We organized the lists in a manner that ensures we have neither repeated nor omitted a list. The first row of the chart contains all the possible lists that begin with 1, the second row those that begin with 2, and so on. Thus there are $4 \times 4 = 16$ length-two lists whose elements are any one of the digits 1 through 4.

**Mathspeak!**

The mathematical use of the word *choice* is strange. If a restaurant has a menu with only one entrée, the mathematician would say that this menu offers one choice. The rest of the world probably would say that the menu offers no choices! The mathematical use of the word *choice* is similar to *option*.

Let's generalize this example a little bit. Suppose we wish to know the number of two-element lists where there are $n$ possible choices for each entry in the list. We may assume the possible elements are the integers 1 through $n$. As before, we organize all the possible lists into a chart.

$$\begin{array}{cccc} (1,1) & (1,2) & \cdots & (1,n) \\ (2,1) & (2,2) & \cdots & (2,n) \\ \vdots & \vdots & \ddots & \vdots \\ (n,1) & (n,2) & \cdots & (n,n) \end{array}$$

The first row contains all the lists that begin with 1, the second those that begin with 2, and so forth. There are $n$ rows in all. Each row has exactly $n$ lists. Therefore there are $n \times n = n^2$ possible lists.

When a list is formed, the options for the second position may be different from the options for the first position. Imagine that a meal is a two-element list consisting of an entrée followed by a dessert. The number of possible entrées might be different from the number of possible desserts.

Therefore let us ask: How many two-element lists are possible in which there are $n$ choices for the first element and $m$ choices for the second element? Suppose that the possible entries in the first position of the list are the integers 1 through $n$, and the possible entries in the second position are 1 through $m$.

We construct a chart of all the possibilities as before.

$$\begin{array}{cccc} (1,1) & (1,2) & \cdots & (1,m) \\ (2,1) & (2,2) & \cdots & (2,m) \\ \vdots & \vdots & \ddots & \vdots \\ (n,1) & (n,2) & \cdots & (n,m) \end{array}$$

There are $n$ rows (for each possible first choice), and each row contains $m$ entries. Thus the number of possible such lists is

$$\underbrace{m + m + \cdots + m}_{n \text{ times}} = m \times n.$$

Sometimes the elements of a list satisfy special properties. In particular, the choice of the second element might depend on what the first element is. For example, suppose we wish to count the number of two-element lists we can form from the integers 1 through 5, in which the two numbers on the list must be different. For example, we want to count $(3,2)$ and $(2,5)$ but not $(4,4)$. We make a chart of the possible lists.

$$\begin{array}{ccccc} - & (1,2) & (1,3) & (1,4) & (1,5) \\ (2,1) & - & (2,3) & (2,4) & (2,5) \\ (3,1) & (3,2) & - & (3,4) & (3,5) \\ (4,1) & (4,2) & (4,3) & - & (4,5) \\ (5,1) & (5,2) & (5,3) & (5,4) & - \end{array}$$

As before, the first row contains all the possible lists that begin with 1, the second row those lists that start with 2, and so on, so there are 5 rows. Notice that each row contains exactly $5 - 1 = 4$ lists, so the number of lists is $5 \times 4 = 20$.

Let us summarize and generalize what we have learned in a general principle.