PART – 7

$$A = \begin{bmatrix} a & b & t_x \\ -b & a & t_y \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} \hat{x}_1 \\ \hat{y}_1 \\ \hat{w}_1 \end{bmatrix} = \begin{bmatrix} a & b & t_x \\ -b & a & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Using eqn $\boxed{Pq = r.}$

$$\hat{x}_1 = ax_1 + by_1 + t_{x_1}$$
$$\hat{y} = -bx_1 + ay_1 + ty_1$$
$$\hat{w}_1 = 1$$

$$\Rightarrow x_1' = \frac{\hat{x}_1}{\hat{w}_1} = \frac{ax_1 + by + t_{x_1}}{1} \Rightarrow \hat{x}_1 = ax_1 + by_1 + t_{x_1} \quad \text{— (1)}$$

$$\Rightarrow y_1' = \frac{\hat{y}_1}{\hat{w}_1} = \frac{-bx_1 + ay_1 + t_{y_1}}{1} \Rightarrow \hat{y}_1 = -bx_1 + ay_1 + t_{y_1} \quad \text{— (2)}$$

From eqn (1) and (2) we build the design matrix.

$$P = \begin{bmatrix} x_1 & y_1 & 1 & 0 \\ y_1 & -x_1 & 0 & 1 \\ x_2 & y_2 & 1 & 0 \\ y_2 & -x_2 & 0 & 1 \end{bmatrix}, \quad q = \begin{bmatrix} a \\ b \\ t_x \\ t_y \end{bmatrix}, \quad r = \begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ y_2' \end{bmatrix}$$

For homogenous system of equations, $Pq = 0$

$$P = \begin{bmatrix} x_1 & y_1 & 1 & 0 & -x_1' \\ y_1 & -x_1 & 0 & 1 & -y_1' \\ x_2 & y_2 & 1 & 0 & -x_2' \\ y_2 & -x_2 & 0 & 1 & -y_2' \end{bmatrix}, \quad q = \begin{bmatrix} a \\ b \\ t_x \\ t_y \\ 1 \end{bmatrix}, \quad r = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

There are –

4 – variables / unknowns

4 – equations

2 – is the least number of correspondences needed for similarity transform.