

Section 8.4

INVERSE TRIG FUNCTIONS

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Arcsine

Definition: The arcsine function, written as $\arcsin(x)$ or $\sin^{-1} x$ is the inverse of sine, restricted to the domain of $[-\frac{\pi}{2}, \frac{\pi}{2}]$

Why this domain?

In other words,

$\arcsin(x) =$ if and only if $\sin(\theta) =$ AND

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Example: Finding inverse sine values

Determine the exact value of the following

(1) $\sin^{-1}(\frac{1}{2}) =$

(2) $\sin^{-1}(-\frac{\sqrt{2}}{2}) =$

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Example: Function composition

Determine the exact value of the following

(1) $\sin\left(\arcsin\left(\frac{1}{3}\right)\right) =$

(2) $\sin^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right) =$

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Example continued

(3) $\sin(\sin^{-1}(\frac{4\pi}{3})) =$

Notice that

$$\sin(\arcsin(x)) = x \text{ for}$$

$$\arcsin(\sin(x)) = x \text{ for}$$

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Arccosine

Definition: The arccosine function, written as $\arccos(x)$ or $\cos^{-1} x$ is the inverse of sine, restricted to the domain of $[0, \pi]$

Why this domain?

In other words,

$$\arccos(x) = \quad \text{if and only if } \cos(\theta) = \quad \text{AND}$$

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Example: Finding inverse sine values

Determine the exact value of the following

(1) $\cos^{-1}\left(\frac{1}{2}\right) =$

(2) $\arccos\left(-\frac{\sqrt{2}}{2}\right) =$

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Example: Function composition

Determine the exact value of the following

(1) $\cos\left(\arccos\left(\frac{1}{2}\right)\right) =$

(2) $\cos^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right) =$

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Example continued

(3) $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right) =$

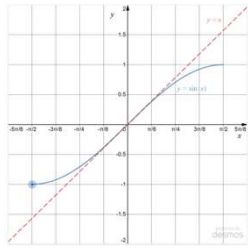
Notice that

$$\cos(\arccos(x)) = x \text{ for}$$

$$\arccos(\cos(x)) = x \text{ for}$$

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Graphing arcsine



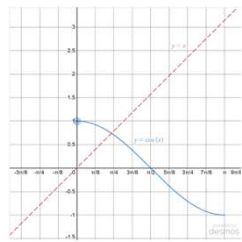
What does $y_2 = \arcsin(x)$ look like?

Domain:

Range:

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Graphing arccosine



What does $y_2 = \arccos(x)$ look like?

Domain:

Range:

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Arctangent

Definition: The arctangent function, written as $\arctan(x)$ or $\tan^{-1}x$ is the inverse of tangent, restricted to the domain of $(-\frac{\pi}{2}, \frac{\pi}{2})$.

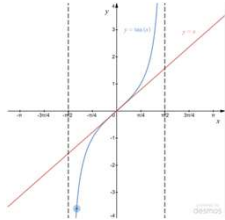
Why this domain?

In other words,

$\arctan(x) =$ if and only if $\tan(\theta) =$ AND

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Graphing arctangent



What does $y_2 = \arctan(x)$ look like?

Domain:

Range:

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Example: Other inverses

Determine the exact value of the following

(1) $\arctan(-\sqrt{3}) =$

(2) $\operatorname{arccsc}(-2) =$

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Example: More function compositions

Determine the exact value of the following

(1) $\sin\left(\arccos\left(-\frac{1}{2}\right)\right) =$

(2) $\operatorname{arccot}\left(\tan\left(-\frac{\pi}{3}\right)\right) =$

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Example: Trig Equations

Find the solutions of the below equations on the interval $[0, 2\pi]$

(1) $\sin(x) = -\frac{1}{\sqrt{3}}\cos(x)$

(2) $\arctan(x) = \frac{\pi}{4}$

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Example: Application

The rim of a standard basketball goal is 10 feet high. A point guards eyes are 6 feet above the floor. She stands 25 feet from a point directly below the front of the rim. At what angle t (measured in degrees) must she incline her eyes to look directly at the front of the rim?

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