

Bayesian Network Recap

Basic Distributions

Binary R.V. \Rightarrow support is $\{0, 1\}$

Bernoulli; (θ) Coin Flip

If $X \sim \text{Bernoulli}(\theta)$

$$P(X=1) = \theta$$

$$P(x=0) = 1 - \theta$$

X	P(X)
0	1 - θ
1	θ

$$\text{Categorical}(\phi) \quad \text{Cat}(\phi)$$

If $X \sim \text{Cat}(\phi)$ then

$$\text{support}(x) = \{1, 2, \dots, \dim(\phi)\}$$

$$P(X=1) = \phi_1$$

$$P(X=2) = \phi_2$$

1
1
6

$$P(X=n) = \phi_n$$

X	$P(X)$
1	ϕ_1
2	ϕ_2
\vdots	\vdots
n	ϕ_n

Causal Bayesian Networks

Each Node: R.V.

Each Edge: Causal relationship

Informally, B is a function of A with some additional uncertainty

Example:



$$B \sim \begin{cases} \text{Bernoulli}(0.2) & \text{if } A=0 \\ \text{Bernoulli}(0.7) & \text{if } A=1 \end{cases}$$

$$P(B=1 | A=1) = 0.7$$

$$P(B=1|A=0) = 0.2$$

If we have a BN with nodes x_1, \dots, x_n

$$\Rightarrow \begin{cases} P(x_1, x_2, \dots, x_n) = P(x_1 | pa(x_1)) P(x_2 | pa(x_2)) \dots P(x_n | pa(x_n)) \\ \quad \quad \quad = \prod_{i \in 1..n} P(x_i | pa(x_i)) \end{cases}$$

X_1	X_2	X_3	$P(X_1, X_2, X_3)$
0	0	0	
0	0	1	

A	B	C	$P(A,B,C)$
0	0	0	θ_1
0	0	1	θ_2
		\vdots	

$2^3 - 1$ indep parameters

7 parameters



$$P(A|pa(A)) = P(A) \quad \text{Bernoulli}(\cdot) \quad \boxed{1 \text{ param}}$$

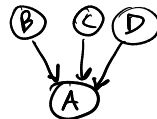
$$P(B|pa(B)) = P(B|A) \quad \text{Bernoulli}(\cdot) \text{ for each possible value of A} \quad \boxed{2 \text{ param}}$$

$$P(C|pa(C)) = P(C|B) \quad \boxed{2 \text{ param}}$$

Since $P(A,B,C)$ is defined by *
Just need $P(X|pa(X))$

5 total parameters

A, B, C, D binary RV



$$P(A|pa(A)) = P(A|B,C,D)$$

$2^3 = 8$ possibilities

8 parameters

Independence

$A \perp B$

$$P(A|B) = P(A)$$

$$P(A,B) = P(A)P(B)$$

$$\frac{P(A,B)}{P(B)} = P(A)$$

$$P(A,B) = P(A)P(B)$$

Conditional Independence

$A \perp B | C$

$$P(A|B,C) = P(A|C)$$

$$P(A,B|C) = P(A|C)P(B|C)$$

$$\frac{P(A,B|C)}{P(B|C)} = P(A|C)$$

$$P(A,B|C) = P(A|C)P(B|C)$$



$A \perp C | B$?

Yes

need to prove that

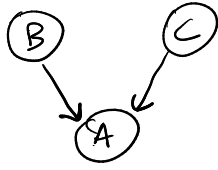
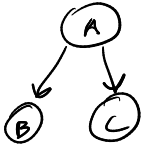
$$P(A,C|B) = P(A|B)P(C|B)$$

$$P(A,B,C) = P(A|pa(A))P(B|pa(B))P(C|pa(C)) \\ = P(A)P(B|A)P(C|B)$$

$$\frac{P(A,B,C)}{P(B)} = \frac{P(A)P(B|A)P(C|B)}{P(B)}$$

$$P(A,C|B) = P(A|B)P(C|B)$$

Similarly can prove $B \perp C | A$



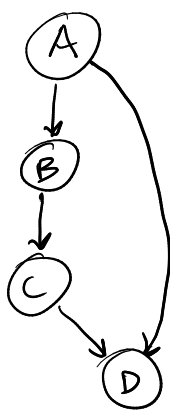
$B \perp C | A$? cannot prove from structure

but you could pick particular parameters where $B \perp C | A$

example: $A \sim \text{Bernoulli}(0.5)$ for any values of

$B \sim \text{" " " " } B, C$

$C \sim \text{" " " " }$



$A \perp C | B$? True

paths

$A \rightarrow B \rightarrow C$ rule 1: d-sep

$A \rightarrow D \leftarrow C$ rule 3: d-sep

all paths are d-sep

$A \perp C | B, D$? Inconclusive

paths

$A \rightarrow B \rightarrow C$ rule 1: d-sep

$A \rightarrow D \leftarrow C$ rule 3: not d-sep

not d-sep