V generator and three loads. Assume that the transmission lines in this power system are lossless, and answer the following questions.

(a) Assume that Load 1 is Y-connected. What are the phase voltage and currents in that load?

(b) Assume that Load 2 is Δ-connected. What are the phase voltage and currents in that load?

(c) What real, reactive, and apparent power does the generator supply when the switch is open?

(d) What is the total line current I_L when the switch is open?

(e) What real, reactive, and apparent power does the generator supply when the switch is closed?

(f) What is the total line current I_L when the switch is closed?

(g) How does the total line current I_L compare to the sum of the three individual currents $I_1 + I_2 + I_3$? If they are not equal, why not?
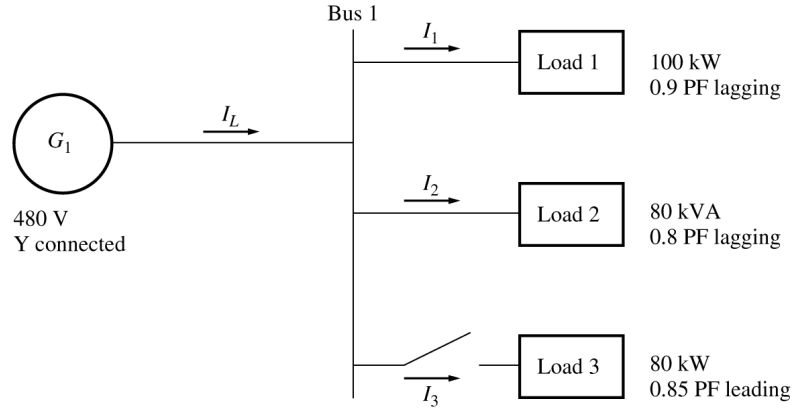
SOLUTION Since the transmission lines are lossless in this power system, the full voltage generated by G_1 will be present at each of the loads.

(a) Since this load is Y-connected, the phase voltage is

$$V_{\phi 1} = \frac{480 \text{ V}}{\sqrt{3}} = 277 \text{ V}$$

The phase current can be derived from the equation $P = 3V_\phi I_\phi \cos\theta$ as follows:

$$I_{\phi 1} = \frac{P}{3V_\phi \cos\theta} = \frac{100 \text{ kW}}{3(277 \text{ V})(0.9)} = 133.7 \text{ A}$$

(b) Since this load is Δ-connected, the phase voltage is

$$V_{\phi 2} = 480 \text{ V}$$

The phase current can be derived from the equation $S = 3V_\phi I_\phi$ as follows:

$$I_{\phi 2} = \frac{S}{3V_\phi} = \frac{80 \text{ kVA}}{3(480 \text{ V})} = 55.56 \text{ A}$$

(c) The real and reactive power supplied by the generator when the switch is open is just the sum of the real and reactive powers of Loads 1 and 2.

$$P_1 = 100 \text{ kW}$$
$$Q_1 = P \tan\theta = P \tan\left(\cos^{-1} \text{PF}\right) = (100 \text{ kW})(\tan 25.84°) = 48.4 \text{ kvar}$$
$$P_2 = S \cos\theta = (80 \text{ kVA})(0.8) = 64 \text{ kW}$$
$$Q_2 = S \sin\theta = (80 \text{ kVA})(0.6) = 48 \text{ kvar}$$
$$P_G = P_1 + P_2 = 100 \text{ kW} + 64 \text{ kW} = 164 \text{ kW}$$
$$Q_G = Q_1 + Q_2 = 48.4 \text{ kvar} + 48 \text{ kvar} = 96.4 \text{ kvar}$$

(d) The line current when the switch is open is given by $I_L = \dfrac{P}{\sqrt{3}\,V_L \cos\theta}$, where $\theta = \tan^{-1}\dfrac{Q_G}{P_G}$.

$$\theta = \tan^{-1}\frac{Q_G}{P_G} = \tan^{-1}\frac{96.4 \text{ kvar}}{164 \text{ kW}} = 30.45°$$

$$I_L = \frac{P}{\sqrt{3}\,V_L \cos\theta} = \frac{164 \text{ kW}}{\sqrt{3}(480 \text{ V})\cos(30.45°)} = 228.8 \text{ A}$$

(e)   The real and reactive power supplied by the generator when the switch is closed is just the sum of the real and reactive powers of Loads 1, 2, and 3. The powers of Loads 1 and 2 have already been calculated. The real and reactive power of Load 3 are:

$$P_3 = 80 \text{ kW}$$

$$Q_3 = P\tan\theta = P\tan\left(\cos^{-1}\text{PF}\right) = (80 \text{ kW})\left[\tan\left(-31.79°\right)\right] = -49.6 \text{ kvar}$$

$$P_G = P_1 + P_2 + P_3 = 100 \text{ kW} + 64 \text{ kW} + 80 \text{ kW} = 244 \text{ kW}$$

$$Q_G = Q_1 + Q_2 + Q_3 = 48.4 \text{ kvar} + 48 \text{ kvar} - 49.6 \text{ kvar} = 46.8 \text{ kvar}$$

(f)   The line current when the switch is closed is given by $I_L = \dfrac{P}{\sqrt{3}\ V_L \cos\theta}$, where $\theta = \tan^{-1}\dfrac{Q_G}{P_G}$.

$$\theta = \tan^{-1}\frac{Q_G}{P_G} = \tan^{-1}\frac{46.8 \text{ kvar}}{244 \text{ kW}} = 10.86°$$

$$I_L = \frac{P}{\sqrt{3}\ V_L\ \cos\theta} = \frac{244 \text{ kW}}{\sqrt{3}(480 \text{ V})\cos(10.86°)} = 298.8 \text{ A}$$

(g)   The total line current from the generator is 298.8 A. The line currents to each individual load are:

$$I_{L1} = \frac{P_1}{\sqrt{3}\ V_L\ \cos\theta_1} = \frac{100 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.9)} = 133.6 \text{ A}$$

$$I_{L2} = \frac{S_2}{\sqrt{3}\ V_L} = \frac{80 \text{ kVA}}{\sqrt{3}(480 \text{ V})} = 96.2 \text{ A}$$

$$I_{L3} = \frac{P_3}{\sqrt{3}\ V_L\ \cos\theta_3} = \frac{80 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.85)} = 113.2 \text{ A}$$

The sum of the three individual line currents is 343 A, while the current supplied by the generator is 298.8 A. These values are *not* the same, because the three loads have different impedance angles. Essentially, Load 3 is supplying some of the reactive power being consumed by Loads 1 and 2, so that it does not have to come from the generator.

2-4.   Prove that the line voltage of a Y-connected generator with an *acb* phase sequence lags the corresponding phase voltage by 30°. Draw a phasor diagram showing the phase and line voltages for this generator.

SOLUTION If the generator has an *acb* phase sequence, then the three phase voltages will be

$$\mathbf{V}_{an} = V_\phi\angle 0°$$

$$\mathbf{V}_{bn} = V_\phi\angle -240°$$

$$\mathbf{V}_{cn} = V_\phi\angle -120°$$

The relationship between line voltage and phase voltage is derived below. By Kirchhoff's voltage law, the line-to-line voltage $\mathbf{V}_{ab}$ is given by

$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b$$

$$\mathbf{V}_{ab} = V_\phi\angle 0° - V_\phi\angle -240°$$

$$\mathbf{V}_{ab} = V_\phi - \left(-\frac{1}{2}V_\phi + j\frac{\sqrt{3}}{2}V_\phi\right) = \frac{3}{2}V_\phi - j\frac{\sqrt{3}}{2}V_\phi$$

$$\mathbf{V}_{ab} = \sqrt{3}V_\phi\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)$$

$$\mathbf{V}_{ab} = \sqrt{3}V_\phi\angle -30°$$

Thus the line voltage *lags* the corresponding phase voltage by 30°. The phasor diagram for this connection is shown below.
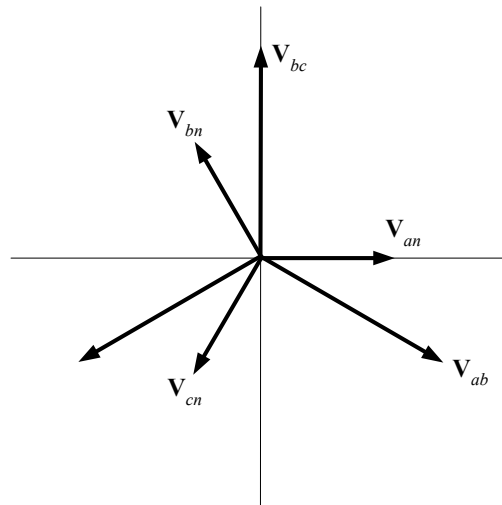
2-5. Find the magnitudes and angles of each line and phase voltage and current on the load shown in Figure P2-3.
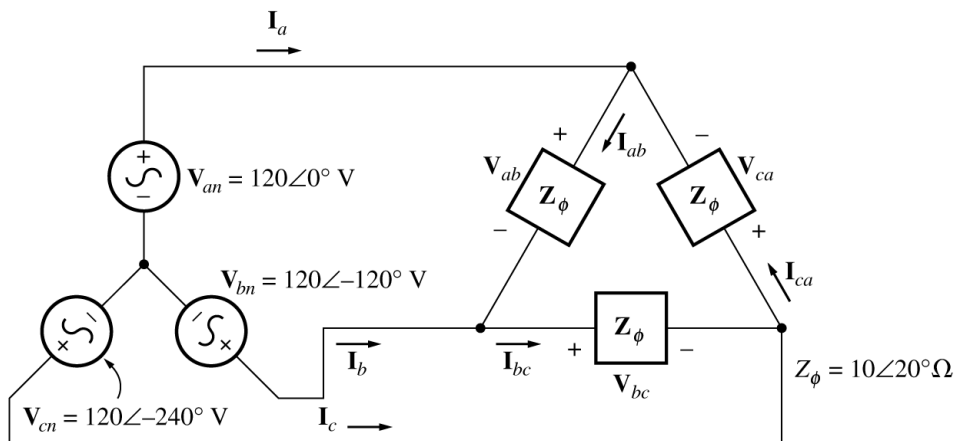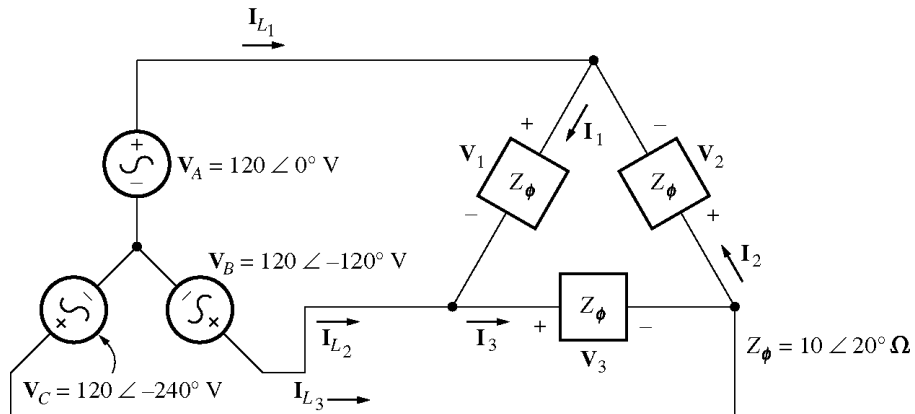
SOLUTION Note that because this load is Δ-connected, the line and phase voltages are identical.

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = 120\angle 0° \text{ V} - 120\angle\text{-}120° \text{ V} = 208\angle 30° \text{ V}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = 120\angle -120° \text{ V} - 120\angle\text{-}240° \text{ V} = 208\angle\text{-}90° \text{ V}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = 120\angle -240° \text{ V} - 120\angle 0° \text{ V} = 208\angle 150° \text{ V}$$

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ab}}{Z_\phi} = \frac{208\angle 30° \text{ V}}{10\angle 20° \text{ }\Omega} = 20.8\angle 10° \text{ A}$$

$$\mathbf{I}_{bc} = \frac{\mathbf{V}_{bc}}{Z_\phi} = \frac{208\angle -90° \text{ V}}{10\angle 20° \text{ }\Omega} = 20.8\angle -110° \text{ A}$$

$$\mathbf{I}_{ca} = \frac{\mathbf{V}_{ca}}{Z_\phi} = \frac{208\angle 150° \text{ V}}{10\angle 20° \text{ }\Omega} = 20.8\angle 130° \text{ A}$$

$$\mathbf{I}_a = \mathbf{I}_{ab} - \mathbf{I}_{ca} = 20.8\angle 10° \text{ A } - \text{ } 20.8\angle 130° \text{ A} = 36\angle -20° \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_{bc} - \mathbf{I}_{ab} = 20.8\angle -110° \text{ A } - \text{ } 20.8\angle 10° \text{ A} = 36\angle -140° \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_{ca} - \mathbf{I}_{bc} = 20.8\angle 130° \text{ A } - \text{ } 20.8\angle -110° \text{ A} = 36\angle 100° \text{ A}$$

2-6. Figure P2-4 shows a small 480-V distribution system. Assume that the lines in the system have zero impedance.
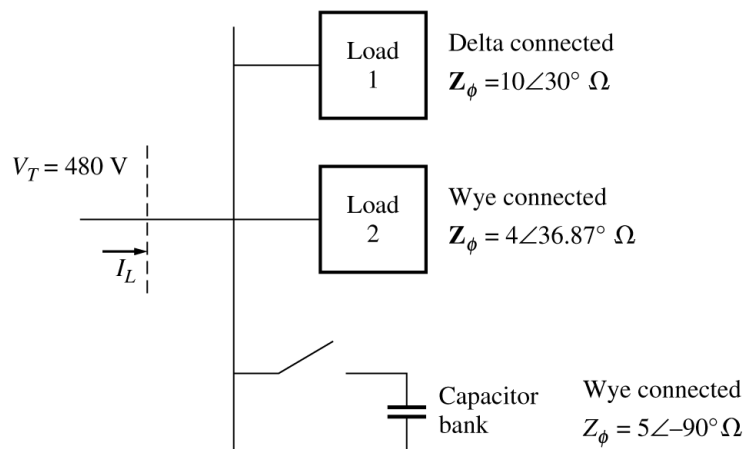
(a) If the switch shown is open, find the real, reactive, and apparent powers in the system. Find the total current supplied to the distribution system by the utility.

(b) Repeat part *(a)* with the switch closed. What happened to the total current supplied? Why?

SOLUTION

(a) With the switch open, the power supplied to each load is

$$P_1 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(480 \text{ V})^2}{10\text{ }\Omega}\cos 30° = 59.86 \text{ kW}$$

$$Q_1 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(480 \text{ V})^2}{10\text{ }\Omega}\sin 30° = 34.56 \text{ kvar}$$

$$P_2 = 3\frac{V_\phi^2}{Z}\cos\theta = 3\frac{(277 \text{ V})^2}{4\text{ }\Omega}\cos 36.87° = 46.04 \text{ kW}$$

$$Q_2 = 3\frac{V_\phi^2}{Z}\sin\theta = 3\frac{(277 \text{ V})^2}{4\text{ }\Omega}\sin 36.87° = 34.53 \text{ kvar}$$

$$P_{TOT} = P_1 + P_2 = 59.86 \text{ kW} + 46.04 \text{ kW} = 105.9 \text{ kW}$$

$$Q_{TOT} = Q_1 + Q_2 = 34.56 \text{ kvar } + \text{ } 34.53 \text{ kvar } = \text{ } 69.09 \text{ kvar}$$

The apparent power supplied by the utility is

$$S_{TOT} = \sqrt{P_{TOT}^2 + Q_{TOT}^2} = 126.4 \text{ kVA}$$

The power factor supplied by the utility is

$$PF = \cos\left[\tan^{-1}\frac{Q_{\text{TOT}}}{P_{\text{TOT}}}\right] = \cos\left[\tan^{-1}\frac{69.09 \text{ kvar}}{105.9 \text{ kW}}\right] = 0.838 \text{ lagging}$$

The current supplied by the utility is

$$I_L = \frac{P_{\text{TOT}}}{\sqrt{3}\ V_T\ \text{PF}} = \frac{105.9 \text{ kW}}{\sqrt{3}\ (480 \text{ V})\ (0.838)} = 152 \text{ A}$$

(b)   With the switch closed, P_3 is added to the circuit. The real and reactive power of P_3 is

$$P_3 = 3\frac{V_\phi^{\ 2}}{Z}\cos\theta = 3\frac{(277 \text{ V})^2}{5\ \Omega}\cos\left(-90°\right) = 0 \text{ kW}$$

$$P_3 = 3\frac{V_\phi^{\ 2}}{Z}\sin\theta = 3\frac{(277 \text{ V})^2}{5\ \Omega}\sin\left(-90°\right) = -46.06 \text{ kvar}$$

$$P_{\text{TOT}} = P_1 + P_2 + P_3 = 59.86 \text{ kW} + 46.04 \text{ kW} + 0 \text{ kW} = 105.9 \text{ kW}$$

$$Q_{\text{TOT}} = Q_1 + Q_2 + Q_3 = 34.56 \text{ kvar} + 34.53 \text{ kvar} - 46.06 \text{ kvar} = 23.03 \text{ kvar}$$

The apparent power supplied by the utility is

$$S_{\text{TOT}} = \sqrt{P_{\text{TOT}}^{\ 2} + Q_{\text{TOT}}^{\ 2}} = 108.4 \text{ kVA}$$

The power factor supplied by the utility is

$$PF = \cos\left[\tan^{-1}\frac{Q_{\text{TOT}}}{P_{\text{TOT}}}\right] = \cos\left[\tan^{-1}\frac{23.03 \text{ kVAR}}{105.9 \text{ kW}}\right] = 0.977 \text{ lagging}$$

The current supplied by the utility is

$$I_L = \frac{P_{\text{TOT}}}{\sqrt{3}\ V_T\ \text{PF}} = \frac{105.9 \text{ kW}}{\sqrt{3}\ (480 \text{ V})\ (0.977)} = 130.4 \text{ A}$$

Chapter 3: *Transformers*

3-1. The secondary winding of a transformer has a terminal voltage of $v_s(t) = 282.8 \sin 377t$ V. The turns ratio of the transformer is 50:200 ($a = 0.25$). If the secondary current of the transformer is $i_s(t) = 7.07 \sin(377t - 36.87°)$ A, what is the primary current of this transformer? What are its voltage regulation and efficiency? The impedances of this transformer referred to the primary side are

$$R_{eq} = 0.05 \ \Omega \qquad\qquad R_C = 75 \ \Omega$$

$$X_{eq} = 0.225 \ \Omega \qquad\qquad X_M = 20 \ \Omega$$

SOLUTION The equivalent circuit of this transformer is shown below. (Since no particular equivalent circuit was specified, we are using the approximate equivalent circuit referred to the primary side.)

(a) $R_{eqP} = R_P + a^2 R_S$

$X_{eqP} = X_P + a^2 X_S$
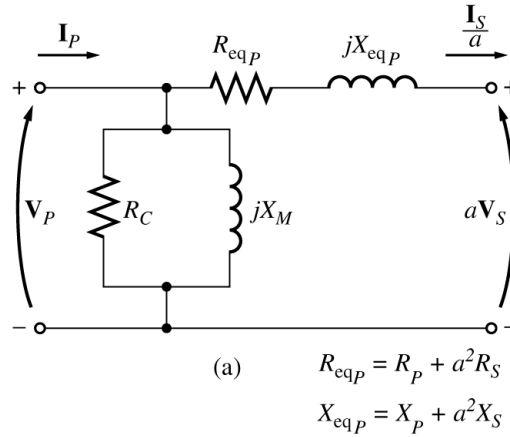
The secondary voltage and current are

$$\mathbf{V}_S = \frac{282.8}{\sqrt{2}} \angle 0° \ \text{V} = 200 \angle 0° \ \text{V}$$

$$\mathbf{I}_S = \frac{7.07}{\sqrt{2}} \angle -36.87° \ \text{A} \ = \ 5\angle -36.87° \ \text{A}$$

The secondary voltage referred to the primary side is

$$\mathbf{V}_S{}' = a\mathbf{V}_S = 50\angle 0° \ \text{V}$$

The secondary current referred to the primary side is

$$\mathbf{I}_S{}' = \frac{\mathbf{I}_S}{a} = 20\angle -36.87° \ \text{A}$$

The primary circuit voltage is given by

$$\mathbf{V}_P = \mathbf{V}_S{}' + \mathbf{I}_S{}' \left(R_{eq} + jX_{eq}\right)$$
$$\mathbf{V}_P = 50\angle 0° \ \text{V} + (20\angle -36.87° \ \text{A})(0.05 \ \Omega + j0.225 \ \Omega) = 53.6\angle 3.2° \ \text{V}$$

The excitation current of this transformer is

$$\mathbf{I}_{EX} = \mathbf{I}_C + \mathbf{I}_M = \frac{53.6\angle 3.2° \ \text{V}}{75 \ \Omega} + \frac{53.6\angle 3.2° \ \text{V}}{j20 \ \Omega} = 0.7145\angle 3.2° + 2.679\angle -86.8°$$

$$\mathbf{I}_{EX} = 2.77\angle -71.9°$$

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Therefore, the total primary current of this transformer is

$$\mathbf{I}_P = \mathbf{I}_S' + \mathbf{I}_{EX} = 20\angle -36.87° + 2.77\angle -71.9° = 22.3\angle -41.0°\ \text{A}$$

The voltage regulation of the transformer at this load is

$$\text{VR} = \frac{V_P - aV_S}{aV_S}\times100\% = \frac{53.6 - 50}{50}\times100\% = 7.2\%$$

The input power to this transformer is

$$P_{IN} = V_P I_P\ \cos\ \theta = (53.6\ \text{V})(22.3\ \text{A})\cos\ \left[3.2° - (-41.0°)\right]$$
$$P_{IN} = (53.6\ \text{V})(22.3\ \text{A})\cos 44.2° = 857\ \text{W}$$

The output power from this transformer is

$$P_{OUT} = V_S I_S\ \cos\ \theta = (200\ \text{V})(5\ \text{A})\cos (36.87°) = 800\ \text{W}$$

Therefore, the transformer's efficiency is

$$\eta = \frac{P_{OUT}}{P_{IN}}\times100\% = \frac{800\ \text{W}}{857\ \text{W}}\times100\% = 93.4\%$$

3-2. A 20-kVA 8000/277-V distribution transformer has the following resistances and reactances:

$$R_P = 32\ \Omega \qquad\qquad R_S = 0.05\ \Omega$$
$$X_P = 45\ \Omega \qquad\qquad X_S = 0.06\ \Omega$$
$$R_C = 250\ \text{k}\Omega \qquad\qquad X_M = 30\ \text{k}\Omega$$

The excitation branch impedances are given referred to the high-voltage side of the transformer.

(a) Find the equivalent circuit of this transformer referred to the high-voltage side.

(b) Find the per-unit equivalent circuit of this transformer.

(c) Assume that this transformer is supplying rated load at 277 V and 0.8 PF lagging. What is this transformer's input voltage? What is its voltage regulation?

(d) What is the transformer's efficiency under the conditions of part *(c)*?

SOLUTION

(a) The turns ratio of this transformer is *a* = 8000/277 = 28.89. Therefore, the secondary impedances referred to the primary side are

$$R_S' = a^2 R_S = (28.89)^2 (0.05\ \Omega) = 41.7\ \Omega$$
$$X_S' = a^2 X_S = (28.89)^2 (0.06\ \Omega) = 50.1\ \Omega$$

The resulting equivalent circuit is
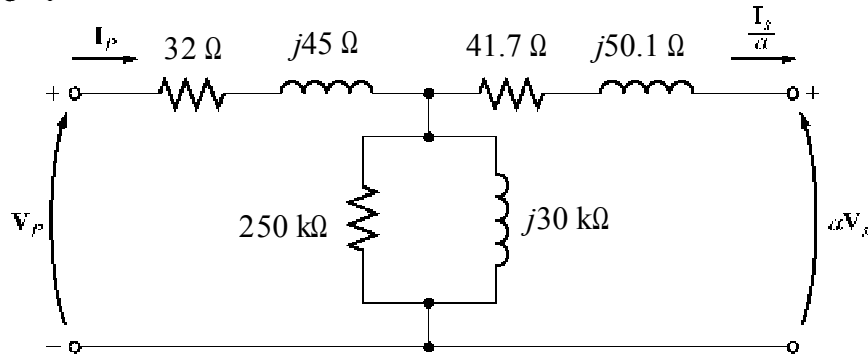
(b)   The rated kVA of the transformer is 20 kVA, and the rated voltage on the primary side is 8000 V, so the rated current in the primary side is 20 kVA/8000 V = 2.5 A. Therefore, the base impedance on the primary side is

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{8000\text{ V}}{2.5\text{ A}} = 3200\ \Omega$$

Since $Z_{pu} = Z_{actual} / Z_{base}$, the resulting per-unit equivalent circuit is as shown below:
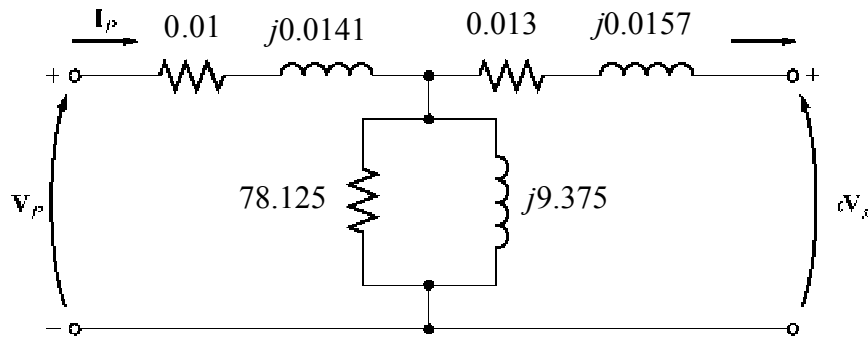
(c) To simplify the calculations, use the simplified equivalent circuit referred to the primary side of the transformer:
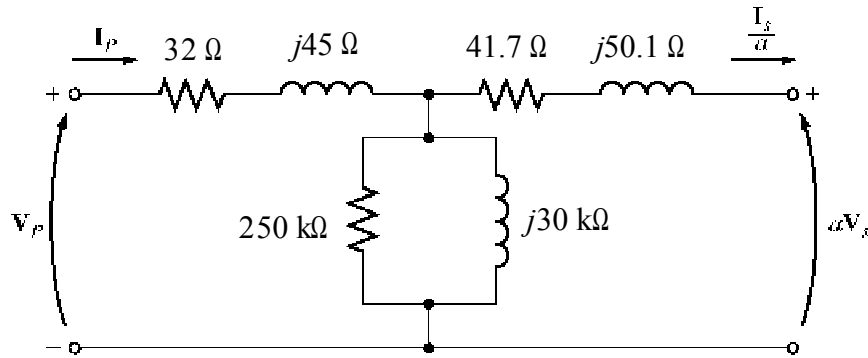
The secondary current in this transformer is

$$\mathbf{I}_S = \frac{20\text{ kVA}}{277\text{ V}} \angle -36.87°\text{ A} = 72.2\angle -36.87°\text{ A}$$

The secondary current referred to the primary side is

$$\mathbf{I}_S{}' = \frac{\mathbf{I}_S}{a} = \frac{72.2\angle -36.87°\text{ A}}{28.89} = 2.50\angle -36.87°\text{ A}$$

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Therefore, the primary voltage on the transformer is

$$\mathbf{V}_P = \mathbf{V}_S{}' + \left(R_{EQ} + jX_{EQ}\right)\mathbf{I}_S{}'$$

$$\mathbf{V}_P = 8000\angle 0° \text{ V} + \left(73.7 + j95.1\right)\left(2.50\angle\text{-}36.87° \text{ A}\right) = 8290\angle 0.55° \text{ V}$$

The voltage regulation of the transformer under these conditions is

$$\text{VR} = \frac{8290 \text{ - } 8000}{8000} \times 100\% = 3.63\%$$

(d) Under the conditions of part *(c)*, the transformer's output power copper losses and core losses are:

$$P_{OUT} = S\cos\theta = \left(20\,\text{kVA}\right)\left(0.8\right) = 16\,\text{kW}$$

$$P_{CU} = \left(I_S{}'\right)^2 R_{EQ} = \left(2.5\right)^2\left(73.7\right) = 461\,\text{W}$$

$$P_{core} = \frac{V_S{}'^2}{R_C} = \frac{8290^2}{250{,}000} = 275\,\text{W}$$

The efficiency of this transformer is

$$\eta = \frac{P_{OUT}}{P_{OUT} + P_{CU} + P_{core}} \times 100\% = \frac{16{,}000}{16{,}000 + 461 + 275} \times 100\% = 95.6\%$$

3-3. A 2000-VA 230/115-V transformer has been tested to determine its equivalent circuit. The results of the tests are shown below.

Open-circuit test	Short-circuit test
V_{OC} = 230 V	V_{SC} = 13.2 V
I_{OC} = 0.45 A	I_{SC} = 6.0 A
P_{OC} = 30 W	P_{SC} = 20.1 W

All data given were taken from the primary side of the transformer.

(a) Find the equivalent circuit of this transformer referred to the low-voltage side of the transformer.

(b) Find the transformer's voltage regulation at rated conditions and (1) 0.8 PF lagging, (2) 1.0 PF, (3) 0.8 PF leading.

(c) Determine the transformer's efficiency at rated conditions and 0.8 PF lagging.

SOLUTION

(a) <u>OPEN CIRCUIT TEST:</u>

$$\left|Y_{EX}\right| = \left|G_C - jB_M\right| = \frac{0.45\,\text{A}}{230\,\text{V}} = 0.001957\ \text{S}$$

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} = \cos^{-1}\frac{30\,\text{W}}{\left(230\,\text{V}\right)\left(0.45\,\text{A}\right)} = 73.15°$$

$$Y_{EX} = G_C - jB_M = 0.001957\angle-73.15°\ \text{S} = 0.000567\text{-}j0.001873\ \text{S}$$

$$R_C = \frac{1}{G_C} = 1763\,\Omega$$

$$X_M = \frac{1}{B_M} = 534\,\Omega$$

<u>SHORT CIRCUIT TEST:</u>

$$\left|Z_{EQ}\right| = \left|R_{EQ} + jX_{EQ}\right| = \frac{13.2\,\text{V}}{6.0\,\text{A}} = 2.20\,\Omega$$

$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} = \cos^{-1}\frac{20.1\,\text{W}}{(13.2\,\text{V})(6\,\text{A})} = 75.3°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 2.20\angle75.3°\,\Omega = 0.558 + j2.128\,\Omega$$

$$R_{EQ} = 0.558\,\Omega$$

$$X_{EQ} = j2.128\,\Omega$$

To convert the equivalent circuit to the secondary side, divide each impedance by the square of the turns ratio ($a = 230/115 = 2$). The resulting equivalent circuit is shown below:

(b) $R_{eq\,S} = \dfrac{R_p}{a^2} + R_S$

$X_{eq\,S} = \dfrac{X_p}{a^2} + X_S$
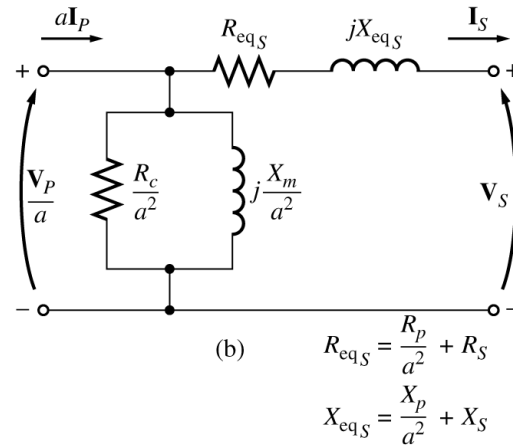
$R_{EQ,S} = 0.140\,\Omega$ $\qquad\qquad$ $X_{EQ,S} = j0.532\,\Omega$

$R_{C,S} = 441\,\Omega$ $\qquad\qquad$ $X_{M,S} = 134\,\Omega$

(b) To find the required voltage regulation, we will use the equivalent circuit of the transformer referred to the secondary side. The rated secondary current is

$$I_S = \frac{1000\,\text{VA}}{115\,\text{V}} = 8.70\,\text{A}$$

We will now calculate the primary voltage referred to the secondary side and use the voltage regulation equation for each power factor.

(1) <u>**0.8 PF Lagging:**</u>

$$\mathbf{V}_P{}' = \mathbf{V}_S + Z_{EQ}\mathbf{I}_S = 115\angle0°\,\text{V} + (0.140 + j0.532\,\Omega)(8.7\angle\text{-}36.87°\,\text{A})$$

$$\mathbf{V}_P{}' = 118.8\angle1.4°\,\text{V}$$

$$\text{VR} = \frac{118.8\,\text{-}\,115}{115}\times100\% = 3.3\%$$

(2) <u>**1.0 PF:**</u>

$$\mathbf{V}_P{}' = \mathbf{V}_S + Z_{EQ}\mathbf{I}_S = 115\angle0°\,\text{V} + (0.140 + j0.532\,\Omega)(8.7\angle0°\,\text{A})$$

$$\mathbf{V}_P{}' = 116.3\angle2.28°\,\text{V}$$

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$$VR = \frac{116.3 - 115}{115} \times 100\% = 1.1\%$$

(3) **0.8 PF Leading:**

$$\mathbf{V}_P{}' = \mathbf{V}_S + Z_{EQ}\mathbf{I}_S = 115\angle 0° \text{ V} + (0.140 + j0.532 \text{ }\Omega)(8.7\angle 36.87° \text{ A})$$

$$\mathbf{V}_P{}' = 113.3\angle 2.24° \text{ V}$$

$$VR = \frac{113.3 - 115}{115} \times 100\% = -1.5\%$$

(c) At rated conditions and 0.8 PF lagging, the output power of this transformer is

$$P_{OUT} = V_S I_S \cos\theta = (115 \text{ V})(8.7 \text{ A})(0.8) = 800 \text{ W}$$

The copper and core losses of this transformer are

$$P_{CU} = I_S{}^2 R_{EQ,S} = (8.7 \text{ A})^2 (0.140 \text{ }\Omega) = 10.6 \text{ W}$$

$$P_{core} = \frac{\left(V_P{}'\right)^2}{R_C} = \frac{(118.8 \text{ V})^2}{441 \text{ }\Omega} = 32.0 \text{ W}$$

Therefore the efficiency of this transformer at these conditions is

$$\eta = \frac{P_{OUT}}{P_{OUT} + P_{CU} + P_{core}} \times 100\% = \frac{800 \text{ W}}{800 \text{ W} + 10.6 \text{ W} + 32.0 \text{ W}} = 94.9\%$$

3-4. A single-phase power system is shown in Figure P3-1. The power source feeds a 100-kVA 14/2.4-kV transformer through a feeder impedance of $38.2 + j140 \text{ }\Omega$. The transformer's equivalent series impedance referred to its low-voltage side is $0.12 + j0.5 \text{ }\Omega$. The load on the transformer is 90 kW at 0.85 PF lagging and 2300 V.

| Source | Feeder (transmission line) | Transformer | Load |
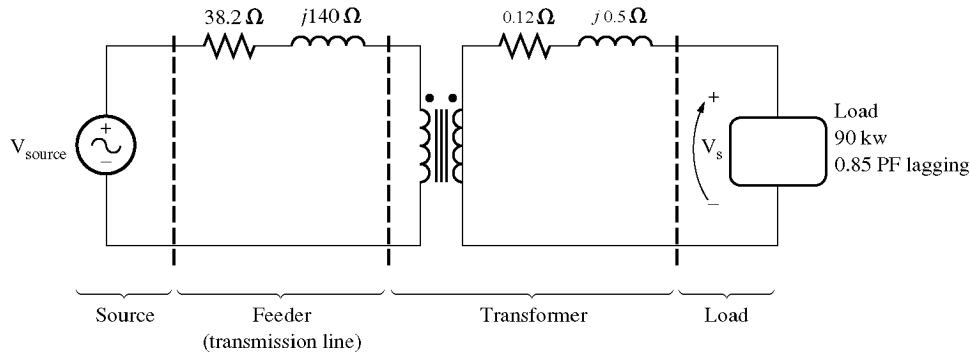
(a) What is the voltage at the power source of the system?

(b) What is the voltage regulation of the transformer?

(c) How efficient is the overall power system?

SOLUTION

To solve this problem, we will refer the circuit to the secondary (low-voltage) side. The feeder's impedance referred to the secondary side is

$$Z_{line}{}' = \left(\frac{2.4 \text{ kV}}{14 \text{ kV}}\right)^2 (38.2 \text{ }\Omega + j140 \text{ }\Omega) = 1.12 + j4.11 \text{ }\Omega$$

The secondary current $\mathbf{I}_S$ is given by

$$I_S = \frac{90\text{ kW}}{(2300\text{ V})(0.9)} = 43.48\text{ A}$$

$$\mathbf{I}_S = 43.48\angle -25.8°\text{ A}$$

(a) The voltage at the power source of this system (referred to the secondary side) is

$$\mathbf{V}_{source}{}' = \mathbf{V}_S + \mathbf{I}_S Z_{line}{}' + \mathbf{I}_S Z_{EQ}$$

$$\mathbf{V}_{source}{}' = 2300\angle 0°\text{ V} + (43.48\angle -25.8°\text{ A})(1.12 + j4.11\ \Omega) + (43.48\angle -25.8°\text{ A})(0.12 + j0.5\ \Omega)$$

$$\mathbf{V}_{source}{}' = 2441\angle 3.7°\text{ V}$$

Therefore, the voltage at the power source is

$$\mathbf{V}_{source} = (2441\angle 3.7°\text{ V})\frac{14\text{ kV}}{2.4\text{ kV}} = 14.24\angle 3.7°\text{ kV}$$

(b) To find the voltage regulation of the transformer, we must find the voltage at the primary side of the transformer (referred to the secondary side) under full load conditions:

$$\mathbf{V}_P{}' = \mathbf{V}_S + \mathbf{I}_S Z_{EQ}$$

$$\mathbf{V}_P{}' = 2300\angle 0°\text{ V} + (43.48\angle -25.8°\text{ A})(0.12 + j0.5\ \Omega) = 2314\angle 0.43°\text{ V}$$

There is a voltage drop of 14 V under these load conditions. Therefore the voltage regulation of the transformer is

$$\text{VR} = \frac{2314 - 2300}{2300}\times 100\% = 0.6\%$$

(c) The power supplied to the load is $P_{OUT} = 90$ kW. The power supplied by the source is

$$P_{IN} = V_{source}{}' I_S \cos\theta = (2441\text{ V})(43.48\text{ A})\cos 29.5° = 92.37\text{ kW}$$

Therefore, the efficiency of the power system is

$$\eta = \frac{P_{OUT}}{P_{IN}}\times 100\% = \frac{90\text{ kW}}{92.37\text{ kW}}\times 100\% = 97.4\%$$

3-5. When travelers from the USA and Canada visit Europe, they encounter a different power distribution system. Wall voltages in North America are 120 V rms at 60 Hz, while typical wall voltages in Europe are 220-240 V at 50 Hz. Many travelers carry small step-up / step-down transformers so that they can use their appliances in the countries that they are visiting. A typical transformer might be rated at 1-kVA and 120/240 V. It has **500**[1] turns of wire on the 120-V side and **1000** turns of wire on the 240-V side. The magnetization curve for this transformer is shown in Figure P3-2, and can be found in file `p32.mag` at this book's Web site.

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[1] Note that this turns ratio was backwards in the first printing of the text. This error should be corrected in all subsequent printings.

Flux (Wb) vs MMF (A-turns)
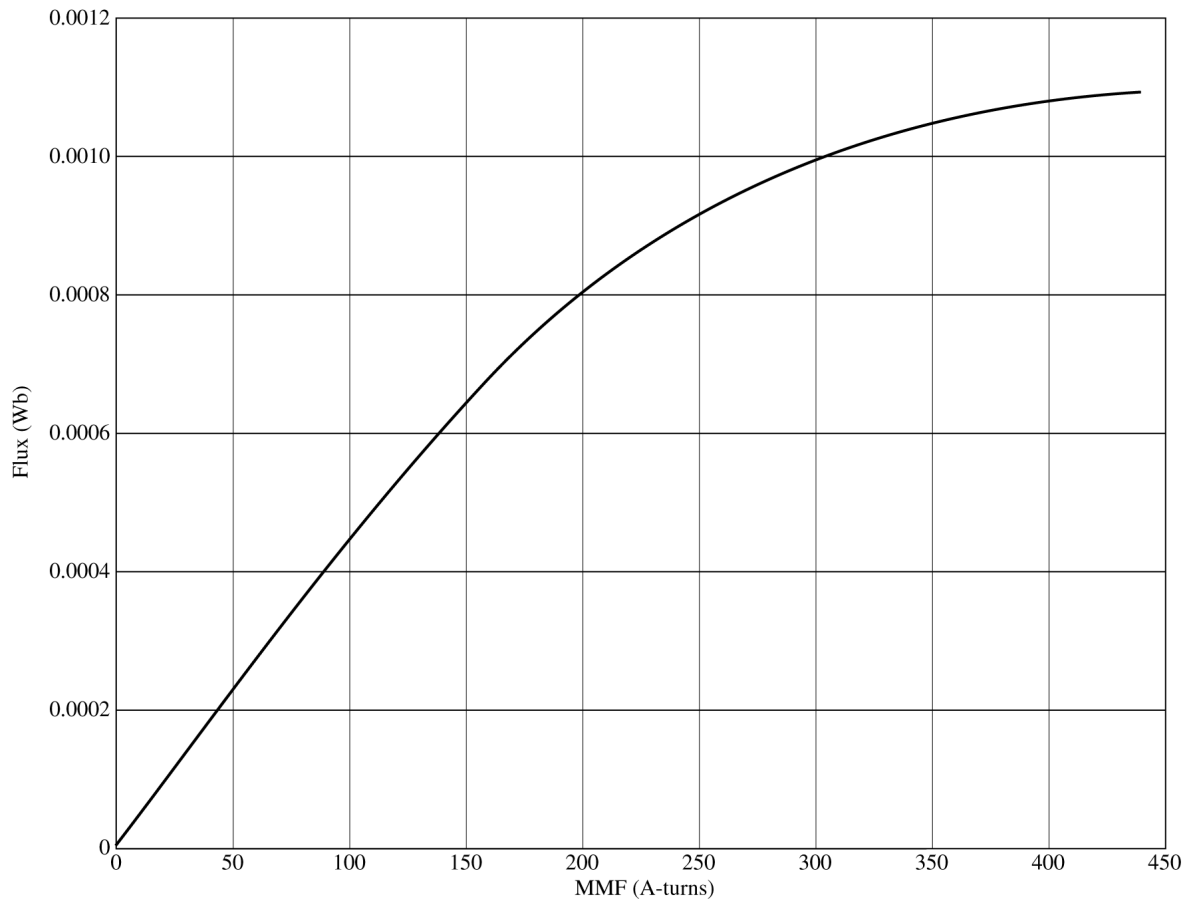
(a) Suppose that this transformer is connected to a 120-V, 60 Hz power source with no load connected to the 240-V side. Sketch the magnetization current that would flow in the transformer. (Use MATLAB to plot the current accurately, if it is available.) What is the rms amplitude of the magnetization current? What percentage of full-load current is the magnetization current?

(b) Now suppose that this transformer is connected to a 240-V, 50 Hz power source with no load connected to the 120-V side. Sketch the magnetization current that would flow in the transformer. (Use MATLAB to plot the current accurately, if it is available.) What is the rms amplitude of the magnetization current? What percentage of full-load current is the magnetization current?

(c) In which case is the magnetization current a higher percentage of full-load current? Why?

SOLUTION

(a) When this transformer is connected to a 120-V 60 Hz source, the flux in the core will be given by the equation

$$\phi(t) = -\frac{V_M}{\omega N_P} \cos \omega t \qquad\qquad (3\text{-}104)$$

The magnetization current required for any given flux level can be found from Figure P3-2, or alternately from the equivalent table in file `p32.mag`. The MATLAB program shown below calculates the flux level at each time, the corresponding magnetization current, and the rms value of the magnetization current.

```
% M-file: prob3_5a.m
% M-file to calculate and plot the magnetization
% current of a 120/240 transformer operating at
```

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```
% 120 volts and 60 Hz.  This program also
% calculates the rms value of the mag. current.

% Load the magnetization curve.  It is in two
% columns, with the first column being mmf and
% the second column being flux.
load p32.mag;
mmf_data = p32(:,1);
flux_data = p32(:,2);

% Initialize values
S = 1000;                  % Apparent power (VA)
Vrms = 120;                % Rms voltage (V)
VM = Vrms * sqrt(2);       % Max voltage (V)
NP = 500;                  % Primary turns

% Calculate angular velocity for 60 Hz
freq = 60;                 % Freq (Hz)
w = 2 * pi * freq;

% Calculate flux versus time
time = 0:1/3000:1/30;      % 0 to 1/30 sec
flux = -VM/(w*NP) * cos(w .* time);

% Calculate the mmf corresponding to a given flux
% using the MATLAB interpolation function.
mmf = interp1(flux_data,mmf_data,flux);

% Calculate the magnetization current
im = mmf / NP;

% Calculate the rms value of the current
irms = sqrt(sum(im.^2)/length(im));
disp(['The rms current at 120 V and 60 Hz is ', num2str(irms)]);

% Calculate the full-load current
i_fl = S / Vrms;

% Calculate the percentage of full-load current
percnt = irms / i_fl * 100;
disp(['The magnetization current is ' num2str(percnt) ...
      '% of full-load current.']);

% Plot the magnetization current.
figure(1)
plot(time,im);
title ('\bfMagnetization Current at 120 V and 60 Hz');
xlabel ('\bfTime (s)');
ylabel ('\bf\itI_{m} \rm(A)');
axis([0 0.04 -0.5 0.5]);
grid on;
```

When this program is executed, the results are

```
» prob3_5a
The rms current at 120 V and 60 Hz is 0.31863
The magnetization current is 3.8236% of full-load current.
```

The rms magnetization current is 0.318 A. Since the full-load current is 1000 VA / 120 V = 8.33 A, the magnetization current is 3.82% of the full-load current. The resulting plot is

Magnetization Current at 120 V and 60 Hz
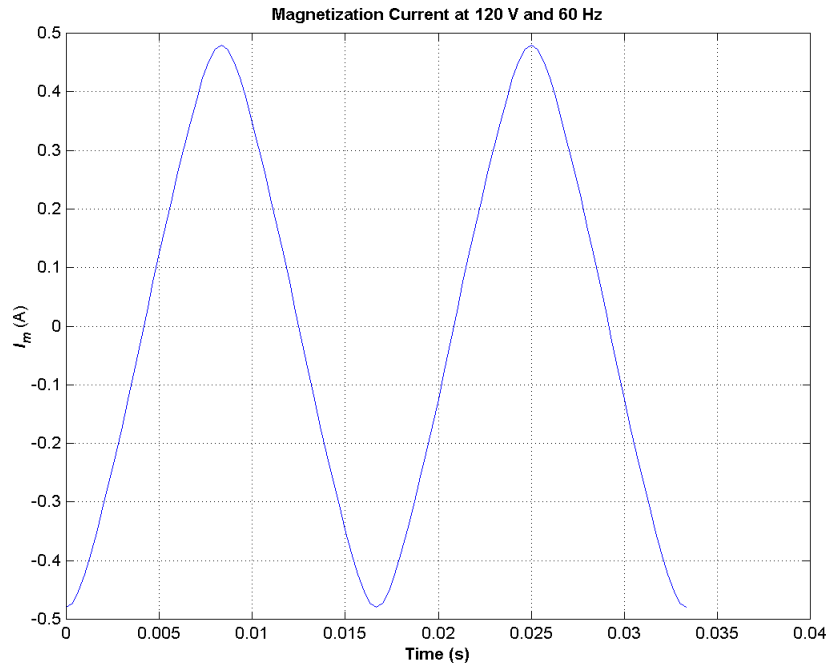
(b)   When this transformer is connected to a 240-V 50 Hz source, the flux in the core will be given by the equation

$$\phi(t) = -\frac{V_M}{\omega N_P} \cos \omega t \qquad (3\text{-}104)$$

The magnetization current required for any given flux level can be found from Figure P3-2, or alternately from the equivalent table in file `p32.mag`. The MATLAB program shown below calculates the flux level at each time, the corresponding magnetization current, and the rms value of the magnetization current.

```
% M-file: prob3_5b.m
% M-file to calculate and plot the magnetization
% current of a 120/240 transformer operating at
% 240 volts and 50 Hz.  This program also
% calculates the rms value of the mag. current.

% Load the magnetization curve.  It is in two
% columns, with the first column being mmf and
% the second column being flux.
load p32.mag;
mmf_data = p32(:,1);
flux_data = p32(:,2);

% Initialize values
S = 1000;                 % Apparent power (VA)
Vrms = 240;               % Rms voltage (V)
VM = Vrms * sqrt(2);      % Max voltage (V)
NP = 1000;                % Primary turns

% Calculate angular velocity for 50 Hz
freq = 50;                % Freq (Hz)
```

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```
w = 2 * pi * freq;

% Calculate flux versus time
time = 0:1/2500:1/25;        % 0 to 1/25 sec
flux = -VM/(w*NP) * cos(w .* time);

% Calculate the mmf corresponding to a given flux
% using the MATLAB interpolation function.
mmf = interp1(flux_data,mmf_data,flux);

% Calculate the magnetization current
im = mmf / NP;

% Calculate the rms value of the current
irms = sqrt(sum(im.^2)/length(im));
disp(['The rms current at 50 Hz is ', num2str(irms)]);

% Calculate the full-load current
i_fl = S / Vrms;

% Calculate the percentage of full-load current
percnt = irms / i_fl * 100;
disp(['The magnetization current is ' num2str(percnt) ...
      '% of full-load current.']);

% Plot the magnetization current.
figure(1);
plot(time,im);
title ('\bfMagnetization Current at 240 V and 50 Hz');
xlabel ('\bfTime (s)');
ylabel ('\bf\itI_{m} \rm(A)');
axis([0 0.04 -0.5 0.5]);
grid on;
```

When this program is executed, the results are

```
» prob3_5b
The rms current at 50 Hz is 0.22973
The magnetization current is 5.5134% of full-load current.
```

The rms magnetization current is 0.318 A. Since the full-load current is 1000 VA / 240 V = 4.17 A, the magnetization current is 5.51% of the full-load current. The resulting plot is shown below.

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Magnetization Current at 240 V and 50 Hz
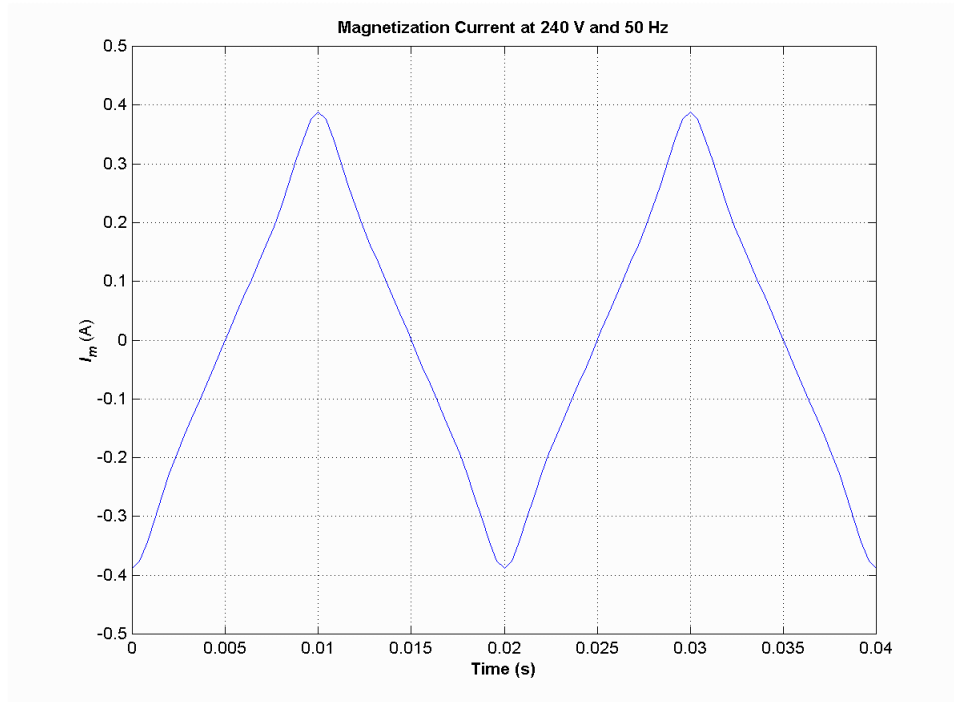
(c) The magnetization current is a higher percentage of the full-load current for the 50 Hz case than for the 60 Hz case. This is true because the peak flux is higher for the 50 Hz waveform, driving the core further into saturation.

3-6. A 15-kVA 8000/230-V distribution transformer has an impedance referred to the primary of $80 + j300\ \Omega$. The components of the excitation branch referred to the primary side are $R_C = 350\ \text{k}\Omega$ and $X_M = 70\ \text{k}\Omega$.

(a) If the primary voltage is 7967 V and the load impedance is $Z_L = 3.2 + j1.5\ \Omega$, what is the secondary voltage of the transformer? What is the voltage regulation of the transformer?

(b) If the load is disconnected and a capacitor of $-j3.5\ \Omega$ is connected in its place, what is the secondary voltage of the transformer? What is its voltage regulation under these conditions?

SOLUTION

(a) The easiest way to solve this problem is to refer all components to the *primary* side of the transformer. The turns ratio is $a = 8000/230 = 34.78$. Thus the load impedance referred to the primary side is

$$Z_L{}' = (34.78)^2 (3.2 + j1.5\ \Omega) = 3871 + j1815\ \Omega$$

The referred secondary current is

$$\mathbf{I}_S{}' = \frac{7967\angle 0^\circ\ \text{V}}{(80 + j300\ \Omega) + (3871 + j1815\ \Omega)} = \frac{7967\angle 0^\circ\ \text{V}}{4481\angle 28.2^\circ\ \Omega} = 1.78\angle - 28.2^\circ\ \text{A}$$

and the referred secondary voltage is

$$\mathbf{V}_S{}' = \mathbf{I}_S{}' Z_L{}' = (1.78\angle - 28.2^\circ\ \text{A})(3871 + j1815\ \Omega) = 7610\angle - 3.1^\circ\ \text{V}$$

The actual secondary voltage is thus

$$\mathbf{V}_S = \frac{\mathbf{V}_S{}'}{a} = \frac{7610\angle - 3.1^\circ\ \text{V}}{34.78} = 218.8\angle - 3.1^\circ\ \text{V}$$

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The voltage regulation is

$$VR = \frac{7967 - 7610}{7610} \times 100\% = 4.7\%$$

(b) As before, the easiest way to solve this problem is to refer all components to the *primary* side of the transformer. The turns ratio is again $a = 34.78$. Thus the load impedance referred to the primary side is

$$Z_L{}' = (34.78)^2 (-j3.5\,\Omega) = -j4234\,\Omega$$

The referred secondary current is

$$\mathbf{I}_S{}' = \frac{7967\angle 0°\text{ V}}{(80 + j300\,\Omega) + (-j4234\,\Omega)} = \frac{7967\angle 0°\text{ V}}{3935\angle -88.8°\,\Omega} = 2.025\angle 88.8°\text{ A}$$

and the referred secondary voltage is

$$\mathbf{V}_S{}' = \mathbf{I}_S{}' Z_L{}' = (2.25\angle 88.8°\text{ A})(-j4234\,\Omega) = 8573\angle -1.2°\text{ V}$$

The actual secondary voltage is thus

$$\mathbf{V}_S = \frac{\mathbf{V}_S{}'}{a} = \frac{8573\angle -1.2°\text{ V}}{34.78} = 246.5\angle -1.2°\text{ V}$$

The voltage regulation is

$$VR = \frac{7967 - 8573}{8573} \times 100\% = -7.07\%$$

3-7. A 5000-kVA 230/13.8-kV single-phase power transformer has a per-unit resistance of 1 percent and a per-unit reactance of 5 percent (data taken from the transformer's nameplate). The open-circuit test performed on the low-voltage side of the transformer yielded the following data:

$$V_{OC} = 13.8\text{ kV} \qquad I_{OC} = 15.1\text{ A} \qquad P_{OC} = 44.9\text{ kW}$$

(a) Find the equivalent circuit referred to the low-voltage side of this transformer.

(b) If the voltage on the secondary side is 13.8 kV and the power supplied is 4000 kW at 0.8 PF lagging, find the voltage regulation of the transformer. Find its efficiency.

SOLUTION

(a) The open-circuit test was performed on the low-voltage side of the transformer, so it can be used to directly find the components of the excitation branch relative to the low-voltage side.

$$|Y_{EX}| = |G_C - jB_M| = \frac{15.1\text{ A}}{13.8\text{ kV}} = 0.0010942$$

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC}I_{OC}} = \cos^{-1}\frac{44.9\text{ kW}}{(13.8\text{ kV})(15.1\text{ A})} = 77.56°$$

$$Y_{EX} = G_C - jB_M = 0.0010942\angle -77.56°\text{ S} = 0.0002358 - j0.0010685\text{ S}$$

$$R_C = \frac{1}{G_C} = 4240\,\Omega$$

$$X_M = \frac{1}{B_M} = 936\,\Omega$$

The base impedance of this transformer referred to the secondary side is

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(13.8 \text{ kV})^2}{5000 \text{ kVA}} = 38.09 \, \Omega$$

so $R_{EQ} = (0.01)(38.09 \, \Omega) = 0.38 \, \Omega$ and $X_{EQ} = (0.05)(38.09 \, \Omega) = 1.9 \, \Omega$. The resulting equivalent circuit is shown below:

$$R_{eq,s} = 0.38 \, \Omega \qquad\qquad X_{eq,s} = j1.9 \, \Omega$$
$$R_{C,s} = 4240 \, \Omega \qquad\qquad X_{M,s} = 936 \, \Omega$$
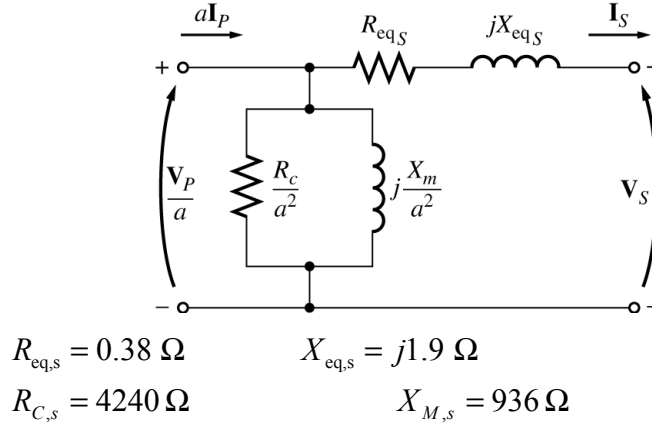
(b) If the load on the secondary side of the transformer is 4000 kW at 0.8 PF lagging and the secondary voltage is 13.8 kV, the secondary current is

$$I_S = \frac{P_{LOAD}}{V_S \text{ PF}} = \frac{4000 \text{ kW}}{(13.8 \text{ kV})(0.8)} = 362.3 \text{ A}$$
$$\mathbf{I}_S = 362.3\angle -36.87° \text{ A}$$

The voltage on the primary side of the transformer (referred to the secondary side) is

$$\mathbf{V}_P{}' = \mathbf{V}_S + \mathbf{I}_S Z_{EQ}$$
$$\mathbf{V}_P{}' = 13,800\angle 0° \text{ V} + (362.3\angle -36.87° \text{ A})(0.38 + j1.9 \, \Omega) = 14,330\angle 1.9° \text{ V}$$

There is a voltage drop of 14 V under these load conditions. Therefore the voltage regulation of the transformer is

$$VR = \frac{14,330 - 13,800}{13,800} \times 100\% = 3.84\%$$

The transformer copper losses and core losses are

$$P_{CU} = I_S{}^2 \, R_{EQ,S} = (362.3 \text{ A})^2 (0.38 \, \Omega) = 49.9 \text{ kW}$$

$$P_{core} = \frac{\left(V_P{}'\right)^2}{R_C} = \frac{(14,330 \text{ V})^2}{4240 \, \Omega} = 48.4 \text{ W}$$

Therefore the efficiency of this transformer at these conditions is

$$\eta = \frac{P_{OUT}}{P_{OUT} + P_{CU} + P_{core}} \times 100\% = \frac{4000 \text{ W}}{4000 \text{ W} + 49.9 \text{ W} + 48.4 \text{ W}} = 97.6\%$$

3-8. A 150-MVA 15/200-kV single-phase power transformer has a per-unit resistance of 1.2 percent and a per-unit reactance of 5 percent (data taken from the transformer's nameplate). The magnetizing impedance is $j100$ per unit.

(a) Find the equivalent circuit referred to the low-voltage side of this transformer.

(b) Calculate the voltage regulation of this transformer for a full-load current at power factor of 0.8 lagging.

(c) Assume that the primary voltage of this transformer is a constant 15 kV, and plot the secondary voltage as a function of load current for currents from no-load to full-load. Repeat this process for power factors of 0.8 lagging, 1.0, and 0.8 leading.

SOLUTION

(a) The base impedance of this transformer referred to the primary (low-voltage) side is

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(15\,\text{kV})^2}{150\,\text{MVA}} = 1.5\,\Omega$$

so

$$R_{EQ} = (0.012)(1.5\,\Omega) = 0.018\,\Omega$$

$$X_{EQ} = (0.05)(1.5\,\Omega) = 0.075\,\Omega$$

$$X_M = (100)(1.5\,\Omega) = 150\,\Omega$$

The equivalent circuit is

(a) $R_{eqP} = R_P + a^2 R_S$

$X_{eqP} = X_P + a^2 X_S$
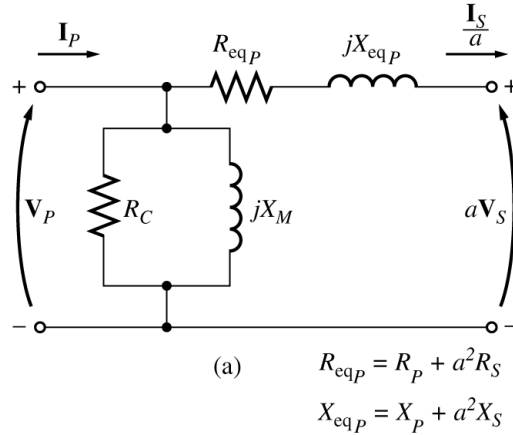
$R_{EQ,P} = 0.018\,\Omega$ $X_{EQ,P} = j0.075\,\Omega$

$R_C = \text{not specified}$ $X_M = 150\,\Omega$

(b) If the load on the *secondary* (high voltage) side of the transformer is 150 MVA at 0.8 PF lagging, and the referred secondary voltage is 15 kV, then the referred secondary current is

$$I_S' = \frac{P_{LOAD}}{V_S\,\text{PF}} = \frac{150\,\text{MVA}}{(15\,\text{kV})(0.8)} = 12,500\,\text{A}$$

$$\mathbf{I}_S' = 12,500\angle -36.87°\,\text{A}$$

The voltage on the primary side of the transformer is

$$\mathbf{V}_P = \mathbf{V}_S' + \mathbf{I}_S' Z_{EQ,P}$$

$$\mathbf{V}_P = 15,000\angle 0°\,\text{V} + (12,500\angle -36.87°\,\text{A})(0.018 + j0.075\,\Omega) = 15,755\angle 2.24°\,\text{V}$$

Therefore the voltage regulation of the transformer is

$$VR = \frac{15,755 - 15,000}{15,000} \times 100\% = 5.03\%$$

(c) This problem is repetitive in nature, and is ideally suited for MATLAB. A program to calculate the secondary voltage of the transformer as a function of load is shown below:

```
% M-file: prob3_8.m
% M-file to calculate and plot the secondary voltage
% of a transformer as a function of load for power
% factors of 0.8 lagging, 1.0, and 0.8 leading.
% These calculations are done using an equivalent
% circuit referred to the primary side.

% Define values for this transformer
VP = 15000;                    % Primary voltage (V)
amps = 0:125:12500;            % Current values (A)
Req = 0.018;                   % Equivalent R (ohms)
Xeq = 0.075;                   % Equivalent X (ohms)

% Calculate the current values for the three
% power factors.  The first row of I contains
% the lagging currents, the second row contains
% the unity currents, and the third row contains
% the leading currents.
I(1,:) = amps .* ( 0.8 - j*0.6);  % Lagging
I(2,:) = amps .* ( 1.0       );  % Unity
I(3,:) = amps .* ( 0.8 + j*0.6);  % Leading

% Calculate VS referred to the primary side
% for each current and power factor.
aVS = VP - (Req.*I + j.*Xeq.*I);

% Refer the secondary voltages back to the
% secondary side using the turns ratio.
VS = aVS * (200/15);

% Plot the secondary voltage versus load
plot(amps,abs(VS(1,:)),'b-','LineWidth',2.0);
hold on;
plot(amps,abs(VS(2,:)),'k--','LineWidth',2.0);
plot(amps,abs(VS(3,:)),'r-.','LineWidth',2.0);
title ('\bfSecondary Voltage Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfSecondary Voltage (%)');
legend('0.8 PF lagging','1.0 PF','0.8 PF leading');
grid on;
hold off;
```

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The resulting plot of secondary voltage versus load is shown below:
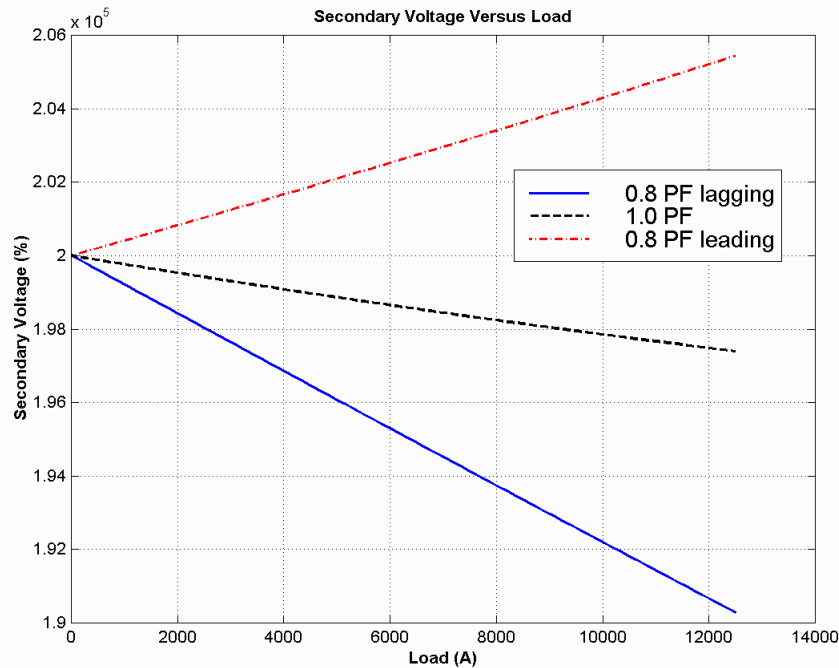
3-9. A three-phase transformer bank is to handle 400 kVA and have a 34.5/13.8-kV voltage ratio. Find the rating of each individual transformer in the bank (high voltage, low voltage, turns ratio, and apparent power) if the transformer bank is connected to *(a)* Y-Y, *(b)* Y-Δ, *(c)* Δ-Y, *(d)* Δ-Δ.

SOLUTION   For these four connections, the apparent power rating of each transformer is 1/3 of the total apparent power rating of the three-phase transformer.

The ratings for **each transformer** in the bank for each connection are given below:

Connection	Primary Voltage	Secondary Voltage	Apparent Power	Turns Ratio
Y-Y	19.9 kV	7.97 kV	133 kVA	2.50:1
Y-Δ	19.9 kV	13.8 kV	133 kVA	1.44:1
Δ-Y	34.5 kV	7.97 kV	133 kVA	4.33:1
Δ-Δ	34.5 kV	13.8 kV	133 kVA	2.50:1

3-10. A Y-connected of three identical 100-kVA 7967/**277**-V[2] transformers is supplied with power directly from a large constant-voltage bus. In the short-circuit test, the recorded values on the high-voltage side for one of these transformers are

$$V_{SC} = 560 \text{ V} \qquad I_{SC} = 12.6 \text{ A} \qquad P_{SC} = 3300 \text{ W}$$

(a) If this bank delivers a rated load at 0.88 PF lagging and rated voltage, what is the line-to-line voltage on the primary of the transformer bank?

(b) What is the voltage regulation under these conditions?

(c) Assume that the primary line voltage of this transformer bank is a constant 13.8 kV, and plot the secondary line voltage as a function of load current for currents from no-load to full-load. Repeat this process for power factors of 0.85 lagging, 1.0, and 0.85 leading.

(d) Plot the voltage regulation of this transformer as a function of load current for currents from no-load to full-load. Repeat this process for power factors of 0.85 lagging, 1.0, and 0.85 leading.

---

[2] This voltage was misprinted as 7967/480-V in the first printing of the text. This error should be corrected in all subsequent printings.

SOLUTION From the short-circuit information, it is possible to determine the per-phase impedance of the transformer bank referred to the high-voltage side. The primary of this transformer is Y-connected, so the short-circuit phase voltage is

$$V_{\phi,\text{SC}} = \frac{V_{\text{SC}}}{\sqrt{3}} = 323.3 \text{ V}$$

the short-circuit phase current is

$$I_{\phi,\text{SC}} = 12.6 \text{ A}$$

and the power per phase is

$$P_{\phi,\text{SC}} = \frac{P_{\text{SC}}}{3} = 1100 \text{ W}$$

Thus the per-phase impedance is

$$\left| Z_{\text{EQ}} \right| = \left| R_{\text{EQ}} + jX_{\text{EQ}} \right| = \frac{323.3 \text{ V}}{12.6 \text{ A}} = 25.66 \, \Omega$$

$$\theta = \cos^{-1} \frac{P_{\text{SC}}}{V_{\text{SC}} I_{\text{SC}}} = \cos^{-1} \frac{1100 \text{ W}}{(323.3 \text{ V})(12.6 \text{ A})} = 74.3°$$

$$Z_{\text{EQ}} = R_{\text{EQ}} + jX_{\text{EQ}} = 25.66 \angle 74.3° \, \Omega = 6.94 + j24.7 \, \Omega$$

$$R_{\text{EQ}} = 6.94 \, \Omega$$

$$X_{\text{EQ}} = j24.7 \, \Omega$$

(a) If this Y-Y transformer bank delivers rated kVA at 0.88 power factor lagging while the secondary voltage is a rated value, then each transformer delivers 33.3 kVA at a voltage of 277 V and 0.88 PF lagging. Referred to the *primary side* of one of the transformers, this load is equivalent to 33.3 kVA at 7967 V and 0.88 PF lagging. The equivalent current flowing in the secondary of one transformer referred to the primary side is

$$I_{\phi,S}{}' = \frac{33.3 \text{ kVA}}{7967 \text{ V}} = 4.184 \text{ A}$$

$$\mathbf{I}_{\phi,S}{}' = 4.184 \angle -28.36° \text{ A}$$

The voltage on the primary side of a single transformer is thus

$$\mathbf{V}_{\phi,P} = \mathbf{V}_{\phi,S}{}' + \mathbf{I}_{\phi,S}{}' Z_{\text{EQ},P}$$

$$\mathbf{V}_{\phi,P} = 7967 \angle 0° \text{ V} + (4.184 \angle -28.36° \text{ A})(6.94 + j24.7 \, \Omega) = 8042 \angle 0.55° \text{ V}$$

The line-to-line voltage on the primary of the transformer is

$$V_{\text{LL},P} = \sqrt{3} \, V_{\phi,P} = \sqrt{3}(8042 \text{ V}) = 13.93 \text{ kV}$$

(b) The voltage regulation of each transformer in the bank, and therefore of the entire transformer bank, is

$$\text{VR} = \frac{8042 - 7967}{7967} \times 100\% = 0.94\%$$

(c)   This sort of repetitive operation is best performed with MATLAB. A suitable MATLAB program is shown below:

```
% M-file: prob3_10c.m
% M-file to calculate and plot the secondary voltage
% of a three-phase Y-Y transformer bank as a function
% of load for power factors of 0.85 lagging, 1.0,
% and 0.85 leading.  These calculations are done using
% an equivalent circuit referred to the primary side.

% Define values for this transformer
VL = 13800;                  % Primary line voltage (V)
VPP = VL / sqrt(3);          % Primary phase voltage (V)
amps = 0:0.04184:4.184;      % Phase current values (A)
Req = 6.94;                  % Equivalent R (ohms)
Xeq = 24.7;                  % Equivalent X (ohms)

% Calculate the current values for the three
% power factors.  The first row of I contains
% the lagging currents, the second row contains
% the unity currents, and the third row contains
% the leading currents.
re = 0.85;
im = sin(acos(re));
I(1,:) = amps .* ( re - j*im);  % Lagging
I(2,:) = amps .* ( 1.0      );  % Unity
I(3,:) = amps .* ( re + j*im);  % Leading

% Calculate secondary phase voltage referred
% to the primary side for each current and
% power factor.
aVSP = VPP - (Req.*I + j.*Xeq.*I);

% Refer the secondary phase voltages back to
% the secondary side using the turns ratio.
% Because this is a delta-connected secondary,
% this is also the line voltage.
VSP = aVSP * (277/7967);

% Convert secondary phase voltage to line
% voltage.
VSL = sqrt(3) * VSP;

% Plot the secondary voltage versus load
plot(amps,abs(VSL(1,:)),'b-','LineWidth',2.0);
hold on;
plot(amps,abs(VSL(2,:)),'k--','LineWidth',2.0);
plot(amps,abs(VSL(3,:)),'r-.','LineWidth',2.0);
title ('\bfSecondary Voltage Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfSecondary Voltage (V)');
legend('0.85 PF lagging','1.0 PF','0.85 PF leading');
```

```
grid on;
hold off;
```

The resulting plot is shown below:

Secondary Voltage Versus Load
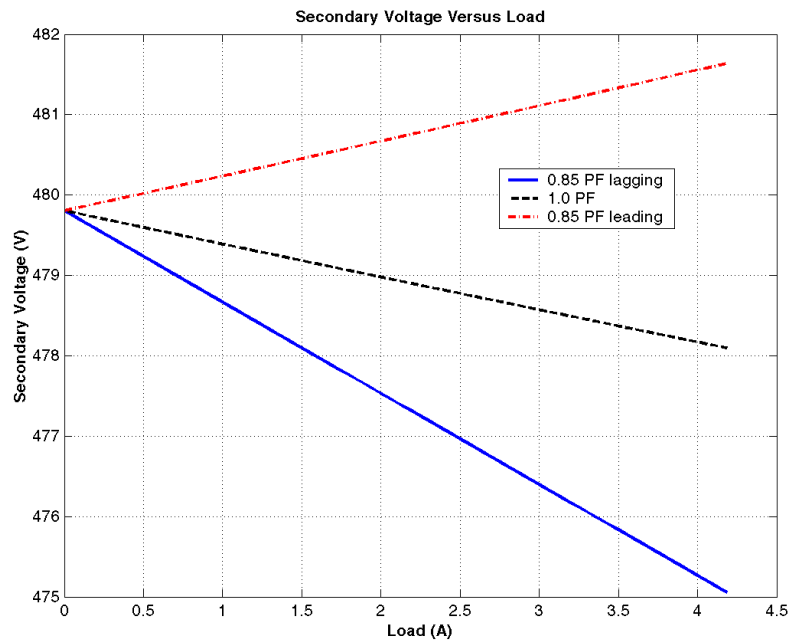
(d) This sort of repetitive operation is best performed with MATLAB. A suitable MATLAB program is shown below:

```
% M-file: prob3_10d.m
% M-file to calculate and plot the voltage regulation
% of a three-phase Y-Y transformer bank as a function
% of load for power factors of 0.85 lagging, 1.0,
% and 0.85 leading.  These calculations are done
% using an equivalent circuit referred to the primary side.

% Define values for this transformer
VL = 13800;                    % Primary line voltage (V)
VPP = VL / sqrt(3);            % Primary phase voltage (V)
amps = 0:0.04184:4.184;        % Phase current values (A)
Req = 6.94;                    % Equivalent R (ohms)
Xeq = 24.7;                    % Equivalent X (ohms)

% Calculate the current values for the three
% power factors.  The first row of I contains
% the lagging currents, the second row contains
% the unity currents, and the third row contains
% the leading currents.
re = 0.85;
im = sin(acos(re));
I(1,:) = amps .* ( re - j*im);  % Lagging
I(2,:) = amps .* ( 1.0      );  % Unity
I(3,:) = amps .* ( re + j*im);  % Leading

% Calculate secondary phase voltage referred
% to the primary side for each current and
% power factor.
```

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```
aVSP = VPP - (Req.*I + j.*Xeq.*I);

% Calculate the voltage regulation.
VR = (VPP - abs(aVSP)) ./ abs(aVSP) .* 100;

% Plot the voltage regulation versus load
plot(amps,VR(1,:),'b-','LineWidth',2.0);
hold on;
plot(amps,VR(2,:),'k--','LineWidth',2.0);
plot(amps,VR(3,:),'r-.','LineWidth',2.0);
title ('\bfVoltage Regulation Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfVoltage Regulation (%)');
legend('0.85 PF lagging','1.0 PF','0.85 PF leading');
grid on;
hold off;
```

The resulting plot is shown below:
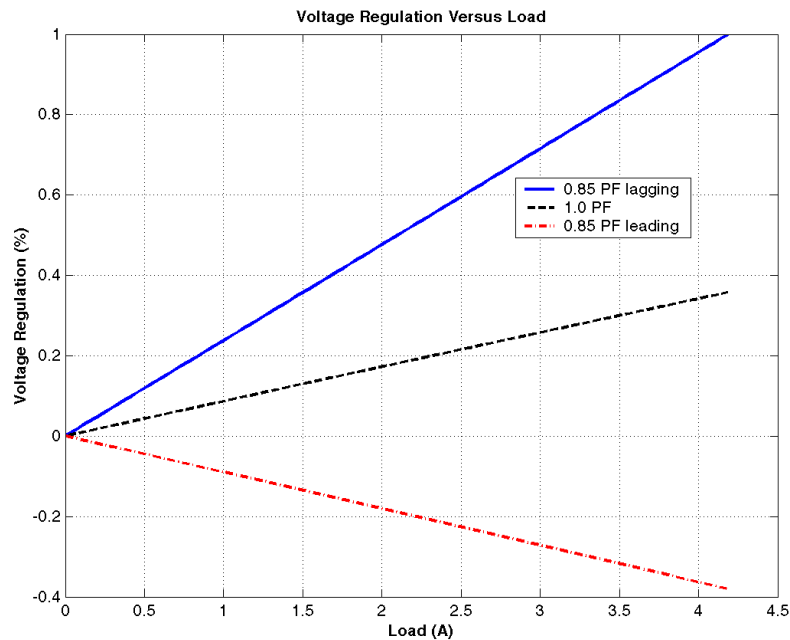
3-11. A 100,000-kVA 230/115-kV Δ-Δ three-phase power transformer has a per-unit resistance of 0.02 pu and a per-unit reactance of 0.055 pu. The excitation branch elements are $R_C = 110$ pu and $X_M = 20$ pu .

(a) If this transformer supplies a load of 80 MVA at 0.85 PF lagging, draw the phasor diagram of one phase of the transformer.

(b) What is the voltage regulation of the transformer bank under these conditions?

(c) Sketch the equivalent circuit referred to the low-voltage side of one phase of this transformer. Calculate all the transformer impedances referred to the low-voltage side.

SOLUTION

(a) The transformer supplies a load of 80 MVA at 0.85 PF lagging. Therefore, the secondary line current of the transformer is

$$I_{LS} = \frac{S}{\sqrt{3}V_{LS}} = \frac{80{,}000{,}000 \text{ VA}}{\sqrt{3}(115{,}000 \text{ V})} = 402 \text{ A}$$

The base value of the secondary line current is

$$I_{LS,\text{base}} = \frac{S_\text{base}}{\sqrt{3}\,V_{LS,\text{base}}} = \frac{100,000,000\text{ VA}}{\sqrt{3}\left(115,000\text{ V}\right)} = 502\text{ A}$$

so the per-unit secondary current is

$$\mathbf{I}_{LS,\text{pu}} = \frac{I_{LS}}{I_{LS,\text{pu}}} = \frac{402\text{ A}}{502\text{ A}}\angle\cos^{-1}\left(0.85\right) = 0.8\angle -31.8°$$

The per-unit phasor diagram is shown below:
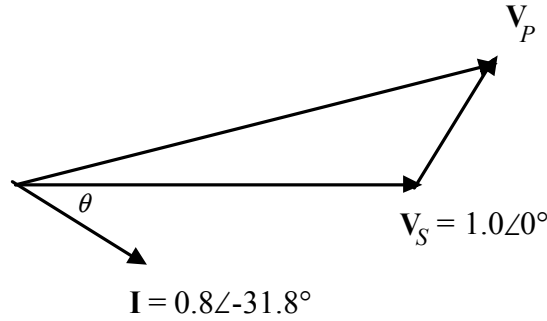
(b) The per-unit primary voltage of this transformer is

$$\mathbf{V}_P = \mathbf{V}_S + \mathbf{I}\,Z_\text{EQ} = 1.0\angle 0° + \left(0.8\angle -31.8°\right)\left(0.02 + j0.055\right) = 1.037\angle 1.6°$$

and the voltage regulation is

$$\text{VR} = \frac{1.037 - 1.0}{1.0}\times 100\% = 3.7\%$$

(c) The base impedance of the transformer referred to the low-voltage side is:

$$Z_\text{base} = \frac{3\,V_{\phi,\text{base}}{}^2}{S_\text{base}} = \frac{2\left(115\text{ kV}\right)^2}{100\text{ MVA}} = 397\ \Omega$$

Each per-unit impedance is converted to actual ohms referred to the low-voltage side by multiplying it by this base impedance. The resulting equivalent circuit is shown below:
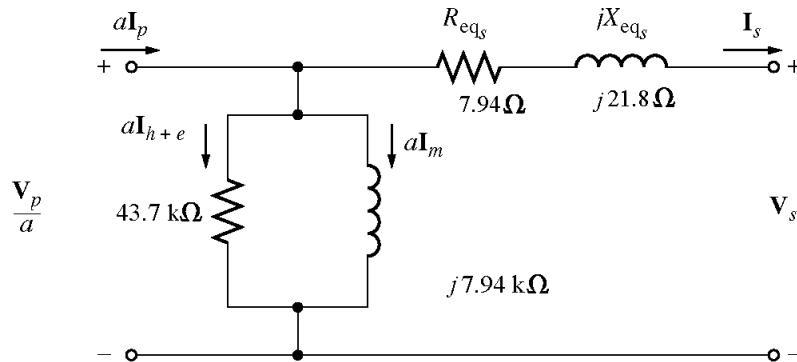
$$R_\text{EQ,S} = \left(0.02\right)\left(397\ \Omega\right) = 7.94\ \Omega$$
$$X_\text{EQ,S} = \left(0.055\right)\left(397\ \Omega\right) = 21.8\ \Omega$$
$$R_C = \left(110\right)\left(397\ \Omega\right) = 43.7\text{ k}\Omega$$
$$X_M = \left(20\right)\left(397\ \Omega\right) = 7.94\text{ k}\Omega$$

3-12. An autotransformer is used to connect a 12.6-kV distribution line to a 13.8-kV distribution line. It must be capable of handling 2000 kVA. There are three phases, connected Y-Y with their neutrals solidly grounded.

(a) What must the N_C / N_{SE} turns ratio be to accomplish this connection?

(b) How much apparent power must the windings of each autotransformer handle?

(c) If one of the autotransformers were reconnected as an ordinary transformer, what would its ratings be?

SOLUTION

(a) The transformer is connected Y-Y, so the primary and secondary phase voltages are the line voltages divided by $\sqrt{3}$. The turns ratio of each autotransformer is given by

$$\frac{V_H}{V_L} = \frac{N_C + N_{SE}}{N_C} = \frac{13.8\,\text{kV}/\sqrt{3}}{12.6\,\text{kV}/\sqrt{3}}$$

$$12.6\,N_C + 12.6\,N_{SE} = 13.8\,N_C$$

$$12.6\,N_{SE} = 1.2\,N_C$$

Therefore, $N_C / N_{SE} = 10.5$.

(b) The power advantage of this autotransformer is

$$\frac{S_{IO}}{S_W} = \frac{N_C + N_{SE}}{N_C} = \frac{N_C + 10.5 N_C}{N_C} = 11.5$$

so 1/11.5 of the power in each transformer goes through the windings. Since 1/3 of the total power is associated with each phase, the windings in each autotransformer must handle

$$S_W = \frac{2000\,\text{kVA}}{(3)(11.5)} = 58\,\text{kVA}$$

(c) The voltages across each phase of the autotransformer are $13.8\,\text{kV}/\sqrt{3} = 7967$ V and $12.6\,\text{kV}/\sqrt{3}$ = 7275 V. The voltage across the common winding (N_C) is 7275 kV, and the voltage across the series winding (N_{SE}) is 7967 kV – 7275 kV = 692 V. Therefore, a single phase of the autotransformer connected as an ordinary transformer would be rated at 7275/692 V and 58 kVA.

3-13. A 12.4-kV single-phase generator supplies power to a load through a transmission line. The load's impedance is $Z_{\text{load}} = 500\angle 36.87°\ \Omega$, and the transmission line's impedance is $Z_{\text{line}} = 60\angle 60°\ \Omega$.

(a)

(b)
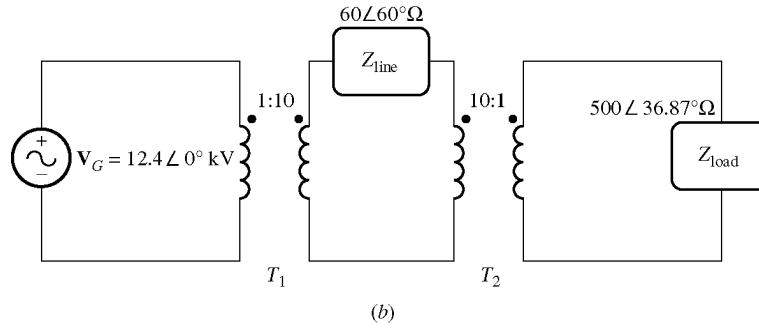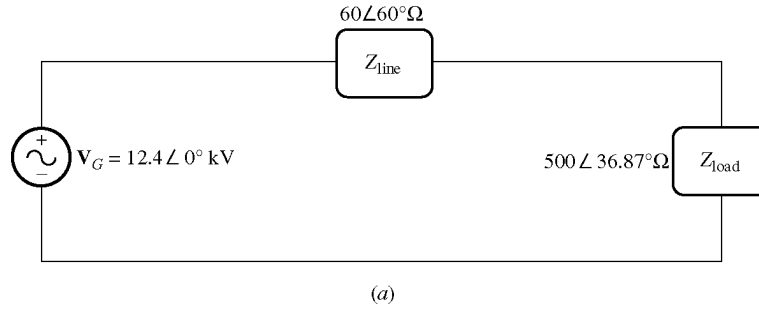
(a) If the generator is directly connected to the load (Figure P3-3a), what is the ratio of the load voltage to the generated voltage? What are the transmission losses of the system?

(b) If a 1:10 step-up transformer is placed at the output of the generator and a 10:1 transformer is placed at the load end of the transmission line, what is the new ratio of the load voltage to the generated voltage (Figure P3-3b)? What are the transmission losses of the system now? (*Note*: The transformers may be assumed to be ideal.)

SOLUTION

(a) In the case of the directly-connected load, the line current is

$$\mathbf{I}_{\text{line}} = \mathbf{I}_{\text{load}} = \frac{12.4\angle 0° \text{ kV}}{60\angle 60° \ \Omega \ + 500\angle 36.87° \ \Omega} = 22.32\angle -39.3° \text{ A}$$

The load voltage is

$$\mathbf{V}_{\text{load}} = \mathbf{I}_{\text{load}}Z_{\text{load}} = \left(22.32\angle -39.3° \text{ A}\right)\left(500\angle 36.87° \ \Omega\right) = 11.16\angle -2.43° \text{ kV}$$

The ratio of the load voltage to the generated voltage is 11.16/12.4 = 0.90. The transmission losses in the system are

$$P_{\text{loss}} = I_{\text{line}}{}^2 R_{\text{line}} = \left(22.32 \text{ A}\right)^2 \left(30 \ \Omega\right) = 14.9 \text{ kW}$$

(b) In this case, a 1:10 step-up transformer precedes the transmission line and a 10:1 step-down transformer follows the transmission line. If the transformers are removed by referring the transmission line to the voltage levels found on either end, then the impedance of the transmission line becomes

$$Z_{\text{line}}{}' = \left(\frac{1}{10}\right)^2 Z_{\text{line}} = \left(\frac{1}{10}\right)^2 \left(60\angle 60° \ \Omega\right) = 0.60\angle 60° \ \Omega$$

The current in the referred transmission line and in the load becomes

$$\mathbf{I}_{\text{line}}{}' = \mathbf{I}_{\text{load}} = \frac{12.4\angle 0° \text{ kV}}{0.60\angle 60° \ \Omega \ + 500\angle 36.87° \ \Omega} = 24.773\angle -36.90° \text{ A}$$

The load voltage is

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$$\mathbf{V}_{\text{load}} = \mathbf{I}_{\text{load}} Z_{\text{load}} = (24.773\angle - 36.90° \text{ A})(500\angle 36.87° \text{ } \Omega) = 12.386\angle - 0.03° \text{ kV}$$

The ratio of the load voltage to the generated voltage is 12.386/12.4 = 0.999. Also, the transmission losses in the system are reduced. The current in the transmission line is

$$I_{\text{line}} = \left(\frac{1}{10}\right) I_{\text{load}} = \left(\frac{1}{10}\right)(24.77 \text{ A}) = 2.477 \text{ A}$$

and the losses in the transmission line are

$$P_{\text{loss}} = I_{\text{line}}{}^2 R_{\text{line}} = (2.477 \text{ A})^2 (30 \text{ } \Omega) = 184 \text{ W}$$

Transmission losses have decreased by a factor of more than 80!

3-14. A 5000-VA 480/120-V conventional transformer is to be used to supply power from a 600-V source to a 120-V load. Consider the transformer to be ideal, and assume that all insulation can handle 600 V.

(a) Sketch the transformer connection that will do the required job.

(b) Find the kilovoltampere rating of the transformer in the configuration.

(c) Find the maximum primary and secondary currents under these conditions.

SOLUTION *(a)* For this configuration, the common winding must be the *smaller* of the two windings, and $N_{\text{SE}} = 4N_C$. The transformer connection is shown below:
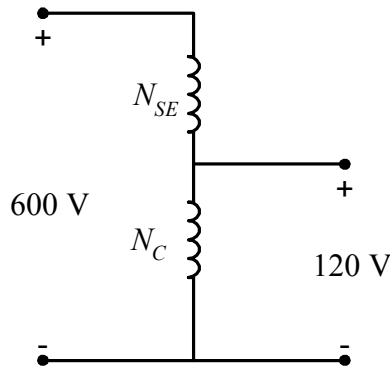
(b) The kVA rating of the autotransformer can be found from the equation

$$S_{\text{IO}} = \frac{N_{\text{SE}} + N_C}{N_{\text{SE}}} S_W = \frac{4N_C + N_C}{4N_C}(5000 \text{ VA}) = 6250 \text{ VA}$$

(c) The maximum primary current for this configuration will be

$$I_P = \frac{S}{V_P} = \frac{6250 \text{ VA}}{600 \text{ V}} = 10.4 \text{ A}$$

and the maximum secondary current is

$$I_S = \frac{S}{V_S} = \frac{6250 \text{ VA}}{120 \text{ V}} = 52.1 \text{ A}$$

3-15. A 5000-VA 480/120-V conventional transformer is to be used to supply power from a 600-V source to a 480-V load. Consider the transformer to be ideal, and assume that all insulation can handle 600 V. Answer the questions of Problem 3-14 for this transformer.

SOLUTION *(a)* For this configuration, the common winding must be the *larger* of the two windings, and $N_C = 4N_{\text{SE}}$. The transformer connection is shown below:
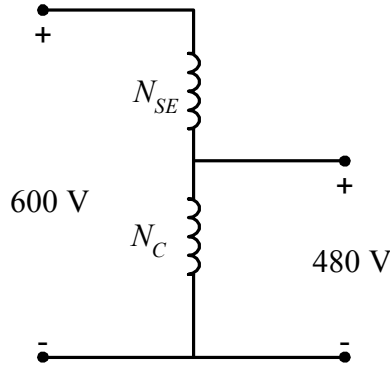
(b) The kVA rating of the autotransformer can be found from the equation

$$S_{IO} = \frac{N_{SE} + N_C}{N_{SE}} S_W = \frac{N_{SE} + 4N_{SE}}{N_{SE}} (5000 \text{ VA}) = 25{,}000 \text{ VA}$$

(c) The maximum primary current for this configuration will be

$$I_P = \frac{S}{V_P} = \frac{25{,}000 \text{ VA}}{600 \text{ V}} = 41.67 \text{ A}$$

and the maximum secondary current is

$$I_S = \frac{S}{V_S} = \frac{25{,}000 \text{ VA}}{480 \text{ V}} = 52.1 \text{ A}$$

Note that the apparent power handling capability of the autotransformer is *much* higher when there is only a small difference between primary and secondary voltages. Autotransformers are normally used when there is a relatively small difference between the two voltage levels.

3-16. Prove the following statement: If a transformer having a series impedance Z_{eq} is connected as an autotransformer, its per-unit series impedance Z'_{eq} as an autotransformer will be

$$Z'_{eq} = \frac{N_{SE}}{N_{SE} + N_C} Z_{eq}$$

Note that this expression is the reciprocal of the autotransformer power advantage.

SOLUTION   The impedance of a transformer can be found by shorting the secondary winding and determining the ratio of the voltage to the current of its primary winding. For the transformer connected as an ordinary transformer, the impedance referred to the primary (N_C) is:
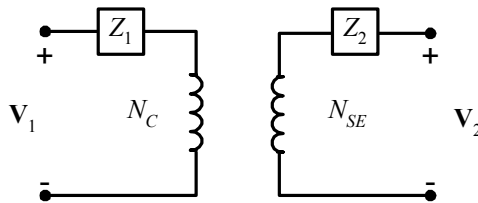
$$Z_{eq} = Z_1 + \left(\frac{N_C}{N_{SE}}\right)^2 Z_2$$

The corresponding equivalent circuit is:
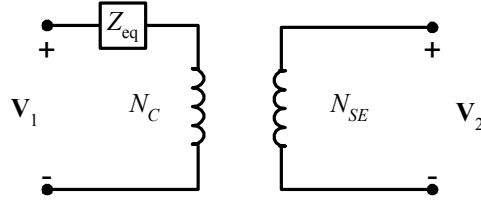
When this transformer is connected as an autotransformer, the circuit is as shown below. If the output windings of the autotransformer are shorted out, the voltages $\mathbf{V}_H$ (and hence $\mathbf{V}_C$) will be zero, and the voltage $\mathbf{V}_L$ will be
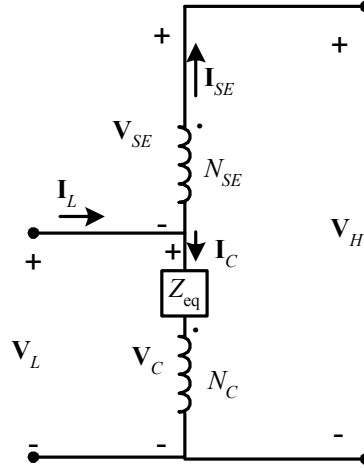
$$\mathbf{V}_L = \mathbf{I}_C Z_{eq}$$

where Z_{eq} is the impedance of the ordinary transformer. However,

$$\mathbf{I}_L = \mathbf{I}_C + \mathbf{I}_{SE} = \mathbf{I}_C + \frac{N_C}{N_{SE}}\mathbf{I}_C = \frac{N_{SE} + N_C}{N_{SE}}\mathbf{I}_C$$

or $\quad \mathbf{I}_C = \dfrac{N_{SE}}{N_{SE} + N_C}\mathbf{I}_L$

so the input voltage can be expressed in terms of the input current as:

$$\mathbf{V}_L = \mathbf{I}_C Z_{eq} = \frac{N_{SE}}{N_{SE} + N_C}\mathbf{I}_L Z_{eq}$$

The input impedance of the autotransformer is *defined* as $Z_{eq} = \mathbf{V}_L / \mathbf{I}_L$, so

$$\boxed{Z_{eq}{}' = \frac{\mathbf{V}_L}{\mathbf{I}_L} = \frac{N_{SE}}{N_{SE} + N_C}Z_{eq}}$$

This is the expression that we were trying to prove.

3-17. Three 25-kVA 24,000/277-V distribution transformers are connected in Δ-Y. The open-circuit test was performed on the low-voltage side of this transformer bank, and the following data were recorded:

$$V_{\text{line,OC}} = 480 \text{ V} \quad I_{\text{line,OC}} = 4.10 \text{ A} \quad\quad P_{3\phi,\text{OC}} = 945 \text{ W}$$

The short-circuit test was performed on the high-voltage side of this transformer bank, and the following data were recorded:

$$V_{\text{line,SC}} = 1400 \text{ V} \quad I_{\text{line,SC}} = 1.80 \text{ A} \qquad P_{3\phi,\text{SC}} = 912 \text{ W}$$

(a) Find the per-unit equivalent circuit of this transformer bank.

(b) Find the voltage regulation of this transformer bank at the rated load and 0.90 PF lagging.

(c) What is the transformer bank's efficiency under these conditions?

SOLUTION *(a)* The equivalent of this three-phase transformer bank can be found just like the equivalent circuit of a single-phase transformer if we work on a per-phase bases. The open-circuit test data on the low-voltage side can be used to find the excitation branch impedances referred to the secondary side of the transformer bank. Since the low-voltage side of the transformer is Y-connected, the per-phase open-circuit quantities are:

$$V_{\phi,\text{OC}} = 277 \text{ V} \qquad I_{\phi,\text{OC}} = 4.10 \text{ A} \qquad P_{\phi,\text{OC}} = 315 \text{ W}$$

The excitation admittance is given by

$$|Y_{EX}| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{4.10 \text{ A}}{277 \text{ V}} = 0.01483 \text{ S}$$

The admittance angle is

$$\theta = -\cos^{-1}\left(\frac{P_{\phi,OC}}{V_{\phi,OC} \, I_{\phi,OC}}\right) = -\cos^{-1}\left(\frac{315 \text{ W}}{(277 \text{ V}) \, (4.10 \text{ A})}\right) = -73.9°$$

Therefore,

$$Y_{EX} = G_C - jB_M = 0.01483\angle -73.9° = 0.00411 - j0.01425$$
$$R_C = 1/G_C = 243 \text{ }\Omega$$
$$X_M = 1/B_M = 70.2 \text{ }\Omega$$

The *base impedance* referred to the low-voltage side is

$$Z_{\text{base,S}} = \frac{\left(V_{\phi,S}\right)^2}{S_\phi} = \frac{(277 \text{ V})^2}{25 \text{ kVA}} = 3.069 \text{ }\Omega$$

so the excitation branch elements can be expressed in per-unit as

$$R_C = \frac{243 \text{ }\Omega}{3.069 \text{ }\Omega} = 79.2 \text{ pu} \qquad\qquad X_M = \frac{70.2 \text{ }\Omega}{3.069 \text{ }\Omega} = 22.9 \text{ pu}$$

The short-circuit test data can be used to find the series impedances referred to the high-voltage side, since the short-circuit test data was taken on the high-voltage side. Note that the high-voltage is Δ-connected, so $V_{\phi,\text{SC}} = V_{\text{SC}} = 1400 \text{ V}$, $I_{\phi,\text{SC}} = I_{\text{SC}}/\sqrt{3} = 1.039 \text{ A}$, and $P_{\phi,\text{SC}} = P_{\text{SC}}/3 = 304 \text{ W}$.

$$|Z_{EQ}| = \frac{V_{\phi,SC}}{I_{\phi,SC}} = \frac{1400 \text{ V}}{1.039 \text{ A}} = 1347 \text{ }\Omega$$

$$\theta = \cos^{-1}\left(\frac{P_{\phi,SC}}{V_{\phi,SC} \, I_{\phi,SC}}\right) = \cos^{-1}\left(\frac{304 \text{ W}}{(1400 \text{ V}) \, (1.039 \text{ A})}\right) = 77.9°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 1347\angle 77.9° = 282 + j1371 \text{ }\Omega$$

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The *base impedance* referred to the high-voltage side is

$$Z_{\text{base},P} = \frac{\left(V_{\phi,P}\right)^2}{S_\phi} = \frac{\left(24{,}000 \text{ V}\right)^2}{25 \text{ kVA}} = 24{,}040 \text{ }\Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{282 \text{ }\Omega}{24{,}040 \text{ }\Omega} = 0.0117 \text{ pu} \qquad\qquad X_{EQ} = \frac{1371 \text{ }\Omega}{24{,}040 \text{ }\Omega} = 0.057 \text{ pu}$$

The per-unit, per-phase equivalent circuit of the transformer bank is shown below:
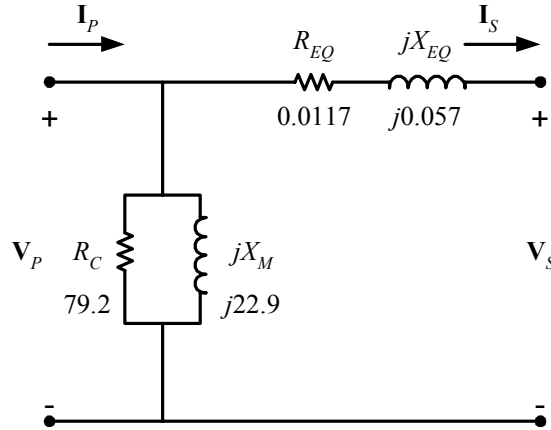
(b)   If this transformer is operating at rated load and 0.90 PF lagging, then current flow will be at an angle of $-\cos^{-1}(0.9)$, or $-25.8°$. The voltage at the primary side of the transformer will be

$$\mathbf{V}_P = \mathbf{V}_S + \mathbf{I}_S Z_{EQ} = 1.0\angle 0° + \left(1.0\angle -25.8°\right)\left(0.0117 + j0.057\right) = 1.037\angle 2.65°$$

The voltage regulation of this transformer bank is

$$\text{VR} = \frac{1.037\text{-}1.0}{1.0} \times 100\% = 3.7\%$$

(c)   The output power of this transformer bank is

$$P_{\text{OUT}} = V_S I_S \cos\theta = (1.0)(1.0)(0.9) = 0.9 \text{ pu}$$

The copper losses are

$$P_{\text{CU}} = I_S{}^2 R_{\text{EQ}} = (1.0)^2 (0.0117) = 0.0117 \text{ pu}$$

The core losses are

$$P_{\text{core}} = \frac{V_P{}^2}{R_C} = \frac{(1.067)^2}{79.2} = 0.014 \text{ pu}$$

Therefore, the total input power to the transformer bank is

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}} = 0.9 + 0.0117 + 0.014 = 0.9257$$

and the efficiency of the transformer bank is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{0.9}{0.9257} \times 100\% = 97.2\%$$

3-18. A 20-kVA 20,000/480-V 60-Hz distribution transformer is tested with the following results:

Open-circuit test (measured from secondary side)	Short-circuit test (measured from primary side)
$V_{OC} = 480$ V	$V_{SC} = 1130$ V
$I_{OC} = 1.51$ A	$I_{SC} = 1.00$ A
$V_{OC} = 271$ W	$P_{SC} = 260$ W

(a) Find the per-unit equivalent circuit for this transformer at 60 Hz.

(b) What would the rating of this transformer be if it were operated on a 50-Hz power system?

(c) Sketch the equivalent circuit of this transformer referred to the primary side *if it is operating at 50 Hz.*

SOLUTION

(a) The base impedance of this transformer referred to the primary side is

$$Z_{base,P} = \frac{(V_P)^2}{S} = \frac{(20{,}000 \text{ V})^2}{20 \text{ kVA}} = 20 \text{ k}\Omega$$

The base impedance of this transformer referred to the secondary side is

$$Z_{base,S} = \frac{(V_S)^2}{S} = \frac{(480 \text{ V})^2}{20 \text{ kVA}} = 11.52 \ \Omega$$

The open circuit test yields the values for the excitation branch (referred to the *secondary* side):

$$|Y_{EX}| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{1.51 \text{ A}}{480 \text{ V}} = 0.00315 \text{ S}$$

$$\theta = -\cos^{-1}\left(\frac{P_{OC}}{V_{OC}\, I_{OC}}\right) = -\cos^{-1}\left(\frac{271 \text{ W}}{(480 \text{ V})\,(1.51 \text{ A})}\right) = -68°$$

$$Y_{EX} = G_C - jB_M = 0.00315\angle - 68° = 0.00118 - j0.00292$$

$$R_C = 1/G_C = 847 \ \Omega$$

$$X_M = 1/B_M = 342 \ \Omega$$

The excitation branch elements can be expressed in per-unit as

$$R_C = \frac{847 \ \Omega}{11.52 \ \Omega} = 73.5 \text{ pu} \qquad\qquad X_M = \frac{342 \ \Omega}{11.52 \ \Omega} = 29.7 \text{ pu}$$

The short circuit test yields the values for the series impedances (referred to the *primary* side):

$$|Z_{EQ}| = \frac{V_{SC}}{I_{SC}} = \frac{1130 \text{ V}}{1.00 \text{ A}} = 1130 \ \Omega$$

$$\theta = \cos^{-1}\left(\frac{P_{SC}}{V_{SC}\, I_{SC}}\right) = \cos^{-1}\left(\frac{260 \text{ W}}{(1130 \text{ V})\,(1.00 \text{ A})}\right) = 76.7°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 1130\angle 76.7° = 260 + j1100 \ \Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{260 \ \Omega}{20{,}000 \ \Omega} = 0.013 \text{ pu} \qquad\qquad X_{EQ} = \frac{1100 \ \Omega}{20{,}000 \ \Omega} = 0.055 \text{ pu}$$

The per-unit equivalent circuit is
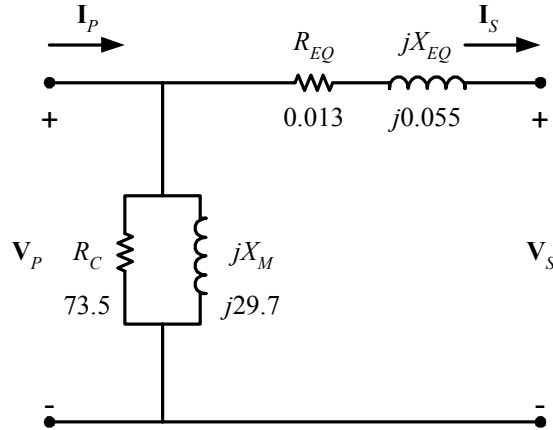
(b) If this transformer were operated at 50 Hz, both the voltage and apparent power would have to be derated by a factor of 50/60, so its ratings would be 16.67 kVA, 16,667/400 V, and 50 Hz.

(c) The transformer parameters referred to the primary side *at 60 Hz* are:

$$R_C = Z_{\text{base}} R_{C,\text{pu}} = (20 \text{ k}\Omega)(73.5) = 1.47 \text{ M}\Omega$$

$$X_M = Z_{\text{base}} X_{M,\text{pu}} = (20 \text{ k}\Omega)(29.7) = 594 \text{ k}\Omega$$

$$R_{\text{EQ}} = Z_{\text{base}} R_{\text{EQ,pu}} = (20 \text{ k}\Omega)(0.013) = 260 \text{ }\Omega$$

$$X_{\text{EQ}} = Z_{\text{base}} X_{\text{EQ,pu}} = (20 \text{ k}\Omega)(0.055) = 1100 \text{ }\Omega$$

At 50 Hz, the resistance will be unaffected but the reactances are reduced in direct proportion to the decrease in frequency. At 50 Hz, the reactances are

$$X_M = 495 \text{ k}\Omega$$
$$X_{\text{EQ}} = 917 \text{ }\Omega$$

The resulting equivalent circuit referred to the primary at 50 Hz is shown below:
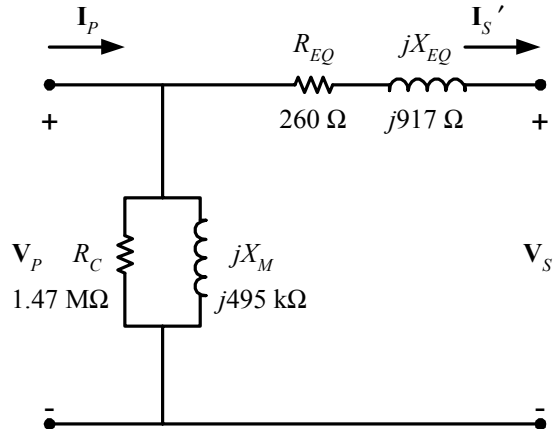
3-19. Prove that the three-phase system of voltages on the secondary of the Y-Δ transformer shown in Figure 3-37*b* lags the three-phase system of voltages on the primary of the transformer by 30°.

SOLUTION The figure is reproduced below:
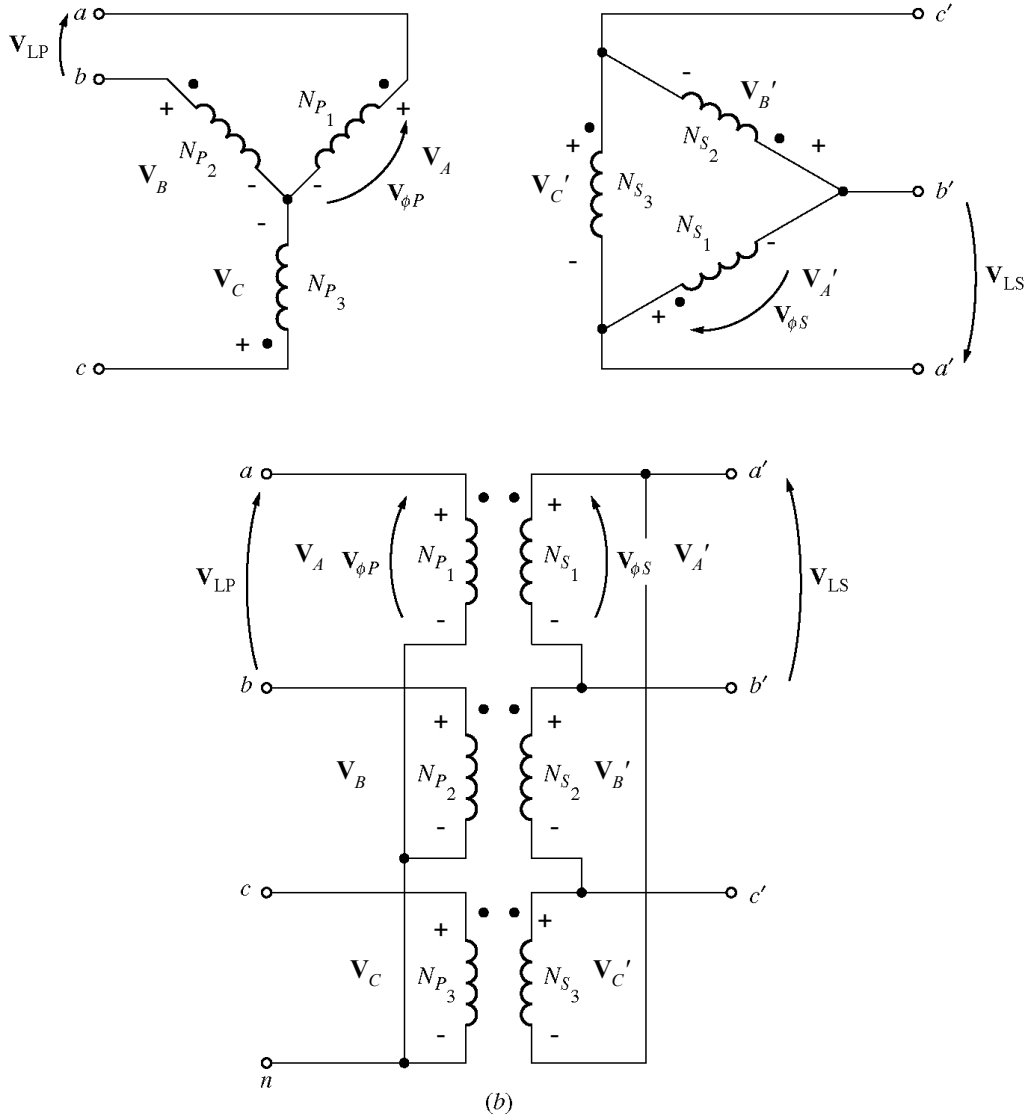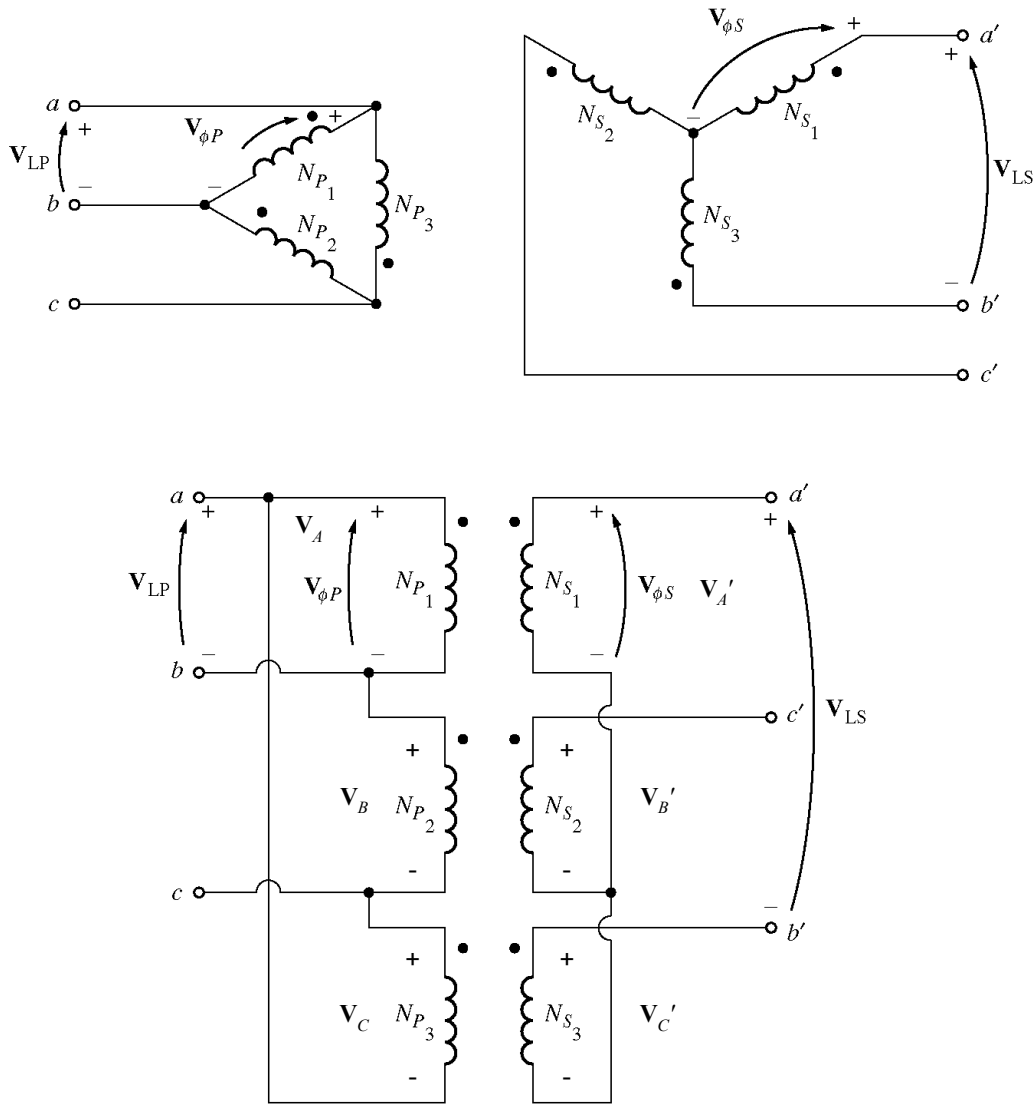
(b)

Assume that the phase voltages on the primary side are given by

$$\mathbf{V}_A = V_{\phi P}\angle 0° \qquad \mathbf{V}_B = V_{\phi P}\angle -120° \qquad \mathbf{V}_C = V_{\phi P}\angle 120°$$

Then the phase voltages on the secondary side are given by

$$\mathbf{V}_A{'} = V_{\phi S}\angle 0° \qquad \mathbf{V}_B{'} = V_{\phi S}\angle -120° \qquad \mathbf{V}_C{'} = V_{\phi S}\angle 120°$$

where $V_{\phi S} = V_{\phi P}/a$. Since this is a Y-$\Delta$ transformer bank, the line voltage $\mathbf{V}_{ab}$ on the primary side is

$$\mathbf{V}_{ab} = \mathbf{V}_A - \mathbf{V}_B = V_{\phi P}\angle 0° - V_{\phi P}\angle -120° = \sqrt{3}V_{\phi P}\angle 30°$$

and the voltage $\mathbf{V}_{a'b'} = \mathbf{V}_A{'} = V_{\phi S}\angle 0°$. *Note that the line voltage on the secondary side lags the line voltage on the primary side by 30°.*

3-20. Prove that the three-phase system of voltages on the secondary of the Δ-Y transformer shown in Figure 3-37*c* lags the three-phase system of voltages on the primary of the transformer by 30°.

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SOLUTION The figure is reproduced below:

Assume that the phase voltages on the primary side are given by

$$\mathbf{V}_A = V_{\phi P}\angle 0° \qquad \mathbf{V}_B = V_{\phi P}\angle -120° \qquad \mathbf{V}_C = V_{\phi P}\angle 120°$$

Then the phase voltages on the secondary side are given by

$$\mathbf{V}_A' = V_{\phi S}\angle 0° \qquad \mathbf{V}_B' = V_{\phi S}\angle -120° \qquad \mathbf{V}_C' = V_{\phi S}\angle 120°$$

where $V_{\phi S} = V_{\phi P}/a$. Since this is a Δ-Y transformer bank, the line voltage $\mathbf{V}_{ab}$ on the primary side is just equal to $\mathbf{V}_A = V_{\phi P}\angle 0°$. The line voltage on the secondary side is given by

$$\mathbf{V}_{a'b'} = \mathbf{V}_A - \mathbf{V}_C = V_{\phi P}\angle 0° - V_{\phi P}\angle 120° = \sqrt{3}V_{\phi P}\angle -30°$$

Note that the line voltage on the secondary side lags the line voltage on the primary side by 30°.

3-21. A single-phase 10-kVA 480/120-V transformer is to be used as an autotransformer tying a 600-V distribution line to a 480-V load. When it is tested as a conventional transformer, the following values are measured on the primary (480-V) side of the transformer:

Open-circuit test	Short-circuit test
V_{OC} = 480 V	V_{SC} = 10.0 V
I_{OC} = 0.41 A	I_{SC} = 10.6 A
V_{OC} = 38 W	P_{SC} = 26 W

(a) Find the per-unit equivalent circuit of this transformer when it is connected in the conventional manner. What is the efficiency of the transformer at rated conditions and unity power factor? What is the voltage regulation at those conditions?

(b) Sketch the transformer connections when it is used as a 600/480-V step-down autotransformer.

(c) What is the kilovoltampere rating of this transformer when it is used in the autotransformer connection?

(d) Answer the questions in *(a)* for the autotransformer connection.

SOLUTION

(a) The base impedance of this transformer referred to the primary side is

$$Z_{base,P} = \frac{(V_P)^2}{S} = \frac{(480 \text{ V})^2}{10 \text{ kVA}} = 23.04 \ \Omega$$

The open circuit test yields the values for the excitation branch (referred to the *primary* side):

$$\left|Y_{EX}\right| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{0.41 \text{ A}}{480 \text{ V}} = 0.000854 \ \text{S}$$

$$\theta = -\cos^{-1}\left(\frac{P_{OC}}{V_{OC} \ I_{OC}}\right) = -\cos^{-1}\left(\frac{38 \text{ W}}{(480 \text{ V}) \ (0.41 \text{ A})}\right) = -78.87°$$

$$Y_{EX} = G_C - jB_M = 0.000854\angle - 78.87° = 0.000165 - j0.000838$$

$$R_C = 1/G_C = 6063 \ \Omega$$

$$X_M = 1/B_M = 1193 \ \Omega$$

The excitation branch elements can be expressed in per-unit as

$$R_C = \frac{6063 \ \Omega}{23.04 \ \Omega} = 263 \text{ pu} \qquad\qquad X_M = \frac{1193 \ \Omega}{23.04 \ \Omega} = 51.8 \text{ pu}$$

The short circuit test yields the values for the series impedances (referred to the *primary* side):

$$\left|Z_{EQ}\right| = \frac{V_{SC}}{I_{SC}} = \frac{10.0 \text{ V}}{10.6 \text{ A}} = 0.943 \ \Omega$$

$$\theta = \cos^{-1}\left(\frac{P_{SC}}{V_{SC} \ I_{SC}}\right) = \cos^{-1}\left(\frac{26 \text{ W}}{(10.0 \text{ V}) \ (10.6 \text{ A})}\right) = 75.8°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 0.943\angle75.8° = 0.231 + j0.915 \ \Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{0.231 \ \Omega}{23.04 \ \Omega} = 0.010 \text{ pu} \qquad\qquad X_{EQ} = \frac{0.915 \ \Omega}{23.04 \ \Omega} = 0.0397 \text{ pu}$$

The per-unit equivalent circuit is
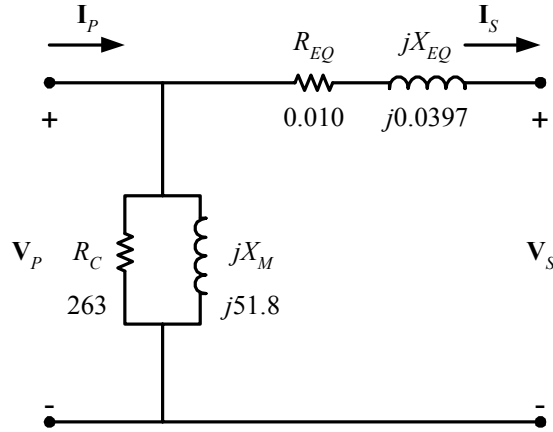
At rated conditions and unity power factor, the input power to this transformer would be $P_{IN} = 1.0$ pu. The core losses (in resistor R_C) would be

$$P_{core} = \frac{V^2}{R_C} = \frac{(1.0)^2}{263} = 0.00380 \text{ pu}$$

The copper losses (in resistor R_{EQ}) would be

$$P_{CU} = I^2 R_{EQ} = (1.0)^2 (0.010) = 0.010 \text{ pu}$$

The output power of the transformer would be

$$P_{OUT} = P_{OUT} - P_{CU} - P_{core} = 1.0 - 0.010 - 0.0038 = 0.986$$

and the transformer efficiency would be

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{0.986}{1.0} \times 100\% = 98.6\%$$

The output voltage of this transformer is

$$\mathbf{V}_{OUT} = \mathbf{V}_{IN} - \mathbf{I}Z_{EQ} = 1.0 - (1.0\angle 0°)(0.01 + j0.0397) = 0.991\angle - 2.3°$$

The voltage regulation of the transformer is

$$VR = \frac{1.0 - 0.991}{0.991} \times 100\% = 0.9\%$$

(b) The autotransformer connection for 600/480 V stepdown operation is
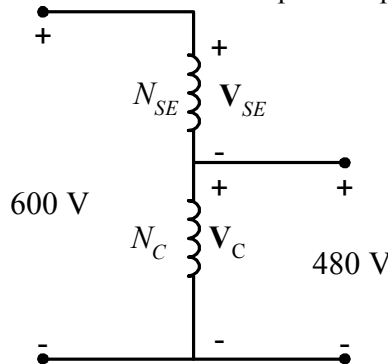
(c) When used as an autotransformer, the kVA rating of this transformer becomes:

$$S_{IO} = \frac{N_C + N_{SE}}{N_{SE}} S_W = \frac{4+1}{1}(10 \text{ kVA}) = 50 \text{ kVA}$$

(d) As an autotransformer, the per-unit series impedance Z_{EQ} is decreased by the reciprocal of the power advantage, so the series impedance becomes

$$R_{EQ} = \frac{0.010}{5} = 0.002 \text{ pu}$$

$$X_{EQ} = \frac{0.0397}{5} = 0.00794 \text{ pu}$$

while the magnetization branch elements are basically unchanged. At rated conditions and unity power factor, the input power to this transformer would be $P_{IN} = 1.0$ pu. The core losses (in resistor R_C) would be

$$P_{core} = \frac{V^2}{R_C} = \frac{(1.0)^2}{263} = 0.00380 \text{ pu}$$

The copper losses (in resistor R_{EQ}) would be

$$P_{CU} = I^2 R_{EQ} = (1.0)^2 (0.002) = 0.002 \text{ pu}$$

The output power of the transformer would be

$$P_{OUT} = P_{OUT} - P_{CU} - P_{core} = 1.0 - 0.002 - 0.0038 = 0.994$$

and the transformer efficiency would be

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{0.994}{1.0} \times 100\% = 99.4\%$$

The output voltage of this transformer is

$$\mathbf{V}_{OUT} = \mathbf{V}_{IN} - \mathbf{I}Z_{EQ} = 1.0 - (1.0\angle 0°)(0.002 + j0.00794) = 0.998\angle -0.5°$$

The voltage regulation of the transformer is

$$VR = \frac{1.0 - 0.998}{0.998} \times 100\% = 0.2\%$$

3-22. Figure P3-4 shows a power system consisting of a three-phase 480-V 60-Hz generator supplying two loads through a transmission line with a pair of transformers at either end.

(a) Sketch the per-phase equivalent circuit of this power system.

(b) With the switch opened, find the real power *P*, reactive power *Q*, and apparent power *S* supplied by the generator. What is the power factor of the generator?

(c) With the switch closed, find the real power *P*, reactive power *Q*, and apparent power *S* supplied by the generator. What is the power factor of the generator?

(d) What are the transmission losses (transformer plus transmission line losses) in this system with the switch open? With the switch closed? What is the effect of adding Load 2 to the system?

Generator
480 V

T_1

Line
480/13,800 V $\mathbf{Z}_L = 1.5 + j\ 10\Omega$ 13,800/480 V
1000 kVA   500 kVA
$R = 0.010$ pu $R = 0.008$ pu
$X = 0.040$ pu $X = 0.040$ pu

T_2

Load 1

$\mathbf{Z}_{\text{Load1}} =$
$0.5\angle 36.87°\ \Omega$
Y–connected

Load 2

$\mathbf{Z}_{\text{Load2}} =$
$-j\ 0.833\ \Omega$
Y–connected

Region 1	Region 2	Region 3
$S_{\text{base1}} = 1000$ kVA	$S_{\text{base2}} = 1000$ kVA	$S_{\text{base3}} = 1000$ kVA
$V_{L,\text{base2}} = 480$ V	$V_{L,\text{base2}} = 13{,}800$ V	$V_{L,\text{base3}} = 480$ V
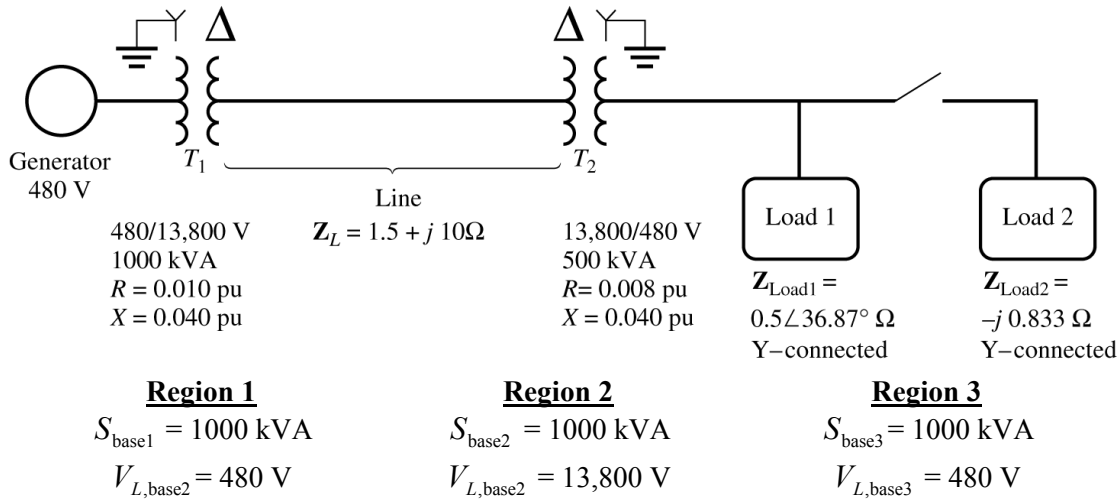
SOLUTION   This problem can best be solved using the per-unit system of measurements. The power system can be divided into three regions by the two transformers. If the per-unit base quantities in Region 1 are chosen to be $S_{\text{base1}} = 1000$ kVA and $V_{L,\text{base1}} = 480$ V, then the base quantities in Regions 2 and 3 will be as shown above. The base impedances of each region will be:

$$Z_{\text{base1}} = \frac{3V_{\phi1}^{\ 2}}{S_{\text{base1}}} = \frac{3(277\text{ V})^2}{1000\text{ kVA}} = 0.238\ \Omega$$

$$Z_{\text{base2}} = \frac{3V_{\phi2}^{\ 2}}{S_{\text{base2}}} = \frac{3(7967\text{ V})^2}{1000\text{ kVA}} = 190.4\ \Omega$$

$$Z_{\text{base3}} = \frac{3V_{\phi3}^{\ 2}}{S_{\text{base3}}} = \frac{3(277\text{ V})^2}{1000\text{ kVA}} = 0.238\ \Omega$$

(a)   To get the per-unit, per-phase equivalent circuit, we must convert each impedance in the system to per-unit on the base of the region in which it is located. The impedance of transformer T_1 is already in per-unit to the proper base, so we don't have to do anything to it:

$$R_{1,\text{pu}} = 0.010$$

$$X_{1,\text{pu}} = 0.040$$

The impedance of transformer T_2 is already in per-unit, but it is per-unit to the base of transformer T_2, so it must be converted to the base of the power system.

$$(R,\ X,\ Z)_{\text{pu on base 2}} = (R,\ X,\ Z)_{\text{pu on base 1}} \frac{(V_{\text{base 1}})^2 (S_{\text{base 2}})}{(V_{\text{base 2}})^2 (S_{\text{base 1}})} \tag{3-66}$$

$$R_{2,\text{pu}} = 0.020 \frac{(7967\text{ V})^2 (1000\text{ kVA})}{(7967\text{ V})^2 (1000\text{ kVA})} = 0.040$$

$$X_{2,\text{pu}} = 0.085 \frac{(7967\text{ V})^2 (1000\text{ kVA})}{(7967\text{ V})^2 (1000\text{ kVA})} = 0.170$$

The per-unit impedance of the transmission line is

$$Z_{\text{line,pu}} = \frac{Z_{\text{line}}}{Z_{\text{base2}}} = \frac{1.5 + j10\ \Omega}{190.4\ \Omega} = 0.00788 + j0.0525$$

The per-unit impedance of Load 1 is

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$$Z_{\text{load1,pu}} = \frac{Z_{\text{load1}}}{Z_{\text{base3}}} = \frac{0.5\angle 36.87° \ \Omega}{0.238 \ \Omega} = 1.681 + j1.261$$

The per-unit impedance of Load 2 is

$$Z_{\text{load2,pu}} = \frac{Z_{\text{load2}}}{Z_{\text{base3}}} = \frac{-j0.833 \ \Omega}{0.238 \ \Omega} = -j3.5$$

The resulting per-unit, per-phase equivalent circuit is shown below:
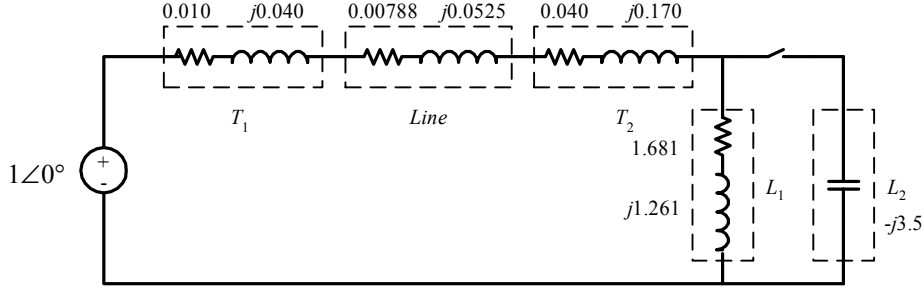
(b) With the switch opened, the equivalent impedance of this circuit is

$$Z_{\text{EQ}} = 0.010 + j0.040 + 0.00788 + j0.0525 + 0.040 + j0.170 + 1.681 + j1.261$$
$$Z_{\text{EQ}} = 1.7389 + j1.5235 = 2.312\angle 41.2°$$

The resulting current is

$$\mathbf{I} = \frac{1\angle 0°}{2.312\angle 41.2°} = 0.4325\angle -41.2°$$

The load voltage under these conditions would be

$$\mathbf{V}_{\text{Load,pu}} = \mathbf{I}\,Z_{\text{Load}} = (0.4325\angle -41.2°)(1.681 + j1.261) = 0.909\angle -4.3°$$
$$V_{\text{Load}} = V_{\text{Load,pu}}V_{\text{base3}} = (0.909)(480 \ \text{V}) = 436 \ \text{V}$$

The power supplied to the load is

$$P_{\text{Load,pu}} = I^2 R_{\text{Load}} = (0.4325)^2 (1.681) = 0.314$$

$$P_{\text{Load}} = P_{\text{Load,pu}}S_{\text{base}} = (0.314)(1000 \ \text{kVA}) = 314 \ \text{kW}$$

The power supplied by the generator is

$$P_{G,\text{pu}} = VI\cos\theta = (1)(0.4325)\cos 41.2° = 0.325$$
$$Q_{G,\text{pu}} = VI\sin\theta = (1)(0.4325)\sin 41.2° = 0.285$$
$$S_{G,\text{pu}} = VI = (1)(0.4325) = 0.4325$$
$$P_G = P_{G,\text{pu}}S_{\text{base}} = (0.325)(1000 \ \text{kVA}) = 325 \ \text{kW}$$
$$Q_G = Q_{G,\text{pu}}S_{\text{base}} = (0.285)(1000 \ \text{kVA}) = 285 \ \text{kVAR}$$
$$S_G = S_{G,\text{pu}}S_{\text{base}} = (0.4325)(1000 \ \text{kVA}) = 432.5 \ \text{kVA}$$

The power factor of the generator is

$$PF = \cos 41.2° = 0.752 \ \text{lagging}$$

(c)   With the switch closed, the equivalent impedance of this circuit is

$$Z_{EQ} = 0.010 + j0.040 + 0.00788 + j0.0525 + 0.040 + j0.170 + \frac{(1.681 + j1.261)(-j3.5)}{1.681 + j1.261 - j3.5}$$

$$Z_{EQ} = 0.010 + j0.040 + 0.00788 + j0.0525 + 0.040 + j0.170 + (2.627 - j0.0011)$$

$$Z_{EQ} = 2.685 + j0.261 = 2.698\angle 5.6°$$

The resulting current is

$$\mathbf{I} = \frac{1\angle 0°}{2.698\angle 5.6°} = 0.371\angle -5.6°$$

The load voltage under these conditions would be

$$\mathbf{V}_{Load,pu} = \mathbf{I}\, Z_{Load} = (0.371\angle -5.6°)(2.627 - j0011) = 0.975\angle -5.6°$$

$$V_{Load} = V_{Load,pu}V_{base3} = (0.975)(480\text{ V}) = 468\text{ V}$$

The power supplied to the two loads is

$$P_{Load,pu} = I^2 R_{Load} = (0.371)^2(2.627) = 0.361$$

$$P_{Load} = P_{Load,pu}S_{base} = (0.361)(1000\text{ kVA}) = 361\text{ kW}$$

The power supplied by the generator is

$$P_{G,pu} = VI\cos\theta = (1)(0.371)\cos 5.6° = 0.369$$

$$Q_{G,pu} = VI\sin\theta = (1)(0.371)\sin 5.6° = 0.036$$

$$S_{G,pu} = VI = (1)(0.371) = 0.371$$

$$P_G = P_{G,pu}S_{base} = (0.369)(1000\text{ kVA}) = 369\text{ kW}$$

$$Q_G = Q_{G,pu}S_{base} = (0.036)(1000\text{ kVA}) = 36\text{ kVAR}$$

$$S_G = S_{G,pu}S_{base} = (0.371)(1000\text{ kVA}) = 371\text{ kVA}$$

The power factor of the generator is

$$PF = \cos 5.6° = 0.995\text{ lagging}$$

(d)   The transmission losses with the switch *open* are:

$$P_{line,pu} = I^2 R_{line} = (0.4325)^2(0.00788) = 0.00147$$

$$P_{line} = P_{line,pu}S_{base} = (0.00147)(1000\text{ kVA}) = 1.47\text{ kW}$$

The transmission losses with the switch *closed* are:

$$P_{line,pu} = I^2 R_{line} = (0.371)^2(0.00788) = 0.00108$$

$$P_{line} = P_{line,pu}S_{base} = (0.00108)(1000\text{ kVA}) = 1.08\text{ kW}$$

Load 2 improved the power factor of the system, increasing the load voltage and the total power supplied to the loads, while simultaneously decreasing the current in the transmission line and the transmission line losses. This problem is a good example of the advantages of power factor correction in power systems.

Chapter 4: *AC Machinery Fundamentals*

4-1. Develop a table showing the speed of magnetic field rotation in ac machines of 2, 4, 6, 8, 10, 12, and 14 poles operating at frequencies of 50, 60, and 400 Hz.

SOLUTION The equation relating the speed of magnetic field rotation to the number of poles and electrical frequency is

$$n_m = \frac{120 f_e}{P}$$

The resulting table is

Number of Poles	f_e = 50 Hz	f_e = 60 Hz	f_e = 400 Hz
2	3000 r/min	3600 r/min	24000 r/min
4	1500 r/min	1800 r/min	12000 r/min
6	1000 r/min	1200 r/min	8000 r/min
8	750 r/min	900 r/min	6000 r/min
10	600 r/min	720 r/min	4800 r/min
12	500 r/min	600 r/min	4000 r/min
14	428.6 r/min	514.3 r/min	3429 r/min

4-2. A three-phase two-pole winding is installed in six slots on a stator. There are 80 turns of wire in each slot of the windings. All coils in each phase are connected in series, and the three phases are connected in Δ. The flux per pole in the machine is 0.060 Wb, and the speed of rotation of the magnetic field is 3600 r/min.

(a) What is the frequency of the voltage produced in this winding?

(b) What are the resulting phase and terminal voltages of this stator?

SOLUTION

(a) The frequency of the voltage produced in this winding is

$$f_e = \frac{n_m P}{120} = \frac{(3600 \text{ r/min})(2 \text{ poles})}{120} = 60 \text{ Hz}$$

(b) There are six slots on this stator, with 80 turns of wire per slot. The voltage in the coils in one phase is

$$E_A = \sqrt{2}\pi N_C \phi f = \sqrt{2}\pi(80 \text{ t})(0.060 \text{ Wb})(60 \text{ Hz}) = 1280 \text{ V}$$

Since the machine is Δ-connected, $V_L = V_\phi = 1280 \text{ V}$

4-3. A three-phase Y-connected 50-Hz two-pole synchronous machine has a stator with 2000 turns of wire per phase. What rotor flux would be required to produce a terminal (line-to-line) voltage of 6 kV?

SOLUTION The phase voltage of this machine should be $V_\phi = V_L / \sqrt{3} = 3464 \text{ V}$. The induced voltage per phase in this machine (which is equal to V_ϕ at no-load conditions) is given by the equation

$$E_A = \sqrt{2}\pi N_C \phi f$$

so

$$\phi = \frac{E_A}{\sqrt{2}\pi N_C f} = \frac{3464 \text{ V}}{\sqrt{2}\pi(2000 \text{ t})(50 \text{ Hz})} = 0.0078 \text{ Wb}$$

4-4. Modify the MATLAB in Example 4-1 by swapping the currents flowing in any two phases. What happens to the resulting net magnetic field?

SOLUTION This modification is very simple—just swap the currents supplied to two of the three phases.

```
% M-file: mag_field2.m
% M-file to calculate the net magetic field produced
% by a three-phase stator.

% Set up the basic conditions
bmax = 1;                   % Normalize bmax to 1
freq = 60;                  % 60 Hz
w = 2*pi*freq;              % angluar velocity (rad/s)

% First, generate the three component magnetic fields
t = 0:1/6000:1/60;
Baa = sin(w*t) .* (cos(0) + j*sin(0));
Bbb = sin(w*t+2*pi/3) .* (cos(2*pi/3) + j*sin(2*pi/3));
Bcc = sin(w*t-2*pi/3) .* (cos(-2*pi/3) + j*sin(-2*pi/3));

% Calculate Bnet
Bnet = Baa + Bbb + Bcc;

% Calculate a circle representing the expected maximum
% value of Bnet
circle = 1.5 * (cos(w*t) + j*sin(w*t));

% Plot the magnitude and direction of the resulting magnetic
% fields.  Note that Baa is black, Bbb is blue, Bcc is
% magneta, and Bnet is red.
for ii = 1:length(t)

   % Plot the reference circle
   plot(circle,'k');
   hold on;

   % Plot the four magnetic fields
   plot([0 real(Baa(ii))],[0 imag(Baa(ii))],'k','LineWidth',2);
   plot([0 real(Bbb(ii))],[0 imag(Bbb(ii))],'b','LineWidth',2);
   plot([0 real(Bcc(ii))],[0 imag(Bcc(ii))],'m','LineWidth',2);
   plot([0 real(Bnet(ii))],[0 imag(Bnet(ii))],'r','LineWidth',3);
   axis square;
   axis([-2 2 -2 2]);
   drawnow;
   hold off;

end
```

When this program executes, the net magnetic field rotates clockwise, instead of counterclockwise.

4-5. If an ac machine has the rotor and stator magnetic fields shown in Figure P4-1, what is the direction of the induced torque in the machine? Is the machine acting as a motor or generator?
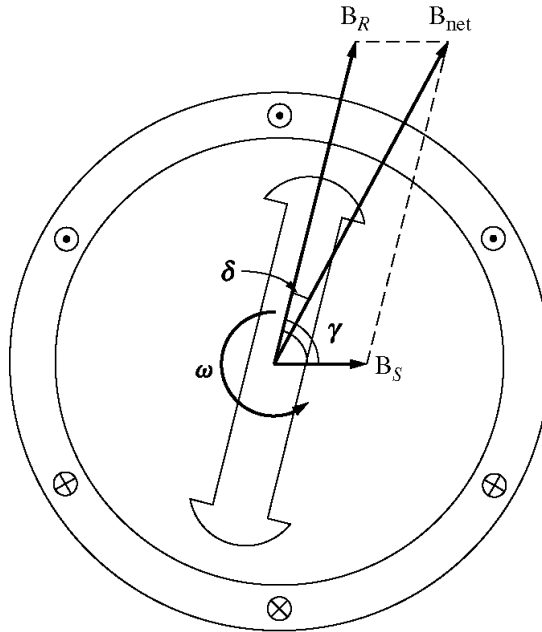
SOLUTION Since $\boldsymbol{\tau}_{\text{ind}} = k\mathbf{B}_R \times \mathbf{B}_{\text{net}}$, the induced torque is *clockwise*, opposite the direction of motion. The machine is acting as a *generator*.

Chapter 5: *Synchronous Machines*

5-1. At a location in Europe, it is necessary to supply 300 kW of 60-Hz power. The only power sources available operate at 50 Hz. It is decided to generate the power by means of a motor-generator set consisting of a synchronous motor driving a synchronous generator. How many poles should each of the two machines have in order to convert 50-Hz power to 60-Hz power?

SOLUTION The speed of a synchronous machine is related to its frequency by the equation

$$n_m = \frac{120 \, f_e}{P}$$

To make a 50 Hz and a 60 Hz machine have the same mechanical speed so that they can be coupled together, we see that

$$n_{\text{sync}} = \frac{120(50 \text{ Hz})}{P_1} = \frac{120(60 \text{ Hz})}{P_2}$$

$$\frac{P_2}{P_1} = \frac{6}{5} = \frac{12}{10}$$

Therefore, a 10-pole synchronous motor must be coupled to a 12-pole synchronous generator to accomplish this frequency conversion.

5-2. A 480-V 200-kVA 0.8-power-factor-lagging 60-Hz two-pole Y-connected synchronous generator has a synchronous reactance of 0.25 Ω and an armature resistance of 0.03 Ω. At 60 Hz, its friction and windage losses are 6 kW, and its core losses are 4 kW. The field circuit has a dc voltage of 200 V, and the maximum I_F is 10 A. The resistance of the field circuit is adjustable over the range from 20 to 200 Ω. The OCC of this generator is shown in Figure P5-1.

(a) How much field current is required to make V_T equal to 480 V when the generator is running at no load?

(b) What is the internal generated voltage of this machine at rated conditions?

(c) How much field current is required to make V_T equal to 480 V when the generator is running at rated conditions?

(d) How much power and torque must the generator's prime mover be capable of supplying?
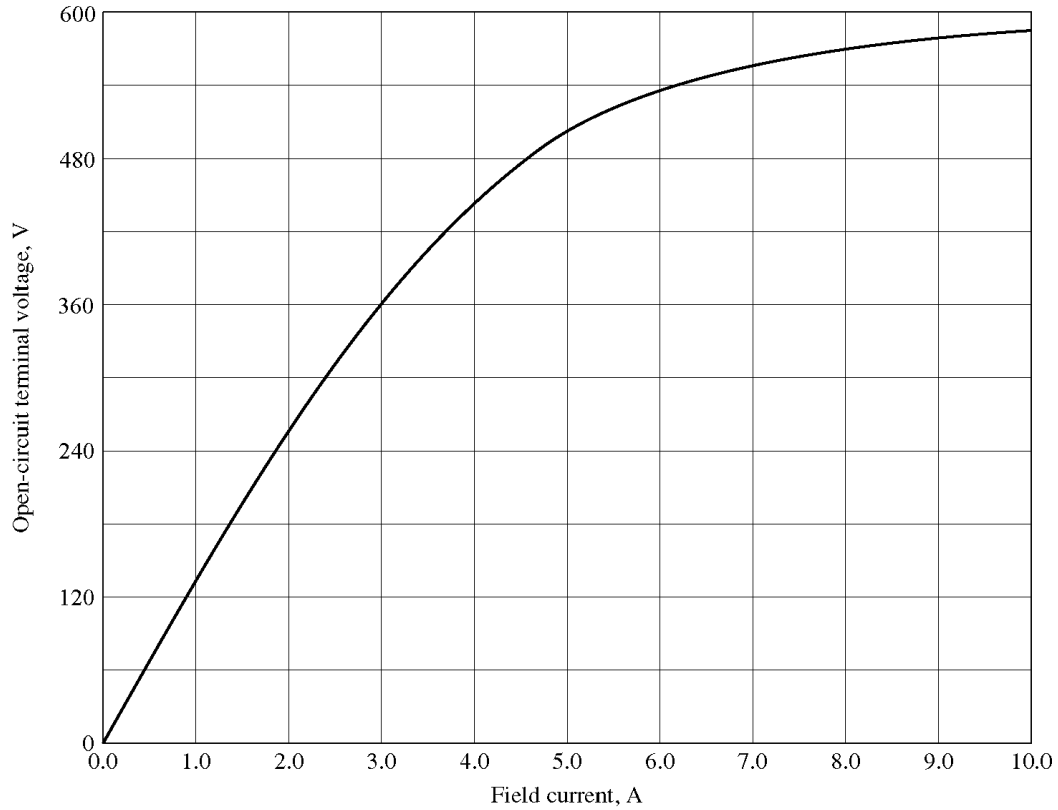
SOLUTION

(a)   If the no-load terminal voltage is 480 V, the required field current can be read directly from the open-circuit characteristic. It is 4.55 A.

(b)   This generator is Y-connected, so $I_L = I_A$. At rated conditions, the line and phase current in this generator is

$$I_A = I_L = \frac{P}{\sqrt{3}\, V_L} = \frac{200\,\text{kVA}}{\sqrt{3}(480\,\text{V})} = 240.6\,\text{A at an angle of } -36.87°$$

The internal generated voltage of the machine is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 277\angle 0° + (0.03\ \Omega)(240.6\angle - 36.87°\ \text{A}) + j(0.25\ \Omega)(240.6\angle - 36.87°\ \text{A})$$
$$\mathbf{E}_A = 322\angle 7.8°\ \text{V}$$

(c)   The equivalent open-circuit terminal voltage corresponding to an E_A of 322 volts is

$$V_{T,\text{oc}} = \sqrt{3}(322\ \text{V}) = 558\ \text{V}$$

From the OCC, the required field current is 7.0 A.

(d)   The input power to this generator is equal to the output power plus losses. The rated output power is

$$P_{\text{OUT}} = (200\ \text{kVA})(0.8) = 160\ \text{kW}$$

$$P_{\text{CU}} = 3I_A{}^2 R_A = 3(240.6\ \text{A})^2(0.03\ \Omega) = 5.2\ \text{kW}$$

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$$P_{F\&W} = 6 \text{ kW}$$

$$P_{core} = 4 \text{ kW}$$

$$P_{stray} = (\text{assumed } 0)$$

$$P_{IN} = P_{OUT} + P_{CU} + P_{F\&W} + P_{core} + P_{stray} = 175.2 \text{ kW}$$

Therefore the prime mover must be capable of supplying 175 kW. Since the generator is a two-pole 60 Hz machine, to must be turning at 3600 r/min. The required torque is

$$\tau_{APP} = \frac{P_{IN}}{\omega_m} = \frac{175.2 \text{ kW}}{(3600 \text{ r/min})\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 465 \text{ N} \cdot \text{m}$$

5-3. Assume that the field current of the generator in Problem 5-2 is adjusted to achieve rated voltage (480 V) at full load conditions in each of the questions below.

(a) What is the efficiency of the generator at rated load?

(b) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with 0.8-PF-lagging loads?

(c) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with 0.8-PF-leading loads?

(d) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with unity-power-factor loads?

(e) Use MATLAB to plot the terminal voltage of the generator as a function of load for all three power factors.

SOLUTION

(a) This generator is Y-connected, so $I_L = I_A$. At rated conditions, the line and phase current in this generator is

$$I_A = I_L = \frac{P}{\sqrt{3} \, V_L} = \frac{200 \text{ kVA}}{\sqrt{3}(480 \text{ V})} = 240.6 \text{ A} \text{ at an angle of } -36.87°$$

The internal generated voltage of the machine at rated conditions is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 277\angle 0° + (0.03 \ \Omega)(240.6\angle - 36.87° \text{ A}) + j(0.25 \ \Omega)(240.6\angle - 36.87° \text{ A})$$
$$\mathbf{E}_A = 322\angle 7.8° \text{ V}$$

The input power to this generator is equal to the output power plus losses. The rated output power is

$$P_{OUT} = (200 \text{ kVA})(0.8) = 160 \text{ kW}$$

$$P_{CU} = 3I_A^2 R_A = 3(240.6 \text{ A})^2(0.03 \ \Omega) = 5.2 \text{ kW}$$

$$P_{F\&W} = 6 \text{ kW}$$

$$P_{core} = 4 \text{ kW}$$

$$P_{\text{stray}} = (\text{assumed } 0)$$

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{stray}} = 175.2 \text{ kW}$$

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{160 \text{ kW}}{175.2 \text{ kW}} \times 100\% = 91.3\%$$

(b) If the generator is loaded to rated kVA with lagging loads, the phase voltage is $\mathbf{V}_\phi = 277\angle 0° \text{ V}$ and the internal generated voltage is $\mathbf{E}_A = 322\angle 7.8° \text{ V}$. Therefore, the phase voltage at no-load would be $\mathbf{V}_\phi = 322\angle 0° \text{ V}$. The voltage regulation would be:

$$VR = \frac{322 - 277}{277} \times 100\% = 16.2\%$$

(c) If the generator is loaded to rated kVA with leading loads, the phase voltage is $\mathbf{V}_\phi = 277\angle 0° \text{ V}$ and the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 277\angle 0° + (0.03 \ \Omega)(240.6\angle +36.87° \text{ A}) + j(0.25 \ \Omega)(240.6\angle +36.87° \text{ A})$$
$$\mathbf{E}_A = 252\angle 12° \text{ V}$$

The voltage regulation would be:

$$VR = \frac{252 - 277}{277} \times 100\% = -9.0\%$$

(d) If the generator is loaded to rated kVA at unity power factor, the phase voltage is $\mathbf{V}_\phi = 277\angle 0° \text{ V}$ and the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 277\angle 0° + (0.03 \ \Omega)(240.6\angle 0° \text{ A}) + j(0.25 \ \Omega)(240.6\angle 0° \text{ A})$$
$$\mathbf{E}_A = 291\angle 12° \text{ V}$$

The voltage regulation would be:

$$VR = \frac{291 - 277}{277} \times 100\% = 5.1\%$$

(e) For this problem, we will assume that the terminal voltage is adjusted to 480 V at no load conditions, and see what happens to the voltage as load increases at 0.8 lagging, unity, and 0.8 leading power factors. Note that the maximum current will be 240.6 A in any case. A phasor diagram representing the situation at lagging power factor is shown below:
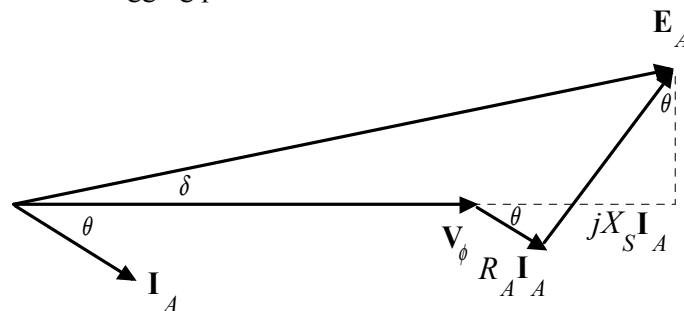
By the Pythagorean Theorem,

$$E_A^2 = \left(V_\phi + R_A I_A \cos\theta + X_S I_A \sin\theta\right)^2 + \left(X_S I_A \cos\theta - R_A I_S \sin\theta\right)^2$$

$$V_\phi = \sqrt{E_A^2 - \left(X_S I_A \cos\theta - R_A I_S \sin\theta\right)^2} - R_A I_A \cos\theta - X_S I_A \sin\theta$$

A phasor diagram representing the situation at leading power factor is shown below:

By the Pythagorean Theorem,

$$E_A^2 = \left(V_\phi + R_A I_A \cos\theta - X_S I_A \sin\theta\right)^2 + \left(X_S I_A \cos\theta + R_A I_S \sin\theta\right)^2$$

$$V_\phi = \sqrt{E_A^2 - \left(X_S I_A \cos\theta + R_A I_S \sin\theta\right)^2} - R_A I_A \cos\theta + X_S I_A \sin\theta$$

A phasor diagram representing the situation at unity power factor is shown below:

By the Pythagorean Theorem,

$$E_A^2 = V_\phi^2 + \left(X_S I_A\right)^2$$

$$V_\phi = \sqrt{E_A^2 - \left(X_S I_A\right)^2}$$

The MATLAB program is shown below takes advantage of this fact.

```
% M-file: prob5_3e.m
% M-file to calculate and plot the terminal voltage
% of a synchronous generator as a function of load
% for power factors of 0.8 lagging, 1.0, and 0.8 leading.

% Define values for this generator
EA = 277;                    % Internal gen voltage
I = 0:2.406:240.6;           % Current values (A)
R = 0.03;                    % R (ohms)
X = 0.25;                    % XS (ohms)

% Calculate the voltage for the lagging PF case
VP_lag  = sqrt( EA^2 - (X.*I.*0.8 - R.*I.*0.6).^2 ) ...
```

```
            - R.*I.*0.8 - X.*I.*0.6;
VT_lag    = VP_lag .* sqrt(3);

% Calculate the voltage for the leading PF case
VP_lead = sqrt( EA^2 - (X.*I.*0.8 + R.*I.*0.6).^2 ) ...
          - R.*I.*0.8 + X.*I.*0.6;
VT_lead = VP_lead .* sqrt(3);

% Calculate the voltage for the unity PF case
VP_unity = sqrt( EA^2 - (X.*I).^2 );
VT_unity = VP_unity .* sqrt(3);

% Plot the terminal voltage versus load
plot(I,abs(VT_lag),'b-','LineWidth',2.0);
hold on;
plot(I,abs(VT_unity),'k--','LineWidth',2.0);
plot(I,abs(VT_lead),'r-.','LineWidth',2.0);
title ('\bfTerminal Voltage Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfTerminal Voltage (V)');
legend('0.8 PF lagging','1.0 PF','0.8 PF leading');
axis([0 260 300 540]);
grid on;
hold off;
```

The resulting plot is shown below:
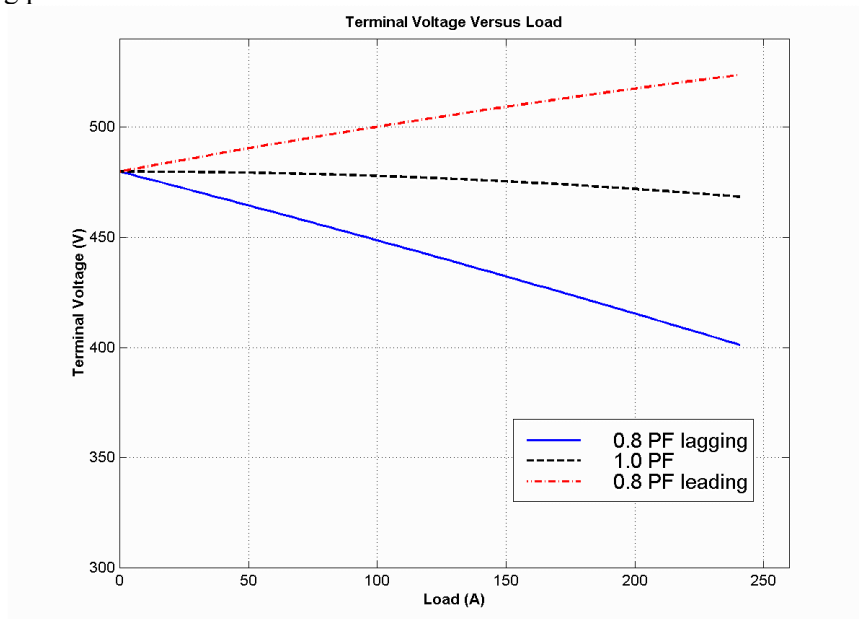
5-4. Assume that the field current of the generator in Problem 5-2 has been adjusted so that it supplies rated voltage when loaded with rated current at unity power factor. (You may ignore the effects of R_A when answering these questions.)

(a) What is the torque angle δ of the generator when supplying rated current at unity power factor?

(b) What is the maximum power that this generator can deliver to a unity power factor load when the field current is adjusted to the current value?

(c) When this generator is running at full load with unity power factor, how close is it to the static stability limit of the machine?

(d) Plot the torque angle δ as a function of the power supplied by the generator for these conditions.

SOLUTION   For the purposes of this problem, we will ignore the armature resistance R_A, so that the maximum power can be calculated from Equation (5-18).

$$P = \frac{3V_\phi E_A \sin \delta}{X_S} \tag{5-18}$$

(a)   The torque δ angle can be found by calculating $\mathbf{E}_A$:

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 277\angle 0° + j(0.25\ \Omega)(240.6\angle 0°\ \text{A})$$
$$\mathbf{E}_A = 283\angle 12°\ \text{V}$$

Thus the torque angle $\delta = 12°$.

(b)   The maximum power that this generator can deliver to a unity power factor load can be found from Equation (5-18):

$$P = \frac{3V_\phi E_A \sin \delta}{X_S}$$
$$P_{max} = \frac{3(277\ \text{V})(283\ \text{V})\sin 90°}{0.25\ \Omega} = 941\ \text{kW}$$

(c)   The current torque angle is $\delta = 12°$, and the maximum angle is 90°, so it is very far from the static stability limit. In terms of power, the generator is currently supplying

$$P = \frac{3V_\phi E_A \sin \delta}{X_S} = \frac{3(277\ \text{V})(283\ \text{V})\sin 12°}{0.25\ \Omega} = 196\ \text{kW}$$

Therefore the general is supplying about 1/5 of the static stability limit.

(d)   A program to plot the torque as a function of δ is shown below:

```
% M-file: prob5_4d.m
% M-file to calculate and plot the power supplied by
% a synchronous generator as a function of the torque
% angle delta.

% Define values for this generator
EA = 283;                  % Internal gen voltage
VP = 277;                  % Phase voltage
XS = 0.25;                 % XS (ohms)

% Calculate the power vs torque angle
delta = 0:1:90;
P = 3 * EA * VP / XS * sin( delta * pi / 180 );

% Plot the output power versus torque angle
plot(delta,P/1000,'b-','LineWidth',2.0);
title ('\bfOutput Power Versus Torque Angle');
xlabel ('\bfTorque Angle (deg)');
ylabel ('\bfOutput Power (kW)');
grid on;
```

The resulting plot is shown below:

Output Power Versus Torque Angle
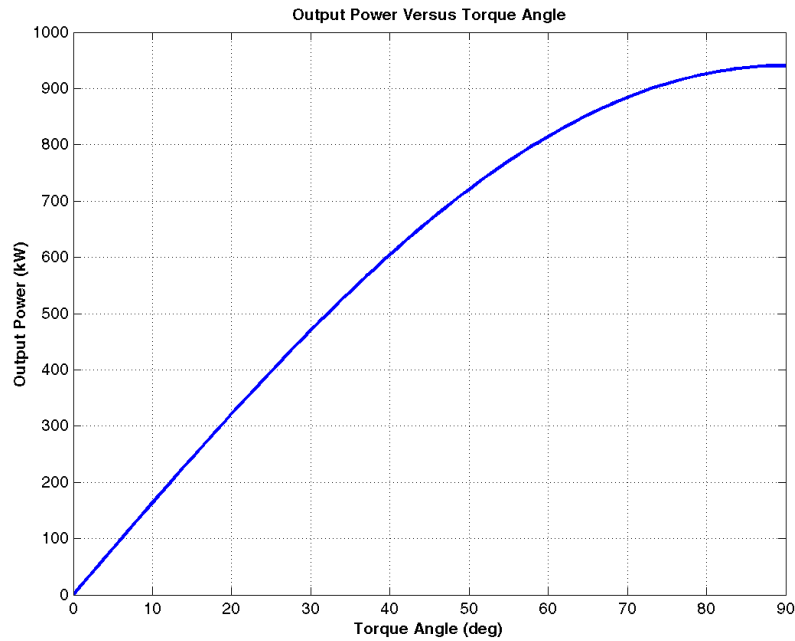
Problems 5-5 to 5-14 refer to a two-pole Y-connected synchronous generator rated at 300 kVA, 480 V, 60 Hz, and 0.85 PF lagging. Its armature resistance R_A is 0.04 Ω. The core losses of this generator at rated conditions are 10 kW, and the friction and windage losses are 13 kW. The open-circuit and short-circuit characteristics are shown in Figure P5-2.
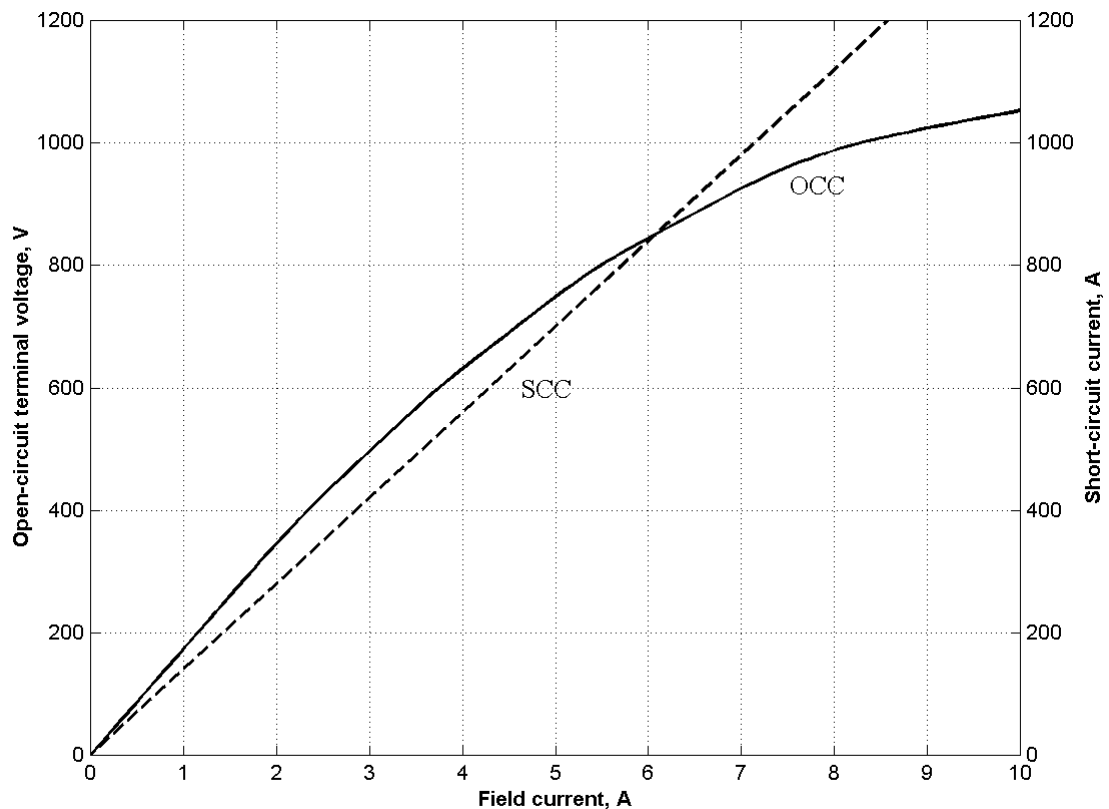
5-5. *(a)* What is the saturated synchronous reactance of this generator at the rated conditions? *(b)* What is the unsaturated synchronous reactance of this generator? *(c)* Plot the saturated synchronous reactance of this generator as a function of load.

SOLUTION

(a) The rated armature current for this generator is

$$I_A = I_L = \frac{S}{\sqrt{3}\,V_T} = \frac{300\,\text{kVA}}{\sqrt{3}\,(480\,\text{V})} = 361\,\text{A}$$

The field current required to produce this much short-circuit current may be read from the SCC. It is 2.58 A. The open circuit voltage at 2.58 A is 435 V, so the open-circuit phase voltage (= E_A) is $435/\sqrt{3}$ = 251 V. The approximate saturated synchronous reactance X_S is

$$X_S = \frac{251\,\text{V}}{361\,\text{A}} = 0.695\,\Omega$$

(b) The unsaturated synchronous reactance X_{Su} is the ratio of the air-gap line to the SCC. At I_F = 2.58 A, the air-gap line voltage is 447 V, and the SCC is 361 A.

$$X_{Su} = \frac{(447\,\text{V})/\sqrt{3}}{361\,\text{A}} = 0.715\,\Omega$$

(c) This task can best be performed with MATLAB. The open-circuit characteristic is available in a file called `p52.occ`, and the short-circuit characteristic is available in a file called `p52.scc`. Each of these files are organized in two columns, where the first column is field current and the second column is either open-circuit terminal voltage or short-circuit current. A program to read these files and calculate and plot X_S is shown below.

```
% M-file: prob5_5c.m
% M-file to calculate and plot the saturated
% synchronous reactance of a synchronous
% generator.

% Load the open-circuit characteristic.  It is in
% two columns, with the first column being field
% current and the second column being terminal
% voltage.
load p52.occ;
if_occ = p52(:,1);
vt_occ = p52(:,2);

% Load the short-circuit characteristic.  It is in
% two columns, with the first column being field
% current and the second column being line current
% (= armature current)
load p52.scc;
if_scc = p52(:,1);
ia_scc = p52(:,2);

% Calculate Xs
If = 0.001:0.1:10;                  % Current steps
vt = interp1(if_occ,vt_occ,If);   % Terminal voltage
ia = interp1(if_scc,ia_scc,If);   % Current
Xs = (vt ./ sqrt(3)) ./ ia;

% Plot the synchronous reactance
figure(1)
```

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```
plot(If,Xs,'LineWidth',2.0);
title ('\bfSaturated Synchronous Reactance \itX_{s} \rm');
xlabel ('\bfField Current (A)');
ylabel ('\bf\itX_{s} \rm\bf(\Omega)');
grid on;
```

The resulting plot is:
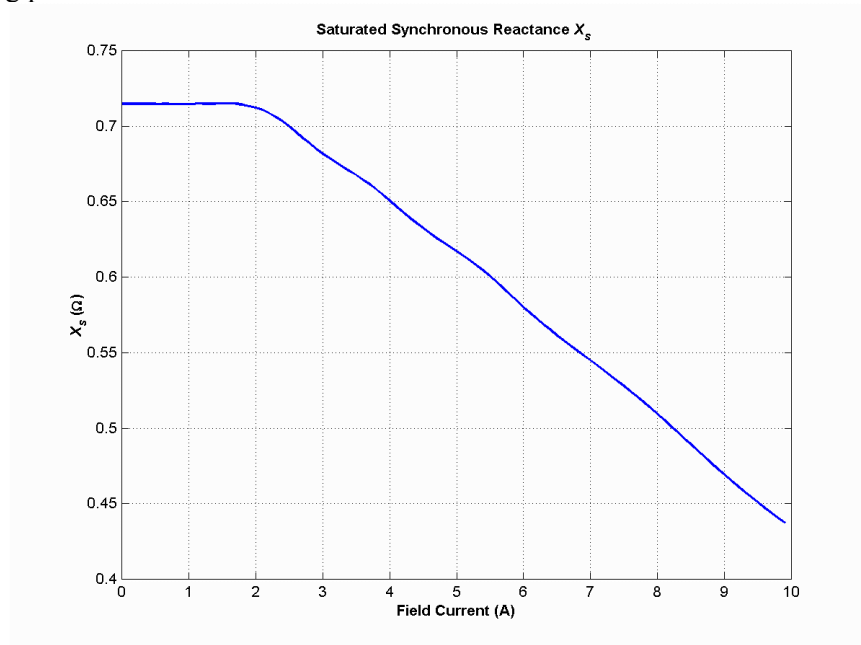
5-6. *(a)* What are the rated current and internal generated voltage of this generator? *(b)* What field current does this generator require to operate at the rated voltage, current, and power factor?

SOLUTION

(a) The rated line and armature current for this generator is

$$I_A = I_L = \frac{S}{\sqrt{3}\ V_T} = \frac{300\ \text{kVA}}{\sqrt{3}(480\ \text{V})} = 361\ \text{A}$$

The power factor is 0.85 lagging, so $\mathbf{I}_A = 361\angle -31.8°\ \text{A}$. The rated phase voltage is V_ϕ = 480 V / $\sqrt{3}$ = 277 V. The saturated synchronous reactance at rated conditions was found to be 0.695 Ω in the previous problem. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 277\angle 0° + (0.04\ \Omega)(361\angle -31.8°\ \text{A}) + j(0.695\ \Omega)(361\angle -31.8°\ \text{A})$$
$$\mathbf{E}_A = 469\angle 26°\ \text{V}$$

(b) This internal generated voltage corresponds to a no-load terminal voltage of $\sqrt{3}(469)$ = 812 V. From the open-circuit characteristic, the required field current would be 5.64 A.

5-7. What is the voltage regulation of this generator at the rated current and power factor?

SOLUTION The voltage regulation is

$$\text{VR} = \frac{V_{T,\text{nl}} - V_{T,\text{fl}}}{V_{T,\text{fl}}} \times 100\% = \frac{812 - 480}{480} \times 100\% = 69.2\%$$

5-8. If this generator is operating at the rated conditions and the load is suddenly removed, what will the terminal voltage be?

SOLUTION From the above calculations, V_T will be 812 V.

5-9. What are the electrical losses in this generator at rated conditions?

SOLUTION The electrical losses are

$$P_{CU} = 3I_A^2 R_A = 3(361 \text{ A})^2 (0.04 \text{ }\Omega) = 15.6 \text{ kW}$$

5-10. If this machine is operating at rated conditions, what input torque must be applied to the shaft of this generator? Express your answer both in newton-meters and in pound-feet.

SOLUTION To get the applied torque, we must know the input power. The input power to this generator is equal to the output power plus losses. The rated output power and the losses are

$$P_{OUT} = (300 \text{ kVA})(0.85) = 255 \text{ kW}$$

$$P_{CU} = 3I_A^2 R_A = 3(361 \text{ A})^2 (0.04 \text{ }\Omega) = 15.6 \text{ kW}$$

$$P_{F\&W} = 13 \text{ kW}$$

$$P_{core} = 10 \text{ kW}$$

$$P_{stray} = (\text{assumed } 0)$$

$$P_{IN} = P_{OUT} + P_{CU} + P_{F\&W} + P_{core} + P_{stray} = 293.6 \text{ kW}$$

Therefore, the applied torque is

$$\tau_{APP} = \frac{P_{IN}}{\omega_m} = \frac{293.6 \text{ kW}}{(3600 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 779 \text{ N} \cdot \text{m}$$

or $\quad \tau_{APP} = \frac{7.04 \ P}{n_m} = \frac{7.04 \ (293.6 \text{ kW})}{3600 \text{ r/min}} = 574 \text{ lb} \cdot \text{ft}$

5-11. What is the torque angle δ of this generator at rated conditions?

SOLUTION From the calculations in Problem 5-6, $\delta = 26°$.

5-12. Assume that the generator field current is adjusted to supply 480 V under rated conditions. What is the static stability limit of this generator? (*Note:* You may ignore R_A to make this calculation easier.) How close is the full-load condition of this generator to the static stability limit?

SOLUTION At rated conditions, $\mathbf{E}_A = 469\angle 26° \text{ V}$. Therefore, the static stability limit is

$$P_{MAX} = \frac{3 \ V_\phi \ E_A}{X_S} = \frac{3 \ (277 \text{ V})(469 \text{ V})}{0.695 \text{ }\Omega} = 561 \text{ kW}$$

The full-load rated power of this generator is less than half of the static stability limit.

5-13. Assume that the generator field current is adjusted to supply 480 V under rated conditions. Plot the power supplied by the generator as a function of the torque angle δ. (*Note:* You may ignore R_A to make this calculation easier.)

SOLUTION We will again ignore R_A to make this calculation easier. The power supplied by the generator is

$$P_G = \frac{3\,V_\phi\,E_A}{X_S}\sin\delta = \frac{3\,(277\text{ V})(469\text{ V})}{0.695\text{ }\Omega}\sin\delta = (561\text{ kW})\,\sin\delta$$

The power supplied as a function of the torque angle δ may be plotted using a simple MATLAB program:

```
% M-file: prob5_13.m
% M-file to plot the power output of a
% synchronous generator as a function of
% the torque angle.

% Calculate Xs
delta = (0:1:90);                        % Torque angle (deg)
Pout = 561 .* sin(delta * pi/180); % Pout

% Plot the output power
figure(1)
plot(delta,Pout,'LineWidth',2.0);
title ('\bfOutput power vs torque angle \delta');
xlabel ('\bfTorque angle \delta (deg)');
ylabel ('\bf\itP_{OUT} \rm\bf(kW)');
grid on;
```

The resulting plot is:
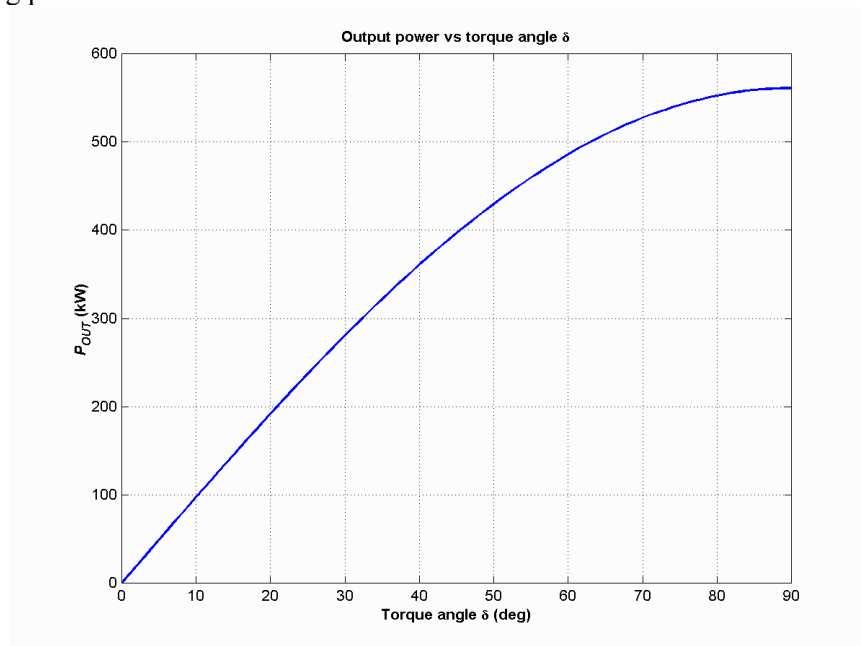
5-14. Assume that the generator's field current is adjusted so that the generator supplies rated voltage at the rated load current and power factor. If the field current and the magnitude of the load current are held constant, how will the terminal voltage change as the load power factor varies from 0.85 PF lagging to 0.85 PF leading? Make a plot of the terminal voltage versus the impedance angle of the load being supplied by this generator.

SOLUTION If the field current is held constant, then the magnitude of $\mathbf{E}_A$ will be constant, although its angle δ will vary. Also, the magnitude of the armature current is constant. Since we also know R_A, X_S,

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and the current angle θ, we know enough to find the phase voltage V_ϕ, and therefore the terminal voltage V_T. At lagging power factors, V_ϕ can be found from the following relationships:
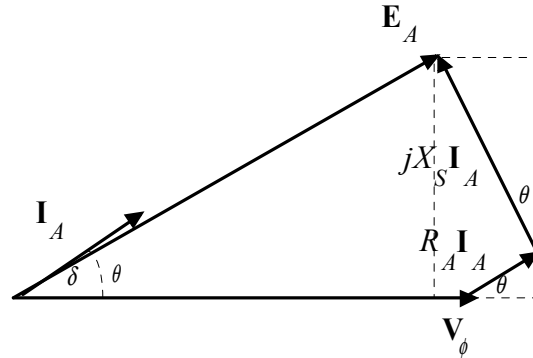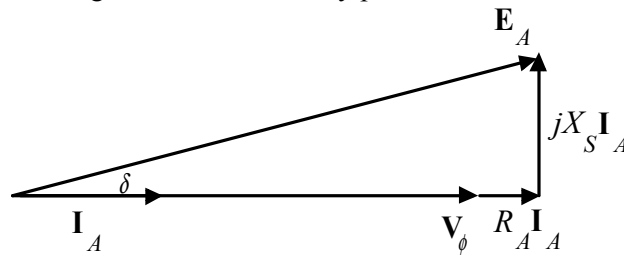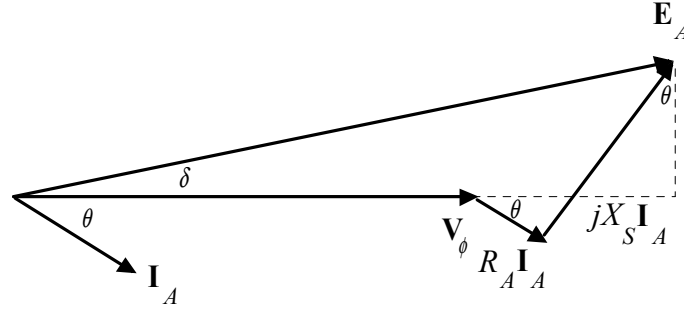

By the Pythagorean Theorem,

$$E_A{}^2 = \left(V_\phi + R_A I_A \cos\theta + X_S I_A \sin\theta\right)^2 + \left(X_S I_A \cos\theta - R_A I_S \sin\theta\right)^2$$
$$V_\phi = \sqrt{E_A{}^2 - \left(X_S I_A \cos\theta - R_A I_S \sin\theta\right)^2} - R_A I_A \cos\theta - X_S I_A \sin\theta$$

At unity power factor, V_ϕ can be found from the following relationships:
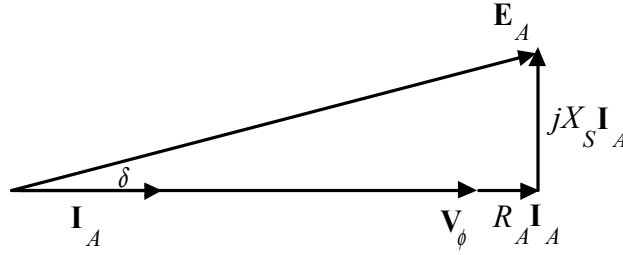
By the Pythagorean Theorem,

$$E_A{}^2 = V_\phi{}^2 + \left(X_S I_A\right)^2$$
$$V_\phi = \sqrt{E_A{}^2 - \left(X_S I_A\right)^2}$$

At leading power factors, V_ϕ can be found from the following relationships:
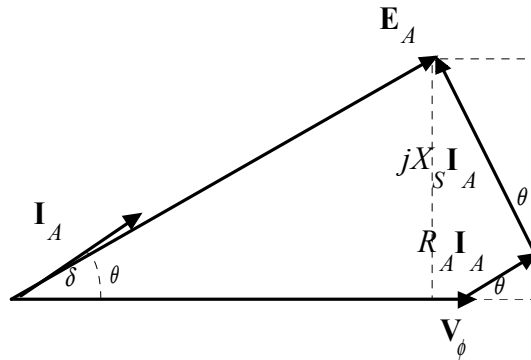
By the Pythagorean Theorem,

$$E_A{}^2 = \left(V_\phi + R_A I_A \cos\theta - X_S I_A \sin\theta\right)^2 + \left(X_S I_A \cos\theta + R_A I_S \sin\theta\right)^2$$
$$V_\phi = \sqrt{E_A{}^2 - \left(X_S I_A \cos\theta + R_A I_S \sin\theta\right)^2} - R_A I_A \cos\theta + X_S I_A \sin\theta$$

If we examine these three cases, we can see that the only difference among them is the sign of the term $\sin\theta$. If θ is taken as positive for lagging power factors and negative for leading power factors (in other words, if θ is the *impedance angle*), then all three cases can be represented by the single equation:

$$V_\phi = \sqrt{E_A^2 - \left(X_S I_A \cos\theta - R_A I_S \sin\theta\right)^2} - R_A I_A \cos\theta - X_S I_A \sin\theta$$

A MATLAB program that calculates terminal voltage as function of impedance angle is shown below:

```
% M-file: prob5_14.m
% M-file to calculate and plot the terminal voltage
% of a synchronous generator as a function of impedance
% angle as PF changes from 0.85 lagging to 0.85
% leading.

% Define values for this generator
EA = 469;                  % Internal gen voltage
I = 361;                   % Current (A)
R = 0.04;                  % R (ohms)
X = 0.695;                 % XS (ohms)

% Calculate impedance angle theta in degrees
theta = -31.8:0.318:31.8;
th = theta * pi/180;       % In radians

% Calculate the phase voltage and terminal voltage
VP  = sqrt( EA^2 - (X.*I.*cos(th) - R.*I.*sin(th)).^2 ) ...
    - R.*I.*cos(th) - X.*I.*sin(th);
VT  = VP .* sqrt(3);

% Plot the terminal voltage versus power factor
figure(1);
plot(theta,abs(VT),'b-','LineWidth',2.0);
title ('\bfTerminal Voltage Versus Load');
xlabel ('\bfImpedance Angle (deg)');
ylabel ('\bfTerminal Voltage (V)');
%axis([0 260 300 540]);
grid on;
hold off;
```

The resulting plot of terminal voltage versus impedance angle (with field and armature currents held constant) is shown below:

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Terminal Voltage Versus Load
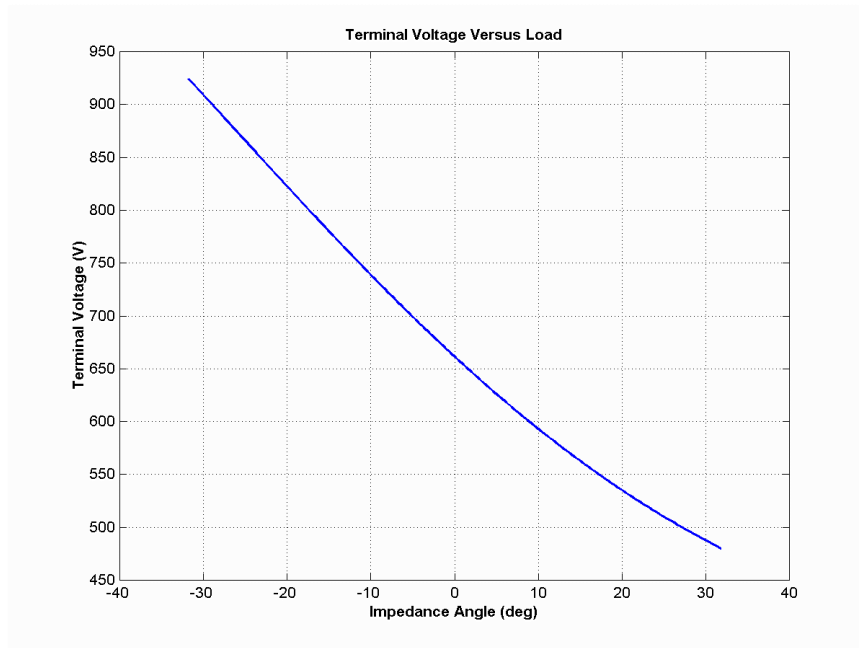
5-15. A 100-MVA 11.5-kV 0.8-PF-lagging 50-Hz two-pole Y-connected synchronous generator has a per-unit synchronous reactance of 0.8 and a per-unit armature resistance of 0.012.

(a) What are its synchronous reactance and armature resistance in ohms?

(b) What is the magnitude of the internal generated voltage E_A at the rated conditions? What is its torque angle δ at these conditions?

(c) Ignoring losses in this generator, what torque must be applied to its shaft by the prime mover at full load?

SOLUTION   The base phase voltage of this generator is $V_{\phi,\text{base}} = 11,500/\sqrt{3} = 6640 \text{ V}$. Therefore, the base impedance of the generator is

$$Z_{\text{base}} = \frac{3\,V_{\phi,\text{base}}^{\,2}}{S_{\text{base}}} = \frac{3(6640 \text{ V})^2}{100,000,000 \text{ VA}} = 1.32 \ \Omega$$

(a)   The generator impedances in ohms are:

$$R_A = (0.012)(1.32 \ \Omega) = 0.0158 \ \Omega$$

$$X_S = (0.8)(1.32 \ \Omega) = 1.06 \ \Omega$$

(b)   The rated armature current is

$$I_A = I_L = \frac{S}{\sqrt{3}\,V_T} = \frac{100 \text{ MVA}}{\sqrt{3}(11.5 \text{ kV})} = 5020 \text{ A}$$

The power factor is 0.8 lagging, so $\mathbf{I}_A = 5020\angle - 36.87°\text{ A}$. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 6640\angle 0° + (0.0158 \ \Omega)(5020\angle - 36.87° \text{ A}) + j(1.06 \ \Omega)(5020\angle - 36.87° \text{ A})$$

$$\mathbf{E}_A = 10,750\angle 23° \text{ V}$$

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Therefore, the magnitude of the internal generated voltage $E_A = 10,750$ V, and the torque angle $\delta = 23°$.

(c) Ignoring losses, the input power would equal the output power. Since

$$P_{\text{OUT}} = (0.80)(100 \text{ MVA}) = 80 \text{ MW}$$

and

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50 \text{ Hz})}{2} = 3000 \text{ r/min}$$

the applied torque would be

$$\tau_{\text{app}} = \tau_{\text{ind}} = \frac{80,000,000 \text{ W}}{(3000 \text{ r/min})(2\pi \text{ rad/r})(1 \text{ min/60 s})} = 255,000 \text{ N} \cdot \text{m}$$

5-16. A three-phase Y-connected synchronous generator is rated 120 MVA, 13.8 kV, 0.85 PF lagging, and 60 Hz. Its synchronous reactance is 0.7 Ω, and its resistance may be ignored.

(a) What is its voltage regulation?

(b) What would the voltage and apparent power rating of this generator be if it were operated at 50 Hz with the same armature and field losses as it had at 60 Hz?

(c) What would the voltage regulation of the generator be at 50 Hz?

SOLUTION

(a) The rated armature current is

$$I_A = I_L = \frac{S}{\sqrt{3} V_T} = \frac{120 \text{ MVA}}{\sqrt{3}(13.8 \text{ kV})} = 5020 \text{ A}$$

The power factor is 0.85 lagging, so $\mathbf{I}_A = 5020\angle - 31.8° \text{ A}$. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 7967\angle 0° + j(0.7 \text{ }\Omega)(5020\angle - 31.8° \text{ A})$$
$$\mathbf{E}_A = 10,260\angle 16.9° \text{ V}$$

The resulting voltage regulation is

$$\text{VR} = \frac{10,260 - 7967}{7967} \times 100\% = 28.8\%$$

(b) If the generator is to be operated at 50 Hz with the same armature and field losses as at 60 Hz (so that the windings do not overheat), then its armature and field currents must not change. Since the voltage of the generator is directly proportional to the speed of the generator, the voltage rating (and hence the apparent power rating) of the generator will be reduced by a factor of 5/6.

$$V_{T,\text{rated}} = \frac{5}{6}(13.8 \text{ kV}) = 11.5 \text{ kV}$$

$$S_{\text{rated}} = \frac{5}{6}(120 \text{ MVA}) = 100 \text{ MVA}$$

Also, the synchronous reactance will be reduced by a factor of 5/6.

$$X_S = \frac{5}{6}(0.7 \ \Omega) = 0.583 \ \Omega$$

(c)   At 50 Hz rated conditions, the armature current would be

$$I_A = I_L = \frac{S}{\sqrt{3} \ V_T} = \frac{100 \ \text{MVA}}{\sqrt{3}(11.5 \ \text{kV})} = 5020 \ \text{A}$$

The power factor is 0.8 lagging, so $\mathbf{I}_A = 5020\angle - 31.8° \ \text{A}$. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 6640\angle 0° + j(0.583 \ \Omega)(5020\angle - 31.8° \ \text{A})$$
$$\mathbf{E}_A = 8550\angle 16.9° \ \text{V}$$

The resulting voltage regulation is

$$\text{VR} = \frac{8550 - 6640}{6640} \times 100\% = 28.8\%$$

Because voltage, apparent power, and synchronous reactance all scale linearly with frequency, the voltage regulation at 50 Hz is the same as that at 60 Hz. Note that this is not *quite* true, if the armature resistance R_A is included, since R_A does not scale with frequency in the same fashion as the other terms.

5-17.   A 25-MVA three-phase 13.8-kV two-pole 60-Hz synchronous generator was tested by the open-circuit test, and its air-gap voltage was extrapolated with the following results:

Open-circuit test

Field current, A	320	365	380	475	570
Line voltage, kV	13.0	13.8	14.1	15.2	16.0
Extrapolated air-gap voltage, kV	15.4	17.5	18.3	22.8	27.4

The short-circuit test was then performed with the following results:

Short-circuit test

Field current, A	320	365	380	475	570
Armature current, A	1040	1190	1240	1550	1885

The armature resistance is 0.24 Ω per phase.

(a) Find the unsaturated synchronous reactance of this generator in ohms per phase and in per-unit.

(b) Find the approximate saturated synchronous reactance X_S at a field current of 380 A. Express the answer both in ohms per phase and in per-unit.

(c) Find the approximate saturated synchronous reactance at a field current of 475 A. Express the answer both in ohms per phase and in per-unit.

SOLUTION

(a)   The unsaturated synchronous reactance of this generator is the same at any field current, so we will look at it at a field current of 380 A. The extrapolated air-gap voltage at this point is 18.3 kV, and the short-circuit current is 1240 A. Since this generator is Y-connected, the phase voltage is $V_\phi = 18.3 \ \text{kV}/\sqrt{3} = 10{,}566 \ \text{V}$ and the armature current is $I_A = 1240 \ \text{A}$. Therefore, the *unsaturated* synchronous reactance is

$$X_{Su} = \frac{10{,}566 \text{ V}}{1240 \text{ A}} = 8.52 \,\Omega$$

The base impedance of this generator is

$$Z_{base} = \frac{3 \, V_{\phi,base}^2}{S_{base}} = \frac{3(7967 \text{ V})^2}{25{,}000{,}000 \text{ VA}} = 7.62 \,\Omega$$

Therefore, the per-unit unsaturated synchronous reactance is

$$X_{Su,pu} = \frac{8.52 \,\Omega}{7.62 \,\Omega} = 1.12$$

(b) The saturated synchronous reactance at a field current of 380 A can be found from the OCC and the SCC. The OCC voltage at I_F = 380 A is 14.1 kV, and the short-circuit current is 1240 A. Since this generator is Y-connected, the corresponding phase voltage is $V_\phi = 14.1 \text{ kV}/\sqrt{3} = 8141 \text{ V}$ and the armature current is $I_A = 1240 \text{ A}$. Therefore, the *saturated* synchronous reactance is

$$X_{Su} = \frac{8141 \text{ V}}{1240 \text{ A}} = 6.57 \,\Omega$$

and the per-unit unsaturated synchronous reactance is

$$X_{Su,pu} = \frac{6.57 \,\Omega}{7.62 \,\Omega} = 0.862$$

(c) The saturated synchronous reactance at a field current of 475 A can be found from the OCC and the SCC. The OCC voltage at I_F = 475 A is 15.2 kV, and the short-circuit current is 1550 A. Since this generator is Y-connected, the corresponding phase voltage is $V_\phi = 15.2 \text{ kV}/\sqrt{3} = 8776 \text{ V}$ and the armature current is $I_A = 1550 \text{ A}$. Therefore, the *saturated* synchronous reactance is

$$X_{Su} = \frac{8776 \text{ V}}{1550 \text{ A}} = 5.66 \,\Omega$$

and the per-unit unsaturated synchronous reactance is

$$X_{Su,pu} = \frac{5.66 \,\Omega}{7.62 \,\Omega} = 0.743$$

5-18. A 480-V, 60 Hz, six-pole synchronous motor draws 80 A from the line at unity power factor and full load. Assuming that the motor is lossless, answer the following questions:

(a) What is the output torque of this motor? Express the answer both in newton-meters and in pound-feet.

(b) What must be done to change the power factor to 0.8 leading? Explain your answer, using phasor diagrams.

(c) What will the magnitude of the line current be if the power factor is adjusted to 0.8 leading?

SOLUTION

(a) If this motor is assumed lossless, then the input power is equal to the output power. The input power to this motor is

$$P_{IN} = \sqrt{3} \, V_T I_L \cos\theta = \sqrt{3}(480 \text{ V})(80 \text{ A})(1.0) = 66.5 \text{ kW}$$

Since this motor is assumed to be lossless, $P_{OUT} = P_{IN}$. The output torque would be

$$\tau_{LOAD} = \frac{P_{OUT}}{\omega_m} = \frac{66.5 \text{ kW}}{(1200 \text{ r/min})\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 529 \text{ N} \cdot \text{m}$$

In English units,

$$\tau_{LOAD} = \frac{7.04 \, P_{OUT}}{n_m} = \frac{(7.04)(66.5 \text{ kW})}{(1200 \text{ r/min})} = 390 \text{ lb} \cdot \text{ft}$$

(b) To change the motor's power factor to 0.8 leading, its field current must be increased. Since the power supplied to the load is independent of the field current level, an increase in field current increases $|\mathbf{E}_A|$ while keeping the distance $E_A \sin \delta$ constant. This increase in E_A changes the angle of the current $\mathbf{I}_A$, eventually causing it to reach a power factor of 0.8 leading.

$Q \propto I_A \sin \theta$
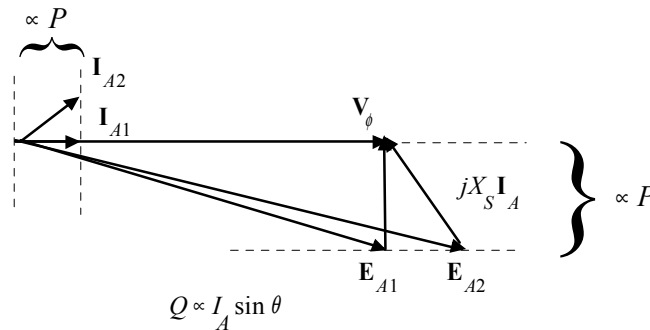
(c) The magnitude of the line current will be

$$I_L = \frac{P}{\sqrt{3} \, V_T \, \text{PF}} = \frac{66.5 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.8)} = 100 \text{ A}$$

5-19. A 480-V, 60 Hz, 400-hp 0.8-PF-leading eight-pole Δ-connected synchronous motor has a synchronous reactance of 1.0 Ω and negligible armature resistance. Ignore its friction, windage, and core losses for the purposes of this problem.

(a) If this motor is initially supplying 400 hp at 0.8 PF lagging, what are the magnitudes and angles of $\mathbf{E}_A$ and $\mathbf{I}_A$?

(b) How much torque is this motor producing? What is the torque angle δ? How near is this value to the maximum possible induced torque of the motor for this field current setting?

(c) If $|\mathbf{E}_A|$ is increased by 15 percent, what is the new magnitude of the armature current? What is the motor's new power factor?

(d) Calculate and plot the motor's V-curve for this load condition.

SOLUTION

(a) If losses are being ignored, the output power is equal to the input power, so the input power will be

$$P_{IN} = (400 \text{ hp})(746 \text{ W/hp}) = 298.4 \text{ kW}$$

This situation is shown in the phasor diagram below:
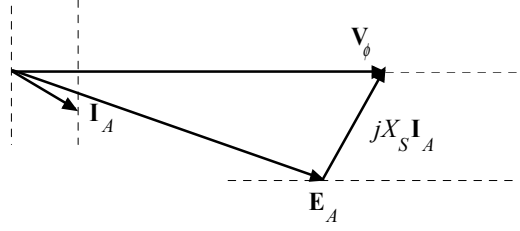
The line current flow under these circumstances is

$$I_L = \frac{P}{\sqrt{3}\ V_T\ \text{PF}} = \frac{298.4\ \text{kW}}{\sqrt{3}(480\ \text{V})(0.8)} = 449\ \text{A}$$

Because the motor is Δ-connected, the corresponding phase current is $I_A = 449/\sqrt{3} = 259\ \text{A}$. The angle of the current is $-\cos^{-1}(0.80) = -36.87°$, so $\mathbf{I}_A = 259\angle -36.87°\ \text{A}$. The internal generated voltage $\mathbf{E}_A$ is

$$\mathbf{E}_A = \mathbf{V}_\phi - jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = (480\angle 0°\ \text{V}) - j(1.0\ \Omega)(259\angle -36.87°\ \text{A}) = 385\angle -32.6°\ \text{V}$$

(b) This motor has 8 poles and an electrical frequency of 60 Hz, so its rotation speed is $n_m = 900$ r/min. The induced torque is

$$\tau_{\text{ind}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{298.4\ \text{kW}}{(900\ \text{r/min})\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)\left(\dfrac{2\pi\ \text{rad}}{1\ \text{r}}\right)} = 3166\ \text{N}\cdot\text{m}$$

The maximum possible induced torque for the motor at this field setting is

$$\tau_{\text{ind,max}} = \frac{3\ V_\phi\ E_A}{\omega_m\ X_S} = \frac{3(480\ \text{V})(385\ \text{V})}{(900\ \text{r/min})\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)\left(\dfrac{2\pi\ \text{rad}}{1\ \text{r}}\right)(1.0\ \Omega)} = 5882\ \text{N}\cdot\text{m}$$

(c) If the magnitude of the internal generated voltage E_A is increased by 15%, the new torque angle can be found from the fact that $E_A \sin\delta \propto P = \text{constant}$.

$$E_{A2} = 1.15\ E_{A1} = 1.15(385\ \text{V}) = 443\ \text{V}$$

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_1\right) = \sin^{-1}\left(\frac{385\ \text{V}}{443\ \text{V}}\sin(-32.6°)\right) = -27.9°$$

The new armature current is

$$\mathbf{I}_{A2} = \frac{\mathbf{V}_\phi - \mathbf{E}_{A2}}{jX_S} = \frac{480\angle 0°\ \text{V} - 443\angle -27.9°\ \text{V}}{j1.0\ \Omega} = 225\angle -23.1°\ \text{A}$$

The magnitude of the armature current is 225 A, and the power factor is cos (-23.1°) = 0.920 lagging.

(d) A MATLAB program to calculate and plot the motor's V-curve is shown below:

```
% M-file: prob5_19d.m
% M-file create a plot of armature current versus Ea
%    for the synchronous motor of Problem 5-19.
```

```
% First, initialize the field current values (21 values
% in the range 3.8-5.8 A)
i_f = (38:1:58) / 10;

% Initialize values
Ea = (1:0.01:1.70)*385;          % Magnitude of Ea volts
Ear = 385;                       % Reference Ea
deltar = -32.6 * pi/180;         % Reference torque angle
Xs = 1.0;                        % Synchronous reactance
Vp = 480;                        % Phase voltage at 0 degrees
Ear = Ear * (cos(deltar) + j * sin(deltar));

% Calculate delta2
delta2 = asin ( abs(Ear) ./ abs(Ea) .* sin(deltar) );

% Calculate the phasor Ea
Ea = Ea .* (cos(delta2) + j .* sin(delta2));

% Calculate Ia
Ia = ( Vp - Ea ) / ( j * Xs);

% Plot the v-curve
figure(1);
plot(abs(Ea),abs(Ia),'b','Linewidth',2.0);
xlabel('\bf\itE_{A}\rm\bf (V)');
ylabel('\bf\itI_{A}\rm\bf (A)');
title ('\bfSynchronous Motor V-Curve');
grid on;
```

The resulting plot is shown below
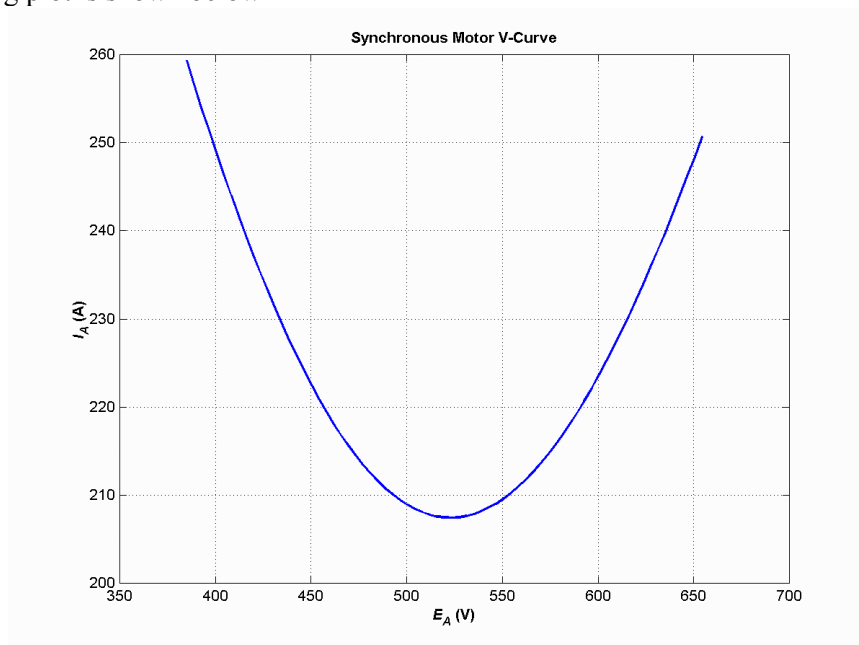
5-20. A 2300-V 2000-hp 0.8-PF leading 60-Hz two-pole Y-connected synchronous motor has a synchronous reactance of 1.5 Ω and an armature resistance of 0.3 Ω. At 60 Hz, its friction and windage losses are 50 kW, and its core losses are 40 kW. The field circuit has a dc voltage of 200 V, and the maximum I_F is

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10 A. The open-circuit characteristic of this motor is shown in Figure P5-3. Answer the following questions about the motor, assuming that it is being supplied by an infinite bus.

(a) How much field current would be required to make this machine operate at unity power factor when supplying full load?

(b) What is the motor's efficiency at full load and unity power factor?

(c) If the field current were increased by 5 percent, what would the new value of the armature current be? What would the new power factor be? How much reactive power is being consumed or supplied by the motor?

(d) What is the maximum torque this machine is theoretically capable of supplying at unity power factor? At 0.8 PF leading?
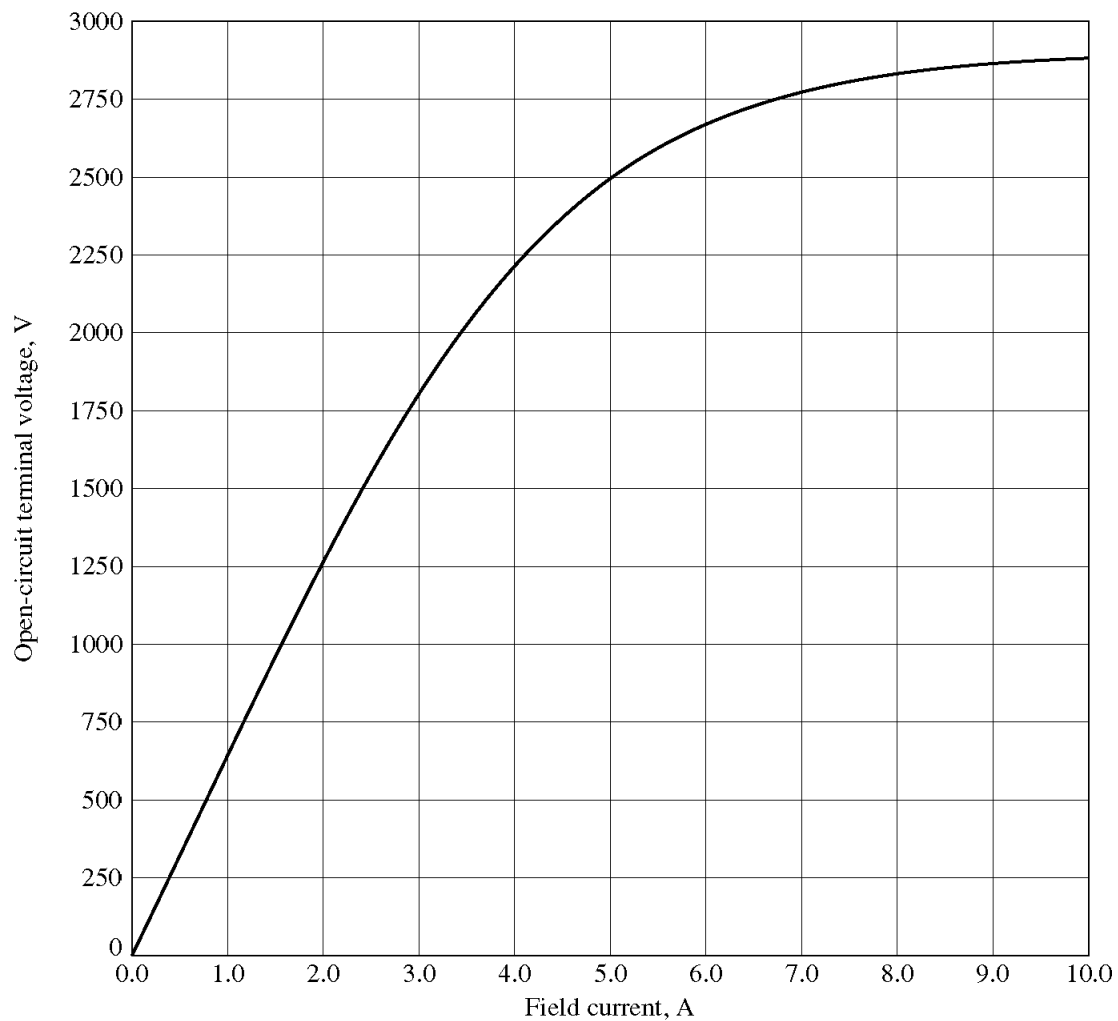
SOLUTION

(a)   At full load, the input power to the motor is

$$P_{IN} = P_{OUT} + P_{mech} + P_{core} + P_{CU}$$

We can't know the copper losses until the armature current is known, so we will find the input power ignoring that term, and then correct the input power after we know it.

$$P_{IN} = (2000 \text{ hp})(746 \text{ W/hp}) + 50 \text{ kW} + 40 \text{ kW} = 1582 \text{ kW}$$

Therefore, the line and phase current at unity power factor is

$$I_A = I_L = \frac{P}{\sqrt{3}\ V_T\ \mathrm{PF}} = \frac{1582\ \mathrm{kW}}{\sqrt{3}\,(2300\ \mathrm{V})(1.0)} = 397\ \mathrm{A}$$

The copper losses due to a current of 397 A are

$$P_{\mathrm{CU}} = 3I_A^{\,2}R_A = 3(397\ \mathrm{A})^2\,(0.3\ \Omega) = 142\ \mathrm{kW}$$

Therefore, a better estimate of the input power at full load is

$$P_{\mathrm{IN}} = (2000\ \mathrm{hp})(746\ \mathrm{W/hp}) + 50\ \mathrm{kW} + 40\ \mathrm{kW} + 142\ \mathrm{kW} = 1724\ \mathrm{kW}$$

and a better estimate of the line and phase current at unity power factor is

$$I_A = I_L = \frac{P}{\sqrt{3}\ V_T\ \mathrm{PF}} = \frac{1724\ \mathrm{kW}}{\sqrt{3}\,(2300\ \mathrm{V})(1.0)} = 433\ \mathrm{A}$$

The phasor diagram of this motor operating a unity power factor is shown below:
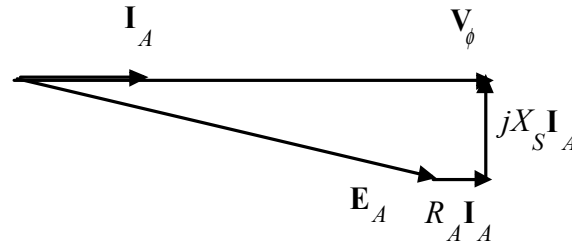
The required internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi - R_A\mathbf{I}_A - jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 1328\angle 0°\ \mathrm{V} - (0.2\ \Omega)(433\angle 0°\ \mathrm{A}) - j(1.5\ \Omega)(433\angle 0°\ \mathrm{A})$$
$$\mathbf{E}_A = 1410\angle -27.6°\ \mathrm{V}$$

This internal generated voltage corresponds to a terminal voltage of $\sqrt{3}\,(1410) = 2442\ \mathrm{V}$. This voltage would require a field current of 4.7 A.

(b) The motor's efficiency at full load an unity power factor is

$$\eta = \frac{P_{\mathrm{OUT}}}{P_{\mathrm{IN}}} \times 100\% = \frac{1492\ \mathrm{kW}}{1724\ \mathrm{kW}} \times 100\% = 86.5\%$$

(c) To solve this problem, we will temporarily ignore the effects of the armature resistance R_A. If R_A is ignored, then $E_A \sin\delta$ is directly proportional to the power supplied by the motor. Since the power supplied by the motor does not change when I_F is changed, this quantity will be a constant.

If the field current is increased by 5%, then the new field current will be 4.94 A, and the new value of the open-circuit terminal voltage will be 2475 V. The new value of E_A will be 2475 V / $\sqrt{3}$ = 1429 V. Therefore, the new torque angle δ will be

$$\delta_2 = \sin^{-1}\!\left(\frac{E_{A1}}{E_{A2}}\sin\delta_1\right) = \sin^{-1}\!\left(\frac{1410\ \mathrm{V}}{1429\ \mathrm{V}}\sin(-27.6°)\right) = -27.2°$$

Therefore, the new armature current will be

$$I_A = \frac{V_\phi - E_A}{R_A + jX_S} = \frac{1328\angle 0° \text{ V} - 1429\angle\text{-}27.2° \text{ V}}{0.3 + j1.5 \text{ }\Omega} = 428.6\angle 6.3° \text{ A}$$

The new current is about the same as before, but the phase angle has become positive. The new power factor is cos 6.3° = 0.994 leading, and the reactive power supplied by the motor is

$$Q = \sqrt{3} \text{ } V_T \text{ } I_L \sin\theta = \sqrt{3}(2300 \text{ V})(428.6 \text{ A})(0.110) = 187 \text{ kVAR}$$

(d) The maximum torque possible at unity power factor (ignoring the effects if R_A) is:

$$\tau_{\text{ind,max}} = \frac{3 \text{ } V_\phi \text{ } E_A}{\omega_m \text{ } X_S} = \frac{3(1328 \text{ V})(1410 \text{ V})}{(3600 \text{ r/min})\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)(1.5 \text{ }\Omega)} = 9940 \text{ N} \cdot \text{m}$$

At a power factor of 0.8 leading, the current flow will be

$$I_A = I_L = \frac{P_{\text{IN}}}{\sqrt{3} \text{ } V_T \text{ PF}} = \frac{1724 \text{ kW}}{\sqrt{3}(2300 \text{ V})(0.8)} = 541 \text{ A}$$

so $I_A = 541\angle 36.87° \text{ A}$. The internal generated voltage at 0.8 PF leading is

$$E_A = V_\phi - R_A I_A - jX_S I_A$$
$$E_A = 1328\angle 0° \text{ V} - (0.2 \text{ }\Omega)(541\angle 36.87° \text{ A}) - j(1.5 \text{ }\Omega)(541\angle 36.87° \text{ A})$$
$$E_A = 1870\angle -22.4° \text{ V}$$

Therefore, the maximum torque at a power factor of 0.8 leading is

$$\tau_{\text{ind,max}} = \frac{3 \text{ } V_\phi \text{ } E_A}{\omega_m \text{ } X_S} = \frac{3(1328 \text{ V})(1870 \text{ V})}{(3600 \text{ r/min})\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)(1.5 \text{ }\Omega)} = 13{,}180 \text{ N} \cdot \text{m}$$

5-21. Plot the V-curves (I_A versus I_F) for the synchronous motor of Problem 5-20 at no-load, half-load, and full-load conditions. (Note that an electronic version of the open-circuit characteristics in Figure P5-3 is available at the book's Web site. It may simplify the calculations required by this problem. Also, you may assume that R_A is negligible for this calculation.)

SOLUTION The input power at no-load, half-load and full-load conditions is given below. Note that we are assuming that R_A is negligible in each case.

$$P_{\text{IN,nl}} = 50 \text{ kW} + 40 \text{ kW} = 90 \text{ kW}$$

$$P_{\text{IN,half}} = (1000 \text{ hp})(746 \text{ W/hp}) + 50 \text{ kW} + 40 \text{ kW} = 836 \text{ kW}$$

$$P_{\text{IN,full}} = (2000 \text{ hp})(746 \text{ W/hp}) + 50 \text{ kW} + 40 \text{ kW} = 1582 \text{ kW}$$

If the power factor is adjusted to unity, then armature currents will be

$$I_{A,\text{nl}} = \frac{P}{\sqrt{3} \text{ } V_T \text{ PF}} = \frac{90 \text{ kW}}{\sqrt{3}(2300 \text{ V})(1.0)} = 22.6 \text{ A}$$

$$I_{A,\text{fl}} = \frac{P}{\sqrt{3} \text{ } V_T \text{ PF}} = \frac{836 \text{ kW}}{\sqrt{3}(2300 \text{ V})(1.0)} = 210 \text{ A}$$

$$I_{A,\text{fl}} = \frac{P}{\sqrt{3}\, V_T\, \text{PF}} = \frac{1582\,\text{kW}}{\sqrt{3}(2300\,\text{V})(1.0)} = 397\,\text{A}$$

The corresponding internal generated voltages at unity power factor are:

$$\mathbf{E}_A = \mathbf{V}_\phi - jX_S \mathbf{I}_A$$

$$\mathbf{E}_{A,\text{nl}} = 1328\angle 0°\,\text{V} - j(1.5\,\Omega)(22.6\angle 0°\,\text{A}) = 1328.4\angle -1.46°\,\text{V}$$

$$\mathbf{E}_{A,\text{half}} = 1328\angle 0°\,\text{V} - j(1.5\,\Omega)(210\angle 0°\,\text{A}) = 1365\angle -13.3°\,\text{V}$$

$$\mathbf{E}_{A,\text{full}} = 1328\angle 0°\,\text{V} - j(1.5\,\Omega)(397\angle 0°\,\text{A}) = 1455\angle -24.1°\,\text{V}$$

These values of E_A and δ at unity power factor can serve as reference points in calculating the synchronous motor V-curves. The MATLAB program to solve this problem is shown below:

```
% M-file: prob5_21.m
% M-file create a plot of armature current versus field
%   current for the synchronous motor of Problem 5-20 at
%   no-load, half-load, and full-load.

% First, initialize the field current values (21 values
% in the range 3.8-5.8 A)
If = 2.5:0.1:8;

% Get the OCC
load p53.occ;
if_values = p53(:,1);
vt_values = p53(:,2);

% Now initialize all other values
Xs = 1.5;                        % Synchronous reactance
Vp = 1328;                       % Phase voltage

% The following values of Ea and delta are for unity
% power factor.  They will serve as reference values
% when calculating the V-curves.
d_nl = -1.46 * pi/180;           % delta at no-load
d_half = -13.3 * pi/180;         % delta at half-load
d_full = -24.1 * pi/180;         % delta at full-load
Ea_nl = 1328.4;                  % Ea at no-load
Ea_half = 1365;                  % Ea at half-load
Ea_full = 1455;                  % Ea at full-load

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate the actual Ea corresponding to each level
% of field current
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Ea = interp1(if_values,vt_values,If) / sqrt(3);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate the armature currents associated with
% each value of Ea for the no-load case.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% First, calculate delta.
delta = asin ( Ea_nl ./ Ea .* sin(d_nl) );
```

```
% Calculate the phasor Ea
Ea2 = Ea .* (cos(delta) + j .* sin(delta));

% Now calculate Ia
Ia_nl = ( Vp - Ea2 ) / (j * Xs);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate the armature currents associated with
% each value of Ea for the half-load case.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% First, calculate delta.
delta = asin ( Ea_half ./ Ea .* sin(d_half) );

% Calculate the phasor Ea
Ea2 = Ea .* (cos(delta) + j .* sin(delta));

% Now calculate Ia
Ia_half = ( Vp - Ea2 ) / (j * Xs);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculate the armature currents associated with
% each value of Ea for the full-load case.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% First, calculate delta.
delta = asin ( Ea_full ./ Ea .* sin(d_full) );

% Calculate the phasor Ea
Ea2 = Ea .* (cos(delta) + j .* sin(delta));

% Now calculate Ia
Ia_full = ( Vp - Ea2 ) / (j * Xs);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Plot the v-curves
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
plot(If,abs(Ia_nl),'k-','Linewidth',2.0);
hold on;
plot(If,abs(Ia_half),'b--','Linewidth',2.0);
plot(If,abs(Ia_full),'r:','Linewidth',2.0);
xlabel('\bfField Current (A)');
ylabel('\bfArmature Current (A)');
title ('\bfSynchronous Motor V-Curve');
grid on;
```

The resulting plot is shown below. The flattening visible to the right of the V-curves is due to magnetic saturation in the machine.

Synchronous Motor V-Curve
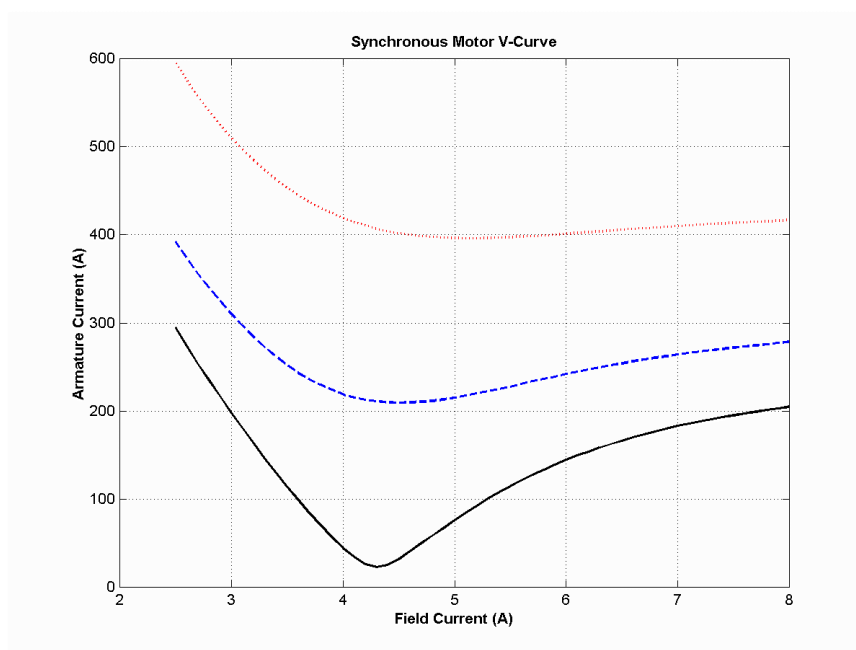
5-22. If a 60-Hz synchronous motor is to be operated at 50 Hz, will its synchronous reactance be the same as at 60 Hz, or will it change? *(Hint:* Think about the derivation of X_S.)

SOLUTION The synchronous reactance represents the effects of the armature reaction voltage $\mathbf{E}_{stat}$ and the armature self-inductance. The armature reaction voltage is caused by the armature magnetic field $\mathbf{B}_S$, and the amount of voltage is directly proportional to the speed with which the magnetic field sweeps over the stator surface. The higher the frequency, the faster $\mathbf{B}_S$ sweeps over the stator, and the higher the armature reaction voltage $\mathbf{E}_{stat}$ is. Therefore, the armature reaction voltage is directly proportional to frequency. Similarly, the reactance of the armature self-inductance is directly proportional to frequency, so the total synchronous reactance X_S is directly proportional to frequency. If the frequency is changed from 60 Hz to 50 Hz, the synchronous reactance will be decreased by a factor of 5/6.

5-23. A 480-V 100-kW 0.85-PF leading 50-Hz six-pole Y-connected synchronous motor has a synchronous reactance of 1.5 Ω and a negligible armature resistance. The rotational losses are also to be ignored. This motor is to be operated over a continuous range of speeds from 300 to 1000 r/min, where the speed changes are to be accomplished by controlling the system frequency with a solid-state drive.

(a) Over what range must the input frequency be varied to provide this speed control range?

(b) How large is E_A at the motor's rated conditions?

(c) What is the maximum power the motor can produce at the rated conditions?

(d) What is the largest E_A could be at 300 r/min?

(e) Assuming that the applied voltage V_ϕ is derated by the same amount as E_A, what is the maximum power the motor could supply at 300 r/min?

(f) How does the power capability of a synchronous motor relate to its speed?

SOLUTION

(a) A speed of 300 r/min corresponds to a frequency of

$$f_e = \frac{n_m P}{120} = \frac{(300 \text{ r/min})(6)}{120} = 15 \text{ Hz}$$

A speed of 1000 r/min corresponds to a frequency of

$$f_e = \frac{n_m P}{120} = \frac{(1000 \text{ r/min})(6)}{120} = 50 \text{ Hz}$$

The frequency must be controlled in the range 15 to 50 Hz.

(b) The armature current at rated conditions is

$$I_A = I_L = \frac{P}{\sqrt{3} \ V_T \ \text{PF}} = \frac{100 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.85)} = 141.5 \text{ A}$$

so $\mathbf{I}_A = 141.5\angle 31.8° \text{ A}$. This machine is Y-connected, so the phase voltage is $V_\phi = 480 / \sqrt{3} = 277 \text{ V}$. The internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi - R_A\mathbf{I}_A - jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 277\angle 0° \text{ V} - j(1.5 \ \Omega)(141.5\angle 31.8° \text{ A})$$
$$\mathbf{E}_A = 429\angle -24.9° \text{ V}$$

So $E_A = 429 \text{ V}$ at rated conditions.

(c) The maximum power that the motor can produce at rated speed with the value of E_A from part *(b)* is

$$P_{\max} = \frac{3V_\phi E_A}{X_S} = \frac{3(277 \text{ V})(429 \text{ V})}{1.5 \ \Omega} = 238 \text{ kW}$$

(d) Since E_A must be decreased linearly with frequency, the maximum value at 300 r/min would be

$$E_{A,300} = \left(\frac{15 \text{ Hz}}{50 \text{ Hz}}\right)(429 \text{ V}) = 129 \text{ V}$$

(e) If the applied voltage V_ϕ is derated by the same amount as E_A, then $V_\phi = (15/50)(277) = 83.1 \text{ V}$. Also, note that $X_S = (15/50)(1.5 \ \Omega) = 0.45 \ \Omega$. The maximum power that the motor could supply would be

$$P_{\max} = \frac{3V_\phi E_A}{X_S} = \frac{3(83.1 \text{ V})(129 \text{ V})}{0.45 \ \Omega} = 71.5 \text{ kW}$$

(f) As we can see by comparing the results of *(c)* and *(e)*, the power-handling capability of the synchronous motor varies *linearly* with the speed of the motor.

5-24. A 208-V Y-connected synchronous motor is drawing 50 A at unity power factor from a 208-V power system. The field current flowing under these conditions is 2.7 A. Its synchronous reactance is 0.8 Ω. Assume a linear open-circuit characteristic.

(a) Find the torque angle δ.

(b) How much field current would be required to make the motor operate at 0.78 PF leading?

(c) What is the new torque angle in part *(b)*?

SOLUTION

(a) The phase voltage of this motor is V_ϕ = 120 V, and the armature current is $\mathbf{I}_A = 50\angle 0°$ A. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi - R_A\mathbf{I}_A - jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 120\angle 0° \text{ V} - j(0.8 \ \Omega)(50\angle 0° \text{ A})$$
$$\mathbf{E}_A = 126.5\angle -18.4° \text{ V}$$

The torque angle δ of this machine is –18.4°.

(b) A phasor diagram of the motor operating at a power factor of 0.78 leading is shown below.
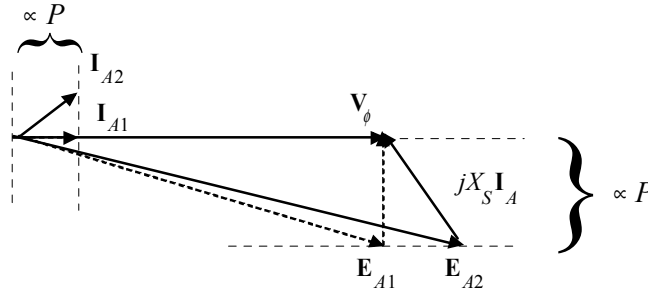
Since the power supplied by the motor is constant, the quantity $I_A \cos\theta$, which is directly proportional to power, must be constant. Therefore,

$$I_{A2}(0.78) = (50 \text{ A})(1.00)$$
$$\mathbf{I}_{A2} = 64.1\angle 38.7° \text{ A}$$

The internal generated voltage required to produce this current would be

$$\mathbf{E}_{A2} = \mathbf{V}_\phi - R_A\mathbf{I}_{A2} - jX_S\mathbf{I}_{A2}$$
$$\mathbf{E}_{A2} = 120\angle 0° \text{ V} - j(0.8 \ \Omega)(64.1\angle 38.7° \text{ A})$$
$$\mathbf{E}_{A2} = 157\angle -14.7° \text{ V}$$

The internal generated voltage E_A is directly proportional to the field flux, and we have assumed in this problem that the flux is directly proportional to the field current. Therefore, the required field current is

$$I_{F2} = \frac{E_{A2}}{E_{A1}} I_{F1} = \frac{157 \text{ V}}{126.5 \text{ V}}(2.7 \text{ A}) = 3.35 \text{ A}$$

(c) The new torque angle δ of this machine is –14.7°.

5-25. A synchronous machine has a synchronous reactance of 2.0 Ω per phase and an armature resistance of 0.4 Ω per phase. If $\mathbf{E}_A$ = 460∠-8° V and $\mathbf{V}_\phi$ = 480∠0° V, is this machine a motor or a generator? How much power P is this machine consuming from or supplying to the electrical system? How much reactive power Q is this machine consuming from or supplying to the electrical system?

SOLUTION This machine is a motor, *consuming* power from the power system, because $\mathbf{E}_A$ is lagging $\mathbf{V}_\phi$. It is also *consuming* reactive power, because $E_A \cos\delta < V_\phi$. The current flowing in this machine is

$$\mathbf{I}_A = \frac{\mathbf{V}_\phi - \mathbf{E}_A}{R_A + jX_S} = \frac{480\angle 0° \text{ V} - 460\angle -8° \text{ V}}{0.4 + j2.0 \ \Omega} = 33.6\angle -9.6° \text{ A}$$

Therefore the real power consumed by this motor is

$$P = 3V_\phi I_A \cos\theta = 3(480 \text{ V})(33.6 \text{ A})\cos 9.6° = 47.7 \text{ kW}$$

and the reactive power consumed by this motor is

$$Q = 3V_\phi I_A \sin\theta = 3(480 \text{ V})(33.6 \text{ A})\sin 9.6° = 8.07 \text{ kVAR}$$

5-26. Figure P5-4 shows a synchronous motor phasor diagram for a motor operating at a leading power factor with no R_A. For this motor, the torque angle is given by

$$\tan\delta = \frac{X_S I_A \cos\theta}{V_\phi + X_S I_A \sin\theta}$$

$$\delta = \tan^{-1}\left(\frac{X_S I_A \cos\theta}{V_\phi + X_S I_A \sin\theta}\right)$$

Derive an equation for the torque angle of the synchronous motor *if the armature resistance is included*.
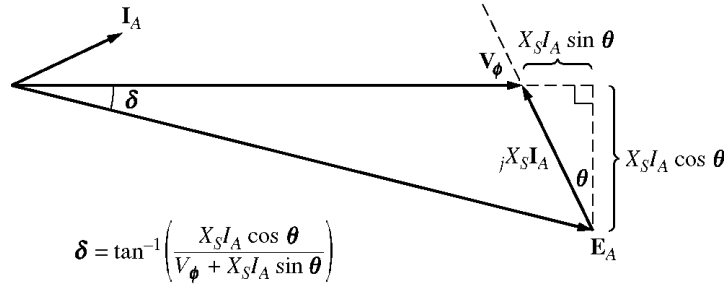
$$\delta = \tan^{-1}\left(\frac{X_S I_A \cos\theta}{V_\phi + X_S I_A \sin\theta}\right)$$

SOLUTION The phasor diagram with the armature resistance considered is shown below.
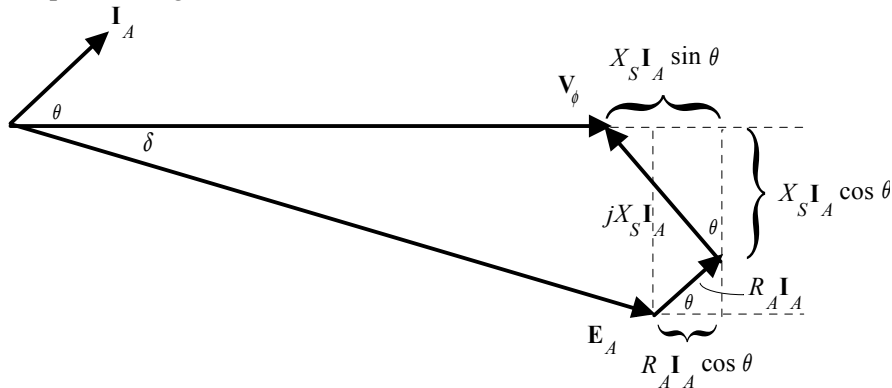
Therefore,

$$\tan\delta = \frac{X_S I_A \cos\theta + R_A I_A \sin\theta}{V_\phi + X_S I_A \sin\theta - R_A I_A \cos\theta}$$

$$\boxed{\delta = \tan^{-1}\left(\frac{X_S I_A \cos\theta + R_A I_A \sin\theta}{V_\phi + X_S I_A \sin\theta - R_A I_A \cos\theta}\right)}$$

5-27. A 2300-V 400-hp 60-Hz eight-pole Y-connected synchronous motor has a rated power factor of 0.85 leading. At full load, the efficiency is 85 percent. The armature resistance is 0.4 Ω, and the synchronous reactance is 4.4 Ω. Find the following quantities for this machine when it is operating at full load:

(a) Output torque

(b) Input power

(c) n_m

(d) $\mathbf{E}_A$

(e) $\left| \mathbf{I}_A \right|$

(f) P_{conv}

(g) $P_{\text{mech}} + P_{\text{core}} + P_{\text{stray}}$

SOLUTION

(a) Since this machine has 8 poles, it rotates at a speed of

$$n_m = \frac{120 f_e}{P} = \frac{120(60 \text{ Hz})}{8} = 900 \text{ r/min}$$

If the output power is 400 hp, the output torque is

$$\tau_{\text{load}} = \frac{P_{\text{out}}}{\omega_m} = \frac{(400 \text{ hp})(746 \text{ W/hp})}{(900 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 3166 \text{ N} \cdot \text{m}$$

(b) The input power is

$$P_{\text{IN}} = \frac{P_{\text{OUT}}}{\eta} = \frac{(400 \text{ hp})(746 \text{ W/hp})}{0.85} = 351 \text{ kW}$$

(c) The mechanical speed is

$$n_m = 900 \text{ r/min}$$

(d) The armature current is

$$I_A = I_L = \frac{P}{\sqrt{3} \, V_T \text{ PF}} = \frac{351 \text{ kW}}{\sqrt{3}(2300 \text{ V})(0.85)} = 104 \text{ A}$$

$$\mathbf{I}_A = 104\angle 31.8° \text{ A}$$

Therefore, $\mathbf{E}_A$ is

$$\mathbf{E}_A = \mathbf{V}_\phi - R_A \mathbf{I}_A - j X_S \mathbf{I}_A$$
$$\mathbf{E}_A = (1328\angle 0° \text{ V}) - (0.4 \text{ }\Omega)(104\angle 31.8° \text{ A}) - j(4.4 \text{ }\Omega)(104\angle 31.8° \text{ A})$$
$$\mathbf{E}_A = 1588\angle -15° \text{ V}$$

(e) The magnitude of the armature current is 104 A.

(f) The power converted from electrical to mechanical form is given by the equation $P_{\text{conv}} = P_{\text{IN}} - P_{\text{CU}}$

$$P_{\text{CU}} = 3 I_A^2 R_A = 3(104 \text{ A})^2 (0.4 \text{ }\Omega) = 13 \text{ kW}$$

$$P_{\text{conv}} = P_{\text{IN}} - P_{\text{CU}} = 351 \text{ kW} - 13 \text{ kW} = 338 \text{ kW}$$

(g) The mechanical, core, and stray losses are given by the equation

$$P_{mech} + P_{core} + P_{stray} = P_{conv} - P_{OUT} = 338 \text{ kW} - 298 \text{ kW} = 40 \text{ kW}$$

5-28. A 440-V three-phase Y-connected synchronous motor has a synchronous reactance of 1.5 Ω per phase. The field current has been adjusted so that the torque angle δ is 28° when the power supplied by the generator is 90 kW.

(a) What is the magnitude of the internal generated voltage $\mathbf{E}_A$ in this machine?

(b) What are the magnitude and angle of the armature current in the machine? What is the motor's power factor?

(c) If the field current remains constant, what is the absolute maximum power this motor could supply?

SOLUTION

(a) The power supplied to the motor is 90 kW. This power is give by the equation

$$P = \frac{3V_\phi E_A}{X_S} \sin \delta$$

so the magnitude of $\mathbf{E}_A$ is

$$E_A = \frac{X_S P}{3V_\phi \sin \delta} = \frac{(1.5 \text{ } \Omega)(90 \text{ kW})}{3(254 \text{ V}) \sin 28°} = 377 \text{ V}$$

(b) The armature current in this machine is given by

$$\mathbf{I}_A = \frac{\mathbf{V}_\phi - \mathbf{E}_A}{jX_S} = \frac{254\angle 0° \text{ V} - 377\angle -28°}{j1.5} = 129\angle 24° \text{ A}$$

The power factor of the motor is PF = cos 24° = 0.914 leading.

(c) The maximum power that the motor could supply at this field current

$$P_{max} = \frac{3V_\phi E_A}{X_S} = \frac{3(254 \text{ V})(377 \text{ V})}{1.5 \Omega} = 191.5 \text{ kW}$$

5-29. A 100-hp 440-V 0.8-PF-leading Δ-connected synchronous motor has an armature resistance of 0.22 Ω and a synchronous reactance of 3.0 Ω. Its efficiency at full load is 89 percent.

(a) What is the input power to the motor at rated conditions?

(b) What is the line current of the motor at rated conditions? What is the phase current of the motor at rated conditions?

(c) What is the reactive power consumed by or supplied by the motor at rated conditions?

(d) What is the internal generated voltage $\mathbf{E}_A$ of this motor at rated conditions?

(e) What are the stator copper losses in the motor at rated conditions?

(f) What is P_{conv} at rated conditions?

(g) If E_A is decreased by 10 percent, how much reactive power will be consumed by or supplied by the motor?

SOLUTION

(a) The input power to the motor at rated conditions is

$$P_{IN} = \frac{P_{OUT}}{\eta} = \frac{(100 \text{ hp})(746 \text{ W/hp})}{0.89} = 83.8 \text{ kW}$$

(b) The line current to the motor at rated conditions is

$$I_L = \frac{P}{\sqrt{3}\, V_T \text{ PF}} = \frac{83.8 \text{ kW}}{\sqrt{3}(440 \text{ V})(0.8)} = 137 \text{ A}$$

The phase current to the motor at rated conditions is

$$I_\phi = \frac{I_L}{\sqrt{3}} = \frac{137 \text{ A}}{\sqrt{3}} = 79.4 \text{ A}$$

(c) The reactive power supplied by this motor to the power system at rated conditions is

$$Q_{\text{rated}} = 3V_\phi I_A \sin\theta = 3(440 \text{ V})(79.4 \text{ A})\sin 36.87° = 62.9 \text{ kVAR}$$

(d) The internal generated voltage at rated conditions is

$$\mathbf{E}_A = \mathbf{V}_\phi - R_A\mathbf{I}_A - jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 440\angle 0° \text{ V} - (0.22\,\Omega)(79.4\angle 36.87° \text{ A}) - j(3.0\,\Omega)(79.4\angle 36.87° \text{ A})$$
$$\mathbf{E}_A = 603\angle -19.5° \text{ V}$$

(e) The stator copper losses at rated conditions are

$$P_{CU} = 3I_A^2 R_A = 3(79.4 \text{ A})^2(0.22\,\Omega) = 4.16 \text{ kW}$$

(f) P_{conv} at rated conditions is

$$P_{\text{conv}} = P_{IN} - P_{CU} = 83.8 \text{ kW} - 4.16 \text{ kW} = 79.6 \text{ kW}$$

(g) If E_A is decreased by 10%, the new value if $E_A = (0.9)(603 \text{ V}) = 543 \text{ V}$. To simplify this part of the problem, we will ignore R_A. Then

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_1\right) = \sin^{-1}\left(\frac{603 \text{ V}}{543 \text{ V}}\sin(-19.5°)\right) = -21.8°$$

Therefore,

$$\mathbf{I}_A = \frac{\mathbf{V}_\phi - \mathbf{E}_A}{jX_S} = \frac{440\angle 0° \text{ V} - 543\angle -21.8°}{j3.0} = 70.5\angle 17.7° \text{ A}$$

and the reactive power supplied by the motor to the power system will be

$$Q = 3V_\phi I_A \sin\theta = 3(440 \text{ V})(70.5 \text{ A})\sin 17.7° = 28.3 \text{ kVAR}$$

5-30. Answer the following questions about the machine of Problem 5-29.

(a) If $\mathbf{E}_A = 430\angle 13.5° \text{ V}$ and $\mathbf{V}_\phi = 440\angle 0° \text{ V}$, is this machine consuming real power from or supplying real power to the power system? Is it consuming reactive power from or supplying reactive power to the power system?

(b) Calculate the real power P and reactive power Q supplied or consumed by the machine under the conditions in part *(a)*. Is the machine operating within its ratings under these circumstances?

(c) If $\mathbf{E}_A = 470\angle{-12°}$ V and $\mathbf{V}_\phi = 440\angle{0°}$ V, is this machine consuming real power from or supplying real power to the power system? Is it consuming reactive power from or supplying reactive power to the power system?

(d) Calculate the real power P and reactive power Q supplied or consumed by the machine under the conditions in part *(c)*. Is the machine operating within its ratings under these circumstances?

SOLUTION

(a) This machine is a generator supplying real power to the power system, because $\mathbf{E}_A$ is ahead of $\mathbf{V}_\phi$. It is consuming reactive power because $E_A \cos\delta < V_\phi$.

(b) This machine is acting as a generator, and the current flow in these conditions is

$$\mathbf{I}_A = \frac{\mathbf{E}_A - \mathbf{V}_\phi}{R_A + jX_S} = \frac{430\angle 13.5° - 440\angle 0° \text{ V}}{0.22 + j3.0} = 34.2\angle 16.5° \text{ A}$$

The real power supplied by this machine is

$$P = 3V_\phi I_A \cos\theta = 3(440 \text{ V})(34.2 \text{ A})\cos(-16.5°) = 43.3 \text{ kW}$$

The reactive power supplied by this machine is

$$Q = 3V_\phi I_A \sin\theta = 3(440 \text{ V})(34.2 \text{ A})\sin(-16.5°) = -12.8 \text{ kVAR}$$

(c) This machine is a motor consuming real power from the power system, because $\mathbf{E}_A$ is behind $\mathbf{V}_\phi$. It is supplying reactive power because $E_A \cos\delta > V_\phi$.

(d) This machine is acting as a motor, and the current flow in these conditions is

$$\mathbf{I}_A = \frac{\mathbf{V}_\phi - \mathbf{E}_A}{R_A + jX_S} = \frac{440\angle 0° \text{ V} - 470\angle -12°}{0.22 + j3.0} = 33.1\angle 15.6° \text{ A}$$

The real power consumed by this machine is

$$P = 3V_\phi I_A \cos\theta = 3(440 \text{ V})(33.1 \text{ A})\cos(15.6°) = 42.1 \text{ kW}$$

The reactive power supplied by this machine is

$$Q = 3V_\phi I_A \sin\theta = 3(440 \text{ V})(33.1 \text{ A})\sin(15.6°) = +11.7 \text{ kVAR}$$

Chapter 6: *Parallel Operation of Synchronous Generators*

6-1. A 480-V 400-kVA 0.85-PF-lagging 50-Hz four-pole Δ-connected generator is driven by a 500-hp diesel engine and is used as a standby or emergency generator. This machine can also be paralleled with the normal power supply (a very large power system) if desired.

(a) What are the conditions required for paralleling the emergency generator with the existing power system? What is the generator's rate of shaft rotation after paralleling occurs?

(b) If the generator is connected to the power system and is initially floating on the line, sketch the resulting magnetic fields and phasor diagram.

(c) The governor setting on the diesel is now increased. Show both by means of house diagrams and by means of phasor diagrams what happens to the generator. How much reactive power does the generator supply now?

(d) With the diesel generator now supplying real power to the power system, what happens to the generator as its field current is increased and decreased? Show this behavior both with phasor diagrams and with house diagrams.

SOLUTION

(a)   To parallel this generator to the large power system, the required conditions are:

1. The generator must have the *same voltage* as the power system.

2. The *phase sequence* of the oncoming generator must be the same as the phase sequence of the power system.

3. The *frequency* of the oncoming generator should be *slightly* higher than the frequency of the running system.

4. The circuit breaker connecting the two systems together should be shut when the above conditions are met and the generator is *in phase* with the power system.

After paralleling, the generator's shaft will be rotating at

$$n_m = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{4} = 1500\text{ r/min}$$

(b)   The magnetic field and phasor diagrams immediately after paralleling are shown below:
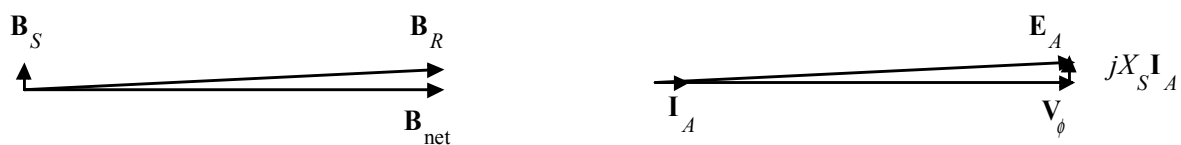
(c)   When the governor setpoints on the generator are increased, the emergency generator begins to supply more power to the loads, as shown below:
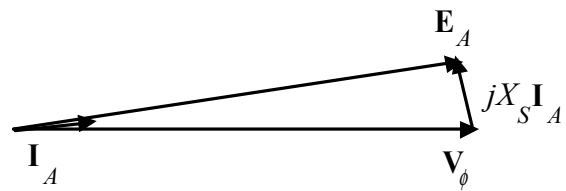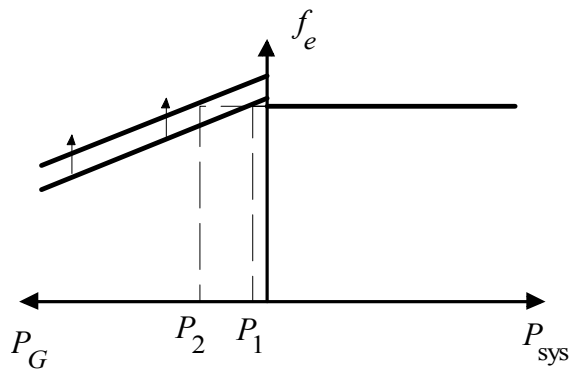
Note that as the load increased with $|\mathbf{E}_A|$ constant, the generator began to consume a small amount of reactive power.

(d) With the generator now supplying power to the system, an increase in field current increases the reactive power supplied to the loads, and a decrease in field current decreases the reactive power supplied to the loads.
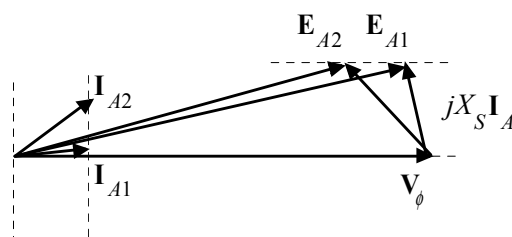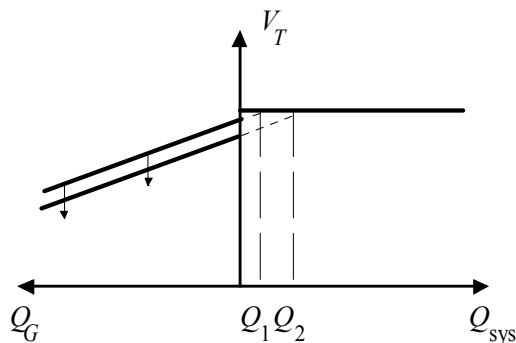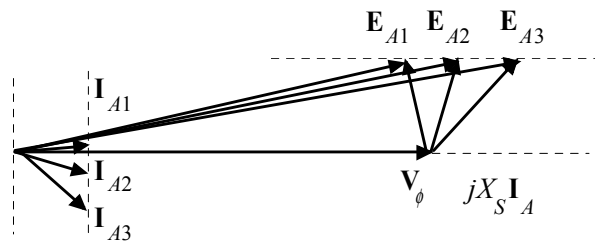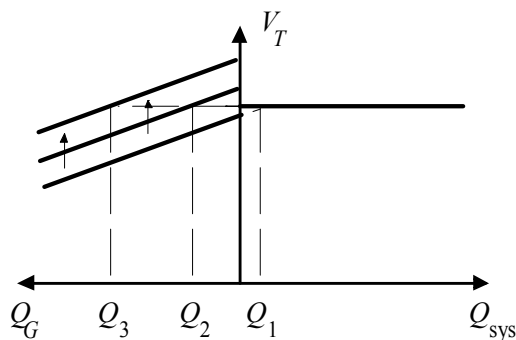
6-2. A 13.5-kV 20-MVA 0.8-PF-lagging 60-Hz two-pole Y-connected steam-turbine generator has a synchronous reactance of 5.0 Ω per phase and an armature resistance of 0.5 Ω per phase. This generator is operating in parallel with a large power system (infinite bus).

(a) What is the magnitude of E_A at rated conditions?

(b) What is the torque angle of the generator at rated conditions?

(c) If the field current is constant, what is the maximum power possible out of this generator? How much reserve power or torque does this generator have at full load?

(d) At the absolute maximum power possible, how much reactive power will this generator be supplying or consuming? Sketch the corresponding phasor diagram. (Assume I_F is still unchanged.)

(a) The phase voltage of this generator at rated conditions is

$$V_\phi = \frac{V_T}{\sqrt{3}} = \frac{13.5 \text{ kV}}{\sqrt{3}} = 7794 \text{ V}$$

The armature current per phase at rated conditions is

$$I_A = \frac{S}{\sqrt{3} \ V_T} = \frac{20,000,000 \text{ VA}}{\sqrt{3}(13,500 \text{ V})} = 855 \text{ A}$$

Therefore, the internal generated voltage at rated conditions is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 7794\angle 0° + (0.5 \ \Omega)(855\angle -36.87° \text{ A}) + j(5.0 \ \Omega)(855\angle -36.87° \text{ A})$$
$$\mathbf{E}_A = 11,160\angle 16.5° \text{ V}$$

The magnitude of E_A is 11,160 V.

(b) The torque angle of the generator at rated conditions is $\delta = 16.5°$.

(c) Ignoring R_A, the *maximum output power* of the generator is given by

$$P_{\text{MAX}} = \frac{3V_\phi E_A}{X_S} = \frac{3(7794 \text{ V})(11,160 \text{ V})}{5 \ \Omega} = 52.2 \text{ MW}$$

(d) The phasor diagram at these conditions is shown below:
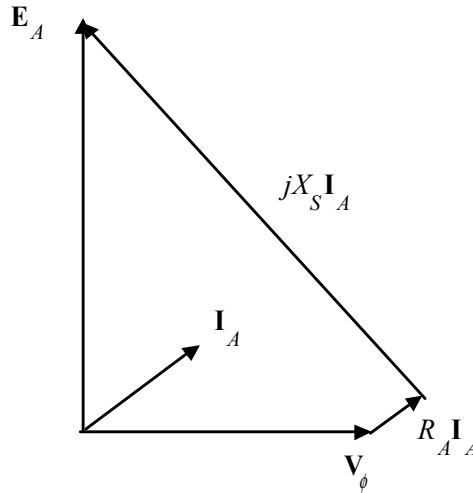
Under these conditions, the armature current is

$$\mathbf{I}_A = \frac{\mathbf{E}_A - \mathbf{V}_\phi}{R_A + jX_S} = \frac{11,650\angle 90° \text{ V} - 7794\angle 0° \text{ V}}{0.5 + j5.0 \ \Omega} = 2790\angle 39.5° \text{ A}$$

The reactive power produced by the generator is

$$Q = 3 \ V_\phi I_A \sin \theta = 3(7794 \text{ V})(2790 \text{ A}) \ \sin \ (0°\text{-}39.5°) = -41.5 \text{ MVAR}$$

The generator is actually consuming reactive power at this time.

6-3. A 480-V 200-kW two-pole three-phase 50-Hz synchronous generator's prime mover has a no-load speed of 3040 r/min and a full-load speed of 2975 r/min. It is operating in parallel with a 480-V 150-kW four-

pole 50-Hz synchronous generator whose prime mover has a no-load speed of 1500 r/min and a full-load speed of 1485 r/min. The loads supplied by the two generators consist of 200 kW at 0.85 PF lagging.

(a) Calculate the speed droops of generator 1 and generator 2.

(b) Find the operating frequency of the power system.

(c) Find the power being supplied by each of the generators in this system.

(d) If V_T is 460 V, what must the generator's operators do to correct for the low terminal voltage?

SOLUTION The no-load frequency of generator 1 corresponds to a frequency of

$$f_{nl1} = \frac{n_m P}{120} = \frac{(3040 \text{ r/min})(2)}{120} = 50.67 \text{ Hz}$$

The full-load frequency of generator 1 corresponds to a frequency of

$$f_{fl1} = \frac{n_m P}{120} = \frac{(2975 \text{ r/min})(2)}{120} = 49.58 \text{ Hz}$$

The no-load frequency of generator 2 corresponds to a frequency of

$$f_{nl2} = \frac{n_m P}{120} = \frac{(1500 \text{ r/min})(4)}{120} = 50.00 \text{ Hz}$$

The full-load frequency of generator 2 corresponds to a frequency of

$$f_{fl2} = \frac{n_m P}{120} = \frac{(1485 \text{ r/min})(4)}{120} = 49.50 \text{ Hz}$$

(a) The speed droop of generator 1 is given by

$$SD_1 = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\% = \frac{3040 \text{ r/min} - 2975 \text{ r/min}}{2975 \text{ r/min}} \times 100\% = 2.18\%$$

The speed droop of generator 2 is given by

$$SD_2 = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\% = \frac{1500 \text{ r/min} - 1485 \text{ r/min}}{1485 \text{ r/min}} \times 100\% = 1.01\%$$

(b) The power supplied by generator 1 is given by

$$P_1 = s_{P1}\left(f_{nl1} - f_{sys}\right)$$

and the power supplied by generator 1 is given by

$$P_2 = s_{P2}\left(f_{nl2} - f_{sys}\right)$$

The power curve's slope for generator 1 is

$$s_{P1} = \frac{P}{f_{nl} - f_{fl}} = \frac{200 \text{ kW}}{50.67 \text{ Hz} - 49.58 \text{ Hz}} = 0.185 \text{ MW/Hz}$$

The power curve's slope for generator 1 is

$$s_{P2} = \frac{P}{f_{nl} - f_{fl}} = \frac{150 \text{ kW}}{50.00 \text{ Hz} - 49.50 \text{ Hz}} = 0.300 \text{ MW/Hz}$$

The no-load frequency of generator 1 is 50.67 Hz and the no-load frequency of generator 2 is 50 Hz. The total power that they must supply is 200 kW, so the system frequency can be found from the equations

$$P_{LOAD} = P_1 + P_2$$

$$P_{LOAD} = s_{P1}\left(f_{nl1} - f_{sys}\right) + s_{P2}\left(f_{nl2} - f_{sys}\right)$$

$$200\,\text{kW} = (0.185\,\text{MW/Hz})(50.67\,\text{Hz} - f_{sys}) + (0.300\,\text{MW/Hz})(50.00\,\text{Hz} - f_{sys})$$

$$200\,\text{kW} = 9374\,\text{kW} - (0.185\,\text{MW/Hz})f_{sys} + 15{,}000\,\text{kW} - (0.300\,\text{MW/Hz})f_{sys}$$

$$(0.485\,\text{MW/Hz})f_{sys} = 9374\,\text{kW} + 15{,}000\,\text{kW} - 200\,\text{kW}$$

$$f_{sys} = \frac{24{,}174\,\text{kW}}{0.485\,\text{MW/Hz}} = 49.843\,\text{Hz}$$

(c) The power supplied by generator 1 is

$$P_1 = s_{P1}\left(f_{nl1} - f_{sys}\right) = (0.185\,\text{MW/Hz})(50.67\,\text{Hz} - 49.843\,\text{Hz}) = 153\,\text{kW}$$

The power supplied by generator 2 is

$$P_2 = s_{P2}\left(f_{nl2} - f_{sys}\right) = (0.300\,\text{MW/Hz})(50.00\,\text{Hz} - 49.843\,\text{Hz}) = 47\,\text{kW}$$

(d) If the terminal voltage is 460 V, the operators of the generators must increase the field currents on both generators simultaneously. That action will increase the terminal voltages of the system without changing the power sharing between the generators.

6-4. Three physically identical synchronous generators are operating in parallel. They are all rated for a full load of 3 MW at 0.8 PF lagging. The no-load frequency of generator A is 61 Hz, and its speed droop is 3.4 percent. The no-load frequency of generator B is 61.5 Hz, and its speed droop is 3 percent. The no-load frequency of generator C is 60.5 Hz, and its speed droop is 2.6 percent.

(a) If a total load consisting of 7 MW is being supplied by this power system, what will the system frequency be and how will the power be shared among the three generators?

(b) Create a plot showing the power supplied by each generator as a function of the total power supplied to all loads (you may use MATLAB to create this plot). At what load does one of the generators exceed its ratings? Which generator exceeds its ratings first?

(c) Is this power sharing in *(a)* acceptable? Why or why not?

(d) What actions could an operator take to improve the real power sharing among these generators?

SOLUTION

(a) Speed droop is defined as

$$SD = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\% = \frac{f_{nl} - f_{fl}}{f_{fl}} \times 100\%$$

so $\qquad f_{fl} = \dfrac{f_{nl}}{\dfrac{SD}{100} + 1}$

Thus, the full-load frequencies of generators A, B, and C are

$$f_{\text{fl},A} = \frac{f_{\text{nl},A}}{\dfrac{SD_A}{100} + 1} = \frac{61\,\text{Hz}}{\dfrac{3.4}{100} + 1} = 59.0\,\text{Hz}$$

$$f_{\text{fl},B} = \frac{f_{\text{nl},B}}{\dfrac{SD_B}{100} + 1} = \frac{61.5\,\text{Hz}}{\dfrac{3.0}{100} + 1} = 59.71\,\text{Hz}$$

$$f_{\text{fl},C} = \frac{f_{\text{nl},C}}{\dfrac{SD_C}{100} + 1} = \frac{60.5\,\text{Hz}}{\dfrac{2.6}{100} + 1} = 58.97\,\text{Hz}$$

and the slopes of the power-frequency curves are:

$$S_{PA} = \frac{3\,\text{MW}}{2\,\text{Hz}} = 1.5\,\text{MW/Hz}$$

$$S_{PB} = \frac{3\,\text{MW}}{1.79\,\text{Hz}} = 1.676\,\text{MW/Hz}$$

$$S_{PC} = \frac{3\,\text{MW}}{1.53\,\text{Hz}} = 1.961\,\text{MW/Hz}$$

(a) The total load is 7 MW, so the system frequency is

$$P_{\text{LOAD}} = s_{PA}\left(f_{\text{nlA}} - f_{\text{sys}}\right) + s_{PB}\left(f_{\text{nlB}} - f_{\text{sys}}\right) + s_{PC}\left(f_{\text{nlC}} - f_{\text{sys}}\right)$$

$$7\,\text{MW} = (1.5)\left(61.0\text{-}f_{\text{sys}}\right) + (1.676)\left(61.5\text{-}f_{\text{sys}}\right) + (1.961)\left(60.5\text{-}f_{\text{sys}}\right)$$

$$7\,\text{MW} = 91.5 - 1.5 f_{\text{sys}} + 103.07 - 1.676 f_{\text{sys}} + 118.64 - 1.961 f_{\text{sys}}$$

$$5.137 f_{\text{sys}} = 306.2$$

$$f_{\text{sys}} = 59.61\,\text{Hz}$$

The power supplied by each generator will be

$$P_A = s_{PA}\left(f_{\text{nlA}} - f_{\text{sys}}\right) = (1.5\,\text{MW/Hz})(61.0\,\text{Hz} - 59.61\,\text{Hz}) = 2.09\,\text{MW}$$
$$P_B = s_{PB}\left(f_{\text{nlB}} - f_{\text{sys}}\right) = (1.676\,\text{MW/Hz})(61.5\,\text{Hz} - 59.61\,\text{Hz}) = 3.17\,\text{MW}$$
$$P_C = s_{PC}\left(f_{\text{nlC}} - f_{\text{sys}}\right) = (1.961\,\text{MW/Hz})(60.5\,\text{Hz} - 59.61\,\text{Hz}) = 1.74\,\text{MW}$$

(b) The equation in part *(a)* can be re-written slightly to express system frequency as a function of load.

$$P_{\text{LOAD}} = (1.5)\left(61.0 - f_{\text{sys}}\right) + (1.676)\left(61.5 - f_{\text{sys}}\right) + (1.961)\left(60.5 - f_{\text{sys}}\right)$$

$$P_{\text{LOAD}} = 91.5 - 1.5 f_{\text{sys}} + 103.07 - 1.676 f_{\text{sys}} + 118.64 - 1.961 f_{\text{sys}}$$

$$5.137 f_{\text{sys}} = 313.2 - P_{\text{LOAD}}$$

$$f_{\text{sys}} = \frac{313.2 - P_{\text{LOAD}}}{5.137}$$

A MATLAB program that uses this equation to determine the power sharing among the generators as a function of load is shown below:

```
% M-file: prob6_4b.m
% M-file to calculate and plot the power sharing among
```

```
% three generators as a function of load.

% Define values for this generator
fnlA = 61.0;                 % No-load freq of Gen A
fnlB = 61.5;                 % No-load freq of Gen B
fnlC = 60.5;                 % No-load freq of Gen C
spA = 1.5;                   % Slope of Gen A (MW/Hz)
spB = 1.676;                 % Slope of Gen B (MW/Hz)
spC = 1.961;                 % Slope of Gen C (MW/Hz)
Pload = 0:0.05:10;           % Load in MW

% Calculate the system frequency
fsys = (313.2 - Pload) ./ 5.137;

% Calculate the power of each generator
PA = spA .* ( fnlA - fsys);
PB = spB .* ( fnlB - fsys);
PC = spC .* ( fnlC - fsys);

% Plot the power sharing versus load
plot(Pload,PA,'b-','LineWidth',2.0);
hold on;
plot(Pload,PB,'k--','LineWidth',2.0);
plot(Pload,PC,'r-.','LineWidth',2.0);
plot([0 10],[3 3],'k','LineWidth',1.0);
plot([0 10],[0 0],'k:');
title ('\bfPower Sharing Versus Total Load');
xlabel ('\bfTotal Load (MW)');
ylabel ('\bfGenerator Power (MW)');
legend('Generator A','Generator B','Generator C','Power Limit');
grid on;
hold off;
```

The resulting plot is shown below:
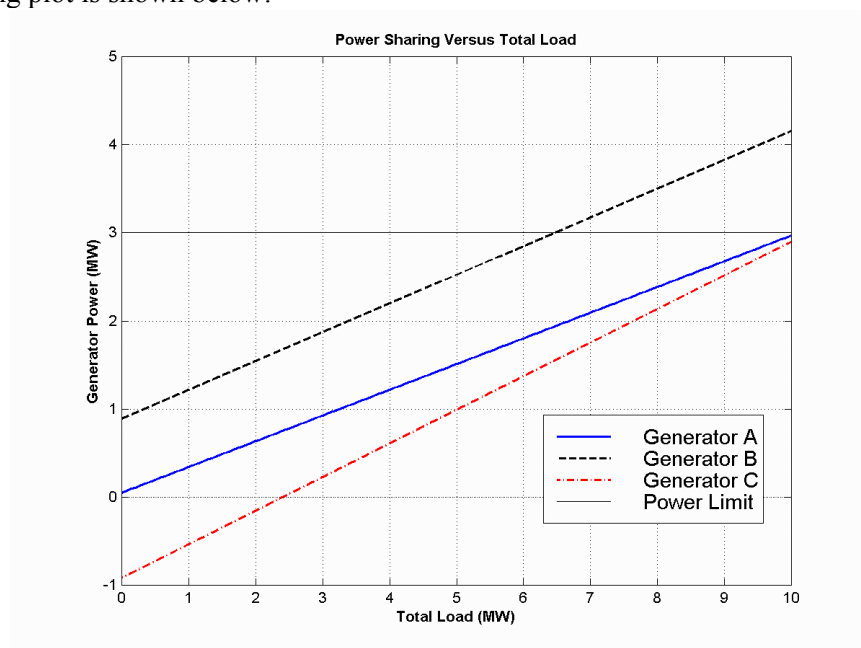
This plot reveals that there are power sharing problems both for high loads and for low loads. Generator B is the first to exceed its ratings as load increases. Its rated power is reached at a total load of 6.45 MW.

On the other hand, Generator C gets into trouble as the total load is reduced. When the total load drops to 2.4 MW, the direction of power flow reverses in Generator C.

(c)   The power sharing in (a) is not acceptable, because Generator 2 has exceeded its power limits.

(d)   To improve the power sharing among the three generators in (a) without affecting the operating frequency of the system, the operator should decrease the governor setpoints on Generator B while simultaneously increasing them in Generators A and C.

6-5.   A paper mill has installed three steam generators (boilers) to provide process steam and also to use some its waste products as an energy source. Since there is extra capacity, the mill has installed three 5-MW turbine generators to take advantage of the situation. Each generator is a 4160-V 6250-kVA 0.85-PF-lagging two-pole Y-connected synchronous generator with a synchronous reactance of 0.75 Ω and an armature resistance of 0.04 Ω. Generators 1 and 2 have a characteristic power-frequency slope s_P of 2.5 MW/Hz, and generator 3 has a slope of 3 MW/Hz.

(a) If the no-load frequency of each of the three generators is adjusted to 61 Hz, how much power will the three machines be supplying when actual system frequency is 60 Hz?

(b) What is the maximum power the three generators can supply in this condition without the ratings of one of them being exceeded? At what frequency does this limit occur? How much power does each generator supply at that point?

(c) What would have to be done to get all three generators to supply their rated real and reactive powers at an overall operating frequency of 60 Hz?

(d) What would the internal generated voltages of the three generators be under this condition?

SOLUTION

(a)   If the system frequency is 60 Hz and the no-load frequencies of the generators are 61 Hz, then the power supplied by the generators will be

$$P_1 = s_{P1}\left(f_{\text{nl1}} - f_{\text{sys}}\right) = (2.5 \text{ MW/Hz})(61 \text{ Hz - } 60 \text{ Hz}) = 2.5 \text{ MW}$$

$$P_2 = s_{P2}\left(f_{\text{nl2}} - f_{\text{sys}}\right) = (2.5 \text{ MW/Hz})(61 \text{ Hz - } 60 \text{ Hz}) = 2.5 \text{ MW}$$

$$P_3 = s_{P3}\left(f_{\text{nl3}} - f_{\text{sys}}\right) = (3.0 \text{ MW/Hz})(61 \text{ Hz - } 60 \text{ Hz}) = 3.0 \text{ MW}$$

Therefore the total power supplied by the generators is 8 MW.

(b)   The maximum power supplied by any one generator is (6250 kVA)(0.85) = 5.31 MW. Generator 3 will be the first machine to reach that limit. Generator 3 will supply this power at a frequency of

$$5.31 \text{ MW} = (3.0 \text{ MW/Hz})\left(61 \text{ Hz - } f_{\text{sys}}\right)$$

$$f_{\text{sys}} = 59.23 \text{ Hz}$$

At this point the power supplied by Generators 1 and 2 is

$$P_1 = P_2 = s_{P1}\left(f_{\text{nl1}} - f_{\text{sys}}\right) = (2.5 \text{ MW/Hz})(61 \text{ Hz - } 59.23 \text{ Hz}) = 4.425 \text{ MW}$$

The total power supplied by the generators at this condition is 14.16 MW.

(c)   To get each of the generators to supply 5.31 MW at 60 Hz, the no-load frequencies of Generator 1 and Generator 2 would have to be adjusted to 62.12 Hz, and the no-load frequency of Generator 3 would have to be adjusted to 61.77 Hz. The field currents of the three generators must then be adjusted to get them supplying a power factor of 0.85 lagging. At that point, each generator will be supplying its rated real and reactive power.

(d) Under the conditions of part *(c)*, which are the rated conditions of the generators, the internal generated voltage would be given by

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$

The phase voltage of the generators is 4160 V / $\sqrt{3}$ = 2402 V, and since the generators are Y-connected, their rated current is

$$I_A = I_L = \frac{S}{\sqrt{3}\, V_T} = \frac{6250\,\text{kVA}}{\sqrt{3}(4160\,\text{V})} = 867\,\text{A}$$

The power factor is 0.85 lagging, so $\mathbf{I}_A = 867\angle -31.8°\,\text{A}$. Therefore,

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 2402\angle 0° + (0.04\,\Omega)(867\angle -31.8°\,\text{A}) + j(0.75\,\Omega)(867\angle -31.8°\,\text{A})$$
$$\mathbf{E}_A = 2825\angle 10.9°\,\text{V}$$

6-6. Two identical 300-kVA 480-V synchronous generators are connected in parallel to supply a load. The prime movers of the two generators happen to have different speed droop characteristics. When the field currents of the two generators are equal, one delivers 200 A at 0.9 PF lagging, while the other delivers 180 A at 0.75 PF lagging.

(a) What are the real power and the reactive power supplied by each generator to the load?

(b) What is the overall power factor of the load?

(c) In what direction must the field current on each generator be adjusted in order for them to operate at the same power factor?

Solution

(a) The real and reactive powers are

$$P_1 = \sqrt{3}\ V_T\ I_L\ \cos\theta = \sqrt{3}(480\,\text{V})(200\,\text{A})(0.9) = 150\,\text{kW}$$

$$Q_1 = \sqrt{3}\ V_T\ I_L\ \sin\theta = \sqrt{3}(480\,\text{V})(200\,\text{A})\sin\left[\cos^{-1}(0.9)\right] = 72.50\,\text{kVAR}$$

$$P_2 = \sqrt{3}\ V_T\ I_L\ \cos\theta = \sqrt{3}(480\,\text{V})(180\,\text{A})(0.75) = 112\,\text{kW}$$

$$Q_2 = \sqrt{3}\ V_T\ I_L\ \sin\theta = \sqrt{3}(480\,\text{V})(180\,\text{A})\sin\left[\cos^{-1}(0.75)\right] = 99\,\text{kVAR}$$

(b) The overall power factor can be found from the total real and reactive power supplied to the load.

$$P_{\text{TOT}} = P_1 + P_2 = 150\,\text{kW} + 112\,\text{kW} = 262\,\text{kW}$$

$$Q_{\text{TOT}} = Q_1 + Q_2 = 72.5\,\text{kVAR} + 99\,\text{kVAR} = 171.5\,\text{kVAR}$$

The overall power factor is

$$\text{PF} = \cos\left[\tan^{-1}\frac{Q_{\text{TOT}}}{P_{\text{TOT}}}\right] = 0.837\,\text{lagging}$$

(c) The field current of generator 1 should be increased, and the field current of generator 2 should be simultaneously decreased.

6-7. A generating station for a power system consists of four 120-MVA 15-kV 0.85-PF-lagging synchronous generators with identical speed droop characteristics operating in parallel. The governors on the

generators' prime movers are adjusted to produce a 3-Hz drop from no load to full load. Three of these generators are each supplying a steady 75 MW at a frequency of 60 Hz, while the fourth generator (called the *swing generator*) handles all incremental load changes on the system while maintaining the system's frequency at 60 Hz.

(a) At a given instant, the total system loads are 260 MW at a frequency of 60 Hz. What are the no-load frequencies of each of the system's generators?

(b) If the system load rises to 290 MW and the generator's governor set points do not change, what will the new system frequency be?

(c) To what frequency must the no-load frequency of the swing generator be adjusted in order to restore the system frequency to 60 Hz?

(d) If the system is operating at the conditions described in part *(c)*, what would happen if the swing generator were tripped off the line (disconnected from the power line)?

SOLUTION

(a) The full-load power of these generators is $(120\,\text{MVA})(0.85) = 102\,\text{MW}$ and the droop from no-load to full-load is 3 Hz. Therefore, the slope of the power-frequency curve for these four generators is

$$s_P = \frac{102\,\text{MW}}{3\,\text{Hz}} = 34\,\text{MW/Hz}$$

If generators 1, 2, and 3 are supplying 75 MW each, then generator 4 must be supplying 35 MW. The no-load frequency of the first three generators is

$$P_1 = s_{P1}\left(f_{\text{nl1}} - f_{\text{sys}}\right)$$

$$75\,\text{MW} = (34\,\text{MW/Hz})\left(f_{\text{nl1}} - 60\,\text{Hz}\right)$$

$$f_{\text{nl1}} = 62.21\,\text{Hz}$$

The no-load frequency of the fourth generator is

$$P_4 = s_{P4}\left(f_{\text{nl4}} - f_{\text{sys}}\right)$$

$$35\,\text{MW} = (34\,\text{MW/Hz})\left(f_{\text{nl1}} - 60\,\text{Hz}\right)$$

$$f_{\text{nl1}} = 61.03\,\text{Hz}$$

(b) The setpoints of generators 1, 2, 3, and 4 do not change, so the new system frequency will be

$$P_{\text{LOAD}} = s_{P1}\left(f_{\text{nl1}} - f_{\text{sys}}\right) + s_{P2}\left(f_{\text{nl2}} - f_{\text{sys}}\right) + s_{P3}\left(f_{\text{nl3}} - f_{\text{sys}}\right) + s_{P4}\left(f_{\text{nl4}} - f_{\text{sys}}\right)$$

$$290\,\text{MW} = (34)\left(62.21 - f_{\text{sys}}\right) + (34)\left(62.21 - f_{\text{sys}}\right) + (34)\left(62.21 - f_{\text{sys}}\right) + (34)\left(61.03 - f_{\text{sys}}\right)$$

$$8.529 = 247.66 - 4 f_{\text{sys}}$$

$$f_{\text{sys}} = 59.78\,\text{Hz}$$

(c) The governor setpoints of the swing generator must be increased until the system frequency rises back to 60 Hz. At 60 Hz, the other three generators will be supplying 75 MW each, so the swing generator must supply 290 MW – 3(75 MW) = 65 MW at 60 Hz. Therefore, the swing generator's setpoints must be set to

$$P_4 = s_{P4}\left(f_{\text{nl4}} - f_{\text{sys}}\right)$$

$$65 \text{ MW} = \left(34 \text{ MW/Hz}\right)\left(f_{\text{nl1}} - 60 \text{ Hz}\right)$$

$$f_{\text{nl1}} = 61.91 \text{ Hz}$$

(d) If the swing generator trips off the line, the other three generators would have to supply all 290 MW of the load. Therefore, the system frequency will become

$$P_{\text{LOAD}} = s_{P1}\left(f_{\text{nl1}} - f_{\text{sys}}\right) + s_{P2}\left(f_{\text{nl2}} - f_{\text{sys}}\right) + s_{P3}\left(f_{\text{nl3}} - f_{\text{sys}}\right)$$

$$290 \text{ MW} = \left(34\right)\left(62.21 - f_{\text{sys}}\right) + \left(34\right)\left(62.21 - f_{\text{sys}}\right) + \left(34\right)\left(62.21 - f_{\text{sys}}\right)$$

$$8.529 = 186.63 - 3 f_{\text{sys}}$$

$$f_{\text{sys}} = 59.37 \text{ Hz}$$

Each generator will supply 96.7 MW to the loads.

6-8. Suppose that you were an engineer planning a new electric co-generation facility for a plant with excess process steam. You have a choice of either two 10 MW turbine-generators or a single 20 MW turbine generator. What would be the advantages and disadvantages of each choice?

SOLUTION A single 20 MW generator will probably be cheaper and more efficient than two 10 MW generators, but if the 20 MW generator goes down all 20 MW of generation would be lost at once. If two 10 MW generators are chosen, one of them could go down for maintenance and some power could still be generated.

Chapter 7: *Induction Motors*

7-1. A dc test is performed on a 460-V Δ-connected 100-hp induction motor. If V_{DC} = 21 V and I_{DC} = 72 A, what is the stator resistance R_1? *Why is this so?*

SOLUTION If this motor's armature is connected in delta, then there will be two phases in parallel with one phase between the lines tested.
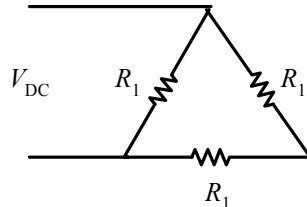
Therefore, the stator resistance R_1 will be

$$\frac{V_{DC}}{I_{DC}} = \frac{R_1(R_1 + R_1)}{R_1 + (R_1 + R_1)} = \frac{2}{3}R_1$$

$$R_1 = \frac{3}{2}\frac{V_{DC}}{I_{DC}} = \frac{3}{2}\left(\frac{21\text{ V}}{72\text{ A}}\right) = 0.438\,\Omega$$

7-2. A 220-V three-phase six-pole 50-Hz induction motor is running at a slip of 3.5 percent. Find:

(a) The speed of the magnetic fields in revolutions per minute

(b) The speed of the rotor in revolutions per minute

(c) The slip speed of the rotor

(d) The rotor frequency in hertz

SOLUTION

(a) The speed of the magnetic fields is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{6} = 1000\text{ r/min}$$

(b) The speed of the rotor is

$$n_m = (1-s)\,n_{\text{sync}} = (1-0.035)(1000\text{ r/min}) = 965\text{ r/min}$$

(c) The slip speed of the rotor is

$$n_{\text{slip}} = sn_{\text{sync}} = (0.035)(1000\text{ r/min}) = 35\text{ r/min}$$

(d) The rotor frequency is

$$f_r = \frac{n_{\text{slip}}P}{120} = \frac{(35\text{ r/min})(6)}{120} = 1.75\text{ Hz}$$

7-3. Answer the questions in Problem 7-2 for a 480-V three-phase four-pole 60-Hz induction motor running at a slip of 0.025.

SOLUTION

(a) The speed of the magnetic fields is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(60 \text{ Hz})}{4} = 1800 \text{ r/min}$$

(b)   The speed of the rotor is

$$n_m = (1-s) \, n_{sync} = (1-0.025)(1800 \text{ r/min}) = 1755 \text{ r/min}$$

(c)   The slip speed of the rotor is

$$n_{slip} = s n_{sync} = (0.025)(1800 \text{ r/min}) = 45 \text{ r/min}$$

(d)   The rotor frequency is

$$f_r = \frac{n_{slip}P}{120} = \frac{(45 \text{ r/min})(4)}{120} = 1.5 \text{ Hz}$$

7-4.   A three-phase 60-Hz induction motor runs at 715 r/min at no load and at 670 r/min at full load.

(a) How many poles does this motor have?

(b) What is the slip at rated load?

(c) What is the speed at one-quarter of the rated load?

(d) What is the rotor's electrical frequency at one-quarter of the rated load?

SOLUTION

(a)   This machine has 10 poles, which produces a synchronous speed of

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(60 \text{ Hz})}{10} = 720 \text{ r/min}$$

(b)   The slip at rated load is

$$s = \frac{n_{sync} - n_m}{n_{sync}} \times 100\% = \frac{720 - 670}{720} \times 100\% = 6.94\%$$

(c)   The motor is operating in the linear region of its torque-speed curve, so the slip at ¼ load will be

$$s = 0.25(0.0694) = 0.0174$$

The resulting speed is

$$n_m = (1-s) \, n_{sync} = (1-0.0174)(720 \text{ r/min}) = 707 \text{ r/min}$$

(d)   The electrical frequency at ¼ load is

$$f_r = s f_e = (0.0174)(60 \text{ Hz}) = 1.04 \text{ Hz}$$

7-5.   A 50-kW 440-V 50-Hz two-pole induction motor has a slip of 6 percent when operating at full-load conditions. At full-load conditions, the friction and windage losses are 520 W, and the core losses are 500 W. Find the following values for full-load conditions:

(a) The shaft speed n_m

(b) The output power in watts

(c) The load torque τ_{load} in newton-meters

(d) The induced torque τ_{ind} in newton-meters

(e) The rotor frequency in hertz

SOLUTION

(a) The synchronous speed of this machine is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50 \text{ Hz})}{2} = 3000 \text{ r/min}$$

Therefore, the shaft speed is

$$n_m = (1-s) \, n_{\text{sync}} = (1-0.06)(3000 \text{ r/min}) = 2820 \text{ r/min}$$

(b) The output power in watts is 50 kW (stated in the problem).

(c) The load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{50 \text{ kW}}{(2820 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 169.3 \text{ N} \cdot \text{m}$$

(d) The induced torque can be found as follows:

$$P_{\text{conv}} = P_{\text{OUT}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{misc}} = 50 \text{ kW} + 520 \text{ W} + 500 \text{ W} = 51.2 \text{ kW}$$

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{51.2 \text{ kW}}{(2820 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 173.4 \text{ N} \cdot \text{m}$$

(e) The rotor frequency is

$$f_r = sf_e = (0.06)(50 \text{ Hz}) = 3.00 \text{ Hz}$$

7-6. A three-phase 60-Hz two-pole induction motor runs at a no-load speed of 3580 r/min and a full-load speed of 3440 r/min. Calculate the slip and the electrical frequency of the rotor at no-load and full-load conditions. What is the speed regulation of this motor [Equation (4-57)]?

SOLUTION The synchronous speed of this machine is 3600 r/min. The slip and electrical frequency at no-load conditions is

$$s_{\text{nl}} = \frac{n_{\text{sync}} - n_{\text{nl}}}{n_{\text{sync}}} \times 100\% = \frac{3600 - 3580}{3600} \times 100\% = 0.56\%$$

$$f_{r,\text{nl}} = sf_e = (0.0056)(60 \text{ Hz}) = 0.33 \text{ Hz}$$

The slip and electrical frequency at full load conditions is

$$s_{\text{fl}} = \frac{n_{\text{sync}} - n_{\text{nl}}}{n_{\text{sync}}} \times 100\% = \frac{3600 - 3440}{3600} \times 100\% = 4.44\%$$

$$f_{r,\text{fl}} = sf_e = (0.0444)(60 \text{ Hz}) = 2.67 \text{ Hz}$$

The speed regulation is

$$\text{SR} = \frac{n_{\text{nl}} - n_{\text{fl}}}{n_{\text{fl}}} \times 100\% = \frac{3580 - 3440}{3440} \times 100\% = 4.1\%$$

7-7. A 208-V four-pole 60-Hz Y-connected wound-rotor induction motor is rated at 15 hp. Its equivalent circuit components are

$$R_1 = 0.220 \ \Omega \qquad\qquad R_2 = 0.127 \ \Omega \qquad\qquad X_M = 15.0 \ \Omega$$

$$X_1 = 0.430 \ \Omega \qquad\qquad X_2 = 0.430 \ \Omega$$

$$P_{mech} = 300 \ \text{W} \qquad\qquad P_{misc} \approx 0 \qquad\qquad P_{core} = 200 \ \text{W}$$

For a slip of 0.05, find

(a) The line current I_L

(b) The stator copper losses P_{SCL}

(c) The air-gap power P_{AG}

(d) The power converted from electrical to mechanical form P_{conv}

(e) The induced torque τ_{ind}

(f) The load torque τ_{load}

(g) The overall machine efficiency

(h) The motor speed in revolutions per minute and radians per second

SOLUTION The equivalent circuit of this induction motor is shown below:
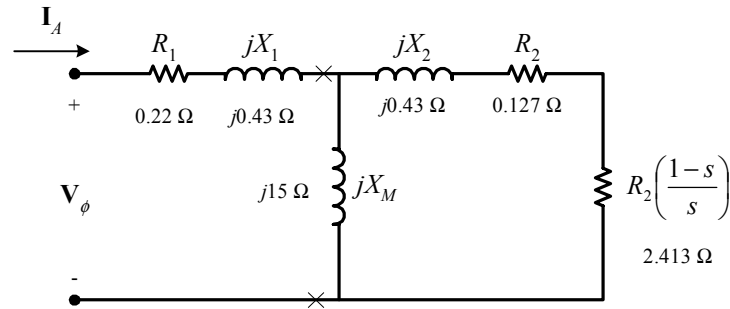
(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.

The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \cfrac{1}{\cfrac{1}{jX_M} + \cfrac{1}{Z_2}} = \cfrac{1}{\cfrac{1}{j15 \ \Omega} + \cfrac{1}{2.54 + j0.43}} = 2.337 + j0.803 = 2.47\angle19° \ \Omega$$

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The phase voltage is $208/\sqrt{3} = 120$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle0° \text{ V}}{0.22\,\Omega + j0.43\,\Omega + 2.337\,\Omega + j0.803\,\Omega}$$

$$I_L = I_A = 42.3\angle-25.7° \text{ A}$$

(b)   The stator copper losses are

$$P_{\text{SCL}} = 3I_A{}^2 R_1 = 3(42.3 \text{ A})^2(0.22\,\Omega) = 1180 \text{ W}$$

(c)   The air gap power is $P_{\text{AG}} = 3I_2{}^2 \dfrac{R_2}{s} = 3I_A{}^2 R_F$

(Note that $3I_A{}^2 R_F$ is equal to $3I_2{}^2 \dfrac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F.   The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{\text{AG}} = 3I_2{}^2 \frac{R_2}{s} = 3I_A{}^2 R_F = 3(42.3 \text{ A})^2(2.337\,\Omega) = 12.54 \text{ kW}$$

(d)   The power converted from electrical to mechanical form is

$$P_{\text{conv}} = (1-s)P_{\text{AG}} = (1-0.05)(12.54 \text{ kW}) = 11.92 \text{ kW}$$

(e)   The induced torque in the motor is

$$\tau_{\text{ind}} = \frac{P_{\text{AG}}}{\omega_{\text{sync}}} = \frac{12.54 \text{ kW}}{(1800 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 66.5 \text{ N} \cdot \text{m}$$

(f)   The output power of this motor is

$$P_{\text{OUT}} = P_{\text{conv}} - P_{\text{mech}} - P_{\text{core}} - P_{\text{misc}} = 11.92 \text{ kW} - 300 \text{ W} - 200 \text{ W} - 0 \text{ W} = 11.42 \text{ kW}$$

The output speed is

$$n_m = (1-s)n_{\text{sync}} = (1-0.05)(1800 \text{ r/min}) = 1710 \text{ r/min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{11.42 \text{ kW}}{(1710 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 63.8 \text{ N} \cdot \text{m}$$

(g)   The overall efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{P_{\text{OUT}}}{3V_\phi I_A \cos\theta} \times 100\%$$

$$\eta = \frac{11.42 \text{ kW}}{3(120 \text{ V})(42.3 \text{ A})\cos 25.7°} \times 100\% = 83.2\%$$

(h)   The motor speed in revolutions per minute is 1710 r/min. The motor speed in radians per second is

$$\omega_m = (1710 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 179 \text{ rad/s}$$

7-8. For the motor in Problem 7-7, what is the slip at the pullout torque? What is the pullout torque of this motor?

SOLUTION The slip at pullout torque is found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model.

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + jX_1 + jX_M} \mathbf{V}_\phi$$

$$V_{TH} = \frac{X_M}{\sqrt{R_1{}^2 + (X_1 + X_M)^2}} \mathbf{V}_\phi$$

(a)

$$Z_{TH} = \frac{jX_M (R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

(b)

(c)

$$Z_{TH} = \frac{jX_M (R_1 + jX_1)}{R_1 + j(X_1 + X_M)} = \frac{(j15 \ \Omega)(0.22 \ \Omega + j0.43 \ \Omega)}{0.22 \ \Omega + j(0.43 \ \Omega + 15 \ \Omega)} = 0.208 + j0.421 \ \Omega = 0.470\angle 63.7° \ \Omega$$

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + j(X_1 + X_M)} \mathbf{V}_\phi = \frac{(j15 \ \Omega)}{0.22 \ \Omega + j(0.43 \ \Omega + 15 \ \Omega)}(120\angle 0° \text{ V}) = 116.7\angle 0.8° \text{ V}$$

The slip at pullout torque is

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}{}^2 + (X_{TH} + X_2)^2}}$$

$$s_{max} = \frac{0.127 \ \Omega}{\sqrt{(0.208 \ \Omega)^2 + (0.421 \ \Omega + 0.430 \ \Omega)^2}} = 0.145$$

The pullout torque of the motor is

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$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}\right]}$$

$$\tau_{max} = \frac{3\left(116.7\ \text{V}\right)^2}{2\left(188.5\ \text{rad/s}\right)\left[0.208\ \Omega + \sqrt{\left(0.208\ \Omega\right)^2 + \left(0.421\ \Omega\ + 0.430\ \Omega\right)^2}\right]}$$

$$\tau_{max} = 100\ \text{N}\cdot\text{m}$$

7-9. *(a)* Calculate and plot the torque-speed characteristic of the motor in Problem 7-7. *(b)* Calculate and plot the output power versus speed curve of the motor in Problem 7-7.

SOLUTION

(a) A MATLAB program to calculate the torque-speed characteristic is shown below.

```
% M-file: prob7_9a.m
% M-file create a plot of the torque-speed curve of the
%    induction motor of Problem 7-7.

% First, initialize the values needed in this program.
r1 = 0.220;                % Stator resistance
x1 = 0.430;                % Stator reactance
r2 = 0.127;                % Rotor resistance
x2 = 0.430;                % Rotor reactance
xm = 15.0;                 % Magnetization branch reactance
v_phase = 208 / sqrt(3);   % Phase voltage
n_sync = 1800;             % Synchronous speed (r/min)
w_sync = 188.5;            % Synchronous speed (rad/s)

% Calculate the Thevenin voltage and impedance from Equations
% 7-38 and 7-41.
v_th = v_phase * ( xm / sqrt(r1^2 + (x1 + xm)^2) );
z_th = ((j*xm) * (r1 + j*x1)) / (r1 + j*(x1 + xm));
r_th = real(z_th);
x_th = imag(z_th);

% Now calculate the torque-speed characteristic for many
% slips between 0 and 1.  Note that the first slip value
% is set to 0.001 instead of exactly 0 to avoid divide-
% by-zero problems.
s = (0:1:50) / 50;         % Slip
s(1) = 0.001;
nm = (1 - s) * n_sync;     % Mechanical speed

% Calculate torque versus speed
for ii = 1:51
   t_ind(ii) = (3 * v_th^2 * r2 / s(ii)) / ...
           (w_sync * ((r_th + r2/s(ii))^2 + (x_th + x2)^2) );
end

% Plot the torque-speed curve
figure(1);
plot(nm,t_ind,'k-','LineWidth',2.0);
xlabel('\bf\itn_{m}');
ylabel('\bf\tau_{ind}');
```

```
title ('\bfInduction Motor Torque-Speed Characteristic');
grid on;
```

The resulting plot is shown below:
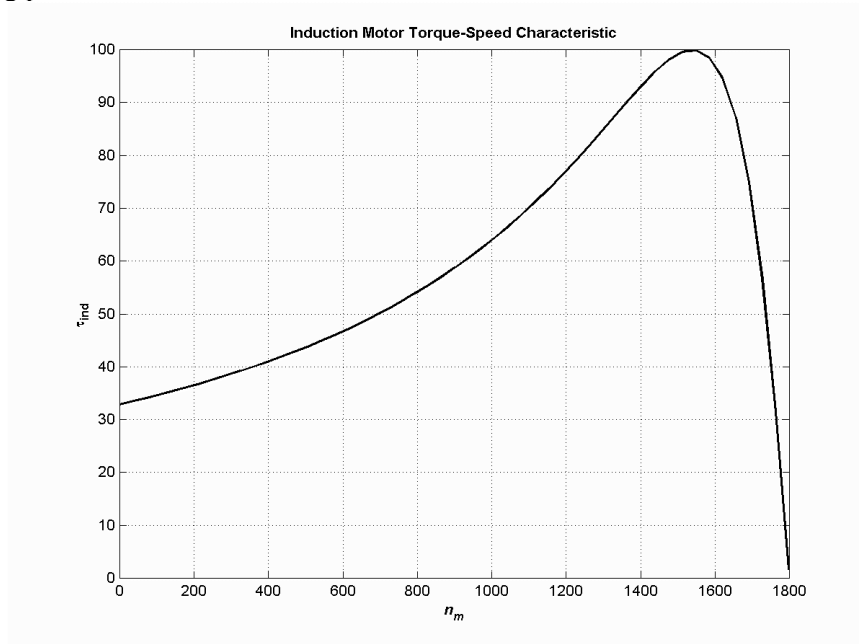
(b)   A MATLAB program to calculate the output-power versus speed curve is shown below.

```
% M-file: prob7_9b.m
% M-file create a plot of the output pwer versus speed
%    curve of the induction motor of Problem 7-7.

% First, initialize the values needed in this program.
r1 = 0.220;                 % Stator resistance
x1 = 0.430;                 % Stator reactance
r2 = 0.127;                 % Rotor resistance
x2 = 0.430;                 % Rotor reactance
xm = 15.0;                  % Magnetization branch reactance
v_phase = 208 / sqrt(3);    % Phase voltage
n_sync = 1800;              % Synchronous speed (r/min)
w_sync = 188.5;             % Synchronous speed (rad/s)

% Calculate the Thevenin voltage and impedance from Equations
% 7-38 and 7-41.
v_th = v_phase * ( xm / sqrt(r1^2 + (x1 + xm)^2) );
z_th = ((j*xm) * (r1 + j*x1)) / (r1 + j*(x1 + xm));
r_th = real(z_th);
x_th = imag(z_th);

% Now calculate the torque-speed characteristic for many
% slips between 0 and 1.  Note that the first slip value
% is set to 0.001 instead of exactly 0 to avoid divide-
% by-zero problems.
s = (0:1:50) / 50;          % Slip
s(1) = 0.001;
nm = (1 - s) * n_sync;      % Mechanical speed (r/min)
wm = (1 - s) * w_sync;      % Mechanical speed (rad/s)
```

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```
% Calculate torque and output power versus speed
for ii = 1:51
    t_ind(ii) = (3 * v_th^2 * r2 / s(ii)) / ...
            (w_sync * ((r_th + r2/s(ii))^2 + (x_th + x2)^2) );
    p_out(ii) = t_ind(ii) * wm(ii);
end

% Plot the torque-speed curve
figure(1);
plot(nm,p_out/1000,'k-','LineWidth',2.0);
xlabel('\bf\itn_{m}   \rm\bf(r/min)');
ylabel('\bf\itP_{OUT}   \rm\bf(kW)');
title ('\bfInduction Motor Ouput Power versus Speed');
grid on;
```

The resulting plot is shown below:
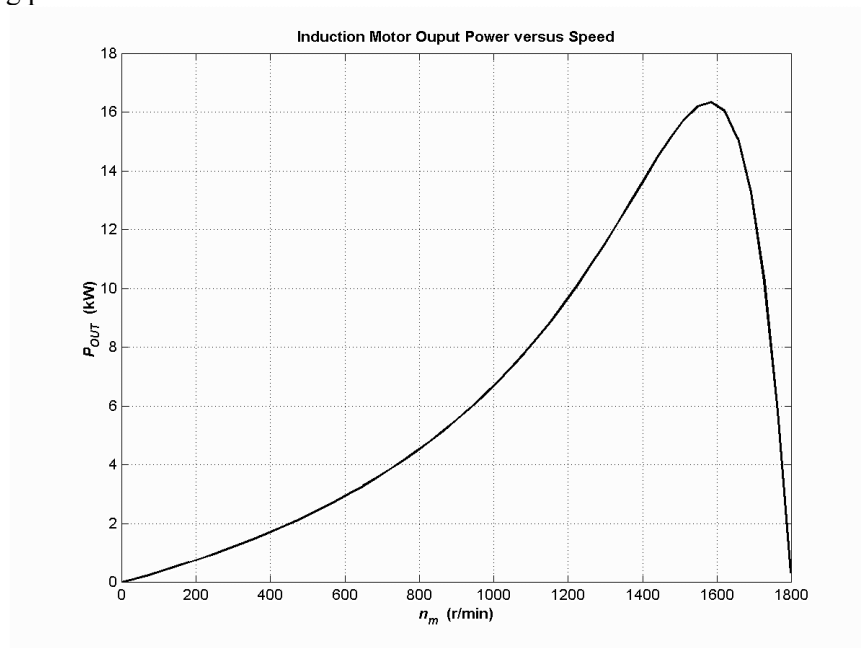
7-10. For the motor of Problem 7-7, how much additional resistance (referred to the stator circuit) would it be necessary to add to the rotor circuit to make the maximum torque occur at starting conditions (when the shaft is not moving)? Plot the torque-speed characteristic of this motor with the additional resistance inserted.

SOLUTION To get the maximum torque at starting, the s_{max} must be 1.00. Therefore,

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$1.00 = \frac{R_2}{\sqrt{(0.208\ \Omega)^2 + (0.421\ \Omega + 0.430\ \Omega)^2}}$$

$$R_2 = 0.876\ \Omega$$

Therefore, an additional 0.749 Ω must be added to the rotor circuit. The resulting torque-speed characteristic is:

Induction Motor Torque-Speed Characteristic

With extra rotor resistance
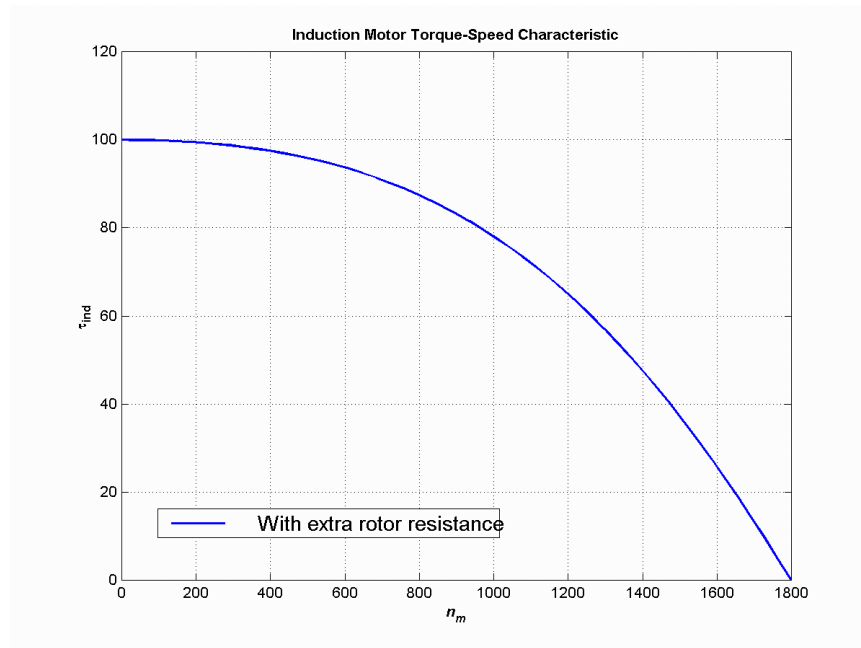
7-11. If the motor in Problem 7-7 is to be operated on a 50-Hz power system, what must be done to its supply voltage? Why? What will the equivalent circuit component values be at 50 Hz? Answer the questions in Problem 7-7 for operation at 50 Hz with a slip of 0.05 and the proper voltage for this machine.

SOLUTION If the input frequency is decreased to 50 Hz, then the applied voltage must be decreased by 5/6 also. If this were not done, the flux in the motor would go into saturation, since

$$\phi = \frac{1}{N}\int_T v\ dt$$

and the period T would be increased. At 50 Hz, the resistances will be unchanged, but the reactances will be reduced to 5/6 of their previous values. The equivalent circuit of the induction motor at 50 Hz is shown below:
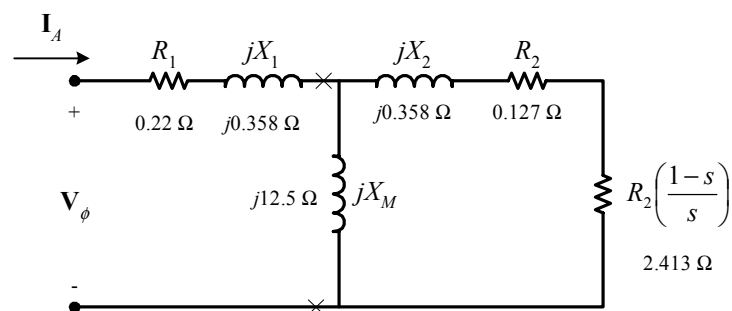
(a)   The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.

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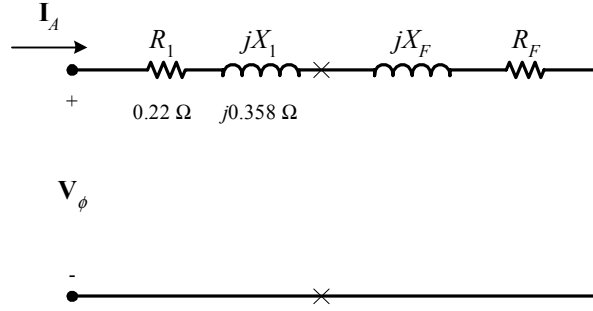
The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\dfrac{1}{jX_M} + \dfrac{1}{Z_2}} = \frac{1}{\dfrac{1}{j12.5\,\Omega} + \dfrac{1}{2.54 + j0.358}} = 2.310 + j0.804 = 2.45\angle 19.2°\,\Omega$$

The line voltage must be derated by 5/6, so the new line voltage is $V_T = 173.3\,\text{V}$. The phase voltage is $173.3/\sqrt{3} = 100\,\text{V}$, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{100\angle 0°\,\text{V}}{0.22\,\Omega + j0.358\,\Omega + 2.310\,\Omega + j0.804\,\Omega}$$

$$I_L = I_A = 35.9\angle -24.7°\,\text{A}$$

(b) The stator copper losses are

$$P_{SCL} = 3I_A^2 R_1 = 3(35.9\,\text{A})^2(0.22\,\Omega) = 851\,\text{W}$$

(c) The air gap power is $P_{AG} = 3I_2^2\,\dfrac{R_2}{s} = 3I_A^2 R_F$

(Note that $3I_A^2 R_F$ is equal to $3I_2^2\,\dfrac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{AG} = 3I_2^2\,\frac{R_2}{s} = 3I_A^2 R_F = 3(35.9\,\text{A})^2(2.310\,\Omega) = 8.93\,\text{kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{conv} = (1-s)P_{AG} = (1-0.05)(8.93\,\text{kW}) = 8.48\,\text{kW}$$

(e) The induced torque in the motor is

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{8.48\,\text{kW}}{(1500\,\text{r/min})\left(\dfrac{2\pi\,\text{rad}}{1\,\text{r}}\right)\left(\dfrac{1\,\text{min}}{60\,\text{s}}\right)} = 54.0\,\text{N}\cdot\text{m}$$

(f) In the absence of better information, we will treat the mechanical and core losses as constant despite the change in speed. This is not true, but we don't have reason for a better guess. Therefore, the output power of this motor is

$$P_{OUT} = P_{conv} - P_{mech} - P_{core} - P_{misc} = 8.48\,\text{kW} - 300\,\text{W} - 200\,\text{W} - 0\,\text{W} = 7.98\,\text{kW}$$

The output speed is

$$n_m = (1-s)\, n_{\text{sync}} = (1-0.05)(1500 \text{ r/min}) = 1425 \text{ r/min}$$

Therefore the load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{7.98 \text{ kW}}{(1425 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 53.5 \text{ N} \cdot \text{m}$$

(g)   The overall efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{P_{\text{OUT}}}{3 V_\phi I_A \cos\theta} \times 100\%$$

$$\eta = \frac{7.98 \text{ kW}}{3(100 \text{ V})(35.9 \text{ A})\cos 24.7°} \times 100\% = 81.6\%$$

(h)   The motor speed in revolutions per minute is 1425 r/min. The motor speed in radians per second is

$$\omega_m = (1425 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 149.2 \text{ rad/s}$$

7-12.   Figure 7-16*a* shows a simple circuit consisting of a voltage source, a resistor, and two reactances. Find the Thevenin equivalent voltage and impedance of this circuit at the terminals.   Then derive the expressions for the magnitude of $\mathbf{V}_{\text{TH}}$ and for R_{TH} given in Equations (7-38) and (7-42).

(a)
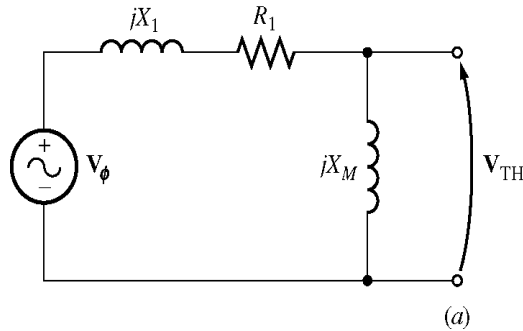
SOLUTION The Thevenin voltage of this circuit is

$$\mathbf{V}_{\text{TH}} = \frac{jX_M}{R_1 + j(X_1 + X_M)}\mathbf{V}_\phi$$

The *magnitude* of this voltage is

$$V_{\text{TH}} = \frac{X_M}{\sqrt{R_1^2 + (X_1 + X_M)^2}}V_\phi$$

If $X_M \gg X_1$, then $R_1^2 + (X_1 + X_M)^2 \approx (X_1 + X_M)^2$, so

$$\boxed{V_{\text{TH}} \approx \frac{X_M}{X_1 + X_M}V_\phi}$$

The Thevenin impedance of this circuit is

$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)}$$

$$Z_{TH} = \frac{jX_M(R_1 + jX_1)[R_1 - j(X_1 + X_M)]}{[R_1 + j(X_1 + X_M)][R_1 - j(X_1 + X_M)]}$$

$$Z_{TH} = \frac{\left[-R_1X_1X_M + R_1X_1X_M + R_1X_M^2\right] + j\left[R_1^2X_M + X_1^2X_M + X_1X_M^2\right]}{R_1^2 + (X_1 + X_M)^2}$$

$$Z_{TH} = R_{TH} + jX_{TH} = \frac{R_1X_M^2}{R_1^2 + (X_1 + X_M)^2} + j\frac{R_1^2X_M + X_1^2X_M + X_1X_M^2}{R_1^2 + (X_1 + X_M)^2}$$

The Thevenin resistance is $R_{TH} = \dfrac{R_1X_M^2}{R_1^2 + (X_1 + X_M)^2}$. If $X_M \gg R_1$, then

$R_1^2 + (X_1 + X_M)^2 \approx (X_1 + X_M)^2$, so

$$\boxed{R_{TH} \approx R_1\left(\frac{X_M}{X_1 + X_M}\right)^2}$$

The Thevenin reactance is $X_{TH} = \dfrac{R_1^2X_M + X_1^2X_M + X_1X_M^2}{R_1^2 + (X_1 + X_M)^2}$.

If $X_M \gg R_1$ and $X_M \gg X_1$ then $X_1X_M^2 \gg R_1^2X_M + X_1^2X_M$ and $(X_1 + X_M)^2 \approx X_M^2 \gg R_1^2$, so

$$\boxed{X_{TH} \approx \frac{X_1X_M^2}{X_M^2} = X_1}$$

7-13. Figure P7-1 shows a simple circuit consisting of a voltage source, two resistors, and two reactances in series with each other. If the resistor R_L is allowed to vary but all the other components are constant, at what value of R_L will the maximum possible power be supplied to it? *Prove* your answer. (*Hint:* Derive an expression for load power in terms of V, R_S, X_S, R_L and X_L and take the partial derivative of that expression with respect to R_L.) Use this result to derive the expression for the pullout torque [Equation (7-52)].
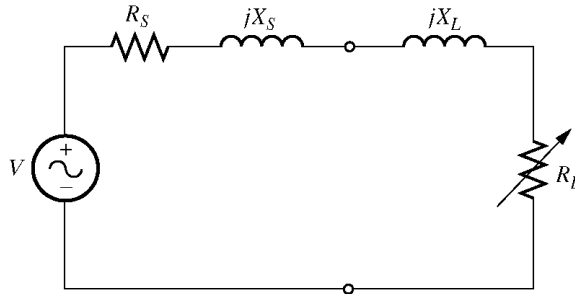
SOLUTION The current flowing in this circuit is given by the equation

$$\mathbf{I}_L = \frac{\mathbf{V}}{R_S + jX_S + R_L + jX_L}$$

$$I_L = \frac{V}{\sqrt{(R_S + R_L)^2 + (X_S + X_L)^2}}$$

The power supplied to the load is

$$P = I_L^2 R_L = \frac{V^2 R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{\left[(R_S + R_L)^2 + (X_S + X_L)^2\right] V^2 - V^2 R_L \left[2(R_S + R_L)\right]}{\left[(R_S + R_L)^2 + (X_S + X_L)^2\right]^2}$$

To find the point of maximum power supplied to the load, set $\partial P / \partial R_L = 0$ and solve for R_L.

$$\left[(R_S + R_L)^2 + (X_S + X_L)^2\right] V^2 - V^2 R_L \left[2(R_S + R_L)\right] = 0$$

$$\left[(R_S + R_L)^2 + (X_S + X_L)^2\right] = 2R_L (R_S + R_L)$$

$$R_S^2 + 2R_S R_L + R_L^2 + (X_S + X_L)^2 = 2R_S R_L + 2R_L^2$$

$$R_S^2 + R_L^2 + (X_S + X_L)^2 = 2R_L^2$$

$$R_S^2 + (X_S + X_L)^2 = R_L^2$$

Therefore, for maximum power transfer, the load resistor should be

$$\boxed{R_L = \sqrt{R_S^2 + (X_S + X_L)^2}}$$

7-14. A 440-V 50-Hz six-pole Y-connected induction motor is rated at 75 kW. The equivalent circuit parameters are

$R_1 = 0.082 \ \Omega$	$R_2 = 0.070 \ \Omega$	$X_M = 7.2 \ \Omega$
$X_1 = 0.19 \ \Omega$	$X_2 = 0.18 \ \Omega$	
$P_{F\&W} = 1.3$ kW	$P_{misc} = 150$ W	$P_{core} = 1.4$ kW

For a slip of 0.04, find
(a) The line current I_L

(b) The stator power factor

(c) The rotor power factor

(d) The stator copper losses P_{SCL}

(e) The air-gap power P_{AG}

(f) The power converted from electrical to mechanical form P_{conv}

(g) The induced torque τ_{ind}

(h) The load torque τ_{load}

(i) The overall machine efficiency η

(j) The motor speed in revolutions per minute and radians per second

SOLUTION The equivalent circuit of this induction motor is shown below:

(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.

The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\dfrac{1}{jX_M} + \dfrac{1}{Z_2}} = \frac{1}{\dfrac{1}{j7.2\,\Omega} + \dfrac{1}{1.75 + j0.18}} = 1.557 + j0.550 = 1.67\angle 19.2°\,\Omega$$

The phase voltage is $440/\sqrt{3} = 254$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{254\angle 0°\ \text{V}}{0.082\,\Omega + j0.19\,\Omega + 1.557\,\Omega + j0.550\,\Omega}$$

$$I_L = I_A = 141\angle -24.3°\ \text{A}$$

(b) The stator power factor is

$$\text{PF} = \cos 24.3° = 0.911\ \text{lagging}$$

(c) To find the rotor power factor, we must find the impedance angle of the rotor

$$\theta_R = \tan^{-1}\frac{X_2}{R_2/s} = \tan^{-1}\frac{0.18}{1.75} = 5.87°$$

Therefore the rotor power factor is

$$\text{PF}_R = \cos 5.87° = 0.995\ \text{lagging}$$

(d) The stator copper losses are

$$P_{\text{SCL}} = 3I_A^2 R_1 = 3(141\ \text{A})^2 (0.082\ \Omega) = 4890\ \text{W}$$

(e)   The air gap power is $P_{AG} = 3I_2^2 \dfrac{R_2}{s} = 3I_A^2 R_F$

(Note that $3I_A^2 R_F$ is equal to $3I_2^2 \dfrac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{AG} = 3I_2^2 \frac{R_2}{s} = 3I_A^2 R_F = 3(141\ \text{A})^2 (1.557\ \Omega) = 92.6\ \text{kW}$$

(f)   The power converted from electrical to mechanical form is

$$P_{conv} = (1-s)P_{AG} = (1-0.04)(92.6\ \text{kW}) = 88.9\ \text{kW}$$

(g)   The synchronous speed of this motor is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(50\ \text{Hz})}{6} = 1000\ \text{r/min}$$

$$\omega_{sync} = (1000\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 104.7\ \text{rad/s}$$

Therefore the induced torque in the motor is

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{92.6\ \text{kW}}{(1000\ \text{r/min})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)} = 884\ \text{N}\cdot\text{m}$$

(h)   The output power of this motor is

$$P_{OUT} = P_{conv} - P_{mech} - P_{core} - P_{misc} = 88.9\ \text{kW} - 1.3\ \text{kW} - 1.4\ \text{kW} - 300\ \text{W} = 85.9\ \text{kW}$$

The output speed is

$$n_m = (1-s)n_{sync} = (1-0.04)(1000\ \text{r/min}) = 960\ \text{r/min}$$

Therefore the load torque is

$$\tau_{load} = \frac{P_{OUT}}{\omega_m} = \frac{85.9\ \text{kW}}{(960\ \text{r/min})\left(\dfrac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)} = 854\ \text{N}\cdot\text{m}$$

(i)   The overall efficiency is

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{P_{OUT}}{3V_\phi I_A \cos\theta} \times 100\%$$

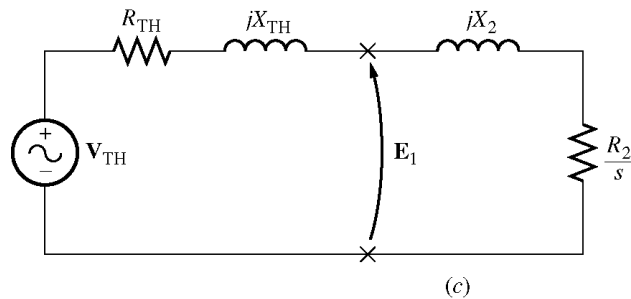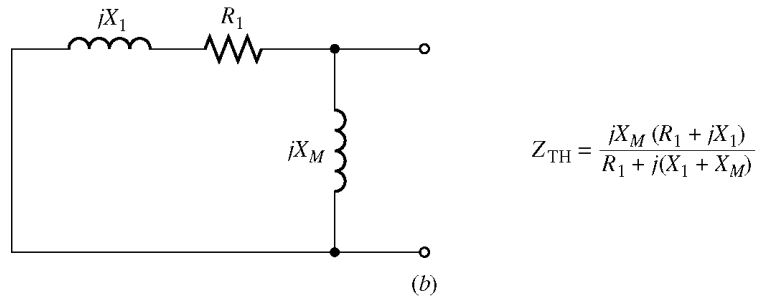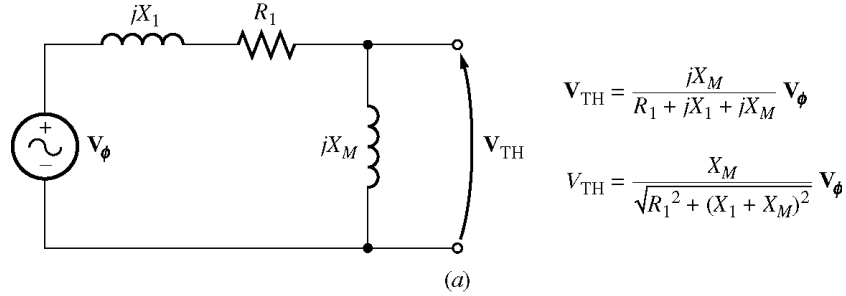$$\eta = \frac{85.9\ \text{kW}}{3(254\ \text{V})(141\ \text{A})\cos 24.3°} \times 100\% = 87.7\%$$

(j)   The motor speed in revolutions per minute is 960 r/min. The motor speed in radians per second is

$$\omega_m = (960\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 100.5\ \text{rad/s}$$

7-15. For the motor in Problem 7-14, what is the pullout torque? What is the slip at the pullout torque? What is the rotor speed at the pullout torque?

SOLUTION The slip at pullout torque is found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model.

$$Z_{TH} = \frac{jX_M (R_1 + jX_1)}{R_1 + j(X_1 + X_M)} = \frac{(j7.2\,\Omega)(0.082\,\Omega + j0.19\,\Omega)}{0.082\,\Omega + j(0.19\,\Omega + 7.2\,\Omega)} = 0.0778 + j0.1860\,\Omega = 0.202\angle67.3°\,\Omega$$

$$V_{TH} = \frac{jX_M}{R_1 + j(X_1 + X_M)}V_\phi = \frac{(j7.2\,\Omega)}{0.082\,\Omega + j(0.19\,\Omega + 7.2\,\Omega)}(254\angle0°\text{ V}) = 247.5\angle0.6°\text{ V}$$

The slip at pullout torque is

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$s_{max} = \frac{0.070\,\Omega}{\sqrt{(0.0778\,\Omega)^2 + (0.186\,\Omega + 0.180\,\Omega)^2}} = 0.187$$

The pullout torque of the motor is

$$\tau_{max} = \frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]}$$

$$\tau_{max} = \frac{3(247.5\text{ V})^2}{2(104.7\text{ rad/s})\left[0.0778\,\Omega + \sqrt{(0.0778\,\Omega)^2 + (0.186\,\Omega + 0.180\,\Omega)^2}\right]}$$

$$\tau_{max} = 1941\text{ N}\cdot\text{m}$$

7-16. If the motor in Problem 7-14 is to be driven from a 440-V 60-Hz power supply, what will the pullout torque be? What will the slip be at pullout?

SOLUTION If this motor is driven from a 60 Hz source, the resistances will be unchanged and the reactances will be increased by a ratio of 6/5. The resulting equivalent circuit is shown below.
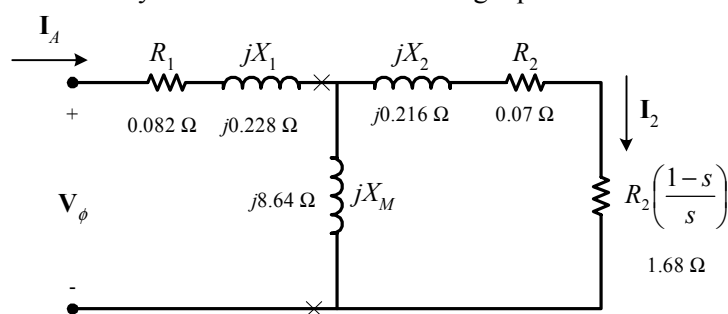
The slip at pullout torque is found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model.

$$Z_{TH} = \frac{jX_M (R_1 + jX_1)}{R_1 + j(X_1 + X_M)} = \frac{(j8.64\,\Omega)(0.082\,\Omega + j0.228\,\Omega)}{0.082\,\Omega + j(0.228\,\Omega + 8.64\,\Omega)} = 0.0778 + j0.223\,\Omega = 0.236\angle70.7°\,\Omega$$

$$V_{TH} = \frac{jX_M}{R_1 + j(X_1 + X_M)}V_\phi = \frac{j8.64\,\Omega}{0.082\,\Omega + j(0.228\,\Omega + 8.64\,\Omega)}(254\angle0°\text{ V}) = 247.5\angle0.5°\text{ V}$$

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The slip at pullout torque is

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$s_{max} = \frac{0.070\,\Omega}{\sqrt{(0.0778\,\Omega)^2 + (0.223\,\Omega + 0.216\,\Omega)^2}} = 0.157$$

The synchronous speed of this motor is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{6} = 1200\text{ r/min}$$

$$\omega_{sync} = (1200\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 125.7\text{ rad/s}$$

Therefore the pullout torque of the motor is

$$\tau_{max} = \frac{3 V_{TH}^2}{2\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}\right]}$$

$$\tau_{max} = \frac{3(247.5\text{ V})^2}{2(125.7\text{ rad/s})\left[0.0778\,\Omega + \sqrt{(0.0778\,\Omega)^2 + (0.223\,\Omega + 0.216\,\Omega)^2}\right]}$$

$$\tau_{max} = 1396\text{ N}\cdot\text{m}$$

7-17. Plot the following quantities for the motor in Problem 7-14 as slip varies from 0% to 10%: *(a)* τ_{ind} *(b)* P_{conv} *(c)* P_{out} *(d)* Efficiency η. At what slip does P_{out} equal the rated power of the machine?

SOLUTION This problem is ideally suited to solution with a MATLAB program. An appropriate program is shown below. It follows the calculations performed for Problem 7-14, but repeats them at many values of slip, and then plots the results. Note that it plots all the specified values versus n_m, which varies from 900 to 1000 r/min, corresponding to a range of 0 to 10% slip.

```
% M-file: prob7_17.m
% M-file create a plot of the induced torque, power
%    converted, power out, and efficiency of the induction
%    motor of Problem 7-14 as a function of slip.

% First, initialize the values needed in this program.
r1 = 0.082;                % Stator resistance
x1 = 0.190;                % Stator reactance
r2 = 0.070;                % Rotor resistance
x2 = 0.180;                % Rotor reactance
xm = 7.2;                  % Magnetization branch reactance
v_phase = 440 / sqrt(3);   % Phase voltage
n_sync = 1000;             % Synchronous speed (r/min)
w_sync = 104.7;            % Synchronous speed (rad/s)
p_mech = 1300;             % Mechanical losses (W)
p_core = 1400;             % Core losses (W)
p_misc = 150;              % Miscellaneous losses (W)

% Calculate the Thevenin voltage and impedance from Equations
```

```matlab
% 7-38 and 7-41.
v_th = v_phase * ( xm / sqrt(r1^2 + (x1 + xm)^2) );
z_th = ((j*xm) * (r1 + j*x1)) / (r1 + j*(x1 + xm));
r_th = real(z_th);
x_th = imag(z_th);

% Now calculate the torque-speed characteristic for many
% slips between 0 and 0.1.  Note that the first slip value
% is set to 0.001 instead of exactly 0 to avoid divide-
% by-zero problems.
s = (0:0.001:0.1);          % Slip
s(1) = 0.001;
nm = (1 - s) * n_sync;      % Mechanical speed
wm = nm * 2*pi/60;          % Mechanical speed

% Calculate torque, P_conv, P_out, and efficiency
% versus speed
for ii = 1:length(s)

   % Induced torque
   t_ind(ii) = (3 * v_th^2 * r2 / s(ii)) / ...
           (w_sync * ((r_th + r2/s(ii))^2 + (x_th + x2)^2) );

   % Power converted
   p_conv(ii) = t_ind(ii) * wm(ii);

   % Power output
   p_out(ii) = p_conv(ii) - p_mech - p_core - p_misc;

   % Power input
   zf = 1 / ( 1/(j*xm) + 1/(r2/s(ii)+j*x2) );
   ia = v_phase / ( r1 + j*x1 + zf );
   p_in(ii) = 3 * v_phase * abs(ia) * cos(atan(imag(ia)/real(ia)));

   % Efficiency
   eff(ii) = p_out(ii) / p_in(ii) * 100;

end

% Plot the torque-speed curve
figure(1);
plot(nm,t_ind,'b-','LineWidth',2.0);
xlabel('\bf\itn_{m}  \rm\bf(r/min)');
ylabel('\bf\tau_{ind}  \rm\bf(N-m)');
title ('\bfInduced Torque versus Speed');
grid on;

% Plot power converted versus speed
figure(2);
plot(nm,p_conv/1000,'b-','LineWidth',2.0);
xlabel('\bf\itn_{m}  \rm\bf(r/min)');
ylabel('\bf\itP\rm\bf_{conv}  (kW)');
title ('\bfPower Converted versus Speed');
grid on;

% Plot output power versus speed
```

```
figure(3);
plot(nm,p_out/1000,'b-','LineWidth',2.0);
xlabel('\bf\itn_{m}  \rm\bf(r/min)');
ylabel('\bf\itP\rm\bf_{out}  (kW)');
title ('\bfOutput Power versus Speed');
axis([900 1000 0 160]);
grid on;

% Plot the efficiency
figure(4);
plot(nm,eff,'b-','LineWidth',2.0);
xlabel('\bf\itn_{m}  \rm\bf(r/min)');
ylabel('\bf\eta  (%)');
title ('\bfEfficiency versus Speed');
grid on;
```

The four plots are shown below:

Power Converted versus Speed

Output Power versus Speed

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Efficiency versus Speed
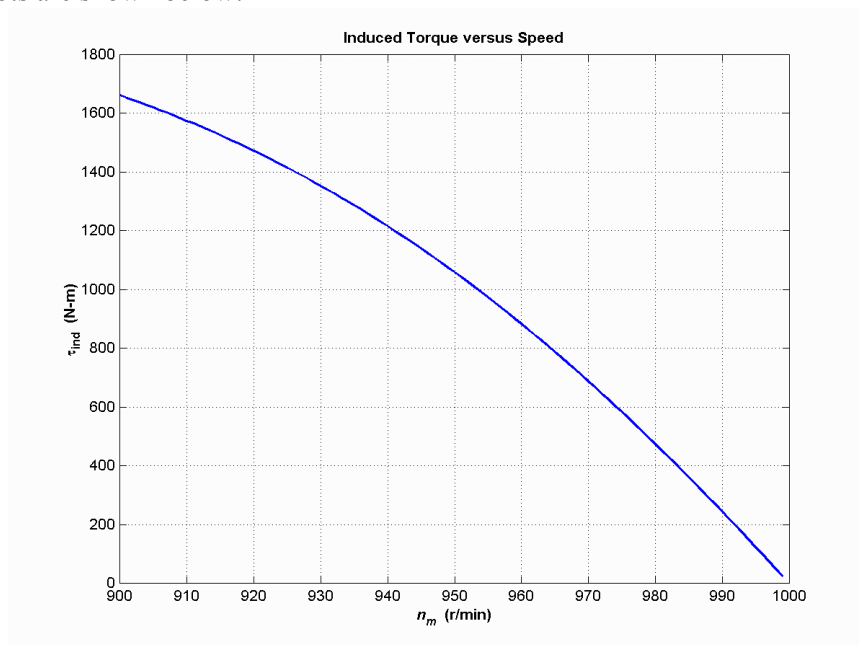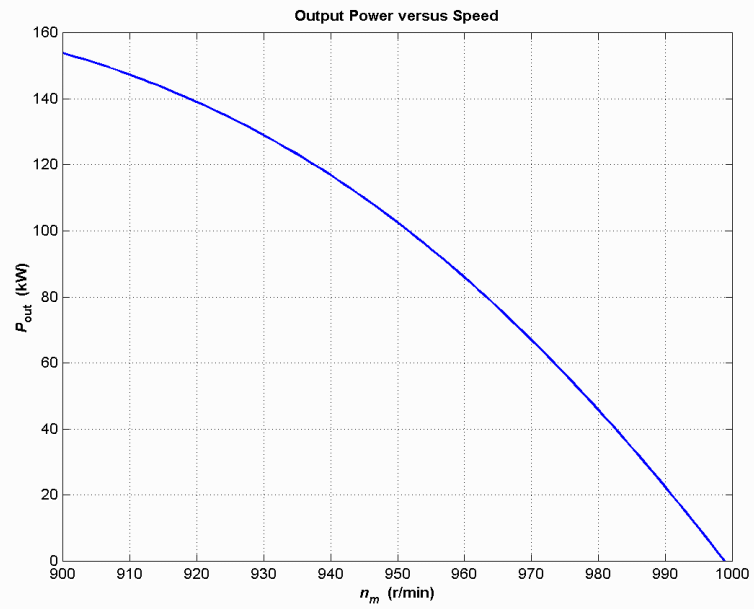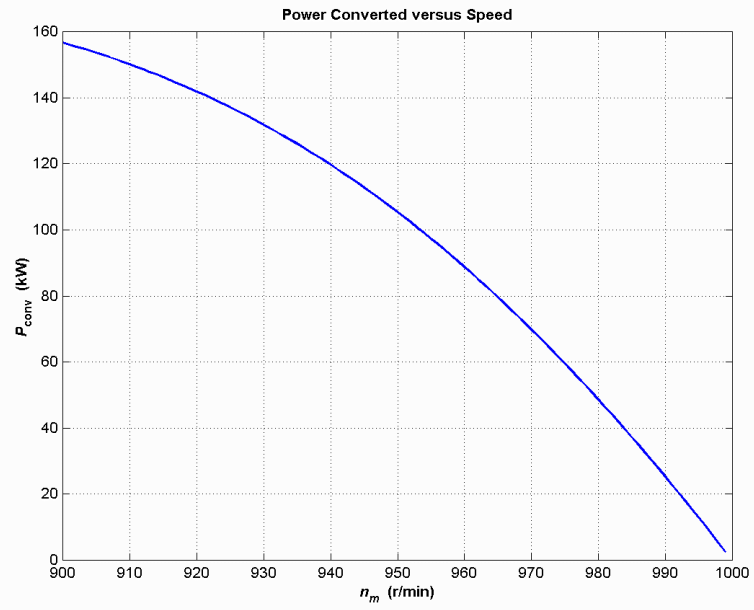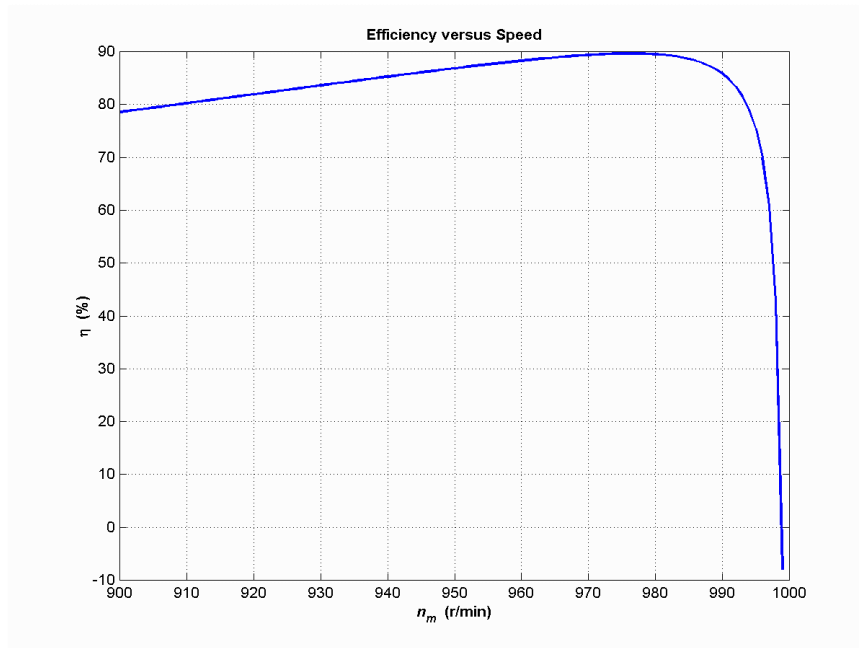
This machine is rated at 75 kW. It produces an output power of 75 kW at 3.4% slip, or a speed of 966 r/min.

7-18. A 208-V, 60 Hz, six-pole Y-connected 25-hp design class B induction motor is tested in the laboratory, with the following results:

No load: 208 V, 22.0 A, 1200 W, 60 Hz

Locked rotor: 24.6 V, 64.5 A, 2200 W, 15 Hz

DC test: 13.5 V, 64 A

Find the equivalent circuit of this motor, and plot its torque-speed characteristic curve.

SOLUTION From the DC test,

$$2R_1 = \frac{13.5\ \text{V}}{64\ \text{A}} \qquad \Rightarrow \qquad R_1 = 0.105\ \Omega$$
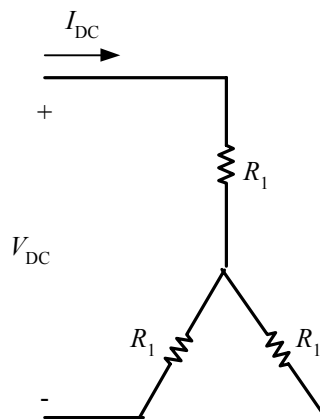
In the no-load test, the line voltage is 208 V, so the phase voltage is 120 V. Therefore,

$$X_1 + X_M = \frac{V_\phi}{I_{A,\text{nl}}} = \frac{120\ \text{V}}{22.0\ \text{A}} = 5.455\ \Omega \quad @ \quad 60\ \text{Hz}$$

In the locked-rotor test, the line voltage is 24.6 V, so the phase voltage is 14.2 V. From the locked-rotor test *at 15 Hz*,

$$\left|Z'_{\text{LR}}\right| = \left|R_{\text{LR}} + jX'_{\text{LR}}\right| = \frac{V_\phi}{I_{A,\text{LR}}} = \frac{14.2\ \text{V}}{64.5\ \text{A}} = 0.2202\ \Omega$$

$$\theta'_{\text{LR}} = \cos^{-1}\frac{P_{\text{LR}}}{S_{\text{LR}}} = \cos^{-1}\left(\frac{2200\ \text{W}}{\sqrt{3}(24.6\ \text{V})(64.5\ \text{A})}\right) = 36.82°$$

Therefore,

$$R_{\text{LR}} = \left|Z'_{\text{LR}}\right|\cos\theta_{\text{LR}} = (0.2202\ \Omega)\cos 36.82° = 0.176\ \Omega$$
$$\Rightarrow \quad R_1 + R_2 = 0.176\ \Omega$$
$$\Rightarrow \quad R_2 = 0.071\ \Omega$$

$$X'_{\text{LR}} = \left|Z'_{\text{LR}}\right|\sin\theta_{\text{LR}} = (0.2202\ \Omega)\sin 36.82° = 0.132\ \Omega$$

At a frequency of 60 Hz,

$$X_{\text{LR}} = \left(\frac{60\ \text{Hz}}{15\ \text{Hz}}\right)X'_{\text{LR}} = 0.528\ \Omega$$

For a Design Class B motor, the split is $X_1 = 0.211\ \Omega$ and $X_2 = 0.317\ \Omega$. Therefore,

$$X_M = 5.455\ \Omega - 0.211\ \Omega = 5.244\ \Omega$$

The resulting equivalent circuit is shown below:
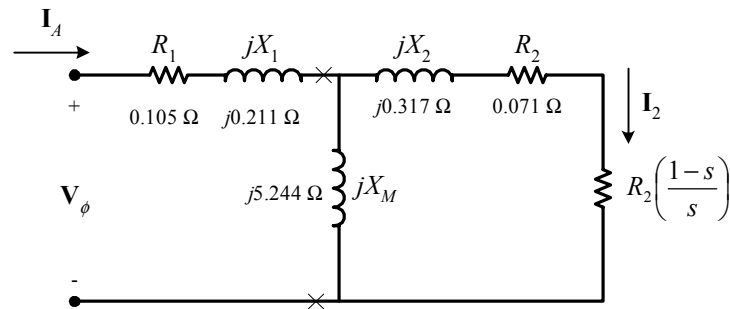
A MATLAB program to calculate the torque-speed characteristic of this motor is shown below:

```
% M-file: prob7_18.m
% M-file create a plot of the torque-speed curve of the
%   induction motor of Problem 7-18.

% First, initialize the values needed in this program.
r1 = 0.105;               % Stator resistance
x1 = 0.211;               % Stator reactance
r2 = 0.071;               % Rotor resistance
x2 = 0.317;               % Rotor reactance
xm = 5.244;               % Magnetization branch reactance
v_phase = 208 / sqrt(3);  % Phase voltage
n_sync = 1200;            % Synchronous speed (r/min)
w_sync = 125.7;           % Synchronous speed (rad/s)

% Calculate the Thevenin voltage and impedance from Equations
% 7-38 and 7-41.
```

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```
v_th = v_phase * ( xm / sqrt(r1^2 + (x1 + xm)^2) );
z_th = ((j*xm) * (r1 + j*x1)) / (r1 + j*(x1 + xm));
r_th = real(z_th);
x_th = imag(z_th);

% Now calculate the torque-speed characteristic for many
% slips between 0 and 1.  Note that the first slip value
% is set to 0.001 instead of exactly 0 to avoid divide-
% by-zero problems.
s = (0:1:50) / 50;          % Slip
s(1) = 0.001;
nm = (1 - s) * n_sync;      % Mechanical speed

% Calculate torque versus speed
for ii = 1:51
   t_ind(ii) = (3 * v_th^2 * r2 / s(ii)) / ...
           (w_sync * ((r_th + r2/s(ii))^2 + (x_th + x2)^2) );
end

% Plot the torque-speed curve
figure(1);
plot(nm,t_ind,'b-','LineWidth',2.0);
xlabel('\bf\itn_{m}');
ylabel('\bf\tau_{ind}');
title ('\bfInduction Motor Torque-Speed Characteristic');
grid on;
```

The resulting plot is shown below:

7-19. A 208-V four-pole 10-hp 60-Hz Y-connected three-phase induction motor develops its full-load induced torque at 3.8 percent slip when operating at 60 Hz and 208 V. The per-phase circuit model impedances of the motor are

$$R_1 = 0.33 \ \Omega \qquad\qquad X_M = 16 \ \Omega$$

$$X_1 = 0.42 \ \Omega \qquad\qquad X_2 = 0.42 \ \Omega$$

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Mechanical, core, and stray losses may be neglected in this problem.

(a) Find the value of the rotor resistance R_2.

(b) Find τ_{max}, s_{max}, and the rotor speed at maximum torque for this motor.

(c) Find the starting torque of this motor.

(d) What code letter factor should be assigned to this motor?

SOLUTION The equivalent circuit for this motor is
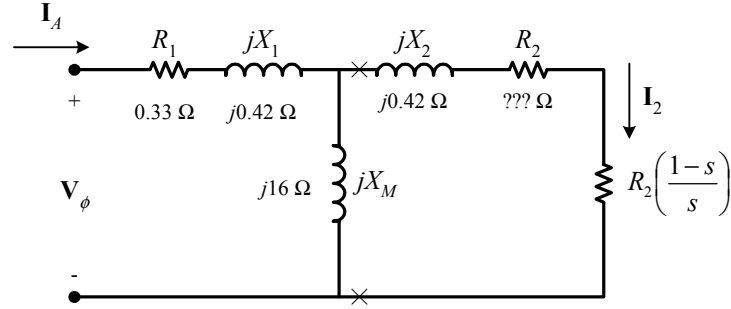
The Thevenin equivalent of the input circuit is:

$$Z_{TH} = \frac{jX_M(R_1 + jX_1)}{R_1 + j(X_1 + X_M)} = \frac{(j16\ \Omega)(0.33\ \Omega + j0.42\ \Omega)}{0.33\ \Omega + j(0.42\ \Omega + 16\ \Omega)} = 0.313 + j0.416\ \Omega = 0.520\angle 53°\ \Omega$$

$$V_{TH} = \frac{jX_M}{R_1 + j(X_1 + X_M)} V_\phi = \frac{(j16\ \Omega)}{0.33\ \Omega + j(0.42\ \Omega + 16\ \Omega)}(120\angle 0°\ V) = 116.9\angle 1.2°\ V$$

(a) If losses are neglected, the induced torque in a motor is equal to its load torque. At full load, the output power of this motor is 10 hp and its slip is 3.8%, so the induced torque is

$$n_m = (1 - 0.038)(1800\ r/min) = 1732\ r/min$$

$$\tau_{ind} = \tau_{load} = \frac{(10\ hp)(746\ W/hp)}{(1732\ r/min)\left(\dfrac{2\pi\ rad}{1\ r}\right)\left(\dfrac{60\ s}{1\ min}\right)} = 41.1\ N \cdot m$$

The induced torque is given by the equation

$$\tau_{ind} = \frac{3V_{TH}^2 R_2 / s}{\omega_{sync}\left[(R_{TH} + R_2 / s)^2 + (X_{TH} + X_2)^2\right]}$$

Substituting known values and solving for R_2 / s yields

$$41.1\ N \cdot m = \frac{3(116.9\ V)^2 R_2 / s}{(188.5\ rad/s)\left[(0.313 + R_2 / s)^2 + (0.416 + 0.42)^2\right]}$$

$$7,747 = \frac{40,997\ R_2 / s}{\left[(0.313 + R_2 / s)^2 + 0.699\right]}$$

$$\left[(0.313 + R_2 / s)^2 + 0.699\right] = 5.292\ R_2 / s$$

$$\left[0.098+0.626R_2/s+\left(R_2/s\right)^2+0.699\right]=5.292\ R_2/s$$

$$\left(\frac{R_2}{s}\right)^2-4.666\left(\frac{R_2}{s}\right)+0.797=0$$

$$\left(\frac{R_2}{s}\right)=0.178,\ \ 4.488$$

$$R_2=0.0067\ \Omega,\ 0.17\ \Omega$$

These two solutions represent two situations in which the torque-speed curve would go through this specific torque-speed point. The two curves are plotted below. As you can see, only the 0.17 Ω solution is realistic, since the 0.0067 Ω solution passes through this torque-speed point at an unstable location on the back side of the torque-speed curve.
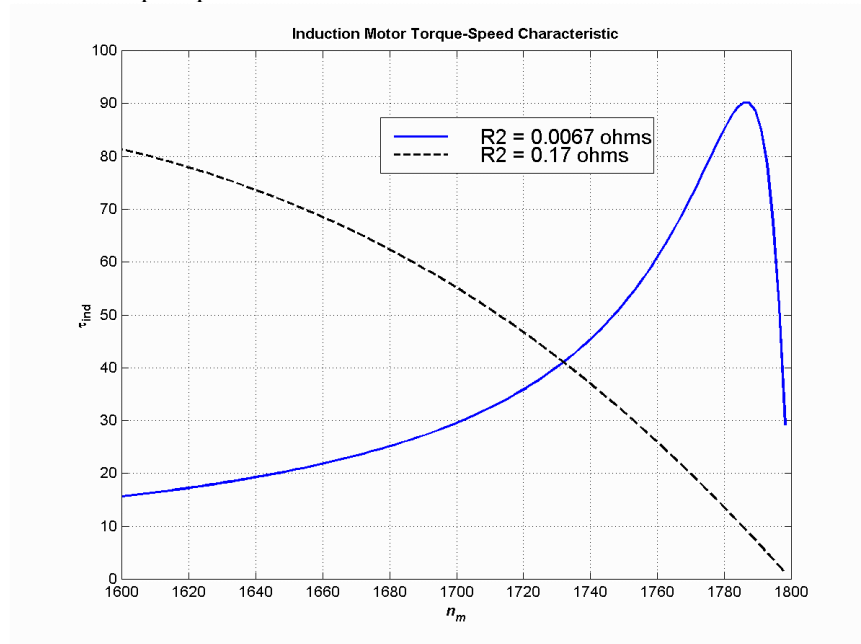
(b) The slip at pullout torque can be found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model. The Thevenin equivalent of the input circuit was calculate in part *(a)*. The slip at pullout torque is

$$s_{max}=\frac{R_2}{\sqrt{R_{TH}^{\ 2}+\left(X_{TH}+X_2\right)^2}}$$

$$s_{max}=\frac{0.17\ \Omega}{\sqrt{\left(0.313\ \Omega\right)^2+\left(0.416\ \Omega\ +0.420\ \Omega\right)^2}}=0.190$$

The rotor speed a maximum torque is

$$n_{pullout}=(1-s)\ n_{sync}=(1-0.190)(1800\ \text{r/min})=1457\ \text{r/min}$$

and the pullout torque of the motor is

$$\tau_{max}=\frac{3V_{TH}^2}{2\omega_{sync}\left[R_{TH}+\sqrt{R_{TH}^2+\left(X_{TH}+X_2\right)^2}\right]}$$

$$\tau_{\max} = \frac{3\,(116.9\text{ V})^2}{2(188.5\text{ rad/s})\left[0.313\,\Omega + \sqrt{(0.313\,\Omega)^2 + (0.416\,\Omega + 0.420\,\Omega)^2}\,\right]}$$

$$\tau_{\max} = 90.2\text{ N}\cdot\text{m}$$

(c) The starting torque of this motor is the torque at slip $s = 1$. It is

$$\tau_{\text{ind}} = \frac{3V_{\text{TH}}^2 R_2 / s}{\omega_{\text{sync}}\left[\left(R_{\text{TH}} + R_2/s\right)^2 + \left(X_{\text{TH}} + X_2\right)^2\right]}$$

$$\tau_{\text{ind}} = \frac{3(116.9\text{ V})^2\,(0.17\ \Omega)}{(188.5\text{ rad/s})\left[(0.313 + 0.17\ \Omega)^2 + (0.416 + 0.420)^2\right]} = 38.3\text{ N}\cdot\text{m}$$

(d) To determine the starting code letter, we must find the locked-rotor kVA per horsepower, which is equivalent to finding the starting kVA per horsepower. The easiest way to find the line current (or armature current) at starting is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M at starting conditions, and then calculate the starting current as the phase voltage divided by the sum of the series impedances, as shown below.
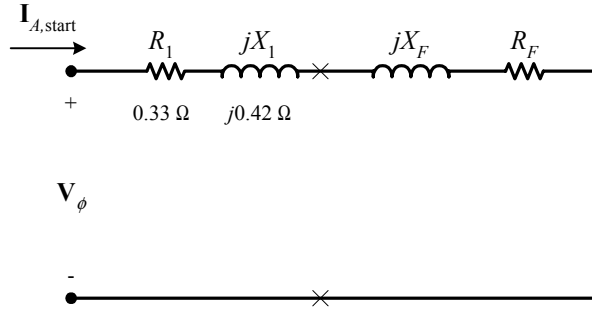
The equivalent impedance of the rotor circuit in parallel with jX_M at starting conditions ($s = 1.0$) is:

$$Z_{F,\text{start}} = \frac{1}{\dfrac{1}{jX_M} + \dfrac{1}{Z_2}} = \frac{1}{\dfrac{1}{j16\,\Omega} + \dfrac{1}{0.17 + j0.42}} = 0.161 + j0.411 = 0.442\angle 68.6°\ \Omega$$

The phase voltage is $208/\sqrt{3} = 120$ V, so line current $\mathbf{I}_{L,\text{start}}$ is

$$\mathbf{I}_{L,\text{start}} = \mathbf{I}_A = \frac{\mathbf{V}_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0°\text{ V}}{0.33\,\Omega + j0.42\,\Omega + 0.161\,\Omega + j0.411\,\Omega}$$

$$\mathbf{I}_{L,\text{start}} = \mathbf{I}_A = 124\angle -59.4°\text{ A}$$

Therefore, the locked-rotor kVA of this motor is

$$S = \sqrt{3}\,V_T\,I_{L,\text{rated}} = \sqrt{3}(208\text{ V})(124\text{ A}) = 44.7\text{ kVA}$$

and the kVA per horsepower is

$$\text{kVA/hp} = \frac{44.7\text{ kVA}}{10\text{ hp}} = 4.47\text{ kVA/hp}$$

This motor would have **starting code letter D**, since letter D covers the range 4.00-4.50.

7-20. Answer the following questions about the motor in Problem 7-19.

(a) If this motor is started from a 208-V infinite bus, how much current will flow in the motor at starting?

(b) If transmission line with an impedance of $0.50 + j0.35\ \Omega$ per phase is used to connect the induction motor to the infinite bus, what will the starting current of the motor be? What will the motor's terminal voltage be on starting?

(c) If an ideal 1.2:1 step-down autotransformer is connected between the transmission line and the motor, what will the current be in the transmission line during starting? What will the voltage be at the motor end of the transmission line during starting?

SOLUTION

(a) The equivalent circuit of this induction motor is shown below:
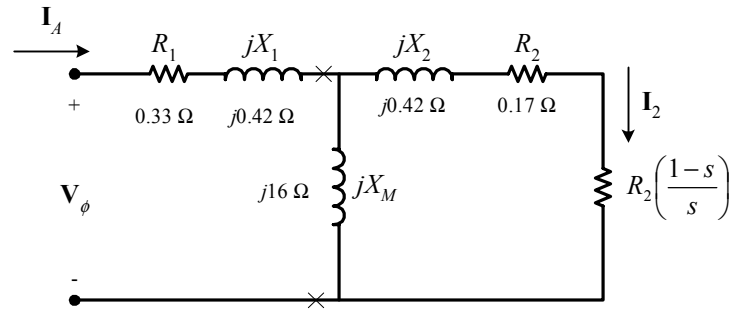
The easiest way to find the line current (or armature current) at starting is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M at starting conditions, and then calculate the starting current as the phase voltage divided by the sum of the series impedances, as shown below.
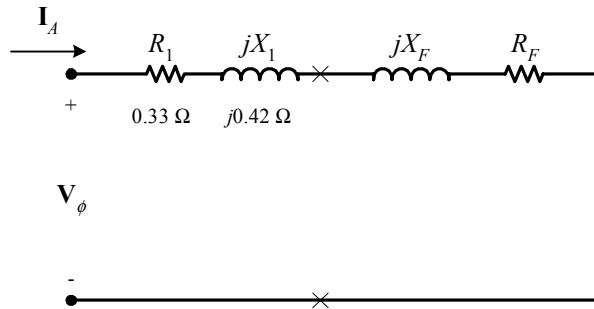
The equivalent impedance of the rotor circuit in parallel with jX_M at starting conditions ($s = 1.0$) is:

$$Z_F = \cfrac{1}{\cfrac{1}{jX_M} + \cfrac{1}{Z_2}} = \cfrac{1}{\cfrac{1}{j16\ \Omega} + \cfrac{1}{0.17 + j0.42}} = 0.161 + j0.411 = 0.442\angle 68.6°\ \Omega$$

The phase voltage is $208/\sqrt{3} = 120$ V, so line current $\mathbf{I}_L$ is

$$\mathbf{I}_L = \mathbf{I}_A = \frac{\mathbf{V}_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0°\ \text{V}}{0.33\ \Omega + j0.42\ \Omega + 0.161\ \Omega + j0.411\ \Omega}$$

$$\mathbf{I}_L = \mathbf{I}_A = 124\angle -59.4°\ \text{A}$$

(b) If a transmission line with an impedance of $0.50 + j0.35\ \Omega$ per phase is used to connect the induction motor to the infinite bus, its impedance will be in series with the motor's impedances, and the starting current will be

$$\mathbf{I}_L = \mathbf{I}_A = \frac{\mathbf{V}_{\phi,\text{bus}}}{R_{\text{line}} + jX_{\text{line}} + R_1 + jX_1 + R_F + jX_F}$$

$$\mathbf{I}_L = \mathbf{I}_A = \frac{120\angle 0° \text{ V}}{0.50 \text{ Ω} + j0.35 \text{ Ω} + 0.33 \text{ Ω} + j0.42 \text{ Ω} + 0.161 \text{ Ω} + j0.411 \text{ Ω}}$$

$$\mathbf{I}_L = \mathbf{I}_A = 77.8\angle - 50.0° \text{ A}$$

The voltage at the terminals of the motor will be

$$\mathbf{V}_\phi = \mathbf{I}_A \left(R_1 + jX_1 + R_F + jX_F \right)$$

$$\mathbf{V}_\phi = \left(77.8\angle - 50.0° \text{ A} \right) \left(0.33 \text{ Ω} + j0.42 \text{ Ω} + 0.161 \text{ Ω} + j0.411 \text{ Ω} \right)$$

$$\mathbf{V}_\phi = 75.1\angle 9.4° \text{ V}$$

Therefore, the terminal voltage will be $\sqrt{3}\,(75.1 \text{ V}) = 130 \text{ V}$. Note that the terminal voltage sagged by 37.5% during motor starting, which would be unacceptable.

(c) If an ideal 1.2:1 step-down autotransformer is connected between the transmission line and the motor, the motor's impedances will be referred across the transformer by the square of the turns ratio *a* = 1.2. The referred impedances are

$$R_1' = a^2 R_1 = 1.44(0.33 \text{ Ω}) = 0.475 \text{ Ω}$$

$$X_1' = a^2 X_1 = 1.44(0.42 \text{ Ω}) = 0.605 \text{ Ω}$$

$$R_F' = a^2 R_F = 1.44(0.161 \text{ Ω}) = 0.232 \text{ Ω}$$

$$X_F' = a^2 X_F = 1.44(0.411 \text{ Ω}) = 0.592 \text{ Ω}$$

Therefore, the starting current referred to the primary side of the transformer will be

$$\mathbf{I}_L' = \mathbf{I}_A' = \frac{\mathbf{V}_{\phi,\text{bus}}}{R_{\text{line}} + jX_{\text{line}} + R_1' + jX_1' + R_F' + jX_F'}$$

$$\mathbf{I}_L' = \mathbf{I}_A' = \frac{120\angle 0° \text{ V}}{0.50 \text{ Ω} + j0.35 \text{ Ω} + 0.475 \text{ Ω} + j0.605 \text{ Ω} + 0.232 \text{ Ω} + j0.592 \text{ Ω}}$$

$$\mathbf{I}_L' = \mathbf{I}_A' = 61.2\angle - 52° \text{ A}$$

The voltage at the motor end of the transmission line would be the same as the referred voltage at the terminals of the motor

$$\mathbf{V}_\phi' = \mathbf{I}_A' \left(R_1' + jX_1' + R_F' + jX_F' \right)$$

$$\mathbf{V}_\phi = \left(61.2\angle - 52° \text{ A} \right) \left(0.475 \text{ Ω} + j0.605 \text{ Ω} + 0.232 \text{ Ω} + j0.592 \text{ Ω} \right)$$

$$\mathbf{V}_\phi = 85.0\angle 7.4° \text{ V}$$

Therefore, the line voltage at the motor end of the transmission line will be $\sqrt{3}\,(85 \text{ V}) = 147.3 \text{ V}$. Note that this voltage sagged by 29.2% during motor starting, which is less than the 37.5% sag with case of across-the-line starting. Since the sag is still large, it might be possible to use a bigger autotransformer turns ratio on the starter.

7-21. In this chapter, we learned that a step-down autotransformer could be used to reduce the starting current drawn by an induction motor. While this technique works, an autotransformer is relatively expensive. A much less expensive way to reduce the starting current is to use a device called *Y-Δ starter*. If an induction motor is normally Δ-connected, it is possible to reduce its phase voltage V_ϕ (and hence its starting current) by simply re-connecting the stator windings in Y during starting, and then restoring the connections to Δ when the motor comes up to speed. Answer the following questions about this type of starter.

(a) How would the phase voltage at starting compare with the phase voltage under normal running conditions?

(b) How would the starting current of the Y-connected motor compare to the starting current if the motor remained in a Δ-connection during starting?

SOLUTION

(a) The phase voltage at starting would be $1 / \sqrt{3}$ = 57.7% of the phase voltage under normal running conditions.

(b) Since the phase voltage decreases to $1 / \sqrt{3}$ = 57.7% of the normal voltage, the starting phase current will also decrease to 57.7% of the normal starting current. However, since the line current for the original delta connection was $\sqrt{3}$ times the phase current, while the line current for the Y starter connection is equal to its phase current, *the line current is reduced by a factor of 3 in a Y-Δ starter.*

For the Δ-connection: $I_{L,\Delta} = \sqrt{3}\, I_{\phi,\Delta}$

For the Y-connection: $I_{L,Y} = I_{\phi,Y}$

But $I_{\phi,\Delta} = \sqrt{3} I_{\phi,Y}$, so $\boxed{I_{L,\Delta} = 3 I_{L,Y}}$

7-22. A 460-V 50-hp six-pole Δ-connected 60-Hz three-phase induction motor has a full-load slip of 4 percent, an efficiency of 91 percent, and a power factor of 0.87 lagging. At start-up, the motor develops 1.75 times the full-load torque but draws 7 times the rated current at the rated voltage. This motor is to be started with an autotransformer reduced voltage starter.

(a) What should the output voltage of the starter circuit be to reduce the starting torque until it equals the rated torque of the motor?

(b) What will the motor starting current and the current drawn from the supply be at this voltage?

SOLUTION

(a) The starting torque of an induction motor is proportional to the square of V_{TH} ,

$$\frac{\tau_{start2}}{\tau_{start1}} = \left(\frac{V_{TH2}}{V_{TH1}}\right)^2 = \left(\frac{V_{T2}}{V_{T1}}\right)^2$$

$$\frac{\tau_{start2}}{\tau_{start1}} = \left(\frac{V_{TH2}}{V_{TH1}}\right)^2 = \left(\frac{V_{T2}}{V_{T1}}\right)^2$$

If a torque of 1.75 τ_{rated} is produced by a voltage of 460 V, then a torque of 1.00 τ_{rated} would be produced by a voltage of

$$\frac{1.00\,\tau_{rated}}{1.75\,\tau_{rated}} = \left(\frac{V_{T2}}{460\text{ V}}\right)^2$$

$$V_{T2} = \sqrt{\frac{(460\text{ V})^2}{1.75}} = 348\text{ V}$$

(b) The motor starting current is directly proportional to the starting voltage, so

$$I_{L2} = \left(\frac{348\text{ V}}{460\text{ V}}\right) I_{L1} = (0.756) I_{L1} = (0.756)(7 I_{rated}) = 5.296\, I_{rated}$$

The input power to this motor is

$$P_{IN} = \frac{P_{OUT}}{\eta} = \frac{(50 \text{ hp})(746 \text{ W/hp})}{0.91} = 40.99 \text{ kW}$$

The rated current is equal to

$$I_{rated} = \frac{P_{IN}}{\sqrt{3} \ V_T \ \text{PF}} = \frac{(40.99 \text{ kW})}{\sqrt{3}(460 \text{ V})(0.87)} = 59.1 \text{ A}$$

Therefore, the motor starting current is

$$I_{L2} = 5.843 \ I_{rated} = (5.296)(59.1 \text{ A}) = 313 \text{ A}$$

The turns ratio of the autotransformer that produces this starting voltage is

$$\frac{N_{SE} + N_C}{N_C} = \frac{460 \text{ V}}{348 \text{ V}} = 1.32$$

so the current drawn from the supply will be

$$I_{line} = \frac{I_{start}}{1.32} = \frac{313 \text{ A}}{1.32} = 237 \text{ A}$$

7-23. A wound-rotor induction motor is operating at rated voltage and frequency with its slip rings shorted and with a load of about 25 percent of the rated value for the machine. If the rotor resistance of this machine is doubled by inserting external resistors into the rotor circuit, explain what happens to the following:

(a) Slip s

(b) Motor speed n_m

(c) The induced voltage in the rotor

(d) The rotor current

(e) τ_{ind}

(f) P_{out}

(g) P_{RCL}

(h) Overall efficiency η

SOLUTION

(a) The slip s will increase.

(b) The motor speed n_m will decrease.

(c) The induced voltage in the rotor will increase.

(d) The rotor current will increase.

(e) The induced torque will adjust to supply the load's torque requirements at the new speed. This will depend on the shape of the load's torque-speed characteristic. For most loads, the induced torque will decrease.

Induction Motor Torque-Speed Characteristic
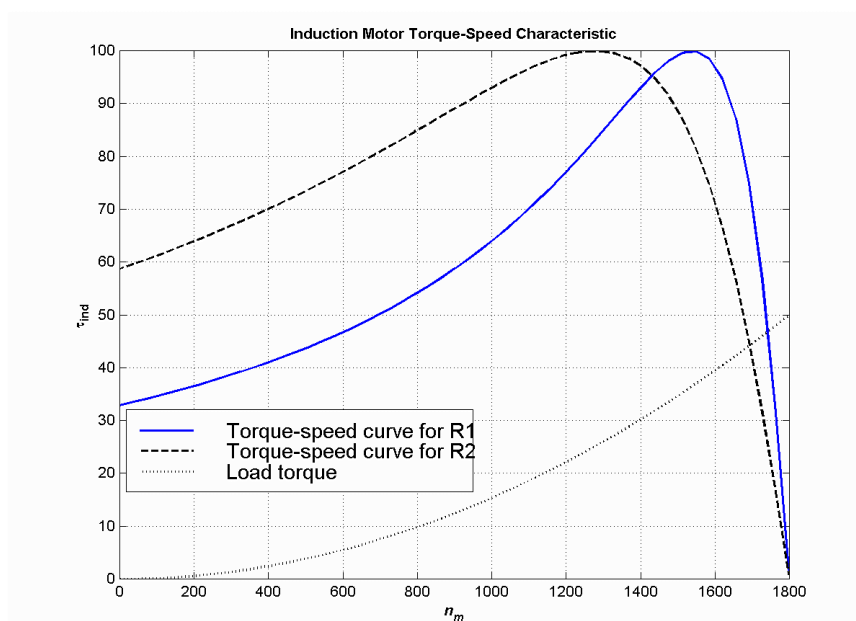
(f) The output power will generally decrease: $P_{\text{OUT}} = \tau_{\text{ind}} \downarrow \omega_m \downarrow$

(g) The rotor copper losses (including the external resistor) will increase.

(h) The overall efficiency η will decrease.

7-24. Answer the following questions about a 460-V Δ-connected two-pole 100-hp 60-Hz starting code letter F induction motor:

(a) What is the maximum current starting current that this machine's controller must be designed to handle?

(b) If the controller is designed to switch the stator windings from a Δ connection to a Y connection during starting, what is the maximum starting current that the controller must be designed to handle?

(c) If a 1.25:1 step-down autotransformer starter is used during starting, what is the maximum starting current that will be drawn from the line?

SOLUTION

(a) The maximum starting kVA of this motor is

$$S_{\text{start}} = (100 \text{ hp})(5.60) = 560 \text{ kVA}$$

Therefore,

$$I_{\text{start}} = \frac{S}{\sqrt{3} \, V_T} = \frac{560 \text{ kVA}}{\sqrt{3} \, (460 \text{ V})} = 703 \text{ A}$$

(b) The line voltage will still be 460 V when the motor is switched to the Y-connection, but now the phase voltage will be $460 / \sqrt{3} = 265.6$ V.

Before (in Δ):

$$I_{\phi,\Delta} = \frac{460 \text{ V}}{\left(R_{\text{TH}} + R_2\right) + j\left(X_{\text{TH}} + X_2\right)}$$

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$$I_{L,\Delta} = \sqrt{3}I_{\phi,\Delta} = \frac{797 \text{ V}}{(R_{TH} + R_2) + j(X_{TH} + X_2)}$$

After (in Y):

$$I_{L,Y} = I_{\phi,Y} = \frac{265.6 \text{ V}}{(R_{TH} + R_2) + j(X_{TH} + X_2)}$$

Therefore the line current will decrease by a factor of 3! The starting current with a Δ-Y starter is

$$I_{\text{start}} = \frac{703 \text{ A}}{3} = 234 \text{ A}$$

(c) A 1.25:1 step-down autotransformer reduces the phase voltage on the motor by a factor 0.8. This reduces the phase current and line current in the motor (and on the secondary side of the transformer) by a factor of 0.8. However, the current on the primary of the autotransformer will be reduced by another factor of 0.8, so the total starting current drawn from the line will be 64% of its original value. Therefore, the maximum starting current drawn from the line will be

$$I_{\text{start}} = (0.64)(703 \text{ A}) = 450 \text{ A}$$

7-25. When it is necessary to stop an induction motor very rapidly, many induction motor controllers reverse the direction of rotation of the magnetic fields by switching any two stator leads. When the direction of rotation of the magnetic fields is reversed, the motor develops an induced torque opposite to the current direction of rotation, so it quickly stops and tries to start turning in the opposite direction. If power is removed from the stator circuit at the moment when the rotor speed goes through zero, then the motor has been stopped very rapidly. This technique for rapidly stopping an induction motor is called *plugging*. The motor of Problem 7-19 is running at rated conditions and is to be stopped by plugging.

(a) What is the slip s before plugging?

(b) What is the frequency of the rotor before plugging?

(c) What is the induced torque τ_{ind} before plugging?

(d) What is the slip s immediately after switching the stator leads?

(e) What is the frequency of the rotor immediately after switching the stator leads?

(f) What is the induced torque τ_{ind} immediately after switching the stator leads?

SOLUTION

(a) The slip before plugging is 0.038 (see Problem 7-19).

(b) The frequency of the rotor before plugging is $f_r = sf_e = (0.038)(60 \text{ Hz}) = 2.28 \text{ Hz}$

(c) The induced torque before plugging is 41.1 N·m in the direction of motion (see Problem 7-19).

(d) After switching stator leads, the synchronous speed becomes –1800 r/min, while the mechanical speed initially remains 1732 r/min. Therefore, the slip becomes

$$s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} = \frac{-1800 - 1732}{-1800} = 1.962$$

(e) The frequency of the rotor after plugging is $f_r = sf_e = (1.962)(60 \text{ Hz}) = 117.72 \text{ Hz}$

(f) The induced torque immediately after switching the stator leads is

$$\tau_{\mathrm{ind}} = \frac{3V_{\mathrm{TH}}^2 R_2 / s}{\omega_{\mathrm{sync}} \left[\left(R_{\mathrm{TH}} + R_2 / s \right)^2 + \left(X_{\mathrm{TH}} + X_2 \right)^2 \right]}$$

$$\tau_{\mathrm{ind}} = \frac{3(116.9 \text{ V})^2 (0.17 \text{ }\Omega/1.962)}{(188.5 \text{ rad/s}) \left[(0.313 + 0.17 \text{ }\Omega/1.962)^2 + (0.416 + 0.420)^2 \right]}$$

$$\tau_{\mathrm{ind}} = \frac{3(116.9 \text{ V})^2 (0.0866)}{(188.5 \text{ rad/s}) \left[(0.313 + 0.0866)^2 + (0.416 + 0.420)^2 \right]}$$

$\tau_{\mathrm{ind}} = 21.9 \text{ N} \cdot \text{m}$, opposite the direction of motion

Chapter 8: *DC Motors*

8-1. The following information is given about the simple rotating loop shown in Figure 8-6:

$$B = 0.4 \text{ T} \qquad\qquad V_B = 48 \text{ V}$$

$$l = 0.5 \text{ m} \qquad\qquad R = 0.4 \ \Omega$$

$$r = 0.25 \text{ m} \qquad\qquad \omega = 500 \text{ rad/s}$$

(a) Is this machine operating as a motor or a generator? Explain.

(b) What is the current i flowing into or out of the machine? What is the power flowing into or out of the machine?

(c) If the speed of the rotor were changed to 550 rad/s, what would happen to the current flow into or out of the machine?

(d) If the speed of the rotor were changed to 450 rad/s, what would happen to the current flow into or out of the machine?
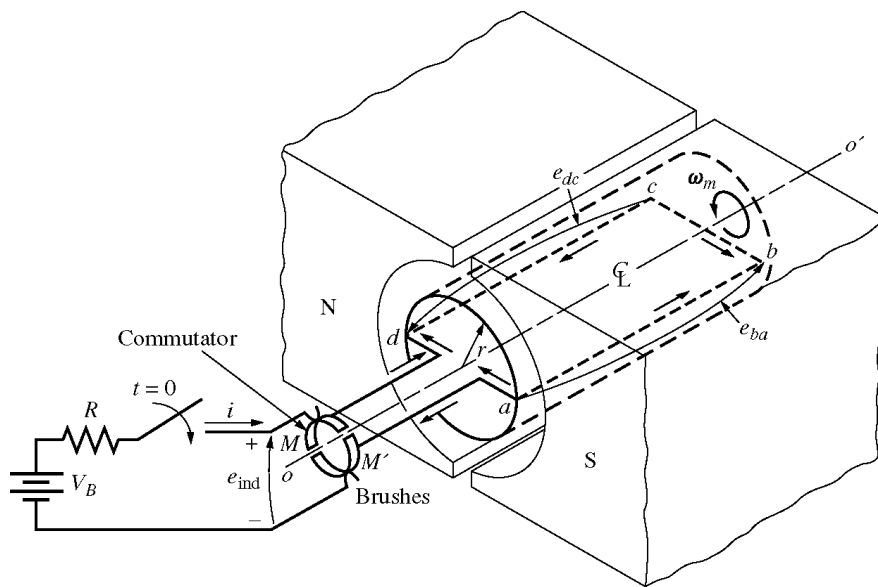
(a)
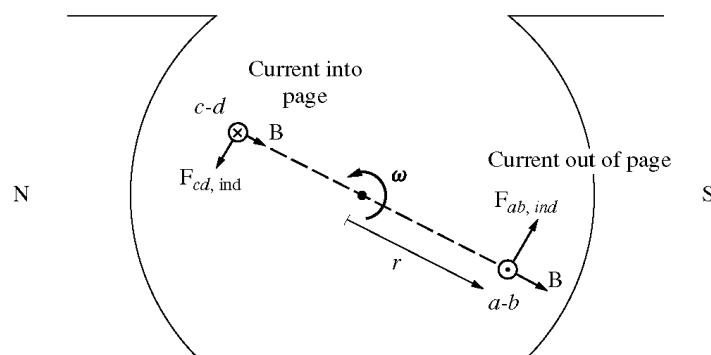
(b)

SOLUTION

(a) If the speed of rotation ω of the shaft is 500 rad/s, then the voltage induced in the rotating loop will be

$$e_{\text{ind}} = 2rlB\omega$$

$$e_{\text{ind}} = 2(0.25 \text{ m})(0.5 \text{ m})(0.4 \text{ T})(500 \text{ rad/s}) = 50 \text{ V}$$

Since the external battery voltage is only 48 V, this machine is operating as a *generator*, charging the battery.

(b) The current flowing out of the machine is approximately

$$i = \frac{e_{\text{ind}} - V_B}{R} = \left(\frac{50 \text{ V - 48 V}}{0.4 \, \Omega} \right) = 5.0 \text{ A}$$

(*Note* that this value is the current flowing while the loop is under the pole faces. When the loop goes beyond the pole faces, e_{ind} will momentarily fall to 0 V, and the current flow will momentarily reverse. Therefore, the *average* current flow over a complete cycle will be somewhat less than 5.0 A.)

(c) If the speed of the rotor were increased to 550 rad/s, the induced voltage of the loop would increase to

$$e_{\text{ind}} = 2rlB\omega$$

$$e_{\text{ind}} = 2(0.25 \text{ m})(0.5 \text{ m})(0.4 \text{ T})(550 \text{ rad/s}) = 55 \text{ V}$$

and the current flow out of the machine will increase to

$$i = \frac{e_{\text{ind}} - V_B}{R} = \left(\frac{55 \text{ V - 48 V}}{0.4 \, \Omega} \right) = 17.5 \text{ A}$$

(d) If the speed of the rotor were decreased to 450 rad/s, the induced voltage of the loop would fall to

$$e_{\text{ind}} = 2rlB\omega$$

$$e_{\text{ind}} = 2(0.25 \text{ m})(0.5 \text{ m})(0.4 \text{ T})(450 \text{ rad/s}) = 45 \text{ V}$$

Here, e_{ind} is less than V_B, so current flows into the loop and the machine is acting as a motor. The current flow into the machine would be

$$i = \frac{V_B - e_{\text{ind}}}{R} = \left(\frac{48 \text{ V - 45 V}}{0.4 \, \Omega} \right) = 7.5 \text{ A}$$

8-2. The power converted from one form to another within a dc motor was given by

$$P_{\text{conv}} = E_A I_A = \tau_{\text{ind}} \omega_m$$

Use the equations for E_A and τ_{ind} [Equations (8-30) and (8-31)] to prove that $E_A I_A = \tau_{\text{ind}} \omega_m$; that is, prove that the electric power disappearing at the point of power conversion is exactly equal to the mechanical power appearing at that point.

SOLUTION

$$P_{\text{conv}} = E_A I_A$$

Substituting Equation (8-30) for E_A

$$P_{\text{conv}} = (K \phi \omega) I_A$$

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$$P_{conv} = (K\phi I_A)\omega$$

But from Equation (8-31), $\tau_{ind} = K\phi I_A$, so

$$P_{conv} = \tau_{ind}\,\omega$$

Problems 8-3 to 8-14 refer to the following dc motor:

P_{rated} = 30 hp	$I_{L,rated}$ = 110 A
V_T = 240 V	N_F = 2700 turns per pole
n_{rated} = 1200 r/min	N_{SE} = 12 turns per pole
R_A = 0.19 Ω	R_F = 75 Ω
R_S = 0.02 Ω	R_{adj} = 100 to 400 Ω

Rotational losses = 3550 W at full load
Magnetization curve as shown in Figure P8-1
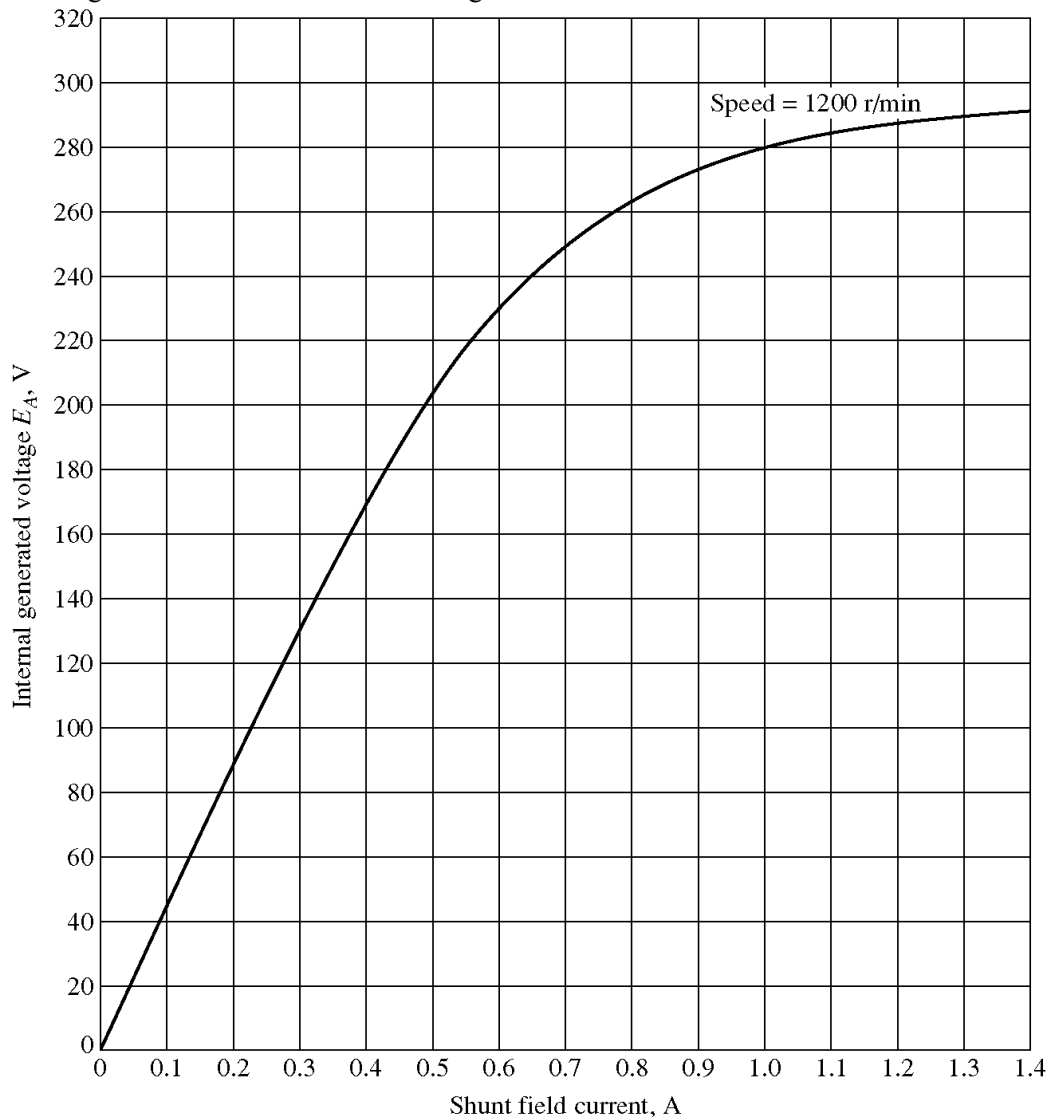
In Problems 8-3 through 8-9, assume that the motor described above can be connected in shunt. The equivalent circuit of the shunt motor is shown in Figure P8-2.
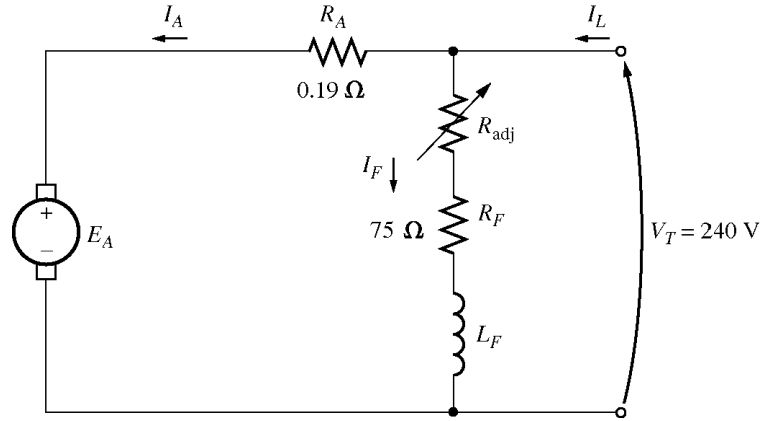
8-3. If the resistor R_{adj} is adjusted to 175 Ω what is the rotational speed of the motor at no-load conditions?

SOLUTION At no-load conditions, $E_A = V_T = 240$ V . The field current is given by

$$I_F = \frac{V_T}{R_{\text{adj}} + R_F} = \frac{240 \text{ V}}{175\,\Omega + 75\,\Omega} = \frac{240 \text{ V}}{250\,\Omega} = 0.96 \text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 277 V at a speed n_o of 1200 r/min. Therefore, the speed n with a voltage of 240 V would be

$$\frac{E_A}{E_{Ao}} = \frac{n}{n_o}$$

$$n = \left(\frac{E_A}{E_{Ao}}\right) n_o = \left(\frac{240 \text{ V}}{277 \text{ V}}\right)(1200 \text{ r/min}) = 1040 \text{ r/min}$$

8-4. Assuming no armature reaction, what is the speed of the motor at full load? What is the speed regulation of the motor?

SOLUTION At full load, the armature current is

$$I_A = I_L - I_F = 110 \text{ A} - 0.96 \text{ A} = 109 \text{ A}$$

The internal generated voltage E_A is

$$E_A = V_T - I_A R_A = 240 \text{ V} - (109 \text{ A})(0.19\,\Omega) = 219.3 \text{ V}$$

The field current is the same as before, and there is no armature reaction, so E_{Ao} is still 277 V at a speed n_o of 1200 r/min. Therefore,

$$n = \left(\frac{E_A}{E_{Ao}}\right) n_o = \left(\frac{219.3 \text{ V}}{277 \text{ V}}\right)(1200 \text{ r/min}) = 950 \text{ r/min}$$

The speed regulation is

$$\text{SR} = \frac{n_{\text{nl}} - n_{\text{fl}}}{n_{\text{fl}}} \times 100\% = \frac{1040 \text{ r/min} - 950 \text{ r/min}}{950 \text{ r/min}} \times 100\% = 9.5\%$$

8-5. If the motor is operating at full load and if its variable resistance R_{adj} is increased to 250 Ω, what is the new speed of the motor? Compare the full-load speed of the motor with R_{adj} = 175 Ω to the full-load speed with R_{adj} = 250 Ω. (Assume no armature reaction, as in the previous problem.)

SOLUTION If R_{adj} is set to 250 Ω, the field current is now

$$I_F = \frac{V_T}{R_{adj} + R_F} = \frac{240 \text{ V}}{250 \, \Omega + 75 \, \Omega} = \frac{240 \text{ V}}{250 \, \Omega} = 0.739 \text{ A}$$

Since the motor is still at full load, E_A is still 219.3 V. From the magnetization curve (Figure P8-1), this current would produce a voltage E_{Ao} of 256 V at a speed n_o of 1200 r/min. Therefore,

$$n = \left(\frac{E_A}{E_{Ao}}\right) n_o = \left(\frac{219.3 \text{ V}}{256 \text{ V}}\right)(1200 \text{ r/min}) = 1028 \text{ r/min}$$

Note that R_{adj} has increased, and as a result the speed of the motor n increased.

8-6. Assume that the motor is operating at full load and that the variable resistor R_{adj} is again 175 Ω. If the armature reaction is 1200 A·turns at full load, what is the speed of the motor? How does it compare to the result for Problem 8-5?

SOLUTION The field current is again 0.96 A, and the motor is again at full load conditions. However, this time there is an armature reaction of 1200 A·turns, and the *effective* field current is

$$I_F^{\ *} = I_F - \frac{AR}{N_F} = 0.96 \text{ A} - \frac{1200 \text{ A} \cdot \text{turns}}{2700 \text{ turns}} = 0.516 \text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 210 V at a speed n_o of 1200 r/min. The actual internal generated voltage E_A at these conditions is

$$E_A = V_T - I_A R_A = 240 \text{ V} - (109 \text{ A})(0.19 \, \Omega) = 219.3 \text{ V}$$

Therefore, the speed n with a voltage of 240 V would be

$$n = \left(\frac{E_A}{E_{Ao}}\right) n_o = \left(\frac{219.3 \text{ V}}{210 \text{ V}}\right)(1200 \text{ r/min}) = 1253 \text{ r/min}$$

If all other conditions are the same, the motor with armature reaction runs at a higher speed than the motor without armature reaction.

8-7. If R_{adj} can be adjusted from 100 to 400 Ω, what are the maximum and minimum no-load speeds possible with this motor?

SOLUTION The minimum speed will occur when R_{adj} = 100 Ω, and the maximum speed will occur when R_{adj} = 400 Ω. The field current when R_{adj} = 100 Ω is:

$$I_F = \frac{V_T}{R_{adj} + R_F} = \frac{240 \text{ V}}{100 \, \Omega + 75 \, \Omega} = \frac{240 \text{ V}}{175 \, \Omega} = 1.37 \text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 289 V at a speed n_o of 1200 r/min. Therefore, the speed n with a voltage of 240 V would be

$$\frac{E_A}{E_{Ao}} = \frac{n}{n_o}$$

$$n = \left(\frac{E_A}{E_{Ao}}\right)n_o = \left(\frac{240\text{ V}}{289\text{ V}}\right)(1200\text{ r/min}) = 997\text{ r/min}$$

The field current when $R_{adj} = 400\ \Omega$ is:

$$I_F = \frac{V_T}{R_{adj} + R_F} = \frac{240\text{ V}}{400\ \Omega + 75\ \Omega} = \frac{240\text{ V}}{475\ \Omega} = 0.505\text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 207 V at a speed n_o of 1200 r/min. Therefore, the speed n with a voltage of 240 V would be

$$\frac{E_A}{E_{Ao}} = \frac{n}{n_o}$$

$$n = \left(\frac{E_A}{E_{Ao}}\right)n_o = \left(\frac{240\text{ V}}{207\text{ V}}\right)(1200\text{ r/min}) = 1391\text{ r/min}$$

8-8. What is the starting current of this machine if it is started by connecting it directly to the power supply V_T? How does this starting current compare to the full-load current of the motor?

SOLUTION The starting current of this machine (ignoring the small field current) is

$$I_{L,\text{start}} = \frac{V_T}{R_A} = \frac{240\text{ V}}{0.19\ \Omega} = 1263\text{ A}$$

The rated current is 110 A, so the starting current is 11.5 times greater than the full-load current. This much current is extremely likely to damage the motor.

8-9. Plot the torque-speed characteristic of this motor assuming no armature reaction, and again assuming a full-load armature reaction of 1200 A·turns.

SOLUTION This problem is best solved with MATLAB, since it involves calculating the torque-speed values at many points. A MATLAB program to calculate and display both torque-speed characteristics is shown below.

```
% M-file: prob8_9.m
% M-file to create a plot of the torque-speed curve of the
%   the shunt dc motor with and without armature reaction.

% Get the magnetization curve.  This file contains the
% three variables if_value, ea_value, and n_0.
load p81.mat

% First, initialize the values needed in this program.
v_t = 240;              % Terminal voltage (V)
r_f = 75;               % Field resistance (ohms)
r_adj = 175;            % Adjustable resistance (ohms)
r_a = 0.19;             % Armature resistance (ohms)
i_l = 0:2:110;          % Line currents (A)
n_f = 2700;             % Number of turns on field
f_ar0 = 1200;           % Armature reaction @ 110 A (A-t/m)
```

```matlab
% Calculate the armature current for each load.
i_a = i_l - v_t / (r_f + r_adj);

% Now calculate the internal generated voltage for
% each armature current.
e_a = v_t - i_a * r_a;

% Calculate the armature reaction MMF for each armature
% current.
f_ar = (i_a / 110) * f_ar0;

% Calculate the effective field current with and without
% armature reaction.  Ther term i_f_ar is the field current
% with armature reaction, and the term i_f_noar is the
% field current without armature reaction.
i_f_ar = v_t / (r_f + r_adj) - f_ar / n_f;
i_f_noar = v_t / (r_f + r_adj);

% Calculate the resulting internal generated voltage at
% 1200 r/min by interpolating the motor's magnetization
% curve.
e_a0_ar = interp1(if_values,ea_values,i_f_ar);
e_a0_noar = interp1(if_values,ea_values,i_f_noar);

% Calculate the resulting speed from Equation (8-38).
n_ar = ( e_a ./ e_a0_ar ) * n_0;
n_noar = ( e_a ./ e_a0_noar ) * n_0;

% Calculate the induced torque corresponding to each
% speed from Equations (8-28) and (8-29).
t_ind_ar = e_a .* i_a ./ (n_ar * 2 * pi / 60);
t_ind_noar = e_a .* i_a ./ (n_noar * 2 * pi / 60);

% Plot the torque-speed curves
figure(1);
plot(t_ind_noar,n_noar,'b-','LineWidth',2.0);
hold on;
plot(t_ind_ar,n_ar,'k--','LineWidth',2.0);
xlabel('\bf\tau_{ind} (N-m)');
ylabel('\bf\itn_{m} \rm\bf(r/min)');
title ('\bfShunt DC Motor Torque-Speed Characteristic');
legend('No armature reaction','With armature reaction');
axis([ 0 250 800 1250]);
grid on;
hold off;
```

The resulting plot is shown below:
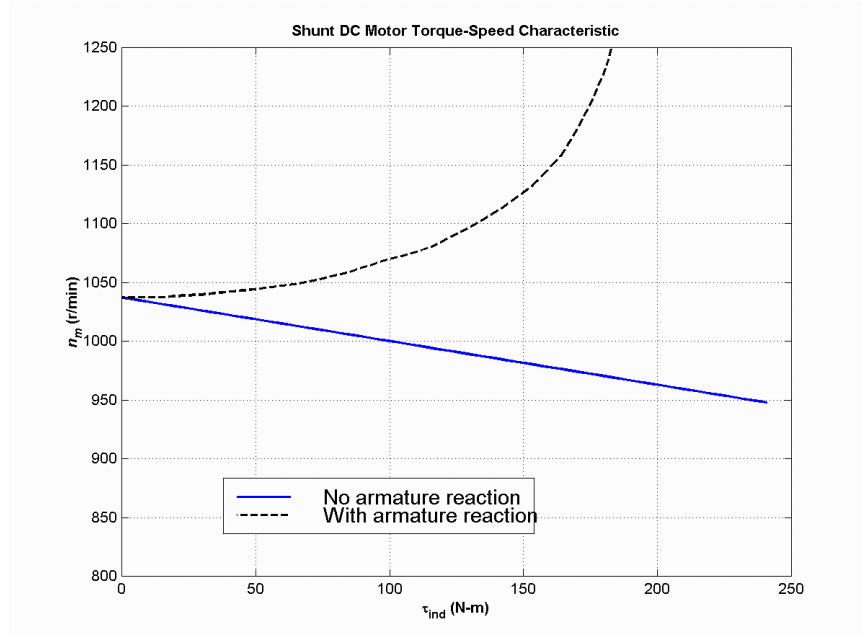
For Problems 8-10 and 8-11, the shunt dc motor is reconnected separately excited, as shown in Figure P8-3. It has a fixed field voltage V_F of 240 V and an armature voltage V_A that can be varied from 120 to 240 V.
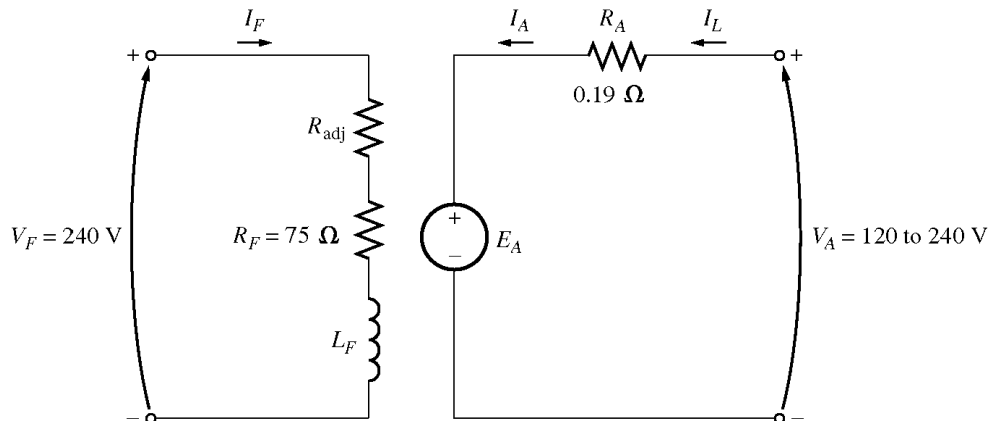
8-10. What is the no-load speed of this separately excited motor when $R_{adj} = 175 \ \Omega$ and *(a)* $V_A = 120$ V, *(b)* $V_A = 180$ V, *(c)* $V_A = 240$ V?

SOLUTION At no-load conditions, $E_A = V_A$. The field current is given by

$$I_F = \frac{V_F}{R_{adj} + R_F} = \frac{240 \ \text{V}}{175 \ \Omega + 75 \ \Omega} = \frac{240 \ \text{V}}{250 \ \Omega} = 0.96 \ \text{A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 277 V at a speed n_o of 1200 r/min. Therefore, the speed n with a voltage of 240 V would be

$$\frac{E_A}{E_{Ao}} = \frac{n}{n_o}$$

$$n = \left(\frac{E_A}{E_{Ao}} \right) n_o$$

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(a) If V_A = 120 V, then E_A = 120 V, and

$$n = \left(\frac{120\text{ V}}{277\text{ V}}\right)(1200\text{ r/min}) = 520\text{ r/min}$$

(a) If V_A = 180 V, then E_A = 180 V, and

$$n = \left(\frac{180\text{ V}}{277\text{ V}}\right)(1200\text{ r/min}) = 780\text{ r/min}$$

(a) If V_A = 240 V, then E_A = 240 V, and

$$n = \left(\frac{240\text{ V}}{277\text{ V}}\right)(1200\text{ r/min}) = 1040\text{ r/min}$$

8-11. For the separately excited motor of Problem 8-10:

(a) What is the maximum no-load speed attainable by varying both V_A and R_{adj}?

(b) What is the minimum no-load speed attainable by varying both V_A and R_{adj}?

SOLUTION

(a) The maximum speed will occur with the maximum V_A and the maximum R_{adj}. The field current when R_{adj} = 400 Ω is:

$$I_F = \frac{V_T}{R_{adj} + R_F} = \frac{240\text{ V}}{400\,\Omega + 75\,\Omega} = \frac{240\text{ V}}{475\,\Omega} = 0.505\text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 207 V at a speed n_o of 1200 r/min. At no-load conditions, the maximum internal generated voltage $E_A = V_A$ = 240 V. Therefore, the speed n with a voltage of 240 V would be

$$\frac{E_A}{E_{Ao}} = \frac{n}{n_o}$$

$$n = \left(\frac{E_A}{E_{Ao}}\right)n_o = \left(\frac{240\text{ V}}{207\text{ V}}\right)(1200\text{ r/min}) = 1391\text{ r/min}$$

(b) The minimum speed will occur with the minimum V_A and the minimum R_{adj}. The field current when R_{adj} = 100 Ω is:

$$I_F = \frac{V_T}{R_{adj} + R_F} = \frac{240\text{ V}}{100\,\Omega + 75\,\Omega} = \frac{240\text{ V}}{175\,\Omega} = 1.37\text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 289 V at a speed n_o of 1200 r/min. At no-load conditions, the minimum internal generated voltage $E_A = V_A$ = 120 V. Therefore, the speed n with a voltage of 120 V would be

$$\frac{E_A}{E_{Ao}} = \frac{n}{n_o}$$

$$n = \left(\frac{E_A}{E_{Ao}}\right) n_o = \left(\frac{120 \text{ V}}{289 \text{ V}}\right)(1200 \text{ r/min}) = 498 \text{ r/min}$$

8-12. If the motor is connected cumulatively compounded as shown in Figure P8-4 and if $R_{adj} = 175\ \Omega$, what is its no-load speed? What is its full-load speed? What is its speed regulation? Calculate and plot the torque-speed characteristic for this motor. (Neglect armature effects in this problem.)
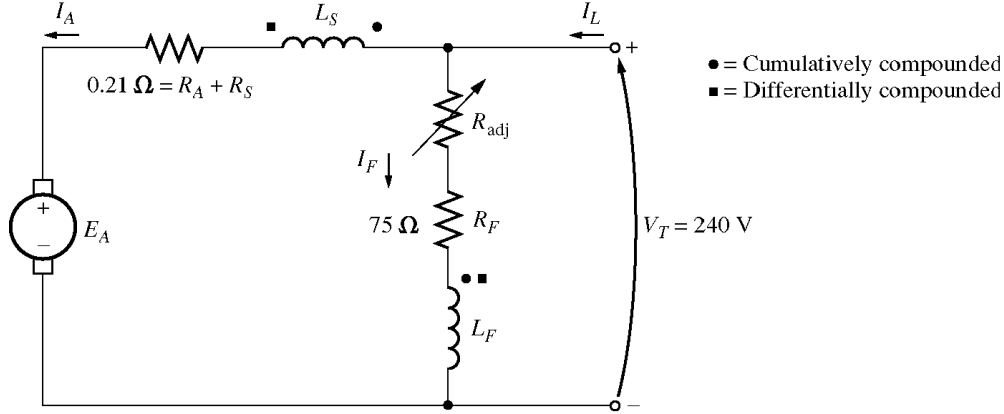
SOLUTION *At no-load conditions*, $E_A = V_T = 240 \text{ V}$. The field current is given by

$$I_F = \frac{V_T}{R_{adj} + R_F} = \frac{240 \text{ V}}{175\ \Omega + 75\ \Omega} = \frac{240 \text{ V}}{250\ \Omega} = 0.96 \text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 277 V at a speed n_o of 1200 r/min. Therefore, the speed n with a voltage of 240 V would be

$$\frac{E_A}{E_{Ao}} = \frac{n}{n_o}$$

$$n = \left(\frac{E_A}{E_{Ao}}\right) n_o = \left(\frac{240 \text{ V}}{277 \text{ V}}\right)(1200 \text{ r/min}) = 1040 \text{ r/min}$$

At full load conditions, the armature current is

$$I_A = I_L - I_F = 110 \text{ A} - 0.96 \text{ A} = 109 \text{ A}$$

The internal generated voltage E_A is

$$E_A = V_T - I_A R_A = 240 \text{ V} - (109 \text{ A})(0.21\ \Omega) = 217.1 \text{ V}$$

The equivalent field current is

$$I_F^* = I_F + \frac{N_{SE}}{N_F} I_A = 0.96 \text{ A} + \frac{12 \text{ turns}}{2700 \text{ turns}}(109 \text{ A}) = 1.44 \text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 290 V at a speed n_o of 1200 r/min. Therefore,

$$n = \left(\frac{E_A}{E_{Ao}}\right) n_o = \left(\frac{217.1 \text{ V}}{290 \text{ V}}\right)(1200 \text{ r/min}) = 898 \text{ r/min}$$

The speed regulation is

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$$\text{SR} = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\% = \frac{1040 \text{ r/min} - 898 \text{ r/min}}{898 \text{ r/min}} \times 100\% = 15.8\%$$

The torque-speed characteristic can best be plotted with a MATLAB program. An appropriate program is shown below.

```
% M-file: prob8_12.m
% M-file to create a plot of the torque-speed curve of the
%    a cumulatively compounded dc motor without
%    armature reaction.

% Get the magnetization curve.  This file contains the
% three variables if_values, ea_values, and n_0.
load p81.mat

% First, initialize the values needed in this program.
v_t = 240;              % Terminal voltage (V)
r_f = 75;               % Field resistance (ohms)
r_adj = 175;            % Adjustable resistance (ohms)
r_a = 0.21;             % Armature + series resistance (ohms)
i_l = 0:2:110;          % Line currents (A)
n_f = 2700;             % Number of turns on shunt field
n_se = 12;              % Number of turns on series field

% Calculate the armature current for each load.
i_a = i_l - v_t / (r_f + r_adj);

% Now calculate the internal generated voltage for
% each armature current.
e_a = v_t - i_a * r_a;

% Calculate the effective field current for each armature
% current.
i_f = v_t / (r_f + r_adj) + (n_se / n_f) * i_a;

% Calculate the resulting internal generated voltage at
% 1200 r/min by interpolating the motor's magnetization
% curve.
e_a0 = interp1(if_values,ea_values,i_f);

% Calculate the resulting speed from Equation (8-38).
n = ( e_a ./ e_a0 ) * n_0;

% Calculate the induced torque corresponding to each
% speed from Equations (8-28) and (8-29).
t_ind = e_a .* i_a ./ (n * 2 * pi / 60);

% Plot the torque-speed curves
figure(1);
plot(t_ind,n,'b-','LineWidth',2.0);
xlabel('\bf\tau_{ind} (N-m)');
ylabel('\bf\itn_{m} \rm\bf(r/min)');
title ('\bfCumulatively-Compounded DC Motor Torque-Speed Characteristic');
axis([0 250 800 1250]);
grid on;
```

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The resulting plot is shown below:

Cumulatively-Compounded DC Motor Torque-Speed Characteristic

Compare this torque-speed curve to that of the shunt motor in Problem 8-9. (Both curves are plotted on the same scale to facilitate comparison.)

8-13. The motor is connected cumulatively compounded and is operating at full load. What will the new speed of the motor be if R_{adj} is increased to 250 Ω? How does the new speed compared to the full-load speed calculated in Problem 8-12?

SOLUTION If R_{adj} is increased to 250 Ω, the field current is given by

$$I_F = \frac{V_T}{R_{\text{adj}} + R_F} = \frac{240 \text{ V}}{250\,\Omega + 75\,\Omega} = \frac{240 \text{ V}}{325\,\Omega} = 0.739 \text{ A}$$

At full load conditions, the armature current is

$$I_A = I_L - I_F = 110 \text{ A} - 0.738 \text{ A} = 109.3 \text{ A}$$

The internal generated voltage E_A is

$$E_A = V_T - I_A R_A = 240 \text{ V} - (109.3 \text{ A})(0.21\,\Omega) = 217 \text{ V}$$

The equivalent field current is

$$I_F^{\;*} = I_F + \frac{N_{\text{SE}}}{N_F} I_A = 0.739 \text{ A} + \frac{12 \text{ turns}}{2700 \text{ turns}} (109.3 \text{ A}) = 1.06 \text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 288 V at a speed n_o of 1200 r/min. Therefore,

$$n = \left(\frac{E_A}{E_{Ao}}\right) n_o = \left(\frac{217 \text{ V}}{288 \text{ V}}\right)(1200 \text{ r/min}) = 904 \text{ r/min}$$

The new full-load speed is higher than the full-load speed in Problem 8-12.

8-14. The motor is now connected differentially compounded.

(a) If R_{adj} = 175 Ω, what is the no-load speed of the motor?

(b) What is the motor's speed when the armature current reaches 20 A? 40 A? 60 A?

(c) Calculate and plot the torque-speed characteristic curve of this motor.

SOLUTION

(a) At no-load conditions, $E_A = V_T = 240$ V. The field current is given by

$$I_F = \frac{V_T}{R_{adj} + R_F} = \frac{240\text{ V}}{175\,\Omega + 75\,\Omega} = \frac{240\text{ V}}{250\,\Omega} = 0.96\text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 277 V at a speed n_o of 1200 r/min. Therefore, the speed n with a voltage of 240 V would be

$$\frac{E_A}{E_{Ao}} = \frac{n}{n_o}$$

$$n = \left(\frac{E_A}{E_{Ao}}\right) n_o = \left(\frac{240\text{ V}}{277\text{ V}}\right)(1200\text{ r/min}) = 1040\text{ r/min}$$

(b) At I_A = 20A, the internal generated voltage E_A is

$$E_A = V_T - I_A R_A = 240\text{ V} - (20\text{ A})(0.21\,\Omega) = 235.8\text{ V}$$

The equivalent field current is

$$I_F^* = I_F - \frac{N_{SE}}{N_F} I_A = 0.96\text{ A} - \frac{12\text{ turns}}{2700\text{ turns}}(20\text{ A}) = 0.871\text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 271 V at a speed n_o of 1200 r/min. Therefore,

$$n = \left(\frac{E_A}{E_{Ao}}\right) n_o = \left(\frac{235.8\text{ V}}{271\text{ V}}\right)(1200\text{ r/min}) = 1044\text{ r/min}$$

At I_A = 40A, the internal generated voltage E_A is

$$E_A = V_T - I_A R_A = 240\text{ V} - (40\text{ A})(0.21\,\Omega) = 231.6\text{ V}$$

The equivalent field current is

$$I_F^* = I_F - \frac{N_{SE}}{N_F} I_A = 0.96\text{ A} - \frac{12\text{ turns}}{2700\text{ turns}}(40\text{ A}) = 0.782\text{ A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 261 V at a speed n_o of 1200 r/min. Therefore,

$$n = \left(\frac{E_A}{E_{Ao}}\right) n_o = \left(\frac{231.6\text{ V}}{261\text{ V}}\right)(1200\text{ r/min}) = 1065\text{ r/min}$$

At I_A = 60A, the internal generated voltage E_A is

$$E_A = V_T - I_A R_A = 240\text{ V} - (60\text{ A})(0.21\,\Omega) = 227.4\text{ V}$$

The equivalent field current is

$$I_F^* = I_F - \frac{N_{SE}}{N_F} I_A = 0.96\,\text{A} - \frac{12\,\text{turns}}{2700\,\text{turns}}\left(60\,\text{A}\right) = 0.693\,\text{A}$$

From Figure P8-1, this field current would produce an internal generated voltage E_{Ao} of 249 V at a speed n_o of 1200 r/min. Therefore,

$$n = \left(\frac{E_A}{E_{Ao}}\right)n_o = \left(\frac{227.4\,\text{V}}{249\,\text{V}}\right)\left(1200\,\text{r/min}\right) = 1099\,\text{r/min}$$

(c) The torque-speed characteristic can best be plotted with a MATLAB program. An appropriate program is shown below.

```
% M-file: prob8_14.m
% M-file to create a plot of the torque-speed curve of the
%    a differentially compounded dc motor withwithout
%    armature reaction.

% Get the magnetization curve.  This file contains the
% three variables if_value, ea_value, and n_0.
load p81.mat

% First, initialize the values needed in this program.
v_t = 240;                % Terminal voltage (V)
r_f = 75;                 % Field resistance (ohms)
r_adj = 175;              % Adjustable resistance (ohms)
r_a = 0.21;               % Armature + series resistance (ohms)
i_l = 0:2:110;            % Line currents (A)
n_f = 2700;               % Number of turns on shunt field
n_se = 12;                % Number of turns on series field

% Calculate the armature current for each load.
i_a = i_l - v_t / (r_f + r_adj);

% Now calculate the internal generated voltage for
% each armature current.
e_a = v_t - i_a * r_a;

% Calculate the effective field current for each armature
% current.
i_f = v_t / (r_f + r_adj) - (n_se / n_f) * i_a;

% Calculate the resulting internal generated voltage at
% 1200 r/min by interpolating the motor's magnetization
% curve.
e_a0 = interp1(if_values,ea_values,i_f);

% Calculate the resulting speed from Equation (8-38).
n = ( e_a ./ e_a0 ) * n_0;

% Calculate the induced torque corresponding to each
% speed from Equations (8-28) and (8-29).
t_ind = e_a .* i_a ./ (n * 2 * pi / 60);
```

```
% Plot the torque-speed curves
figure(1);
plot(t_ind,n,'b-','LineWidth',2.0);
xlabel('\bf\tau_{ind} (N-m)');
ylabel('\bf\itn_{m} \rm\bf(r/min)');
title ('\bfDifferentially-Compounded DC Motor Torque-Speed Characteristic');
axis([0 250 800 1350]);
grid on;
```

The resulting plot is shown below:
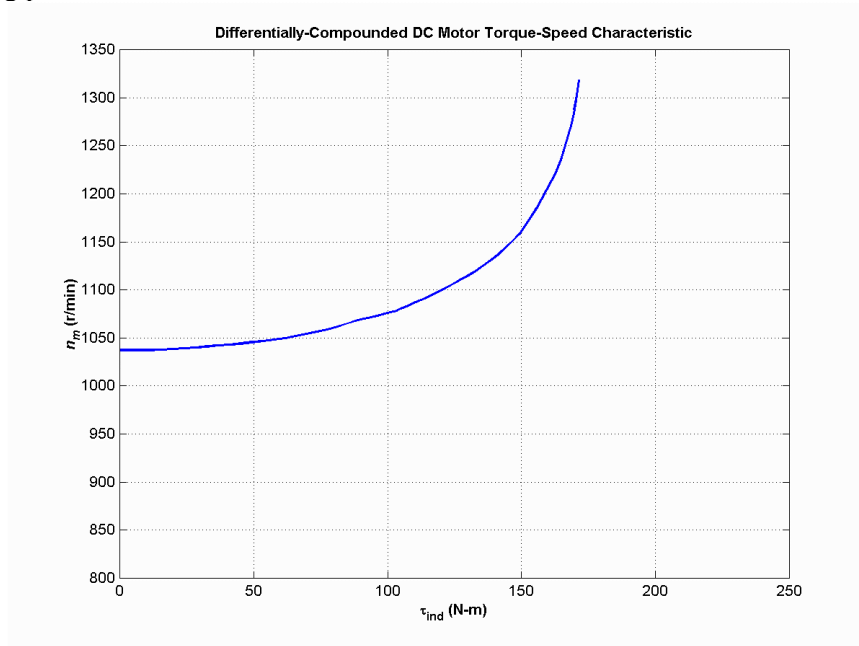
Compare this torque-speed curve to that of the shunt motor in Problem 8-9 and the cumulatively-compounded motor in Problem 8-12. (Note that this plot has a larger vertical scale to accommodate the speed runaway of the differentially-compounded motor.)

8-15. A 7.5-hp 120-V series dc motor has an armature resistance of 0.2 Ω and a series field resistance of 0.16 Ω. At full load, the current input is 58 A, and the rated speed is 1050 r/min. Its magnetization curve is shown in Figure P8-5. The core losses are 200 W, and the mechanical losses are 240 W at full load. Assume that the mechanical losses vary as the cube of the speed of the motor and that the core losses are constant.
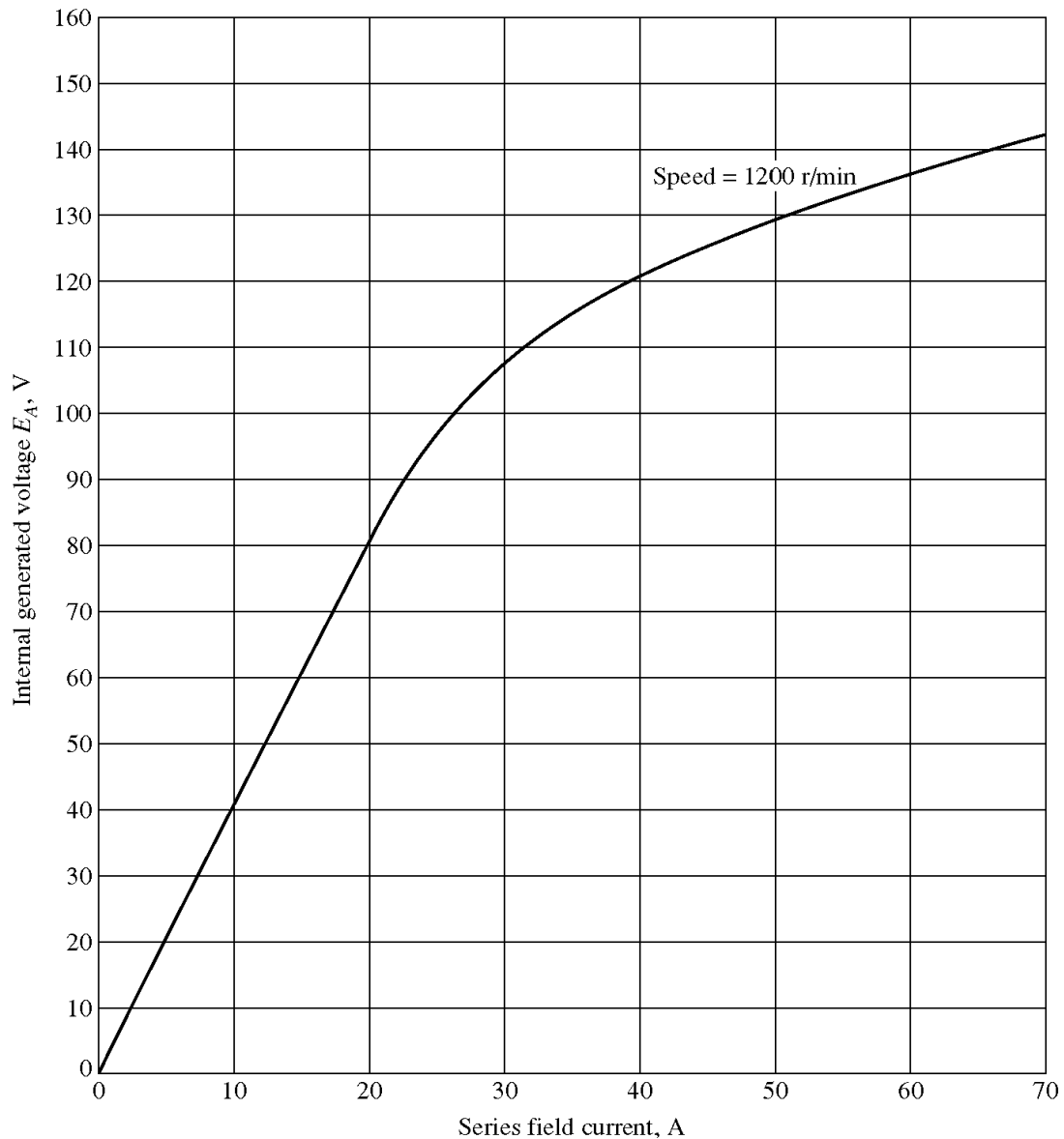
(a) What is the efficiency of the motor at full load?

(b) What are the speed and efficiency of the motor if it is operating at an armature current of 35 A?

(c) Plot the torque-speed characteristic for this motor.

SOLUTION

(a)   The output power of this motor at full load is

$$P_{\text{OUT}} = (7.5\,\text{hp})(746\,\text{W/hp}) = 5595\,\text{W}$$

The input power is

$$P_{\text{IN}} = V_T I_L = (120\,\text{V})(58\,\text{A}) = 6960\,\text{W}$$

Therefore the efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{5595\,\text{W}}{6960\,\text{W}} \times 100\% = 80.4\%$$

(b)   If the armature current is 35 A, then the input power to the motor will be

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$$P_{IN} = V_T I_L = (120 \text{ V})(35 \text{ A}) = 4200 \text{ W}$$

The internal generated voltage at this condition is

$$E_{A2} = V_T - I_A(R_A + R_S) = 120 \text{ V} - (35 \text{ A})(0.20 \, \Omega + 0.16 \, \Omega) = 107.4 \text{ V}$$

and the internal generated voltage at rated conditions is

$$E_{A1} = V_T - I_A(R_A + R_S) = 120 \text{ V} - (58 \text{ A})(0.20 \, \Omega + 0.16 \, \Omega) = 99.1 \text{ V}$$

The final speed is given by the equation

$$\frac{E_{A2}}{E_{A1}} = \frac{K \phi_2 \, \omega_2}{K \phi_2 \, \omega_2} = \frac{E_{Ao,2} \, n_2}{E_{Ao,1} \, n_1}$$

since the ratio $E_{Ao,2} / E_{Ao,1}$ is the same as the ratio ϕ_2 / ϕ_1. Therefore, the final speed is

$$n_2 = \frac{E_{A2}}{E_{A1}} \frac{E_{Ao,1}}{E_{Ao,2}} n_1$$

The internal generated voltage $E_{Ao,2}$ for a current of 35 A and a speed of $n_o = 1200$ r/min is $E_{Ao,2} = 115$ V, and the internal generated voltage $E_{Ao,1}$ for a current of 58 A and a speed of $n_o = 1200$ r/min is $E_{Ao,1} = 134$ V.

$$n_2 = \frac{E_{A2}}{E_{A1}} \frac{E_{Ao,1}}{E_{Ao,2}} n_1 = \left(\frac{107.4 \text{ V}}{99.1 \text{ V}} \right) \left(\frac{134 \text{ V}}{115 \text{ V}} \right) (1050 \text{ r/min}) = 1326 \text{ r/min}$$

The power converted from electrical to mechanical form is

$$P_{conv} = E_A I_A = (107.4 \text{ V})(35 \text{ A}) = 3759 \text{ W}$$

The core losses in the motor are 200 W, and the mechanical losses in the motor are

$$P_{mech} = \left(\frac{n_2}{n_1} \right)^3 (240 \text{ W}) = \left(\frac{1326 \text{ r/min}}{1050 \text{ r/min}} \right)^3 (240 \text{ W}) = 483 \text{ W}$$

Therefore, the output power is

$$P_{OUT} = P_{conv} - P_{mech} - P_{core} = 3759 \text{ W} - 483 \text{ W} - 200 \text{ W} = 3076 \text{ W}$$

and the efficiency is

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{3076 \text{ W}}{4200 \text{ W}} \times 100\% = 73.2\%$$

(c) A MATLAB program to plot the torque-speed characteristic of this motor is shown below:

```
% M-file: prob8_15.m
% M-file to create a plot of the torque-speed curve of the
%   the series dc motor in Problem 8-15.

% Get the magnetization curve.  This file contains the
% three variables if_value, ea_value, and n_0.
load p85.mat

% First, initialize the values needed in this program.
```

```
v_t = 120;                  % Terminal voltage (V)
r_a = 0.36;                 % Armature + field resistance (ohms)
i_a = 9:1:58;               % Armature (line) currents (A)

% Calculate the internal generate voltage e_a.
e_a = v_t - i_a * r_a;

% Calculate the resulting internal generated voltage at
% 1200 r/min by interpolating the motor's magnetization
% curve.
e_a0 = interp1(if_values,ea_values,i_a,'spline');

% Calculate the motor's speed from Equation (8-38).
n = (e_a ./ e_a0) * n_0;

% Calculate the induced torque corresponding to each
% speed from Equations (8-28) and (8-29).
t_ind = e_a .* i_a ./ (n * 2 * pi / 60);

% Plot the torque-speed curve
figure(1);
plot(t_ind,n,'b-','LineWidth',2.0);
hold on;
xlabel('\bf\tau_{ind} (N-m)');
ylabel('\bf\itn_{m} \rm\bf(r/min)');
title ('\bfSeries DC Motor Torque-Speed Characteristic');
grid on;
hold off;
```

The resulting torque-speed characteristic is shown below:
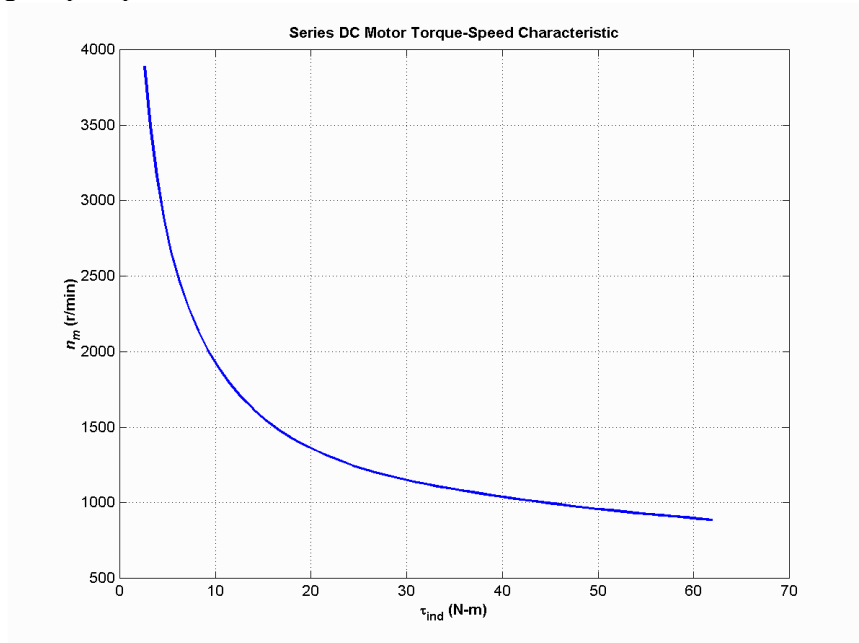
8-16. A 20-hp 240-V 76-A 900 r/min series motor has a field winding of 33 turns per pole. Its armature resistance is 0.09 Ω, and its field resistance is 0.06 Ω. The magnetization curve expressed in terms of magnetomotive force versus E_A at 900 r/min is given by the following table:

E_A, V	95	150	188	212	229	243

$$\mathcal{F}, \text{A·turns} \quad | \quad 500 \quad | \quad 1000 \quad | \quad 1500 \quad | \quad 2000 \quad | \quad 2500 \quad | \quad 3000$$

Armature reaction is negligible in this machine.

(a) Compute the motor's torque, speed, and output power at 33, 67, 100, and 133 percent of full-load armature current. (Neglect rotational losses.)

(b) Plot the terminal characteristic of this machine.

SOLUTION Note that this magnetization curve has been stored in a file called `prob8_16.mat`. The contents of the file are an array of `mmf_values`, an array of `ea_values`, and the term `n_0`. Because the data in the file is relatively sparse, it is important that interpolation be done using smooth curves, so be sure to specify the `'spline'` option in the MATLAB `interp1` function:

```
Eao = interp1(mmf_values,ea_values,mmf,'spline')
```

(a) This calculation must be performed for armature currents of 25.3 A, 50.7 A, 76 A, and 101.3 A.

If $I_A = 23.3$ A, then

$$E_A = V_T - I_A(R_A + R_S) = 240 \text{ V} - (25.3 \text{ A})(0.09 \,\Omega + 0.06 \,\Omega) = 236.2 \text{ V}$$

The magnetomotive force is $\mathcal{F} = NI_A = (33 \text{ turns})(25.3 \text{ A}) = 835 \text{ A·turns}$, which produces a voltage E_{Ao} of 134 V at $n_o = 900$ r/min. Therefore the speed of the motor at these conditions is

$$n = \frac{E_A}{E_{Ao}} n_o = \frac{236.2 \text{ V}}{134 \text{ V}} (900 \text{ r/min}) = 1586 \text{ r/min}$$

The power converted from electrical to mechanical form is

$$P_{\text{conv}} = E_A I_A = (236.2 \text{ V})(25.3 \text{ A}) = 5976 \text{ W}$$

Since the rotational losses are ignored, this is also the output power of the motor. The induced torque is

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{5976 \text{ W}}{(1586 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 36 \text{ N·m}$$

If $I_A = 50.7$ A, then

$$E_A = V_T - I_A(R_A + R_S) = 240 \text{ V} - (50.7 \text{ A})(0.09 \,\Omega + 0.06 \,\Omega) = 232.4 \text{ V}$$

The magnetomotive force is $\mathcal{F} = NI_A = (33 \text{ turns})(50.7 \text{ A}) = 1672 \text{ A·turns}$, which produces a voltage E_{Ao} of 197 V at $n_o = 900$ r/min. Therefore the speed of the motor at these conditions is

$$n = \frac{E_A}{E_{Ao}} n_o = \frac{232.4 \text{ V}}{197 \text{ V}} (900 \text{ r/min}) = 1062 \text{ r/min}$$

The power converted from electrical to mechanical form is

$$P_{\text{conv}} = E_A I_A = (232.4 \text{ V})(50.7 \text{ A}) = 11{,}780 \text{ W}$$

Since the rotational losses are ignored, this is also the output power of the motor. The induced torque is

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{11{,}780 \text{ W}}{(1062 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 106 \text{ N} \cdot \text{m}$$

If $I_A = 76$ A, then

$$E_A = V_T - I_A(R_A + R_S) = 240 \text{ V} - (76 \text{ A})(0.09\,\Omega + 0.06\,\Omega) = 228.6 \text{ V}$$

The magnetomotive force is $\mathcal{F} = NI_A = (33 \text{ turns})(76 \text{ A}) = 2508 \text{ A} \cdot \text{turns}$, which produces a voltage E_{Ao} of 229 V at $n_o = 900$ r/min. Therefore the speed of the motor at these conditions is

$$n = \frac{E_A}{E_{Ao}} n_o = \frac{228.6 \text{ V}}{229 \text{ V}}(900 \text{ r/min}) = 899 \text{ r/min}$$

The power converted from electrical to mechanical form is

$$P_{conv} = E_A I_A = (228.6 \text{ V})(76 \text{ A}) = 17{,}370 \text{ W}$$

Since the rotational losses are ignored, this is also the output power of the motor. The induced torque is

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{17{,}370 \text{ W}}{(899 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 185 \text{ N} \cdot \text{m}$$

If $I_A = 101.3$ A, then

$$E_A = V_T - I_A(R_A + R_S) = 240 \text{ V} - (101.3 \text{ A})(0.09\,\Omega + 0.06\,\Omega) = 224.8 \text{ V}$$

The magnetomotive force is $\mathcal{F} = NI_A = (33 \text{ turns})(101.3 \text{ A}) = 3343 \text{ A} \cdot \text{turns}$, which produces a voltage E_{Ao} of 252 V at $n_o = 900$ r/min. Therefore the speed of the motor at these conditions is

$$n = \frac{E_A}{E_{Ao}} n_o = \frac{224.8 \text{ V}}{252 \text{ V}}(900 \text{ r/min}) = 803 \text{ r/min}$$

The power converted from electrical to mechanical form is

$$P_{conv} = E_A I_A = (224.8 \text{ V})(101.3 \text{ A}) = 22{,}770 \text{ W}$$

Since the rotational losses are ignored, this is also the output power of the motor. The induced torque is

$$\tau_{ind} = \frac{P_{conv}}{\omega_m} = \frac{22{,}770 \text{ W}}{(803 \text{ r/min})\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)} = 271 \text{ N} \cdot \text{m}$$

(b) A MATLAB program to plot the torque-speed characteristic of this motor is shown below:

```
% M-file: prob8_16.m
% M-file to create a plot of the torque-speed curve of the
%    the series dc motor in Problem 8-16.

% Get the magnetization curve.  This file contains the
% three variables mmf_values, ea_values, and n_0.
load prob8_16.mat

% First, initialize the values needed in this program.
```

```
v_t = 240;                    % Terminal voltage (V)
r_a = 0.15;                   % Armature + field resistance (ohms)
i_a = 15:1:76;                % Armature (line) currents (A)
n_s = 33;                     % Number of series turns on field

% Calculate the MMF for each load
f = n_s * i_a;

% Calculate the internal generate voltage e_a.
e_a = v_t - i_a * r_a;

% Calculate the resulting internal generated voltage at
% 900 r/min by interpolating the motor's magnetization
% curve.  Specify cubic spline interpolation to provide
% good results with this sparse magnetization curve.
e_a0 = interp1(mmf_values,ea_values,f,'spline');

% Calculate the motor's speed from Equation (8-38).
n = (e_a ./ e_a0) * n_0;

% Calculate the induced torque corresponding to each
% speed from Equations (8-28) and (8-29).
t_ind = e_a .* i_a ./ (n * 2 * pi / 60);

% Plot the torque-speed curve
figure(1);
plot(t_ind,n,'b-','LineWidth',2.0);
hold on;
xlabel('\bf\tau_{ind} (N-m)');
ylabel('\bf\itn_{m} \rm\bf(r/min)');
title ('\bfSeries DC Motor Torque-Speed Characteristic');
%axis([ 0 700 0 5000]);
grid on;
hold off;
```

The resulting torque-speed characteristic is shown below:

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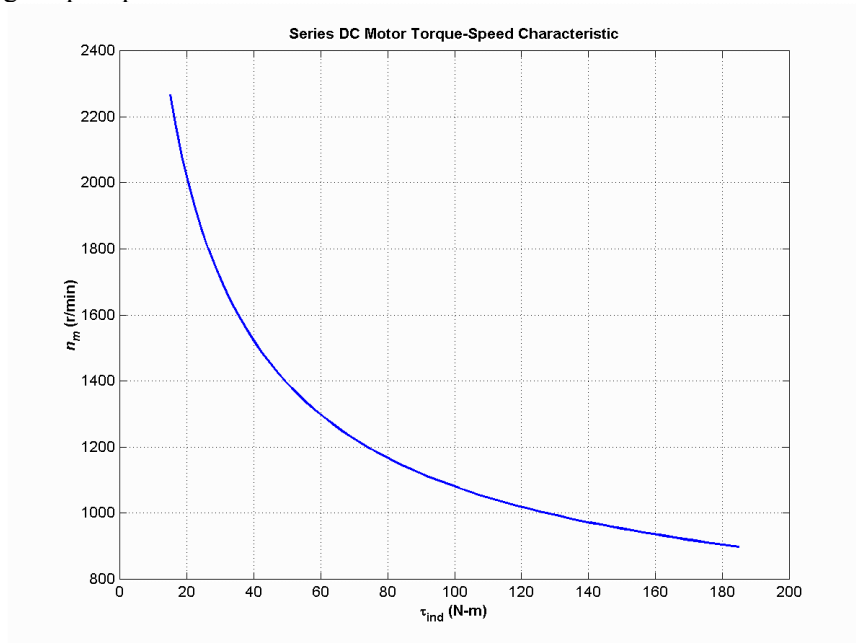
8-17. A 300-hp 440-V 560-A, 863 r/min shunt dc motor has been tested, and the following data were taken:

Blocked-rotor test:

$V_A = 16.3$ V   exclusive of brushes $V_F = 440$ V

$I_A = 500$ A $I_F = 8.86$ A

No-load operation:

$V_A = 16.3$ V   including brushes $I_F = 8.76$ A

$I_A = 23.1$ A $n = 863$ r/min

What is this motor's efficiency at the rated conditions? [*Note*: Assume that (1) the brush voltage drop is 2 V; (2) the core loss is to be determined at an armature voltage equal to the armature voltage under full load; and (3) stray load losses are 1 percent of full load.]

SOLUTION The armature resistance of this motor is

$$R_A = \frac{V_{A,\text{br}}}{I_{A,\text{br}}} = \frac{16.3\text{ V}}{500\text{ A}} = 0.0326\,\Omega$$

Under no-load conditions, the core and mechanical losses taken together (that is, the *rotational losses*) of this motor are equal to the product of the internal generated voltage E_A and the armature current I_A, since this is no output power from the motor at no-load conditions. Therefore, the rotational losses at rated speed can be found as

$$E_A = V_A - V_{\text{brush}} - I_A R_A = 442\text{ V} - 2\text{ V} - (23.1\text{ A})(0.0326\,\Omega) = 439.2\text{ V}$$

$$P_{\text{rot}} = P_{\text{conv}} = E_A I_A = (439.2\text{ V})(23.1\text{ A}) = 10.15\text{ kW}$$

The input power to the motor *at full load* is

$$P_{\text{IN}} = V_T I_L = (440\text{ V})(560\text{ A}) = 246.4\text{ kW}$$

The output power from the motor at full load is

$$P_{\text{OUT}} = P_{\text{IN}} - P_{\text{CU}} - P_{\text{rot}} - P_{\text{brush}} - P_{\text{stray}}$$

The copper losses are

$$P_{\text{CU}} = I_A{}^2 R_A + V_F I_F = (560\text{ A})^2 (0.0326\,\Omega) + (440\text{ V})(8.86\text{ A}) = 14.1\text{ kW}$$

The brush losses are

$$P_{\text{brush}} = V_{\text{brush}} I_A = (2\text{ V})(560\text{ A}) = 1120\text{ W}$$

Therefore,

$$P_{\text{OUT}} = P_{\text{IN}} - P_{\text{CU}} - P_{\text{rot}} - P_{\text{brush}} - P_{\text{stray}}$$
$$P_{\text{OUT}} = 246.4\text{ kW} - 14.1\text{ kW} - 10.15\text{ kW} - 1.12\text{ kW} - 2.46\text{ kW} = 218.6\text{ kW}$$

The motor's efficiency at full load is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{218.6\text{ kW}}{246.4\text{ kW}} \times 100\% = 88.7\%$$

Problems 8-18 to 8-21 refer to a 240-V 100-A dc motor which has both shunt and series windings. Its characteristics are

$$R_A = 0.14 \ \Omega \qquad\qquad\qquad N_F = 1500 \text{ turns}$$
$$R_S = 0.05 \ \Omega \qquad\qquad\qquad N_{SE} = 10 \text{ turns}$$
$$R_F = 200 \ \Omega \qquad\qquad\qquad n_m = 1800 \text{ r/min}$$
$$R_{adj} = 0 \text{ to } 300 \ \Omega, \text{ currently set to } 120 \ \Omega$$

This motor has compensating windings and interpoles. The magnetization curve for this motor at 1800 r/min is shown in Figure P8-6.
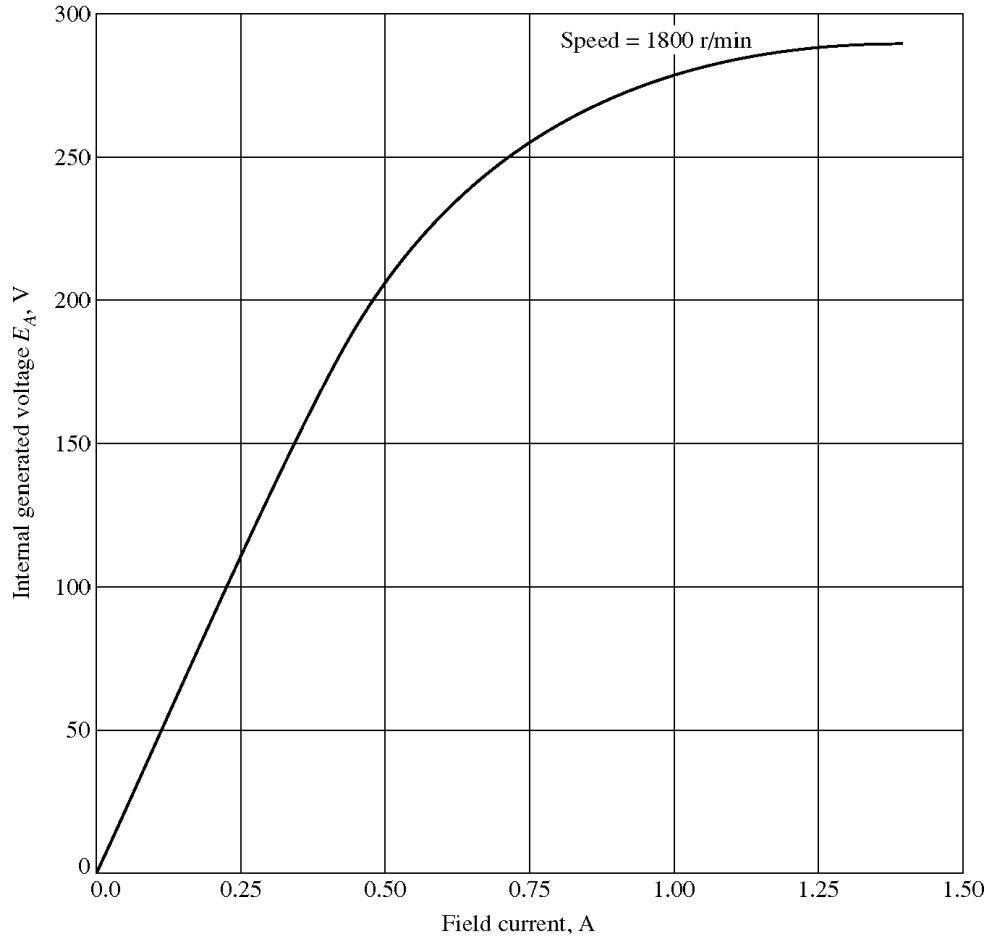
8-18. The motor described above is connected in *shunt*.

(a) What is the no-load speed of this motor when $R_{adj} = 120 \ \Omega$?

(b) What is its full-load speed?

(c) Under no-load conditions, what range of possible speeds can be achieved by adjusting R_{adj}?

SOLUTION

(a) If $R_{adj} = 120 \ \Omega$, the total field resistance is 320 Ω, and the resulting field current is

$$I_F = \frac{V_T}{R_F + R_{adj}} = \frac{240 \text{ V}}{200 \ \Omega + 120 \ \Omega} = 0.75 \text{ A}$$

This field current would produce a voltage E_{Ao} of 256 V at a speed of $n_o = 1800$ r/min. The actual E_A is 240 V, so the actual speed will be

$$n = \frac{E_A}{E_{Ao}} n_o = \frac{240\text{ V}}{256\text{ V}} (1800\text{ r/min}) = 1688\ \text{r/min}$$

(b) At full load, $I_A = I_L - I_F = 100\text{ A} - 0.75\text{ A} = 99.25\text{ A}$, and

$$E_A = V_T - I_A R_A = 240\text{ V} - (99.25\text{ A})(0.14\ \Omega) = 226.1\text{ V}$$

Therefore, the speed at full load will be

$$n = \frac{E_A}{E_{Ao}} n_o = \frac{226.1\text{ V}}{256\text{ V}} (1800\text{ r/min}) = 1590\text{ r/min}$$

(c) If R_{adj} is maximum at no-load conditions, the total resistance is 500 Ω, and

$$I_F = \frac{V_T}{R_F + R_{\text{adj}}} = \frac{240\text{ V}}{200\ \Omega + 300\ \Omega} = 0.48\text{ A}$$

This field current would produce a voltage E_{Ao} of 200 V at a speed of $n_o = 1800$ r/min. The actual E_A is 240 V, so the actual speed will be

$$n = \frac{E_A}{E_{Ao}} n_o = \frac{240\text{ V}}{200\text{ V}} (1800\text{ r/min}) = 2160\text{ r/min}$$

If R_{adj} is minimum at no-load conditions, the total resistance is 200 Ω, and

$$I_F = \frac{V_T}{R_F + R_{\text{adj}}} = \frac{240\text{ V}}{200\ \Omega + 0\ \Omega} = 1.2\text{ A}$$

This field current would produce a voltage E_{Ao} of 287 V at a speed of $n_o = 1800$ r/min. The actual E_A is 240 V, so the actual speed will be

$$n = \frac{E_A}{E_{Ao}} n_o = \frac{240\text{ V}}{287\text{ V}} (1800\text{ r/min}) = 1505\text{ r/min}$$

8-19. This machine is now connected as a cumulatively compounded dc motor with $R_{\text{adj}} = 120\ \Omega$.

(a) What is the full-load speed of this motor?

(b) Plot the torque-speed characteristic for this motor.

(c) What is its speed regulation?

SOLUTION

(a) At full load, $I_A = I_L - I_F = 100\text{ A} - 0.75\text{ A} = 99.25\text{ A}$, and

$$E_A = V_T - I_A (R_A + R_S) = 240\text{ V} - (99.25\text{ A})(0.14\ \Omega + 0.05\ \Omega) = 221.1\text{ V}$$

The actual field current will be

$$I_F = \frac{V_T}{R_F + R_{\text{adj}}} = \frac{240\text{ V}}{200\ \Omega + 120\ \Omega} = 0.75\text{ A}$$

and the effective field current will be

$$I_F{}^* = I_F + \frac{N_{SE}}{N_F} I_A = 0.75 \, \text{A} + \frac{8 \, \text{turns}}{1500 \, \text{turns}} (99.25 \, \text{A}) = 1.28 \, \text{A}$$

This field current would produce a voltage E_{Ao} of 288 V at a speed of n_o = 1800 r/min. The actual E_A is 240 V, so the actual speed at full load will be

$$n = \frac{E_A}{E_{Ao}} n_o = \frac{221.1 \, \text{V}}{288 \, \text{V}} (1800 \, \text{r/min}) = 1382 \, \text{r/min}$$

(b) A MATLAB program to calculate the torque-speed characteristic of this motor is shown below:

```
% M-file: prob8_19.m
% M-file to create a plot of the torque-speed curve of the
%   a cumulatively compounded dc motor.

% Get the magnetization curve.  This file contains the
% three variables if_values, ea_values, and n_0.
load p86.mat

% First, initialize the values needed in this program.
v_t = 240;              % Terminal voltage (V)
r_f = 200;              % Field resistance (ohms)
r_adj = 120;            % Adjustable resistance (ohms)
r_a = 0.19;            % Armature + series resistance (ohms)
i_l = 0:2:100;          % Line currents (A)
n_f = 1500;            % Number of turns on shunt field
n_se = 8;              % Number of turns on series field

% Calculate the armature current for each load.
i_a = i_l - v_t / (r_f + r_adj);

% Now calculate the internal generated voltage for
% each armature current.
e_a = v_t - i_a * r_a;

% Calculate the effective field current for each armature
% current.
i_f = v_t / (r_f + r_adj) + (n_se / n_f) * i_a;

% Calculate the resulting internal generated voltage at
% 1800 r/min by interpolating the motor's magnetization
% curve.
e_a0 = interp1(if_values,ea_values,i_f);

% Calculate the resulting speed from Equation (8-38).
n = ( e_a ./ e_a0 ) * n_0;

% Calculate the induced torque corresponding to each
% speed from Equations (8-28) and (8-29).
t_ind = e_a .* i_a ./ (n * 2 * pi / 60);

% Plot the torque-speed curves
figure(1);
plot(t_ind,n,'b-','LineWidth',2.0);
xlabel('\bf\tau_{ind} (N-m)');
```

```
ylabel('\bf\itn_{m} \rm\bf(r/min)');
title ('\bfCumulatively-Compounded DC Motor Torque-Speed Characteristic');
axis([0 160 1000 2000]);
grid on;
```

The resulting torque-speed characteristic is shown below:

(c) The no-load speed of this machine is the same as the no-load speed of the corresponding shunt dc motor with R_{adj} = 120 Ω, which is 1688 r/min. The speed regulation of this motor is thus

$$\text{SR} = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\% = \frac{1688 \text{ r/min} - 1382 \text{ r/min}}{1382 \text{ r/min}} \times 100\% = 22.1\%$$

8-20. The motor is reconnected differentially compounded with R_{adj} = 120 Ω. Derive the shape of its torque-speed characteristic.

SOLUTION A MATLAB program to calculate the torque-speed characteristic of this motor is shown below:

```
% M-file: prob8_20.m
% M-file to create a plot of the torque-speed curve of the
%   a differentially compounded dc motor.

% Get the magnetization curve.  This file contains the
% three variables if_values, ea_values, and n_0.
load p86.mat

% First, initialize the values needed in this program.
v_t = 240;              % Terminal voltage (V)
r_f = 200;              % Field resistance (ohms)
r_adj = 120;            % Adjustable resistance (ohms)
r_a = 0.19;             % Armature + series resistance (ohms)
i_l = 0:2:45;           % Line currents (A)
n_f = 1500;             % Number of turns on shunt field
n_se = 8;               % Number of turns on series field

% Calculate the armature current for each load.
```

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```
i_a = i_l - v_t / (r_f + r_adj);

% Now calculate the internal generated voltage for
% each armature current.
e_a = v_t - i_a * r_a;

% Calculate the effective field current for each armature
% current.
i_f = v_t / (r_f + r_adj) - (n_se / n_f) * i_a;

% Calculate the resulting internal generated voltage at
% 1800 r/min by interpolating the motor's magnetization
% curve.
e_a0 = interp1(if_values,ea_values,i_f);

% Calculate the resulting speed from Equation (8-38).
n = ( e_a ./ e_a0 ) * n_0;

% Calculate the induced torque corresponding to each
% speed from Equations (8-28) and (8-29).
t_ind = e_a .* i_a ./ (n * 2 * pi / 60);

% Plot the torque-speed curves
figure(1);
plot(t_ind,n,'b-','LineWidth',2.0);
xlabel('\bf\tau_{ind} (N-m)');
ylabel('\bf\itn_{m} \rm\bf(r/min)');
title ('\bfDifferentially-Compounded DC Motor Torque-Speed Characteristic');
axis([0 160 1000 2000]);
grid on;
```

The resulting torque-speed characteristic is shown below:

This curve is plotted on the same scale as the torque-speed curve in Problem 8-19. Compare the two curves.

8-21. A series motor is now constructed from this machine by leaving the shunt field out entirely. Derive the torque-speed characteristic of the resulting motor.

SOLUTION This motor will have extremely high speeds, since there are only a few series turns, and the flux in the motor will be very small. A MATLAB program to calculate the torque-speed characteristic of this motor is shown below:

```
% M-file: prob8_21.m
% M-file to create a plot of the torque-speed curve of the
%   a series dc motor.  This motor was formed by removing
%   the shunt field from the cumulatively-compounded machine
%   of Problem 8-19.

% Get the magnetization curve.  This file contains the
% three variables if_values, ea_values, and n_0.
load p86.mat

% First, initialize the values needed in this program.
v_t = 240;              % Terminal voltage (V)
r_f = 200;              % Field resistance (ohms)
r_adj = 120;            % Adjustable resistance (ohms)
r_a = 0.19;             % Armature + series resistance (ohms)
i_l = 20:1:45;          % Line currents (A)
n_f = 1500;             % Number of turns on shunt field
n_se = 8;               % Number of turns on series field

% Calculate the armature current for each load.
i_a = i_l;

% Now calculate the internal generated voltage for
% each armature current.
e_a = v_t - i_a * r_a;

% Calculate the effective field current for each armature
% current.  (Note that the magnetization curve is defined
% in terms of shunt field current, so we will have to
% translate the series field current into an equivalent
% shunt field current.
i_f = (n_se / n_f) * i_a;

% Calculate the resulting internal generated voltage at
% 1800 r/min by interpolating the motor's magnetization
% curve.
e_a0 = interp1(if_values,ea_values,i_f);

% Calculate the resulting speed from Equation (8-38).
n = ( e_a ./ e_a0 ) * n_0;

% Calculate the induced torque corresponding to each
% speed from Equations (8-28) and (8-29).
t_ind = e_a .* i_a ./ (n * 2 * pi / 60);

% Plot the torque-speed curves
figure(1);
plot(t_ind,n,'b-','LineWidth',2.0);
xlabel('\bf\tau_{ind} (N-m)');
```

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```
ylabel('\bf\itn_{m} \rm\bf(r/min)');
title ('\bfSeries DC Motor Torque-Speed Characteristic');
grid on;
```

The resulting torque-speed characteristic is shown below:

Series DC Motor Torque-Speed Characteristic

The extreme speeds in this characteristic are due to the very light flux in the machine. To make a practical series motor out of this machine, it would be necessary to include 20 to 30 series turns instead of 8.

8-22. An automatic starter circuit is to be designed for a shunt motor rated at 20 hp, 240 V, and 80 A. The armature resistance of the motor is 0.12 Ω, and the shunt field resistance is 40 Ω. The motor is to start with no more than 250 percent of its rated armature current, and as soon as the current falls to rated value, a starting resistor stage is to be cut out. How many stages of starting resistance are needed, and how big should each one be?

SOLUTION The rated line current of this motor is 80 A, and the rated armature current is $I_A = I_L - I_F =$ 80 A – 6 A = 74 A. The maximum desired starting current is (2.5)(74 A) = 185 A. Therefore, the total initial starting resistance must be

$$R_A + R_{\text{start},1} = \frac{240 \text{ V}}{185 \text{ A}} = 1.297 \, \Omega$$

$$R_{\text{start},1} = 1.297 \, \Omega - 0.12 \, \Omega = 1.177 \, \Omega$$

The current will fall to rated value when E_A rises to

$$E_A = 240 \text{ V} - (1.297 \, \Omega)(74 \text{ A}) = 144 \text{ V}$$

At that time, we want to cut out enough resistance to get the current back up to 185 A. Therefore,

$$R_A + R_{\text{start},2} = \frac{240 \text{ V} - 144 \text{ V}}{185 \text{ A}} = 0.519 \, \Omega$$

$$R_{\text{start},2} = 0.519 \, \Omega - 0.12 \, \Omega = 0.399 \, \Omega$$

With this resistance in the circuit, the current will fall to rated value when E_A rises to

$$E_A = 240 \text{ V} - (0.519 \, \Omega)(74 \text{ A}) = 201.6 \text{ V}$$

At that time, we want to cut out enough resistance to get the current back up to 185 A. Therefore,

$$R_A + R_{\text{start},3} = \frac{240 \text{ V} - 201.6 \text{ V}}{185 \text{ A}} = 0.208 \, \Omega$$

$$R_{\text{start},3} = 0.208 \, \Omega - 0.12 \, \Omega = 0.088 \, \Omega$$

With this resistance in the circuit, the current will fall to rated value when E_A rises to

$$E_A = 240 \text{ V} - (0.208 \, \Omega)(74 \text{ A}) = 224.6 \text{ V}$$

If the resistance is cut out when E_A reaches 224.6 V, the resulting current is

$$I_A = \frac{240 \text{ V} - 224.6 \text{ V}}{0.12 \, \Omega} = 128.3 \text{ A} < 185 \text{ A},$$

so there are only three stages of starting resistance. The three stages of starting resistance can be found from the resistance in the circuit at each state during starting.

$$R_{\text{start},1} = R_1 + R_2 + R_3 = 1.177 \, \Omega$$

$$R_{\text{start},2} = R_2 + R_3 = 0.399 \, \Omega$$

$$R_{\text{start},3} = R_3 = 0.088 \, \Omega$$

Therefore, the starting resistances are

$$R_1 = 0.778 \, \Omega$$

$$R_2 = 0.311 \, \Omega$$

$$R_3 = 0.088 \, \Omega$$

Chapter 9:   Transmission Lines

9-1.   Calculate the dc resistance in ohms per kilometer for an aluminum conductor with a 3 cm diameter.

SOLUTION The resistance per meter of aluminum conductor is given by Equation (9-2):

$$r_{DC} = \frac{\rho}{A} = \frac{\rho}{\pi r^2}$$

where $\rho = 2.83 \times 10^{-8}$ Ω-m. This value is

$$r_{DC} = \frac{\left(2.83 \times 10^{-8} \text{ Ω-m}\right)}{\pi \left(0.015 \text{ m}\right)^2} = 4.004 \times 10^{-5} \text{ Ω/m}$$

Therefore the total DC resistance per kilometer would be

$$R_{DC} = \left(4.004 \times 10^{-5} \text{ Ω/m}\right) \left(\frac{1000 \text{ m}}{1 \text{ km}}\right) = 0.040 \text{ Ω/km}$$

9-2.   Calculate the dc resistance in ohms per *mile* for a hard-drawn copper conductor with a 1 inch diameter. (Note that 1 mile = 1.609 km).

SOLUTION The resistance per meter of hard-drawn copper conductor is given by Equation (9-2):

$$r_{DC} = \frac{\rho}{A} = \frac{\rho}{\pi r^2}$$

where $\rho = 1.77 \times 10^{-8}$ Ω-m. Note that the radius in this equation must be in units of meters. This value is

$$r_{DC} = \frac{\left(1.77 \times 10^{-8} \text{ Ω-m}\right)}{\pi \left[\left(0.5 \text{ in}\right)\left(\frac{0.0254 \text{ m}}{1 \text{ in}}\right)\right]^2} = 3.493 \times 10^{-5} \text{ Ω/m}$$

Therefore the total DC resistance per mile would be $1000 \, r_{DC} = 0.03367 \, \Omega$.

$$R_{DC} = \left(3.493 \times 10^{-5} \text{ Ω/m}\right) \left(\frac{1609 \text{ m}}{1 \text{ mile}}\right) = 0.0562 \text{ Ω/mile}$$

Problems 9-3 through 9-7 refer to a single phase, 8 kV, 50-Hz, 50 km-long transmission line consisting of two aluminum conductors with a 3 cm diameter separated by a spacing of 2 meters.

9-3.   Calculate the inductive reactance of this line in ohms.

SOLUTION The series inductance per meter of this transmission line is given by Equation (9-22).

$$l = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{D}{r}\right) \quad \text{H/m} \tag{9-22}$$

where $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m .

$$l = \frac{\mu_0}{\pi}\left(\frac{1}{4} + \ln\frac{2.0 \text{ m}}{0.015 \text{ m}}\right) = \frac{4\pi \times 10^{-7} \text{ H/m}}{\pi}\left(\frac{1}{4} + \ln\frac{2.0 \text{ m}}{0.015 \text{ m}}\right) = 2.057 \times 10^{-6} \text{ H/m}$$

Therefore the inductance of this transmission line will be

$$L = \left(2.057 \times 10^{-6} \text{ H/m}\right)\left(50,000 \text{ m}\right) = 0.1029 \text{ H}$$

The inductive reactance of this transmission line is

$$X = j\omega L = j2\pi f L = j2\pi\left(50 \text{ Hz}\right)\left(0.1029 \text{ H}\right) = j32.3 \, \Omega$$

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9-4. Assume that the 50 Hz ac resistance of the line is 5% greater than its dc resistance, and calculate the series impedance of the line in ohms per km.

SOLUTION The DC resistance per meter of this transmission line is given by Equation (9-22).

$$r_{DC} = \frac{\rho}{A} = \frac{\rho}{\pi r^2}$$

where $\rho = 2.83 \times 10^{-8}$ Ω-m. This value is

$$r_{DC} = \frac{\left(2.83 \times 10^{-8} \text{ Ω-m}\right)}{\pi \left(0.015 \text{ m}\right)^2} = 4.004 \times 10^{-5} \text{ Ω/m}$$

Therefore the total DC resistance of the line would be

$$R_{DC} = \left(4.004 \times 10^{-5} \text{ Ω/m}\right)\left(50,000 \text{ m}\right) = 2.0 \text{ Ω}$$

The AC resistance of the line would be

$$R_{AC} = \left(2.0 \text{ Ω}\right)\left(1.05\right) = 2.1 \text{ Ω}$$

The total series impedance of this line would be $Z = 2.1 + j30.5$ Ω, so the impedance per kilometer would be

$$Z = \left(2.1 + j32.3 \text{ Ω}\right)/\left(50 \text{ km}\right) = 0.042 + j0.646 \text{ Ω/km}$$

9-5. Calculate the shunt admittance of the line in siemens per km.

SOLUTION The shunt capacitance per meter of this transmission line is given by Equation (9-41).

$$c = \frac{\pi\varepsilon}{\ln\left(\dfrac{D}{r}\right)} \tag{9-41}$$

$$c = \frac{\pi\left(8.854 \times 10^{-12} \text{ F/m}\right)}{\ln\left(\dfrac{2.0}{0.015}\right)} = 5.69 \times 10^{-12} \text{ F/m}$$

Therefore the capacitance per kilometer will be

$$c = 5.69 \times 10^{-9} \text{ F/km}$$

The shunt admittance of this transmission line per kilometer will be

$$y_{sh} = j2\pi f c = j2\pi\left(50 \text{ Hz}\right)\left(5.69 \times 10^{-9} \text{ F/km}\right) = j1.79 \times 10^{-6} \text{ S/km}$$

Therefore the total shunt admittance will be

$$Y_{sh} = \left(j1.79 \times 10^{-6} \text{ S/km}\right)\left(50 \text{ km}\right) = j8.95 \times 10^{-5} \text{ S}$$

9-6. The single-phase transmission line is operating with the receiving side of the line open-circuited. The sending end voltage is 8 kV at 50 Hz. How much charging current is flowing in the line?

SOLUTION Although this line is in the "short" range of lengths, we will treat it as a medium-length line, because we must include the capacitances if we wish to calculate charging currents. The appropriate transmission line model is shown below.
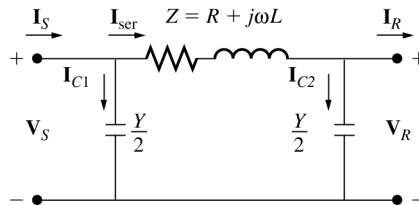
The charging current can be calculated by open-circuiting the output of the transmission line and calculating $\mathbf{I}_S$:

$$\mathbf{I}_S = \frac{Y\mathbf{V}_S}{2} + \frac{\mathbf{V}_S}{Z + \dfrac{1}{Y/2}}$$

$$\mathbf{I}_S = \frac{\left(j8.95 \times 10^{-5}\ \text{S}\right)\left(8000\angle 0°\ \text{V}\right)}{2} + \frac{8000\angle 0°\ \text{V}}{\left(2.1\ + j32.3\ \Omega\right) + \dfrac{1}{j8.95 \times 10^{-5}\ \text{S}/2}}$$

$$\mathbf{I}_S = 0.358\angle 90°\ \text{A} + 0.358\angle 90°\ \text{A} = 0.716\angle 90°\ \text{A}$$

9-7. The single-phase transmission line is now supplying 8 kV to an 800 kVA, 0.9 PF lagging single-phase load.
  (a) What is the sending end voltage and current of this transmission line?
  (b) What is the efficiency of the transmission line under these conditions?
  (c) What is the voltage regulation of the transmission line under these conditions?

SOLUTION At 50 km length, we can treat this transmission line as a "short" line and ignore the effects of the shunt admittance. The corresponding equivalent circuit is shown below.
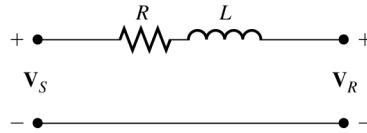
The transmission line is supplying a voltage of 8 kV at the load, so the magnitude of the current flowing to the load is

$$I = \frac{S}{V} = \frac{800\ \text{kVA}}{8\ \text{kV}} = 100\ \text{A}$$

(a) If we assume that the voltage at the load is arbitrarily assigned to be at 0° phase, and the power factor of the load is 0.9 lagging, the phasor current flowing to the load is $\mathbf{I} = 100\angle -25.8°\ \text{A}$. The voltage at the sending end of the transmission line is then

$$\mathbf{V}_S = \mathbf{V}_R + Z\mathbf{I}$$

$$\mathbf{V}_S = 8000\angle 0°\ \text{V} + \left(2.1\ + j32.3\ \Omega\right)\left(100\angle -25.8°\ \text{A}\right)$$

$$\mathbf{V}_S = 10000\angle 16.4°\ \text{V}$$

(b) The complex output power from the transmission line is

$$\mathbf{S}_{\text{OUT}} = \mathbf{V}_R \mathbf{I}^* = \left(8000\angle 0°\ \text{V}\right)\left(100\angle -25.8°\ \text{A}\right)^* = 800,000\angle 25.8°\ \text{VA}$$

Therefore the output power is

$$P_{\text{OUT}} = 800\ \cos 25.8°\ = 720\ \text{kW}$$

The complex input power to the transmission line is

$$\mathbf{S}_{\text{IN}} = \mathbf{V}_S \mathbf{I}^* = \left(10000\angle 16.4°\ \text{V}\right)\left(100\angle -25.8°\ \text{A}\right)^* = 1,000,000\angle 42.2°\ \text{VA}$$

Therefore the input power is

$$P_{\text{IN}} = 1000\ \cos 42.2°\ = 741\ \text{kW}$$

The resulting efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{720\ \text{kW}}{741\ \text{kW}} \times 100\% = 97.2\%$$

(c) The voltage regulation of the transmission line is
$$\text{VR} = \frac{V_S - V_R}{V_R} \times 100\% = \frac{10000 - 8000}{8000} \times 100\% = 25\%$$

Problems 9-8 through 9-10 refer to a single phase, 8 kV, 50-Hz, 50 km-long underground cable consisting of two aluminum conductors with a 3 cm diameter separated by a spacing of 15 cm.

9-8. The single-phase transmission line referred to in Problems 9-3 through 9-7 is to be replaced by an underground cable. The cable consists of two aluminum conductors with a 3 cm diameter, separated by a center-to-center spacing of 15 cm. As before, assume that the 50 Hz ac resistance of the line is 5% greater than its dc resistance, and calculate the series impedance and shunt admittance of the line in ohms per km and siemens per km. Also, calculate the total impedance and admittance for the entire line.

SOLUTION The series inductance per meter of this transmission line is given by Equation (9-22).
$$l = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{D}{r}\right) \quad \text{H/m} \tag{9-22}$$

where $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m .
$$l = \frac{\mu_0}{\pi}\left(\frac{1}{4} + \ln\frac{0.15 \text{ m}}{0.015 \text{ m}}\right) = \frac{4\pi \times 10^{-7} \text{ H/m}}{\pi}\left(\frac{1}{4} + \ln\frac{0.15 \text{ m}}{0.015 \text{ m}}\right) = 1.021 \times 10^{-6} \text{ H/m}$$

Therefore the inductance of this transmission line will be
$$L = \left(1.021 \times 10^{-6} \text{ H/m}\right)\left(50{,}000 \text{ m}\right) = 0.0511 \text{ H}$$

The inductive reactance of this transmission line is
$$X = j\omega L = j2\pi fL = j2\pi(50 \text{ Hz})(0.0511 \text{ H}) = j16.05 \ \Omega$$

The resistance of this transmission line is the same as for the overhead transmission line calculated previously: $R_{AC} = 2.1 \ \Omega$. The total series impedance of this entire line would be $Z = 2.1 + j16.05 \ \Omega$, so the impedance per kilometer would be
$$Z = \left(2.1 + j16.05 \ \Omega\right)/\left(50 \text{ km}\right) = 0.042 + j0.321 \ \Omega/\text{km}$$

The shunt capacitance per meter of this transmission line is given by Equation (9-41).
$$c = \frac{\pi\varepsilon}{\ln\left(\dfrac{D}{r}\right)} \tag{9-41}$$

$$c = \frac{\pi\left(8.854 \times 10^{-12} \text{ F/m}\right)}{\ln\left(\dfrac{0.15}{0.015}\right)} = 1.21 \times 10^{-11} \text{ F/m}$$

Therefore the capacitance per kilometer will be
$$c = 1.21 \times 10^{-8} \text{ F/km}$$

The shunt admittance of this transmission line per kilometer will be
$$y_{sh} = j2\pi fc = j2\pi(50 \text{ Hz})\left(1.21 \times 10^{-8} \text{ F/km}\right) = j3.80 \times 10^{-6} \text{ S/km}$$

Therefore the total shunt admittance will be
$$Y_{sh} = \left(j3.80 \times 10^{-6} \text{ S/km}\right)\left(50 \text{ km}\right) = j1.90 \times 10^{-4} \text{ S}$$

9-9. The underground cable is operating with the receiving side of the line open-circuited. The sending end voltage is 8 kV at 50 Hz. How much charging current is flowing in the line? How does this charging current in the cable compare to the charging current of the overhead transmission line?

SOLUTION Although this line is in the "short" range of lengths, we will treat it as a medium-length line, because we must include the capacitances if we wish to calculate charging currents. The appropriate transmission line model is shown below.
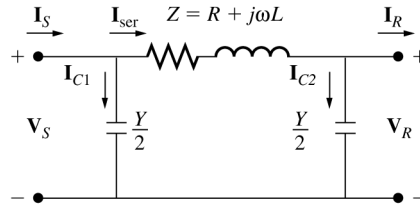
The charging current can be calculated by open-circuiting the output of the transmission line and calculating $\mathbf{I}_S$:

$$\mathbf{I}_S = \frac{Y\mathbf{V}_S}{2} + \frac{\mathbf{V}_S}{Z + \dfrac{1}{Y/2}}$$

$$\mathbf{I}_S = \frac{\left(j1.90\times10^{-4}\ \text{S}\right)\left(8000\angle0°\ \text{V}\right)}{2} + \frac{8000\angle0°\ \text{V}}{\left(2.1 + j32.3\ \Omega\right) + \dfrac{1}{j1.90\times10^{-4}\ \text{S}/2}}$$

$$\mathbf{I}_S = 0.760\angle90°\ \text{A} + 0.762\angle90°\ \text{A} = 1.522\angle90°\ \text{A}$$

Since the shunt admittance of the underground cable is more than twice as large as shunt admittance of the overhead transmission line, the charging current of the underground cable is more than twice as large.

9-10.   The underground cable is now supplying 8 kV to an 800 kVA, 0.9 PF lagging single-phase load.
   (a)   What is the sending end voltage and current of this transmission line?
   (b)   What is the efficiency of the transmission line under these conditions?
   (c)   What is the voltage regulation of the transmission line under these conditions?

SOLUTION At 50 km length, we can treat this transmission line as a "short" line and ignore the effects of the shunt admittance. (Note, however, that this assumption is not as good as it was for the overhead transmission line. The higher shunt admittance makes its effects harder to ignore.) The transmission line is supplying a voltage of 8 kV at the load, so the magnitude of the current flowing to the load is

$$I = \frac{S}{V} = \frac{800\ \text{kVA}}{8\ \text{kV}} = 100\ \text{A}$$

(a)   If we assume that the voltage at the load is arbitrarily assigned to be at 0° phase, and the power factor of the load is 0.9 lagging, the phasor current flowing to the load is $\mathbf{I} = 100\angle-25.8°\ \text{A}$. The voltage at the sending end of the transmission line is then

$$\mathbf{V}_S = \mathbf{V}_R + Z\mathbf{I}$$

$$\mathbf{V}_S = 8000\angle0°\ \text{V} + \left(2.1 + j16.05\ \Omega\right)\left(100\angle-25.8°\ \text{A}\right)$$

$$\mathbf{V}_S = 8990\angle8.7°\ \text{V}$$

(b)   The complex output power from the transmission line is

$$\mathbf{S}_{\text{OUT}} = \mathbf{V}_R\mathbf{I}^* = \left(8000\angle0°\ \text{V}\right)\left(100\angle-25.8°\ \text{A}\right)^* = 800,000\angle25.8°\ \text{VA}$$

Therefore the output power is

$$P_{\text{OUT}} = 800\ \cos25.8° = 720\ \text{kW}$$

The complex input power to the transmission line is

$$\mathbf{S}_{\text{IN}} = \mathbf{V}_S \mathbf{I}^* = (8990\angle 8.7° \text{ V})(100\angle -25.8° \text{ A})^* = 899{,}000\angle 34.5° \text{ VA}$$

Therefore the input power is

$$P_{\text{IN}} = 899 \ \cos 34.5° \ = 740.9 \text{ kW}$$

The resulting efficiency is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{720 \text{ kW}}{740.9 \text{ kW}} \times 100\% = 97.2\%$$

(c)   The voltage regulation of the transmission line is

$$\text{VR} = \frac{V_S - V_R}{V_R} \times 100\% = \frac{8990 - 8000}{8000} \times 100\% = 12.4\%$$

9-11.   A 138 kV, 200 MVA, 60 Hz, three-phase, power transmission line is 100 km long, and has the following characteristics:

$$r = 0.103 \ \Omega/\text{km}$$

$$x = 0.525 \ \Omega/\text{km}$$

$$y = 3.3 \times 10^{-6} \text{ S/km}$$

(a)   What is per phase series impedance and shunt admittance of this transmission line?
(b)   Should it be modeled as a short, medium, or long transmission line?
(c)   Calculate the *ABCD* constants of this transmission line.
(d)   Sketch the phasor diagram of this transmission line when the line is supplying rated voltage and apparent power at a 0.90 power factor lagging.
(e)   Calculate the sending end voltage if the line is supplying rated voltage and apparent power at 0.90 PF lagging.
(f)   What is the voltage regulation of the transmission line for the conditions in *(e)*?
(g)   What is the efficiency of the transmission line for the conditions in *(e)*?

SOLUTION

(a)   The per-phase series impedance of this transmission line is

$$Z = (0.103 + j0.525 \ \Omega/\text{km})(100 \text{ km}) = 10.3 + j52.5 \ \Omega$$

The per-phase shunt admittance of this transmission line is

$$Y = (j3.3 \times 10^{-6} \text{ S/km})(100 \text{ km}) = j0.00033 \text{ S}$$

(b)   This transmission line should be modeled as a medium length transmission line.

(c)   The ABCD constants for a medium length line are given by the following equations:

$$A = \frac{ZY}{2} + 1 \qquad B = Z$$

$$C = Y\left(\frac{ZY}{4} + 1\right) \quad D = \frac{ZY}{2} + 1 \tag{9-73}$$

$$A = \frac{ZY}{2} + 1 = \frac{(10.3 + j52.5 \ \Omega)(j0.00033 \text{ S})}{2} + 1 = 0.9913\angle 0.1°$$

$$B = Z = 10.3 + j52.5 \ \Omega = 53.5\angle 78.9° \ \Omega$$

$$C = Y\left(\frac{ZY}{4} + 1\right) = (j0.00033 \text{ S})\left[\frac{(10.3 + j52.5 \ \Omega)(j0.00033 \text{ S})}{4} + 1\right]$$

$$C = 3.286 \times 10^{-4} \angle 90° \text{ S}$$

$$D = \frac{ZY}{2} + 1 = \frac{(10.3 + j52.5 \text{ } \Omega)(j0.00033 \text{ S})}{2} + 1 = 0.9913\angle 0.1°$$

(d) The phasor diagram is shown below:
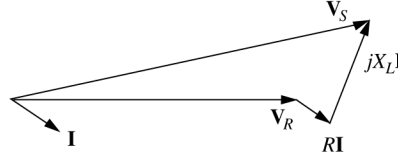
(e) The rated line voltage is 138 kV, so the rated phase voltage is 138 kV / $\sqrt{3}$ = 79.67 kV, and the rated current is

$$I_L = \frac{S_{\text{out}}}{\sqrt{3}V_{LL}} = \frac{200,000,000 \text{ VA}}{\sqrt{3}(138,000 \text{ V})} = 837 \text{ A}$$

If the phase voltage at the receiving end is assumed to be at a phase angle of 0°, then the phase voltage at the receiving end will be $V_R = 79.67\angle 0°$ kV, and the phase current at the receiving end will be $I_R = 837\angle -25.8°$ A. The current and voltage at the sending end of the transmission line are given by the following equations:

$$V_S = AV_R + BI_R$$

$$V_S = (0.9913\angle 0.1°)(79.67\angle 0° \text{ kV}) + (53.5\angle 78.9° \text{ } \Omega)(837\angle -25.8° \text{ A})$$

$$V_S = 111.8\angle 18.75° \text{ kV}$$

$$I_S = CV_R + DI_R$$

$$I_S = (3.286 \times 10^{-4} \angle 90° \text{ S})(79.67\angle 0° \text{ kV}) + (0.9913\angle 0.1°)(837\angle -25.8° \text{ A})$$

$$I_S = 818.7\angle -24.05° \text{ A}$$

(f) The voltage regulation of the transmission line is

$$\text{VR} = \frac{V_S - V_R}{V_R} \times 100\% = \frac{111.8 \text{ kV} - 79.67 \text{ kV}}{79.67 \text{ kV}} \times 100\% = 40.3\%$$

(g) The output power from the transmission line is

$$P_{\text{OUT}} = S \text{ PF} = (200 \text{ MVA})(0.9) = 180 \text{ MW}$$

The input power to the transmission line is

$$P_{\text{IN}} = 3V_{\phi,S}I_{\phi,S} \cos\theta = 3(111.8 \text{ kV})(818.7 \text{ A})\cos(42.8°) = 201.5 \text{ MW}$$

The resulting efficiency is

$$\eta = \frac{P_{\text{IN}}}{P_{\text{OUT}}} \times 100\% = \frac{180 \text{ kW}}{201.5 \text{ kW}} \times 100\% = 89.3\%$$

9-12. If the series resistance and shunt admittance of the transmission line in Problem 9-11 are ignored, what would the value of the angle δ be at rated conditions and 0.90 PF lagging?

SOLUTION If the series resistance and shunt admittance are ignored, then the sending end voltage of the transmission line would be

$$\mathbf{V}_S = \mathbf{V}_R + jX\mathbf{I}_R$$

$$\mathbf{V}_S = 79.67\angle 0° \text{ kV} + \left(j52.5 \ \Omega\right)\left(837\angle -25.8° \text{ A}\right)$$

$$\mathbf{V}_S = 106.4\angle 21.8° \text{ kV}$$

Since δ is the angle between $\mathbf{V}_S$ and $\mathbf{V}_R$, $\delta = 21.8°$.

9-13. If the series resistance and shunt admittance of the transmission line in Problem 9-11 are *not* ignored, what would the value of the angle δ be at rated conditions and 0.90 PF lagging?

SOLUTION   From Problem 9-11, $\mathbf{V}_S = 111.8\angle 18.75° \text{ kV}$ and $\mathbf{V}_R = 79.67\angle 0° \text{ kV}$, so the angle δ is $18.75°$ when the series resistance and shunt admittance are also considered.

9-14. Assume that the transmission line of Problem 9-11 is to supply a load at 0.90 PF lagging with no more than a 5% voltage drop and a torque angle $\delta \le 30°$. Treat the line as a medium-length transmission line. What is the maximum power that this transmission line can supply without violating one of these constraints? Which constraint is violated first?

SOLUTION   A MATLAB program to calculate the voltage regulation and angle δ as a function of the power supplied to a load an a power factor of 0.9 lagging is shown below.

```
% M-file: prob9_13.m
% M-file to calculate the voltage drop and angle delta
%   for a transmission line as load is increased.

% First, initialize the values needed in this program.
v_r = p2r(79670,0);     % Receiving end voltage
v_s = 0;                % Sending end voltage (will calculate)
r = 0.103;              % Resistance in ohms/km
x = 0.525;              % Reactance in ohms/km
y = 3.3e-6;             % Shunt admittance in S/km
l = 100;                % Line length (k)

% Calculate series impedance and shunt admittance
Z = (r + j*x) * l;
Y = y * l;

% Calculate ABCD constants
A = Y*Z/2 + 1;
B = Z;
C = Y*(Y*Z/4 + 1);
D = Y*Z/2 + 1;

% Calculate the transmitted power for various currents
% assuming a power factor of 0.9 lagging (-25.8 degrees).
% This calculation uses the equation for complex power
% P = 3 * v_r * conj(i_r).
i_r = (0:10:300) * p2r(1,-25.8);
power = real( 3 * v_r * conj(i_r));

% Calculate sending end voltage and current for the
% various loads
v_s = A * v_r + B * i_r;
i_s = C * v_r + D * i_r;
```

```
% Calculate VR for each current step
VR = ( abs(v_s) - abs(v_r) ) ./ abs(v_r);

% Calculate angle delta for each current step
[v_r_mag, v_r_angle] = r2p(v_r);
for ii = 1:length(i_r)
   [v_s_mag, v_s_angle] = r2p(v_s(ii));
   delta(ii) = v_s_angle - v_r_angle;
end

% Which limit is reached first?
for ii = 1:length(i_r)
   if VR(ii) > 0.05

      % VR exceeded first--tell user
      str = ['VR exceeds 5% at ' num2str(power(ii)/1e6) ' MW'];
      disp(str);
      break;

   elseif abs(delta) > 30

      % Delta exceeded first--tell user
      str = ['delta exceeds 30 deg at ' num2str(power(ii)/1e6) ' MW'];
      disp(str);
      break;

   end
end

% Plot the VR and delta vs power
figure(1);
plot(power/1e6,VR*100,'b-','LineWidth',2.0);
xlabel('\bfPower (MW)');
ylabel('\bfVoltage Regulation (%)');
title ('\bfVoltage Regulation vs Power Supplied');
grid on;

figure(2);
plot(power/1e6,delta,'b-','LineWidth',2.0);
xlabel('\bfPower (MW)');
ylabel('\bfAngle \delta (deg)');
title ('\bfAngle \delta vs Power Supplied');
grid on;
```

When this program is executed, the results are as shown below. Note that the voltage regulation is the first limit to be exceeded by this transmission line of the load is at 0.9 PF lagging. Try the same calculation with other power factors. How does the maximum power supplied vary?

```
>> prob9_13
VR exceeds 5% at 25.8222 MW
```

Voltage Regulation vs Power Supplied

Angle δ vs Power Supplied
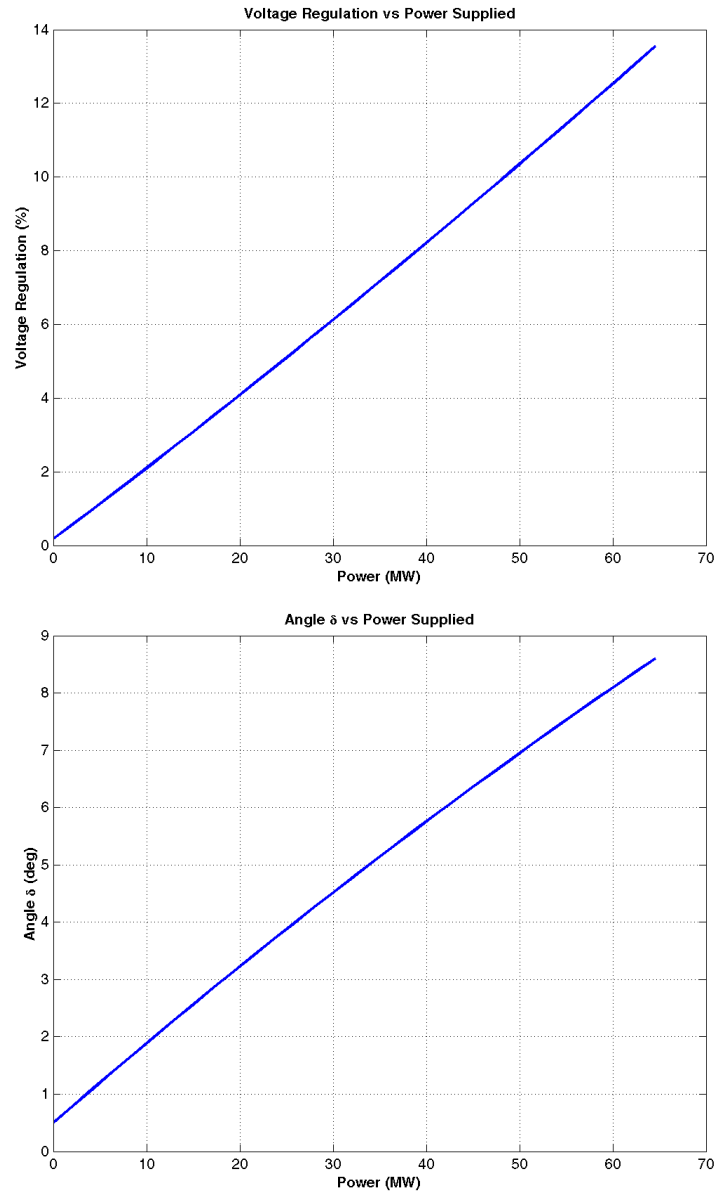
9-15. The transmission line of Problem 9-11 is connected between two infinite busses, as shown in Figure P9-1. Answer the following questions about this transmission line.
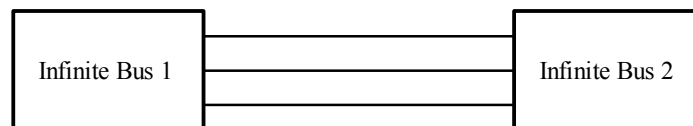
Figure P9-1   A three-phase transmission line connecting two infinite busses together.

(a) If the per-phase (line-to-neutral) voltage on the sending infinite bus is $80\angle 10°$ kV and the per-phase voltage on the receiving infinite bus is $76\angle 0°$ kV, how much real and reactive power are being supplied by the transmission line to the receiving bus?

(b) If the per-phase voltage on the sending infinite bus is changed to $82\angle 10°$ kV, how much real and reactive power are being supplied by the transmission line to the receiving bus? Which changed more, the real or the reactive power supplied to the load?

(c) If the per-phase voltage on the sending infinite bus is changed to $80\angle15°$ kV, how much real and reactive power are being supplied by the transmission line to the receiving bus? Compared to the conditions in part *(a)*, which changed more, the real or the reactive power supplied to the load?

(d) From the above results, how could real power flow be controlled in a transmission line? How could reactive power flow be controlled in a transmission line?

SOLUTION

(a) If the shunt admittance of the transmission line is ignored, the relationship between the voltages and currents on this transmission line is

$$\mathbf{V}_S = \mathbf{V}_R + R\mathbf{I} + jX\mathbf{I}$$

where $\mathbf{I}_S = \mathbf{I}_R = \mathbf{I}$. Therefore we can calculate the current in the transmission line as

$$\mathbf{I} = \frac{\mathbf{V}_S - \mathbf{V}_R}{R + jX}$$

$$\mathbf{I} = \frac{80,000\angle10° - 76,000\angle0°}{10.3 + j52.5 \ \Omega} = 265\angle-0.5° \text{ A}$$

The real and reactive power supplied by this transmission line is

$$P = 3V_{\phi,R}I_\phi \cos\theta = 3(76 \text{ kV})(265 \text{ A})\cos(0.5°) = 60.4 \text{ MW}$$

$$Q = 3V_{\phi,R}I_\phi \sin\theta = 3(76 \text{ kV})(265 \text{ A})\sin(0.5°) = 0.53 \text{ MVAR}$$

(b) If the sending end voltage is changed to $82\angle10°$ kV, the current is

$$\mathbf{I} = \frac{82,000\angle10° - 76,000\angle0°}{10.3 + j52.5 \ \Omega} = 280\angle-7.7° \text{ A}$$

The real and reactive power supplied by this transmission line is

$$P = 3V_{\phi,R}I_\phi \cos\theta = 3(76 \text{ kV})(280 \text{ A})\cos(7.7°) = 63.3 \text{ MW}$$

$$Q = 3V_{\phi,R}I_\phi \sin\theta = 3(76 \text{ kV})(280 \text{ A})\sin(7.7°) = 8.56 \text{ MVAR}$$

In this case, there was a relatively small change in P (3 MW) and a relatively large change in Q (8 MVAR) supplied to the receiving bus.

(c) If the sending end voltage is changed to $82\angle10°$ kV, the current is

$$\mathbf{I} = \frac{80,000\angle15° - 76,000\angle0°}{10.3 + j52.5 \ \Omega} = 388\angle7.2° \text{ A}$$

The real and reactive power supplied by this transmission line is

$$P = 3V_{\phi,R}I_\phi \cos\theta = 3(76 \text{ kV})(388 \text{ A})\cos(-7.2°) = 87.8 \text{ MW}$$

$$Q = 3V_{\phi,R}I_\phi \sin\theta = 3(76 \text{ kV})(388 \text{ A})\sin(-7.2°) = -11 \text{ MVAR}$$

In this case, there was a relatively large change in P (27.4 MW) and a relatively small change in Q (11.5 MVAR) supplied to the receiving bus.

(d) From the above results, we can see that real power flow can be adjusted by changing the phase angle between the two voltages at the two ends of the transmission line, while reactive power flow can be changed by changing the relative magnitude of the two voltages on either side of the transmission line.

9-16. A 50 Hz three phase transmission line is 300 km long. It has a total series impedance of $23 + j75 \ \Omega$ and a shunt admittance of $j500 \ \mu S$. It delivers 150 MW at 220 kV, with a power factor of 0.88 lagging. Find the voltage at the sending end using *(a)* the short line approximation. *(b)* The medium-length line

approximation. *(c)* The long line equation. How accurate are the short and medium-length line approximations for this case?

SOLUTION

(a) In the short line approximation, the shunt admittance is ignored. The *ABCD* constants for this line are:

$$A = 1 \quad B = Z$$
$$C = 0 \quad D = 1 \tag{9-67}$$

$$A = 1$$

$$B = Z = 23 + j75 \ \Omega = 78.4\angle 73° \ \Omega$$

$$C = 0 \ \text{S}$$

$$D = 1$$

The receiving end line voltage is 220 kV, so the rated phase voltage is 220 kV / $\sqrt{3}$ = 127 kV, and the current is

$$I_L = \frac{S_{\text{out}}}{\sqrt{3}V_{LL}} = \frac{150,000,000 \ \text{W}}{\sqrt{3}\,(220,000 \ \text{V})} = 394 \ \text{A}$$

If the phase voltage at the receiving end is assumed to be at a phase angle of 0°, then the phase voltage at the receiving end will be $\mathbf{V}_R = 127\angle 0° \ \text{kV}$, and the phase current at the receiving end will be $\mathbf{I}_R = 394\angle -28.4° \ \text{A}$. The current and voltage at the sending end of the transmission line are given by the following equations:

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R$$

$$\mathbf{V}_S = (1)(127\angle 0° \ \text{kV}) + (78.4\angle 73° \ \Omega)(394\angle -28.4° \ \text{A})$$

$$\mathbf{V}_S = 151\angle 8.2° \ \text{kV}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R$$

$$\mathbf{I}_S = (0 \ \text{S})(133\angle 0° \ \text{kV}) + (1)(394\angle -28.4° \ \text{A})$$

$$\mathbf{I}_S = 394\angle -28.4° \ \text{A}$$

(b) In the medium length line approximation, the shunt admittance divided into two equal pieces at either end of the line. The *ABCD* constants for this line are:

$$A = \frac{ZY}{2} + 1 \qquad B = Z$$
$$C = Y\left(\frac{ZY}{4} + 1\right) \quad D = \frac{ZY}{2} + 1 \tag{9-73}$$

$$A = \frac{ZY}{2} + 1 = \frac{(23 + j75 \ \Omega)(j0.0005 \ \text{S})}{2} + 1 = 0.9813\angle 0.34°$$

$$B = Z = 23 + j75 \ \Omega = 78.4\angle 73° \ \Omega$$

$$C = Y\left(\frac{ZY}{4} + 1\right) = (j0.0005 \ \text{S})\left[\frac{(23 + j75 \ \Omega)(j0.0005 \ \text{S})}{4} + 1\right]$$

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$$C = 4.953 \times 10^{-4} \angle 90.2° \text{ S}$$

$$D = \frac{ZY}{2} + 1 = \frac{(23 + j75 \; \Omega)(j0.0005 \; \text{S})}{2} + 1 = 0.9813 \angle 0.34°$$

The receiving end line voltage is 220 kV, so the rated phase voltage is 220 kV / $\sqrt{3}$ = 127 kV, and the current is

$$I_L = \frac{S_{\text{out}}}{\sqrt{3} V_{LL}} = \frac{150,000,000 \; \text{W}}{\sqrt{3}(220,000 \; \text{V})} = 394 \; \text{A}$$

If the phase voltage at the receiving end is assumed to be at a phase angle of 0°, then the phase voltage at the receiving end will be $\mathbf{V}_R = 127 \angle 0° \; \text{kV}$, and the phase current at the receiving end will be $\mathbf{I}_R = 394 \angle -28.4° \; \text{A}$. The current and voltage at the sending end of the transmission line are given by the following equations:

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R$$

$$\mathbf{V}_S = 148.4 \angle 8.7° \; \text{kV}$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R$$

$$\mathbf{I}_S = 361 \angle -19.2° \; \text{A}$$

(c) In the long transmission line, the *ABCD* constants are based on modified impedances and admittances:

$$Z' = Z \frac{\sinh \gamma d}{\gamma d} \tag{9-74}$$

$$Y' = Y \frac{\tanh(\gamma d / 2)}{\gamma d / 2} \tag{9-75}$$

and the corresponding *ABCD* constants are

$$A = \frac{Z'Y'}{2} + 1 \qquad B = Z'$$

$$C = Y'\left(\frac{Z'Y'}{4} + 1\right) \quad D = \frac{Z'Y'}{2} + 1 \tag{9-76}$$

The propagation constant of this transmission line is $\gamma = \sqrt{yz}$

$$\gamma = \sqrt{yz} = \sqrt{\left(\frac{j500 \times 10^{-6} \; \text{S}}{300 \; \text{km}}\right)\left(\frac{23 + j75 \; \Omega}{300 \; \text{km}}\right)} = 0.00066 \angle 81.5°$$

$$\gamma d = (0.00066 \angle 81.5°)(300 \; \text{km}) = 0.198 \angle 81.5°$$

The modified parameters are

$$Z' = Z \frac{\sinh \gamma d}{\gamma d} = (23 + j75 \; \Omega)\frac{\sinh(0.198 \angle 81.5°)}{0.198 \angle 81.5°} = 77.9 \angle 73° \; \Omega$$

$$Y' = Y \frac{\tanh(\gamma d / 2)}{\gamma d / 2} = 5.01 \times 10^{-4} \angle 89.9° \; \text{S}$$

and the *ABCD* constants are

$$A = \frac{Z'Y'}{2} + 1 = 0.983\angle 0.33°$$

$$B = Z' = 77.9\angle 73° \ \Omega$$

$$C = Y'\left(\frac{Z'Y'}{4} + 1\right) = 4.97\angle 90.1° \ S$$

$$D = \frac{Z'Y'}{2} + 1 = 0.983\angle 0.33°$$

The receiving end line voltage is 220 kV, so the rated phase voltage is 220 kV / $\sqrt{3}$ = 127 kV, and the current is

$$I_L = \frac{S_{out}}{\sqrt{3}V_{LL}} = \frac{150{,}000{,}000 \ W}{\sqrt{3}\,(220{,}000 \ V)} = 394 \ A$$

If the phase voltage at the receiving end is assumed to be at a phase angle of 0°, then the phase voltage at the receiving end will be $\mathbf{V}_R = 127\angle 0° \ kV$, and the phase current at the receiving end will be $\mathbf{I}_R = 394\angle -28.4° \ A$. The current and voltage at the sending end of the transmission line are given by the following equations:

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R$$

$$\mathbf{V}_S = 148.2\angle 8.7° \ kV$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R$$

$$\mathbf{I}_S = 361.2\angle -19.2° \ A$$

The short transmission line approximate was rather inaccurate, but the medium and long line models were both in good agreement with each other.

9-17. A 60 Hz, three phase, 110 kV transmission line has a length of 100 miles and a series impedance of $0.20 + j0.85$ Ω/mile and a shunt admittance of 6×10^{-6} S/mile. The transmission line is supplying 60 MW at a power factor of 0.85 lagging, and the receiving end voltage is 110 kV.
   (a) What are the voltage, current, and power factor at the receiving end of this line?
   (b) What are the voltage, current, and power factor at the sending end of this line?
   (c) How much power is being lost in this transmission line?
   (d) What is the current angle δ of this transmission line? How close is the transmission line to its steady-state stability limit?

SOLUTION This transmission line may be considered to be a medium-line line. The impedance Z and admittance Y of this line are:

$$Z = (0.20 + j0.85 \ \Omega\text{/mile})(100 \text{ miles}) = 20 + j85 \ \Omega$$

$$Y = \left(j6\times 10^{-6}\text{/mile}\right)(100 \text{ miles}) = j0.0006 \ S$$

The *ABCD* constants for this line are:

$$A = \frac{ZY}{2} + 1 = \frac{(20 + j85 \ \Omega)(j0.0006 \ S)}{2} + 1 = 0.9745\angle 0.35°$$

$$B = Z = 20 + j85 \ \Omega = 87.3\angle 76.8° \ \Omega$$

$$C = Y\left(\frac{ZY}{4} + 1\right) = \left(j0.0006 \ S\right)\left[\frac{(20 + j85 \ \Omega)(j0.0006 \ S)}{4} + 1\right] = 0.00059\angle 90.2° \ S$$

$$D = \frac{ZY}{2} + 1 = \frac{(20 + j85 \ \Omega)(j0.0006 \ S)}{2} + 1 = 0.9745\angle 0.35°$$

(a) Assuming that the receiving end voltage is at 0°, the receiving end phase voltage and current are.

$$\mathbf{V}_R = 110\angle 0° \ kV/\sqrt{3} = 63.5\angle 0° \ kV$$

$$I_R = \frac{P}{\sqrt{3}V \cos\theta} = \frac{60 \ MW}{\sqrt{3}(110 \ kV)(0.85)} = 370 \ A$$

$$\mathbf{I}_R = 370\angle -31.7° \ A$$

The receiving end power factor is 0.85 lagging. The receiving end line voltage and current are 110 kV and 370 A, respectively.

(b) The sending end voltage and current are given by

$$\mathbf{V}_S = A\mathbf{V}_R + B\mathbf{I}_R = A(63.5\angle 0° \ kV) + B(370\angle -31.7° \ A)$$

$$\mathbf{V}_S = 87.8\angle 15.35° \ kV$$

$$\mathbf{I}_S = C\mathbf{V}_R + D\mathbf{I}_R = C(63.5\angle 0° \ kV) + D(370\angle -31.7° \ A)$$

$$\mathbf{I}_S = 342.4\angle -26° \ A$$

The sending end power factor is $\cos[15.35° - (-26°)] = \cos(41.4°) = 0.751$ lagging. The sending end line voltage and current are $\sqrt{3}(87.8 \ kV) = 152 \ kV$ and 342 A, respectively.

(c) The power at the sending end of the transmission line is

$$P_S = 3V_{\phi,S}I_{\phi,S}\cos\theta = 3(87,800)(342)(0.751) = 67.7 \ MW$$

The power at the receiving end of the transmission line is

$$P_R = 3V_{\phi,R}I_{\phi,R}\cos\theta = 3(63,500)(370)(0.85) = 59.9 \ MW$$

Therefore the losses in the transmission line are approximately 7.8 MW.

(d) The angle δ is 15.35°. It is about 1/4 of the way to the line's static stability limit.

Chapter 10: *Power System Representation and Equations*

10-1. Sketch the per-phase, per-unit equivalent circuit of the power system in Figure 10-2. (Treat each load on the systems as a resistance in series with a reactance.) Note that you do not have enough information to actually calculate the values of components in the equivalent circuit.
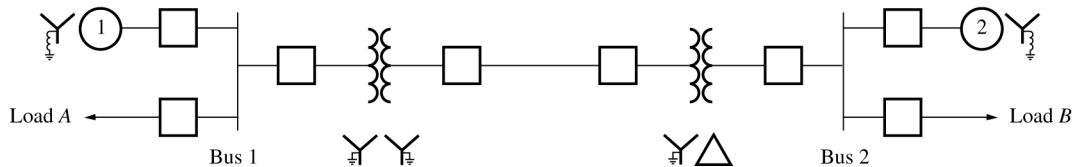
SOLUTION The per-phase, per-unit equivalent circuit would be:
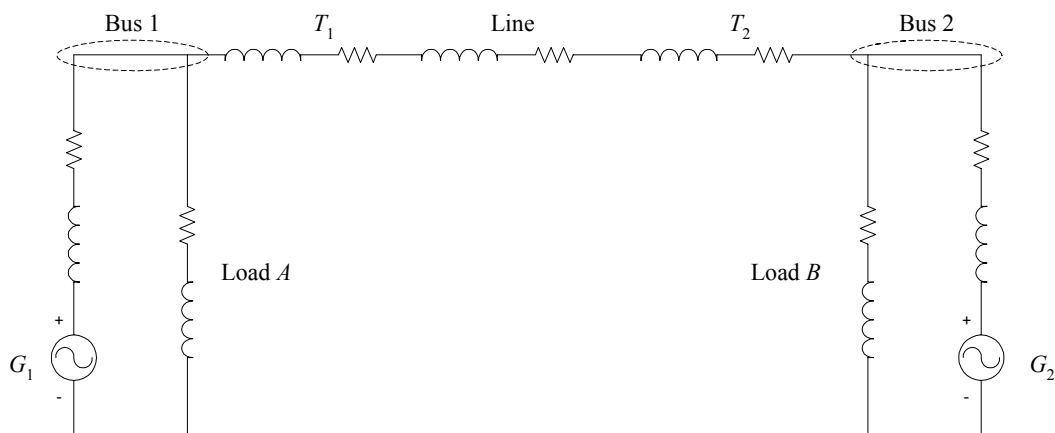
10-2. A 20,000 kVA, 110/13.8 kV, Y-Δ three phase transformer has a series impedance of $0.02 + j0.08$ pu. Find the per-unit impedance of this transformer in a power system with a base apparent power of 500 MVA and a base voltage on the high side of 120 kV.

SOLUTION The per-unit impedance to the new base would be:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}} \right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}} \right) \tag{10-8}$$

$$\text{per-unit } Z_{\text{new}} = \left(0.02 + j0.08 \right) \left(\frac{110 \text{ kV}}{120 \text{ kV}} \right)^2 \left(\frac{500{,}000 \text{ kVA}}{20{,}000 \text{ kVA}} \right) = 0.42 + j1.68 \text{ pu}$$

10-3. Find the per-phase equivalent circuit of the power system shown in Figure P10-1.
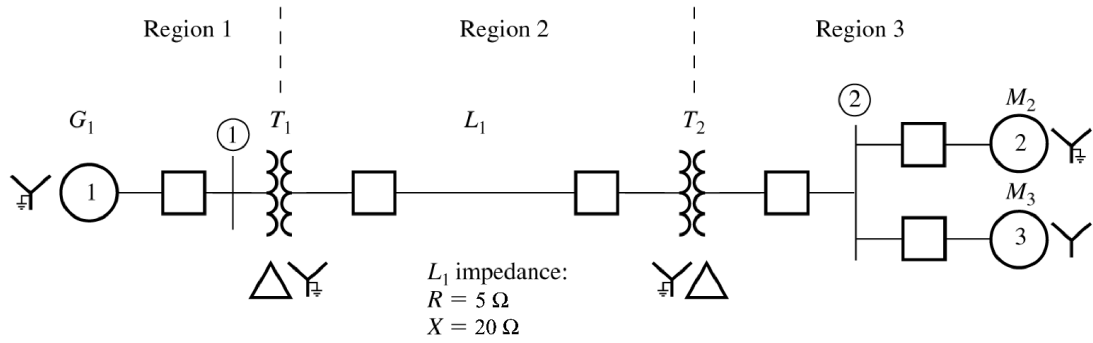
G₁ ratings:	T₁ ratings:		T₂ ratings:	M₂ ratings:	M₃ ratings:
30 MVA	35 MVA		30 MVA	20 MVA	10 MVA
13.8 kV	13.2/115 kV		120/12.5 kV	12.5 kV	12.5 kV
$R = 0.1$ pu	$R = 0.01$ pu		$R = 0.01$ pu	$R = 0.1$ pu	$R = 0.1$ pu
$X_S = 1.0$ pu	$X = 0.10$ pu		$X = 0.08$ pu	$X_S = 1.1$ pu	$X_S = 1.1$ pu

SOLUTION The per-phase equivalent circuit must be created on some system base voltage and apparent power. Since this problem has not specified the system base values, we will use the ratings of generator G_1 as the system base values at that point. Therefore, the system base apparent power is $S_{\text{base}} = 30$ MVA, and the system base voltages in each region are:

$$V_{\text{base},1} = 13.8 \text{ kV}$$

$$V_{\text{base},2} = \left(\frac{115 \text{ kV}}{13.2 \text{ kV}}\right)V_{\text{base},1} = \left(\frac{115 \text{ kV}}{13.2 \text{ kV}}\right)(13.8 \text{ kV}) = 120 \text{ kV}$$

$$V_{\text{base},3} = \left(\frac{12.5 \text{ kV}}{120 \text{ kV}}\right)V_{\text{base},2} = \left(\frac{12.5 \text{ kV}}{120 \text{ kV}}\right)(120 \text{ kV}) = 12.5 \text{ kV}$$

The base impedance of Region 2 is:

$$Z_{\text{base},2} = \frac{\left(V_{LL,\text{base},2}\right)^2}{S_{3\phi,\text{ base}}} = \frac{(120,000 \text{ V})^2}{30,000,000 \text{ VA}} = 480 \text{ }\Omega$$

The per unit resistance and reactance of G_1 are already on the proper base:

$$Z_{G1} = 0.1 + j1.0 \text{ pu}$$

The per unit resistance and reactance of T_1 are:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}}\right)$$

$$Z_{T1} = (0.01 + j0.10)\left(\frac{13.2 \text{ kV}}{13.8 \text{ kV}}\right)^2 \left(\frac{30,000 \text{ kVA}}{35,000 \text{ kVA}}\right) = 0.00784 + j0.0784 \text{ pu}$$

The per unit resistance and reactance of the transmission line are:

$$Z_{\text{line}} = \frac{Z}{Z_{\text{base}}} = \frac{5 + j20 \text{ }\Omega}{480 \text{ }\Omega} = 0.0104 + j0.0417 \text{ pu}$$

The per unit resistance and reactance of T_2 are already on the right base:

$$Z_{T2} = 0.01 + j0.08 \text{ pu}$$

The per unit resistance and reactance of M_1 are:

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$$Z_{M1} = \left(0.1 + j1.1\right)\left(\frac{12.5 \text{ kV}}{12.5 \text{ kV}}\right)^2 \left(\frac{30,000 \text{ kVA}}{20,000 \text{ kVA}}\right) = 0.15 + j1.65 \text{ pu}$$

The per unit resistance and reactance of M_1 are:

$$Z_{M2} = \left(0.1 + j1.1\right)\left(\frac{12.5 \text{ kV}}{12.5 \text{ kV}}\right)^2 \left(\frac{30,000 \text{ kVA}}{10,000 \text{ kVA}}\right) = 0.30 + j3.30 \text{ pu}$$

The resulting per-phase equivalent circuit is:

10-4. Two 4.16 kV three-phase synchronous motors are connected to the same bus. The motor ratings are:

Motor 1: 5,000 hp, 0.8 PF lagging, 95% efficiency, $R = 3\%$, $X_S = 90\%$

Motor 2: 3,000 hp, 1.0 PF, 95% efficiency, $R = 3\%$, $X_S = 90\%$

Calculate the per-unit impedances of these motors to a base of 20 MVA, 4.16 kV. (Note: To calculate these values, you will first have to determine the rated apparent power of each motor considering its rated output power, efficiency, and power factor.)

SOLUTION The rated input power of Motor 1 is

$$P_1 = \frac{P_{\text{out}}}{\eta} = \frac{\left(5,000 \text{ hp}\right)\left(\dfrac{746 \text{ W}}{1 \text{ hp}}\right)}{0.95} = 393 \text{ kW}$$

The apparent power rating is

$$S_1 = \frac{P_1}{\text{PF}} = \frac{393 \text{ kW}}{0.8} = 491 \text{ kVA}$$

The per-unit impedances of Motor 1 are specified to the motor's own base. The impedances converted to the specified base are:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}}\right) \tag{10-8}$$

$$Z_1 = \left(0.03 + j0.90\right)\left(\frac{4.16 \text{ kV}}{4.16 \text{ kV}}\right)^2 \left(\frac{20,000 \text{ kVA}}{491 \text{ kVA}}\right) = 1.22 + j36.7 \text{ pu}$$

The rated input power of Motor 2 is

$$P_2 = \frac{P_{\text{out}}}{\eta} = \frac{(3{,}000 \text{ hp})\left(\dfrac{746 \text{ W}}{1 \text{ hp}}\right)}{0.95} = 236 \text{ kW}$$

The apparent power rating is

$$S_2 = \frac{P_2}{\text{PF}} = \frac{236 \text{ kW}}{1.0} = 236 \text{ kVA}$$

The per-unit impedances of Motor 2 are specified to the motor's own base. The impedances converted to the specified base are:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}}\right) \qquad (10\text{-}8)$$

$$Z_2 = (0.03 + j0.90)\left(\frac{4.16 \text{ kV}}{4.16 \text{ kV}}\right)^2 \left(\frac{20{,}000 \text{ kVA}}{236 \text{ kVA}}\right) = 2.54 + j76.3 \text{ pu}$$

10-5. A Y-connected synchronous generator rated 100 MVA, 13.2 kV has a rated impedance of $R = 5\%$ and $X_S = 80\%$ per-unit. It is connected to a $j10 \ \Omega$ transmission line through a 13.8/120 kV, 100 MVA, Δ-Y transformer with a rated impedance of $R = 2\%$ and $X = 8\%$ per unit. The base for the power system is 200 MVA, 120 kV at the transmission line.
   (a) Sketch the one-line diagram of this power system, with symbols labeled appropriately.
   (b) Find per-unit impedance of the generator on the system base.
   (c) Find per-unit impedance of the transformer on the system base.
   (d) Find per-unit impedance of the transmission line on the system base.
   (e) Find the per-phase, per-unit equivalent circuit of this power system.

SOLUTION The base quantities for this power system in Region 2 are:

$$S_{\text{base}} = 200 \text{ MVA}$$

$$V_{\text{base,2}} = 120 \text{ kV}$$

$$Z_{\text{base,2}} = \frac{\left(V_{LL, \text{ base,2}}\right)^2}{S_{3\phi, \text{ base}}} = \frac{(120{,}000 \text{ V})^2}{200{,}000{,}000 \text{ VA}} = 72 \ \Omega$$

The base quantities for this power system in Region 1 are:

$$S_{\text{base}} = 200 \text{ MVA}$$

$$V_{\text{base,1}} = \left(\frac{13.8 \text{ kV}}{120 \text{ kV}}\right) V_{\text{base,2}} = \left(\frac{13.8 \text{ kV}}{120 \text{ kV}}\right)(120 \text{ kV}) = 13.8 \text{ kV}$$

$$Z_{\text{base,1}} = \frac{\left(V_{LL, \text{ base,1}}\right)^2}{S_{3\phi, \text{ base}}} = \frac{(13{,}800 \text{ V})^2}{200{,}000{,}000 \text{ VA}} = 0.952 \ \Omega$$

(a) The one-line diagram for this power system is shown below:

Region 1   Region 2

G_1 T_1 L_1

L_1 impedance:
$X = j10\ \Omega$

G_1 ratings: T_1 ratings:
100 MVA   100 MVA   <u>Base Values</u>:
13.2 kV   13.8/120 kV   200 MVA
$R = 0.05$ pu $R = 0.02$ pu   120 kV
$X_S = 0.8$ pu $X = 0.08$ pu
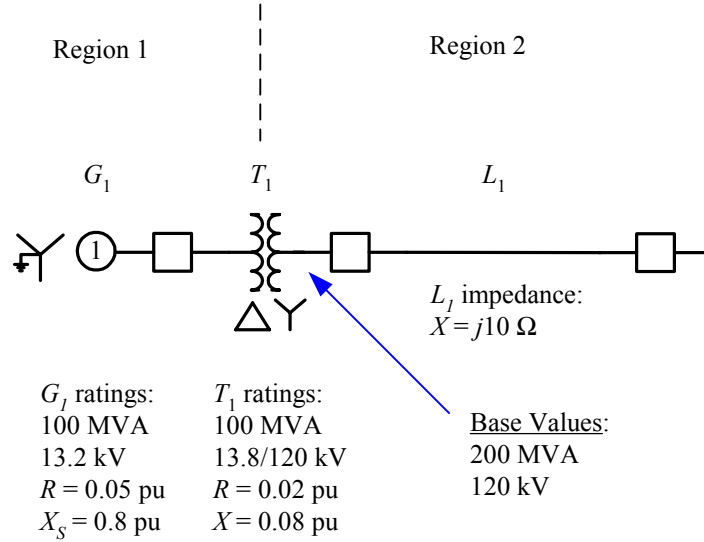
(b) The per-unit impedance of the generator on the system base is:

$$\text{per-unit } Z_{new} = \text{per-unit } Z_{given} \left(\frac{V_{given}}{V_{new}} \right)^2 \left(\frac{S_{new}}{S_{given}} \right) \qquad (10\text{-}8)$$

$$Z_{G1} = (0.05 + j0.80)\left(\frac{13.2 \text{ kV}}{13.8 \text{ kV}} \right)^2 \left(\frac{200 \text{ MVA}}{100 \text{ MVA}} \right) = 0.0915 + j1.464 \text{ pu}$$

(c) The per-unit impedance of the transformer on the system base is:

$$\text{per-unit } Z_{new} = \text{per-unit } Z_{given} \left(\frac{V_{given}}{V_{new}} \right)^2 \left(\frac{S_{new}}{S_{given}} \right) \qquad (10\text{-}8)$$

$$Z_{T1} = (0.02 + j0.08)\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}} \right)^2 \left(\frac{200 \text{ MVA}}{100 \text{ MVA}} \right) = 0.04 + j0.16 \text{ pu}$$

(d) The per unit resistance and reactance of the transmission line are:

$$Z_{line} = \frac{Z}{Z_{base}} = \frac{j10\ \Omega}{72\ \Omega} = j0.139 \text{ pu}$$

(e) The resulting per-phase equivalent circuit is:

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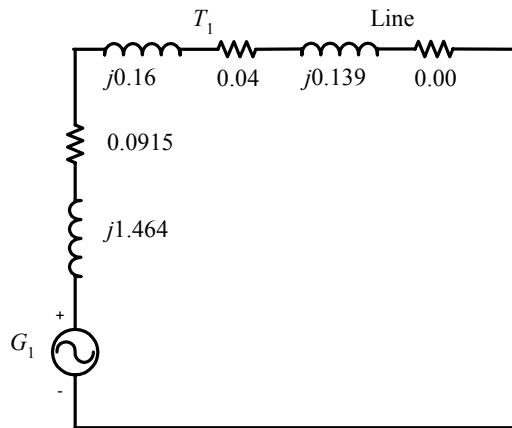
10-6. Assume that the power system of the previous problem is connected to a resistive Y-connected load of 200 Ω per phase. If the internal generated voltage of the generator is $\mathbf{E}_A = 1.10\angle 20°$ per unit, what is the terminal voltage of the generator? How much power is being supplied to the load?

SOLUTION The Y-connected load is connected to the end of the transmission line in Region 2, so $Z_{\text{base},2} = 72\ \Omega$, and the per-unit impedance of the load is

$$Z_{\text{load}} = \frac{Z}{Z_{\text{base}}} = \frac{200\ \Omega}{72\ \Omega} = 2.78\ \text{pu}$$

The resulting current flow is

$$\mathbf{I}_{\text{line}} = \frac{\mathbf{E}_A}{Z_{G1} + Z_{T1} + Z_{\text{line}} + Z_{\text{load}}}$$

$$\mathbf{I}_{\text{line}} = \frac{1.10\angle 20°}{(0.0915 + j1.464) + (0.04 + j0.16) + (j0.139) + (2.78)}$$

$$\mathbf{I}_{\text{line}} = 0.323\angle -11.2°$$

The per-phase terminal voltage of the generator will be

$$\mathbf{V}_\phi = \mathbf{E}_A - \mathbf{I}_A R_A - j\mathbf{I}_A X_S$$

$$\mathbf{V}_\phi = 1.10\angle 20° - (0.323\angle -11.2°)(0.0915) - j(0.323\angle -11.2°)(1.464)$$

$$\mathbf{V}_\phi = 0.917\angle -5.14°$$

Therefore, the terminal voltage will be $(0.917)(13.8\text{ kV}) = 12.7\text{ kV}$. The per-unit power supplied to the load is

$$P_{\text{pu}} = I_{\text{pu}}{}^2 R = (0.323)^2 (2.78) = 0.290$$

Therefore, the total power supplied to the load is $(0.290)(200\text{ MVA}) = 58\text{ MW}$.

10-7. Figure P10-2 shows a one-line diagram of a three-phase power system. The ratings of the various components in the system are:

         Synchronous Generator 1:   40 MVA, 13.8 kV, $R = 3\%$, $X_S = 80\%$

         Synchronous Motor 2:   20 MVA, 13.8 kV, $R = 3\%$, $X_S = 80\%$

         Synchronous Motor 3:   10 MVA, 13.2 kV, $R = 3\%$, $X_S = 100\%$

         Y-Δ Transformers: 20 MVA, 13.8/138 kV, $R = 2\%$, $X = 10\%$

         Line 1: $R = 10\ \Omega$, $X = 50\ \Omega$

         Line 2: $R = 5\ \Omega$, $X = 30\ \Omega$

         Line 3: $R = 5\ \Omega$, $X = 30\ \Omega$

The per-unit system base for this power system is 40 MVA, 128 kV in transmission line 1. Create the per-phase, per-unit equivalent circuit for this power system.
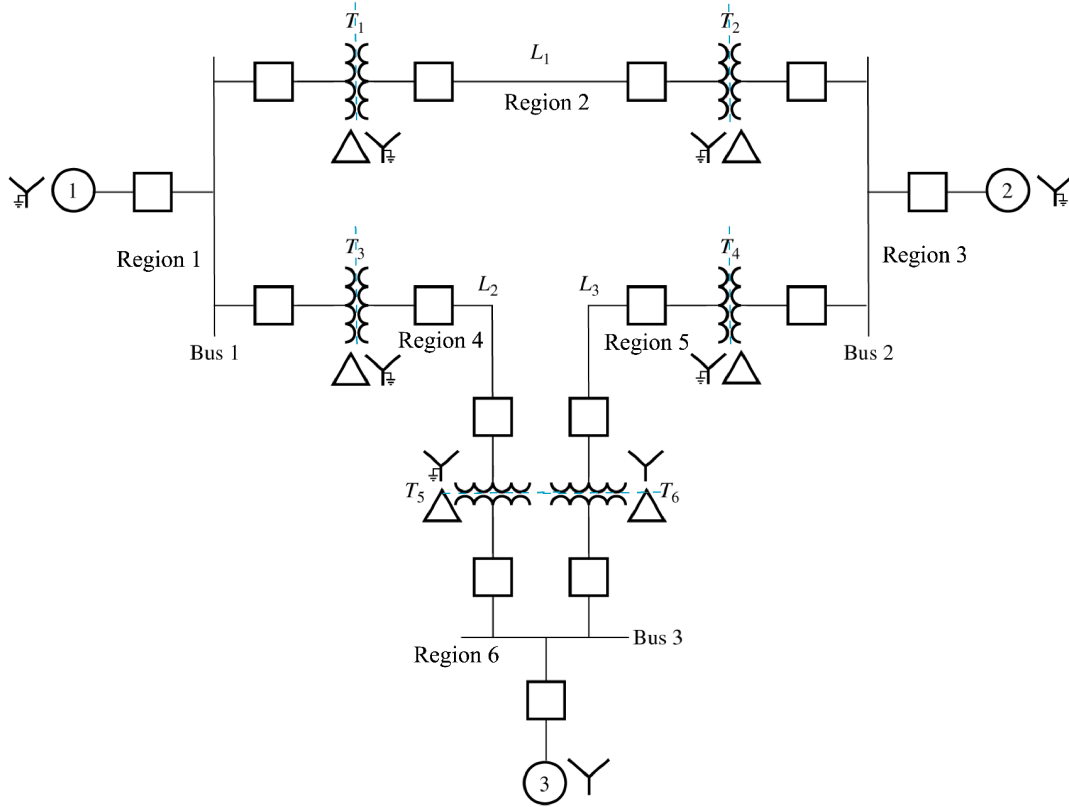
SOLUTION This power system has been divided into regions at the transformers, with the base voltage and apparent power specified in Region 2 to be 128 kV and 40 MVA. The base quantities for this power system in all regions are:

$$S_{base} = 40 \text{ MVA}$$

$$V_{base,1} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right) V_{base,2} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right)(128 \text{ kV}) = 12.8 \text{ kV}$$

$$Z_{base,1} = \frac{\left(V_{LL,\,base,1}\right)^2}{S_{3\phi,\,base}} = \frac{\left(12,800 \text{ V}\right)^2}{40,000,000 \text{ VA}} = 4.096 \text{ }\Omega$$

$$V_{base,2} = 128 \text{ kV}$$

$$Z_{base,2} = \frac{\left(V_{LL,\,base,2}\right)^2}{S_{3\phi,\,base}} = \frac{\left(128,000 \text{ V}\right)^2}{40,000,000 \text{ VA}} = 409.6 \text{ }\Omega$$

$$V_{base,3} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right) V_{base,2} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right)(128 \text{ kV}) = 12.8 \text{ kV}$$

$$Z_{base,3} = \frac{\left(V_{LL,\,base,3}\right)^2}{S_{3\phi,\,base}} = \frac{\left(12,800 \text{ V}\right)^2}{40,000,000 \text{ VA}} = 4.096 \text{ }\Omega$$

$$V_{base,4} = \left(\frac{138 \text{ kV}}{13.8 \text{ kV}}\right) V_{base,1} = \left(\frac{138 \text{ kV}}{13.8 \text{ kV}}\right)(12.8 \text{ kV}) = 128 \text{ kV}$$

$$Z_{base,4} = \frac{\left(V_{LL,\,base,3}\right)^2}{S_{3\phi,\,base}} = \frac{\left(128,000 \text{ V}\right)^2}{40,000,000 \text{ VA}} = 409.6 \text{ }\Omega$$

$$V_{base,5} = \left(\frac{138 \text{ kV}}{13.8 \text{ kV}}\right) V_{base,3} = \left(\frac{138 \text{ kV}}{13.8 \text{ kV}}\right)(12.8 \text{ kV}) = 128 \text{ kV}$$

$$Z_{base,5} = \frac{\left(V_{LL, base,3}\right)^2}{S_{3\phi, base}} = \frac{(128,000 \text{ V})^2}{40,000,000 \text{ VA}} = 409.6 \text{ }\Omega$$

$$V_{base,6} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right) V_{base,4} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right)(128 \text{ kV}) = 12.8 \text{ kV}$$

$$Z_{base,6} = \frac{\left(V_{LL, base,6}\right)^2}{S_{3\phi, base}} = \frac{(12,800 \text{ V})^2}{40,000,000 \text{ VA}} = 4.096 \text{ }\Omega$$

The base quantities for this power system in Region 1 are:

$$S_{base} = 200 \text{ MVA}$$

$$V_{base,1} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right) V_{base,2} = \left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right)(128 \text{ kV}) = 12.8 \text{ kV}$$

$$Z_{base,1} = \frac{\left(V_{LL, base,1}\right)^2}{S_{3\phi, base}} = \frac{(128,000 \text{ V})^2}{40,000,000 \text{ VA}} = 409.6 \text{ }\Omega$$

The per-unit impedances of the various components to the system base are:

$$\text{per-unit } Z_{new} = \text{per-unit } Z_{given} \left(\frac{V_{given}}{V_{new}}\right)^2 \left(\frac{S_{new}}{S_{given}}\right) \tag{10-8}$$

$$Z_{G1} = \left(0.03 + j0.80\right)\left(\frac{13.8 \text{ kV}}{12.8 \text{ kV}}\right)^2 \left(\frac{40 \text{ MVA}}{40 \text{ MVA}}\right) = 0.0349 + j0.930 \text{ pu}$$

$$Z_{M2} = \left(0.03 + j0.80\right)\left(\frac{13.8 \text{ kV}}{12.8 \text{ kV}}\right)^2 \left(\frac{40 \text{ MVA}}{20 \text{ MVA}}\right) = 0.0697 + j1.860 \text{ pu}$$

$$Z_{M3} = \left(0.03 + j1.00\right)\left(\frac{13.8 \text{ kV}}{12.8 \text{ kV}}\right)^2 \left(\frac{40 \text{ MVA}}{10 \text{ MVA}}\right) = 0.140 + j4.65 \text{ pu}$$

$$Z_{T1} = \left(0.02 + j0.10\right)\left(\frac{13.8 \text{ kV}}{12.8 \text{ kV}}\right)^2 \left(\frac{40 \text{ MVA}}{20 \text{ MVA}}\right) = 0.0465 + j0.233 \text{ pu}$$

$$Z_{L1} = \frac{Z}{Z_{base}} = \frac{10 + j50 \text{ }\Omega}{409.6 \text{ }\Omega} = 0.0244 + j0.1221 \text{ pu}$$

$$Z_{L2} = \frac{Z}{Z_{base}} = \frac{5 + j30 \text{ }\Omega}{409.6 \text{ }\Omega} = 0.0122 + j0.0732 \text{ pu}$$

$$Z_{L3} = \frac{Z}{Z_{base}} = \frac{5 + j30 \text{ }\Omega}{409.6 \text{ }\Omega} = 0.0122 + j0.0732 \text{ pu}$$

All transformers in this figure are Y-Δ, and the base quantities are the same for all of them, so the per-unit transformer impedances are:

$$Z_{T1} = Z_{T2} = Z_{T3} = Z_{T4} = Z_{T5} = Z_{T6} = 0.0465 + j0.233 \text{ pu}$$

The resulting per-phase, per-unit equivalent circuit is shown below:
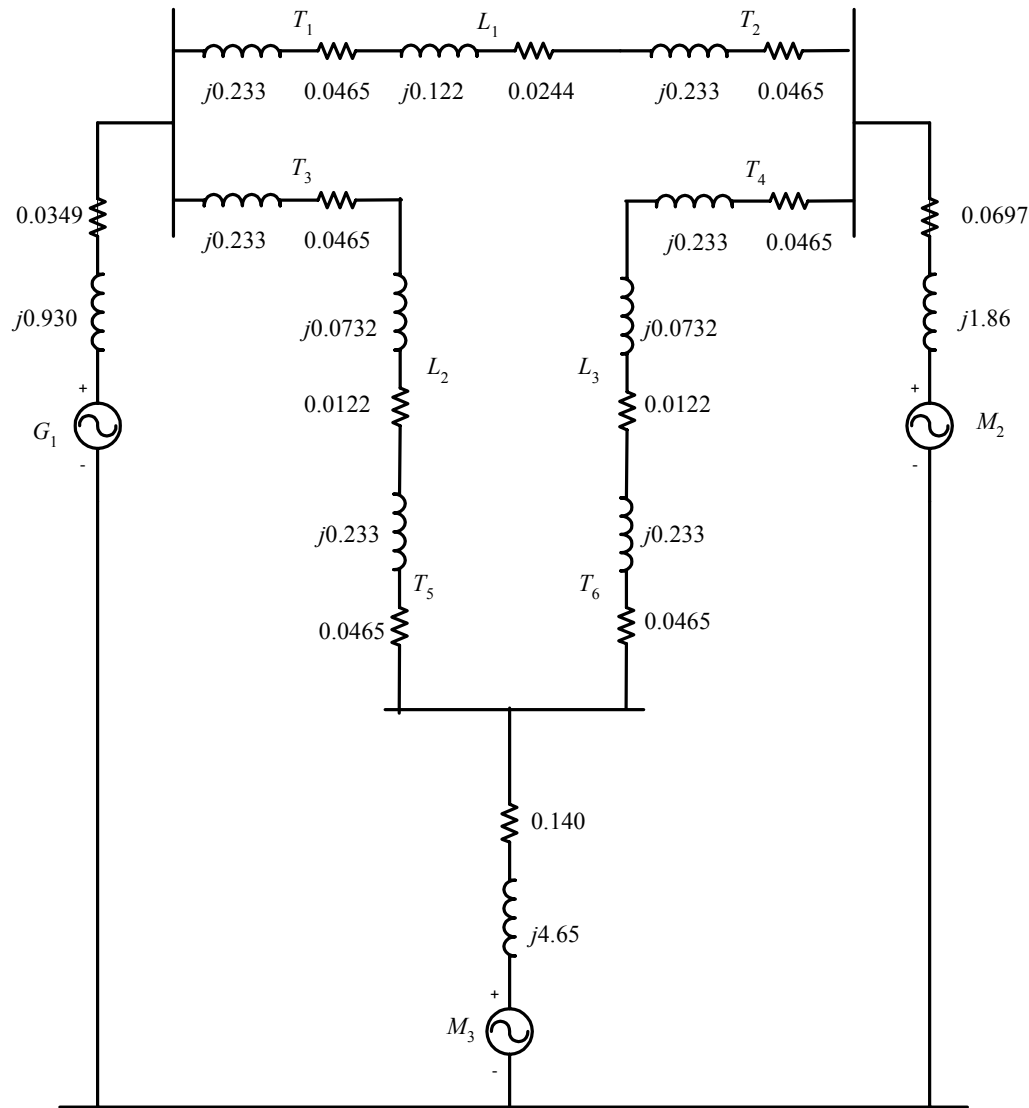
10-8. Calculate the bus admittance matrix $\mathbf{Y}_{bus}$ and the bus impedance matrix $\mathbf{Z}_{bus}$ for the power system shown in Figure P10-2.

SOLUTION The voltage sources can be converted to current sources, and the series impedances between each bus can be replaced by a single admittance, resulting in the system shown below. Note that we have labeled each bus with a number.
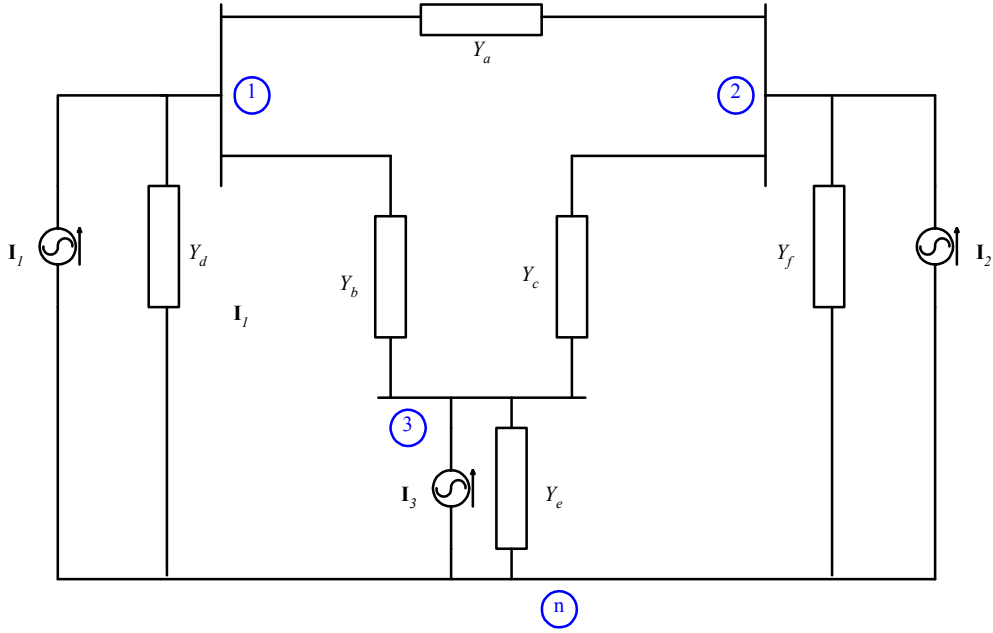
The admittances in this circuit are:

$$Y_a = \frac{1}{Z_{T1} + Z_{L1} + Z_{T2}} = \frac{1}{(0.0465 + j0.233 \text{ pu}) + (0.0244 + j0.122) + (0.0465 + j0.233 \text{ pu})}$$

$$Y_a = 0.3265 - j1.6355 \text{ pu}$$

$$Y_b = \frac{1}{Z_{T3} + Z_{L2} + Z_{T5}} = \frac{1}{(0.0465 + j0.233 \text{ pu}) + (0.0122 + j0.0732) + (0.0465 + j0.233 \text{ pu})}$$

$$Y_b = 0.3486 - j1.7866 \text{ pu}$$

$$Y_c = \frac{1}{Z_{T4} + Z_{L3} + Z_{T6}} = \frac{1}{(0.0465 + j0.233 \text{ pu}) + (0.0122 + j0.0732) + (0.0465 + j0.233 \text{ pu})}$$

$$Y_c = 0.3486 - j1.7866 \text{ pu}$$

$$Y_d = \frac{1}{Z_{G1}} = \frac{1}{0.0349 + j0.930} = 0.3537 - j0.9425 \text{ pu}$$

$$Y_e = \frac{1}{Z_{M3}} = \frac{1}{0.140 + j4.65} = 0.0064 - j0.2149 \text{ pu}$$

$$Y_f = \frac{1}{Z_{M2}} = \frac{1}{0.0697 + j1.860} = 0.0201 - j0.5369 \text{ pu}$$

The bus admittance matrix $\mathbf{Y}_{\text{bus}}$ is:

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} Y_a + Y_b + Y_d & -Y_a & -Y_b \\ -Y_a & Y_a + Y_c + Y_f & -Y_c \\ -Y_b & -Y_c & Y_b + Y_c + Y_e \end{bmatrix}$$

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} 1.0288 - j4.3646 & -0.3265 + j1.6355 & -0.3486 + j1.7866 \\ -0.3265 + j1.6355 & 0.6952 - j3.9590 & -0.3486 + j1.7866 \\ -0.3486 + j1.7866 & -0.3486 + j1.7866 & 0.7036 - j3.7881 \end{bmatrix}$$

The bus impedance matrix $\mathbf{Z}_{\text{bus}}$ is:

$$\mathbf{Z}_{bus} = \mathbf{Y}_{bus}{}^{-1}$$

$$\mathbf{Z}_{bus} = \begin{bmatrix} 0.1532 + j0.6048 & 0.1068 + j0.4875 & 0.1181 + j0.5172 \\ 0.1068 + j0.4875 & 0.1251 + j0.7046 & 0.1045 + j0.5642 \\ 0.1181 + j0.5172 & 0.1045 + j0.5642 & 0.1479 + j0.7670 \end{bmatrix}$$

10-9. Assume that internal generated voltages of the generators and motors in the per-unit equivalent circuit of the previous problem have the following values:

$$\mathbf{E}_{A1} = 1.15\angle 22°$$

$$\mathbf{E}_{A2} = 1.00\angle - 20°$$

$$\mathbf{E}_{A3} = 0.95\angle - 15°$$

(a) Find the per-unit voltages on each bus in the power system.
(b) Find the actual voltages on each bus in the power system.
(c) Find the current flowing in each transmission line in the power system.
(d) Determine the magnitude and direction of the real and reactive power flowing in each transmission line.
(e) Are any of the components in the power system overloaded?

SOLUTION The Norton equivalent currents associated with each voltage source are:

$$\mathbf{I}_1 = \frac{\mathbf{E}_{A1}}{\mathbf{Z}_{G1}} = \frac{1.15\angle 22°}{0.0349 + j0.930} = 0.7832 - j0.8526 = 1.158\angle - 47.4°$$

$$\mathbf{I}_2 = \frac{\mathbf{E}_{A2}}{\mathbf{Z}_{M2}} = \frac{1.00\angle - 20°}{0.0697 + j1.86} = -0.1647 - j0.5114 = 0.5373\angle - 107.9°$$

$$\mathbf{I}_3 = \frac{\mathbf{E}_{A3}}{\mathbf{Z}_{M3}} = \frac{0.95\angle - 15°}{0.14 + j4.65} = -0.0469 - j0.1988 = 0.204\angle - 103.2°$$

The nodal equations for this power system are:

$$\mathbf{Y}_{bus}\mathbf{V} = \mathbf{I}$$

$$\mathbf{V} = \begin{bmatrix} 1.0288 - j4.3646 & -0.3265 + j1.6355 & -0.3486 + j1.7866 \\ -0.3265 + j1.6355 & 0.6952 - j3.9590 & -0.3486 + j1.7866 \\ -0.3486 + j1.7866 & -0.3486 + j1.7866 & 0.7036 - j3.7881 \end{bmatrix}^{-1} \begin{bmatrix} 0.7832 - j0.8526 \\ -0.1647 - j0.5114 \\ -0.0469 - j0.1988 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 0.9646 + j0.1604 \\ 0.9463 + j0.0635 \\ 0.9503 + j0.0926 \end{bmatrix} = \begin{bmatrix} 0.978\angle 9.4° \\ 0.948\angle 3.8° \\ 0.955\angle 5.6° \end{bmatrix}$$

(a) The per-unit voltages at each bus in the power system are:

$$\mathbf{V}_1 = 0.978\angle 9.4°$$

$$\mathbf{V}_2 = 0.948\angle 3.8°$$

$$\mathbf{V}_3 = 0.955\angle 5.6°$$

(b) The actual voltages at each bus in the power system are:

$$V_1 = V_{1,pu}V_{base} = (0.978)(12.8 \text{ kV}) = 12.5 \text{ kV}$$

$$V_2 = V_{2,pu}V_{base} = (0.948)(12.8 \text{ kV}) = 12.1 \text{ kV}$$

$$V_3 = V_{3,pu}V_{base} = (0.955)(12.8 \text{ kV}) = 12.2 \text{ kV}$$

(c) The current flowing in Line 1 is: (start here)

$$\mathbf{I}_{L1} = Y_a\left(\mathbf{V}_1 - \mathbf{V}_2\right) = \left(0.3265 - j1.6355\right)\left(0.978\angle9.4° - 0.948\angle3.8°\right)$$

$$\mathbf{I}_{L1} = 0.1645 + j0.0017 = 0.165\angle0.6°$$

The current flowing in Line 2 is:

$$\mathbf{I}_{L2} = Y_b\left(\mathbf{V}_1 - \mathbf{V}_3\right) = \left(0.3486 - j1.7866\right)\left(0.978\angle9.4° - 0.955\angle5.6°\right)$$

$$\mathbf{I}_{L2} = 0.1263 - j0.0029 = 0.1269\angle - 0.9°$$

The current flowing in Line 3 is:

$$\mathbf{I}_{L3} = Y_c\left(\mathbf{V}_2 - \mathbf{V}_3\right) = \left(0.3486 - j1.7866\right)\left(0.948\angle3.8° - 0.955\angle5.6°\right)$$

$$\mathbf{I}_{L3} = -0.0533 + j0.0029 = 0.0534\angle - 177°$$

(d) The real and reactive power flowing from Bus 1 to Bus 2 in Line 1 is:

$$\mathbf{S}_{L1} = \mathbf{V}_1\mathbf{I}_{L1}{}^* = \left(0.978\angle9.4°\right)\left(0.165\angle0.6°\right)^*$$

$$\mathbf{S}_{L1} = 0.159 + j0.0248 = 0.161\angle8.9°$$

$$P_{L1} = S_{\text{base}}P_{L1,\text{pu}} = \left(40 \text{ MVA}\right)\left(0.159\right) = 6.4 \text{ MW}$$

$$Q_{L1} = S_{\text{base}}Q_{L1,\text{pu}} = \left(40 \text{ MVA}\right)\left(0.0248\right) = 1.0 \text{ MVAR}$$

The real and reactive power flowing from Bus 1 to Bus 3 in Line 2 is:

$$\mathbf{S}_{L2} = \mathbf{V}_1\mathbf{I}_{L2}{}^* = \left(0.978\angle9.4°\right)\left(0.1269\angle - 0.9°\right)^*$$

$$\mathbf{S}_{L2} = 0.1215 + j0.0221 = 0.124\angle10.3°$$

$$P_{L2} = S_{\text{base}}P_{L2,\text{pu}} = \left(40 \text{ MVA}\right)\left(0.1215\right) = 4.86 \text{ MW}$$

$$Q_{L2} = S_{\text{base}}Q_{L2,\text{pu}} = \left(40 \text{ MVA}\right)\left(0.0221\right) = 0.88 \text{ MVAR}$$

The real and reactive power flowing from Bus 2 to Bus 3 in Line 3 is:

$$\mathbf{S}_{L3} = \mathbf{V}_3\mathbf{I}_{L3}{}^* = \left(0.955\angle5.6°\right)\left(0.0534\angle - 177°\right)^*$$

$$\mathbf{S}_{L3} = -0.0509 - j0.0021 = 0.051\angle - 178°$$

$$P_{L3} = S_{\text{base}}P_{L3,\text{pu}} = \left(40 \text{ MVA}\right)\left(-0.0509\right) = -2.04 \text{ MW}$$

$$Q_{L3} = S_{\text{base}}Q_{L3,\text{pu}} = \left(40 \text{ MVA}\right)\left(-0.0021\right) = -0.09 \text{ MVAR}$$

The negative power here means that the power is really flowing from Bus 3 to Bus 2.

(e) None of the components in the power system are even close to being overloaded.

Chapter 11: *Introduction to Power-Flow Studies*

11-1. Find the per-unit currents and the real and reactive power flows in each transmission line in the power system of Example 11-2. Also, calculate the losses in each transmission line.

SOLUTION The power system is shown in below:

Table of Busses:

Bus 1   Slack bus
Bus 2   Load bus
Bus 3   Load bus
Bus 4   Load bus
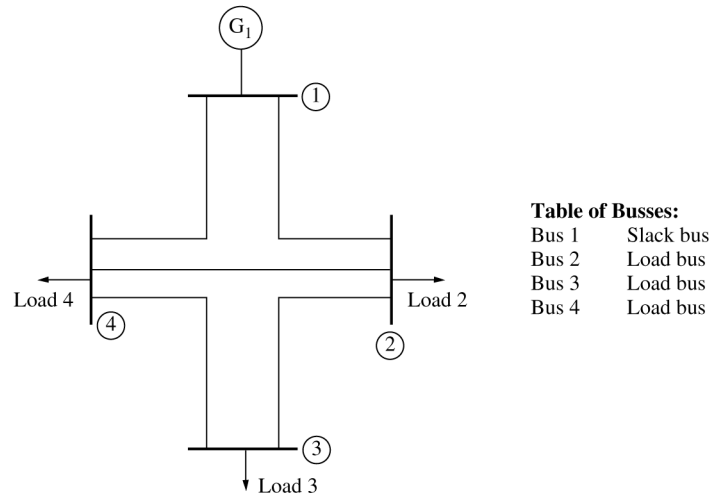
Table 11-1: Per unit series impedances and the corresponding admittances of the transmission lines.

Transmission Line Number	From / To (bus to bus)	Series impedance Z, pu	Series admittance Y, pu
1	1-2	$0.10 + j0.40$	$0.5882 - j2.3529$
2	2-3	$0.10 + j0.50$	$0.3846 - j1.9231$
3	2-4	$0.10 + j0.40$	$0.5882 - j2.3529$
4	3-4	$0.05 + j0.20$	$1.1765 - j4.7059$
5	4-1	$0.05 + j0.20$	$1.1765 - j4.7059$

The impedances of each transmission line are given in Table 11-1, and the resulting bus admittance matrix (ignoring the shunt admittance of the transmission lines) was given in Equation (11-1).

$$\mathbf{Y}_{bus} = \begin{bmatrix} 1.7647 - j7.0588 & -0.5882 + j2.3529 & 0 & -1.1765 + j4.7059 \\ -0.5882 + j2.3529 & 1.5611 - j6.6290 & -0.3846 + j1.9231 & -0.5882 + j2.3529 \\ 0 & -0.3846 + j1.9231 & 1.5611 - j6.6290 & -1.1765 + j4.7059 \\ -1.1765 + j4.7059 & -0.5882 + j2.3529 & -1.1765 + j4.7059 & 2.9412 - j11.7647 \end{bmatrix} \quad (11-1)$$

From Example 11-2, Bus 1 is the slack bus with a voltage $\mathbf{V}_1 = 1.0\angle 0°$ pu. The per-unit real and reactive power *supplied to the loads* from the power system at Busses 2, 3, and 4 are $P_2 = 0.20$ pu, $Q_2 = 0.15$ pu, $P_3 = 0.20$ pu, $Q_3 = 0.15$ pu, $P_4 = 0.20$ pu, and $Q_4 = 0.15$ pu. The resulting voltages at each bus were:

$$\mathbf{V}_{bus}(1) = 1.00000\angle 0° \text{ pu}$$
$$\mathbf{V}_{bus}(2) = 0.89837\angle -5.355° \text{ pu}$$
$$\mathbf{V}_{bus}(3) = 0.87191\angle -7.749° \text{ pu}$$
$$\mathbf{V}_{bus}(4) = 0.91346\angle -4.894° \text{ pu}$$

The per-unit current flowing in transmission lines can be calculated from the following equations

$$\mathbf{I}_{\text{Line 1}} = \frac{\left(\mathbf{V}_1 - \mathbf{V}_2\right)}{Z_{\text{Line 1}}} = \frac{1.00000\angle 0° - 0.89837\angle - 5.355°}{0.10 + j0.40} = 0.3269\angle - 37.5°$$

$$\mathbf{I}_{\text{Line 2}} = \frac{\left(\mathbf{V}_2 - \mathbf{V}_3\right)}{Z_{\text{Line 2}}} = \frac{0.89837\angle - 5.355° - 0.87191\angle - 7.749°}{0.10 + j0.50} = 0.0892\angle - 30.8°$$

$$\mathbf{I}_{\text{Line 3}} = \frac{\left(\mathbf{V}_2 - \mathbf{V}_4\right)}{Z_{\text{Line 3}}} = \frac{0.89837\angle - 5.355° - 0.91346\angle - 4.894°}{0.10 + j0.40} = 0.0406\angle 124.7°$$

$$\mathbf{I}_{\text{Line 4}} = \frac{\left(\mathbf{V}_3 - \mathbf{V}_4\right)}{Z_{\text{Line 4}}} = \frac{0.87191\angle - 7.749° - 0.91346\angle - 4.894°}{0.05 + j0.20} = 0.2952\angle 144.7°$$

$$\mathbf{I}_{\text{Line 5}} = \frac{\left(\mathbf{V}_4 - \mathbf{V}_1\right)}{Z_{\text{Line 5}}} = \frac{0.91346\angle - 4.894° - 1.00000\angle 0°}{0.05 + j0.20} = 0.5770\angle 145.0°$$

The per-unit power flows entering each transmission line are:

$$\mathbf{S}_{\text{Line 1}} = \mathbf{V}_1 \mathbf{I}_{\text{Line 1}}^* = \left(1.00000\angle 0°\right)\left(0.3269\angle - 37.5°\right)^* = 0.2594 + j0.1990$$
$$P_{\text{Line 1}} = 0.2594 \text{ pu}$$
$$Q_{\text{Line 1}} = 0.1990 \text{ pu}$$

$$\mathbf{S}_{\text{Line 2}} = \mathbf{V}_2 \mathbf{I}_{\text{Line 2}}^* = \left(0.89837\angle - 5.355°\right)\left(0.0892\angle - 30.8°\right)^* = 0.0723 + j0.0344$$
$$P_{\text{Line 2}} = 0.0723 \text{ pu}$$
$$Q_{\text{Line 2}} = 0.0344 \text{ pu}$$

$$\mathbf{S}_{\text{Line 3}} = \mathbf{V}_2 \mathbf{I}_{\text{Line 3}}^* = \left(0.89837\angle - 5.355°\right)\left(0.0406\angle 124.7°\right)^* = -0.0235 - j0.0280$$
$$P_{\text{Line 3}} = -0.0235 \text{ pu}$$
$$Q_{\text{Line 3}} = -0.0280 \text{ pu}$$

$$\mathbf{S}_{\text{Line 4}} = \mathbf{V}_3 \mathbf{I}_{\text{Line 4}}^* = \left(0.87191\angle - 7.749°\right)\left(0.2952\angle 144.7°\right)^* = -0.2281 - j0.1192$$
$$P_{\text{Line 4}} = -0.2281 \text{ pu}$$
$$Q_{\text{Line 4}} = -0.1192 \text{ pu}$$

$$\mathbf{S}_{\text{Line 5}} = \mathbf{V}_4 \mathbf{I}_{\text{Line 5}}^* = \left(0.91346\angle - 4.894° \text{ pu}\right)\left(0.5770\angle 145.0°\right)^* = -0.4558 - j0.2647$$
$$P_{\text{Line 5}} = -0.4558 \text{ pu}$$
$$Q_{\text{Line 5}} = -0.2647 \text{ pu}$$

The losses in each transmission line are:

$$P_{\text{Loss 1}} = I_{\text{Line 1}}{}^2 R_{\text{Line 1}} = \left(0.3269\right)^2 \left(0.1\right) = 0.0107 \text{ pu}$$

$$P_{\text{Loss 2}} = I_{\text{Line 2}}{}^2 R_{\text{Line 2}} = \left(0.0892\right)^2 \left(0.1\right) = 0.0008 \text{ pu}$$

$$P_{\text{Loss 3}} = I_{\text{Line 3}}{}^2 R_{\text{Line 3}} = \left(0.0406\right)^2 \left(0.1\right) = 0.00016 \text{ pu}$$

$$P_{\text{Loss 4}} = I_{\text{Line 4}}{}^2 R_{\text{Line 4}} = \left(0.2952\right)^2 \left(0.05\right) = 0.0044 \text{ pu}$$

$$P_{\text{Loss 5}} = I_{\text{Line 5}}{}^2 R_{\text{Line 5}} = (0.5770)^2 (0.05) = 0.0166 \text{ pu}$$

11-2. Assume that a capacitor bank is added to Bus 4 in the power system of Example 11-2. The capacitor bank *consumes* a reactive power of $Q_{\text{cap}} = -0.25$ pu , or alternately, the capacitor *supplies* a reactive power of +0.25 pu to Bus 4. Determine the per-unit bus voltages in the power system after the capacitor is added.

SOLUTION The MATLAB program given in Example 11-2 can be modified by adding the capacitive load to Bus 4, and re-running the program. The modified program is shown below:

```
% M-file: prob11_2.m
% M-file to solve the power-flow problem of Problem 11-2.
% This is a modification of the program used to solve
% Example 11-2.

% Set problem size and initial conditions
n_bus = 4;
swing_bus = 1;

% Create Y-bus
 Ybus = ...
[ 1.7647-j*7.0588 -0.5882+j*2.3529  0                -1.1765+j*4.7059; ...
 -0.5882+j*2.3529  1.5611-j*6.6290 -0.3846+j*1.9231 -0.5882+j*2.3529; ...
  0               -0.3846+j*1.9231  1.5611-j*6.6290 -1.1765+j*4.7059; ...
 -1.1765+j*4.7059 -0.5882+j*2.3529 -1.1765+j*4.7059  2.9412-j*11.7647 ];

% Initialize the real and reactive power supplied to the
% power system at each bus.  Note that the power at the
% swing bus doesn't matter.
P(2) = -0.2;
P(3) = -0.3;
P(4) = -0.2;
Q(2) = -0.15;
Q(3) = -0.15;
Q(4) = -0.10+0.25;        % This is the modified line!

% Initialize the bus voltages to 1.0 at 0 degrees
for ii = 1:n_bus
   Vbus(ii) = 1;
end

% Set convergence criterion
eps = 0.0001;

% Initialize the iteration counter
n_iter = 0;

% Create an infinite loop
while (1)

   % Increment the iteration count
   n_iter = n_iter + 1;

   % Save old bus voltages for comparison purposes
   Vbus_old = Vbus;
```

```matlab
    % Calculate the updated bus voltage
    for ii = 1:n_bus

        % Skip the slack bus!
        if ii ~= swing_bus

            % Calculate updated voltage at bus 'ii'.  First, sum
            % up the current contributions at bus 'ii' from all
            % other busses.
            temp = 0;
            for jj = 1:n_bus
               if ii ~= jj
                   temp = temp - Ybus(ii,jj) * Vbus_old(jj);
               end
            end

            % Add in the current injected at this node
            temp = (P(ii) - j*Q(ii)) / conj(Vbus_old(ii)) + temp;

            % Get updated estimate of Vbus at 'ii'
            Vbus(ii) = 1/Ybus(ii,ii) * temp;
        end
    end

    % Compare the old and new estimate of the voltages.
    % Note that we will compare the real and the imag parts
    % separately, and both must be within tolerances.
    test = 0;
    for ii = 1:n_bus

        % Compare real parts
        if abs( real(Vbus(ii)) - real(Vbus_old(ii)) ) > eps
           test = 1;
        end

        % Compare imaginary parts
        if abs( imag(Vbus(ii)) - imag(Vbus_old(ii)) ) > eps
           test = 1;
        end
    end

    % Did we converge?  If so, get out of the loop.
    if test == 0
       break;
    end
end

% Display results
for ii = 1:n_bus
   [mag, phase] = r2p(Vbus(ii));
   str = ['The voltage at bus ' int2str(ii) ' = ' ...
           num2str(mag) '/' num2str(phase)];
   disp(str);
end

% Display the number of iterations
```

```
str = ['Number of iterations = ' int2str(n_iter) ];
disp(str);
```

When the program is executed, the results are as shown below. Note that the bus voltages have increased as a result of the capacitor being added to the bus.

```
>> prob11_2
The voltage at bus 1 = 1/0
The voltage at bus 2 = 0.9262/-5.498
The voltage at bus 3 = 0.91235/-7.826
The voltage at bus 4 = 0.95653/-5.2766
Number of iterations = 24
```

11-3. Calculate the per-unit currents and the real and reactive power flows in each transmission line in the power system of Problem 11-2 after the capacitor has been added. Did the currents in the lines go up or down as a result of the addition of the capacitor?

SOLUTION The voltages at each bus after the capacitor was added were:

$$\mathbf{V}_{bus}(1) = 1.0000\angle 0° \text{ pu}$$
$$\mathbf{V}_{bus}(2) = 0.9262\angle -5.50° \text{ pu}$$
$$\mathbf{V}_{bus}(3) = 0.9124\angle -7.83° \text{ pu}$$
$$\mathbf{V}_{bus}(4) = 0.9563\angle -5.28° \text{ pu}$$

The per-unit current flowing in transmission lines can be calculated from the following equations

$$\mathbf{I}_{\text{Line 1}} = \frac{(\mathbf{V}_1 - \mathbf{V}_2)}{Z_{\text{Line 1}}} = \frac{1.00000\angle 0° - 0.89837\angle -5.355°}{0.10 + j0.40} = 0.2866\angle -27.3°$$

$$\mathbf{I}_{\text{Line 2}} = \frac{(\mathbf{V}_2 - \mathbf{V}_3)}{Z_{\text{Line 2}}} = \frac{0.9262\angle -5.50° - 0.9124\angle -7.83°}{0.10 + j0.50} = 0.0781\angle -15.7°$$

$$\mathbf{I}_{\text{Line 3}} = \frac{(\mathbf{V}_2 - \mathbf{V}_4)}{Z_{\text{Line 3}}} = \frac{0.9262\angle -5.50° - 0.9563\angle -5.28°}{0.10 + j0.40} = 0.0741\angle 105.5°$$

$$\mathbf{I}_{\text{Line 4}} = \frac{(\mathbf{V}_3 - \mathbf{V}_4)}{Z_{\text{Line 4}}} = \frac{0.9124\angle -7.83° - 0.9563\angle -5.28°}{0.05 + j0.20} = 0.2942\angle 140.7°$$

$$\mathbf{I}_{\text{Line 5}} = \frac{(\mathbf{V}_4 - \mathbf{V}_1)}{Z_{\text{Line 5}}} = \frac{0.9563\angle -5.28° - 1.00000\angle 0°}{0.05 + j0.20} = 0.4850\angle 165.7°$$

The per-unit power flows entering each transmission line are:

$$\mathbf{S}_{\text{Line 1}} = \mathbf{V}_1 \mathbf{I}^*_{\text{Line 1}} = (1.00000\angle 0°)(0.2866\angle -27.3°)^* = 0.2547 + j0.1315$$
$$P_{\text{Line 1}} = 0.2547 \text{ pu}$$
$$Q_{\text{Line 1}} = 0.1315 \text{ pu}$$

$$\mathbf{S}_{\text{Line 2}} = \mathbf{V}_2 \mathbf{I}^*_{\text{Line 2}} = (0.9262\angle -5.50°)(0.0781\angle -15.7°)^* = 0.0712 + j0.0128$$
$$P_{\text{Line 2}} = 0.0712 \text{ pu}$$
$$Q_{\text{Line 2}} = 0.0128 \text{ pu}$$

$$\mathbf{S}_{\text{Line 3}} = \mathbf{V}_2 \mathbf{I}^*_{\text{Line 3}} = (0.9262\angle -5.50°)(0.0741\angle 105.5°)^* = -0.0246 - j0.0641$$

$$P_{\text{Line 3}} = -0.0246 \text{ pu}$$

$$Q_{\text{Line 3}} = -0.0641 \text{ pu}$$

$$\mathbf{S}_{\text{Line 4}} = \mathbf{V}_3 \mathbf{I}^*_{\text{Line 4}} = \left(0.9124\angle -7.83°\right)\left(0.2942\angle140.7°\right)^* = -0.2291 - j0.1400$$

$$P_{\text{Line 4}} = -0.2291 \text{ pu}$$

$$Q_{\text{Line 4}} = -0.1400 \text{ pu}$$

$$\mathbf{S}_{\text{Line 5}} = \mathbf{V}_4 \mathbf{I}^*_{\text{Line 5}} = \left(0.9563\angle -5.28° \text{ pu}\right)\left(0.4850\angle165.7°\right)^* = -0.4581 - j0.0731$$

$$P_{\text{Line 5}} = -0.4581 \text{ pu}$$

$$Q_{\text{Line 5}} = -0.0731 \text{ pu}$$

The current produced by the generator, and the currents in lines 1, 2, and 5, have dropped significantly, because some of the reactive power needed by the loads is being supplied by the capacitor bank at Bus 4.

Problems 11-4 through 11-8 refer to the simple power system with five busses and six transmission lines shown in Figure P11-1. The base apparent power of this power system is 100 MVA, and the tolerance on each bus voltage is 5%. The bus data for the power system is given in Table 11-7, and the line data for the power system is given in Table 11-8.
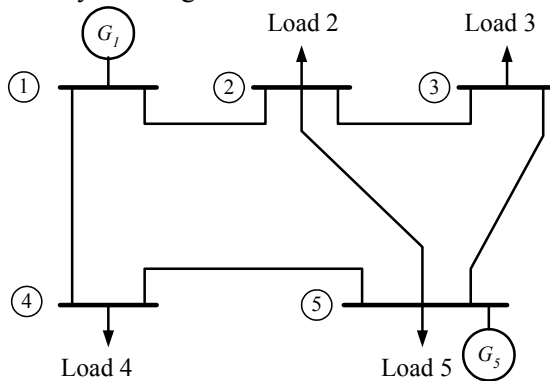
Table of Busses:

Bus 1	Slack Bus
Bus 2	Load Bus
Bus 3	Load Bus
Bus 4	Load Bus
Bus 5	Generator Bus

Figure P11-1   The power system of Problems 11-4 to 11-8.

Table 11-7:   Bus data for the power system in Figure .

Bus Name	Type	V (pu)	Generation		Loads	
			P (MW)	*Q* (Mvar)	*P* (MW)	*Q* (Mvar)
1	SL	$1\angle0°$				
2	PQ	$1\angle0°$			60	35
3	PQ	$1\angle0°$			70	40
4	PQ	$1\angle0°$			80	50
5	PV	$1\angle0°$	190		40	30

Table 11-8: Line data for the power system in Figure P11-1. The shunt admittances of all lines are negligible.

Transmission Line Number	From / To (bus to bus)	Series impedance Z, pu	Rated MVA
1	1-2	$0.0210 + j0.1250$	50
2	1-4	$0.0235 + j0.0940$	100
3	2-3	$0.0250 + j0.1500$	50
4	2-5	$0.0180 + j0.0730$	100
5	3-5	$0.0220 + j0.1100$	70
6	4-5	$0.0190 + j0.0800$	100

11-4. Answer the following questions about this power system:

    (a) Find the voltages at each bus in this power system.

    (b) Find the real and reactive power flows in each transmission line.

    (c) Are any of the bus voltages out of tolerance in this power system?

    (d) Are any of the transmission lines overloaded?

SOLUTION We can solve this problem using program `power_flow.m`. The input file `prob_11_4_input` to calculate the power flows in this power system is given below:

```
% File describing the base case for the power system of
% Problem 11-4.
%
% System data has the form:
%SYSTEM     name         baseMVA     Voltage Tolerance
SYSTEM    Base_case      100              0.05
%
% Bus data has the form:
%BUS name         type   volts   Pgen    Qgen    Pload   Qload   Qcap
BUS One            SL     1.00     0       0       0       0       0
BUS Two            PQ     1.00     0       0       60      35      0
BUS Three          PQ     1.00     0       0       70      40      0
BUS Four           PQ     1.00     0       0       80      50      0
BUS Five           PV     1.00     190     0       40      30      0
%
% Transmission line data has the form:
%LINE    from        to      Rse     Xse     Gsh     Bsh     Rating(MVA)
LINE    One       Two      0.0210  0.1250  0.000   0.000        50
LINE    One       Four     0.0235  0.0940  0.000   0.000       100
LINE    Two       Three    0.0250  0.1500  0.000   0.000        50
LINE    Two       Five     0.0180  0.0730  0.000   0.000       100
LINE    Three     Five     0.0220  0.1100  0.000   0.000        70
LINE    Four      Five     0.0190  0.0800  0.000   0.000       100
```

When this program is executed, the results are as shown below:

```
>> power_flow prob_11_4_input
Input summary statistics:
23 lines in system file
   1 SYSTEM lines
   5 BUS lines
   6 LINE lines
```

Results for Case Base_case

Bus no.	Name	Bus Type	Volts / (pu)	angle (deg)	Generation (MW)	(MVAR)	Load (MW)	(MVAR)	Cap (MVAR)	To Bus	Line Flow (MW)	(MVAR)
1	One	SL	1.000/	0.00	62.55	44.63	0.00	0.00	0.00	Two	26.52	19.76
										Four	36.03	24.88
2	Two	PQ	0.970/	-1.71	0.00	0.00	60.00	35.00	0.00	One	-26.29	-18.39
										Three	15.97	10.10
										Five	-49.52	-26.74
3	Three	PQ	0.951/	-3.04	0.00	0.00	70.00	40.00	0.00	Two	-15.87	-9.54
										Five	-54.07	-30.47
4	Four	PQ	0.969/	-1.66	0.00	0.00	80.00	50.00	0.00	One	-35.58	-23.07
										Five	-44.34	-26.95
5	Five	PV	1.000/	0.14	190.00	123.56	40.00	30.00	0.00	Two	50.13	29.20
										Three	55.01	35.16
										Four	44.88	29.24
Totals					252.55	168.19	250.00	155.00	0.00			

Line Losses

Line no.	From Bus	To Bus	Ploss (MW)	Qloss (MVAR)
1	One	Two	0.23	1.37
2	One	Four	0.45	1.80

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```
3        Two      Three    0.09    0.57
4        Two      Five     0.61    2.46
5        Three    Five     0.94    4.69
6        Four     Five     0.55    2.30
|============================================|
         Totals:           2.86   13.18
|============================================|

              Alerts
|============================================|
NONE

Done in 12 iterations.
```

(a) The voltages in this power system are:

$$\mathbf{V}_1 = 1.000\angle 0.00°$$

$$\mathbf{V}_2 = 0.970\angle -1.71°$$

$$\mathbf{V}_3 = 0.951\angle -3.04°$$

$$\mathbf{V}_4 = 0.969\angle -1.66°$$

$$\mathbf{V}_5 = 1.000\angle 0.14°$$

(b) The real and reactive power flows in each transmission line are given in the output listing above.

(c) No voltages are out of tolerance.

(d) No transmission lines are overloaded.

11-5. Suppose that transmission line 3 in the previous problem (between Busses 2 and 3) is open circuited for maintenance. Find the bus voltages and transmission line powers in the power system with the line removed. Are any of the voltages out of tolerance? Are any of the transmission lines overloaded?

SOLUTION We can solve this problem using program `power_flow.m`. The input file `prob_11_5_input` to calculate the power flows in this power system is given below, with the changes highlighted.

```
% File describing the the case of deleting transmission line 3 from the
% power system of Problem 11-4.
%
% System data has the form:
%SYSTEM     name         baseMVA     Voltage Tolerance
SYSTEM     Line_3_out     100            0.05
%
% Bus data has the form:
%BUS name          type  volts  Pgen    Qgen    Pload  Qload  Qcap
BUS  One            SL    1.00     0       0       0      0      0
BUS  Two            PQ    1.00     0       0      60     35      0
BUS  Three          PQ    1.00     0       0      70     40      0
BUS  Four           PQ    1.00     0       0      80     50      0
BUS  Five           PV    1.00   190       0      40     30      0
%
% Transmission line data has the form:
%LINE     from          to      Rse     Xse      Gsh      Bsh     Rating(MVA)
LINE     One          Two      0.0210  0.1250  0.000    0.000        50
LINE     One          Four     0.0235  0.0940  0.000    0.000        100
%LINE    Two          Three    0.0250  0.1500  0.000    0.000        50
LINE     Two          Five     0.0180  0.0730  0.000    0.000        100
LINE     Three        Five     0.0220  0.1100  0.000    0.000        70
LINE     Four         Five     0.0190  0.0800  0.000    0.000        100
```

When this program is executed, the results are as shown below:

Input summary statistics:
23 lines in system file
1 SYSTEM lines
5 BUS lines
5 LINE lines

Results for Case **Line_3_out**

Bus no.	Bus Name	Type	Volts (pu) /	angle (deg)	Generation (MW)	(MVAR)	Load (MW)	(MVAR)	Cap (MVAR)	To Bus	Line Flow (MW)	(MVAR)
1	One	SL	**1.000/**	**0.00**	62.90	38.85	0.00	0.00	0.00	Two	23.61	14.66
										Four	39.29	24.19
2	Two	PQ	**0.977/**	**-1.55**	0.00	0.00	60.00	35.00	0.00	One	-23.45	-13.69
										Five	-36.46	-21.33
3	Three	PQ	**0.934/**	**-4.40**	0.00	0.00	70.00	40.00	0.00	Five	-69.95	-40.01
4	Four	PQ	**0.969/**	**-1.85**	0.00	0.00	80.00	50.00	0.00	One	-38.79	-22.19
										Five	-41.14	-27.83
5	Five	PV	**1.000/**	**-0.21**	190.00	130.78	40.00	30.00		Two	36.80	22.69
										Three	71.59	48.20
										Four	41.63	29.93
Totals					252.90	169.63	250.00	155.00	0.00			

Line Losses

Line no.	From Bus	To Bus	Ploss (MW)	Qloss (MVAR)
1	One	Two	0.16	0.97
2	One	Four	0.50	2.00
3	Two	Five	0.34	1.36
4	Three	Five	1.64	8.19

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```
   5      Four      Five       0.50      2.10
|=================================================||
                    Totals:     3.14     14.63
|=================================================||

                        Alerts

|=================================================||
ALERT: Voltage on bus Three out of tolerance.
ALERT: Rating on line 4 exceeded: 80.58 MVA > 70.00 MVA.

Done in 10 iterations.
```

The voltages in this power system are given in the listing above. This time, V_3 is out of tolerance, and line 4 is overloaded.

11-6. Suppose that a 40 MVAR capacitor bank is added to Bus 3 of the power system in Problem 11-5. What happens to the bus voltages in this power system? What happens to the apparent powers of the transmission lines? Is this situation better or worse than the one in Problem 11-5?

SOLUTION We can solve this problem using program `power_flow.m`. The input file `prob_11_6_input` to calculate the power flows in this power system is given below, with the changes highlighted.

```
% File describing the the case of deleting transmission line 3 and
% adding 40 MVAR of capacitance to Bus 3 in power system of
% Problem 11-4.
%
% System data has the form:
%SYSTEM    name            baseMVA    Voltage Tolerance
SYSTEM    Line_3_out_cap   100            0.05
%
% Bus data has the form:
%BUS name        type  volts  Pgen   Qgen   Pload  Qload  Qcap
BUS  One          SL   1.00    0      0      0      0      0
BUS  Two          PQ   1.00    0      0      60     35     0
BUS  Three        PQ   1.00    0      0      70     40     40
BUS  Four         PQ   1.00    0      0      80     50     0
BUS  Five         PV   1.00   190     0      40     30     0%
% Transmission line data has the form:
%LINE    from       to      Rse     Xse     Gsh     Bsh     Rating(MVA)
LINE    One      Two      0.0210  0.1250  0.000   0.000       50
LINE    One      Four     0.0235  0.0940  0.000   0.000      100
%LINE    Two      Three    0.0250  0.1500  0.000   0.000       50
LINE    Two      Five     0.0180  0.0730  0.000   0.000      100
LINE    Three    Five     0.0220  0.1100  0.000   0.000       70
LINE    Four     Five     0.0190  0.0800  0.000   0.000      100
```

When this program is executed, the results are as shown below:

>> **power_flow prob_11_6_input**
Input summary statistics:
24 lines in system file
1 SYSTEM lines
5 BUS lines
5 LINE lines

Results for Case **Line_3_out_cap**

Bus no.	Name	Bus Type	Volts / (pu)	angle (deg)	Generation (MW)	(MVAR)	Load (MW)	(MVAR)	Cap (MVAR)	To Bus	Line Flow (MW)	(MVAR)
1	One	SL	1.000/	0.00	62.41	38.94	0.00	0.00	0.00	Two	23.38	14.70
										Four	39.03	24.24
2	Two	PQ	0.977/	-1.53	0.00	0.00	60.00	35.00	0.00	One	-23.22	-13.74
										Five	-36.71	-21.27
3	Three	PQ	0.981/	-4.68	0.00	0.00	70.00	40.00	40.00	Five	-69.95	-0.01
4	Four	PQ	0.969/	-1.83	0.00	0.00	80.00	50.00	0.00	One	-38.53	-22.26
										Five	-41.40	-27.76
5	Five	PV	1.000/	-0.19	190.00	88.09	40.00	30.00	0.00	Two	37.05	22.65
										Three	71.07	5.60
										Four	41.90	29.88
Totals					252.41	127.03	250.00	155.00	40.00			

Line Losses

Line no.	From Bus	To Bus	Ploss (MW)	Qloss (MVAR)
1	One	Two	0.16	0.95
2	One	Four	0.50	1.98
3	Two	Five	0.34	1.38
4	Three	Five	1.12	5.59

5	Four	Five	0.50	2.12
===	======	======	======	======
		Totals:	2.62	12.02
===	======	======	======	======

Alerts
|==========================|
NONE

Done in 10 iterations.

The bus voltages are now back in tolerance, and the apparent powers flowing in the transmission lines have decreased. The power system can not function properly even with transmission line 3 open circuited.

11-7. Assume that the power system is restored to its original configuration. A new plant consuming 20 MW at 0.95 PF lagging is to be added to Bus 4. Will this new load cause any problems for this power system? If this new load will cause problems, what solution could you recommend?

SOLUTION We can solve this problem using program `power_flow.m`. The real power added to Bus 4 is 20 MW, and the reactive load added to Bus 4 is

$$Q = P \tan \theta = P \tan\left(\cos^{-1} PF \right) = (20 \text{ MW}) \tan\left[\cos^{-1}(0.95) \right] = 6.6 \text{ MVAR}$$

The input file `prob_11_7_input` to calculate the power flows in this power system is given below, with the changes highlighted.

```
% File describing the base case plus adding loads to Bus 4.
%
% System data has the form:
%SYSTEM      name           baseMVA     Voltage Tolerance
SYSTEM    Plus_loads       100               0.05
%
% Bus data has the form:
%BUS name         type  volts  Pgen    Qgen    Pload   Qload   Qcap
BUS One            SL    1.00     0       0       0       0       0
BUS Two            PQ    1.00     0       0      60      35       0
BUS Three          PQ    1.00     0       0      70      40       0
BUS Four           PQ    1.00     0       0     100    56.6       0
BUS Five           PV    1.00   190       0      40      30       0
%
% Transmission line data has the form:
%LINE     from        to      Rse     Xse     Gsh     Bsh     Rating(MVA)
LINE    One      Two       0.0210  0.1250  0.000   0.000        50
LINE    One      Four      0.0235  0.0940  0.000   0.000       100
LINE    Two      Three     0.0250  0.1500  0.000   0.000        50
LINE    Two      Five      0.0180  0.0730  0.000   0.000       100
LINE    Three    Five      0.0220  0.1100  0.000   0.000        70
LINE    Four     Five      0.0190  0.0800  0.000   0.000       100
```

When this program is executed, the results are as shown below:

>> **power_flow prob_11_7_input**
Input summary statistics:
22 lines in system file
1 SYSTEM lines
5 BUS lines
6 LINE lines

Results for Case **Plus_loads**

Bus no.	Bus Name	Type	Volts (pu) / angle (deg)	Generation (MW) (MVAR)		Load (MW) (MVAR)		Cap (MVAR)	To Bus	Line Flow (MW) (MVAR)	
1	One	SL	1.000/ 0.00	83.08	46.59	0.00	0.00	0.00	Two	32.01	18.93
									Four	51.07	27.65
2	Two	PQ	0.970/ -2.13	0.00	0.00	60.00	35.00	0.00	One	-31.72	-17.20
									Three	17.15	9.95
									Five	-45.30	-27.77
3	Three	PQ	0.951/ -3.57	0.00	0.00	70.00	40.00	0.00	Two	-17.05	-9.32
									Five	-52.90	-30.69
4	Four	PQ	0.963/ -2.47	0.00	0.00	100.00	56.60	0.00	One	-50.28	-24.48
									Five	-49.65	-32.13
5	Five	PV	1.000/ -0.47	190.00	130.31	40.00	30.00	0.00	Two	45.84	29.96
									Three	53.81	35.24
									Four	50.36	35.15
			Totals	273.08	176.89	270.00	161.60	0.00			

Line Losses

Line no.	From Bus	To Bus	Ploss (MW)	Qloss (MVAR)
1	One	Two	0.29	1.73
2	One	Four	0.79	3.17

3	Two	Three	0.10	0.63
4	Two	Five	0.54	2.19
5	Three	Five	0.91	4.55
6	Four	Five	0.72	3.02
===
	Totals:		3.35	15.28
===

```
                    Alerts
|===========================================|
```
NONE

Done in 15 iterations.

This new load caused the voltage to decrease at Bus 4 and the load on the transmission lines to increase, but neither limit was exceeded. The system could operate indefinitely under these conditions.

11-8. Write your own program to solve for the voltages and currents in the power system instead of using program `power_flow`. You may write the program in any programming language with which you are familiar. How do your answers compare to the ones produced by `power_flow`?

SOLUTION To solve this problem, we must first generate the bus admittance matrix $\mathbf{Y}_{bus}$, and then iterate to a solution using Equation (11-8) for the load busses, and Equations (11-8) and (11-21) for the generator busses.

$$\mathbf{V}_i = \frac{1}{Y_{ii}}\left(\frac{P_i - jQ_i}{\mathbf{V}_i^*} - \sum_{\substack{k=1 \\ (k\neq i)}}^{N} Y_{ik}\mathbf{V}_k \right) \tag{11-8}$$

$$Q_i = -\mathrm{Im}\left(\mathbf{V}_i^*\sum_{k=1}^{N} Y_{ik}\mathbf{V}_k \right) \tag{11-21}$$

The admittance equivalent to the series impedance of each transmission line is given below.

Transmission Line Number	From / To (bus to bus)	Series impedance Z, pu	Series admittance Y, pu
1	1-2	$0.0210 + j0.1250$	$1.3071 - j7.7804$
2	1-4	$0.0235 + j0.0940$	$2.5031 - j10.013$
3	2-3	$0.0250 + j0.1500$	$1.0811 - j6.4865$
4	2-5	$0.0180 + j0.0730$	$3.1842 - j12.914$
5	3-5	$0.0220 + j0.1100$	$1.7483 - j8.7413$
6	4-5	$0.0190 + j0.0800$	$2.8102 - j11.833$

Now the bus admittance matrix $\mathbf{Y}_{bus}$ must be built. The on-diagonal elements are the sum of the admittances of all lines attached to the particular bus, while the off-diagonal elements are the negative of the admittances between the two busses. For example, the on-axis admittance Y_{11} is

$$Y_{11} = Y_{\text{Line }1} + Y_{\text{Line }2} = 3.8102 - j17.7929$$

and the off-axis admittance Y_{12} between Bus 1 and Bus 2 is $-1.3071 + j7.7804$. The complete bus admittance matrix is

$$\mathbf{Y}_{bus} = \begin{bmatrix} 3.8102 - j17.7929 & -1.3071 + j7.7804 & 0 & -2.5031 + j10.0125 & 0 \\ -1.3071 + j7.7804 & 5.5723 - j27.1804 & -1.0811 + j6.4865 & 0 & -3.1842 + j12.9135 \\ 0 & -1.0811 + j6.4865 & 2.8293 - j15.2277 & 0 & -1.7483 + j8.7413 \\ -2.5031 + j10.0125 & 0 & 0 & 5.3134 - j21.8451 & -2.8102 + j11.8326 \\ 0 & -3.1842 + j12.9135 & -1.7483 + j8.7413 & -2.8102 + j11.8326 & 7.7426 - j33.4873 \end{bmatrix}$$

Once we have created $\mathbf{Y}_{bus}$, we can write a program that iteratively calculates the bus voltages until it converges to an answer. As stated above, the program can be in any language that you wish to use. However, it is easiest to write such a program in MATLAB or Fortran, because those languages include a native complex number type. Languages such as C are much harder to use, because the programmer must manually perform all complex calculations. A MATLAB version of the program is shown below:

```
% M-file: prob11_8.m
% M-file to solve the power-flow problem of Problem 11-8.
% This set of equations includes a slack bus, a generator
% bus, and three load busses.

% Set problem size and initial conditions
```

```
n_bus = 5;
swing_bus = 1;
acc_fac = 1.3;

% Specify which busses are generator busses with flags.
% Note that 1 is "true" and 0 is "false" in MATLAB.  In
% this example, Bus 5 is a generator bus, and the others
% are not.
gen_bus = [0 0 0 0 1];

% Create Y-bus
 Ybus = ...
[ 3.8102-j*17.7929 -1.3071+j*7.7804   0                -2.5031+j*10.0125  0;                ...
 -1.3071+j*7.7804   5.5723-j*27.1804 -1.0811+j*6.4865  0                -3.1842+j*12.9135; ...
  0                -1.0811+j*6.4865   2.8293-j*15.2277 0                -1.7483+j*8.7413;  ...
 -2.5031+j*10.0125  0                 0                5.3134-j*21.8451 -2.8102+j*11.8326; ...
  0                -3.1842+j*12.9135 -1.7483+j*8.7413 -2.8102+j*11.8326  7.7426-j*33.4873];

% Initialize the real and reactive power supplied to the
% power system at each bus.  Note that the power at the
% swing bus doesn't matter, and the reactive power at the
% generator bus will be recomputed dynamically.
P(2) = -0.6;
P(3) = -0.7;
P(4) = -0.8;
P(5) =  1.5;
Q(2) = -0.35;
Q(3) = -0.4;
Q(4) = -0.5;
Q(5) = -0.3;

% Initialize the bus voltages to 1.0 at 0 degrees
for ii = 1:n_bus
   Vbus(ii) = 1;
end

% Set convergence criterion
eps = 0.00001;

% Initialize the iteration counter
n_iter = 0;

% Set a maximum number of iterations here so that
% the program will not run in an infinite loop if
% it fails to converge to a solution.
for iter = 1:100

   % Increment the iteration count
   n_iter = n_iter + 1;

   % Save old bus voltages for comparison purposes
   Vbus_old = Vbus;

   % Calculate the updated bus voltage
   for ii = 1:n_bus
```

```
    % Skip the swing bus!
    if ii ~= swing_bus

        % If this is a generator bus, update the reactive
        % power estimate.
        if gen_bus(ii)
            temp = 0;
            for jj = 1:n_bus
                temp = temp + Ybus(ii,jj) * Vbus(jj);
            end
            temp = conj(Vbus(ii)) * temp;
            Q(ii) = -imag(temp);
        end

        % Calculate updated voltage at bus 'ii'.  First, sum
        % up the current contributions at bus 'ii' from all
        % other busses.
        temp = 0;
        for jj = 1:n_bus
            if ii ~= jj
                temp = temp - Ybus(ii,jj) * Vbus(jj);
            end
        end

        % Add in the current injected at this node
        temp = (P(ii) - j*Q(ii)) / conj(Vbus(ii)) + temp;

        % Get updated estimate of Vbus at 'ii'
        Vnew = 1/Ybus(ii,ii) * temp;

        % Apply an acceleration factor to the new voltage estimate
        Vbus(ii) = Vbus_old(ii) + acc_fac * (Vnew - Vbus_old(ii));

        % If this is a generator bus, update the magnitude of the
        % voltage to keep it constant.
        if gen_bus(ii)
            Vbus(ii) = Vbus(ii) * abs(Vbus_old(ii)) / abs(Vbus(ii));
        end
    end
end

% Compare the old and new estimate of the voltages.
% Note that we will compare the real and the imag parts
% separately, and both must be within tolerances.
test = 0;
for ii = 1:n_bus

    % Compare real parts
    if abs( real(Vbus(ii)) - real(Vbus_old(ii)) ) > eps
        test = 1;
    end

    % Compare imaginary parts
    if abs( imag(Vbus(ii)) - imag(Vbus_old(ii)) ) > eps
        test = 1;
    end
```

```matlab
      end

   % Did we converge?  If so, get out of the loop.
   if test == 0
      break;
   end
end

% Did we exceed the maximum number of iterations?
if iter == 100
   disp('Max number of iterations exceeded!');
end

% Display results
for ii = 1:n_bus
   [mag, phase] = r2p(Vbus(ii));
   str = ['The voltage at bus ' int2str(ii) ' = ' ...
           num2str(mag) '/' num2str(phase)];
   disp(str);
end

% Calculate the current flowing and power transmitted
% between each pair of busses
Sline = zeros(n_bus,n_bus);
Pline = zeros(n_bus,n_bus);
Qline = zeros(n_bus,n_bus);
for ii = 1:n_bus
   for jj = 1:n_bus

      % Skip own bus
      if ii == jj
         continue;
      end

      % Calculate line current
      Iline(ii,jj) = Ybus(ii,jj) * ( Vbus(ii) - Vbus(jj) );

      % Calculate complex power flow
      Sline(ii,jj) = Vbus(ii) * conj( Iline(ii,jj) );

      % Calculate real and reactive components
      Pline(ii,jj) = real(Sline(ii,jj));
      Qline(ii,jj) = imag(Sline(ii,jj));
   end
end

% Display currents
for ii = 1:n_bus
   for jj = 1:n_bus

      % Skip own bus
      if ii == jj
         continue;
      end

      [mag, phase] = r2p(Iline(ii,jj));
```

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```
            str = ['The current from ' num2str(ii) ' to ' num2str(jj) ...
                   ' = ' num2str(mag) '/' num2str(phase)];
            disp(str);
        end
    end
end

% Display powers
for ii = 1:n_bus
    for jj = 1:n_bus

        % Skip own bus
        if ii == jj
            continue;
        end

        str = ['The real     power from ' num2str(ii) ' to ' num2str(jj) ...
               ' = ' num2str(Pline(ii,jj))];
        disp(str);
        str = ['The reactive power from ' num2str(ii) ' to ' num2str(jj) ...
               ' = ' num2str(Qline(ii,jj))];
        disp(str);
    end
end

% Display the number of iterations
str = ['Number of iterations = ' int2str(n_iter) ];
disp(str);
```

When this program is executed, the results are

```
>> prob11_8
The voltage at bus 1 = 1/0
The voltage at bus 2 = 0.97017/-1.725
The voltage at bus 3 = 0.95069/-3.0578
The voltage at bus 4 = 0.96855/-1.6666
The voltage at bus 5 = 1/0.1266
The current from 1 to 2 = 0.33185/143.5077
The current from 1 to 3 = 0/0
The current from 1 to 4 = 0.43888/145.5187
The current from 1 to 5 = 0/0
The current from 2 to 1 = 0.33185/-36.4923
The current from 2 to 3 = 0.1949/145.9913
The current from 2 to 4 = 0/0
The current from 2 to 5 = 0.58021/-30.086
The current from 3 to 1 = 0/0
The current from 3 to 2 = 0.1949/-34.0087
The current from 3 to 4 = 0/0
The current from 3 to 5 = 0.65308/-32.4387
The current from 4 to 1 = 0.43888/-34.4813
The current from 4 to 2 = 0/0
The current from 4 to 3 = 0/0
The current from 4 to 5 = 0.53533/-33.0002
The current from 5 to 1 = 0/0
The current from 5 to 2 = 0.58021/149.914
The current from 5 to 3 = 0.65308/147.5613
The current from 5 to 4 = 0.53533/146.9998
The real     power from 1 to 2 = -0.26679
```

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```
The reactive power from 1 to 2 = -0.19736
The real     power from 1 to 3 = 0
The reactive power from 1 to 3 = 0
The real     power from 1 to 4 = -0.36178
The reactive power from 1 to 4 = -0.24847
The real     power from 1 to 5 = 0
The reactive power from 1 to 5 = 0
The real     power from 2 to 1 = 0.26447
The reactive power from 2 to 1 = 0.18359
The real     power from 2 to 3 = -0.15985
The reactive power from 2 to 3 = -0.10099
The real     power from 2 to 4 = 0
The reactive power from 2 to 4 = 0
The real     power from 2 to 5 = 0.49534
The reactive power from 2 to 5 = 0.26739
The real     power from 3 to 1 = 0
The reactive power from 3 to 1 = 0
The real     power from 3 to 2 = 0.15891
The reactive power from 3 to 2 = 0.095294
The real     power from 3 to 4 = 0
The reactive power from 3 to 4 = 0
The real     power from 3 to 5 = 0.54102
The reactive power from 3 to 5 = 0.30461
The real     power from 4 to 1 = 0.35725
The reactive power from 4 to 1 = 0.23036
The real     power from 4 to 2 = 0
The reactive power from 4 to 2 = 0
The real     power from 4 to 3 = 0
The reactive power from 4 to 3 = 0
The real     power from 4 to 5 = 0.44288
The reactive power from 4 to 5 = 0.26963
The real     power from 5 to 1 = 0
The reactive power from 5 to 1 = 0
The real     power from 5 to 2 = -0.5014
The reactive power from 5 to 2 = -0.29197
The real     power from 5 to 3 = -0.5504
The reactive power from 5 to 3 = -0.35153
The real     power from 5 to 4 = -0.44832
The reactive power from 5 to 4 = -0.29256
Number of iterations = 12
```

These results agree with the answers calculated in Problem 11-4 to the accuracy with which we calculated $\mathbf{Y}_{bus}$. Note that the power outputs are in per-unit. They must be converted to MW and MVAR by multiplying system base MVA before comparing with the printout in Problem 11-4.

11-9. Figure P11-2 shows a one-line diagram of a simple power system. Assume that the generator G_1 is supplying a constant 13.8 kV at Bus 1, that load M_2 is consuming 20 MVA at 0.85 PF lagging, and that load M_3 is consuming 10 MVA at 0.90 PF leading. Calculate the bus admittance matrix $\mathbf{Y}_{bus}$ for this system, and then use it to determine the voltage at Bus 2 in this power system. (*Note:* assume that the system base apparent power is 30 MVA.)

G_1 ratings:	T_1 ratings:		T_2 ratings:	M_2 ratings:	M_3 ratings:
30 MVA	35 MVA		30 MVA	20 MVA	10 MVA
13.8 kV	13.2/115 kV		120/12.5 kV	12.5 kV	12.5 kV
$R = 0.1$ pu	$R = 0.01$ pu		$R = 0.01$ pu	$R = 0.1$ pu	$R = 0.1$ pu
$X_S = 1.0$ pu	$X = 0.10$ pu		$X = 0.08$ pu	$X_S = 1.1$ pu	$X_S = 1.1$ pu
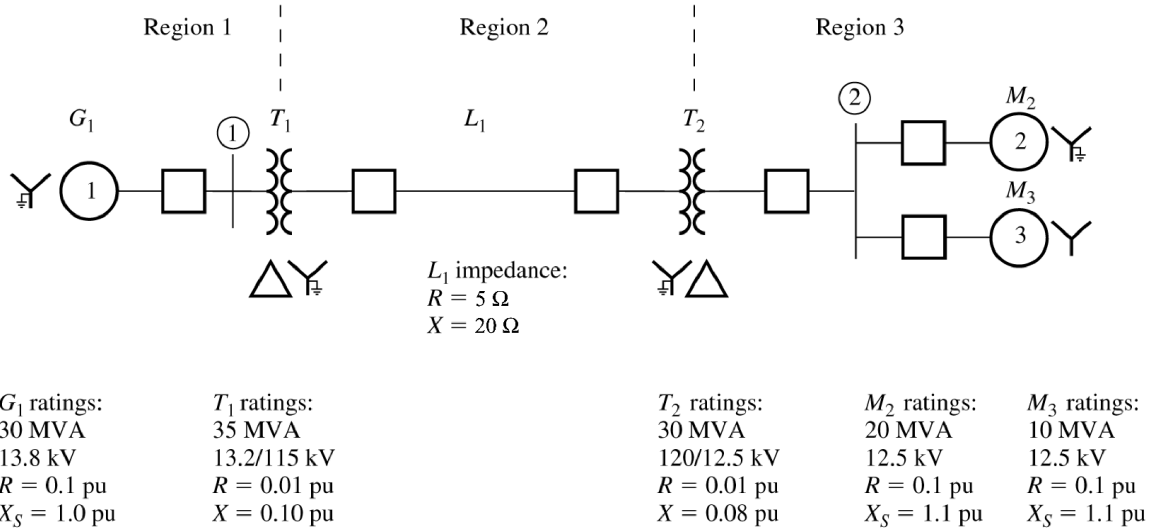
Figure P11-2 One-line diagram of the power system in Problems 11-9 and 11-10.

SOLUTION   The system base apparent power is $S_{\text{base}} = 30$ MVA, and the system base voltages in each region are:

$$V_{\text{base,1}} = 13.8 \text{ kV}$$

$$V_{\text{base,2}} = \left(\frac{115 \text{ kV}}{13.2 \text{ kV}}\right) V_{\text{base,1}} = \left(\frac{115 \text{ kV}}{13.2 \text{ kV}}\right)(13.8 \text{ kV}) = 120 \text{ kV}$$

$$V_{\text{base,3}} = \left(\frac{12.5 \text{ kV}}{120 \text{ kV}}\right) V_{\text{base,2}} = \left(\frac{12.5 \text{ kV}}{120 \text{ kV}}\right)(120 \text{ kV}) = 12.5 \text{ kV}$$

The base impedance of Region 2 is:

$$Z_{\text{base,2}} = \frac{\left(V_{LL,\text{ base,2}}\right)^2}{S_{3\phi,\text{ base}}} = \frac{(120,000 \text{ V})^2}{30,000,000 \text{ VA}} = 480 \ \Omega$$

The per unit resistance and reactance of G_1 are already on the proper base:

$$Z_{G1} = 0.1 + j1.0 \text{ pu}$$

The per unit resistance and reactance of T_1 are:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}} \left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2 \left(\frac{S_{\text{new}}}{S_{\text{given}}}\right)$$

$$Z_{T1} = \left(0.01 + j0.10\right)\left(\frac{13.2 \text{ kV}}{13.8 \text{ kV}}\right)^2 \left(\frac{30,000 \text{ kVA}}{35,000 \text{ kVA}}\right) = 0.00784 + j0.0784 \text{ pu}$$

The per unit resistance and reactance of the transmission line are:

$$Z_{\text{line}} = \frac{Z}{Z_{\text{base}}} = \frac{5 + j20 \ \Omega}{480 \ \Omega} = 0.0104 + j0.0417 \text{ pu}$$

The per unit resistance and reactance of T_2 are already on the right base:

$$Z_{T2} = 0.01 + j0.08 \text{ pu}$$

The per unit resistance and reactance of M_1 are:

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$$Z_{M1} = \left(0.1 + j1.1\right)\left(\frac{12.5 \text{ kV}}{12.5 \text{ kV}}\right)^2 \left(\frac{30,000 \text{ kVA}}{20,000 \text{ kVA}}\right) = 0.15 + j1.65 \text{ pu}$$

The per unit resistance and reactance of M_1 are:

$$Z_{M2} = \left(0.1 + j1.1\right)\left(\frac{12.5 \text{ kV}}{12.5 \text{ kV}}\right)^2 \left(\frac{30,000 \text{ kVA}}{10,000 \text{ kVA}}\right) = 0.30 + j3.30 \text{ pu}$$

The resulting per-phase equivalent circuit is:

If the impedances of the transmission line and the transformers are combined, all voltage sources are converted to their Norton equivalents, and all impedances are converted to admittances, the resulting circuit is:
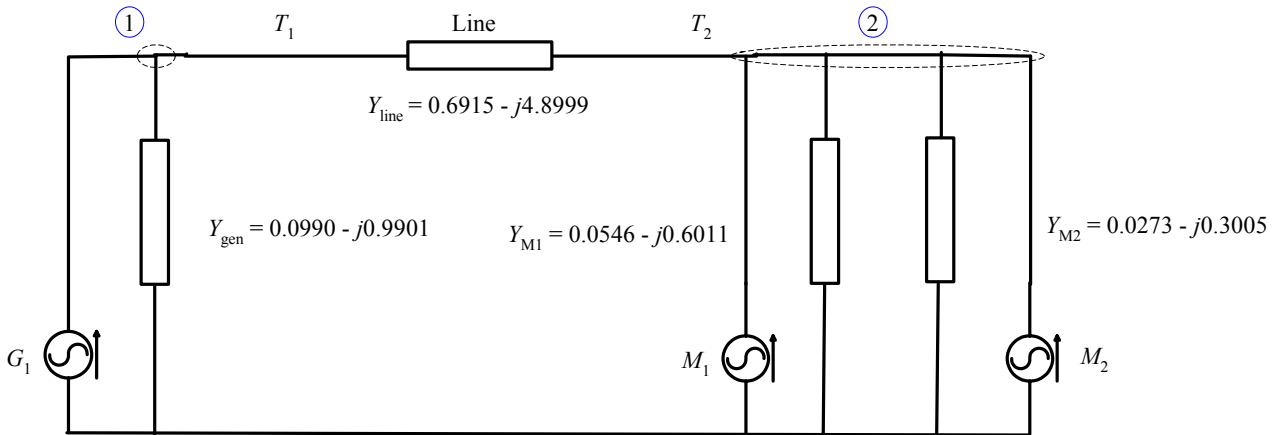
For power flow studies, we ignore the self admittances of each node. There are only two busses in this circuit, so the bus admittance matrix will appear as follows:

$$\mathbf{Y}_{bus} = \begin{bmatrix} 0.6915 - j4.8999 & -0.6915 + j4.8999 \\ -0.6915 + j4.8999 & 0.6915 - j4.8999 \end{bmatrix}$$

The loads on this power system are:

$$P_{M2} = \left(20 \text{ MVA}\right)\left(0.85\right) = 17 \text{ MW} = 0.567 \text{ pu}$$

$$Q_{M2} = \left(20 \text{ MVA}\right)\sin\left[\cos^{-1}\left(0.85\right)\right] = 10.5 \text{ MVAR} = 0.350 \text{ pu}$$

$$P_{M3} = \left(10 \text{ MVA}\right)\left(0.9\right) = 9 \text{ MW} = 0.300 \text{ pu}$$

$$Q_{M3} = (10 \text{ MVA}) \sin\left[-\cos^{-1}(0.9)\right] = -4.36 \text{ MVAR} = -0.145 \text{ pu}$$

The voltages on the busses can be calculated by iteration. A simple program to do so follows:

```
% M-file: prob11_9.m
% M-file to solve the power-flow problem of Problem 11-9.
% This set of equations includes a slack bus and a single
% load bus.

% Set problem size and initial conditions
n_bus = 2;
swing_bus = 1;
acc_fac = 1.3;

% Create Y-bus
 Ybus = ...
[ 0.6915-j*4.8999  -0.6915+j*4.8999; ...
 -0.6915+j*4.8999   0.6915-j*4.8999];

% Initialize the real and reactive power supplied to the
% power system at each bus.  Note that the power at the
% swing bus doesn't matter, and the reactive power at the
% generator bus will be recomputed dynamically.
P(2) = -0.867;
Q(2) = -0.205;

% Initialize the bus voltages to 1.0 at 0 degrees
for ii = 1:n_bus
   Vbus(ii) = 1;
end

% Set convergence criterion
eps = 0.00001;

% Initialize the iteration counter
n_iter = 0;

% Set a maximum number of iterations here so that
% the program will not run in an infinite loop if
% it fails to converge to a solution.
for iter = 1:100

   % Increment the iteration count
   n_iter = n_iter + 1;

   % Save old bus voltages for comparison purposes
   Vbus_old = Vbus;

   % Calculate the updated bus voltage
   for ii = 1:n_bus

      % Skip the swing bus!
      if ii ~= swing_bus

         % Calculate updated voltage at bus 'ii'.  First, sum
         % up the current contributions at bus 'ii' from all
```

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```
            % other busses.
            temp = 0;
            for jj = 1:n_bus
               if ii ~= jj
                  temp = temp - Ybus(ii,jj) * Vbus(jj);
               end
            end

            % Add in the current injected at this node
            temp = (P(ii) - j*Q(ii)) / conj(Vbus(ii)) + temp;

            % Get updated estimate of Vbus at 'ii'
            Vnew = 1/Ybus(ii,ii) * temp;

            % Apply an acceleration factor to the new voltage estimate
            Vbus(ii) = Vbus_old(ii) + acc_fac * (Vnew - Vbus_old(ii));
         end
      end

   % Compare the old and new estimate of the voltages.
   % Note that we will compare the real and the imag parts
   % separately, and both must be within tolerances.
   test = 0;
   for ii = 1:n_bus

      % Compare real parts
      if abs( real(Vbus(ii)) - real(Vbus_old(ii)) ) > eps
         test = 1;
      end

      % Compare imaginary parts
      if abs( imag(Vbus(ii)) - imag(Vbus_old(ii)) ) > eps
         test = 1;
      end
   end

   % Did we converge?  If so, get out of the loop.
   if test == 0
      break;
   end
end

% Did we exceed the maximum number of iterations?
if iter == 100
   disp('Max number of iterations exceeded!');
end

% Display results
for ii = 1:n_bus
   [mag, phase] = r2p(Vbus(ii));
   str = ['The voltage at bus ' int2str(ii) ' = ' ...
            num2str(mag) '/' num2str(phase)];
   disp(str);
end
```

When this program is executed, the results are:

```
>> prob11_9
The voltage at bus 1 = 1/0
The voltage at bus 2 = 0.91101/-10.6076
```

The per-unit voltage at Bus 2 is $0.911\angle -10.6°$. Since the base voltage at Bus 2 is 12.5 kV, the actual voltage at Bus 2 is $V_2 = (0.911)(12.5 \text{ kV}) = 11.4 \text{ kV}$

11-10. Suppose that a 10 MVAR capacitor bank were added to Bus 2 in the power system of Figure P11-2. What is the voltage of Bus 2 now?

SOLUTION The addition of a 10 MVAR capacitor bank at Bus 2 increases the reactive power Q supplied to that bus by (10 MVAR)/(30 MVAR) = 0.3333 pu. This term is added to the reactive power load at Bus 2 in the program that we created for Problem 11-9, as follows:

```
% Initialize the real and reactive power supplied to the
% power system at each bus.  Note that the power at the
% swing bus doesn't matter, and the reactive power at the
% generator bus will be recomputed dynamically.
P(2) = -0.867;
Q(2) = -0.205 + 0.333;   % Add capacitive loads
...
```

When this modified program is executed, the results are:

```
>> prob11_10
The voltage at bus 1 = 1/0
The voltage at bus 2 = 0.98485/-10.3598
```

The voltage at Bus 2 increases to $0.9849\angle -10.4°$ pu, or $V_2 = (0.9849)(12.5 \text{ kV}) = 12.3 \text{ kV}$.

Problems 11-11 through 11-15 refer to the simple power system with six busses and six transmission lines shown in Figure P11-3. The base apparent power of this power system is 100 MVA, and the tolerance on each bus voltage is 5%. Typical daytime bus data for the power system is given in Table 11-9, typical nighttime bus data for the power system is given in Table 11-10. The line data for the power system is given in Table 11-11.
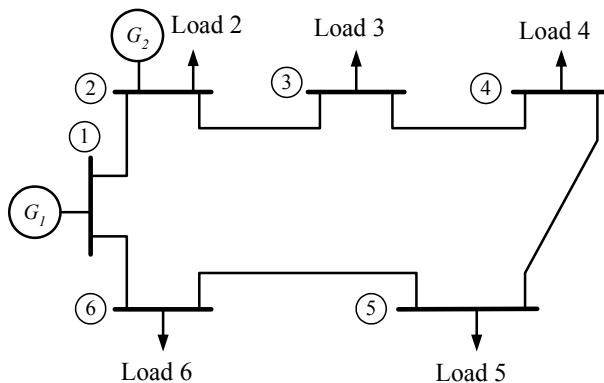
Figure P11-3   The power system of Problems 11-11 to 11-15.

Table 11-9:   Typical daytime bus data for the power system in Figure P11-3.

Bus Name	Type	V (pu)	Generation		Loads	
			P (MW)	Q (Mvar)	P (MW)	Q (Mvar)
1	SL	$1\angle 0°$				
2	PV	$1\angle 0°$	100		60	35
3	PQ	$1\angle 0°$			40	25
4	PQ	$1\angle 0°$			60	40
5	PQ	$1\angle 0°$			30	10
6	PQ	$1\angle 0°$			40	25

Table 11-10:   Typical nighttime bus data for the power system in Figure P11-3.

Bus Name	Type	V (pu)	Generation		Loads	
			P (MW)	Q (Mvar)	P (MW)	Q (Mvar)
1	SL	$1\angle 0°$				
2	PV	$1\angle 0°$	100		40	25
3	PQ	$1\angle 0°$			30	20
4	PQ	$1\angle 0°$			50	35
5	PQ	$1\angle 0°$			15	5
6	PQ	$1\angle 0°$			30	15

Table 11-11:   Line data for the power system in Figure P11-3. The shunt admittances of all lines are negligible.

Transmission Line Number	From / To (bus to bus)	Series impedance Z, pu	Rated MVA
1	1-2	$0.010 + j0.080$	200
2	2-3	$0.010 + j0.080$	200
3	3-4	$0.008 + j0.064$	200
4	4-5	$0.020 + j0.150$	100
5	5-6	$0.008 + j0.064$	200
6	6-1	$0.010 + j0.080$	200

11-11. The bus data given in Table 11-9 represent typical daytime loads on the power system.

(a) Find the voltages at each bus in this power system under typical daytime conditions.

(b) Find the voltages at each bus in this power system under typical nighttime conditions.

(c) Find the real and reactive power flows in each transmission line in each case.

(d) Are any of the bus voltages out of tolerance in this power system?

(e) Are any of the transmission lines overloaded?

SOLUTION We will solve this problem using program `power_flow.m`.

(a) The input file `prob_11_11a_input` to calculate the power flows in this power system is given below.

```
% File describing the base case for the power system of
% Problem 11-11.
%
% System data has the form:
```

```
%SYSTEM    name      baseMVA    Voltage Tolerance
SYSTEM    Daytime      100              0.05
%
% Bus data has the form:
%BUS name        type  volts  Pgen   Qgen   Pload  Qload  Qcap
BUS One          SL    1.00      0      0      0      0      0
BUS Two          PV    1.00    100      0     60     35      0
BUS Three        PQ    1.00      0      0     40     25      0
BUS Four         PQ    1.00      0      0     60     40      0
BUS Five         PQ    1.00      0      0     30     10      0
BUS Six          PQ    1.00      0      0     40     25      0
%
% Transmission line data has the form:
%LINE    from      to      Rse    Xse    Gsh    Bsh    Rating(MVA)
LINE     One       Two    0.010  0.080  0.000  0.000      200
LINE     Two       Three  0.010  0.080  0.000  0.000      200
LINE     Three     Four   0.008  0.064  0.000  0.000      200
LINE     Four      Five   0.020  0.150  0.000  0.000      100
LINE     Five      Six    0.008  0.064  0.000  0.000      200
LINE     Six       One    0.010  0.080  0.000  0.000      200
```

When this program is executed, the results are as shown below:

>> **power_flow prob_11_11a_input**
Input summary statistics:
24 lines in system file
1 SYSTEM lines
6 BUS lines
6 LINE lines

Results for Case Daytime

Bus no.	Name	Bus Type	Volts / (pu)	angle (deg)	Generation (MW)	Generation (MVAR)	Load (MW)	Load (MVAR)	Cap (MVAR)	To Bus	Line Flow (MW)	Line Flow (MVAR)
1	One	SL	1.000/	0.00	132.71	50.20	0.00	0.00	0.00	Two	44.21	-4.72
										Six	88.50	54.92
2	Two	PV	1.000/	-2.05	100.00	109.02	60.00	35.00	0.00	One	-44.01	6.30
										Three	84.14	67.76
3	Three	PQ	0.939/	-5.75	0.00	0.00	40.00	25.00	0.00	Two	-82.98	-58.42
										Four	43.11	33.46
4	Four	PQ	0.913/	-7.41	0.00	0.00	60.00	40.00	0.00	Three	-42.84	-31.30
										Five	-17.13	-8.68
5	Five	PQ	0.932/	-5.80	0.00	0.00	30.00	10.00	0.00	Four	17.22	9.35
										Six	-47.17	-19.33
6	Six	PQ	0.949/	-3.94	0.00	0.00	40.00	25.00	0.00	Five	47.41	21.25
										One	-87.41	-46.24
Totals					232.71	159.22	230.00	135.00	0.00			

Line Losses

Line no.	From Bus	To Bus	Ploss (MW)	Qloss (MVAR)
1	One	Two	0.20	1.58
2	Two	Three	1.17	9.34

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```
     3      Three     Four     0.27     2.16
     4      Four      Five     0.09     0.66
     5      Five      Six      0.24     1.92
     6      Six       One      1.08     8.68
|=================================================|
                   Totals:     3.05    24.34
|=================================================|

                      Alerts
|=================================================|
ALERT: Voltage on bus Three out of tolerance.
ALERT: Voltage on bus Four out of tolerance.
ALERT: Voltage on bus Five out of tolerance.
ALERT: Voltage on bus Six out of tolerance.
|=================================================|

Done in 21 iterations.
```

The voltages on Busses 3, 4, 5, and 6 are out of tolerances, but no power lines are overloaded.

(b) The input file `prob_11_11b_input` to calculate the power flows in this power system is given below.

```
% File describing the nighttime case for the power system of
% Problem 11-11.
%
% System data has the form:
%SYSTEM      name         baseMVA     Voltage Tolerance
SYSTEM    Nighttime       100               0.05
%
% Bus data has the form:
%BUS name        type   volts  Pgen   Qgen   Pload  Qload  Qcap
BUS One           SL    1.00      0      0      0      0      0
BUS Two           PV    1.00    100      0     40     25      0
BUS Three         PQ    1.00      0      0     30     20      0
BUS Four          PQ    1.00      0      0     50     35      0
BUS Five          PQ    1.00      0      0     15      5      0
BUS Six           PQ    1.00      0      0     30     15      0
%
% Transmission line data has the form:
%LINE     from        to     Rse     Xse     Gsh     Bsh     Rating(MVA)
LINE      One        Two    0.010   0.080   0.000   0.000      200
LINE      Two      Three    0.010   0.080   0.000   0.000      200
LINE      Three     Four    0.008   0.064   0.000   0.000      200
LINE      Four      Five    0.020   0.150   0.000   0.000      100
LINE      Five       Six    0.008   0.064   0.000   0.000      200
LINE      Six        One    0.010   0.080   0.000   0.000      200
```

When this program is executed, the results are as shown below:

Input summary statistics:
24 lines in system file
 1 SYSTEM lines
 6 BUS lines
 6 LINE lines

Results for Case Nighttime

Bus no.	Name	Bus Type	Volts / (pu)	angle (deg)	Generation (MW)	(MVAR)	Load (MW)	(MVAR)	Cap (MVAR)	Line Flow To Bus	(MW)	(MVAR)
1	One	SL	**1.000/**	**0.00**	66.25	34.71	0.00	0.00	0.00	Two	7.71	-0.94
										Six	58.54	35.65
2	Two	PV	**1.000/**	**-0.36**	100.00	77.50	40.00	25.00	0.00	One	-7.71	0.99
										Three	67.81	51.54
3	Three	PQ	**0.953/**	**-3.31**	0.00	0.00	30.00	20.00	0.00	Two	-67.09	-45.73
										Four	37.20	25.75
4	Four	PQ	**0.933/**	**-4.71**	0.00	0.00	50.00	35.00	0.00	Three	-37.02	-24.31
										Five	-12.95	-10.68
5	Five	PQ	**0.953/**	**-3.60**	0.00	0.00	15.00	5.00	0.00	Four	13.02	11.17
										Six	-27.98	-16.16
6	Six	PQ	**0.967/**	**-2.57**	0.00	0.00	30.00	15.00	0.00	Five	28.07	16.89
										One	-58.07	-31.89
			Totals		166.25	112.21	165.00	100.00	0.00			

Line Losses

Line no.	From Bus	To Bus	Ploss (MW)	Qloss (MVAR)
1	One	Two	0.01	0.05
2	Two	Three	0.73	5.80

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3	Three	Four	0.18	1.44
4	Four	Five	0.06	0.49
5	Five	Six	0.09	0.74
6	Six	One	0.47	3.76

|=====================================|

Totals: 1.54 12.27

|=====================================|

Alerts

|=====================================|

ALERT: Voltage on bus Four out of tolerance.

|=====================================|

Done in 20 iterations.

Only the voltage at Bus 4 is out of tolerance under nighttime conditions.

(c) The real and reactive power flows in each line were already calculated as a part of the solutions above.

(d) The voltages out of tolerance were calculated as a part of the solutions above.

(e) No transmission lines were overloaded.

11-12. If there are any problems with the bus voltages under typical daytime loads, propose a solution for this problem. Define a set of capacitors at various busses in the system to compensate for out-of-tolerance voltage variations.

SOLUTION There are many possible solutions to this problem, and no single one can be said to be "right". In the daytime configuration, the lowest voltage is at Bus 4. Therefore, as a first cut, we could try to put enough capacitance on that bus to counteract the reactive loads there, and see what happens. The file containing this version of the power system is shown below:

```
% File describing adding capacitors to the power system of
% Problem 11-12.
%
% System data has the form:
%SYSTEM     name         baseMVA     Voltage Tolerance
SYSTEM    Day+Cap         100               0.05
%
% Bus data has the form:
%BUS name         type   volts   Pgen    Qgen    Pload   Qload   Qcap
BUS One            SL     1.00      0       0       0       0       0
BUS Two            PV     1.00    100       0      60      35       0
BUS Three          PQ     1.00      0       0      40      25       0
BUS Four           PQ     1.00      0       0      60      40      40
BUS Five           PQ     1.00      0       0      30      10       0
BUS Six            PQ     1.00      0       0      40      25       0
%
% Transmission line data has the form:
%LINE     from       to      Rse     Xse     Gsh     Bsh     Rating(MVA)
LINE      One        Two    0.010   0.080   0.000   0.000      200
LINE      Two       Three   0.010   0.080   0.000   0.000      200
LINE      Three      Four   0.008   0.064   0.000   0.000      200
LINE      Four       Five   0.020   0.150   0.000   0.000      100
LINE      Five        Six   0.008   0.064   0.000   0.000      200
LINE      Six         One   0.010   0.080   0.000   0.000      200
```

When this program is executed, the results are as shown below:

```
>> power_flow prob_11_12_input
Input summary statistics:
24 lines in system file
1 SYSTEM lines
6 BUS lines
6 LINE lines
```

Results for Case **Day+Cap**

Bus no.	Name	Bus Type	Volts / (pu)	angle (deg)	Generation (MW)	(MVAR)	Load (MW)	(MVAR)	Cap (MVAR)	To Bus	Line Flow (MW)	(MVAR)
1	One	SL	**1.000/**	**0.00**	132.12	35.25	0.00	0.00	0.00	Two	43.90	-4.70
										Six	88.21	39.94
2	Two	PV	**1.000/**	**-2.04**	100.00	78.90	60.00	35.00	0.00	One	-43.71	6.26
										Three	83.81	37.67
3	Three	PQ	**0.964/**	**-5.81**	0.00	0.00	40.00	25.00	0.00	Two	-82.97	-30.92
										Four	43.08	5.94
4	Four	PQ	**0.956/**	**-7.49**	0.00	0.00	60.00	40.00	40.00	Three	-42.92	-4.63
										Five	-17.05	4.64
5	Five	PQ	**0.953/**	**-5.82**	0.00	0.00	30.00	10.00	0.00	Four	17.12	-4.13
										Six	-47.08	-5.86
6	Six	PQ	**0.962/**	**-3.97**	0.00	0.00	40.00	25.00	0.00	Five	47.27	7.44
										One	-87.27	-32.44
Totals					232.12	114.15	230.00	135.00	40.00			

Line Losses

Line no.	From Bus	To Bus	Ploss (MW)	Qloss (MVAR)
1	One	Two	0.19	1.56
2	Two	Three	0.84	6.76

```
3        Three        0.16    1.30
4        Four         0.07    0.51
5        Five         0.20    1.59
6        Six          0.94    7.50
|===============================================|
         Totals:      2.41   19.22
|===============================================|


                 Alerts
|===============================================|
```
NONE

Done in 22 iterations.

This choice appears to have worked, since all of the voltages are now in tolerance. However, some of the voltages are pretty marginal—they are just barely in tolerance. Perhaps another solution would be better. Feel free to experiment and see what you come up with.

11-13. What happens to the bus voltages under nighttime conditions of the capacitors proposed in Problem 11-12 are left attached to the system at night? Is it ok to permanently connect the capacitors to the power system, or must they be switched?

SOLUTION As before, there are many "right" answers to this problem. If the same 40 MVAR capacitive load is added to Bus 4 and night, the resulting power system file is:

```
% File describing the nighttime case for the power system of
% Problem 11-13.
%
% System data has the form:
%SYSTEM      name          baseMVA     Voltage Tolerance
SYSTEM    Night+Cap         100              0.05
%
% Bus data has the form:
%BUS name          type  volts  Pgen   Qgen   Pload  Qload  Qcap
BUS One             SL    1.00     0      0      0      0      0
BUS Two             PV    1.00   100      0     40     25      0
BUS Three           PQ    1.00     0      0     30     20      0
BUS Four            PQ    1.00     0      0     50     35     40
BUS Five            PQ    1.00     0      0     15      5      0
BUS Six             PQ    1.00     0      0     30     15      0
%
% Transmission line data has the form:
%LINE     from         to     Rse     Xse     Gsh     Bsh     Rating(MVA)
LINE      One         Two    0.010   0.080   0.000   0.000       200
LINE      Two        Three   0.010   0.080   0.000   0.000       200
LINE      Three       Four   0.008   0.064   0.000   0.000       200
LINE      Four        Five   0.020   0.150   0.000   0.000       100
LINE      Five         Six   0.008   0.064   0.000   0.000       200
LINE      Six         One    0.010   0.080   0.000   0.000       200
```

When this program is executed, the results are as shown below:

>> **power_flow prob_11_13_input**
Input summary statistics:
24 lines in system file
 1 SYSTEM lines
 6 BUS lines
 6 LINE lines

Results for Case Night+Cap

Bus no.	Name	Bus Type	Volts / (pu)	angle (deg)	Generation (MW)	(MVAR)	Load (MW)	(MVAR)	Cap (MVAR)	To Bus	Line Flow (MW)	(MVAR)
1	One	SL	1.000/	0.00	65.79	20.43	0.00	0.00	0.00	Two	7.58	-0.92
										Six	58.20	21.35
2	Two	PV	1.000/	-0.35	100.00	48.45	40.00	25.00	0.00	One	-7.58	0.97
										Three	67.70	22.50
3	Three	PQ	0.977/	-3.40	0.00	0.00	30.00	20.00	0.00	Two	-67.19	-18.43
										Four	37.32	-1.56
4	Four	PQ	0.975/	-4.84	0.00	0.00	50.00	35.00	40.00	Three	-37.20	2.50
										Five	-12.76	2.51
5	Five	PQ	0.974/	-3.66	0.00	0.00	15.00	5.00	0.00	Four	12.80	-2.24
										Six	-27.75	-2.75
6	Six	PQ	0.978/	-2.60	0.00	0.00	30.00	15.00	0.00	Five	27.82	3.28
										One	-57.82	-18.28
Totals					165.79	68.87	165.00	100.00	40.00			

Line Losses

Line no.	From Bus	To Bus	Ploss (MW)	Qloss (MVAR)
1	One	Two	0.01	0.05
2	Two	Three	0.51	4.07

```
3      Three    0.12    0.94
4      Four     0.04    0.27
5      Five     0.07    0.52
6      Six      0.38    3.07
|===================================|
           Totals:   1.12    8.92
|===================================|


                    Alerts
|===================================|
NONE

Done in 20 iterations.
```

In the case that we created in Problem 11-12, the capacitors required for daytime voltage correction could be left in for the typical nighttime load. *This is not always the case.* If you made a different assumption about the size and distribution of capacitors in Problem 11-12, you might need to switch some or all of the capacitors out ant night. Also, if the nighttime loads can fall well below the "typical" nighttime levels, you might need to switch some capacitors out.

11-14. Suppose that the power system is operating under typical daytime loads, and the transmission line between Bus 4 and Bus 5 open circuits. What happens to the voltages on each bus now? Are any of the transmission lines overloaded?

SOLUTION   In this case, we will start with the typical daytime loads, and remove the transmission line between Busses 4 and 5. The resulting input file is shown below:

```
% File describing the case where the line between busses
% 4 and 5 open circuits.
%
% System data has the form:
%SYSTEM     name        baseMVA    Voltage Tolerance
SYSTEM   Open_line      100             0.05
%
% Bus data has the form:
%BUS name         type  volts  Pgen   Qgen   Pload  Qload  Qcap
BUS One            SL   1.00     0      0      0      0      0
BUS Two            PV   1.00    100     0     60     35      0
BUS Three          PQ   1.00     0      0     40     25      0
BUS Four           PQ   1.00     0      0     60     40      0
BUS Five           PQ   1.00     0      0     30     10      0
BUS Six            PQ   1.00     0      0     40     25      0
%
% Transmission line data has the form:
%LINE      from        to     Rse    Xse    Gsh    Bsh    Rating(MVA)
LINE       One        Two    0.010  0.080  0.000  0.000     200
LINE       Two        Three  0.010  0.080  0.000  0.000     200
LINE       Three      Four   0.008  0.064  0.000  0.000     200
%LINE      Four       Five   0.020  0.150  0.000  0.000     100
LINE       Five       Six    0.008  0.064  0.000  0.000     200
LINE       Six        One    0.010  0.080  0.000  0.000     200
```

When this program is executed, the results are as shown below:

Input summary statistics:
24 lines in system file
 1 SYSTEM lines
 6 BUS lines
 5 LINE lines

Results for Case **Open_line**

Bus no.	Bus Name	Type	Volts / (pu)	angle (deg)	Generation (MW)	(MVAR)	Load (MW)	(MVAR)	Cap (MVAR)	To Bus	Line Flow (MW)	(MVAR)
1	One	SL	1.000/	0.00	133.19	34.85	0.00	0.00	0.00	Two	62.44	-6.21
										Six	70.76	41.06
2	Two	PV	1.000/	-2.90	100.00	127.36	60.00	35.00	0.00	One	-62.04	9.35
										Three	102.15	83.03
3	Three	PQ	0.926/	-7.44	0.00	0.00	40.00	25.00	0.00	Two	-100.42	-69.17
										Four	60.53	44.18
4	Four	PQ	0.891/	-9.89	0.00	0.00	60.00	40.00	0.00	Three	-60.00	-39.99
5	Five	PQ	0.952/	-4.28	0.00	0.00	30.00	10.00	0.00	Six	-30.00	-10.00
6	Six	PQ	0.962/	-3.13	0.00	0.00	40.00	25.00	0.00	Five	30.09	10.71
										One	-70.09	-35.71
Totals					233.19	162.21	230.00	135.00	0.00			

Line Losses

Line no.	From Bus	To Bus	Ploss (MW)	Qloss (MVAR)
1	One	Two	0.39	3.15
2	Two	Three	1.73	13.86
3	Three	Four	0.52	4.19
4	Five	Six	0.09	0.71

```
5        Six      One       0.67      5.35
|=====================================================|
                  Totals:   3.41     27.26
|=====================================================|

                   Alerts
|=====================================================|
ALERT: Voltage on bus Three out of tolerance.
ALERT: Voltage on bus Four out of tolerance.

Done in 24 iterations.
```

When the line between Bus 4 and Bus 5 is open circuited, all real and reactive power flow to Bus 4 must be through Bus 3. This causes the voltages at both Bus 3 and Bus 4 to drop substantially.

11-15. Suppose that a new transmission line is to be added to the power system between Bus 1 and Bus 4. The line is rated at 200 MVA, and its series impedance is $0.008 + j0.064$ pu. Assume that no capacitors have been added to the system, and determine the daytime and nighttime voltages at every bus. Did the new transmission line resolve the voltage problems?

SOLUTION We will add the new transmission line to the power systems of Problem 11-11 *(a)* for daytime conditions and 11-11 *(b)* for nighttime conditions, and see what effect that the new line has on the voltages in the system. The two resulting power system files are shown below:

```
% File describing the daytime case with a new transmission
% line between Busses 1 and 4.
%
% System data has the form:
%SYSTEM      name         baseMVA     Voltage Tolerance
SYSTEM    Day+Line         100              0.05
%
% Bus data has the form:
%BUS name          type  volts  Pgen   Qgen   Pload  Qload  Qcap
BUS One             SL    1.00     0      0      0      0      0
BUS Two             PV    1.00   100      0     60     35      0
BUS Three           PQ    1.00     0      0     40     25      0
BUS Four            PQ    1.00     0      0     60     40      0
BUS Five            PQ    1.00     0      0     30     10      0
BUS Six             PQ    1.00     0      0     40     25      0
%
% Transmission line data has the form:
%LINE      from        to     Rse    Xse     Gsh     Bsh     Rating(MVA)
LINE      One         Two    0.010  0.080   0.000   0.000       200
LINE      Two        Three   0.010  0.080   0.000   0.000       200
LINE      Three       Four   0.008  0.064   0.000   0.000       200
LINE      Four        Five   0.020  0.150   0.000   0.000       100
LINE      Five        Six    0.008  0.064   0.000   0.000       200
LINE      Six         One    0.010  0.080   0.000   0.000       200
LINE      One         Four   0.008  0.064   0.000   0.000       200


% File describing the nighttime case with a new transmission
% line between Busses 1 and 4.
%
% System data has the form:
%SYSTEM      name         baseMVA     Voltage Tolerance
SYSTEM    Night+Line       100              0.05
%
% Bus data has the form:
%BUS name          type  volts  Pgen   Qgen   Pload  Qload  Qcap
BUS One             SL    1.00     0      0      0      0      0
BUS Two             PV    1.00   100      0     40     25      0
BUS Three           PQ    1.00     0      0     30     20      0
BUS Four            PQ    1.00     0      0     50     35      0
BUS Five            PQ    1.00     0      0     15      5      0
BUS Six             PQ    1.00     0      0     30     15      0
%
% Transmission line data has the form:
```

```
%LINE     from       to      Rse     Xse     Gsh     Bsh     Rating(MVA)
LINE      One        Two     0.010   0.080   0.000   0.000      200
LINE      Two        Three   0.010   0.080   0.000   0.000      200
LINE      Three      Four    0.008   0.064   0.000   0.000      200
LINE      Four       Five    0.020   0.150   0.000   0.000      100
LINE      Five       Six     0.008   0.064   0.000   0.000      200
LINE      Six        One     0.010   0.080   0.000   0.000      200
LINE      One        Four    0.008   0.064   0.000   0.000      200
```

When these programs are executed, the results are as shown below:

```
>> power_flow prob_11_15a_input
Input summary statistics:
25 lines in system file
 1 SYSTEM lines
 6 BUS lines
 7 LINE lines
```

Results for Case **Day+Line**

Bus no.	Name	Bus Type	Volts / (pu)	angle (deg)	Generation (MW)	(MVAR)	Load (MW)	(MVAR)	Cap (MVAR)	To Bus	Line Flow (MW)	(MVAR)
1	One	SL	1.000/	0.00	131.14	78.64	0.00	0.00	0.00	Two	4.89	-0.60
										Six	59.38	35.88
										Four	66.88	43.36
2	Two	PV	1.000/	-0.23	100.00	67.13	60.00	35.00	0.00	One	-4.88	0.62
										Three	44.97	31.54
3	Three	PQ	0.971/	-2.17	0.00	0.00	40.00	25.00	0.00	Two	-44.67	-29.13
										Four	4.73	4.15
4	Four	PQ	0.968/	-2.33	0.00	0.00	60.00	40.00	0.00	Three	-4.73	-4.13
										Five	11.13	3.44
										One	-66.37	-39.30
5	Five	PQ	0.960/	-3.32	0.00	0.00	30.00	10.00	0.00	Four	-11.10	-3.22
										Six	-18.86	-6.75
6	Six	PQ	0.966/	-2.60	0.00	0.00	40.00	25.00	0.00	Five	18.90	7.03
										One	-58.90	-32.03
	Totals				231.14	145.77	230.00	135.00	0.00			

Line Losses

Line no.	From Bus	To Bus	Ploss (MW)	Qloss (MVAR)

1	One	Two	0.00	0.02
2	Two	Three	0.30	2.41
3	Three	Four	0.00	0.03
4	Four	Five	0.03	0.22
5	Five	Six	0.03	0.28
6	Six	One	0.48	3.85
7	One	Four	0.51	4.07
		Totals:	1.36	10.87

Alerts

NONE

Done in 12 iterations.

>> **power_flow prob_11_15b_input**
Input summary statistics:
25 lines in system file
1 SYSTEM lines
6 BUS lines
7 LINE lines

Results for Case **Night+Line**

Bus no.	Name	Bus Type	Volts / (pu)	angle (deg)	--Generation-- (MW)	(MVAR)	----Load---- (MW)	(MVAR)	--Cap-- (MVAR)	To Bus	---Line Flow--- (MW)	(MVAR)
1	One	SL	**1.000/**	**0.00**	65.51	59.06	0.00	0.00	0.00	Two	-17.65	2.33
										Six	39.27	22.23
										Four	43.89	34.49
2	Two	PV	**1.000/**	**0.82**	100.00	46.86	40.00	25.00	0.00	One	17.68	-2.08
										Three	42.42	23.97
3	Three	PQ	**0.977/**	**-1.03**	0.00	0.00	30.00	20.00	0.00	Two	-42.18	-22.08

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4	Four	PQ	**0.975/**	**-1.49**	0.00	0.00	50.00	35.00	0.00	Four	12.25	2.10
										Three	-12.24	-1.99
										Five	5.92	-0.49
										One	-43.65	-32.50
5	Five	PQ	**0.974/**	**-2.03**	0.00	0.00	15.00	5.00	0.00	Four	-5.91	0.55
										Six	-9.05	-5.53
6	Six	PQ	**0.979/**	**-1.71**	0.00	0.00	30.00	15.00	0.00	Five	9.06	5.61
										One	-39.06	-20.60
	Totals				165.51	105.92	165.00	100.00	0.00			

Line Losses

Line no.	From Bus	To Bus	Ploss (MW)	Qloss (MVAR)
1	One	Two	0.03	0.25
2	Two	Three	0.24	1.90
3	Three	Four	0.01	0.10
4	Four	Five	0.01	0.06
5	Five	Six	0.01	0.08
6	Six	One	0.20	1.63
7	One	Four	0.25	1.99
		Totals:	0.75	6.01

Alerts

NONE

Done in 11 iterations.

For both daytime and nighttime cases, the addition of the new transmission line made significant improvements in the voltages at each bus. Now the voltages are in tolerance day or night, and no transmission lines are overloaded day or night.

Chapter 12: *Symmetrical Faults*

12-1. A 200 MVA, 20 kV, 60 Hz, three-phase synchronous generator is connected through a 200 MVA, 20/138 kV, Δ-Y transformer to a 138 kV transmission line. The generator reactances to the machine's own base are

$$X_S = 1.40 \qquad\qquad X' = 0.30 \qquad\qquad X'' = 0.15$$

The initial transient dc component in this machine averages 50 percent of the initial symmetrical ac component. The transformer's series reactance is 0.10 pu, and the resistance of both the generator and the transformer may be ignored. Assume that a symmetrical three-phase fault occurs on the 138 kV transmission line near to the point where it is the transformer.

(a) What is the ac component of current in this generator the instant after the fault occurs?

(b) What is the total current (ac plus dc) flowing in the generator right after the fault occurs?

(c) What is the transient fault current I'_f in this fault?

(d) What is the steady-state fault current I_f in this fault?

SOLUTION We will define the base values at the location of the generator. as

$$S_{base,1} = 200 \text{ MVA}$$

$$V_{base,1} = 20 \text{ kV}$$

$$I_{base,1} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base\ 1}} = \frac{200,000,000 \text{ VA}}{\sqrt{3}(20,000 \text{ V})} = 5774 \text{ A}$$

Therefore the base values on the high-voltage side of the transformer will be

$$S_{base,2} = 200 \text{ MVA}$$

$$V_{base,2} = 20 \text{ kV}\left(\frac{138 \text{ kV}}{20 \text{ kV}}\right) = 138 \text{ kV}$$

$$I_{base,2} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base\ 2}} = \frac{200,000,000 \text{ VA}}{\sqrt{3}(138,000 \text{ V})} = 836.7 \text{ A}$$

In this case, the per-unit values of both the synchronous generator and the transformer are already at the right base, and the resulting equivalent circuit is shown below.
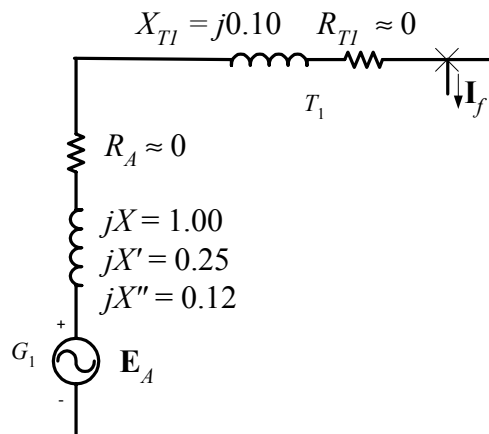
(a) The per-unit ac fault current immediately after the fault occurs will be the current flowing through the subtransient reactance X''.

$$\mathbf{I}_F'' = \frac{\mathbf{E}_A}{jX'' + jX_{T1}} = \frac{1\angle 0°}{j0.12 + j0.10} = 4.545\angle - 90° \text{ pu}$$

The actual ac component of current is $(4.545)(836.7 \text{ A}) = 3803 \text{ A}$

(b) The total current (ac + dc) right after the fault occurs is $(1.5)(3803 \text{ A}) = 5705 \text{ A}$.

(c) The per-unit transient fault current will be the current flowing through the transient reactance X'.

$$\mathbf{I}_F' = \frac{\mathbf{E}_A}{jX' + jX_{T1}} = \frac{1\angle 0°}{j0.25 + j0.10} = 2.857\angle - 90° \text{ pu}$$

The actual transient fault current is $(2.857)(836.7 \text{ A}) = 2390 \text{ A}$

(d) The per-unit steady-state fault current will be the current flowing through the synchronous reactance X_S.

$$\mathbf{I}_F = \frac{\mathbf{E}_A}{jX_S + jX_{T1}} = \frac{1\angle 0°}{j1.00 + j0.10} = 0.909\angle - 90° \text{ pu}$$

The actual steady-state fault current is $(0.909)(836.7 \text{ A}) = 761 \text{ A}$

12-2. A simple three-phase power system is shown in Figure P12-1. Assume that the ratings of the various devices in this system are as follows:

Generator G_1: 250 MVA, 13.8 kV, $R = 0.1$ pu, $X_S = 1.0$ pu, $X'' = 0.18$ pu, $X' = 0.40$ pu

Generator G_2: 500 MVA, 20.0 kV, $R = 0.1$ pu, $X_S = 1.2$ pu, $X'' = 0.15$ pu, $X' = 0.35$ pu

Generator G_3: 250 MVA, 13.8 kV, $R = 0.15$ pu, $X_S = 1.0$ pu, $X'' = 0.20$ pu, $X' = 0.40$ pu

Transformer T_1: 250 MVA, 13.8-Δ/240-Y kV, $R = 0.01$ pu, $X = 0.10$ pu

Transformer T_2: 500 MVA, 20.0-Δ/240-Y kV, $R = 0.01$ pu, $X = 0.08$ pu

Transformer T_3: 250 MVA, 13.8-Δ/240-Y kV, $R = 0.01$ pu, $X = 0.10$ pu

Each Line: $R = 8\ \Omega, X = 40\ \Omega$

Assume that the power system is initially unloaded, and that the voltage at Bus 4 is 250 kV, and that *all resistances may be neglected*.

(a) Convert this power system to per-unit on a base of 500 MVA at 20 kV at generator G_2.

(b) Calculate $\mathbf{Y}_{bus}$ and $\mathbf{Z}_{bus}$ for this power system using the generator subtransient reactances.

(c) Suppose that a three-phase symmetrical fault occurs at Bus 4. What is the subtransient fault current? What is the voltage on each bus in the power system? What is the subtransient current flowing in each of the three transmission lines?

(d) Which circuit breaker in the power system sees the highest instantaneous current when a fault occurs at Bus 4?

(e) What is the transient fault current when a fault occurs at Bus 4? What is the voltage on each bus in the power system? What is the transient current flowing in each of the three transmission lines?

(f) What is the steady-state fault current when a fault occurs at Bus 4? What is the voltage on each bus in the power system? What is the steady-state current flowing in each of the three transmission lines?

(g) Determine the subtransient short-circuit MVA of this power system at Bus 4.
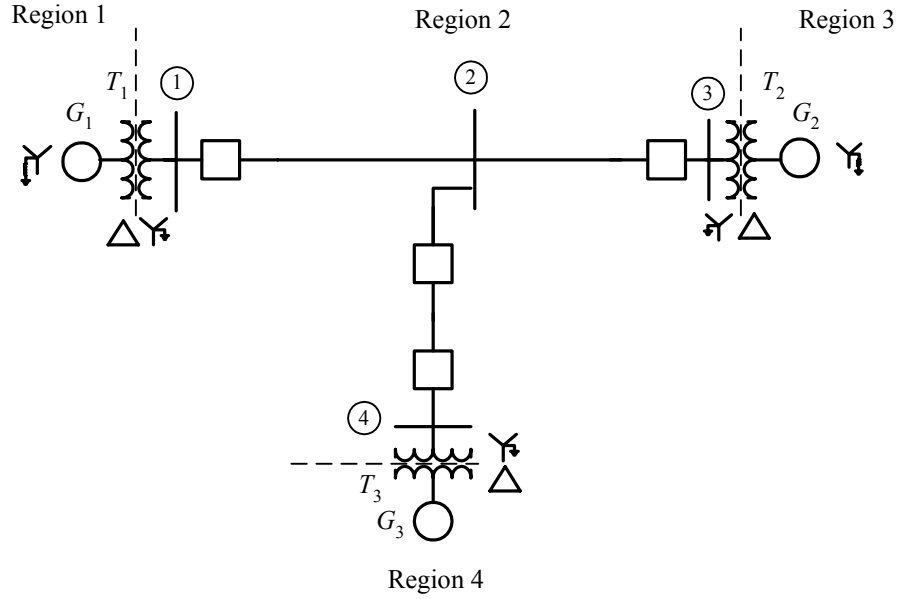
Figure P12-1   The simple power system of Problem 12-2.

SOLUTION The base quantities for this power system are 500 MVA at 20 kV at generator G_2, which is in Region 3. Therefore, the base quantities are:

$$S_{\text{base},3} = 500 \text{ MVA}$$

$$V_{\text{base},3} = 20 \text{ kV}$$

$$I_{\text{base},3} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base 3}}} = \frac{500,000,000 \text{ VA}}{\sqrt{3}(20,000 \text{ V})} = 14,430 \text{ A}$$

$$Z_{\text{base},3} = \frac{(V_{LL,\text{base 3}})^2}{S_{3\phi,\text{base}}} = \frac{(20,000 \text{ V})^2}{500,000,000 \text{ VA}} = 0.80 \ \Omega$$

$$S_{\text{base},2} = 500 \text{ MVA}$$

$$V_{\text{base},2} = 20 \text{ kV}\left(\frac{240 \text{ kV}}{20 \text{ kV}}\right) = 240 \text{ kV}$$

$$I_{\text{base},2} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base 2}}} = \frac{500,000,000 \text{ VA}}{\sqrt{3}(240,000 \text{ V})} = 1203 \text{ A}$$

$$Z_{\text{base},2} = \frac{(V_{LL,\text{base 2}})^2}{S_{3\phi,\text{base}}} = \frac{(240,000 \text{ V})^2}{500,000,000 \text{ VA}} = 115.2 \ \Omega$$

$$S_{\text{base},1} = 500 \text{ MVA}$$

$$V_{\text{base},1} = 240 \text{ kV}\left(\frac{13.8 \text{ kV}}{240 \text{ kV}}\right) = 13.8 \text{ kV}$$

$$I_{base,1} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base\ 1}} = \frac{500,000,000\ \text{VA}}{\sqrt{3}(13,800\ \text{V})} = 20,910\ \text{A}$$

$$Z_{base,1} = \frac{\left(V_{LL,base\ 1}\right)^2}{S_{3\phi,base}} = \frac{(13,800\ \text{V})^2}{500,000,000\ \text{VA}} = 0.381\ \Omega$$

$$S_{base,4} = 500\ \text{MVA}$$

$$V_{base,4} = 240\ \text{kV}\left(\frac{13.8\ \text{kV}}{240\ \text{kV}}\right) = 13.8\ \text{kV}$$

$$I_{base,4} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base\ 4}} = \frac{500,000,000\ \text{VA}}{\sqrt{3}(13,800\ \text{V})} = 20,910\ \text{A}$$

$$Z_{base,4} = \frac{\left(V_{LL,base\ 4}\right)^2}{S_{3\phi,base}} = \frac{(13,800\ \text{V})^2}{500,000,000\ \text{VA}} = 0.381\ \Omega$$

(a) The per-unit impedances expressed on the system base are:

$$\text{per-unit}\ Z_{new} = \text{per-unit}\ Z_{given}\left(\frac{V_{given}}{V_{new}}\right)^2\left(\frac{S_{new}}{S_{given}}\right) \tag{10-8}$$

$$R_{G1} = (0.10\ \text{pu})\left(\frac{13.8\ \text{kV}}{13.8\ \text{kV}}\right)^2\left(\frac{500\ \text{MVA}}{250\ \text{MVA}}\right) = 0.20\ \text{pu}$$

$$X''_{G1} = (0.18\ \text{pu})\left(\frac{13.8\ \text{kV}}{13.8\ \text{kV}}\right)^2\left(\frac{500\ \text{MVA}}{250\ \text{MVA}}\right) = 0.36\ \text{pu}$$

$$X'_{G1} = (0.40\ \text{pu})\left(\frac{13.8\ \text{kV}}{13.8\ \text{kV}}\right)^2\left(\frac{500\ \text{MVA}}{250\ \text{MVA}}\right) = 0.80\ \text{pu}$$

$$X_{S,G1} = (1.00\ \text{pu})\left(\frac{13.8\ \text{kV}}{13.8\ \text{kV}}\right)^2\left(\frac{500\ \text{MVA}}{250\ \text{MVA}}\right) = 2.00\ \text{pu}$$

$$R_{G2} = (0.10\ \text{pu})\left(\frac{20.0\ \text{kV}}{20.0\ \text{kV}}\right)^2\left(\frac{500\ \text{MVA}}{500\ \text{MVA}}\right) = 0.10\ \text{pu}$$

$$X''_{G2} = (0.15\ \text{pu})\left(\frac{20.0\ \text{kV}}{20.0\ \text{kV}}\right)^2\left(\frac{500\ \text{MVA}}{500\ \text{MVA}}\right) = 0.15\ \text{pu}$$

$$X'_{G2} = (0.35\ \text{pu})\left(\frac{20.0\ \text{kV}}{20.0\ \text{kV}}\right)^2\left(\frac{500\ \text{MVA}}{500\ \text{MVA}}\right) = 0.35\ \text{pu}$$

$$X_{S,G2} = (1.20\ \text{pu})\left(\frac{20.0\ \text{kV}}{20.0\ \text{kV}}\right)^2\left(\frac{500\ \text{MVA}}{500\ \text{MVA}}\right) = 1.20\ \text{pu}$$

$$R_{G3} = (0.15\ \text{pu})\left(\frac{13.8\ \text{kV}}{13.8\ \text{kV}}\right)^2\left(\frac{500\ \text{MVA}}{250\ \text{MVA}}\right) = 0.30\ \text{pu}$$

$$X''_{G3} = (0.20\ \text{pu})\left(\frac{13.8\ \text{kV}}{13.8\ \text{kV}}\right)^2\left(\frac{500\ \text{MVA}}{250\ \text{MVA}}\right) = 0.40\ \text{pu}$$

$$X'_{G3} = (0.40 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.80 \text{ pu}$$

$$X_{S,G3} = (1.00 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 2.00 \text{ pu}$$

$$R_{T1} = (0.01 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.02 \text{ pu}$$

$$X_{T1} = (0.10 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.20 \text{ pu}$$

$$R_{T2} = (0.01 \text{ pu})\left(\frac{20.0 \text{ kV}}{20.0 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{500 \text{ MVA}}\right) = 0.01 \text{ pu}$$

$$X_{T2} = (0.08 \text{ pu})\left(\frac{20.0 \text{ kV}}{20.0 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{500 \text{ MVA}}\right) = 0.08 \text{ pu}$$

$$R_{T3} = (0.01 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.02 \text{ pu}$$

$$X_{T3} = (0.10 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.20 \text{ pu}$$

$$R_{\text{line,pu}} = \frac{R_{\text{line},\Omega}}{Z_{\text{base}}} = \frac{8 \ \Omega}{115.2 \ \Omega} = 0.0694 \text{ pu}$$

$$X_{\text{line,pu}} = \frac{X_{\text{line},\Omega}}{Z_{\text{base}}} = \frac{40 \ \Omega}{115.2 \ \Omega} = 0.3472 \text{ pu}$$

The assumption that the voltage at Bus 4 is 250 kV with an unloaded system means that the internal generated voltages of all generators are (250 kV/240 kV) = 1.042∠0° before the fault. The resulting per-unit equivalent circuit of the power system is shown below. Note that all three generator reactances are shown for each generator.
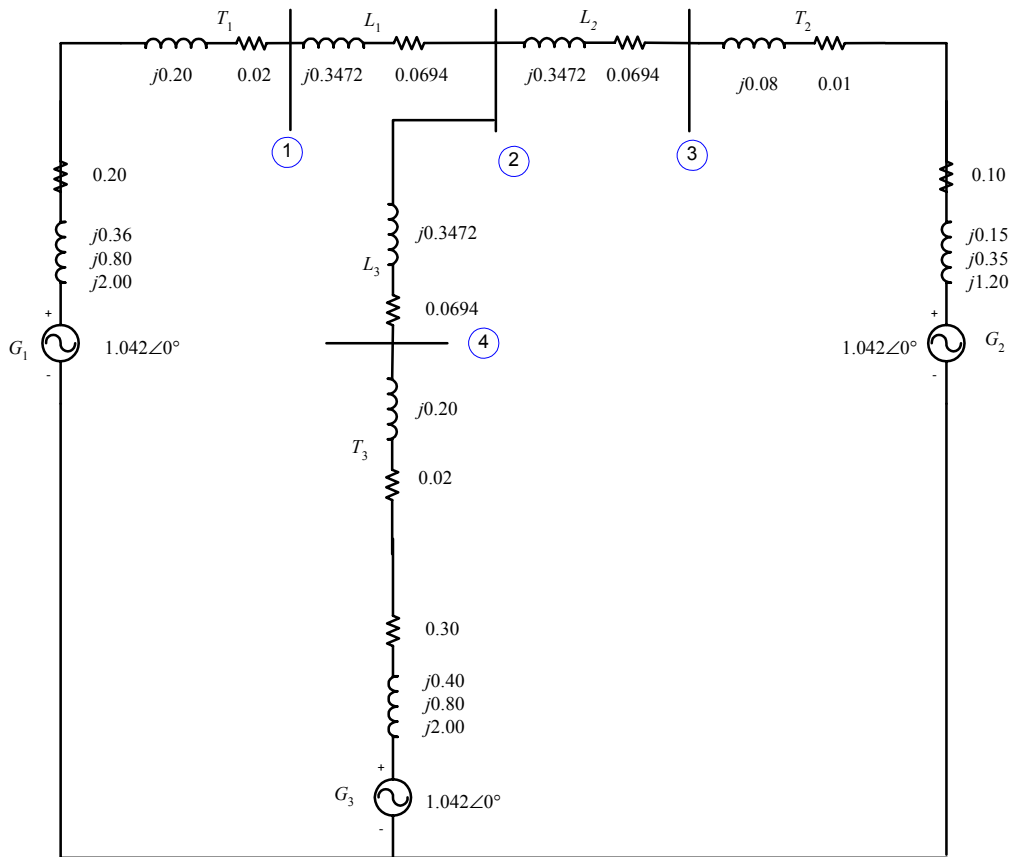
(b) For fault current studies, we include the generator impedances in $\mathbf{Y}_{bus}$ and $\mathbf{Z}_{bus}$. Also, this problem specified that we exclude the resistance in the calculation and use the subtransient reactances of the generators. If resistances are ignored and the impedances are converted to admittances, the resulting power system is:
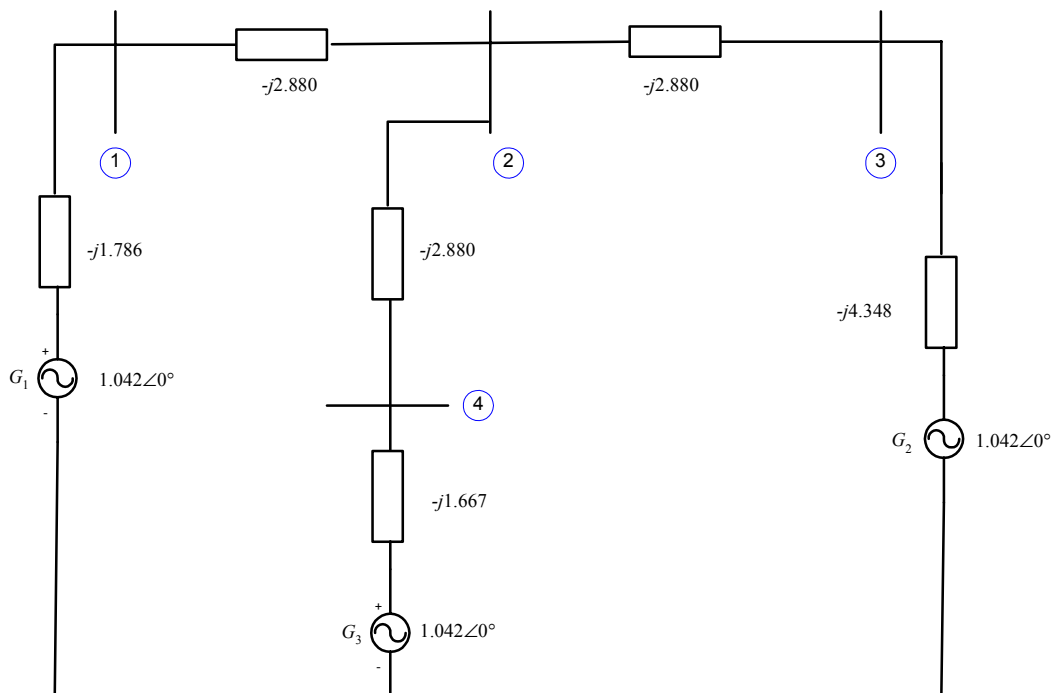
The resulting bus admittance matrix is:

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j4.666 & j2.880 & 0 & 0 \\ j2.880 & -j8.640 & j2.880 & j2.880 \\ 0 & j2.880 & -j7.228 & 0 \\ 0 & j2.880 & 0 & -j4.547 \end{bmatrix}$$

The resulting bus impedance matrix is:

$$\mathbf{Z}_{bus} = \mathbf{Y}_{bus}{}^{-1} = \begin{bmatrix} j0.3128 & j0.1589 & j0.0633 & j0.1007 \\ j0.1589 & j0.2572 & j0.1025 & j0.1629 \\ j0.0633 & j0.1025 & j0.1792 & j0.0649 \\ j0.1007 & j0.1629 & j0.0649 & j0.3231 \end{bmatrix}$$

(c) If a symmetrical three-phase fault occurs at Bus 4, the subtransient fault current will be

$$\mathbf{I}''_f = \frac{\mathbf{V}_f}{Z_{44}} = \frac{1.042\angle 0°}{j0.3231} = 3.225\angle -90°$$

The actual fault current is

$$I''_f = (3.225)(1203 \text{ A}) = 3880 \text{ A}$$

The voltages at each bus in the power system will be

$$\mathbf{V}_j = \left(1 - \frac{Z_{ji}}{Z_{ii}}\right)\mathbf{V}_f \qquad (12\text{-}23)$$

$$\mathbf{V}_1 = \left(1 - \frac{Z_{14}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.1007}{j0.3231}\right)(1.042\angle 0°) = 0.7172\angle 0°$$

$$\mathbf{V}_2 = \left(1 - \frac{Z_{24}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.1629}{j0.3231}\right)(1.042\angle 0°) = 0.5156\angle 0°$$

$$\mathbf{V}_3 = \left(1 - \frac{Z_{34}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.0649}{j0.3231}\right)(1.042\angle 0°) = 0.8327\angle 0°$$

$$\mathbf{V}_1 = \left(1 - \frac{Z_{44}}{Z_{44}}\right)\mathbf{V}_f = 0\angle 0°$$

The actual bus voltages are

$$V_1 = (0.7172)(240 \text{ kV}) = 172 \text{ kV}$$
$$V_2 = (0.5156)(240 \text{ kV}) = 124 \text{ kV}$$
$$V_3 = (0.8327)(240 \text{ kV}) = 200 \text{ kV}$$
$$V_4 = 0 \text{ kV}$$

The subtransient current in the transmission lines will be

Line 1: $\mathbf{I}_{12} = -Y_{12}(\mathbf{V}_1 - \mathbf{V}_2) = (-j2.880)(0.7172\angle 0° - 0.5156\angle 0°) = 0.581\angle -90°$

Line 2: $\mathbf{I}_{23} = -Y_{23}(\mathbf{V}_2 - \mathbf{V}_3) = (-j2.880)(0.5156\angle 0° - 0.8327\angle 0°) = 0.913\angle 90°$

Line 3: $\mathbf{I}_{24} = -Y_{24}(\mathbf{V}_2 - \mathbf{V}_4) = (-j2.880)(0.5156\angle 0° - 0.0\angle 0°) = 1.485\angle -90°$

The actual line currents are

$$I_{\text{line 1}} = (0.581)(1203 \text{ A}) = 699 \text{ A}$$

$$I_{\text{line 2}} = (0.913)(1203 \text{ A}) = 1099 \text{ A}$$

$$I_{\text{line 3}} = (1.485)(1203 \text{ A}) = 1787 \text{ A}$$

(d) The circuit breakers are all located on either side of the transmission line. In this case, the circuit breakers on Line 3 (between Bus 2 and Bus 4) will see the highest subtransient fault current.

(e) This problem specified that we exclude the resistance in the calculation and use the transient reactances of the generators. If resistances are ignored and the impedances are converted to admittances, the resulting power system is:
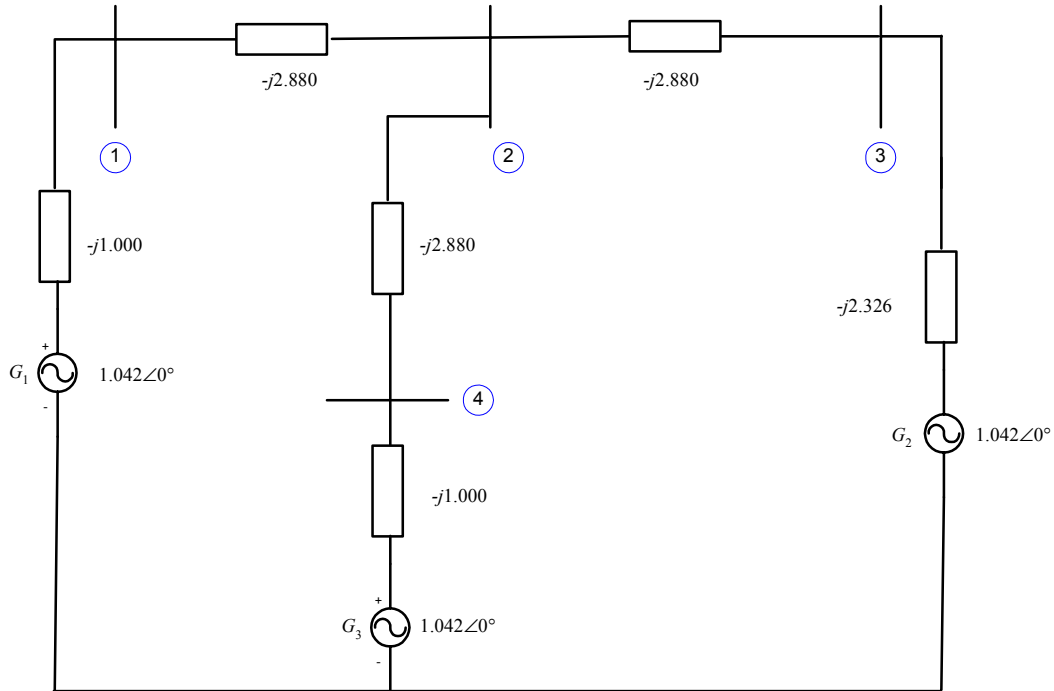
The resulting bus admittance matrix is:

$$\mathbf{Y}_{\text{bus}} = \begin{bmatrix} -j3.880 & j2.880 & 0 & 0 \\ j2.880 & -j8.640 & j2.880 & j2.880 \\ 0 & j2.880 & -j5.206 & 0 \\ 0 & j2.880 & 0 & -j3.880 \end{bmatrix}$$

The resulting bus impedance matrix is:

$$\mathbf{Z}_{\text{bus}} = \mathbf{Y}_{\text{bus}}^{-1} = \begin{bmatrix} j0.4565 & j0.2678 & j0.1482 & j0.1988 \\ j0.2678 & j0.3608 & j0.1996 & j0.2678 \\ j0.1482 & j0.1996 & j0.3025 & j0.1482 \\ j0.1988 & j0.2678 & j0.1482 & j0.4565 \end{bmatrix}$$

If a symmetrical three-phase fault occurs at Bus 4, the transient fault current will be

$$\mathbf{I}'_f = \frac{\mathbf{V}_f}{Z_{44}} = \frac{1.042\angle 0°}{j0.4565} = 2.283\angle -90°$$

The actual fault current is

$$I_f' = (2.283)(1203 \text{ A}) = 2746 \text{ A}$$

The transient voltages at each bus in the power system will be

$$\mathbf{V}_j = \left(1 - \frac{Z_{ji}}{Z_{ii}}\right)\mathbf{V}_f \tag{12-23}$$

$$\mathbf{V}_1 = \left(1 - \frac{Z_{14}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.1988}{j0.4565}\right)(1.042\angle 0°) = 0.5882\angle 0°$$

$$\mathbf{V}_2 = \left(1 - \frac{Z_{24}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.2768}{j0.4565}\right)(1.042\angle 0°) = 0.4307\angle 0°$$

$$\mathbf{V}_3 = \left(1 - \frac{Z_{34}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.1482}{j0.4565}\right)(1.042\angle 0°) = 0.7037\angle 0°$$

$$\mathbf{V}_1 = \left(1 - \frac{Z_{44}}{Z_{44}}\right)\mathbf{V}_f = 0\angle 0°$$

The actual bus voltages are

$$V_1 = (0.5882)(240 \text{ kV}) = 141 \text{ kV}$$
$$V_2 = (0.4307)(240 \text{ kV}) = 103 \text{ kV}$$
$$V_3 = (0.7037)(240 \text{ kV}) = 177 \text{ kV}$$
$$V_4 = 0 \text{ kV}$$

The transient current in the transmission lines will be

Line 1: $\mathbf{I}_{12} = -Y_{12}(\mathbf{V}_1 - \mathbf{V}_2) = (-j2.880)(0.5882\angle 0° - 0.4307\angle 0°) = 0.454\angle -90°$

Line 2: $\mathbf{I}_{23} = -Y_{23}(\mathbf{V}_2 - \mathbf{V}_3) = (-j2.880)(0.4307\angle 0° - 0.7037\angle 0°) = 0.786\angle 90°$

Line 3: $\mathbf{I}_{24} = -Y_{24}(\mathbf{V}_2 - \mathbf{V}_4) = (-j2.880)(0.4307\angle 0° - 0.0\angle 0°) = 1.240\angle -90°$

The actual line currents are

$$I_{\text{line 1}} = (0.454)(1203 \text{ A}) = 546 \text{ A}$$
$$I_{\text{line 2}} = (0.786)(1203 \text{ A}) = 946 \text{ A}$$
$$I_{\text{line 3}} = (1.240)(1203 \text{ A}) = 1492 \text{ A}$$

(f) This time we will use the synchronous reactances of the generators. If resistances are ignored and the impedances are converted to admittances, the resulting power system is:
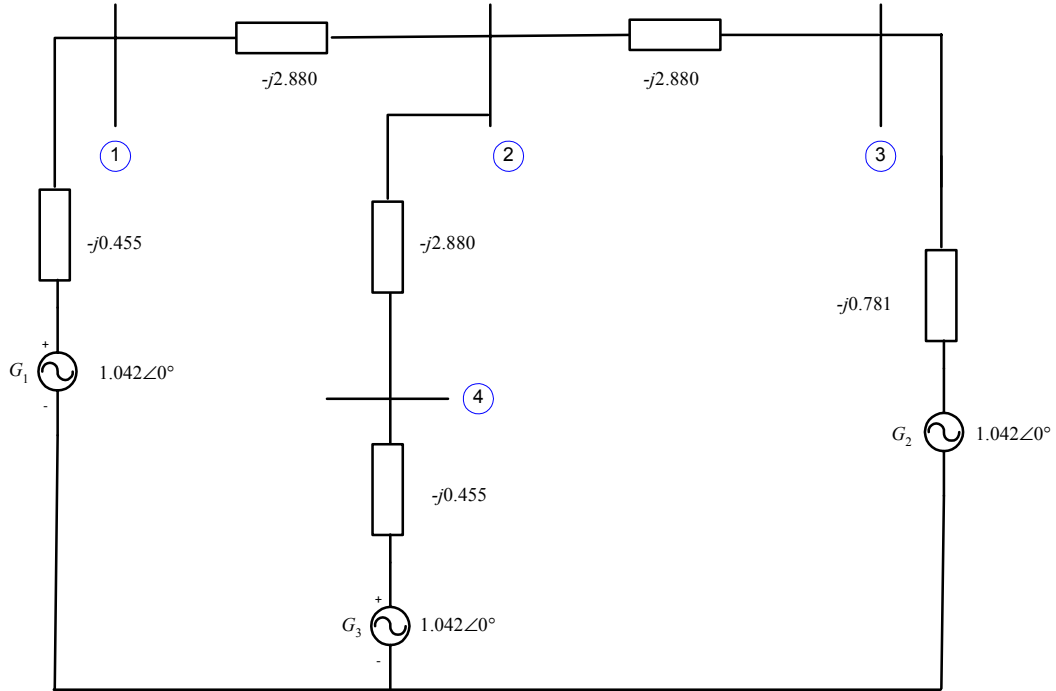
The resulting bus admittance matrix is:

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j3.335 & j2.880 & 0 & 0 \\ j2.880 & -j8.640 & j2.880 & j2.880 \\ 0 & j2.880 & -j3.661 & 0 \\ 0 & j2.880 & 0 & -j3.335 \end{bmatrix}$$

The resulting bus impedance matrix is:

$$\mathbf{Z}_{bus} = \mathbf{Y}_{bus}^{-1} = \begin{bmatrix} j0.8324 & j0.6167 & j0.4852 & j0.5326 \\ j0.6167 & j0.7142 & j0.5618 & j0.6167 \\ j0.4852 & j0.5618 & j0.7151 & j0.4852 \\ j0.5326 & j0.6167 & j0.4852 & j0.8324 \end{bmatrix}$$

If a symmetrical three-phase fault occurs at Bus 4, the steady-state fault current will be

$$\mathbf{I}_f = \frac{\mathbf{V}_f}{Z_{44}} = \frac{1.042\angle 0°}{j0.8324} = 1.2518\angle -90°$$

The actual fault current is

$$I_f = (1.2518)(1203\ \text{A}) = 1506\ \text{A}$$

The steady-state voltages at each bus in the power system will be

$$\mathbf{V}_j = \left(1 - \frac{Z_{ji}}{Z_{ii}}\right)\mathbf{V}_f \tag{12-23}$$

$$\mathbf{V}_1 = \left(1 - \frac{Z_{14}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.5326}{j0.8324}\right)(1.042\angle 0°) = 0.3753\angle 0°$$

$$\mathbf{V}_2 = \left(1 - \frac{Z_{24}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.6167}{j0.8324}\right)(1.042\angle 0°) = 0.2700\angle 0°$$

$$\mathbf{V}_3 = \left(1 - \frac{Z_{34}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.4852}{j0.8324}\right)(1.042\angle 0°) = 0.4346\angle 0°$$

$$\mathbf{V}_1 = \left(1 - \frac{Z_{44}}{Z_{44}}\right)\mathbf{V}_f = 0\angle 0°$$

The actual bus voltages are

$$V_1 = (0.3753)(240 \text{ kV}) = 90 \text{ kV}$$
$$V_2 = (0.2700)(240 \text{ kV}) = 65 \text{ kV}$$
$$V_3 = (0.4346)(240 \text{ kV}) = 104 \text{ kV}$$
$$V_4 = 0 \text{ kV}$$

The steady-state current in the transmission lines will be

Line 1: $\mathbf{I}_{12} = -Y_{12}(\mathbf{V}_1 - \mathbf{V}_2) = (-j2.880)(0.3753\angle 0° - 0.2700\angle 0°) = 0.303\angle -90°$

Line 2: $\mathbf{I}_{23} = -Y_{23}(\mathbf{V}_2 - \mathbf{V}_3) = (-j2.880)(0.2700\angle 0° - 0.4346\angle 0°) = 0.474\angle 90°$

Line 3: $\mathbf{I}_{24} = -Y_{24}(\mathbf{V}_2 - \mathbf{V}_4) = (-j2.880)(0.2700\angle 0° - 0.0\angle 0°) = 0.778\angle -90°$

The actual line currents are

$$I_{\text{line 1}} = (0.303)(1203 \text{ A}) = 365 \text{ A}$$
$$I_{\text{line 2}} = (0.474)(1203 \text{ A}) = 570 \text{ A}$$
$$I_{\text{line 3}} = (0.778)(1203 \text{ A}) = 936 \text{ A}$$

(g) The per-unit short-circuit MVA at Bus 4 is just equal to the per-unit fault current at that bus.

$$\text{Short-circuit MVA}_{\text{pu}} = I_{SC,\text{pu}} = 2.283$$

Therefore, the subtransient short-circuit MVA at Bus 4 is

$$\text{Short-circuit MVA} = \left(\text{Short-circuit MVA}_{\text{pu}}\right)\left(S_{\text{base}}\right)$$

$$\text{Short-circuit MVA} = (2.283)(500 \text{ MVA}) = 1142 \text{ MVA}$$

Note: This problem could also be solved using program `faults`, it we treat the terminals of each generator as an additional bus. The input file for this power system is shown below.

```
% File describing the power system of Problem 12-2.
%
% System data has the form:
%SYSTEM    name              baseMVA
SYSTEM  Prob12_2            500
%
% Bus data has the form:
%BUS name        volts
BUS One          1.042
BUS Two          1.042
BUS Three        1.042
```

```
BUS Four          1.042
BUS G1            1.042
BUS G2            1.042
BUS G3            1.042
%
% Transmission line data has the form:
%LINE    from        to        Rse     Xse     Gsh     Bsh     X0     Vis
LINE    One       Two       0.0000  0.3472  0.000   0.000   0.000   0
LINE    Two       Three     0.0000  0.3472  0.000   0.000   0.000   0
LINE    Two       Four      0.0000  0.3472  0.000   0.000   0.000   0
LINE    G1        One       0.0000  0.2000  0.000   0.000   0.000   0
LINE    Three     G2        0.0000  0.0800  0.000   0.000   0.000   0
LINE    Four      G3        0.0000  0.2000  0.000   0.000   0.000   0
%
% Generator data has the form:
%GENERATOR   bus       R       Xs      Xp      Xpp     X2      X0
GENERATOR   G1        0.00    2.00    0.80    0.36    0.00    0.00
GENERATOR   G2        0.00    1.20    0.35    0.15    0.00    0.00
GENERATOR   G3        0.00    2.00    0.80    0.40    0.00    0.00
%
% type data has the form:
%FAULT    bus       Calc Type   Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT    Four      3P            0
```

The resulting outputs are shown below. Note that the answers agree well with the calculations above.
The slight differences are due to round-off errors in our manual calculation of the bus admittance matrix
$\mathbf{Y}_{bus}$.

```
>> faults prob_12_2_fault
Input summary statistics:
  34 lines in system file
   1 SYSTEM lines
   7 BUS lines
   6 LINE lines
   3 GENERATOR lines
   0 MOTOR lines
   1 TYPE lines


                         Results for Case Prob12_2


Symmetrical Three-Phase Fault at Bus Four
Calculating Subtransient Currents
|====================Bus Information============|=====Line Information=====|
Bus             Volts / angle    Amps / angle  |   To  |  Amps /  angle  |
no.    Name     (pu)    (deg)    (pu)    (deg)  |  Bus  |  (pu)   (deg)   |
|==============================================================================|
  1      One   0.718/   0.00    0.000/   0.00        Two    0.579/ -90.00
                                                     G1     0.579/  90.00
  2      Two   0.517/   0.00    0.000/   0.00        One    0.579/  90.00
                                                     Three  0.910/  90.00
                                                     Four   1.489/ -90.00
  3    Three   0.833/   0.00    0.000/   0.00        Two    0.910/ -90.00
                                                     G2     0.910/  90.00
  4     Four   0.000/   0.00    3.225/ -90.00        Two    1.489/  90.00
                                                     G3     1.737/  90.00
  5       G1   0.834/   0.00    0.000/   0.00        One    0.579/ -90.00
```

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```
  6            G2    0.906/    0.00    0.000/    0.00       Three   0.910/  -90.00
  7            G3    0.347/    0.00    0.000/    0.00       Four    1.737/  -90.00
 |========================================================================|


                        Results for Case Prob12_2


Symmetrical Three-Phase Fault at Bus Four
Calculating Transient Currents
 |===================Bus Information============|=====Line Information=====|
Bus               Volts / angle    Amps / angle  |   To   | Amps /  angle  |
no.      Name     (pu)    (deg)    (pu)   (deg)   |  Bus   |  (pu)   (deg)  |
 |========================================================================|
  1           One    0.588/    0.00    0.000/    0.00       Two     0.454/  -90.00
                                                            G1      0.454/   90.00
  2           Two    0.431/    0.00    0.000/    0.00       One     0.454/   90.00
                                                            Three   0.787/   90.00
                                                            Four    1.240/  -90.00
  3         Three    0.704/    0.00    0.000/    0.00       Two     0.787/  -90.00
                                                            G2      0.787/   90.00
  4          Four    0.000/    0.00    2.282/  -90.00       Two     1.240/   90.00
                                                            G3      1.042/   90.00
  5            G1    0.679/    0.00    0.000/    0.00       One     0.454/  -90.00
  6            G2    0.767/    0.00    0.000/    0.00       Three   0.787/  -90.00
  7            G3    0.208/    0.00    0.000/    0.00       Four    1.042/  -90.00
 |========================================================================|


                        Results for Case Prob12_2


Symmetrical Three-Phase Fault at Bus Four
Calculating Steady-State Currents
 |===================Bus Information============|=====Line Information=====|
Bus               Volts / angle    Amps / angle  |   To   | Amps /  angle  |
no.      Name     (pu)    (deg)    (pu)   (deg)   |  Bus   |  (pu)   (deg)  |
 |========================================================================|
  1           One    0.375/    0.00    0.000/    0.00       Two     0.303/  -90.00
                                                            G1      0.303/   90.00
  2           Two    0.270/    0.00    0.000/    0.00       One     0.303/   90.00
                                                            Three   0.474/   90.00
                                                            Four    0.778/  -90.00
  3         Three    0.435/    0.00    0.000/    0.00       Two     0.474/  -90.00
                                                            G2      0.474/   90.00
  4          Four    0.000/    0.00    1.251/  -90.00       Two     0.778/   90.00
                                                            G3      0.474/   90.00
  5            G1    0.436/    0.00    0.000/    0.00       One     0.303/  -90.00
  6            G2    0.473/    0.00    0.000/    0.00       Three   0.474/  -90.00
  7            G3    0.095/    0.00    0.000/    0.00       Four    0.474/  -90.00
 |========================================================================|
```

12-3. Calculate the subtransient fault current at Bus 4 *if the power system resistances are not neglected.* How much difference does including the resistances make to the amount of fault current flowing?

SOLUTION This time, we will use program `faults` to calculate the subtransient currents. The input file for this power system is shown below.

```
% File describing the power system of Problem 12-3.
%
% System data has the form:
%SYSTEM      name              baseMVA
SYSTEM  Prob12_3          500
%
% Bus data has the form:
%BUS name      volts
BUS One        1.042
BUS Two        1.042
BUS Three      1.042
BUS Four       1.042
BUS G1         1.042
BUS G2         1.042
BUS G3         1.042
%
% Transmission line data has the form:
%LINE     from        to      Rse     Xse     Gsh     Bsh     X0    Vis
LINE   One     Two        0.0000  0.3472  0.000   0.000   0.000  0
LINE   Two     Three      0.0000  0.3472  0.000   0.000   0.000  0
LINE   Two     Four       0.0000  0.3472  0.000   0.000   0.000  0
LINE   G1      One        0.0000  0.2000  0.000   0.000   0.000  0
LINE   Three   G2         0.0000  0.0800  0.000   0.000   0.000  0
LINE   Four    G3         0.0000  0.2000  0.000   0.000   0.000  0
%
% Generator data has the form:
%GENERATOR   bus       R       Xs      Xp      Xpp     X2      X0
GENERATOR    G1     0.00    2.00    0.80    0.36    0.00    0.00
GENERATOR    G2     0.00    1.20    0.35    0.15    0.00    0.00
GENERATOR    G3     0.00    2.00    0.80    0.40    0.00    0.00
%
% type data has the form:
%FAULT    bus      Calc Type   Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT    Four      3P            0
```

The resulting outputs are shown below. Note that the answers agree well with the calculations above.
The slight differences are due to round-off errors in our manual calculation of the bus admittance matrix
$\mathbf{Y}_{bus}$.


```
>> faults prob_12_3_fault
Input summary statistics:
  34 lines in system file
   1 SYSTEM lines
   7 BUS lines
   6 LINE lines
   3 GENERATOR lines
   0 MOTOR lines
   1 TYPE lines


                        Results for Case Prob12_3


Symmetrical Three-Phase Fault at Bus Four
Calculating Subtransient Currents
|====================Bus Information===========|=====Line Information====|
Bus             Volts / angle    Amps / angle  |   To  | Amps / angle  |
no.     Name     (pu)   (deg)   (pu)    (deg)   |  Bus  |  (pu)   (deg) |
```

```
|========================================================================|
  1        One    0.709/    3.20    0.000/    0.00      Two    0.560/ -75.30
                                                         G1    0.560/ 104.70
  2        Two    0.510/    3.13    0.000/    0.00      One    0.560/ 104.70
                                                       Three   0.882/ 104.27
                                                        Four   1.442/ -75.57
  3      Three    0.823/    3.07    0.000/    0.00      Two    0.882/ -75.73
                                                         G2    0.882/ 104.27
  4       Four    0.000/  -14.04    2.953/  -68.54      Two    1.442/ 104.43
                                                         G3    1.532/ 118.07
  5         G1    0.821/    3.99    0.000/    0.00      One    0.560/ -75.30
  6         G2    0.894/    3.39    0.000/    0.00    Three    0.882/ -75.73
  7         G3    0.308/   22.36    0.000/    0.00     Four    1.532/ -61.93
|========================================================================|
```

When resistances are included, the per-unit three-phase subtransient fault current at Bus 4 is $2.953\angle-68.5°$, and the actual current is

$$I_f'' = (2.953)(1203\text{ A}) = 3553\text{ A}$$

12-4. Suppose that a three-bus power system has the subtransient bus impedance matrix given below, and that the power system is initially unloaded with a pre-fault bus voltage of 0.98 pu. Assume that a symmetrical three-phase fault occurs at Bus 1, and find:
  (a) The current flowing in the transmission line from Bus 2 to Bus 1 during the subtransient period.
  (b) The current flowing in the transmission line from Bus 3 to Bus 1 during the subtransient period.

$$\mathbf{Z}_{\text{bus}} = \begin{bmatrix} j0.20 & j0.10 & j0.15 \\ j0.10 & j0.50 & j0.30 \\ j0.15 & j0.30 & j0.80 \end{bmatrix} \text{ per unit}$$

SOLUTION   The first step in solving this problem is to determine the per-unit subtransient three-phase current flowing in the fault at Bus 1.

$$\mathbf{I}_f'' = \frac{\mathbf{V}_f}{Z_{44}} = \frac{0.98\angle0°}{j0.20} = 4.90\angle-90°$$

The per-unit subtransient voltages at the three busses are:

$$\mathbf{V}_j = \left(1 - \frac{Z_{ji}}{Z_{ii}}\right)\mathbf{V}_f \qquad\qquad (12\text{-}23)$$

$$\mathbf{V}_1 = \left(1 - \frac{Z_{11}}{Z_{11}}\right)\mathbf{V}_f = \left(1 - \frac{j0.20}{j0.20}\right)(0.98\angle0°) = 0.0\angle0°$$

$$\mathbf{V}_2 = \left(1 - \frac{Z_{21}}{Z_{11}}\right)\mathbf{V}_f = \left(1 - \frac{j0.10}{j0.20}\right)(0.98\angle0°) = 0.490\angle0°$$

$$\mathbf{V}_3 = \left(1 - \frac{Z_{21}}{Z_{11}}\right)\mathbf{V}_f = \left(1 - \frac{j0.15}{j0.20}\right)(0.98\angle0°) = 0.245\angle0°$$

To find the current flow in each transmission line, we can invert the bus impedance matrix $\mathbf{Z}_{\text{bus}}$ to get the bus admittance matrix $\mathbf{Y}_{\text{bus}}$. The cross terms of $\mathbf{Y}_{\text{bus}}$ will contain the admittances of the lines between each pair of busses in the system.

$$\mathbf{Y}_{bus} = \mathbf{Z}_{bus}^{-1} = \begin{bmatrix} -j5.9903 & j0.6763 & j0.8696 \\ j0.6763 & -j2.6570 & j0.8696 \\ j0.8696 & j0.8696 & -j1.7391 \end{bmatrix} \text{ per unit}$$

$$\mathbf{I}_{21} = -Y_{21}(\mathbf{V}_2 - \mathbf{V}_1) = (-j0.6763)(0.490\angle 0° - 0.0\angle 0°) = 0.3314\angle -90°$$

$$\mathbf{I}_{31} = -Y_{31}(\mathbf{V}_3 - \mathbf{V}_1) = (-j0.8696)(0.245\angle 0° - 0.0\angle 0°) = 0.2131\angle -90°$$

12-5. A 500 kVA, 480 V, 60 Hz, Y-connected synchronous generator has a subtransient reactance X'' of 0.10 pu, a transient reactance X' of 0.22 pu, and a synchronous reactance X_S of 1.0 pu. Suppose that this machine is supplying rated load at rated voltage and 0.9 PF lagging.
  (a) What is the magnitude of the internal generated voltage E_A of this machine?
  (b) A fault occurs at the terminals of the generator. What is the magnitude of the voltage behind subtransient reactance E_A'' of this machine?
  (c) What is the subtransient current I_f'' flowing from this generator?
  (d) What is the magnitude of the voltage behind transient reactance E_A' of this machine?
  (e) What is the transient current I_f' flowing from this generator during the transient period?
  (f) Suppose that we ignore the loads on the generator and assume that it was unloaded before the fault occurred. What is the subtransient current I_f'' flowing from this generator? How much error is caused by ignoring the initial loading of the generator?

SOLUTION Since this machine is Y-connected, its phase voltage is $(480 \text{ V})/\sqrt{3} = 277 \text{ V}$.

Base values of this generator are

$$S_{base} = 500 \text{ kVA}$$
$$V_{LL,base} = 480 \text{ V}$$
$$V_{\phi,base} = 277 \text{ V}$$
$$I_{base} = \frac{S_{base}}{\sqrt{3}V_{\phi,base}} = \frac{500,000 \text{ VA}}{\sqrt{3}(480 \text{ V})} = 602 \text{ A}$$
$$Z_{base} = \frac{(V_{LL,base})}{S_{3\phi,base}} = \frac{(480 \text{ V})^2}{500,000 \text{ VA}} = 0.4608 \text{ } \Omega$$

(a) The power factor is 0.9 lagging, so the impedance angle $\theta = \cos^{-1}(0.9) = 25.84°$. Therefore, the per-unit phasor current is

$$\mathbf{I}_A = 1.0\angle -25.84°$$

The internal generated voltage $\mathbf{E}_A$ of this machine is given by

$$\mathbf{E}_A = \mathbf{V}_\phi + jX_S\mathbf{I}_A = 1\angle 0° + (j1.0)(1\angle -25.84°) = 1.695\angle 32.1° \text{ pu}$$

or $\mathbf{E}_A = (1.695\angle 32.1°)(277 \text{ V}) = 469.5\angle 32.1° \text{ V}$

Therefore, the magnitude E_A is 469.5 V.

(b) The voltage behind subtransient reactance E_A'' is given by

$$\mathbf{E}''_A = \mathbf{V}_\phi + jX''\mathbf{I}_A = 1\angle 0° + (j0.10)(1\angle - 25.84°) = 1.048\angle 4.9° \text{ pu}$$

or $\mathbf{E}''_A = (1.048\angle 4.9°)(277 \text{ V}) = 290\angle 4.9° \text{ V}$

(c) The per-unit subtransient fault current I''_f is given by

$$\mathbf{I}''_f = \frac{\mathbf{E}''_A}{jX''} = \frac{1.048\angle 4.9°}{j0.10} = 10.48\angle - 85.1° \text{ pu}$$

Therefore, the actual subtransient fault current is $I''_f = (10.48 \text{ pu})(602 \text{ A}) = 6309 \text{ A}$.

(d) The voltage behind transient reactance $\mathbf{E}'_A$ is given by

$$\mathbf{E}'_A = \mathbf{V}_\phi + jX'\mathbf{I}_A = 1\angle 0° + (j0.22)(1\angle - 25.84°) = 1.136\angle 10.2° \text{ pu}$$

or $\mathbf{E}''_A = (1.136\angle 10.2°)(277 \text{ V}) = 315\angle 10.2° \text{ V}$

(e) The per-unit subtransient fault current I''_f is given by

$$\mathbf{I}'_f = \frac{\mathbf{E}'_A}{jX'} = \frac{1.136\angle 10.2°}{j0.22} = 5.164\angle - 79.8° \text{ pu}$$

Therefore, the actual subtransient fault current is $I''_f = (5.164 \text{ pu})(602 \text{ A}) = 3109 \text{ A}$.

(f) If we assume that the generator was initially unloaded before the fault, the per-unit subtransient fault current I''_f is given by

$$\mathbf{I}''_f = \frac{\mathbf{E}''_A}{jX''} = \frac{1.00\angle 0°}{j0.10} = 10.0\angle - 90° \text{ pu}$$

The error caused by ignoring the initial loads is $(10.48 - 10) / 10 \times 100\% = 4.8\%$.

12-6. A synchronous generator is connected through a transformer and a transmission line to a synchronous motor. The rated voltage and apparent power of the generator serve as the base quantities for this power system. On the system base, the per unit subtransient reactances of the generator and the motor are 0.15 and 0.40 respectively, the series reactance of the transformer is 0.10, and the series reactance of the transmission line is 0.20. All resistances may be ignored. The generator is supplying rated apparent power and voltage at a power factor of 0.85 lagging, when a three-phase synchronous fault occurs at the terminals of the motor. Find the per-unit subtransient current at the fault, in the generator, and in the motor.

SOLUTION It simplify this solution, we will assume that the power system was initially unloaded. This will make a small error due to ignoring the voltages behind subtransient reactance in the generator and the motor, but the differences will be small. A per-phase, per-unit equivalent circuit for this power system is shown below.
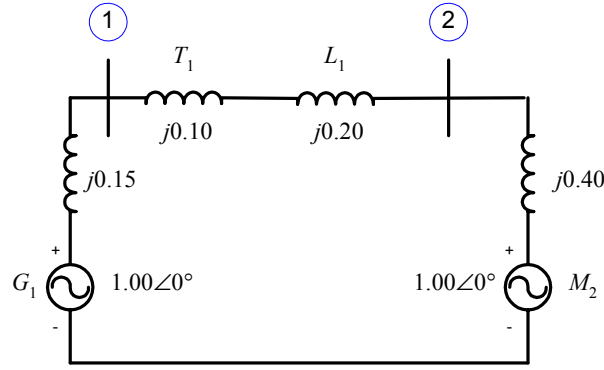
We will define two busses in this system, one at the terminals of the generator and one at the terminals of the motor. The resulting bus admittance matrix can be found by converting the impedances in the system into admittances.
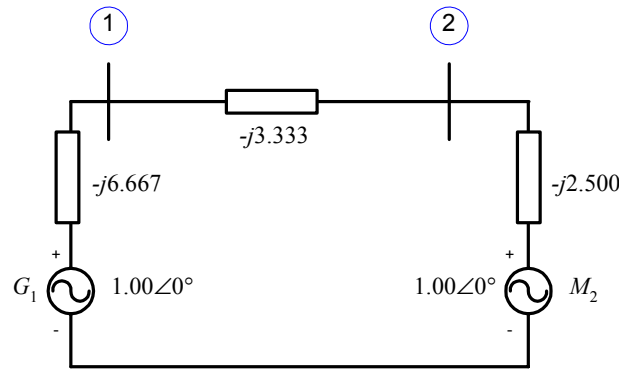
The bus admittance matrix is

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j10.000 & j3.333 \\ j3.333 & -j5.833 \end{bmatrix}$$

and the corresponding bus impedance matrix is

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.1235 & j0.0706 \\ j0.0706 & j0.2118 \end{bmatrix}$$

The current in the fault is

$$\mathbf{I}''_f = \frac{\mathbf{V}_f}{Z_{22}} = \frac{1.00\angle 0°}{j0.2118} = 4.721\angle -90°$$

The voltages at the two busses are

$$\mathbf{V}_j = \left(1 - \frac{Z_{ji}}{Z_{ii}}\right)\mathbf{V}_f \qquad (12\text{-}23)$$

$$\mathbf{V}_1 = \left(1 - \frac{Z_{12}}{Z_{22}}\right)\mathbf{V}_f = \left(1 - \frac{j0.0706}{j0.2118}\right)(1.00\angle 0°) = 0.667\angle 0°$$

$$\mathbf{V}_2 = \left(1 - \frac{Z_{22}}{Z_{22}}\right)\mathbf{V}_f = \left(1 - \frac{j0.2118}{j0.2118}\right)(1.00\angle 0°) = 0.0\angle 0°$$

The subtransient current flowing in the generator is

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$$\mathbf{I}_G'' = \frac{\mathbf{E}_{A,G} - \mathbf{V}_\phi}{jX_G''} = \frac{1.00\angle 0° - 0.667\angle 0°}{j0.15} = 2.222\angle -90°$$

The subtransient current flowing in the motor is

$$\mathbf{I}_M'' = \frac{\mathbf{E}_{A,M} - \mathbf{V}_\phi}{jX_M''} = \frac{1.00\angle 0° - 0.0\angle 0°}{j0.40} = 2.500\angle -90°$$

12-7. Assume that a symmetrical three-phase fault occurs on the high-voltage side of transformer T_2 in the power system shown in Figure P12-2. Make the assumption that the generator is operating at rated voltage, and that the power system is initially unloaded.

  (a) Calculate the subtransient, transient, and steady-state fault current, generator current, and motor current *ignoring resistances*.
  (b) Calculate the subtransient, transient, and steady-state fault current, generator current, and motor current *including resistances*.
  (c) How much effect does the inclusion of resistances have on the fault current calculations?
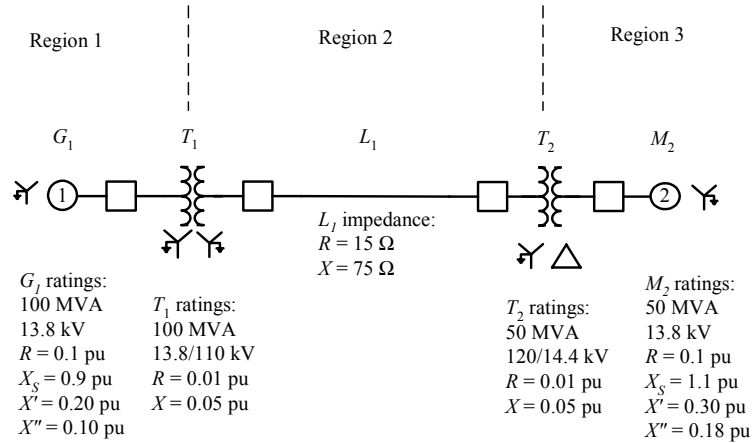
Figure P12-2   One-line diagram of the power system in Problem 12-7.

SOLUTION   To simplify this problem, we will pick the base quantities for this power system to be 100 MVA and 13.8 kV at generator G_1, which is in Region 1. Therefore, the base quantities are:

$$S_{\text{base},1} = 100 \text{ MVA}$$

$$V_{\text{base},1} = 13.8 \text{ kV}$$

$$I_{\text{base},1} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base 1}}} = \frac{100{,}000{,}000 \text{ VA}}{\sqrt{3}(13{,}800 \text{ V})} = 4184 \text{ A}$$

$$Z_{\text{base},1} = \frac{\left(V_{LL,\text{base 1}}\right)^2}{S_{3\phi,\text{base}}} = \frac{(13{,}800 \text{ V})^2}{100{,}000{,}000 \text{ VA}} = 1.904 \text{ }\Omega$$

$$S_{\text{base},2} = 100 \text{ MVA}$$

$$V_{\text{base},2} = 13.8 \text{ kV}\left(\frac{110 \text{ kV}}{13.8 \text{ kV}}\right) = 110 \text{ kV}$$

$$I_{\text{base},2} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base 2}}} = \frac{100{,}000{,}000 \text{ VA}}{\sqrt{3}(110{,}000 \text{ V})} = 524.9 \text{ A}$$

$$Z_{\text{base},2} = \frac{\left(V_{LL,\text{base }2}\right)^2}{S_{3\phi,\text{base}}} = \frac{(110,000 \text{ V})^2}{100,000,000 \text{ VA}} = 121 \ \Omega$$

$$S_{\text{base},3} = 100 \text{ MVA}$$

$$V_{\text{base},3} = 110 \text{ kV}\left(\frac{14.4 \text{ kV}}{120 \text{ kV}}\right) = 13.2 \text{ kV}$$

$$I_{\text{base},3} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base }3}} = \frac{100,000,000 \text{ VA}}{\sqrt{3}(13,200 \text{ V})} = 4374 \text{ A}$$

$$Z_{\text{base},3} = \frac{\left(V_{LL,\text{base }3}\right)^2}{S_{3\phi,\text{base}}} = \frac{(13,200 \text{ V})^2}{100,000,000 \text{ VA}} = 1.742 \ \Omega$$

The per-unit impedances expressed on the system base are:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}}\left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2\left(\frac{S_{\text{new}}}{S_{\text{given}}}\right) \qquad (10\text{-}8)$$

$$R_{G1} = 0.10 \text{ pu}$$
$$X''_{G1} = 0.10 \text{ pu}$$
$$X'_{G1} = 0.20 \text{ pu}$$
$$X_{S,G1} = 0.90 \text{ pu}$$
$$R_{T1} = 0.01 \text{ pu}$$
$$X_{T1} = 0.05 \text{ pu}$$

$$R_{T2} = (0.01 \text{ pu})\left(\frac{120 \text{ kV}}{110 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.0238 \text{ pu}$$

$$X_{T2} = (0.05 \text{ pu})\left(\frac{120 \text{ kV}}{110 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.1190 \text{ pu}$$

$$R_{M2} = (0.10 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.2 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.2186 \text{ pu}$$

$$X''_{M2} = (0.18 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.2 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.3935 \text{ pu}$$

$$X'_{M2} = (0.30 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.2 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.6558 \text{ pu}$$

$$X_{S,M2} = (1.10 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.2 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 2.4045 \text{ pu}$$

$$R_{\text{line,pu}} = \frac{R_{\text{line},\Omega}}{Z_{\text{base}}} = \frac{15 \ \Omega}{121 \ \Omega} = 0.124 \text{ pu}$$

$$X_{\text{line,pu}} = \frac{X_{\text{line},\Omega}}{Z_{\text{base}}} = \frac{75 \ \Omega}{121 \ \Omega} = 0.6198 \text{ pu}$$

The assumption that the system is initially unloaded means that the internal generated voltages of the generator and motor are = $1.00\angle0°$ before the fault. The resulting per-unit equivalent circuit of the power system is shown below. Note that all three synchronous machine reactances are shown for each synchronous machine.
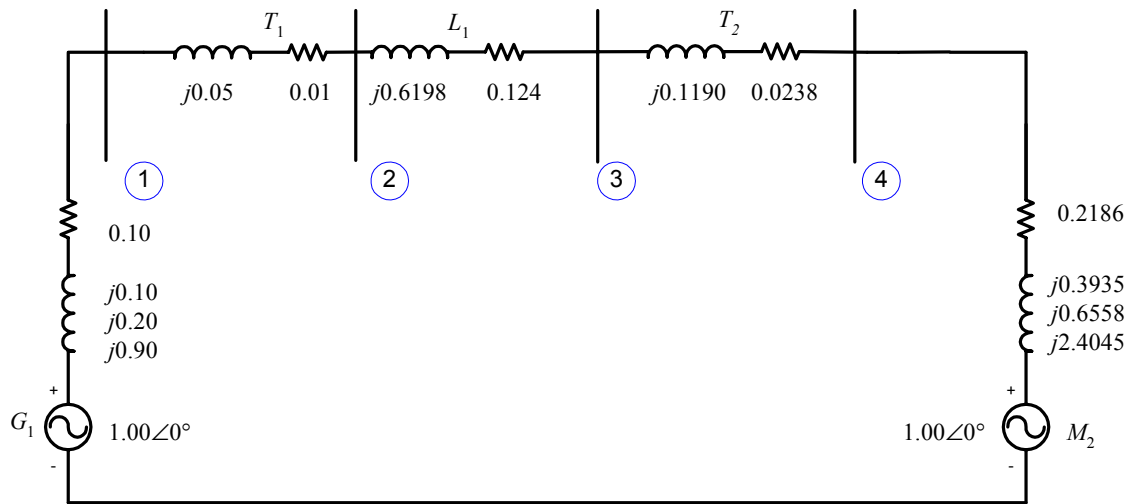
(a) Note that the fault occurs at "Bus 3", which is the high-voltage side of transformer T_2. The input file for this power system *ignoring resistances* is shown below.

```
% File describing the power system of Problem 12-7, ignoring
% resistances.
%
% System data has the form:
%SYSTEM     name               baseMVA
SYSTEM  Prob12_7a            100
%
% Bus data has the form:
%BUS name        volts
BUS One          1.00
BUS Two          1.00
BUS Three        1.00
BUS Four         1.00
%
% Note that transformers T1 and T2 are treated as "transmission lines"
% here.  Transmission line data has the form:
%LINE      from         to      Rse      Xse      Gsh      Bsh      X0     Vis
LINE    One        Two        0.0000   0.0500   0.000    0.000    0.000   0
LINE    Two        Three      0.0000   0.6198   0.000    0.000    0.000   0
LINE    Three      Four       0.0000   0.1190   0.000    0.000    0.000   0
%
% Generator data has the form:
%GENERATOR   bus        R        Xs       Xp       Xpp      X2       X0
GENERATOR    One      0.00     0.90     0.20     0.10     0.00     0.00
%
% Motor data has the form:
%MOTOR       bus        R        Xs       Xp       Xpp      X2       X0
MOTOR        Four     0.0000   2.4045   0.6558   0.3935   0.00     0.00
%
% type data has the form:
```

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```
%FAULT    bus      Calc Type   Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT     Three    3P          0
```

The resulting outputs are shown below.

>> **faults prob_12_7a_fault**
```
Input summary statistics:
  32 lines in system file
   1 SYSTEM lines
   4 BUS lines
   3 LINE lines
   1 GENERATOR lines
   1 MOTOR lines
   1 TYPE lines
```


                          Results for Case Prob12_7a

Symmetrical Three-Phase Fault at Bus Three
Calculating Subtransient Currents

Bus		Volts /	angle	Amps /	angle		To		Amps /	angle
no.	Name	(pu)	(deg)	(pu)	(deg)		Bus		(pu)	(deg)
===	===	===	===	===	===	===	===	===	===	===
1	One	0.870/	0.00	0.000/	0.00		**Two**		**1.299/**	**-90.00**
2	Two	0.805/	0.00	0.000/	0.00		One		1.299/	90.00
							Three		1.299/	-90.00
3	Three	0.000/	0.00	**3.250/**	**-90.00**		Two		1.299/	90.00
							Four		1.951/	90.00
4	Four	0.232/	0.00	0.000/	0.00		**Three**		**1.951/**	**-90.00**

                          Results for Case Prob12_7a

Symmetrical Three-Phase Fault at Bus Three
Calculating Transient Currents

Bus		Volts /	angle	Amps /	angle		To		Amps /	angle
no.	Name	(pu)	(deg)	(pu)	(deg)		Bus		(pu)	(deg)
===	===	===	===	===	===	===	===	===	===	===
1	One	0.770/	0.00	0.000/	0.00		**Two**		**1.150/**	**-90.00**
2	Two	0.713/	0.00	0.000/	0.00		One		1.150/	90.00
							Three		1.150/	-90.00
3	Three	0.000/	0.00	**2.440/**	**-90.00**		Two		1.150/	90.00
							Four		1.291/	90.00
4	Four	0.154/	0.00	0.000/	0.00		**Three**		**1.291/**	**-90.00**

                          Results for Case Prob12_7a

Symmetrical Three-Phase Fault at Bus Three
Calculating Steady-State Currents

Bus		Volts /	angle	Amps /	angle		To		Amps /	angle
no.	Name	(pu)	(deg)	(pu)	(deg)		Bus		(pu)	(deg)
===	===	===	===	===	===	===	===	===	===	===

```
1       One     0.427/   0.00   0.000/   0.00      Two    0.637/  -90.00
2       Two     0.395/   0.00   0.000/   0.00      One    0.637/   90.00
                                                 Three    0.637/  -90.00
3       Three   0.000/   0.00   1.033/  -90.00     Two    0.637/   90.00
                                                  Four    0.396/   90.00
4       Four    0.047/   0.00   0.000/   0.00    Three    0.396/  -90.00
|=========================================================================|
```

The subtransient, transient, and steady-state fault currents are given in per-unit above. The actual fault currents are found by multiplying by the base current in Region 2:

$$\mathbf{I}''_f = (3.250\angle -90°)(524.9 \text{ A}) = 1706\angle -90° \text{ A}$$

$$\mathbf{I}'_f = (2.440\angle -90°)(524.9 \text{ A}) = 1281\angle -90° \text{ A}$$

$$\mathbf{I}_f = (1.033\angle -90°)(524.9 \text{ A}) = 542\angle -90° \text{ A}$$

The subtransient, transient, and steady-state generator currents will be the same as the current flowing from Bus 1 to Bus 2. These currents are given in per-unit above. The actual generator currents are found by multiplying by the base current in Region 1:

$$\mathbf{I}''_{G1} = (1.299\angle -90°)(4184 \text{ A}) = 5435\angle -90° \text{ A}$$

$$\mathbf{I}'_{G1} = (1.150\angle -90°)(4184 \text{ A}) = 4812\angle -90° \text{ A}$$

$$\mathbf{I}_{G1} = (0.637\angle -90°)(4184 \text{ A}) = 2665\angle -90° \text{ A}$$

The subtransient, transient, and steady-state motor currents will be the same as the current flowing from Bus 4 to Bus 3. These currents are given in per-unit above. The actual motor currents are found by multiplying by the base current in Region 3:

$$\mathbf{I}''_{M2} = (1.951\angle -90°)(4374 \text{ A}) = 8534\angle -90° \text{ A}$$

$$\mathbf{I}'_{M2} = (1.291\angle -90°)(4374 \text{ A}) = 5647\angle -90° \text{ A}$$

$$\mathbf{I}_{M2} = (0.396\angle -90°)(4374 \text{ A}) = 1732\angle -90° \text{ A}$$

(b) The input file for this power system *including resistances* is shown below.

```
% File describing the power system of Problem 12-7, including
% resistances.
%
% System data has the form:
%SYSTEM   name            baseMVA
SYSTEM  Prob12_7b          100
%
% Bus data has the form:
%BUS name        volts
BUS One          1.00
BUS Two          1.00
BUS Three        1.00
BUS Four         1.00
%
% Note that transformers T1 and T2 are treated as "transmission lines"
% here.  Transmission line data has the form:
%LINE     from        to      Rse      Xse      Gsh      Bsh      X0    Vis
LINE    One         Two       0.0100   0.0500   0.000    0.000    0.000  0
LINE    Two         Three     0.1240   0.6198   0.000    0.000    0.000  0
LINE    Three       Four      0.0238   0.1190   0.000    0.000    0.000  0
%
```

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```
% Generator data has the form:
%GENERATOR  bus         R       Xs      Xp      Xpp     X2      X0
GENERATOR   One        0.10    0.90    0.20    0.10    0.00    0.00
%
% Motor data has the form:
%MOTOR      bus         R       Xs      Xp      Xpp     X2      X0
MOTOR       Four       0.2186  2.4045  0.6558  0.3935  0.00    0.00
%
% type data has the form:
%FAULT      bus     Calc Type   Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT       Three       3P          0
```

The resulting outputs are shown below.

```
>> faults prob_12_7b_fault
Input summary statistics:
  32 lines in system file
   1 SYSTEM lines
   4 BUS lines
   3 LINE lines
   1 GENERATOR lines
   1 MOTOR lines
   1 TYPE lines


                        Results for Case Prob12_7b


Symmetrical Three-Phase Fault at Bus Three
Calculating Subtransient Currents
|==================Bus Information============|=====Line Information=====|
Bus              Volts / angle  Amps / angle  |  To  |  Amps / angle   |
no.      Name    (pu)    (deg)  (pu)    (deg)  | Bus  |  (pu)    (deg)  |
|============================================================================|
  1       One    0.849/   5.59  0.000/   0.00     Two    1.243/ -73.09
  2       Two    0.786/   5.59  0.000/   0.00     One    1.243/ 106.91
                                                 Three   1.243/ -73.09
  3      Three   0.000/   0.00  2.999/ -68.16     Two    1.243/ 106.91
                                                 Four    1.764/ 115.31
  4      Four    0.214/  14.00  0.000/   0.00    Three   1.764/ -64.69
|============================================================================|



                        Results for Case Prob12_7b


Symmetrical Three-Phase Fault at Bus Three
Calculating Transient Currents
|==================Bus Information============|=====Line Information=====|
Bus              Volts / angle  Amps / angle  |  To  |  Amps / angle   |
no.      Name    (pu)    (deg)  (pu)    (deg)  | Bus  |  (pu)    (deg)  |
|============================================================================|
  1       One    0.758/   3.74  0.000/   0.00     Two    1.110/ -74.94
  2       Two    0.702/   3.74  0.000/   0.00     One    1.110/ 105.06
                                                 Three   1.110/ -74.94
  3      Three   0.000/   0.00  2.342/ -73.72     Two    1.110/ 105.06
                                                 Four    1.232/ 107.37
  4      Four    0.149/   6.06  0.000/   0.00    Three   1.232/ -72.63
|============================================================================|
```

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```
Symmetrical Three-Phase Fault at Bus Three
Calculating Steady-State Currents
|====================Bus Information===========|=====Line Information=====|
Bus              Volts / angle   Amps / angle  |   To  |  Amps / angle   |
no.     Name     (pu)    (deg)   (pu)    (deg)  |  Bus  |  (pu)    (deg)  |
|=============================================================================|
  1      One    0.430/  -2.83   0.000/   0.00     Two    0.630/ -81.52
  2      Two    0.398/  -2.84   0.000/   0.00     One    0.630/  98.48
                                                  Three  0.630/ -81.52
  3      Three  0.000/ -90.00   1.024/ -82.67     Two    0.630/  98.48
                                                  Four   0.394/  95.49
  4      Four   0.048/  -5.82   0.000/   0.00     Three  0.394/ -84.51
|=============================================================================|
```

The subtransient, transient, and steady-state fault currents are given in per-unit above. The actual fault currents are found by multiplying by the base current in Region 2:

$$\mathbf{I}_f'' = (2.999\angle - 68.2°)(524.9\text{ A}) = 1574\angle - 68.2°\text{ A}$$

$$\mathbf{I}_f' = (2.342\angle - 73.7°)(524.9\text{ A}) = 1229\angle - 73.7°\text{ A}$$

$$\mathbf{I}_f = (1.024\angle - 82.7°)(524.9\text{ A}) = 537\angle - 82.7°\text{ A}$$

The subtransient, transient, and steady-state generator currents will be the same as the current flowing from Bus 1 to Bus 2. These currents are given in per-unit above. The actual generator currents are found by multiplying by the base current in Region 1:

$$\mathbf{I}_{G1}'' = (1.243\angle - 73.1°)(4184\text{ A}) = 5201\angle - 73.1°\text{ A}$$

$$\mathbf{I}_{G1}' = (1.110\angle - 68.2°)(4184\text{ A}) = 4644\angle - 68.2°\text{ A}$$

$$\mathbf{I}_{G1} = (0.630\angle - 81.5°)(4184\text{ A}) = 2636\angle - 81.5°\text{ A}$$

The subtransient, transient, and steady-state motor currents will be the same as the current flowing from Bus 4 to Bus 3. These currents are given in per-unit above. The actual motor currents are found by multiplying by the base current in Region 3:

$$\mathbf{I}_{M2}'' = (1.764\angle - 64.7°)(4374\text{ A}) = 7716\angle - 64.7°\text{ A}$$

$$\mathbf{I}_{M2}' = (1.232\angle - 72.6°)(4374\text{ A}) = 5389\angle - 72.6°\text{ A}$$

$$\mathbf{I}_{M2} = (0.394\angle - 84.5°)(4374\text{ A}) = 1723\angle - 84.5°\text{ A}$$

(c) The inclusion of the resistances changed the subtransient fault current by (3.250-2.999)/2.999 × 100% = **8.4%**. It looks like we could neglect the resistances and still be within 10% of the proper fault current value.

12-8. Assume that a symmetrical three-phase fault occurs at the terminals of motor M_1 on the low-voltage side of transformer T_1 in the power system shown in Figure P12-3. Make the assumption that the power system is operating at rated voltage, and that it is initially unloaded. Calculate the subtransient, transient, and steady-state fault current on the high-voltage side of the transformer, the low-voltage side of the transformer, and in the motor.

Region 1   Region 2

Power System:
$V = 138$ kV
short-circuit MVA = 500 MVA

T_1 ratings:
50 MVA
138/13.8 kV
$R = 0.01$ pu
$X = 0.05$ pu

M_1 ratings:
50 MVA
13.8 kV
$R = 0.1$ pu
$X_S = 1.1$ pu
$X' = 0.30$ pu
$X'' = 0.18$ pu
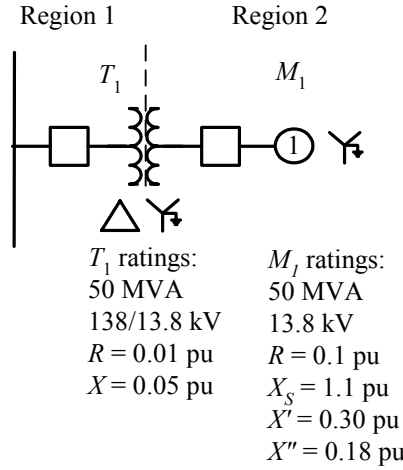
Figure P12-3   One-line diagram of the power system in Problem 12-8.

SOLUTION To simplify this problem, we will pick the base quantities for this power system to be 50 MVA and 138 kV at the high-voltage side of transformer T_1, which is in Region 1. Therefore, the base quantities are:

$$S_{\text{base},1} = 50 \text{ MVA}$$

$$V_{\text{base},1} = 138 \text{ kV}$$

$$I_{\text{base},1} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base 1}}} = \frac{50,000,000 \text{ VA}}{\sqrt{3}(138,000 \text{ V})} = 209.2 \text{ A}$$

$$Z_{\text{base},1} = \frac{\left(V_{LL,\text{base 1}}\right)^2}{S_{3\phi,\text{base}}} = \frac{(138,000 \text{ V})^2}{50,000,000 \text{ VA}} = 380.9 \ \Omega$$

$$S_{\text{base},2} = 50 \text{ MVA}$$

$$V_{\text{base},2} = 138 \text{ kV}\left(\frac{13.8 \text{ kV}}{138 \text{ kV}}\right) = 13.8 \text{ kV}$$

$$I_{\text{base},2} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base 2}}} = \frac{50,000,000 \text{ VA}}{\sqrt{3}(13,800 \text{ V})} = 2092 \text{ A}$$

$$Z_{\text{base},2} = \frac{\left(V_{LL,\text{base 2}}\right)^2}{S_{3\phi,\text{base}}} = \frac{(13,800 \text{ V})^2}{50,000,000 \text{ VA}} = 3.809 \ \Omega$$

The per-unit impedances given in the problem are all correctly expressed on the system base without conversion. The per-unit impedance of the power system can be calculated from Equation (12-26):

$$\text{Short-circuit MVA}_{\text{pu}} = V_{\text{nominal,pu}}I_{SC,\text{pu}} \tag{12-26}$$

Since the voltage is assumed to be equal to the rated value, $V_{\text{nominal,pu}} = 1.00$. Therefore, the short-circuit current is

$$I_{SC,\text{pu}} = \text{Short-circuit MVA}_{\text{pu}} = \frac{500 \text{ MVA}}{50 \text{ MVA}} = 10.0$$

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and $\quad |Z_{TH}| = \dfrac{|\mathbf{V}|}{|\mathbf{I}_{SC}|} = \dfrac{1.00}{10.0} = 0.10 \ \text{pu}$ $\hspace{4cm}$ (12-28)

If the impedance is treated as a pure reactance, $Z_{TH} = j0.10 \ \text{pu}$. The resulting per-unit per-phase equivalent circuit is shown below:
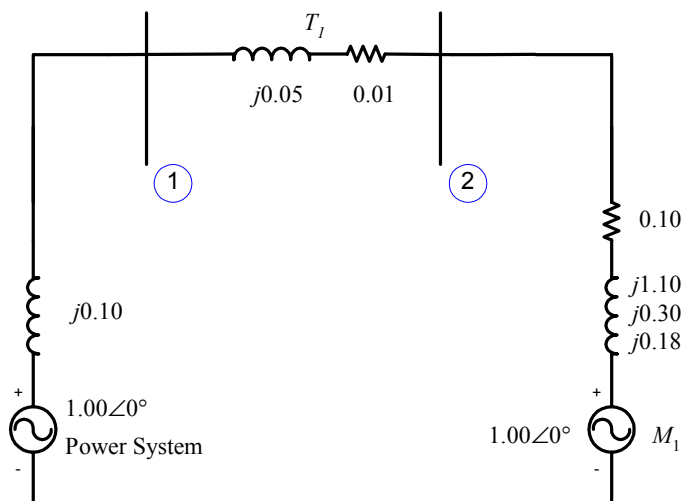
The input file for this power system is shown below.

```
% File describing the power system of Problem 12-8.
%
% System data has the form:
%SYSTEM    name              baseMVA
SYSTEM  Prob12_8            100
%
% Bus data has the form:
%BUS name       volts
BUS One         1.00
BUS Two         1.00
%
% Note that transformer T1 is treated as a "transmission line"
% here.  Transmission line data has the form:
%LINE     from        to      Rse      Xse      Gsh      Bsh      X0     Vis
LINE    One         Two        0.0100   0.0500   0.000    0.000    0.000  0
%
% Generator data has the form:
%GENERATOR   bus        R       Xs       Xp       Xpp      X2       X0
GENERATOR    One     0.10     0.90     0.20     0.10     0.00     0.00
%
% Motor data has the form:
%MOTOR       bus        R       Xs       Xp       Xpp      X2       X0
MOTOR       Two     0.10     1.10     0.30     0.18     0.00     0.00
%
% type data has the form:
%FAULT    bus     Calc Type    Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT    Two        3P              0
```

The resulting outputs are shown below.

```
>> faults prob_12_8_fault
Input summary statistics:
  27 lines in system file
   1 SYSTEM lines
   2 BUS lines
   1 LINE lines
   1 GENERATOR lines
   1 MOTOR lines
   1 TYPE lines
```

                        Results for Case Prob12_8

Symmetrical Three-Phase Fault at Bus Two
Calculating Subtransient Currents
```
|===================Bus Information===========|=====Line Information=====|
Bus             Volts / angle   Amps / angle  |  To  |  Amps / angle  |
no.     Name    (pu)   (deg)    (pu)   (deg)   | Bus  | (pu)    (deg)  |
|=============================================================================|
  1      One   0.274/  24.94   0.000/   0.00     Two   5.376/ -53.75
  2      Two   0.000/   0.00  10.212/ -57.16     One   5.376/ 126.25
|=============================================================================|
```

                        Results for Case Prob12_8

Symmetrical Three-Phase Fault at Bus Two
Calculating Transient Currents
```
|===================Bus Information===========|=====Line Information=====|
Bus             Volts / angle   Amps / angle  |  To  |  Amps / angle  |
no.     Name    (pu)   (deg)    (pu)   (deg)   | Bus  | (pu)    (deg)  |
|=============================================================================|
  1      One   0.187/  12.44   0.000/   0.00     Two   3.661/ -66.25
  2      Two   0.000/  90.00   6.816/ -68.71     One   3.661/ 113.75
|=============================================================================|
```

                        Results for Case Prob12_8

Symmetrical Three-Phase Fault at Bus Two
Calculating Steady-State Currents
```
|===================Bus Information===========|=====Line Information=====|
Bus             Volts / angle   Amps / angle  |  To  |  Amps / angle  |
no.     Name    (pu)   (deg)    (pu)   (deg)   | Bus  | (pu)    (deg)  |
|=============================================================================|
  1      One   0.053/  -4.71   0.000/   0.00     Two   1.046/ -83.40
  2      Two   0.000/   0.00   1.046/ -83.40     One   1.046/  96.60
|=============================================================================|
```

The subtransient, transient, and steady-state fault currents on the high-voltage side of the transformer are the currents flowing from Bus 1 to Bus 2, referred to the high-voltage side. They are calculated by the program above. The actual fault currents are found by multiplying by the base current in Region 1:

$$\mathbf{I}''_{T1} = (5.376\angle - 53.8°)(209.2 \text{ A}) = 1125\angle - 53.8° \text{ A}$$
$$\mathbf{I}'_{T1} = (3.661\angle - 66.3°)(209.2 \text{ A}) = 766\angle - 66.3° \text{ A}$$

$$\mathbf{I}_{T1} = (1.046\angle - 83.4°)(209.2 \text{ A}) = 219\angle - 83.4° \text{ A}$$

The subtransient, transient, and steady-state fault currents on the low-voltage side of the transformer are the fault currents flowing at Bus 2. They are calculated by the program above. The actual fault currents are found by multiplying by the base current in Region 2:

$$\mathbf{I}''_f = (10.212\angle - 57.2°)(2092 \text{ A}) = 21{,}360\angle - 57.2° \text{ A}$$
$$\mathbf{I}'_f = (6.816\angle - 68.7°)(2092 \text{ A}) = 14{,}260\angle - 68.7° \text{ A}$$
$$\mathbf{I}_f = (1.046\angle - 83.4°)(2092 \text{ A}) = 2188\angle - 83.4° \text{ A}$$

The subtransient, transient, and steady-state motor currents will be the currents that would flow in the motor if its terminals were short circuited. These per-unit currents will be 1.00/Z for the subtransient and transient cases, and zero for the steady-state case. (The current from the motor is 0 at steady state because there is nothing to keep the motor turning of the electrical input to the motor is shorted out.)

$$\mathbf{I}''_{M2} = \frac{1.00\angle 0°}{0.10 + j0.18} = 4.856\angle - 60.9° \text{ A}$$
$$\mathbf{I}''_{M2} = \frac{1.00\angle 0°}{0.10 + j0.30} = 3.162\angle - 71.6° \text{ A}$$
$$\mathbf{I}_{M2} = 0 \text{ A}$$

The actual motor currents are found by multiplying by the base current in Region 2:

$$\mathbf{I}''_{M2} = (4.856\angle - 60.9°)(2092 \text{ A}) = 10{,}160\angle - 60.9° \text{ A}$$
$$\mathbf{I}'_{M2} = (3.162\angle - 71.6°)(2092 \text{ A}) = 6615\angle - 71.6° \text{ A}$$
$$\mathbf{I}_{M2} = 0 \text{ A}$$

Chapter 13: *Unsymmetrical Faults*

Problems 13-1 to 13-5 refer to a 200 MVA, 20 kV, 60 Hz, Y-connected, solidly-grounded three-phase synchronous generator connected through a 200 MVA, 20/138 kV, Y-Y transformer to a 138 kV transmission line. The generator reactances to the machine's own base are

$$X_S = 1.40 \qquad X' = 0.30 \qquad X'' = 0.15 \qquad X_2 = 0.15 \qquad X_{g0} = 0.10$$

Both of the transformer's Y-connections are solidly grounded, and its positive-, negative-, and zero-sequence series reactances are all 0.10 pu. The resistance of both the generator and the transformer may be ignored.

13-1. Assume that a symmetrical three-phase fault occurs on the 138 kV transmission line near to the point where it is connected to the transformer.
  (a) How much current flows at the point of the fault during the subtransient period?
  (b) What is the voltage at the terminals of the generator during the subtransient period?
  (c) How much current flows at the point of the fault during the transient period?
  (d) What is the voltage at the terminals of the generator during the transient period?

SOLUTION   We will define the base quantities for this power system to 200 MVA and 20 kV at the synchronous generator. With this definition, the base voltage at the transmission line is 128 kV, and all per-unit impedances are to the correct base. The base currents on the low-voltage and high-voltage sides of the transformer are

$$I_{\text{base},1} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base 1}}} = \frac{200{,}000{,}000 \text{ VA}}{\sqrt{3}(20{,}000 \text{ V})} = 5774 \text{ A}$$

$$I_{\text{base},2} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base 2}}} = \frac{200{,}000{,}000 \text{ VA}}{\sqrt{3}(138{,}000 \text{ V})} = 836.7 \text{ A}$$

Only the positive-sequence diagram will be needed to solve this problem. This diagram is shown below:
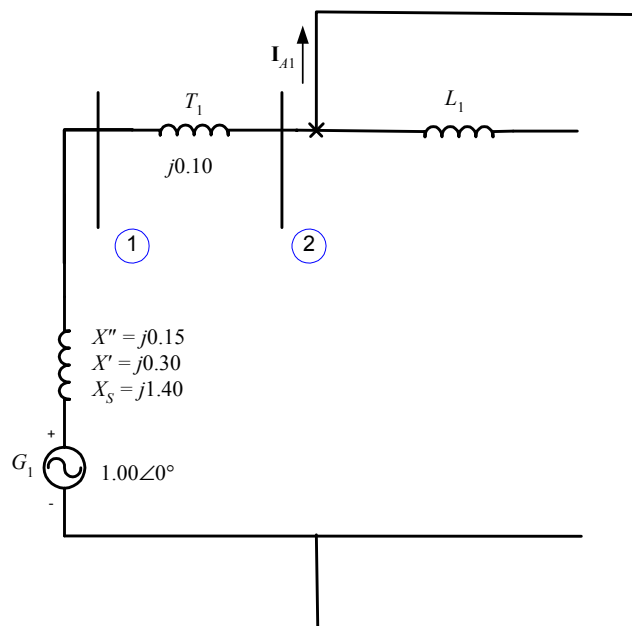
(a) The current flowing in the fault during the subtransient period can be calculated directly from the positive sequence diagram shown above.

$$\mathbf{I}_A = \mathbf{I}_{A1} = \frac{1.00\angle 0°}{j0.15 + j0.10} = 4.000\angle - 90°$$

The resulting *subtransient* fault current in amps is

$$I_f'' = (4.000)(836.7 \text{ A}) = 3347 \text{ A}$$

(b) The per-unit voltage at the terminals of the generator during the subtransient period is

$$\mathbf{V}_\phi = \mathbf{V}_1 = \mathbf{I}_{A1}(jX_T) = (4.000\angle - 90°)(j0.10) = 0.400\angle 0°$$

The resulting *subtransient* voltage at the terminals is

$$V_T'' = (0.400)(20 \text{ kV}) = 8 \text{ kV}$$

(c) The current flowing in the fault during the transient period can be calculated directly from the positive sequence diagram shown above.

$$\mathbf{I}_A = \mathbf{I}_{A1} = \frac{1.00\angle 0°}{j0.30 + j0.10} = 2.500\angle - 90°$$

The resulting *transient* fault current in amps is

$$I_f' = (2.500)(836.7 \text{ A}) = 2092 \text{ A}$$

(d) The per-unit voltage at the terminals of the generator during the transient period is

$$\mathbf{V}_\phi = \mathbf{V}_1 = \mathbf{I}_{A1}(jX_T) = (2.500\angle - 90°)(j0.10) = 0.250\angle 0°$$

The resulting *subtransient* voltage at the terminals is

$$V_T' = (0.250)(20 \text{ kV}) = 5 \text{ kV}$$

13-2. Assume that a single-line-to-ground fault occurs on the 138 kV transmission line near to the point where it is connected to the transformer.
  (a) How much current flows at the point of the fault during the subtransient period?
  (b) What is the voltage at the terminals of the generator during the subtransient period?
  (c) How much current flows at the point of the fault during the transient period?
  (d) What is the voltage at the terminals of the generator during the transient period?

SOLUTION All three sequence diagrams will be needed to solve this problem. These diagrams must be connected in series, as shown below:

(a) If we assume that phase *a* is the one shorted out, the current flowing in the fault during the subtransient period can be calculated directly from the network of sequence diagrams shown above.

$$\mathbf{I}_{A1} = \mathbf{I}_{A2} = \mathbf{I}_{A0} = \frac{1.00\angle 0°}{\left(j0.15 + j0.10\right) + \left(j0.15 + j0.10\right) + \left(j0.10 + j0.10\right)} = 1.429\angle -90°$$

The resulting *subtransient* fault current in amps is

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-23}$$

$$\mathbf{I}_A = 1.429\angle -90° + 1.429\angle -90° + 1.429\angle -90° = 4.287\angle -90°$$

$$\mathbf{I}_B = a^2\mathbf{I}_{A1} + a\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-24}$$

$$\mathbf{I}_B = a^2\left(1.429\angle -90°\right) + a\left(1.429\angle -90°\right) + 1.429\angle -90° = 0.0\angle 0°$$

$$\mathbf{I}_C = a\mathbf{I}_{A1} + a^2\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-25}$$

$$\mathbf{I}_C = a\left(1.429\angle -90°\right) + a^2\left(1.429\angle -90°\right) + 1.429\angle -90° = 0.0\angle 0°$$

In amps, the subtransient fault current in phase *a* is

$$I_f'' = \left(4.287\right)\left(836.7 \text{ A}\right) = 3587 \text{ A}$$

(b) The per-unit sequence voltages at the terminals of this generator during the subtransient period are

$$\mathbf{V}_{A1} = \mathbf{E}_{A1} - \mathbf{I}_{A1}Z_1 \tag{13-38}$$

$$\mathbf{V}_{A1} = 1.00\angle 0° - \left(1.429\angle -90°\right)\left(j0.15\right) = 0.7856\angle 0°$$

$$\mathbf{V}_{A2} = -\mathbf{I}_{A2}Z_2 \tag{13-39}$$

$$\mathbf{V}_{A2} = -\left(1.429\angle -90°\right)\left(j0.15\right) = 0.2144\angle 180°$$

$$\mathbf{V}_{A0} = -\mathbf{I}_{A0}Z_0 \tag{13-40}$$

$$\mathbf{V}_{A0} = -\left(1.429\angle -90°\right)\left(j0.10\right) = 0.1429\angle 180°$$

The resulting *subtransient* voltages at the terminals are

$$\mathbf{V}_A = \mathbf{V}_{A1} + \mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-8}$$

$$\mathbf{V}_A = 0.7826\angle 0° + 0.2144\angle 180° + 0.1429\angle 180° = 0.425\angle 0°$$

$$\mathbf{V}_B = a^2\mathbf{V}_{A1} + a\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-9}$$

$$\mathbf{V}_B = a^2\left(0.7826\angle 0°\right) + a\left(0.2144\angle 180°\right) + 0.1429\angle 180° = 0.966\angle -116.3°$$

$$\mathbf{V}_C = a\mathbf{V}_{A1} + a^2\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-10}$$

$$\mathbf{V}_C = a\left(0.7826\angle 0°\right) + a^2\left(0.2144\angle 180°\right) + 0.1429\angle 180° = 0.966\angle 116.3°$$

The line-to-neutral base voltage at the low-voltage side of the transformer is 20 kV / $\sqrt{3}$ = 11.55 kV, so the actual phase voltages are:

$$\mathbf{V}_A = \left(0.425\angle 0°\right)\left(11.55 \text{ kV}\right) = 4.909\angle 0° \text{ kV}$$

$$\mathbf{V}_B = \left(0.966\angle -116.3°\right)\left(11.55 \text{ kV}\right) = 11.16\angle -116.3° \text{ kV}$$

$$\mathbf{V}_C = \left(0.966\angle 116.3°\right)\left(11.55 \text{ kV}\right) = 11.16\angle 116.3° \text{ kV}$$

(c) The current flowing in the fault during the transient period can be calculated directly from the sequence network shown above.

$$\mathbf{I}_{A1} = \mathbf{I}_{A2} = \mathbf{I}_{A0} = \frac{1.00\angle 0°}{\left(j0.30 + j0.10\right) + \left(j0.15 + j0.10\right) + \left(j0.10 + j0.10\right)} = 1.177\angle -90°$$

The resulting *transient* fault current in amps is

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} + \mathbf{I}_{A0} \qquad (13\text{-}23)$$

$$\mathbf{I}_A = 1.177\angle -90° + 1.177\angle -90° + 1.177\angle -90° = 3.529\angle -90°$$

$$\mathbf{I}_B = a^2\mathbf{I}_{A1} + a\mathbf{I}_{A2} + \mathbf{I}_{A0} \qquad (13\text{-}24)$$

$$\mathbf{I}_B = a^2\left(1.177\angle -90°\right) + a\left(1.177\angle -90°\right) + 1.177\angle -90° = 0.0\angle 0°$$

$$\mathbf{I}_C = a\mathbf{I}_{A1} + a^2\mathbf{I}_{A2} + \mathbf{I}_{A0} \qquad (13\text{-}25)$$

$$\mathbf{I}_C = a\left(1.177\angle -90°\right) + a^2\left(1.177\angle -90°\right) + 1.177\angle -90° = 0.0\angle 0°$$

In amps, the transient fault current in phase *a* is

$$I'_f = (3.529)(836.7 \text{ A}) = 2953 \text{ A}$$

(d) The per-unit sequence voltages at the terminals of this generator during the transient period are

$$\mathbf{V}_{A1} = \mathbf{E}_{A1} - \mathbf{I}_{A1}Z_1 \qquad (13\text{-}38)$$

$$\mathbf{V}_{A1} = 1.00\angle 0° - \left(1.177\angle -90°\right)(j0.30) = 0.6471\angle 0°$$

$$\mathbf{V}_{A2} = -\mathbf{I}_{A2}Z_2 \qquad (13\text{-}39)$$

$$\mathbf{V}_{A2} = -\left(1.177\angle -90°\right)(j0.15) = 0.1765\angle 180°$$

$$\mathbf{V}_{A0} = -\mathbf{I}_{A0}Z_0 \qquad (13\text{-}40)$$

$$\mathbf{V}_{A0} = -\left(1.177\angle -90°\right)(j0.10) = 0.1176\angle 180°$$

The resulting *transient* voltages at the terminals are

$$\mathbf{V}_A = \mathbf{V}_{A1} + \mathbf{V}_{A2} + \mathbf{V}_{A0} \qquad (13\text{-}8)$$

$$\mathbf{V}_A = 0.6471\angle 0° + 0.1765\angle 180° + 0.1176\angle 180° = 0.387\angle 0°$$

$$\mathbf{V}_B = a^2\mathbf{V}_{A1} + a\mathbf{V}_{A2} + \mathbf{V}_{A0} \qquad (13\text{-}9)$$

$$\mathbf{V}_B = a^2\left(0.6471\angle 0°\right) + a\left(0.1765\angle 180°\right) + 0.1176\angle 180° = 0.778\angle -121.6°$$

$$\mathbf{V}_C = a\mathbf{V}_{A1} + a^2\mathbf{V}_{A2} + \mathbf{V}_{A0} \qquad (13\text{-}10)$$

$$\mathbf{V}_C = a\left(0.6471\angle 0°\right) + a^2\left(0.1765\angle 180°\right) + 0.1176\angle 180° = 0.778\angle 121.6°$$

The line-to-neutral base voltage at the low-voltage side of the transformer is 20 kV / $\sqrt{3}$ = 11.55 kV, so the actual phase voltages are:

$$\mathbf{V}_A = \left(0.387\angle 0°\right)(11.55 \text{ kV}) = 4.470\angle 0° \text{ kV}$$

$$\mathbf{V}_B = \left(0.778\angle -121.6°\right)(11.55 \text{ kV}) = 8.986\angle -121.6° \text{ kV}$$

$$\mathbf{V}_C = \left(0.778\angle 121.6°\right)(11.55 \text{ kV}) = 8.986\angle 121.6° \text{ kV}$$

13-3. Assume that a line-to-line fault occurs on the 138 kV transmission line near to the point where it is connected to the transformer.
  (a) How much current flows at the point of the fault during the subtransient period?
  (b) What is the voltage at the terminals of the generator during the subtransient period?
  (c) How much current flows at the point of the fault during the transient period?
  (d) What is the voltage at the terminals of the generator during the transient period?

SOLUTION The positive and negative sequence diagrams will be needed to solve this problem. These diagrams must be connected in parallel, as shown below:
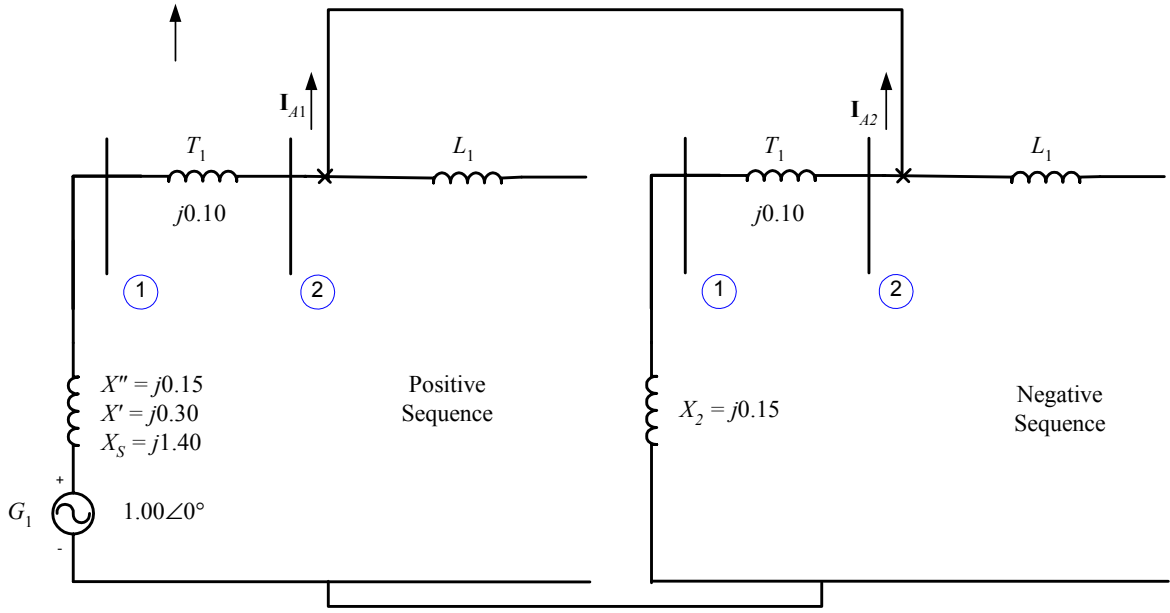
(a) If we assume that phases *a* and *c* are the ones shorted out, the current flowing in the fault during the subtransient period can be calculated directly from the network of sequence diagrams shown above.

$$\mathbf{I}_{A1} = \frac{1.00\angle 0°}{\left(j0.15 + j0.10 \right) + \left(j0.15 + j0.10 \right)} = 2.000\angle -90°$$

$$\mathbf{I}_{A2} = -\mathbf{I}_{A1} = 2.000\angle 90°$$

$$\mathbf{I}_{A0} = 0.0\angle 0°$$

The resulting *subtransient* fault current in amps is

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-23}$$

$$\mathbf{I}_A = 2.000\angle -90° + 2.000\angle 90° + 0.0\angle 0° = 0.0\angle 0°$$

$$\mathbf{I}_B = a^2 \mathbf{I}_{A1} + a\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-24}$$

$$\mathbf{I}_B = a^2 \left(2.000\angle -90° \right) + a\left(2.000\angle -90° \right) + 0.0\angle 0° = 3.464\angle 180°$$

$$\mathbf{I}_C = a\mathbf{I}_{A1} + a^2 \mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-25}$$

$$\mathbf{I}_C = a\left(2.000\angle -90° \right) + a^2 \left(2.000\angle -90° \right) + 0.0\angle 0° = 3.464\angle 0°$$

In amps, the subtransient fault current in phases *b* and *c* is

$$I_f'' = \left(3.464 \right)\left(836.7 \text{ A} \right) = 2898 \text{ A}$$

(b) The per-unit sequence voltages at the terminals of this generator during the subtransient period are

$$\mathbf{V}_{A1} = \mathbf{E}_{A1} - \mathbf{I}_{A1}Z_1 \tag{13-38}$$

$$\mathbf{V}_{A1} = 1.00\angle 0° - \left(2.000\angle -90° \right)\left(j0.15 \right) = 0.7000\angle 0°$$

$$\mathbf{V}_{A2} = -\mathbf{I}_{A2}Z_2 \tag{13-39}$$

$$\mathbf{V}_{A2} = -\left(2.000\angle 90° \right)\left(j0.15 \right) = 0.3000\angle 0°$$

$$\mathbf{V}_{A0} = -\mathbf{I}_{A0}Z_0 \tag{13-40}$$

$$\mathbf{V}_{A0} = 0.0\angle 0°$$

The resulting *subtransient* voltages at the terminals are

$$\mathbf{V}_A = \mathbf{V}_{A1} + \mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-8}$$

$$\mathbf{V}_A = 0.7000\angle 0° + 0.3000\angle 0° + 0.0\angle 0° = 1.000\angle 0°$$

$$\mathbf{V}_B = a^2\mathbf{V}_{A1} + a\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-9}$$

$$\mathbf{V}_B = a^2\left(0.7000\angle 0°\right) + a\left(0.3000\angle 0°\right) + 0.0\angle 0° = 0.608\angle -145.3°$$

$$\mathbf{V}_C = a\mathbf{V}_{A1} + a^2\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-10}$$

$$\mathbf{V}_C = a\left(0.7000\angle 0°\right) + a^2\left(0.3000\angle 180°\right) + 0.0\angle 0° = 0.608\angle 145.3°$$

The line-to-neutral base voltage at the low-voltage side of the transformer is 20 kV / $\sqrt{3}$ = 11.55 kV, so the actual phase voltages are:

$$\mathbf{V}_A = \left(1.000\angle 0°\right)\left(11.55 \text{ kV}\right) = 11.55\angle 0° \text{ kV}$$

$$\mathbf{V}_B = \left(0.608\angle -145.3°\right)\left(11.55 \text{ kV}\right) = 7.02\angle -145.3° \text{ kV}$$

$$\mathbf{V}_C = \left(0.608\angle 145.3°\right)\left(11.55 \text{ kV}\right) = 7.02\angle 145.3° \text{ kV}$$

(c) The current flowing in the fault during the transient period can be calculated directly from the sequence network shown above.

$$\mathbf{I}_{A1} = \frac{1.00\angle 0°}{\left(j0.30 + j0.10\right) + \left(j0.15 + j0.10\right)} = 1.539\angle -90°$$

$$\mathbf{I}_{A2} = -\mathbf{I}_{A1} = 1.539\angle 90°$$

$$\mathbf{I}_{A0} = 0.0\angle 0°$$

The resulting *transient* fault current in amps is

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-23}$$

$$\mathbf{I}_A = 1.539\angle -90° + 1.539\angle 90° + 0.0\angle 0° = 0.0\angle 0°$$

$$\mathbf{I}_B = a^2\mathbf{I}_{A1} + a\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-24}$$

$$\mathbf{I}_B = a^2\left(1.539\angle -90°\right) + a\left(1.539\angle -90°\right) + 0.0\angle 0° = 2.666\angle 180°$$

$$\mathbf{I}_C = a\mathbf{I}_{A1} + a^2\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-25}$$

$$\mathbf{I}_C = a\left(1.539\angle -90°\right) + a^2\left(1.539\angle -90°\right) + 0.0\angle 0° = 2.666\angle 0°$$

In amps, the subtransient fault current in phases *b* and *c* is

$$I_f' = \left(2.666\right)\left(836.7 \text{ A}\right) = 2231 \text{ A}$$

(d) The per-unit sequence voltages at the terminals of this generator during the transient period are

$$\mathbf{V}_{A1} = \mathbf{E}_{A1} - \mathbf{I}_{A1}Z_1 \tag{13-38}$$

$$\mathbf{V}_{A1} = 1.00\angle 0° - \left(1.539\angle -90°\right)\left(j0.30\right) = 0.5383\angle 0°$$

$$\mathbf{V}_{A2} = -\mathbf{I}_{A2}Z_2 \tag{13-39}$$

$$\mathbf{V}_{A2} = -\left(1.539\angle 90°\right)\left(j0.15\right) = 0.2308\angle 0°$$

$$\mathbf{V}_{A0} = -\mathbf{I}_{A0}Z_0 \tag{13-40}$$

$$\mathbf{V}_{A0} = 0.0\angle 0°$$

The resulting *transient* voltages at the terminals are

$$\mathbf{V}_A = \mathbf{V}_{A1} + \mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-8}$$

$$\mathbf{V}_A = 0.5383\angle0° + 0.1765\angle0° + 0.0\angle0° = 0.715\angle0°$$

$$\mathbf{V}_B = a^2\mathbf{V}_{A1} + a\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-9}$$

$$\mathbf{V}_B = a^2(0.5383\angle0°) + a(0.1765\angle0°) + 0.0\angle0° = 0.475\angle-138.8°$$

$$\mathbf{V}_C = a\mathbf{V}_{A1} + a^2\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-10}$$

$$\mathbf{V}_C = a(0.5383\angle0°) + a^2(0.1765\angle0°) + 0.0\angle0° = 0.475\angle138.8°$$

The line-to-neutral base voltage at the low-voltage side of the transformer is 20 kV / $\sqrt{3}$ = 11.55 kV, so the actual phase voltages are:

$$\mathbf{V}_A = (0.715\angle0°)(11.55 \text{ kV}) = 8.258\angle0° \text{ kV}$$

$$\mathbf{V}_B = (0.475\angle-138.8°)(11.55 \text{ kV}) = 5.486\angle-138.8° \text{ kV}$$

$$\mathbf{V}_C = (0.475\angle138.8°)(11.55 \text{ kV}) = 5.486\angle138.8° \text{ kV}$$

13-4. Assume that a double-line-to-ground fault occurs on the 138 kV transmission line near to the point where it is connected to the transformer.
   (a) How much current flows at the point of the fault during the subtransient period?
   (b) What is the voltage at the terminals of the generator during the subtransient period?
   (c) How much current flows at the point of the fault during the transient period?
   (d) What is the voltage at the terminals of the generator during the transient period?

SOLUTION The positive, negative, and zero sequence diagrams will be needed to solve this problem. These diagrams must be connected in parallel, as shown below:
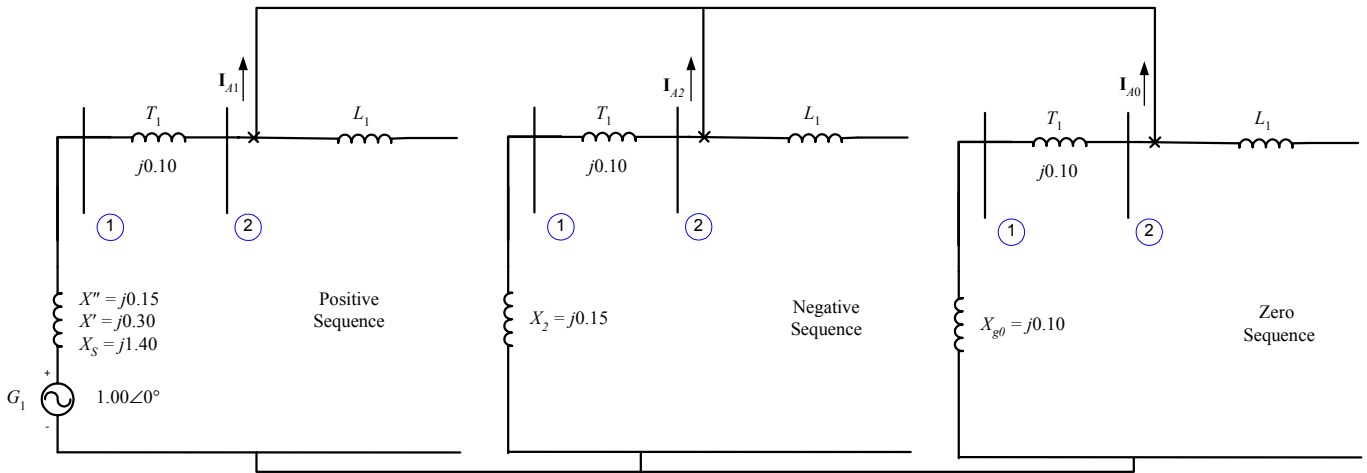
(a) If we assume that phases *a* and *c* are the ones shorted out, the current flowing in the fault during the subtransient period can be calculated directly from the network of sequence diagrams shown above.

$$\mathbf{I}_{A1} = \frac{1.00\angle0°}{Z_1 + \dfrac{Z_2 Z_0}{Z_2 + Z_0}}$$

$$\mathbf{I}_{A1} = \frac{1.00\angle0°}{(j0.25) + \dfrac{(j0.25)(j0.20)}{(j0.25)+(j0.20)}} = 2.7692\angle-90°$$

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$$\mathbf{I}_{A2} = -\mathbf{I}_{A1}\frac{Z_0}{Z_2 + Z_0} = -(2.7692\angle - 90°)\frac{j0.20}{j0.25 + j0.20} = 1.2308\angle 90°$$

$$\mathbf{I}_{A0} = -\mathbf{I}_{A1}\frac{Z_2}{Z_2 + Z_0} = -(2.7692\angle - 90°)\frac{j0.25}{j0.25 + j0.20} = 1.5385\angle 90°$$

The resulting *subtransient* fault current in amps is

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-23}$$

$$\mathbf{I}_A = 2.7692\angle - 90° + 1.2308\angle 90° + 1.5385\angle 90° = 0.0\angle 0°$$

$$\mathbf{I}_B = a^2\mathbf{I}_{A1} + a\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-24}$$

$$\mathbf{I}_B = a^2(2.7692\angle - 90°) + a(1.2308\angle 90°) + 1.5385\angle 90° = 4.162\angle 146.3°$$

$$\mathbf{I}_C = a\mathbf{I}_{A1} + a^2\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-25}$$

$$\mathbf{I}_C = a(2.7692\angle - 90°) + a^2(1.2308\angle 90°) + 1.5385\angle 90° = 4.162\angle 33.7°$$

In amps, the subtransient fault current in phases *b* and *c* is

$$I_f'' = (4.162)(836.7 \text{ A}) = 3482 \text{ A}$$

(b) The per-unit sequence voltages at the terminals of this generator during the subtransient period are

$$\mathbf{V}_{A1} = \mathbf{E}_{A1} - \mathbf{I}_{A1}Z_1 \tag{13-38}$$

$$\mathbf{V}_{A1} = 1.00\angle 0° - (2.7692\angle - 90°)(j0.15) = 0.5846\angle 0°$$

$$\mathbf{V}_{A2} = -\mathbf{I}_{A2}Z_2 \tag{13-39}$$

$$\mathbf{V}_{A2} = -(1.2308\angle 90°)(j0.15) = 0.1846\angle 0°$$

$$\mathbf{V}_{A0} = -\mathbf{I}_{A0}Z_0 \tag{13-40}$$

$$\mathbf{V}_{A0} = -(1.5385\angle 90°)(j0.10) = 0.1539\angle 0°$$

The resulting *subtransient* voltages at the terminals are

$$\mathbf{V}_A = \mathbf{V}_{A1} + \mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-8}$$

$$\mathbf{V}_A = 0.5846\angle 0° + 0.1846\angle 0° + 0.1539\angle 0° = 0.923\angle 0°$$

$$\mathbf{V}_B = a^2\mathbf{V}_{A1} + a\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-9}$$

$$\mathbf{V}_B = a^2(0.5846\angle 0°) + a(0.1846\angle 0°) + 0.1539\angle 0° = 0.416\angle - 123.7°$$

$$\mathbf{V}_C = a\mathbf{V}_{A1} + a^2\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-10}$$

$$\mathbf{V}_C = a(0.5846\angle 0°) + a^2(0.1846\angle 0°) + 0.1539\angle 0° = 0.416\angle 123.7°$$

The line-to-neutral base voltage at the low-voltage side of the transformer is 20 kV / $\sqrt{3}$ = 11.55 kV, so the actual phase voltages are:

$$\mathbf{V}_A = (0.923\angle 0°)(11.55 \text{ kV}) = 10.66\angle 0° \text{ kV}$$

$$\mathbf{V}_B = (0.416\angle - 123.7°)(11.55 \text{ kV}) = 4.81\angle - 145.3° \text{ kV}$$

$$\mathbf{V}_C = (0.416\angle 123.7°)(11.55 \text{ kV}) = 4.81\angle 123.7° \text{ kV}$$

(c) The current flowing in the fault during the transient period can be calculated directly from the sequence network shown above.

$$\mathbf{I}_{A1} = \frac{1.00\angle 0°}{Z_1 + \dfrac{Z_2 Z_0}{Z_2 + Z_0}}$$

$$\mathbf{I}_{A1} = \frac{1.00\angle 0°}{(j0.40) + \dfrac{(j0.25)(j0.20)}{(j0.25)+(j0.20)}} = 1.9565\angle -90°$$

$$\mathbf{I}_{A2} = -\mathbf{I}_{A1}\frac{Z_0}{Z_2 + Z_0} = -(1.9565\angle -90°)\frac{j0.20}{j0.25 + j0.20} = 0.8696\angle 90°$$

$$\mathbf{I}_{A0} = -\mathbf{I}_{A1}\frac{Z_2}{Z_2 + Z_0} = -(1.9565\angle -90°)\frac{j0.25}{j0.25 + j0.20} = 1.0870\angle 90°$$

The resulting *transient* fault current in amps is

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-23}$$
$$\mathbf{I}_A = 1.9565\angle -90° + 0.8696\angle 90° + 1.0870\angle 90° = 0.0\angle 0°$$
$$\mathbf{I}_B = a^2 \mathbf{I}_{A1} + a\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-24}$$
$$\mathbf{I}_B = a^2 (1.9565\angle -90°) + a(0.8696\angle 90°) + 1.0870\angle 90° = 2.941\angle 146.3°$$
$$\mathbf{I}_C = a\mathbf{I}_{A1} + a^2\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-25}$$
$$\mathbf{I}_C = a(1.9565\angle -90°) + a^2 (0.8696\angle 90°) + 1.0870\angle 90° = 2.941\angle 33.7°$$

In amps, the subtransient fault current in phases *b* and *c* is

$$I'_f = (2.941)(836.7 \text{ A}) = 2461 \text{ A}$$

(d) The per-unit sequence voltages at the terminals of this generator during the transient period are

$$\mathbf{V}_{A1} = \mathbf{E}_{A1} - \mathbf{I}_{A1}Z_1 \tag{13-38}$$
$$\mathbf{V}_{A1} = 1.00\angle 0° - (1.9565\angle -90°)(j0.30) = 0.4130\angle 0°$$
$$\mathbf{V}_{A2} = -\mathbf{I}_{A2}Z_2 \tag{13-39}$$
$$\mathbf{V}_{A2} = -(0.8696\angle 90°)(j0.15) = 0.1304\angle 0°$$
$$\mathbf{V}_{A0} = -\mathbf{I}_{A0}Z_0 \tag{13-40}$$
$$\mathbf{V}_{A0} = -(1.0870\angle 90°)(j0.10) = 0.1087\angle 0°$$

The resulting *transient* voltages at the terminals are

$$\mathbf{V}_A = \mathbf{V}_{A1} + \mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-8}$$
$$\mathbf{V}_A = 0.4130\angle 0° + 0.1304\angle 0° + 0.1087\angle 0° = 0.652\angle 0°$$
$$\mathbf{V}_B = a^2\mathbf{V}_{A1} + a\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-9}$$
$$\mathbf{V}_B = a^2 (0.4130\angle 0°) + a(0.1304\angle 0°) + 0.1087\angle 0° = 0.294\angle -123.7°$$
$$\mathbf{V}_C = a\mathbf{V}_{A1} + a^2\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-10}$$
$$\mathbf{V}_C = a(0.4130\angle 0°) + a^2 (0.1304\angle 0°) + 0.1087\angle 0° = 0.294\angle 123.7°$$

The line-to-neutral base voltage at the low-voltage side of the transformer is 20 kV / $\sqrt{3}$ = 11.55 kV, so the actual phase voltages are:

$$\mathbf{V}_A = (0.652\angle 0°)(11.55 \text{ kV}) = 7.531\angle 0° \text{ kV}$$

$$\mathbf{V}_B = (0.294\angle -123.7°)(11.55 \text{ kV}) = 3.396\angle -123.7° \text{ kV}$$

$$\mathbf{V}_C = (0.294\angle 123.7°)(11.55 \text{ kV}) = 3.396\angle 123.7° \text{ kV}$$

13-5. Suppose the transformer in this power system is changed from a Y-Y to a Y-Δ, with the Y-connection solidly grounded. Nothing else changes in the power system. Now suppose that a single-line-to-ground fault occurs on the 138 kV transmission line near to the point where it is connected to the transformer.

(a) How much current flows at the point of the fault during the subtransient period?
(b) What is the voltage at the terminals of the generator during the subtransient period?
(c) How much current flows at the point of the fault during the transient period?
(d) What is the voltage at the terminals of the generator during the transient period?

SOLUTION All three sequence diagrams will be needed to solve this problem. The positive and negative sequence diagrams are the same as in the previous three problems. The zero sequence diagram is different because the connectivity of the transformer has changed. These three sequence diagrams must be connected in series, as shown below:
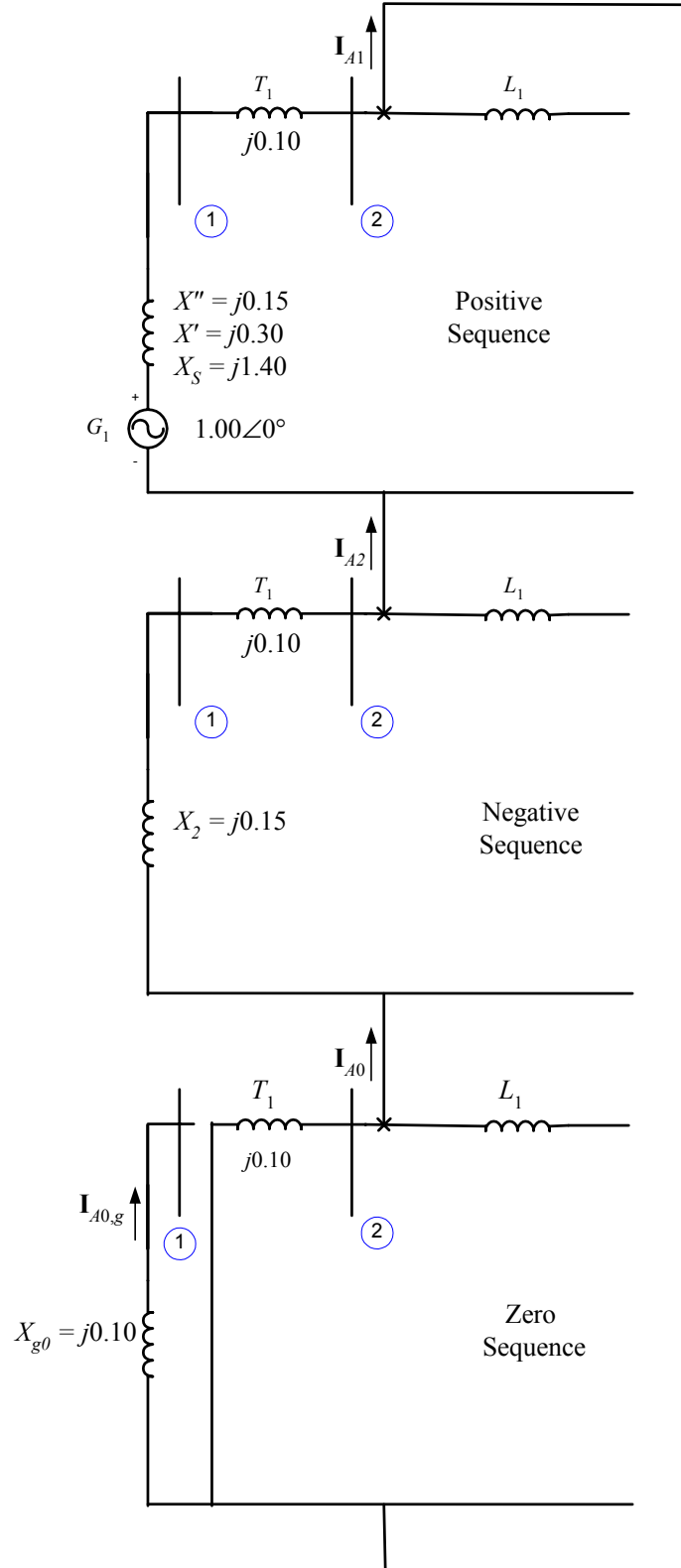
(a) If we assume that phase *a* is the one shorted out, the current flowing in the fault during the subtransient period can be calculated directly from the network of sequence diagrams shown above.

$$\mathbf{I}_{A1} = \mathbf{I}_{A2} = \mathbf{I}_{A0} = \frac{1.00\angle 0°}{\left(j0.15 + j0.10\right) + \left(j0.15 + j0.10\right) + \left(j0.10\right)} = 1.6667\angle -90°$$

The resulting *subtransient* fault current in amps is

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} + \mathbf{I}_{A0} \qquad (13\text{-}23)$$

$$\mathbf{I}_A = 1.6667\angle -90° + 1.6667\angle -90° + 1.6667\angle -90° = 5.000\angle -90°$$

$$\mathbf{I}_B = a^2\mathbf{I}_{A1} + a\mathbf{I}_{A2} + \mathbf{I}_{A0} \qquad (13\text{-}24)$$

$$\mathbf{I}_B = a^2\left(1.6667\angle -90°\right) + a\left(1.6667\angle -90°\right) + 1.6667\angle -90° = 0.0\angle 0°$$

$$\mathbf{I}_C = a\mathbf{I}_{A1} + a^2\mathbf{I}_{A2} + \mathbf{I}_{A0} \qquad (13\text{-}25)$$

$$\mathbf{I}_C = a\left(1.6667\angle -90°\right) + a^2\left(1.6667\angle -90°\right) + 1.6667\angle -90° = 0.0\angle 0°$$

In amps, the subtransient fault current in phase *a* is

$$I_f'' = (5.000)(836.7 \text{ A}) = 4184 \text{ A}$$

(b) The per-unit sequence voltages at the terminals of this generator during the subtransient period are given below. Note that the zero sequence current we need here is the zero sequence current *through the generator*, which is zero.

$$\mathbf{V}_{A1} = \mathbf{E}_{A1} - \mathbf{I}_{A1}Z_1 \qquad (13\text{-}38)$$

$$\mathbf{V}_{A1} = 1.00\angle 0° - (1.6667\angle -90°)(j0.15) = 0.7500\angle 0°$$

$$\mathbf{V}_{A2} = -\mathbf{I}_{A2}Z_2 \qquad (13\text{-}39)$$

$$\mathbf{V}_{A2} = -(1.6667\angle -90°)(j0.15) = 0.2500\angle 180°$$

$$\mathbf{V}_{A0} = -\mathbf{I}_{A0,g}Z_0 \qquad (13\text{-}40)$$

$$\mathbf{V}_{A0} = 0.0\angle 0°$$

The resulting *subtransient* voltages at the terminals are

$$\mathbf{V}_A = \mathbf{V}_{A1} + \mathbf{V}_{A2} + \mathbf{V}_{A0} \qquad (13\text{-}8)$$

$$\mathbf{V}_A = 0.7500\angle 0° + 0.2500\angle 180° + 0.0\angle 0° = 0.500\angle 0°$$

$$\mathbf{V}_B = a^2\mathbf{V}_{A1} + a\mathbf{V}_{A2} + \mathbf{V}_{A0} \qquad (13\text{-}9)$$

$$\mathbf{V}_B = a^2\left(0.7500\angle 0°\right) + a\left(0.2500\angle 180°\right) + 0.0\angle 0° = 0.9014\angle -106.1°$$

$$\mathbf{V}_C = a\mathbf{V}_{A1} + a^2\mathbf{V}_{A2} + \mathbf{V}_{A0} \qquad (13\text{-}10)$$

$$\mathbf{V}_C = a\left(0.7500\angle 0°\right) + a^2\left(0.2500\angle 180°\right) + 0.0\angle 0° = 0.9014\angle 106.1°$$

The line-to-neutral base voltage at the low-voltage side of the transformer is 20 kV / $\sqrt{3}$ = 11.55 kV, so the actual phase voltages are:

$$\mathbf{V}_A = (0.500\angle 0°)(11.55 \text{ kV}) = 5.775\angle 0° \text{ kV}$$

$$\mathbf{V}_B = (0.9014\angle -106.1°)(11.55 \text{ kV}) = 10.41\angle -106.1° \text{ kV}$$

$$\mathbf{V}_C = (0.9014\angle 106.1°)(11.55 \text{ kV}) = 10.41\angle 106.1° \text{ kV}$$

(c) The current flowing in the fault during the transient period can be calculated directly from the sequence network shown above.

$$\mathbf{I}_{A1} = \mathbf{I}_{A2} = \mathbf{I}_{A0} = \frac{1.00\angle 0°}{(j0.30 + j0.10) + (j0.15 + j0.10) + (j0.10)} = 1.3333\angle -90°$$

The resulting *transient* fault current in amps is

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} + \mathbf{I}_{A0} \qquad (13\text{-}23)$$

$$\mathbf{I}_A = 1.3333\angle - 90° + 1.3333\angle - 90° + 1.3333\angle - 90° = 4.0000\angle - 90°$$

$$\mathbf{I}_B = a^2\mathbf{I}_{A1} + a\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-24}$$

$$\mathbf{I}_B = a^2\left(1.3333\angle - 90°\right) + a\left(1.3333\angle - 90°\right) + 1.3333\angle - 90° = 0.0\angle 0°$$

$$\mathbf{I}_C = a\mathbf{I}_{A1} + a^2\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-25}$$

$$\mathbf{I}_C = a\left(1.3333\angle - 90°\right) + a^2\left(1.3333\angle - 90°\right) + 1.3333\angle - 90° = 0.0\angle 0°$$

In amps, the transient fault current in phase a is

$$I_f' = (4.000)(836.7 \text{ A}) = 3347 \text{ A}$$

(d) The per-unit sequence voltages at the terminals of this generator during the transient period are given below. Note that the zero sequence current we need here is the zero sequence current *through the generator*, which is zero.

$$\mathbf{V}_{A1} = \mathbf{E}_{A1} - \mathbf{I}_{A1}Z_1 \tag{13-38}$$

$$\mathbf{V}_{A1} = 1.00\angle 0° - \left(1.3333\angle - 90°\right)\left(j0.30\right) = 0.6000\angle 0°$$

$$\mathbf{V}_{A2} = -\mathbf{I}_{A2}Z_2 \tag{13-39}$$

$$\mathbf{V}_{A2} = -\left(1.3333\angle - 90°\right)\left(j0.15\right) = 0.4000\angle 180°$$

$$\mathbf{V}_{A0} = -\mathbf{I}_{A0,g}Z_0 \tag{13-40}$$

$$\mathbf{V}_{A0} = 0.0\angle 0°$$

The resulting *transient* voltages at the terminals are

$$\mathbf{V}_A = \mathbf{V}_{A1} + \mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-8}$$

$$\mathbf{V}_A = 0.6000\angle 0° + 0.4000\angle 180° + 0.0\angle 0° = 0.2000\angle 0°$$

$$\mathbf{V}_B = a^2\mathbf{V}_{A1} + a\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-9}$$

$$\mathbf{V}_B = a^2\left(0.6000\angle 0°\right) + a\left(0.4000\angle 180°\right) + 0.0\angle 0° = 0.8718\angle - 96.6°$$

$$\mathbf{V}_C = a\mathbf{V}_{A1} + a^2\mathbf{V}_{A2} + \mathbf{V}_{A0} \tag{13-10}$$

$$\mathbf{V}_C = a\left(0.6000\angle 0°\right) + a^2\left(0.4000\angle 180°\right) + 0.0\angle 0° = 0.8718\angle 96.6°$$

The line-to-neutral base voltage at the low-voltage side of the transformer is 20 kV / $\sqrt{3}$ = 11.55 kV, so the actual phase voltages are:

$$\mathbf{V}_A = \left(0.2000\angle 0°\right)\left(11.55 \text{ kV}\right) = 2.310\angle 0° \text{ kV}$$

$$\mathbf{V}_B = \left(0.8718\angle - 96.6°\right)\left(11.55 \text{ kV}\right) = 10.07\angle - 96.6° \text{ kV}$$

$$\mathbf{V}_C = \left(0.8718\angle 96.6°\right)\left(11.55 \text{ kV}\right) = 10.07\angle 96.6° \text{ kV}$$

13-6. A simple three-phase power system is shown in Figure P13-1. Assume that the ratings of the various devices in this system are as follows:

Generator G_1: 250 MVA, 13.8 kV, $R = 0.1$ pu, $X'' = 0.18$ pu, $X' = 0.40$ pu,
$X_2 = 0.15$ pu, $X_{g0} = 0.10$ pu. This generator is grounded
through an impedance $Z_N = j0.20$ pu.

Generator G_2: 500 MVA, 20.0 kV, $R = 0.1$ pu, $X'' = 0.15$ pu, $X' = 0.35$ pu
$X_2 = 0.15$ pu, $X_{g0} = 0.10$ pu. This generator is grounded
through an impedance $Z_N = j0.20$ pu.

Generator G_3: 250 MVA, 13.8 kV, $R = 0.15$ pu, $X'' = 0.20$ pu, $X' = 0.40$ pu

$X_2 = 0.20$ pu, $X_{g0} = 0.15$ pu. This generator is grounded

through an impedance $Z_N = j0.25$ pu.

Transformer T_1: 250 MVA, 13.8-Δ/240-Y kV, $R = 0.01$ pu, $X_1 = X_2 = X_0 = 0.10$ pu

Transformer T_2: 500 MVA, 20.0-Δ/240-Y kV, $R = 0.01$ pu, $X_1 = X_2 = X_0 = 0.08$ pu

Transformer T_3: 250 MVA, 13.8-Δ/240-Y kV, $R = 0.01$ pu, $X_1 = X_2 = X_0 = 0.10$ pu

Each Line: $R = 8\ \Omega$, $X_1 = X_2 = 40\ \Omega$, $X_0 = 80\ \Omega$.

Assume that the power system is initially unloaded, and that the voltage at Bus 4 is 250 kV, and that *all resistances may be neglected*.

(a) Convert this power system to per-unit on a base of 500 MVA at 20 kV at generator G_2. Create the positive-, negative-, and zero-phase sequence diagrams.

(b) Calculate $\mathbf{Y}_{bus}$ and $\mathbf{Z}_{bus}$ for this power system using the generator subtransient reactances.

(c) Suppose that a three-phase symmetrical fault occurs at Bus 4. What is the subtransient fault current? What is the voltage on each bus in the power system? What is the subtransient current flowing in each of the three transmission lines?

(d) Suppose that a single-line-to-ground fault occurs at Bus 4. What is the subtransient fault current? What is the voltage on each bus in the power system? What is the subtransient current flowing in each of the three transmission lines?
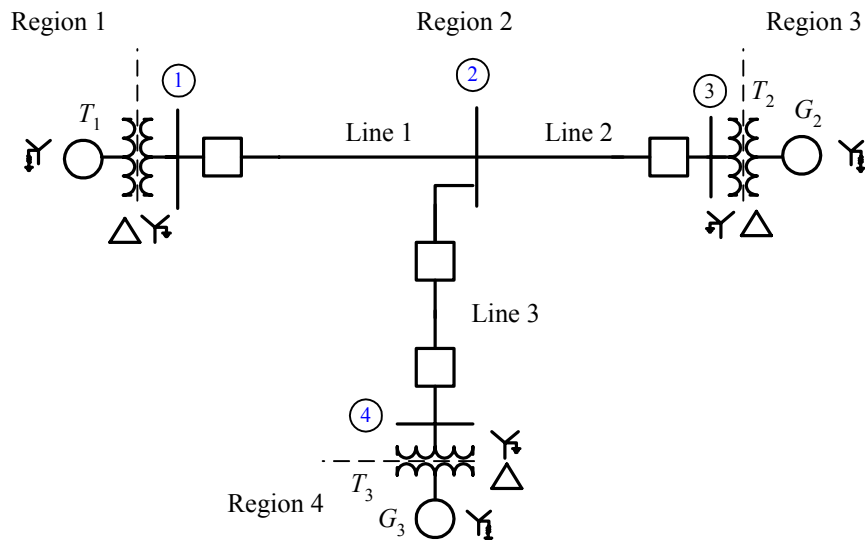
Figure P13-1   The simple power system of Problem 13-6.

SOLUTION The base quantities for this power system are 500 MVA at 20 kV at generator G_2, which is in Region 3. Therefore, the base quantities are:

$$S_{base,3} = 500 \text{ MVA}$$

$$V_{base,3} = 20 \text{ kV}$$

$$I_{base,3} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base\ 3}} = \frac{500,000,000 \text{ VA}}{\sqrt{3}(20,000 \text{ V})} = 14,430 \text{ A}$$

$$Z_{base,3} = \frac{\left(V_{LL,base\ 3}\right)^2}{S_{3\phi,base}} = \frac{(20,000 \text{ V})^2}{500,000,000 \text{ VA}} = 0.80\ \Omega$$

$$S_{base,2} = 500 \text{ MVA}$$

$$V_{base,2} = 20 \text{ kV} \left(\frac{240 \text{ kV}}{20 \text{ kV}} \right) = 240 \text{ kV}$$

$$I_{base,2} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base\ 2}} = \frac{500,000,000 \text{ VA}}{\sqrt{3}(240,000 \text{ V})} = 1203 \text{ A}$$

$$Z_{base,2} = \frac{\left(V_{LL,base\ 2}\right)^2}{S_{3\phi,base}} = \frac{(240,000 \text{ V})^2}{500,000,000 \text{ VA}} = 115.2 \text{ }\Omega$$

$$S_{base,1} = 500 \text{ MVA}$$

$$V_{base,1} = 240 \text{ kV} \left(\frac{13.8 \text{ kV}}{240 \text{ kV}} \right) = 13.8 \text{ kV}$$

$$I_{base,1} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base\ 1}} = \frac{500,000,000 \text{ VA}}{\sqrt{3}(13,800 \text{ V})} = 20,910 \text{ A}$$

$$Z_{base,1} = \frac{\left(V_{LL,base\ 1}\right)^2}{S_{3\phi,base}} = \frac{(13,800 \text{ V})^2}{500,000,000 \text{ VA}} = 0.381 \text{ }\Omega$$

$$S_{base,4} = 500 \text{ MVA}$$

$$V_{base,4} = 240 \text{ kV} \left(\frac{13.8 \text{ kV}}{240 \text{ kV}} \right) = 13.8 \text{ kV}$$

$$I_{base,4} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base\ 4}} = \frac{500,000,000 \text{ VA}}{\sqrt{3}(13,800 \text{ V})} = 20,910 \text{ A}$$

$$Z_{base,4} = \frac{\left(V_{LL,base\ 4}\right)^2}{S_{3\phi,base}} = \frac{(13,800 \text{ V})^2}{500,000,000 \text{ VA}} = 0.381 \text{ }\Omega$$

(a) The zero-sequence impedances of the three generators to their own bases are:

$$Z_{0,G1} = Z_{g0} + 3Z_N = j0.10 + 3(j0.20) = j0.70$$
$$Z_{0,G2} = Z_{g0} + 3Z_N = j0.10 + 3(j0.20) = j0.70$$
$$Z_{0,G3} = Z_{g0} + 3Z_N = j0.15 + 3(j0.25) = j0.90$$

The per-unit impedances expressed on the system base are:

$$\text{per-unit } Z_{new} = \text{per-unit } Z_{given} \left(\frac{V_{given}}{V_{new}} \right)^2 \left(\frac{S_{new}}{S_{given}} \right) \tag{10-8}$$

$$R_{G1} = (0.10 \text{ pu}) \left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}} \right)^2 \left(\frac{500 \text{ MVA}}{250 \text{ MVA}} \right) = 0.20 \text{ pu}$$

$$X''_{G1} = (0.18 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.36 \text{ pu}$$

$$X'_{G1} = (0.40 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.80 \text{ pu}$$

$$X_{2,G1} = (0.15 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.30 \text{ pu}$$

$$X_{0,G1} = (0.70 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 1.40 \text{ pu}$$

$$R_{G2} = (0.10 \text{ pu})\left(\frac{20.0 \text{ kV}}{20.0 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{500 \text{ MVA}}\right) = 0.10 \text{ pu}$$

$$X''_{G2} = (0.15 \text{ pu})\left(\frac{20.0 \text{ kV}}{20.0 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{500 \text{ MVA}}\right) = 0.15 \text{ pu}$$

$$X'_{G2} = (0.35 \text{ pu})\left(\frac{20.0 \text{ kV}}{20.0 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{500 \text{ MVA}}\right) = 0.35 \text{ pu}$$

$$X_{2,G2} = (0.15 \text{ pu})\left(\frac{20.0 \text{ kV}}{20.0 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{500 \text{ MVA}}\right) = 0.15 \text{ pu}$$

$$X_{0,G2} = (0.70 \text{ pu})\left(\frac{20.0 \text{ kV}}{20.0 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{500 \text{ MVA}}\right) = 0.70 \text{ pu}$$

$$R_{G3} = (0.15 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.30 \text{ pu}$$

$$X''_{G3} = (0.20 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.40 \text{ pu}$$

$$X'_{G3} = (0.40 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.80 \text{ pu}$$

$$X_{2,G3} = (0.20 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.40 \text{ pu}$$

$$X_{0,G3} = (0.90 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 1.80 \text{ pu}$$

$$R_{T1} = (0.01 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.02 \text{ pu}$$

$$X_{1,T1} = X_{2,T1} = X_{0,T1} = (0.10 \text{ pu})\left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{250 \text{ MVA}}\right) = 0.20 \text{ pu}$$

$$R_{T2} = (0.01 \text{ pu})\left(\frac{20.0 \text{ kV}}{20.0 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{500 \text{ MVA}}\right) = 0.01 \text{ pu}$$

$$X_{1,T2} = X_{2,T2} = X_{0,T2} = (0.08 \text{ pu})\left(\frac{20.0 \text{ kV}}{20.0 \text{ kV}}\right)^2\left(\frac{500 \text{ MVA}}{500 \text{ MVA}}\right) = 0.08 \text{ pu}$$

$$R_{T3} = (0.01 \text{ pu}) \left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}} \right)^2 \left(\frac{500 \text{ MVA}}{250 \text{ MVA}} \right) = 0.02 \text{ pu}$$

$$X_{1,T3} = X_{2,T3} = X_{0,T3} = (0.10 \text{ pu}) \left(\frac{13.8 \text{ kV}}{13.8 \text{ kV}} \right)^2 \left(\frac{500 \text{ MVA}}{250 \text{ MVA}} \right) = 0.20 \text{ pu}$$

$$R_{\text{line,pu}} = \frac{R_{\text{line},\Omega}}{Z_{\text{base}}} = \frac{8 \ \Omega}{115.2 \ \Omega} = 0.0694 \text{ pu}$$

$$X_{1,\text{line,pu}} = X_{2,\text{line,pu}} = \frac{X_{\text{line},\Omega}}{Z_{\text{base}}} = \frac{40 \ \Omega}{115.2 \ \Omega} = 0.3472 \text{ pu}$$

$$X_{0,\text{line,pu}} = \frac{X_{0,\text{line},\Omega}}{Z_{\text{base}}} = \frac{80 \ \Omega}{115.2 \ \Omega} = 0.6944 \text{ pu}$$

The assumption that the voltage at Bus 4 is 250 kV with an unloaded system means that the internal generated voltages of all generators are (250 kV/240 kV) = 1.042∠0° before the fault. The resulting positive, negative, and zero sequence diagrams of the power system are shown below. Note that resistances have been neglected, as specified in the problem.
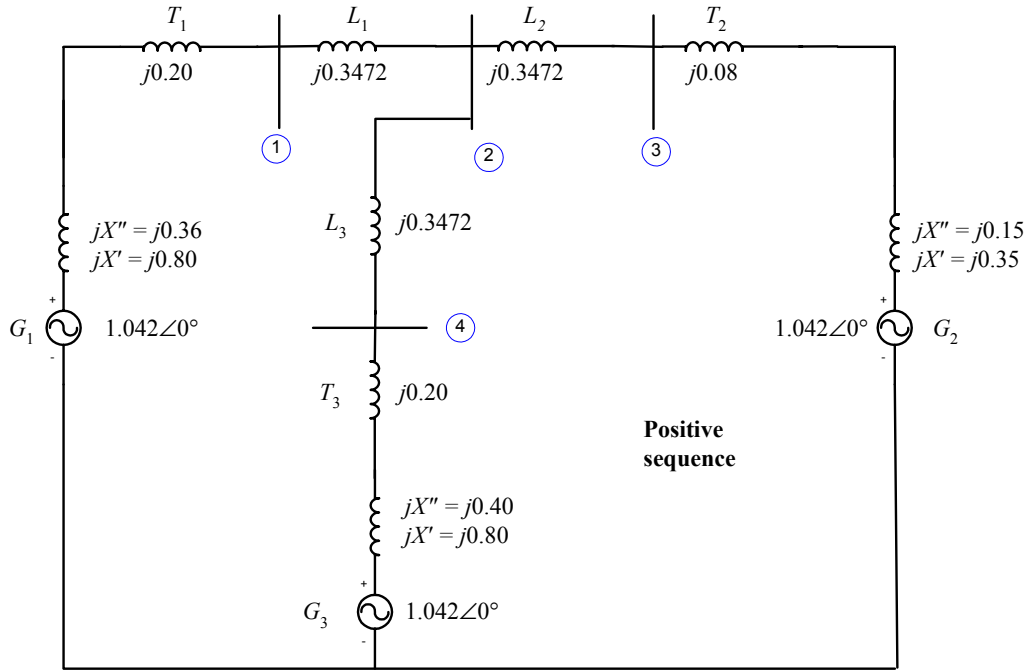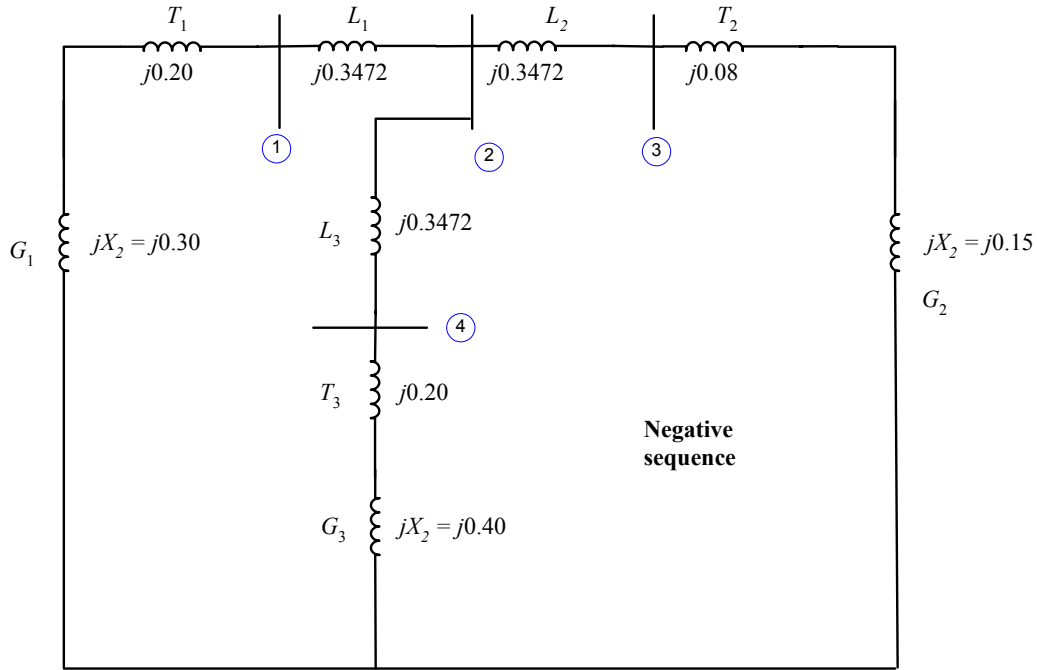
The zero sequence diagram shows the transformers connected to ground from the high-voltage side, and open circuited from the low-voltage side, because of the Δ-Y connections with grounded Y's.
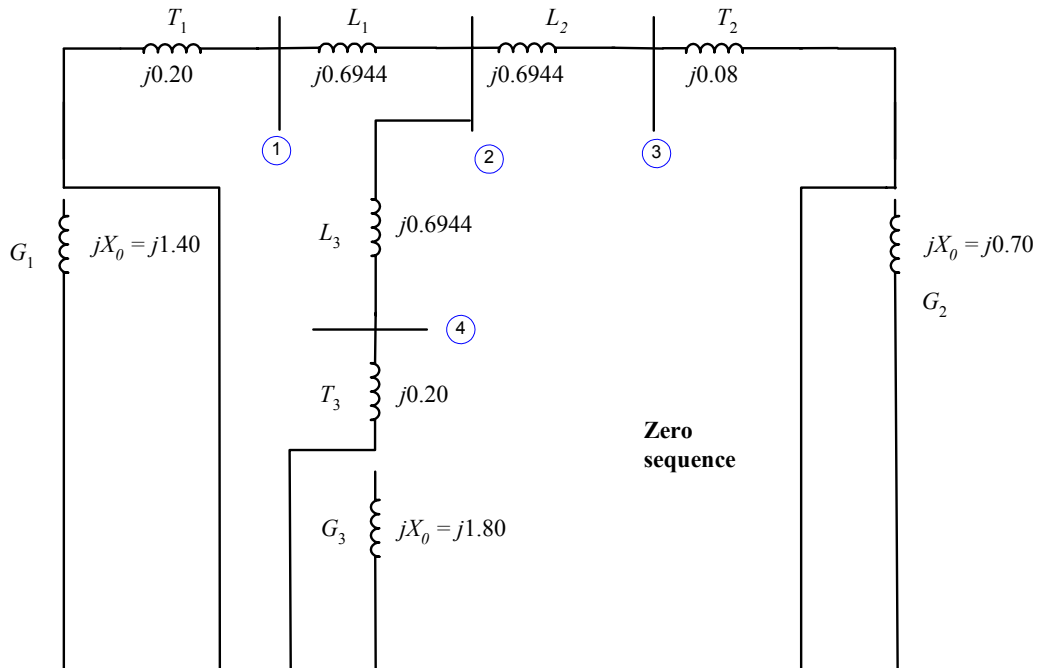
(b) The bus admittance matrices $\mathbf{Y}_{bus}$ can be calculated by converting each impedance to an admittance, and summing the admittances at each node for on-diagonal terms, while using the negative of the admittances between each node for the off-diagonal terms. The sequence diagrams with impedances converted to admittances are shown below:

Positive sequence

Negative sequence
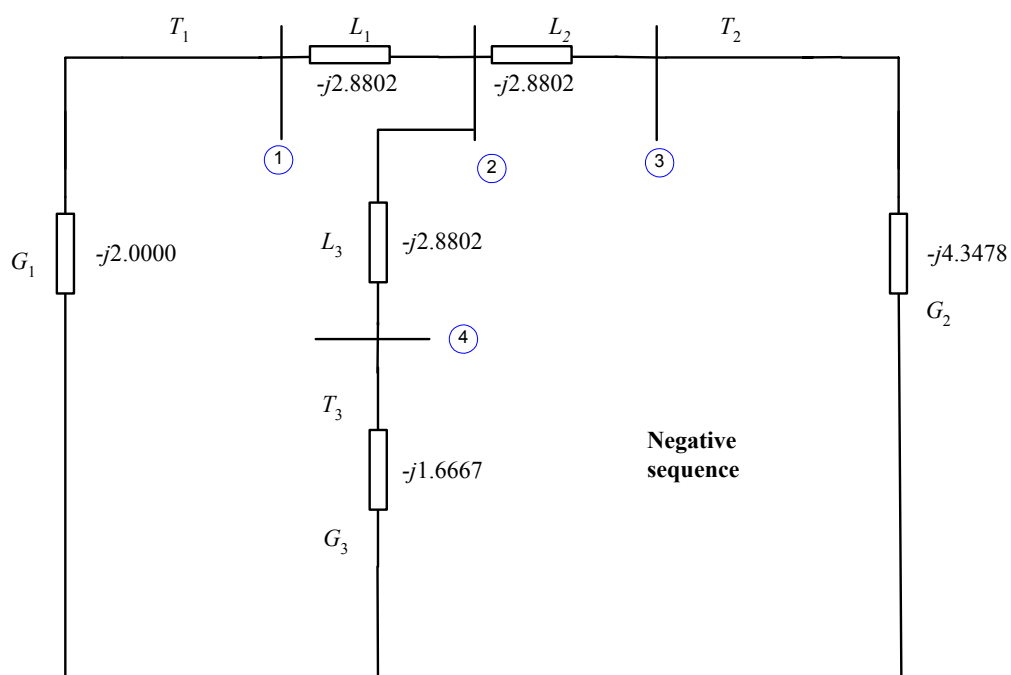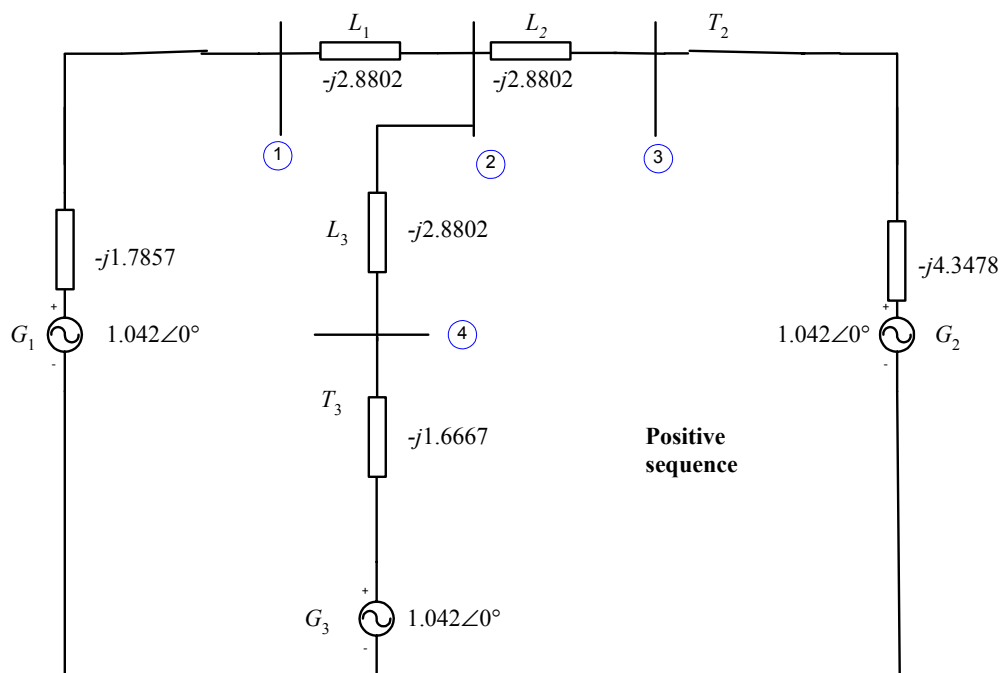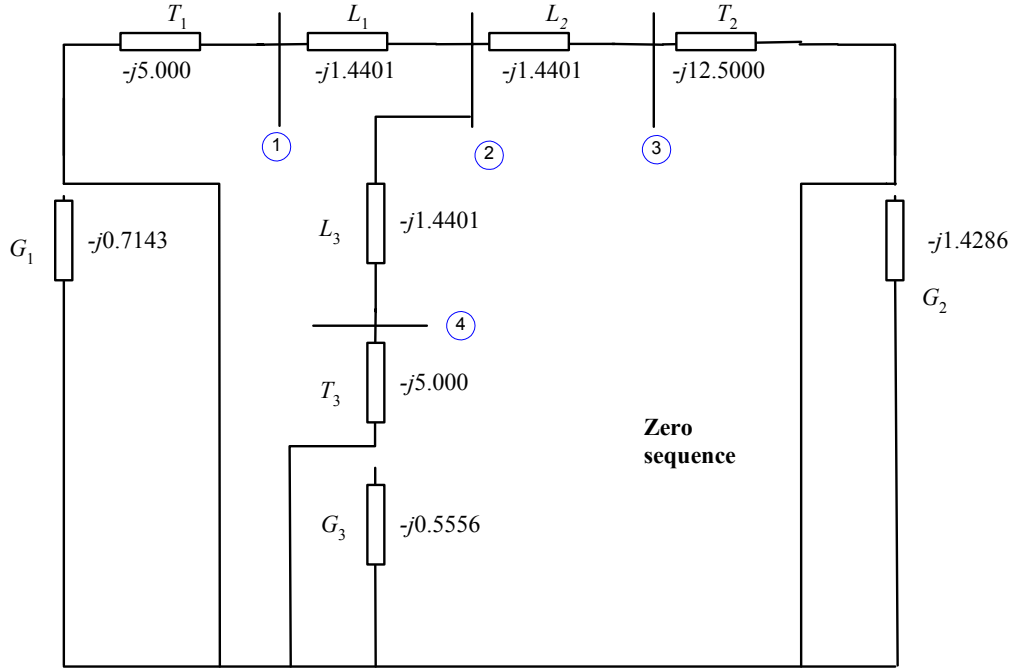
The resulting bus admittance matrices are:

$$\mathbf{Y}_{\text{bus},1} = \begin{bmatrix} -j4.6659 & j2.8802 & 0 & 0 \\ j2.8802 & -j8.6406 & j2.8802 & j2.8802 \\ 0 & j2.8802 & -j7.2280 & 0 \\ 0 & j2.8802 & 0 & -j4.5469 \end{bmatrix}$$

$$\mathbf{Y}_{\text{bus},2} = \begin{bmatrix} -j4.8802 & j2.8802 & 0 & 0 \\ j2.8802 & -j8.6406 & j2.8802 & j2.8802 \\ 0 & j2.8802 & -j7.2280 & 0 \\ 0 & j2.8802 & 0 & -j4.5469 \end{bmatrix}$$

$$\mathbf{Y}_{\text{bus},0} = \begin{bmatrix} -j6.4401 & j1.4401 & 0 & 0 \\ j1.4401 & -j4.3203 & j1.4401 & j1.4401 \\ 0 & j1.4401 & -j13.9401 & 0 \\ 0 & j1.4401 & 0 & -j6.4401 \end{bmatrix}$$

The corresponding bus impedance matrices are:

$$\mathbf{Z}_{\text{bus},1} = \mathbf{Y}_{\text{bus},1}{}^{-1} = \begin{bmatrix} j0.3123 & j0.1587 & j0.0632 & j0.1005 \\ j0.1587 & j0.2570 & j0.1024 & j0.1628 \\ j0.0632 & j0.1024 & j0.1792 & j0.0649 \\ j0.1005 & j0.1628 & j0.0649 & j0.3231 \end{bmatrix}$$

$$\mathbf{Z}_{\text{bus},2} = \mathbf{Y}_{\text{bus},2}{}^{-1} = \begin{bmatrix} j0.2927 & j0.1487 & j0.0593 & j0.0942 \\ j0.1487 & j0.2520 & j0.1004 & j0.1596 \\ j0.0593 & j0.1004 & j0.1784 & j0.0636 \\ j0.0942 & j0.1596 & j0.0636 & j0.3210 \end{bmatrix}$$

$$\mathbf{Z}_{bus,0} = \mathbf{Y}_{bus,0}^{-1} = \begin{bmatrix} j0.1695 & j0.0634 & j0.0065 & j0.0142 \\ j0.0634 & j0.2835 & j0.0293 & j0.0634 \\ j0.0065 & j0.0293 & j0.0748 & j0.0065 \\ j0.0142 & j0.0634 & j0.0065 & j0.1695 \end{bmatrix}$$

(c) If a symmetrical three-phase fault occurs at Bus 4, the subtransient fault current can be calculated using the positive sequence network only. It is:

$$\mathbf{I}_f'' = \frac{\mathbf{V}_f}{Z_{44}} = \frac{1.042\angle 0°}{j0.3231} = 3.225\angle - 90°$$

The actual fault current is

$$I_f'' = (3.225)(1203 \text{ A}) = 3880 \text{ A}$$

The voltages at each bus in the power system will be

$$\mathbf{V}_j = \left(1 - \frac{Z_{ji}}{Z_{ii}}\right)\mathbf{V}_f \qquad (12\text{-}23)$$

$$\mathbf{V}_1 = \left(1 - \frac{Z_{14}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.1007}{j0.3231}\right)(1.042\angle 0°) = 0.7172\angle 0°$$

$$\mathbf{V}_2 = \left(1 - \frac{Z_{24}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.1629}{j0.3231}\right)(1.042\angle 0°) = 0.5156\angle 0°$$

$$\mathbf{V}_3 = \left(1 - \frac{Z_{34}}{Z_{44}}\right)\mathbf{V}_f = \left(1 - \frac{j0.0649}{j0.3231}\right)(1.042\angle 0°) = 0.8327\angle 0°$$

$$\mathbf{V}_1 = \left(1 - \frac{Z_{44}}{Z_{44}}\right)\mathbf{V}_f = 0\angle 0°$$

The actual bus voltages are

$$V_1 = (0.7172)(240 \text{ kV}) = 172 \text{ kV}$$
$$V_2 = (0.5156)(240 \text{ kV}) = 124 \text{ kV}$$
$$V_3 = (0.8327)(240 \text{ kV}) = 200 \text{ kV}$$
$$V_4 = 0 \text{ kV}$$

The subtransient current in the transmission lines will be

Line 1: $\mathbf{I}_{12} = -Y_{12}(\mathbf{V}_1 - \mathbf{V}_2) = (-j2.880)(0.7172\angle 0° - 0.5156\angle 0°) = 0.581\angle - 90°$

Line 2: $\mathbf{I}_{23} = -Y_{23}(\mathbf{V}_2 - \mathbf{V}_3) = (-j2.880)(0.5156\angle 0° - 0.8327\angle 0°) = 0.913\angle 90°$

Line 3: $\mathbf{I}_{24} = -Y_{24}(\mathbf{V}_2 - \mathbf{V}_4) = (-j2.880)(0.5156\angle 0° - 0.0\angle 0°) = 1.485\angle - 90°$

The actual line currents are

$$I_{\text{line 1}} = (0.581)(1203 \text{ A}) = 699 \text{ A}$$
$$I_{\text{line 2}} = (0.913)(1203 \text{ A}) = 1099 \text{ A}$$
$$I_{\text{line 3}} = (1.485)(1203 \text{ A}) = 1787 \text{ A}$$

(d) If a single line to ground fault occurs at Bus 4, the subtransient fault current can be calculated using the all three sequence networks connected in series. The resulting current is:

$$\mathbf{I}_{A1} = \mathbf{I}_{A2} = \mathbf{I}_{A0} = \frac{\mathbf{V}_f}{Z_{44,1} + Z_{44,2} + Z_{44,0}} = \frac{1.00\angle 0°}{j0.3231 + j0.3210 + j0.1695} = 1.2291\angle -90°$$

The resulting *subtransient* fault current in amps is

$$\mathbf{I}_A = \mathbf{I}_{A1} + \mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-23}$$

$$\mathbf{I}_A = 1.2291\angle -90° + 1.2291\angle -90° + 1.2291\angle -90° = 3.6873\angle -90°$$

$$\mathbf{I}_B = a^2\mathbf{I}_{A1} + a\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-24}$$

$$\mathbf{I}_B = a^2\left(1.2291\angle -90°\right) + a\left(1.2291\angle -90°\right) + 1.2291\angle -90° = 0.0\angle 0°$$

$$\mathbf{I}_C = a\mathbf{I}_{A1} + a^2\mathbf{I}_{A2} + \mathbf{I}_{A0} \tag{13-25}$$

$$\mathbf{I}_C = a1.2291\angle -90° + a^2\left(1.2291\angle -90°\right) + 1.2291\angle -90° = 0.0\angle 0°$$

In amps, the transient fault current in phase *a* is

$$I''_f = \left(3.6873\right)\left(1203 \text{ A}\right) = 4436 \text{ A}$$

The voltages and line currents could be calculated by hand as we did in Problems 13-2 through 13-5, but that task would be very tedious for a system with four busses and three transmission lines. To speed up the effort, we will use now program `faults` to perform the calculations. We will treat the terminals of each generator as an additional bus, and the transformers as "transmission lines" visible from one or the other end in the zero-sequence diagrams.

```
% File describing the power system of Problem 13-6.  Note that
% we are treating the terminals of each generator as an
% additional "bus" and the transformers as additional "trans-
% mission lines".
%
% System data has the form:
%SYSTEM     name              baseMVA
SYSTEM  Prob13_6               500
%
% Bus data has the form:
%BUS name         volts
BUS One           1.00
BUS Two           1.00
BUS Three         1.00
BUS Four          1.00
BUS G1            1.00
BUS G2            1.00
BUS G3            1.00
%
% Transmission line data has the form:
%LINE    from         to       Rse      Xse      Gsh      Bsh      X0     Vis
LINE    One          Two       0.0000   0.3472   0.000    0.000    0.6944  3
LINE    Two          Three     0.0000   0.3472   0.000    0.000    0.6944  3
LINE    Two          Four      0.0000   0.3472   0.000    0.000    0.6944  3

% The following threee "lines" are really transformers.  Note that
% the zero-sequence impedances are "visible" from only one end of
% the lines.
LINE    G1           One       0.0000   0.2000   0.000    0.000    0.2000  2
```

```
LINE    Three   G2              0.0000  0.0800  0.000  0.000  0.0800  1
LINE    Four    G3              0.0000  0.2000  0.000  0.000  0.2000  1
%
% Generator data has the form:
%GENERATOR  bus        R       Xs      Xp     Xpp     X2     X0
GENERATOR   G1        0.00    2.00    0.80    0.36    0.30   1.40
GENERATOR   G2        0.00    1.20    0.35    0.15    0.15   0.70
GENERATOR   G3        0.00    2.00    0.80    0.40    0.40   1.80
%
% type data has the form:
%FAULT    bus       Calc Type   Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT     Four      SLG            1
```

The resulting outputs are shown below. Note that the answer for the fault current at Bus 4 agrees with the hand calculations above.

```
>> faults prob_13_6_fault
Input summary statistics:
Input summary statistics:
  39 lines in system file
   1 SYSTEM lines
   7 BUS lines
   6 LINE lines
   3 GENERATOR lines
   0 MOTOR lines
   1 TYPE lines


                        Results for Case Prob13_6


Single Line-to-Ground Fault at Bus Four
Calculating Subtransient Currents
|=====================Bus Information=============|======Line Information=====|
Bus           P   Volts / angle   Amps / angle  |   To  | P  Amps / angle   |
no.    Name   h   (pu)    (deg)    (pu)   (deg)  |  Bus  | h  (pu)    (deg)   |
|==========================================================================|
  1      One a  0.743/   0.00   0.000/   0.00
             b  0.947/-114.84   0.000/   0.00
             c  0.947/ 114.84   0.000/   0.00
                                                  Two   a 0.539/ -90.00
                                                        b 0.139/  86.09
                                                        c 0.139/  93.91
                                                  G1    a 0.452/  90.00
                                                        b 0.226/ -92.41
                                                        c 0.226/ -87.59
  2      Two a  0.526/   0.00   0.000/   0.00
             b  0.943/-113.76   0.000/   0.00
             c  0.943/ 113.76   0.000/   0.00
                                                  One   a 0.539/  90.00
                                                        b 0.139/ -93.91
                                                        c 0.139/ -86.09
                                                  Three a 0.787/  90.00
                                                        b 0.243/ -88.61
                                                        c 0.243/ -91.39
                                                  Four  a 1.327/ -90.00
                                                        b 0.382/  89.46
                                                        c 0.382/  90.54
```

```
3       Three a   0.834/    0.00   0.000/    0.00
              b   0.965/-116.39   0.000/    0.00
              c   0.965/ 116.39   0.000/    0.00
                                                Two    a 0.787/  -90.00
                                                       b 0.243/   91.39
                                                       c 0.243/   88.61
                                                G2     a 0.687/   90.00
                                                       b 0.343/  -89.01
                                                       c 0.343/  -90.99
4       Four  a   0.000/    0.00   3.687/  -90.00
              b   0.919/-109.88   0.000/    0.00
              c   0.919/ 109.88   0.000/    0.00
                                                Two    a 1.327/   90.00
                                                       b 0.382/  -90.54
                                                       c 0.382/  -89.46
                                                G3     a 1.320/   90.00
                                                       b 0.660/  -89.69
                                                       c 0.660/  -90.31
5       G1    a   0.851/    0.00   0.000/    0.00
              b   0.957/-116.40   0.000/    0.00
              c   0.957/ 116.40   0.000/    0.00
                                                One    a 0.452/  -90.00
                                                       b 0.226/   87.59
                                                       c 0.226/   92.41
6       G2    a   0.897/    0.00   0.000/    0.00
              b   0.974/-117.40   0.000/    0.00
              c   0.974/ 117.40   0.000/    0.00
                                                Three  a 0.687/  -90.00
                                                       b 0.343/   90.99
                                                       c 0.343/   89.01
7       G3    a   0.472/    0.00   0.000/    0.00
              b   0.896/-105.27   0.000/    0.00
              c   0.896/ 105.27   0.000/    0.00
                                                Four   a 1.320/  -90.00
                                                       b 0.660/   90.31
                                                       c 0.660/   89.69
|=============================================================================|
```

The actual phase voltages at each bus in the power system will be

$$\mathbf{V}_{1,A} = \mathbf{V}_{1,A,\text{pu}} V_{\text{base},2} = (0.743\angle 0°)(240\text{ kV}) = 178.3\angle 0°\text{ kV}$$

$$\mathbf{V}_{1,B} = (0.947\angle -114.8°)(240\text{ kV}) = 227.3\angle -114.8°\text{ kV}$$

$$\mathbf{V}_{1,C} = (0.947\angle 114.8°)(240\text{ kV}) = 227.3\angle 114.8°\text{ kV}$$

$$\mathbf{V}_{2,A} = (0.526\angle 0°)(240\text{ kV}) = 126.2\angle 0°\text{ kV}$$

$$\mathbf{V}_{2,B} = (0.943\angle -113.8°)(240\text{ kV}) = 226.3\angle -113.8°\text{ kV}$$

$$\mathbf{V}_{2,C} = (0.943\angle 113.8°)(240\text{ kV}) = 226.3\angle 113.8°\text{ kV}$$

$$\mathbf{V}_{3,A} = (0.834\angle 0°)(240\text{ kV}) = 200.2\angle 0°\text{ kV}$$

$$\mathbf{V}_{3,B} = (0.965\angle -116.4°)(240\text{ kV}) = 231.6\angle -116.4°\text{ kV}$$

$$\mathbf{V}_{3,B} = (0.965\angle 116.4°)(240\text{ kV}) = 231.6\angle 116.4°\text{ kV}$$

$$\mathbf{V}_{4,A} = (0.000\angle 0°)(240\text{ kV}) = 0\angle 0°\text{ kV}$$

$$\mathbf{V}_{4,B} = (0.919\angle -109.9°)(240 \text{ kV}) = 220.6\angle -109.9° \text{ kV}$$

$$\mathbf{V}_{4,C} = (0.919\angle 109.9°)(240 \text{ kV}) = 220.6\angle 109.9° \text{ kV}$$

The subtransient currents in each phase of each transmission line will be

Line 1: $\mathbf{I}_{12,A} = \mathbf{I}_{12,A,\text{pu}} I_{\text{base},2} = (0.539\angle -90°)(1203 \text{ A}) = 648\angle -90° \text{ A}$

Line 1: $\mathbf{I}_{12,B} = (0.139\angle 86.1°)(1203 \text{ A}) = 155\angle 86.1° \text{ A}$

Line 1: $\mathbf{I}_{12,C} = (0.139\angle 93.9°)(1203 \text{ A}) = 155\angle 93.9° \text{ A}$

Line 2: $\mathbf{I}_{32,A} = (0.787\angle -90°)(1203 \text{ A}) = 947\angle -90° \text{ A}$

Line 2: $\mathbf{I}_{32,B} = (0.243\angle 91.4°)(1203 \text{ A}) = 292\angle 91.4° \text{ A}$

Line 2: $\mathbf{I}_{32,C} = (0.243\angle 88.6°)(1203 \text{ A}) = 292\angle 88.6° \text{ A}$

Line 3: $\mathbf{I}_{24,A} = (1.327\angle -90°)(1203 \text{ A}) = 1596\angle -90° \text{ A}$

Line 3: $\mathbf{I}_{24,B} = (0.382\angle 89.5°)(1203 \text{ A}) = 460\angle 89.5° \text{ A}$

Line 3: $\mathbf{I}_{24,C} = (0.382\angle 90.5°)(1203 \text{ A}) = 460\angle 90.5° \text{ A}$

The actual line currents are

$$I_{\text{line 1}} = (0.581)(1203 \text{ A}) = 699 \text{ A}$$

$$I_{\text{line 2}} = (0.913)(1203 \text{ A}) = 1099 \text{ A}$$

$$I_{\text{line 3}} = (1.485)(1203 \text{ A}) = 1787 \text{ A}$$

13-7. For the power system of Problem 13-6, calculate the subtransient fault current for a single-line-to-ground fault at Bus 4 *if the neutrals of the three generators are all solidly grounded*. How much difference did the inclusion of the impedances to ground in the generator neutrals make to the total fault current? Why would a power company wish to ground the neutrals of their generators through an impedance?

SOLUTION If the neutrals of the generators are solidly grounded, then the zero-sequence impedance of each generator will be much smaller. However, *this will have no effect on the current in a single-line-to-ground fault at Bus 4*, because the connections of the Y-Δ transformers effectively decouple the generators from Bus 4 in the zero-sequence network.

However, if the power system were configured differently so that the generators were connected to Bus 4 in the zero sequence network, the presence of the impedance in the ground line would reduce the current flowing in a single-line-to-ground or double-line-to-ground fault. Under normal conditions, the current flowing in the ground of a generator is very small, because the loads on the generator are nearly balanced. Therefore, an impedance in the ground line has little or no effect on steady-state operations, while reducing fault currents for single-line-to-ground or double-line-to-ground faults. Since single-line-to-ground faults are the most common kind, it makes sense for a power company to include an impedance in the grounds of its generators.

13-8. Assume that a fault occurs on the high-voltage side of transformer T_2 in the power system shown in Figure P13-2. Make the assumption that the generator is operating at rated voltage, and that the power system is initially unloaded.
  (a) Calculate the subtransient fault current, generator current, and motor current for a symmetrical three-phase fault.
  (b) Calculate the subtransient fault current, generator current, and motor current for a single-line-to-ground fault.
  (c) How will the answers in part *(b)* change if the Y-connection of transformer T_2 is solidly grounded?
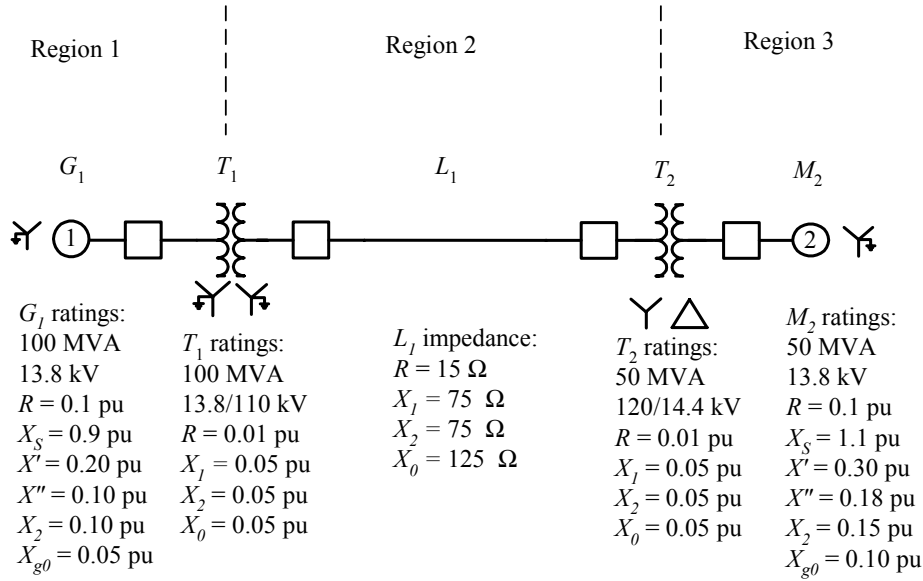
Figure P13-2   One-line diagram of the power system in Problem 13-8.

SOLUTION   To simplify this problem, we will pick the base quantities for this power system to be 100 MVA and 13.8 kV at generator G_1, which is in Region 1. Therefore, the base quantities are:

$$S_{\text{base},1} = 100 \text{ MVA}$$

$$V_{\text{base},1} = 13.8 \text{ kV}$$

$$I_{\text{base},1} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base }1}} = \frac{100,000,000 \text{ VA}}{\sqrt{3}(13,800 \text{ V})} = 4184 \text{ A}$$

$$Z_{\text{base},1} = \frac{\left(V_{LL,\text{base }1}\right)^2}{S_{3\phi,\text{base}}} = \frac{(13,800 \text{ V})^2}{100,000,000 \text{ VA}} = 1.904 \text{ }\Omega$$

$$S_{\text{base},2} = 100 \text{ MVA}$$

$$V_{\text{base},2} = 13.8 \text{ kV}\left(\frac{110 \text{ kV}}{13.8 \text{ kV}}\right) = 110 \text{ kV}$$

$$I_{\text{base},2} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base }2}} = \frac{100,000,000 \text{ VA}}{\sqrt{3}(110,000 \text{ V})} = 524.9 \text{ A}$$

$$Z_{\text{base},2} = \frac{\left(V_{LL,\text{base }2}\right)^2}{S_{3\phi,\text{base}}} = \frac{(110,000 \text{ V})^2}{100,000,000 \text{ VA}} = 121 \text{ }\Omega$$

$$S_{\text{base},3} = 100 \text{ MVA}$$

$$V_{\text{base},3} = 110 \text{ kV}\left(\frac{14.4 \text{ kV}}{120 \text{ kV}}\right) = 13.2 \text{ kV}$$

$$I_{\text{base},3} = \frac{S_{3\phi,\text{base}}}{\sqrt{3}V_{LL,\text{base }3}} = \frac{100,000,000 \text{ VA}}{\sqrt{3}(13,200 \text{ V})} = 4374 \text{ A}$$

$$Z_{\text{base},3} = \frac{\left(V_{LL,\text{base }3}\right)^2}{S_{3\phi,\text{base}}} = \frac{\left(13,200 \text{ V}\right)^2}{100,000,000 \text{ VA}} = 1.742 \text{ }\Omega$$

The per-unit impedances expressed on the system base are:

$$\text{per-unit } Z_{\text{new}} = \text{per-unit } Z_{\text{given}}\left(\frac{V_{\text{given}}}{V_{\text{new}}}\right)^2\left(\frac{S_{\text{new}}}{S_{\text{given}}}\right) \tag{10-8}$$

$R_{G1} = 0.10 \text{ pu}$

$X''_{G1} = 0.10 \text{ pu}$

$X'_{G1} = 0.20 \text{ pu}$

$X_{S,G1} = 0.90 \text{ pu}$

$X_{2,G1} = 0.10 \text{ pu}$

$X_{g0,G1} = 0.05 \text{ pu}$

$R_{T1} = 0.01 \text{ pu}$

$X_{1,T1} = 0.05 \text{ pu}$

$X_{2,T1} = 0.05 \text{ pu}$

$X_{0,T1} = 0.05 \text{ pu}$

$$R_{T2} = \left(0.01 \text{ pu}\right)\left(\frac{120 \text{ kV}}{110 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.0238 \text{ pu}$$

$$X_{1,T2} = \left(0.05 \text{ pu}\right)\left(\frac{120 \text{ kV}}{110 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.1190 \text{ pu}$$

$$X_{2,T2} = \left(0.05 \text{ pu}\right)\left(\frac{120 \text{ kV}}{110 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.1190 \text{ pu}$$

$$X_{0,T2} = \left(0.05 \text{ pu}\right)\left(\frac{120 \text{ kV}}{110 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.1190 \text{ pu}$$

$$R_{M2} = \left(0.10 \text{ pu}\right)\left(\frac{13.8 \text{ kV}}{13.2 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.2186 \text{ pu}$$

$$X''_{M2} = \left(0.18 \text{ pu}\right)\left(\frac{13.8 \text{ kV}}{13.2 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.3935 \text{ pu}$$

$$X'_{M2} = \left(0.30 \text{ pu}\right)\left(\frac{13.8 \text{ kV}}{13.2 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.6558 \text{ pu}$$

$$X_{S,M2} = \left(1.10 \text{ pu}\right)\left(\frac{13.8 \text{ kV}}{13.2 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 2.4045 \text{ pu}$$

$$X_{2,M2} = \left(0.15 \text{ pu}\right)\left(\frac{13.8 \text{ kV}}{13.2 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.3279 \text{ pu}$$

$$X_{g0,M2} = \left(0.10 \text{ pu}\right)\left(\frac{13.8 \text{ kV}}{13.2 \text{ kV}}\right)^2\left(\frac{100 \text{ MVA}}{50 \text{ MVA}}\right) = 0.2186 \text{ pu}$$

$$R_{line,pu} = \frac{R_{line,\Omega}}{Z_{base}} = \frac{15\ \Omega}{121\ \Omega} = 0.124\ \text{pu}$$

$$X_{1,line,pu} = X_{2,line,pu} = \frac{X_{1,line,\Omega}}{Z_{base}} = \frac{75\ \Omega}{121\ \Omega} = 0.6198\ \text{pu}$$

$$X_{0,line,pu} = \frac{X_{0,line,\Omega}}{Z_{base}} = \frac{125\ \Omega}{121\ \Omega} = 1.0331\ \text{pu}$$

The assumption that the system is initially unloaded means that the internal generated voltages of the generator and motor are $= 1.00\angle 0°$ before the fault. The resulting per-unit sequence diagrams of the power system is shown below. Note that the effect of transformer T_2 on the zero sequence diagram.
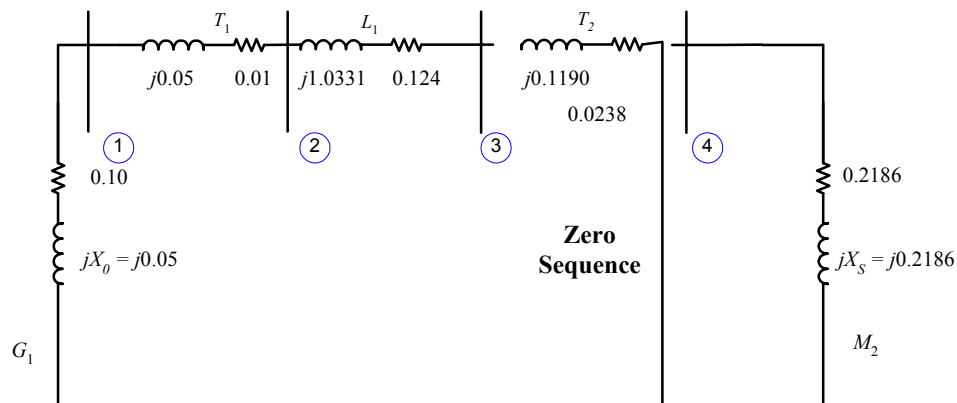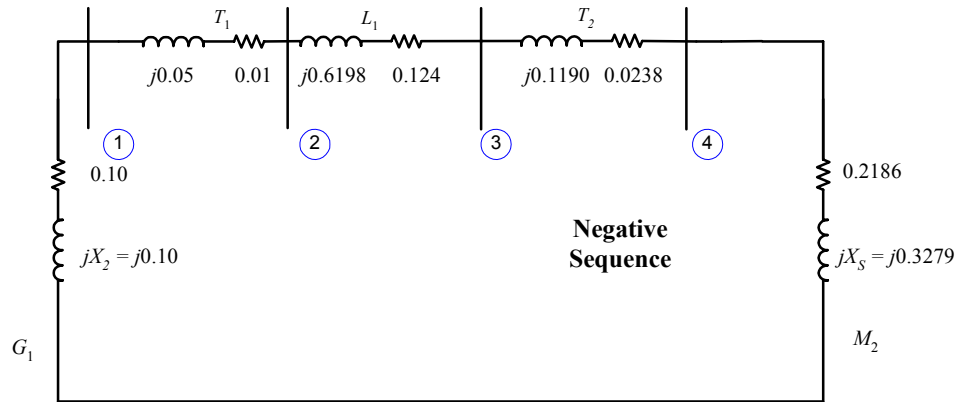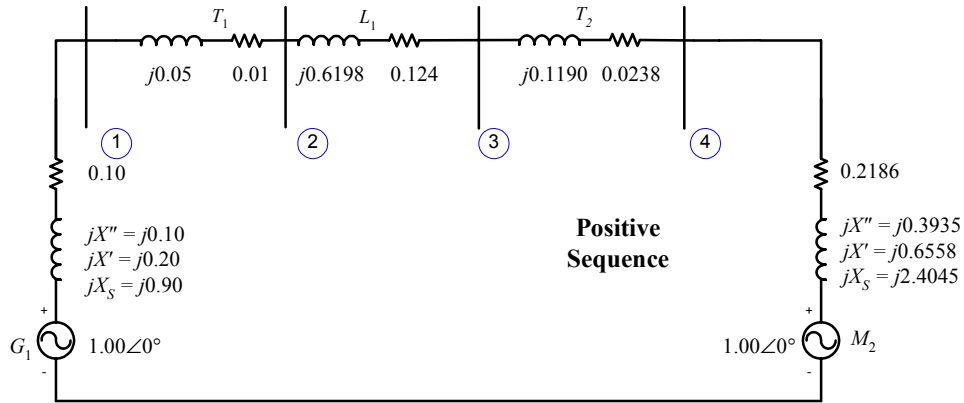
(a) Note that the fault occurs at "Bus 3", which is the high-voltage side of transformer T_2. For the symmetrical three phase fault, we only need to short the positive sequence network at Bus 3 and calculate the current flows. The input file required for program `faults` to calculate the fault current is shown below.

```
% File describing the power system of Problem 13-8a, including
% resistances.
%
% System data has the form:
%SYSTEM     name              baseMVA
SYSTEM  Prob13_8a           100
%
% Bus data has the form:
%BUS name        volts
BUS One          1.00
BUS Two          1.00
BUS Three        1.00
BUS Four         1.00
%
% Note that transformers T1 and T2 are treated as "transmission lines"
% here.  Transmission line data has the form:
%LINE     from        to        Rse      Xse      Gsh      Bsh      X0      Vis
LINE    One        Two        0.0100   0.0500   0.000    0.000    0.0500   3
LINE    Two        Three      0.1240   0.6198   0.000    0.000    1.0331   3
LINE    Three      Four       0.0238   0.1190   0.000    0.000    0.1190   0
%
% Generator data has the form:
%GENERATOR  bus        R        Xs        Xp        Xpp       X2        X0
GENERATOR   One     0.10      0.90      0.20      0.10      0.10      0.05
%
% Motor data has the form:
%MOTOR       bus        R        Xs        Xp        Xpp       X2        X0
MOTOR       Four   0.2186   2.4045  0.6558  0.3935  0.3279  0.2186
%
% type data has the form:
%FAULT     bus      Calc Type     Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT     Three        3P               1
```

The resulting outputs are shown below.

```
>> faults prob_13_8a_fault
Input summary statistics:
  32 lines in system file
   1 SYSTEM lines
   4 BUS lines
   3 LINE lines
   1 GENERATOR lines
   1 MOTOR lines
   1 TYPE lines


                      Results for Case Prob13_8a


Symmetrical Three-Phase Fault at Bus Three
Calculating Subtransient Currents
|====================Bus Information============|=====Line Information=====|
Bus              Volts / angle    Amps / angle  |    To  |  Amps / angle  |
```

no.	Name	(pu)	(deg)	(pu)	(deg)		Bus		(pu)	(deg)	
=====	======	======	=======	======	=======	===	======	===	======	=======	===
1	One	0.849/	5.59	0.000/	0.00		Two		**1.243/**	**-73.09**	
2	Two	0.786/	5.59	0.000/	0.00		One		1.243/	106.91	
							Three		1.243/	-73.09	
3	Three	0.000/	0.00	**2.999/**	**-68.16**		Two		1.243/	106.91	
							Four		1.764/	115.31	
4	Four	0.214/	14.00	0.000/	0.00		Three		**1.764/**	**-64.69**	
=====	======	======	=======	======	=======	===	======	===	======	=======	===

The subtransient fault current is given in per-unit above. The actual fault current is found by multiplying by the base current in Region 2:

The subtransient, transient, and steady-state fault currents are given in per-unit above. The actual fault currents are found by multiplying by the base current in Region 2:

$$\mathbf{I}''_f = \left(2.999\angle - 68.2°\right)\left(524.9 \text{ A}\right) = 1574\angle - 68.2° \text{ A}$$

The subtransient generator current will be the same as the current flowing from Bus 1 to Bus 2. This current is given in per-unit above. The actual generator current is found by multiplying by the base current in Region 1:

$$\mathbf{I}''_{G1} = \left(1.243\angle - 73.1°\right)\left(4184 \text{ A}\right) = 5201\angle - 73.1° \text{ A}$$

The subtransient motor currents will be the same as the current flowing from Bus 4 to Bus 3. This current is given in per-unit above. The actual motor current is found by multiplying by the base current in Region 3:

$$\mathbf{I}''_{M2} = \left(1.764\angle - 64.7°\right)\left(4374 \text{ A}\right) = 7716\angle - 64.7° \text{ A}$$

(b) For a single-line-to-ground fault, we need to connect the three sequence networks in series at Bus 3, and calculate the resulting current flows. The input file required for program `faults` to calculate the results of a single-line-to-ground fault at "Bus 3" is shown below.

```
% File describing the power system of Problem 13-8b, including
% resistances.
%
% System data has the form:
%SYSTEM     name               baseMVA
SYSTEM  Prob13_8b             100
%
% Bus data has the form:
%BUS name         volts
BUS One            1.00
BUS Two            1.00
BUS Three          1.00
BUS Four           1.00
%
% Note that transformers T1 and T2 are treated as "transmission lines"
% here.  Transmission line data has the form:
%LINE     from          to        Rse     Xse     Gsh     Bsh      X0    Vis
LINE    One         Two         0.0100  0.0500  0.000   0.000   0.0500  3
LINE    Two         Three       0.1240  0.6198  0.000   0.000   1.0331  3
LINE    Three       Four        0.0238  0.1190  0.000   0.000   0.1190  0
%
% Generator data has the form:
%GENERATOR  bus       R       Xs       Xp       Xpp    X2      X0
GENERATOR   One       0.10    0.90     0.20     0.10   0.10    0.05
```

```
%
% Motor data has the form:
%MOTOR        bus     R      Xs     Xp     Xpp    X2     X0
MOTOR        Four   0.2186 2.4045 0.6558 0.3935 0.3279 0.2186
%
% type data has the form:
%FAULT   bus     Calc Type   Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT    Three      SLG            1
```

The resulting outputs are shown below.

```
>> faults prob_13_8b_fault
Input summary statistics:
  32 lines in system file
   1 SYSTEM lines
   4 BUS lines
   3 LINE lines
   1 GENERATOR lines
   1 MOTOR lines
   1 TYPE lines


                        Results for Case Prob13_8b


Single Line-to-Ground Fault at Bus Three
Calculating Subtransient Currents
|====================Bus Information============|======Line Information=====|
Bus           P  Volts / angle   Amps / angle  |  To  | P  Amps / angle  |
no.    Name   h  (pu)   (deg)     (pu)  (deg)   | Bus  | h  (pu)   (deg)  |
|==========================================================================|
  1     One  a  0.915/   5.16    0.000/   0.00
             b  0.970/-122.15    0.000/   0.00
             c  1.033/ 120.76    0.000/   0.00
                                                 Two   a 0.986/ -77.25
                                                       b 0.411/ -84.28
                                                       c 0.417/ -58.76
  2     Two  a  0.864/   5.38    0.000/   0.00
             b  0.980/-123.25    0.000/   0.00
             c  1.038/ 121.91    0.000/   0.00
                                                 One   a 0.986/ 102.75
                                                       b 0.411/  95.72
                                                       c 0.417/ 121.24
                                                 Three a 0.986/ -77.25
                                                       b 0.411/ -84.28
                                                       c 0.417/ -58.76
  3    Three a  0.000/ -29.36    1.791/ -74.63
             b  1.344/-140.27    0.000/   0.00
             c  1.263/ 144.95    0.000/   0.00
                                                 Two   a 0.986/ 102.75
                                                       b 0.411/  95.72
                                                       c 0.417/ 121.24
                                                 Four  a 0.808/ 108.57
                                                       b 0.411/ -84.28
                                                       c 0.417/ -58.76
  4    Four  a  0.789/   4.14    0.000/   0.00
             b  0.901/-115.95    0.000/   0.00
             c  0.849/ 117.54    0.000/   0.00
                                                 Three a 0.808/ -71.43
```

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|==|

The subtransient, transient, and steady-state fault currents are given in per-unit above. The actual fault currents are found by multiplying by the base current in Region 2:

$$\mathbf{I}_{f,A} = \mathbf{I}_{f,A,\text{pu}} I_{\text{base},2} = \left(1.791\angle - 74.6°\right)\left(524.9 \text{ A}\right) = 940\angle - 74.6° \text{ A}$$

$$\mathbf{I}_{f,B} = \mathbf{I}_{f,B,\text{pu}} I_{\text{base},2} = 0\angle 0.0° \text{ A}$$

$$\mathbf{I}_{f,C} = \mathbf{I}_{f,C,\text{pu}} I_{\text{base},2} = 0\angle 0.0° \text{ A}$$

The subtransient generator current will be the same as the current flowing from Bus 1 to Bus 2. This current is given in per-unit above. The actual generator current is found by multiplying by the base current in Region 1:

$$\mathbf{I}_{12,A} = \mathbf{I}_{12,A,\text{pu}} I_{\text{base},2} = \left(0.986\angle - 77.3°\right)\left(4184 \text{ A}\right) = 4125\angle - 77.3° \text{ A}$$

$$\mathbf{I}_{12,B} = \mathbf{I}_{12,B,\text{pu}} I_{\text{base},2} = \left(0.411\angle - 84.3°\right)\left(4184 \text{ A}\right) = 1720\angle - 84.3° \text{ A}$$

$$\mathbf{I}_{12,C} = \mathbf{I}_{12,C,\text{pu}} I_{\text{base},2} = \left(0.417\angle - 58.8°\right)\left(4184 \text{ A}\right) = 1745\angle - 58.8° \text{ A}$$

The subtransient motor current will be the same as the current flowing from Bus 4 to Bus 3. This current is given in per-unit above. The actual motor current is found by multiplying by the base current in Region 3:

$$\mathbf{I}_{43,A} = \mathbf{I}_{43,A,\text{pu}} I_{\text{base},2} = \left(0.808\angle - 71.4°\right)\left(4374 \text{ A}\right) = 3534\angle - 71.4° \text{ A}$$

$$\mathbf{I}_{43,B} = \mathbf{I}_{43,B,\text{pu}} I_{\text{base},2} = \left(0.411\angle 95.7°\right)\left(4374 \text{ A}\right) = 1798\angle 95.7° \text{ A}$$

$$\mathbf{I}_{43,C} = \mathbf{I}_{43,C,\text{pu}} I_{\text{base},2} = \left(0.417\angle 121.2°\right)\left(4374 \text{ A}\right) = 1824\angle 121.2° \text{ A}$$

(c) If the Y-connection of transformer T_2 is solidly grounded, there is a zero-sequence connection through transformer T_2 on the high-voltage side of the transformer. The new zero-sequence network is shown below:
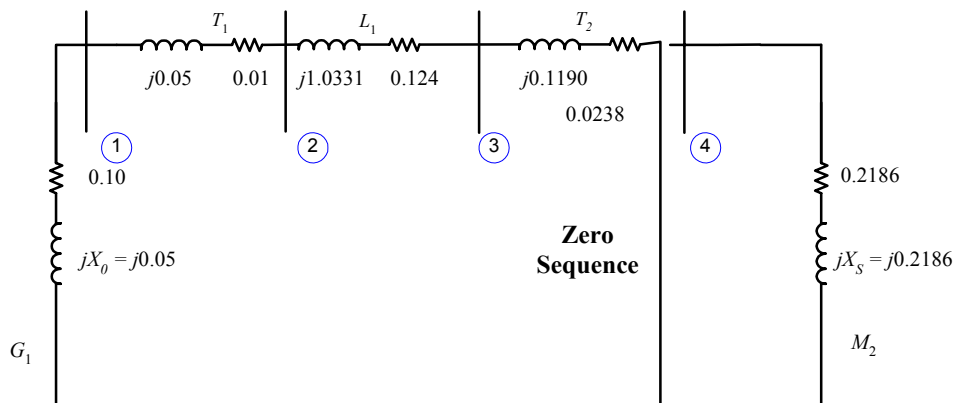
This extra connection will result in a substantial increase in fault current for a single-line-to-ground fault. The modified input file required for program `faults` to calculate the results of a single-line-to-ground fault at "Bus 3" is shown below.

```
% File describing the power system of Problem 13-8c, including
% resistances.
%
% System data has the form:
```

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```
%SYSTEM      name              baseMVA
SYSTEM  Prob13_8c          100
%
% Bus data has the form:
%BUS name          volts
BUS One            1.00
BUS Two            1.00
BUS Three          1.00
BUS Four           1.00
%
% Note that transformers T1 and T2 are treated as "transmission lines"
% here.   Transmission line data has the form:
%LINE     from         to      Rse     Xse     Gsh     Bsh     X0     Vis
LINE    One       Two        0.0100  0.0500  0.000   0.000   0.0500  3
LINE    Two       Three      0.1240  0.6198  0.000   0.000   1.0331  3
LINE    Three     Four       0.0238  0.1190  0.000   0.000   0.1190  1
%
% Generator data has the form:
%GENERATOR   bus        R       Xs       Xp       Xpp      X2       X0
GENERATOR   One     0.10    0.90    0.20    0.10    0.10    0.05
%
% Motor data has the form:
%MOTOR       bus        R       Xs       Xp       Xpp     X2      X0
MOTOR       Four    0.2186  2.4045  0.6558  0.3935  0.3279  0.2186
%
% type data has the form:
%FAULT    bus     Calc Type    Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT    Three       SLG           1
```

The resulting outputs are shown below.

```
>> faults prob_13_8c_fault
Input summary statistics:
  32 lines in system file
   1 SYSTEM lines
   4 BUS lines
   3 LINE lines
   1 GENERATOR lines
   1 MOTOR lines
   1 TYPE lines


                         Results for Case Prob13_8c


Single Line-to-Ground Fault at Bus Three
Calculating Subtransient Currents
|====================Bus Information=============|======Line Information======|
Bus          P  Volts / angle   Amps / angle  |    To  | P  Amps / angle  |
no.    Name  h  (pu)   (deg)    (pu)   (deg)   |  Bus   | h  (pu)   (deg)  |
|===============================================================================|
  1      One a  0.866/   5.60   0.000/   0.00
             b  0.964/-116.92   0.000/   0.00
             c  0.932/ 119.76   0.000/   0.00
                                                  Two   a 1.180/ -74.31
                                                        b 0.400/ 138.43
                                                        c 0.476/  72.03
  2      Two a  0.805/   5.69   0.000/   0.00
             b  0.946/-116.38   0.000/   0.00
```

```
                   c    0.911/ 118.97    0.000/    0.00
                                                             One    a 1.180/ 105.69
                                                                    b 0.400/ -41.57
                                                                    c 0.476/-107.97
                                                             Three  a 1.180/ -74.31
                                                                    b 0.400/ 138.43
                                                                    c 0.476/  72.03
  3      Three a    0.000/  14.04    4.737/ -68.56
               b    0.769/-111.04    0.000/    0.00
               c    0.670/ 110.64    0.000/    0.00
                                                             Two    a 1.180/ 105.69
                                                                    b 0.400/ -41.57
                                                                    c 0.476/-107.97
                                                             Four   a 2.137/ 114.64
                                                                    b 1.088/ -78.21
                                                                    c 1.103/ -52.69
  4      Four  a    0.433/  12.04    0.000/    0.00
               b    0.728/-109.02    0.000/    0.00
               c    0.626/ 107.24    0.000/    0.00
                                                             Three  a 2.137/ -65.36
                                                                    b 1.088/ 101.79
                                                                    c 1.103/ 127.31
|===============================================================================|
```

The subtransient, transient, and steady-state fault currents are given in per-unit above. The actual fault currents are found by multiplying by the base current in Region 2:

$$\mathbf{I}_{f,A} = \mathbf{I}_{f,A,\text{pu}} I_{\text{base},2} = (4.737\angle -68.6^\circ)(524.9\text{ A}) = 2487\angle -68.6^\circ\text{ A}$$
$$\mathbf{I}_{f,B} = \mathbf{I}_{f,B,\text{pu}} I_{\text{base},2} = 0\angle 0.0^\circ\text{ A}$$
$$\mathbf{I}_{f,C} = \mathbf{I}_{f,C,\text{pu}} I_{\text{base},2} = 0\angle 0.0^\circ\text{ A}$$

The subtransient generator current will be the same as the current flowing from Bus 1 to Bus 2. This current is given in per-unit above. The actual generator current is found by multiplying by the base current in Region 1:

$$\mathbf{I}_{12,A} = \mathbf{I}_{12,A,\text{pu}} I_{\text{base},2} = (1.180\angle -74.3^\circ)(4184\text{ A}) = 4937\angle -74.3^\circ\text{ A}$$
$$\mathbf{I}_{12,B} = \mathbf{I}_{12,B,\text{pu}} I_{\text{base},2} = (0.400\angle 138.4^\circ)(4184\text{ A}) = 1674\angle -138.4^\circ\text{ A}$$
$$\mathbf{I}_{12,C} = \mathbf{I}_{12,C,\text{pu}} I_{\text{base},2} = (0.476\angle 72.0^\circ)(4184\text{ A}) = 1992\angle 72.0^\circ\text{ A}$$

The subtransient motor current will be the same as the current flowing from Bus 4 to Bus 3. This current is given in per-unit above. The actual motor current is found by multiplying by the base current in Region 3:

$$\mathbf{I}_{43,A} = \mathbf{I}_{43,A,\text{pu}} I_{\text{base},2} = (2.137\angle -65.4^\circ)(4374\text{ A}) = 9347\angle -65.4^\circ\text{ A}$$
$$\mathbf{I}_{43,B} = \mathbf{I}_{43,B,\text{pu}} I_{\text{base},2} = (1.088\angle 101.8^\circ)(4374\text{ A}) = 4759\angle 101.8^\circ\text{ A}$$
$$\mathbf{I}_{43,C} = \mathbf{I}_{43,C,\text{pu}} I_{\text{base},2} = (1.103\angle 127.3^\circ)(4374\text{ A}) = 4825\angle 127.3^\circ\text{ A}$$

13-9. The Ozzie Outback Electric Power (OOEP) Company maintains the power system shown in Figure P13-3. This power system contains five busses, with generators attached to two of them and loads attached to the remaining ones. The power system has six transmission lines connecting the busses together, with the characteristics shown in Table 13-1. There are generators at busses Bunya and Mulga, and loads at all other busses. The characteristics of the two generators are shown in Table 13-2. Note that the neutrals of

the two generators are both grounded through an inductive reactance of 0.60 pu. Note that all values are in per-unit on a 100 MVA base.

(a) Assume that a single-line-to-ground fault occurs at the Mallee bus. What is the per-unit fault current during the subtransient period? during the transient period?

(b) Assume that the neutrals of the generators are now solidly grounded, and that a single-line-to-ground fault occurs at the Mallee bus. What is the per-unit fault current during the subtransient period now? during the transient period?

(c) Assume that a line-to-line fault occurs at the Mallee bus. What is the per-unit fault current during the subtransient period? during the transient period?

(d) How much does the per-unit fault current change for a line-to-line fault if the generators are either solidly grounded or grounded through an impedance?

Table 13-1: Transmission lines in the OOEP system.

From	To	R_{SE}, pu	X_1, pu	X_2, pu	X_0, pu
Bunya	Mulga	0.011	0.051	0.051	0.090
Mulga	Satinay	0.007	0.035	0.035	0.090
Bunya	Myall	0.007	0.035	0.035	0.060
Myall	Satinay	0.007	0.035	0.035	0.060
Myall	Mallee	0.011	0.051	0.051	0.110
Mallee	Satinay	0.011	0.051	0.051	0.090

Table 13-2: Generators in the OOEP system.

Name	Bus	R (pu)	X_S (pu)	X' (pu)	X'' (pu)	X_2 (pu)	X_{g0} (pu)	X_N (pu)
G1	Bunya	0.02	1.5	0.35	0.20	0.20	0.10	0.60
G2	Mulga	0.01	1.0	0.25	0.12	0.12	0.05	0.60
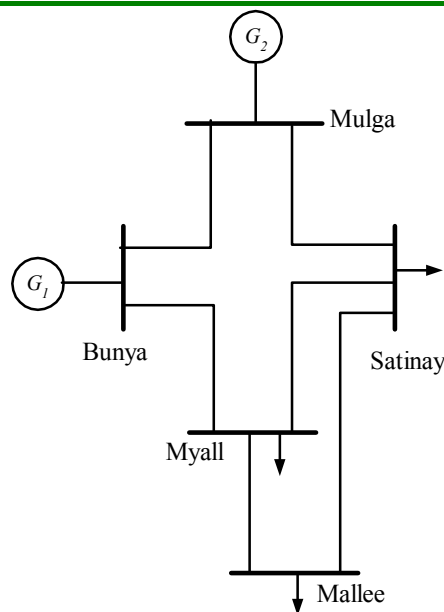
Figure P13-3   The Ozzie Outback Electric Power Company system.

SOLUTION

(a) Note that the impedance to ground for the two generators will be equally to $Z_{g0} + 3Z_N$. The input file required for program `faults` is:

```
% File describing a possible fault at bus Mallee on the
% Ozzie Outback Electric Power system.
%
% System data has the form:
%SYSTEM     name            baseMVA
SYSTEM   OOEP_p13a          100
%
% Bus data has the form:
%BUS name       volts
BUS Bunya       1.00
BUS Mulga       1.00
BUS Mallee      1.00
BUS Myall       1.00
BUS Satinay     1.00
%
% Transmission line data has the form:
%LINE     from        to      Rse    Xse    Gsh    Bsh    X0    Vis
LINE    Bunya     Mulga      0.011  0.051  0.000  0.000  0.090  3
LINE    Mulga     Satinay    0.007  0.035  0.000  0.000  0.090  3
LINE    Bunya     Myall      0.007  0.035  0.000  0.000  0.060  3
LINE    Myall     Satinay    0.007  0.035  0.000  0.000  0.060  3
LINE    Myall     Mallee     0.011  0.051  0.000  0.000  0.110  3
LINE    Mallee    Satinay    0.011  0.051  0.000  0.000  0.090  3
%
% Generator data has the form:
%GENERATOR   bus       R      Xs     Xp     Xpp    X2     X0
GENERATOR   Bunya   0.02   1.5    0.35   0.20   0.20   1.90
GENERATOR   Mulga   0.01   1.0    0.25   0.12   0.00   1.85
%
% type data has the form:
%FAULT    bus     Calc Type   Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT   Mallee      SLG            0
```

The resulting fault currents are shown below.

```
>> faults prob_13_9a_fault
Input summary statistics:
  32 lines in system file
   1 SYSTEM lines
   5 BUS lines
   6 LINE lines
   2 GENERATOR lines
   0 MOTOR lines
   1 TYPE lines


                        Results for Case OOEP_p13a


Single Line-to-Ground Fault at Bus Mallee
Calculating Subtransient Currents
|====================Bus Information============|======Line Information=====|
Bus           P  Volts / angle   Amps / angle  |   To  | P  Amps / angle   |
no.    Name   h  (pu)    (deg)    (pu)    (deg) |  Bus  | h  (pu)    (deg)  |
|==============================================================================|
  1    Bunya  a  0.137/  -6.64   0.000/   0.00
              b  1.515/-145.69   0.000/   0.00
              c  1.471/ 147.65   0.000/   0.00
                                                   Mulga   a 0.333/  97.32
                               321
```

```
                                                           b 0.188/-134.02
                                                           c 0.176/ -25.69
                                                  Myall    a 1.136/ -87.88
                                                           b 0.142/-116.14
                                                           c 0.134/ -46.85
  2     Mulga a  0.156/  -6.31   0.000/   0.00
              b  1.514/-146.04   0.000/   0.00
              c  1.471/ 147.96   0.000/   0.00
                                                  Bunya    a 0.333/ -82.68
                                                           b 0.188/  45.98
                                                           c 0.176/ 154.31
                                                  Satinay  a 1.377/ -87.25
                                                           b 0.142/  63.86
                                                           c 0.134/ 133.15
  3     Mallee a  0.000/-176.42   2.512/ -87.54
              b  1.546/-146.58   0.000/ 180.00
              c  1.502/ 148.59   0.000/ 180.00
                                                  Myall    a 1.193/  92.18
                                                           b 0.021/   0.45
                                                           c 0.020/-157.29
                                                  Satinay  a 1.319/  92.72
                                                           b 0.021/-179.55
                                                           c 0.020/  22.71
  4     Myall a  0.085/  -6.71   0.000/   0.00
             b  1.527/-146.09   0.000/   0.00
             c  1.483/ 148.07   0.000/   0.00
                                                  Bunya    a 1.136/  92.12
                                                           b 0.142/  63.86
                                                           c 0.134/ 133.15
                                                  Satinay  a 0.057/  93.44
                                                           b 0.134/-108.17
                                                           c 0.129/ -55.24
                                                  Mallee   a 1.193/ -87.82
                                                           b 0.021/-179.55
                                                           c 0.020/  22.71
  5     Satinay a  0.086/  -7.01   0.000/   0.00
               b  1.530/-146.28   0.000/   0.00
               c  1.487/ 148.25   0.000/   0.00
                                                  Mulga    a 1.377/  92.75
                                                           b 0.142/-116.14
                                                           c 0.134/ -46.85
                                                  Myall    a 0.057/ -86.56
                                                           b 0.134/  71.83
                                                           c 0.129/ 124.76
                                                  Mallee   a 1.319/ -87.28
                                                           b 0.021/   0.45
                                                           c 0.020/-157.29
|=========================================================================|
```

Results for Case OOEP_p13a

Single Line-to-Ground Fault at Bus Mallee
Calculating Transient Currents
```
|====================Bus Information============|======Line Information=====|
Bus          P  Volts / angle   Amps / angle  |    To  | P  Amps / angle  |
```

```
no.      Name  h   (pu)     (deg)    (pu)    (deg)  |   Bus  | h  (pu)     (deg)  |
|=============================================================================|
  1     Bunya a   0.130/   -6.78   0.000/   0.00
              b   1.431/-145.82   0.000/   0.00
              c   1.390/ 147.50   0.000/   0.00
                                                        Mulga   a 0.297/  97.37
                                                                b 0.184/-139.40
                                                                c 0.175/ -20.80
                                                        Myall   a 1.082/ -88.00
                                                                b 0.134/-120.31
                                                                c 0.129/ -42.84
  2     Mulga a   0.147/   -6.43   0.000/   0.00
              b   1.429/-146.18   0.000/   0.00
              c   1.389/ 147.84   0.000/   0.00
                                                        Bunya   a 0.297/ -82.63
                                                                b 0.184/  40.60
                                                                c 0.175/ 159.20
                                                        Satinay a 1.290/ -87.39
                                                                b 0.134/  59.69
                                                                c 0.129/ 137.16
  3     Mallee a  0.000/  -90.00   2.372/ -87.67
              b   1.460/-146.71   0.000/   0.00
              c   1.418/ 148.46   0.000/   0.00
                                                        Myall   a 1.129/  92.05
                                                                b 0.022/  -2.48
                                                                c 0.021/-156.13
                                                        Satinay a 1.243/  92.58
                                                                b 0.022/ 177.52
                                                                c 0.021/  23.87
  4     Myall a   0.081/   -6.84   0.000/   0.00
              b   1.442/-146.23   0.000/   0.00
              c   1.400/ 147.93   0.000/   0.00
                                                        Bunya   a 1.082/  92.00
                                                                b 0.134/  59.69
                                                                c 0.129/ 137.16
                                                        Satinay a 0.047/  93.32
                                                                b 0.125/-111.48
                                                                c 0.122/ -52.07
                                                        Mallee  a 1.129/ -87.95
                                                                b 0.022/ 177.52
                                                                c 0.021/  23.87
  5     Satinay a  0.081/   -7.14   0.000/   0.00
              b   1.445/-146.41   0.000/   0.00
              c   1.404/ 148.12   0.000/   0.00
                                                        Mulga   a 1.290/  92.61
                                                                b 0.134/-120.31
                                                                c 0.129/ -42.84
                                                        Myall   a 0.047/ -86.68
                                                                b 0.125/  68.52
                                                                c 0.122/ 127.93
                                                        Mallee  a 1.243/ -87.42
                                                                b 0.022/  -2.48
                                                                c 0.021/-156.13
|=============================================================================|
```

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Single Line-to-Ground Fault at Bus Mallee
Calculating Steady-State Currents

Bus			Volts / angle		Amps / angle			To		P	Amps / angle	
no.	Name	h	(pu)	(deg)	(pu)	(deg)		Bus		h	(pu)	(deg)
1	Bunya	a	0.095/	-7.41	0.000/	0.00						
		b	1.053/	-146.44	0.000/	0.00						
		c	1.023/	146.89	0.000/	0.00						
								Mulga		a	0.226/	96.85
										b	0.132/	-137.19
										c	0.126/	-24.42
								Myall		a	0.792/	-88.66
										b	0.098/	-118.63
										c	0.094/	-45.86
2	Mulga	a	0.108/	-7.04	0.000/	0.00						
		b	1.052/	-146.79	0.000/	0.00						
		c	1.022/	147.22	0.000/	0.00						
								Bunya		a	0.226/	-83.15
										b	0.132/	42.81
										c	0.126/	155.58
								Satinay		a	0.953/	-87.98
										b	0.098/	61.37
										c	0.094/	134.14
3	Mallee	a	0.000/	180.00	1.746/	-88.29						
		b	1.074/	-147.33	0.000/	0.00						
		c	1.043/	147.84	0.000/	0.00						
								Myall		a	0.830/	91.42
										b	0.015/	-1.95
										c	0.015/	-157.88
								Satinay		a	0.916/	91.97
										b	0.015/	178.05
										c	0.015/	22.12
4	Myall	a	0.059/	-7.47	0.000/	0.00						
		b	1.061/	-146.85	0.000/	0.00						
		c	1.030/	147.31	0.000/	0.00						
								Bunya		a	0.792/	91.34
										b	0.098/	61.37
										c	0.094/	134.14
								Satinay		a	0.037/	93.19
										b	0.092/	-110.25
										c	0.090/	-54.59
								Mallee		a	0.830/	-88.58
										b	0.015/	178.05
										c	0.015/	22.12
5	Satinay	a	0.060/	-7.75	0.000/	0.00						
		b	1.063/	-147.03	0.000/	0.00						
		c	1.033/	147.50	0.000/	0.00						
								Mulga		a	0.953/	92.02
										b	0.098/	-118.63
										c	0.094/	-45.86
								Myall		a	0.037/	-86.81
										b	0.092/	69.75
										c	0.090/	125.41

```
                                     Mallee    a 0.916/ -88.03
                                               b 0.015/  -1.95
                                               c 0.015/-157.88
|==============================================================================|
```

The subtransient fault current at the Mallee bus for a single-line-to-ground fault is $\mathbf{I}''_f = 2.512\angle -87.5 \text{ pu}$, and the transient fault current at the Mallee bus for a single-line-to-ground fault is $\mathbf{I}'_f = 2.372\angle -87.7 \text{ pu}$.

(b) If the generators are solidly grounded, their zero sequence impedances will become just Z_{g0}. This smaller zero sequence impedance will result in a much higher fault current for a single-line-to-ground fault. The input file required for program `faults` is:

```
% File describing a possible fault at bus Mallee on the
% Ozzie Outback Electric Power system.
%
% System data has the form:
%SYSTEM     name               baseMVA
SYSTEM  OOEP_p13b              100
%
% Bus data has the form:
%BUS name        volts
BUS Bunya        1.00
BUS Mulga        1.00
BUS Mallee       1.00
BUS Myall        1.00
BUS Satinay      1.00
%
% Transmission line data has the form:
%LINE      from        to       Rse    Xse    Gsh    Bsh    X0    Vis
LINE    Bunya      Mulga      0.011  0.051  0.000  0.000  0.090  3
LINE    Mulga      Satinay    0.007  0.035  0.000  0.000  0.090  3
LINE    Bunya      Myall      0.007  0.035  0.000  0.000  0.060  3
LINE    Myall      Satinay    0.007  0.035  0.000  0.000  0.060  3
LINE    Myall      Mallee     0.011  0.051  0.000  0.000  0.110  3
LINE    Mallee     Satinay    0.011  0.051  0.000  0.000  0.090  3
%
% Generator data has the form:
%GENERATOR  bus        R       Xs     Xp     Xpp    X2      X0
GENERATOR   Bunya    0.02    1.5    0.35   0.20   0.20   0.10
GENERATOR   Mulga    0.01    1.0    0.25   0.12   0.00   0.05
%
% type data has the form:
%FAULT    bus     Calc Type   Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT    Mallee      SLG           0
```

The resulting fault currents are shown below.

```
>> faults prob_13_9b_fault
Input summary statistics:
  32 lines in system file
   1 SYSTEM lines
   5 BUS lines
   6 LINE lines
   2 GENERATOR lines
   0 MOTOR lines
```

```
   1 TYPE lines


                          Results for Case OOEP_p13b


Single Line-to-Ground Fault at Bus Mallee
Calculating Subtransient Currents
|====================Bus Information=============|======Line Information=====|
Bus             P  Volts / angle   Amps / angle  |   To  | P  Amps / angle  |
no.     Name    h  (pu)    (deg)    (pu)   (deg)  |  Bus  | h  (pu)   (deg)  |
|===============================================================================|
  1     Bunya   a  0.544/   0.80   0.000/   0.00
                b  0.863/-127.07   0.000/   0.00
                c  0.813/ 122.57   0.000/   0.00
                                                    Mulga   a 1.565/ 103.75
                                                            b 0.621/-140.86
                                                            c 0.645/  -0.41
                                                    Myall   a 4.489/ -80.38
                                                            b 0.483/-113.77
                                                            c 0.476/ -30.88
  2     Mulga   a  0.641/   1.13   0.000/   0.00
                b  0.838/-128.38   0.000/   0.00
                c  0.784/ 123.31   0.000/   0.00
                                                    Bunya   a 1.565/ -76.25
                                                            b 0.621/  39.14
                                                            c 0.645/ 179.59
                                                    Satinay a 5.664/ -79.89
                                                            b 0.483/  66.23
                                                            c 0.476/ 149.12
  3     Mallee  a  0.000/-172.87  10.153/ -80.11
                b  0.966/-133.70   0.000/   0.00
                c  0.868/ 131.99   0.000/   0.00
                                                    Myall   a 4.797/  99.63
                                                            b 0.086/  -7.81
                                                            c 0.093/-136.64
                                                    Satinay a 5.356/ 100.13
                                                            b 0.086/ 172.19
                                                            c 0.093/  43.36
  4     Myall   a  0.342/   0.72   0.000/   0.00
                b  0.897/-130.03   0.000/   0.00
                c  0.827/ 126.50   0.000/   0.00
                                                    Bunya   a 4.489/  99.62
                                                            b 0.483/  66.23
                                                            c 0.476/ 149.12
                                                    Satinay a 0.309/  99.72
                                                            b 0.467/-103.54
                                                            c 0.459/ -42.12
                                                    Mallee  a 4.797/ -80.37
                                                            b 0.086/ 172.19
                                                            c 0.093/  43.36
  5     Satinay a  0.349/   0.41   0.000/   0.00
                b  0.905/-131.17   0.000/   0.00
                c  0.828/ 127.90   0.000/   0.00
                                                    Mulga   a 5.664/ 100.11
                                                            b 0.483/-113.77
                                                            c 0.476/ -30.88
                                                    Myall   a 0.309/ -80.28
```

```
                                                 b  0.467/  76.46
                                                 c  0.459/ 137.88
                                      Mallee     a  5.356/ -79.87
                                                 b  0.086/  -7.81
                                                 c  0.093/-136.64
|===========================================================================|


                      Results for Case OOEP_p13b

Single Line-to-Ground Fault at Bus Mallee
Calculating Transient Currents
|===================Bus Information=============|======Line Information=====|
Bus           P  Volts / angle   Amps / angle  |   To  | P  Amps / angle  |
no.    Name   h   (pu)    (deg)   (pu)    (deg) |  Bus  | h  (pu)    (deg) |
|===========================================================================|
  1    Bunya  a  0.442/  -1.09   0.000/   0.00
              b  0.700/-128.94   0.000/   0.00
              c  0.660/ 120.68   0.000/   0.00
                                                  Mulga    a 1.205/ 101.96
                                                           b 0.538/-148.25
                                                           c 0.569/   1.86
                                                  Myall    a 3.664/ -82.25
                                                           b 0.395/-120.39
                                                           c 0.396/ -28.27
  2    Mulga  a  0.517/  -0.76   0.000/   0.00
              b  0.677/-130.28   0.000/   0.00
              c  0.633/ 121.43   0.000/   0.00
                                                  Bunya    a 1.205/ -78.04
                                                           b 0.538/  31.75
                                                           c 0.569/-178.14
                                                  Satinay  a 4.550/ -81.79
                                                           b 0.395/  59.61
                                                           c 0.396/ 151.73
  3    Mallee a  0.000/ 180.00   8.214/ -82.00
              b  0.781/-135.58   0.000/ 180.00
              c  0.702/ 130.11   0.000/ 180.00
                                                  Myall    a 3.889/  97.74
                                                           b 0.078/ -10.81
                                                           c 0.084/-138.69
                                                  Satinay  a 4.324/  98.24
                                                           b 0.078/ 169.19
                                                           c 0.084/  41.31
  4    Myall  a  0.277/  -1.16   0.000/   0.00
              b  0.726/-131.91   0.000/   0.00
              c  0.670/ 124.60   0.000/   0.00
                                                  Bunya    a 3.664/  97.75
                                                           b 0.395/  59.61
                                                           c 0.396/ 151.73
                                                  Satinay  a 0.225/  97.72
                                                           b 0.376/-109.12
                                                           c 0.375/ -40.32
                                                  Mallee   a 3.889/ -82.26
                                                           b 0.078/ 169.19
                                                           c 0.084/  41.31
  5    Satinay a 0.282/  -1.47   0.000/   0.00
```

```
           b  0.732/-133.07   0.000/   0.00
           c  0.670/ 126.02   0.000/   0.00
                                              Mulga   a 4.550/  98.21
                                                      b 0.395/-120.39
                                                      c 0.396/ -28.27
                                              Myall   a 0.225/ -82.28
                                                      b 0.376/  70.88
                                                      c 0.375/ 139.68
                                              Mallee  a 4.324/ -81.76
                                                      b 0.078/ -10.81
                                                      c 0.084/-138.69
|==============================================================================|
```

Results for Case OOEP_p13b

Single Line-to-Ground Fault at Bus Mallee
Calculating Steady-State Currents

```
|====================Bus Information============|======Line Information=====|
Bus           P  Volts / angle   Amps / angle  |   To  | P  Amps / angle  |
no.    Name   h   (pu)   (deg)    (pu)   (deg)  |  Bus  | h  (pu)    (deg)  |
|==============================================================================|
  1    Bunya  a  0.197/  -5.54   0.000/   0.00
              b  0.312/-133.40   0.000/   0.00
              c  0.294/ 116.23   0.000/   0.00
                                              Mulga   a 0.554/  97.64
                                                      b 0.230/-149.89
                                                      c 0.243/  -5.19
                                              Myall   a 1.629/ -86.74
                                                      b 0.174/-122.18
                                                      c 0.175/ -35.35
  2    Mulga  a  0.231/  -5.19   0.000/   0.00
              b  0.303/-134.71   0.000/   0.00
              c  0.283/ 116.99   0.000/   0.00
                                              Bunya   a 0.554/ -82.36
                                                      b 0.230/  30.11
                                                      c 0.243/ 174.81
                                              Satinay a 2.041/ -86.20
                                                      b 0.174/  57.82
                                                      c 0.175/ 144.65
  3    Mallee a  0.000/ -90.00   3.670/ -86.44
              b  0.349/-140.03   0.000/   0.00
              c  0.314/ 125.66   0.000/   0.00
                                              Myall   a 1.736/  93.29
                                                      b 0.033/ -15.05
                                                      c 0.035/-143.43
                                              Satinay a 1.934/  93.81
                                                      b 0.033/ 164.95
                                                      c 0.035/  36.57
  4    Myall  a  0.124/  -5.61   0.000/   0.00
              b  0.324/-136.36   0.000/   0.00
              c  0.299/ 120.16   0.000/   0.00
                                              Bunya   a 1.629/  93.26
                                                      b 0.174/  57.82
                                                      c 0.175/ 144.65
                                              Satinay a 0.107/  93.71
```

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```
                                                                b 0.168/-111.43
                                                                c 0.167/ -46.90
                                                     Mallee    a 1.736/ -86.71
                                                                b 0.033/ 164.95
                                                                c 0.035/  36.57
   5    Satinay a   0.126/  -5.91    0.000/   0.00
               b   0.327/-137.50    0.000/   0.00
               c   0.299/ 121.58    0.000/   0.00
                                                     Mulga     a 2.041/  93.80
                                                                b 0.174/-122.18
                                                                c 0.175/ -35.35
                                                     Myall     a 0.107/ -86.29
                                                                b 0.168/  68.57
                                                                c 0.167/ 133.10
                                                     Mallee    a 1.934/ -86.19
                                                                b 0.033/ -15.05
                                                                c 0.035/-143.43
|========================================================================|
```

The subtransient fault current at the Mallee bus for a single-line-to-ground fault is $\mathbf{I}''_f = 10.153\angle -80.1 \text{ pu}$, and the transient fault current at the Mallee bus for a single-line-to-ground fault is $\mathbf{I}'_f = 8.214\angle -82.0 \text{ pu}$. These numbers are about 5 times larger than before. Clearly, the reactances in the neutrals of the generators help to reduce the fault current.

(c) The input file required for a line-to-line fault is:

```
% File describing a possible fault at bus Mallee on the
% Ozzie Outback Electric Power system.
%
% System data has the form:
%SYSTEM     name               baseMVA
SYSTEM  OOEP_p13c            100
%
% Bus data has the form:
%BUS name       volts
BUS Bunya       1.00
BUS Mulga       1.00
BUS Mallee      1.00
BUS Myall       1.00
BUS Satinay     1.00
%
% Transmission line data has the form:
%LINE     from         to      Rse    Xse    Gsh    Bsh    X0    Vis
LINE    Bunya     Mulga      0.011  0.051  0.000  0.000  0.090  3
LINE    Mulga     Satinay    0.007  0.035  0.000  0.000  0.090  3
LINE    Bunya     Myall      0.007  0.035  0.000  0.000  0.060  3
LINE    Myall     Satinay    0.007  0.035  0.000  0.000  0.060  3
LINE    Myall     Mallee     0.011  0.051  0.000  0.000  0.110  3
LINE    Mallee    Satinay    0.011  0.051  0.000  0.000  0.090  3
%
% Generator data has the form:
%GENERATOR   bus      R     Xs     Xp     Xpp    X2     X0
GENERATOR   Bunya  0.02   1.5    0.35   0.20   0.20   1.90
GENERATOR   Mulga  0.01   1.0    0.25   0.12   0.00   1.85
%
% type data has the form:
```

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```
%FAULT    bus     Calc Type   Calc_time (0=all;1=sub;2=trans;3=ss)
FAULT   Mallee      LL          0
```

The resulting fault currents are shown below.

>> **faults prob_13_9c_fault**
```
Input summary statistics:
  32 lines in system file
   1 SYSTEM lines
   5 BUS lines
   6 LINE lines
   2 GENERATOR lines
   0 MOTOR lines
   1 TYPE lines
```

```
                         Results for Case OOEP_p13c


Line-to-Line Fault at Bus Mallee
```
Calculating Subtransient Currents
```
|====================Bus Information============|======Line Information======|
Bus           P  Volts / angle    Amps / angle  |   To  | P  Amps / angle  |
no.    Name   h   (pu)   (deg)     (pu)   (deg)  |  Bus  | h  (pu)   (deg)  |
|===============================================================================|
  1    Bunya a  0.656/  -8.08   0.000/   0.00
             b  0.480/-133.36   0.000/   0.00
             c  0.545/ 125.92   0.000/   0.00
                                                   Mulga   a 1.178/ -69.30
                                                           b 1.811/  35.43
                                                           c 1.893/ 178.44
                                                   Myall   a 0.627/ -70.10
                                                           b 4.146/-173.60
                                                           c 4.046/  15.07
  2    Mulga a  0.597/  -9.77   0.000/   0.00
             b  0.525/-123.85   0.000/   0.00
             c  0.613/ 118.87   0.000/   0.00
                                                   Bunya   a 1.178/ 110.70
                                                           b 1.811/-144.57
                                                           c 1.893/  -1.56
                                                   Satinay  a 0.627/ 109.90
                                                           b 5.916/-164.72
                                                           c 5.999/   9.29
  3    Mallee a  0.626/  -8.88   0.000/   0.00
             b  0.313/ 171.12  **10.032/-168.38**
             c  0.313/ 171.12   10.032/  11.62
                                                   Myall   a 0.160/ 110.54
                                                           b 4.792/  10.44
                                                           c 4.767/-167.67
                                                   Satinay  a 0.160/ -69.46
                                                           b 5.242/  12.70
                                                           c 5.267/-169.02
  4    Myall a  0.634/  -8.66   0.000/   0.00
             b  0.376/-147.54   0.000/   0.00
             c  0.429/ 136.23   0.000/   0.00
                                                   Bunya   a 0.627/ 109.90
                                                           b 4.146/   6.40
                                                           c 4.046/-164.93
                                                   Satinay  a 0.467/ -70.32
```

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```
                                                    b 0.719/  34.41
                                                    c 0.751/ 177.42
                                        Mallee    a 0.160/ -69.46
                                                    b 4.792/-169.56
                                                    c 4.767/  12.33
  5    Satinay a  0.618/  -9.11   0.000/   0.00
             b  0.381/-143.73   0.000/   0.00
             c  0.443/ 133.22   0.000/   0.00
                                        Mulga     a 0.627/ -70.10
                                                    b 5.916/  15.28
                                                    c 5.999/-170.71
                                        Myall     a 0.467/ 109.68
                                                    b 0.719/-145.59
                                                    c 0.751/  -2.58
                                        Mallee    a 0.160/ 110.54
                                                    b 5.242/-167.30
                                                    c 5.267/  10.98
 |=============================================================================|


                          Results for Case OOEP_p13c

Line-to-Line Fault at Bus Mallee
Calculating Transient Currents
|====================Bus Information=============|======Line Information=====|
Bus           P  Volts / angle   Amps / angle  |   To  | P  Amps / angle   |
no.    Name   h  (pu)    (deg)   (pu)    (deg)  |  Bus  | h  (pu)    (deg)  |
|=============================================================================|
  1    Bunya  a  0.470/ -11.37   0.000/   0.00
             b  0.344/-136.60   0.000/   0.00
             c  0.391/ 122.55   0.000/   0.00
                                        Mulga     a 0.931/ -73.35
                                                    b 1.236/  35.47
                                                    c 1.286/ 172.17
                                        Myall     a 0.496/ -74.15
                                                    b 3.000/-177.32
                                                    c 2.927/  12.18
  2    Mulga  a  0.423/ -13.17   0.000/   0.00
             b  0.372/-127.22   0.000/   0.00
             c  0.435/ 115.51   0.000/   0.00
                                        Bunya     a 0.931/ 106.65
                                                    b 1.236/-144.53
                                                    c 1.286/  -7.83
                                        Satinay   a 0.496/ 105.85
                                                    b 4.175/-167.70
                                                    c 4.235/   5.58
  3    Mallee a  0.446/ -12.23   0.000/   0.00
             b  0.223/ 167.77   7.151/-171.72
             c  0.223/ 167.77   7.151/   8.28
                                        Myall     a 0.126/ 106.49
                                                    b 3.427/   7.00
                                                    c 3.408/-170.90
                                        Satinay   a 0.126/ -73.51
                                                    b 3.726/   9.45
                                                    c 3.743/-172.47
  4    Myall  a  0.453/ -11.99   0.000/   0.00
```

```
             b  0.268/-150.82   0.000/   0.00
             c  0.307/ 132.85   0.000/   0.00
                                             Bunya    a 0.496/ 105.85
                                                      b 3.000/   2.68
                                                      c 2.927/-167.82
                                             Satinay  a 0.370/ -74.37
                                                      b 0.490/  34.44
                                                      c 0.510/ 171.15
                                             Mallee   a 0.126/ -73.51
                                                      b 3.427/-173.00
                                                      c 3.408/   9.10
  5    Satinay a  0.440/ -12.47   0.000/   0.00
             b  0.271/-147.13   0.000/   0.00
             c  0.315/ 129.91   0.000/   0.00
                                             Mulga    a 0.496/ -74.15
                                                      b 4.175/  12.30
                                                      c 4.235/-174.42
                                             Myall    a 0.370/ 105.63
                                                      b 0.490/-145.56
                                                      c 0.510/  -8.85
                                             Mallee   a 0.126/ 106.49
                                                      b 3.726/-170.55
                                                      c 3.743/   7.53
|=============================================================================|
```

Results for Case OOEP_p13c

```
Line-to-Line Fault at Bus Mallee
Calculating Steady-State Currents
|====================Bus Information=============|======Line Information======|
Bus         P  Volts / angle   Amps / angle  |   To  | P  Amps / angle   |
no.   Name  h   (pu)   (deg)    (pu)  (deg)   |  Bus  | h  (pu)   (deg)   |
|=============================================================================|
  1    Bunya a  0.163/ -16.83   0.000/   0.00
             b  0.120/-142.09   0.000/   0.00
             c  0.136/ 117.13   0.000/   0.00
                                             Mulga    a 0.306/ -78.74
                                                      b 0.443/  28.25
                                                      c 0.459/ 168.64
                                             Myall    a 0.163/ -79.55
                                                      b 1.036/ 177.44
                                                      c 1.012/   6.46
  2    Mulga a  0.148/ -18.52   0.000/   0.00
             b  0.130/-132.59   0.000/   0.00
             c  0.152/ 110.13   0.000/   0.00
                                             Bunya    a 0.306/ 101.26
                                                      b 0.443/-151.75
                                                      c 0.459/ -11.36
                                             Satinay  a 0.163/ 100.45
                                                      b 1.464/-173.28
                                                      c 1.484/   0.42
  3    Mallee a  0.156/ -17.63   0.000/   0.00
             b  0.078/ 162.37   2.493/-177.13
             c  0.078/ 162.37   2.493/   2.87
                                             Myall    a 0.042/ 101.10
```

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```
                                                          b 1.192/    1.64
                                                          c 1.186/-176.38
                                        Satinay   a 0.042/ -78.90
                                                          b 1.301/    4.00
                                                          c 1.307/-177.81
  4     Myall a   0.158/ -17.41    0.000/    0.00
            b   0.093/-156.27    0.000/    0.00
            c   0.107/ 127.46    0.000/    0.00
                                        Bunya     a 0.163/ 100.45
                                                          b 1.036/   -2.56
                                                          c 1.012/-173.54
                                        Satinay   a 0.121/ -79.77
                                                          b 0.176/   27.23
                                                          c 0.182/ 167.62
                                        Mallee    a 0.042/ -78.90
                                                          b 1.192/-178.36
                                                          c 1.186/    3.62
  5     Satinay a  0.154/ -17.86    0.000/    0.00
            b   0.094/-152.50    0.000/    0.00
            c   0.110/ 124.49    0.000/    0.00
                                        Mulga     a 0.163/ -79.55
                                                          b 1.464/    6.72
                                                          c 1.484/-179.58
                                        Myall     a 0.121/ 100.23
                                                          b 0.176/-152.77
                                                          c 0.182/ -12.38
                                        Mallee    a 0.042/ 101.10
                                                          b 1.301/-176.00
                                                          c 1.307/    2.19
|=============================================================================|
```

The subtransient fault current at the Mallee bus for a line-to-line fault is $\mathbf{I}''_f = 10.032\angle -168.4 \text{ pu}$, and the transient fault current at the Mallee bus for a single-line-to-ground fault is $\mathbf{I}'_f = 7.151\angle -171.7 \text{ pu}$.

(d) The line-to-line fault current is not affected by whether or not the neutrals of the generators are solidly grounded, because there are no zero-phase components in a line-to-line fault.