CHAPTER

2 Collections

This chapter deals with collections. We consider two types of collections: ordered collections (lists) and unordered collections (sets).

8 Lists

A *list* is an ordered sequence of objects. We write lists by starting with an open parenthesis, following with the elements of the list separated by commas, and finishing with a close parenthesis. For example, $(1, 2, \mathbb{Z})$ is a list whose first element is the number 1, whose second element is the number 2, and whose third element is the set of integers.

The order in which elements appear in a list is significant. The list $(1, 2, 3)$ is not the same as the list $(3, 2, 1)$.

Elements in a list might be repeated, as in $(3, 3, 2)$.

The number of elements in a list is called its *length*. For example, the list $(1, 1, 2, 1)$ is a list of length four.

A list of length two has a special name; it is called an *ordered pair*.

A list of length zero is called the *empty list* and is denoted $()$.

What it means for two lists to be equal.

Two lists are *equal* provided they have the same length, and elements in the corresponding positions on the two lists are equal. Lists (a, b, c) and (x, y, z) are equal iff $a = x$, $b = y$, and $c = z$.

Mathspeak!

Another word mathematicians use for lists is *tuple*. A list of n elements is known as an n-tuple.

Lists are all-pervasive in mathematics and beyond. A point in the plane is often specified by an ordered pair of real numbers (x, y). A natural number, when written in standard notation, is a list of digits; you can think of the number 172 as the list $(1, 7, 2)$. An English word is a list of letters. An identifier in a computer program is a list of letters and digits (where the first element of the list is a letter).

Counting Two-Element Lists

In this section, we address questions of the form "How many lists can we make?"

Example 8.1 Suppose we wish to make a two-element list where the entries in the list may be any of the digits 1, 2, 3, and 4. How many such lists are possible?

The most direct approach to answering this question is to write out all the possibilities.

$$\begin{array}{cccc} (1,1) & (1,2) & (1,3) & (1,4) \\ (2,1) & (2,2) & (2,3) & (2,4) \\ (3,1) & (3,2) & (3,3) & (3,4) \\ (4,1) & (4,2) & (4,3) & (4,4) \end{array}$$

There are 16 such lists.