EE347 HW4 Nick Porter

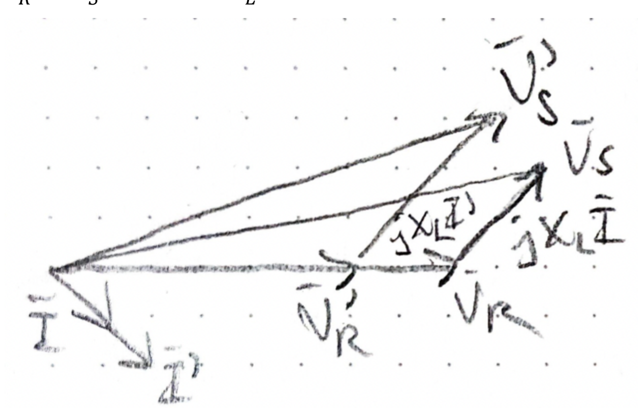
Text Questions:

9-2.
What is the skin effect? How does skin effect change the resistance of an AC transmission line?

The skin effect occurs in AC lines due to eddy currents within the lines caused by the oscillating current. The magnitude of the skin effect is proportional to the AC frequency, as well as the diameter of the conductor. Current oscillating in the line has a corresponding magnetic field around the line, which induces eddy currents that flow against the AC current closer to the inside of the conductor, and with the AC current towards the outside of the conductor. These eddy currents cause higher power losses than DC current.

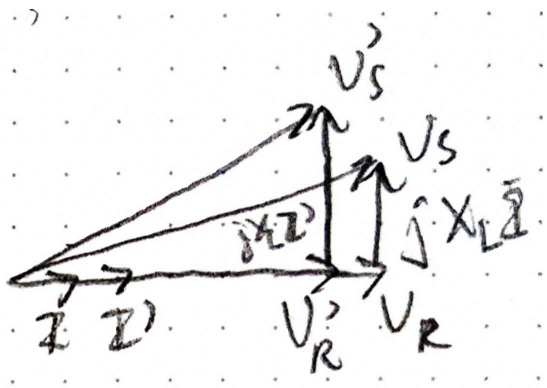
9-9.
What happens to the receiving end voltage as the load on a transmission line is increased if the load has a lagging power factor? Sketch a phasor diagram showing the resulting behavior.

Assuming the PF remains the same, the receiving voltage will sag according to
$\tilde{V}_R = \tilde{V}_S - R\tilde{I} - jX_L\tilde{I}$



9-10.
What happens to the receiving end voltage as the load on a transmission line is increased if the load has unity power factor? Sketch a phasor diagram showing the resulting behavior.

According to the same properties used in the last question, the receiving voltage will sag, but not nearly as much since the unity power factor will mean that the current is in phase with the receiving voltage.



9-12.
What is the significance of the angle sigma between Vs and Vr in a transmission line?

The angle between Vs and Vr, sigma, is dictated by the load on the receiving end, and relates how much power from Vs is transmitted to Vr by the equation: $P = \frac{3V_S V_R \sin\delta}{X_L}$

Text Problem 9-16

$Z = 23 + 75j\Omega = 78.45\Omega\angle 73°$

$Y = j500\mu S$

$S = 150MVA \qquad PF = 0.88$

$V_{LL} = 220kV$

$d = 300km$

$V_\phi = \frac{V_{LL}}{\sqrt{3}} = \frac{220kV}{\sqrt{3}} = 127\ kV \qquad I_L = \frac{S}{\sqrt{3}\cdot V_{LL}} = \frac{150MVA}{\sqrt{3}\cdot 220kV} = 394\ A$

$V_R = V_\phi = 127\ kV$

$I_R = I_L\measuredangle - cos^{-1}(PF) = 394A\measuredangle - 28°$

(a) Short line approx:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 220kV \\ 394A\angle -28.4° \end{bmatrix}$$

$V_S = 150.1\ kV\measuredangle 8.3° \qquad I_S = 394A\measuredangle - 28°$

(b) Medium line approx:

$A = \frac{YZ}{2} + 1 = 0.98\measuredangle 0.34° \qquad B = Z = 78.45\measuredangle 73°$

$C = Y\left(\frac{YZ}{4} + 1\right) = 495 \times 10^{-6}\measuredangle 90.17° \quad D = A$

$V_S = 148.3\ kV\measuredangle 8.7° \qquad I_S = 360.84A\measuredangle - 19.18°$

(c) Long line eq:

$y = Y/d = \frac{j500\mu S}{300km} = 2 \times 10^{-6}\measuredangle 90°$

$z = Z/d = \frac{78.45\Omega\measuredangle 73°}{300km} = 0.26\measuredangle 73°$

$\gamma = \sqrt{yz} = 660 \times 10^{-6}\measuredangle 81.5°$

$Z' = Z \cdot \frac{sinh(\gamma d)}{\gamma d} = 78\Omega\measuredangle 73°$

$Y' = Y \cdot \frac{tanh(\gamma d/2)}{\gamma d/2} = 502\mu S\measuredangle 90°$

$A = \frac{Y'Z'}{2} + 1 = 0.98\measuredangle 0.33° \qquad B = Z' = 78\measuredangle 73°$

$C = Y'\left(\frac{Y'Z'}{4} + 1\right) = 497 \times 10^{-6}\measuredangle 90.11° \quad D = A$

$V_S = 148.1\ kV\measuredangle 8.66° \qquad I_S = 360.85A\measuredangle - 19.15°$

Percent error between short model and long equation: 1.64%

Percent error between medium model and long equation: 0.12%

Problem 1:

In order to test this function, ACSR values for the given specifications were stored as python dictionaries in the form:

```
# ACSR dicts for AC 60Hz @ 50C
ibis = {
      "name": "ibis",
      "resistance": 0.0481,      # ohms per 1000 ft
      "ind_reactance": 0.0835,   # ohms per 1000 ft
      "capacitance": 0.539,      # Mohms per 1000 ft
      "GMR": 0.0265              # ft
}
```

The starting values are:

```
V_rated = 500e3            # nominal voltage
Vr = V_rated/np.sqrt(3)    # receiving phase voltage
S_rated = 150e6            # 150 MVA
S = S_rated/3              # single phase power
d = 95                # length of transmission line
D = 8                 # distance between conductors (ft)
temp_nom = 50         # nominal operating temp, celsius
```

And the function is defined:

```
def findACSR(bird,d,D,Vr,S):
      mi_to_ft_conv = 5.28       # 1000 ft
      gmd = np.cbrt(D**3)        # equilateral triangle formation
      Ir = S/(np.sqrt(3)*Vr)     # receiving current

      # impedance values
      R = bird["resistance"] * d * mi_to_ft_conv
      Xl = (bird["ind_reactance"] * d * mi_to_ft_conv *
            np.log(gmd/bird["GMR"]))
      Xc = (bird["capacitance"]*1e6 / (d * mi_to_ft_conv) *
            np.log(gmd/bird["GMR"]))
      Y = 1/Xc

      # matrix coefficients
      # medium model
      Z = complex(R,Xl)
      Y = complex(0,Y)

      A = (Y*Z)/2 + 1
      B = Z
      C = Y*((Y*Z)/4 + 1)
      D = A
```

```
# calculate Vs, Is, voltage regulation
a = np.array([[A,B],[C,D]])
b = np.array([Vr,Ir])
send = a @ b         # matrix multiplication

Vs = send[0]
Is = send[1]
VR = abs((Vs-Vr)/Vr) * 100
```

The function is called by:
`findACSR(bird,d,D,Vr,S)`
Where `bird` is the conductor size name, `d` is the length of the conductor, `D` is the distance between conductors in a bundle (in an equilateral triangle formation), `Vr` is the receiving side voltage, and `S` is the rated apparent power for a single phase.

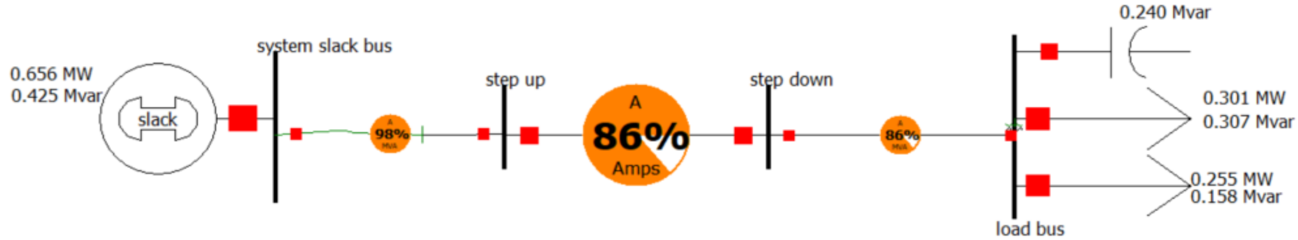
Within the function are included print statements to verify the output (not included here for brevity). The function outputs:

```
drake conductor:

AC resistance per 1000 ft: 0.0242 ohms
inductive reactance per 1000 ft: 0.0756 ohms
capacitive admittance per 1000 ft: 0.4820 Megaohms

Z = 12.14 + j203.36 ohms
Y = j0.0001941 S

A = 0.9803, 0.069 degrees
B = 203.7266, 86.584 degrees
C = 0.000192, 90.034 degrees
D = 0.9803, 0.069 degrees

Vs = 284944.18 V, 4.16 degrees
Is = 112.66 A, 29.56 degrees
voltage regulation = 7.33%

skylark conductor:

AC resistance per 1000 ft: 0.0159 ohms
inductive reactance per 1000 ft: 0.0720 ohms
capacitive admittance per 1000 ft: 0.4550 Megaohms

Z = 7.98 + j188.99 ohms
Y = j0.0002107 S
```

```
A = 0.9801, 0.049 degrees
B = 189.1590, 87.584 degrees
C = 0.000209, 90.024 degrees
D = 0.9801, 0.049 degrees

Vs = 284370.98 V, 3.86 degrees
Is = 115.05 A, 31.61 degrees
voltage regulation = 6.85%

jorea conductor:

AC resistance per 1000 ft: 0.0087 ohms
inductive reactance per 1000 ft: 0.0640 ohms
capacitive admittance per 1000 ft: 0.3990 Megaohms

Z = 4.36 + j155.97 ohms
Y = j0.0002588 S

A = 0.9798, 0.033 degrees
B = 156.0290, 88.397 degrees
C = 0.000256, 90.016 degrees
D = 0.9798, 0.033 degrees

Vs = 283724.49 V, 3.18 degrees
Is = 122.77 A, 37.07 degrees
voltage regulation = 5.77%
```

These values show that the given specifications require conductors towards the lower end of the ACSR list, with even the jorea conductor only being marginally within an appropriate voltage regulation tolerance at 5.77%.

Problem 2:



The step-up transformer is at about 98% amperage capacity, with the rest of transmission at about 86% capacity. This is with a compensating capacitive load of 240kVAr.

The real power provided by the generator remains constant, while its reactive power decreased with the addition of the purely capacitive load.

FE Problems

FE Problem 1:

$$f_0 = \sqrt{\frac{1}{4\pi^2 LC}} = 1678\ Hz$$

(C) = 1.68 x 10^3

FE Problem 2:

$$\frac{N_1}{N_2} = a = \sqrt{\frac{R_1}{R_2}} = 15$$

(A) = 15:1