Functions Refer Wikipedia

Newton's method for approximate root finding

Given a function $f : \mathbb{R} \to \mathbb{R}$

find x : $f(x) = 0$

Input

Find a root of $x^7 + x^6 - x^3 + 24$

output: real number s.t $f(x) = 0$

Newton : function $\longrightarrow$ ~~real~~ rational

$x^2 - 2$ $\qquad$ $\underbrace{\pm\sqrt{2}}_{\text{irrational}}$

Two key Properties

- very close approx.
- very fast convergence

f has to be differentiable.

$f'(x)$ exists.

1) Find square roots of numbers.

2) Find roots of arbitrary functions.

$f(x)$ $f'(x)$

$x_0 \leftarrow$ initial guess

Iterative method

$$x_1 \leftarrow x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_{n+1} \leftarrow x_n - \frac{f(x_n)}{f'(x_n)}$$

We want to compute $\sqrt{a}$.

What should be f?

$$f(x) = x^2 - a \qquad x_{n+1} \leftarrow \underbrace{x_n - \frac{x_n^2 - a}{2x_n}}$$

$$f'(x) = 2x$$

I will choose $\underline{\underline{\frac{1}{2}\left(x_n + \frac{a}{x_n}\right)}}$

When should we stop?

$$|x_{n+1} - x_n| \leq \underline{\underline{\varepsilon}}$$

Newton's method (math) $x_0, x_1, x_2 \cdots x_{10}, x_{11} \cdots$

(Programming)

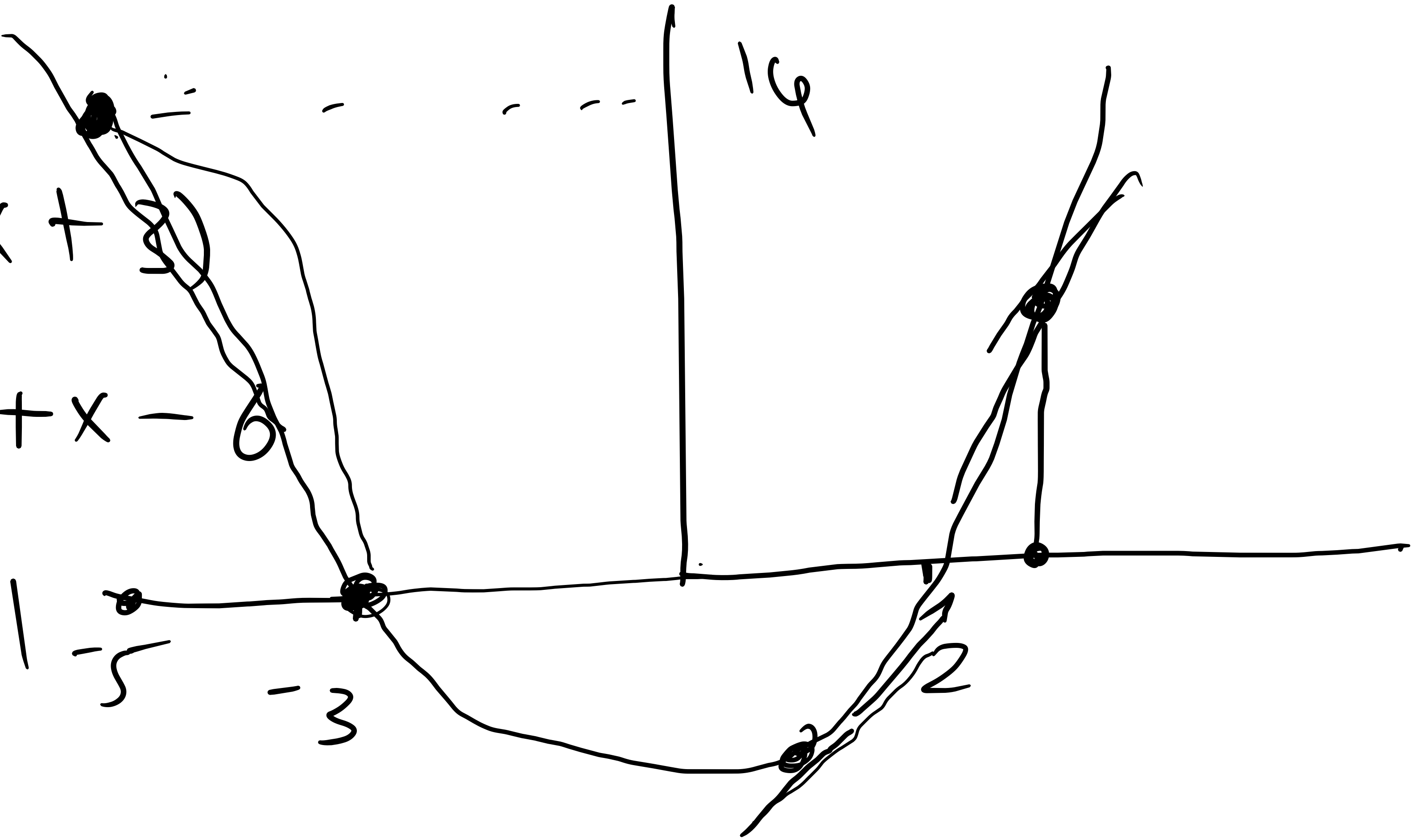
$$\begin{bmatrix} g_0 & g_1 & & & \\ & g_0 & g_1 & & \\ & & g_0 & g_1 & \\ & & & & \ddots \end{bmatrix}$$

time

$$f(x) = (x-2)(x+3)$$

$$= x^2 + x - 6$$

$$f'(x) = 2x + 1$$



Given f, g

The sum: $h = f + g$

$h(x) = f(x) + g(x)$

Composition $(f \circ g)(x) = f(g(x))$

What we want:

$$\text{sqrt\_f} : \text{number} \longrightarrow \text{function}$$

5	$x^2 - 5$
6	$x^2 - 6$
n	$x^2 - n$

derivative : function $\rightarrow$ function

$$\text{derivative}(f)(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$